

Equation (25) in Moelans paper [1] defined the mobility as

$$L_{\alpha,\beta} = \frac{\sqrt{2}mg(\gamma)}{\kappa I_\phi(\gamma)\zeta_{\alpha,\beta}}, \quad (1)$$

using $\gamma = 1.5$ leads to $g(\gamma) = \sqrt{2}/3$ and $I_\phi(\gamma) = 1/2$. Inserting this gives

$$L_{\alpha,\beta} = \frac{4m}{3\kappa\zeta_{\alpha,\beta}}, \quad (2)$$

where m and κ are constants that depends on the surface energy and the width of the interface region which I assume are the same for all interfaces. The $\zeta_{\alpha,\beta}$ is taken as

$$\zeta_{\alpha,\beta} = \frac{(x_\alpha^{eq} - x_\beta^{eq})^2}{0.5(M_\alpha + M_\beta)}, \quad (3)$$

with x_α^{eq} and x_β^{eq} being the equilibrium composition of Sn in the α and β phases and

$$M_\alpha = \frac{D_\alpha}{\frac{d^2 f_\alpha}{dx_\alpha^2}}, \quad M_\beta = \frac{D_\beta}{\frac{d^2 f_\beta}{dx_\beta^2}} \quad (4)$$

are the diffusion mobilities.

Using the parabolic approximation of the free energy $f_\rho = \frac{A_\rho}{2}(x_\rho - x_{\rho,0})^2 + C_\rho$ the diffusion mobility becomes

$$M_\rho = \frac{D_\rho}{A_\rho}. \quad (5)$$

For the numerical examples in the paper the diffusion coefficients are taken as $D_{\text{Cu}} = 1 \times 10^{-25}$, $D_{\text{Cu}_6\text{Sn}_5} = 1 \times 10^{-16}$ and $D_{\text{Sn}} = 1 \times 10^{-14} \text{ m}^2 \text{ s}^{-1}$. The energy coefficients are $A_{\text{Cu}} = 1 \times 10^8$, $A_{\text{Cu}_6\text{Sn}_5} = 1 \times 10^9$, and $A_{\text{Sn}} = 1 \times 10^9 \text{ J m}^{-3}$. The huge difference in diffusion coefficients is what creates the difference in mobilities. For the interface between Cu and Cu_6Sn_5 I get $\zeta_{\text{Cu,Cu}_6\text{Sn}_5} = 3.23 \times 10^{24}$ and for the interface between Cu_6Sn_5 and Sn I get $\zeta_{\text{Cu}_6\text{Sn}_5,\text{Sn}} = 5.42 \times 10^{22}$. This leads to the mobility of the interface between Cu_6Sn_5 and Sn being about 500 times higher than the mobility between Cu and Cu_6Sn_5 .

In the simulation I use eV and nm instead of J and m so the numbers are a bit nicer but their ratio is the same.

References

- [1] Nele Moelans. A quantitative and thermodynamically consistent phase-field interpolation function for multi-phase systems. *Acta Materialia*, 59(3):1077–1086, 2011.