Solution

Problem

Given an array of numbers with duplicated nums :$A = \{n_0 ...\}$ Given a target number: t Given a target length: y

Find all the set S_y^t which stasfied the following condition:

1. $|S_y^t| = y$
2. $s \subseteqq S_y^t \to s \subseteqq A$
3. $\sum S_y^t = t$

Solution

1. **Recursive**

Train of thought:

Given a number n_x :

$$S_y^t = \{n_x\} \bigcup S_{y-1}^{t-n_x} \, (eq1)$$

So we can get a recursive function $eq2$:

1. $F(y,t) = x * F(y-1, t-x) \;\; (y > 1) \;\; (for\ unique(x)\ in\ A)$
2. $F(y,t) = t \;\; (y = 1)$

However this function will get duplicated sets This can be prevented by sort the set while selecting elements from A Which means: if we selected x_{i_0} (i_0 represents the order of the element in the set) $\vee x_i < x_{i_0} (i > i_0)$

Proof:

1. Prove the set from the recursive function meet the requirements and are distinct: a. $\vee S_1, S_2 \; s_i^1 \neq s_i^2$($i$ represent the order in the set. This is guaraenteed by iterating through unique element on each selection) b. Given S is orderd. if $S_1 = S_2$ then $s_i^1 = s_i^2$ which conflicts with a. Thus $\vee S_i \neq S_j$ c. $\because eq1 \therefore \vee \sum S = t$
2. Prove all the S are generated from $eq2$: a. if $\exists S_0$ not generated from $eq2$ then $\exists s_i^0 \not\subseteq A$ $\because$ all the numbers is selected from A. $\therefore \nexists S_0$