

Not Alone

Not Alone is a puzzle appearing starting 7/20/2024 in the New York Times Magazine under Prasanna Seshadri's byline. Here we apply the Metropolis algorithm to generate solutions to these puzzles. The puzzle currently consists of an 8×8 grid (it started out as 6×6) populated with a handful of black circles and white circles. You are to populate the remainder of the grid with black and white circles in such a way that: (i) each row and each column has four black and four white circles; and (ii) no circle is sandwiched between two circles of the other color, either horizontally or vertically. The September 22 puzzle, together with its solution, are shown below.

Puzzle

			○				
					●		
	○						
			○				○
	○			●			●
			○				

Solution

○	●	●	○	○	○	●	●
●	○	○	●	●	●	○	○
●	○	○	●	●	●	○	○
●	○	○	○	●	●	●	○
○	○	○	○	●	●	●	●
○	●	●	○	○	○	●	●
○	●	●	●	○	○	○	●
●	●	●	●	○	○	○	○

Our **state space** S will consist of all possible arrangements of 64 circles where each row contains exactly four black circles. There are $\binom{8}{4}^8 \approx 5.76 \times 10^{14}$ such configurations in S . Two configurations x and y will be **neighbors** if y can be obtained from x by swapping a black and white circle *in the same row* — thus preserving the row-count of four black and four white circles. This is easily seen to adhere to the good neighbor rules of Chapter 3 of “The Metropolis Algorithm: Theory and Examples” (C Douglas Howard, FE Press, 2024). In particular, each configuration has $8 \times 4 \times 4 = 128$ neighbors. One randomly chooses a configuration's neighbor by: (i) randomly choosing a row; and (ii) randomly choosing a black and a white circle in that row and swapping those circles. The **energy function** $E(x)$ will count deviations from the puzzle's rules. The solution must concur with the clues, so starting with $E(x) = 0$ augment $E(x)$ by 5 (this penalty is arbitrary) for

each grid site with a clue whose circle's color differs from the circle's color in the x configuration. Further augment $E(x)$ to reflect each column's deviation from the exactly-four-black-circle rule. Specifically, if a column in x has n black circles it contributes $|n - 4|$ to $E(x)$. Further augment $E(x)$ by 1 for each circle in x that is horizontally sandwiched between two circles of the other color. Finally, do the same for each vertical violation of the sandwich rule. The solution will have an energy equal to 0. This is implemented in `NotAlone.cpp`, where the `.txt` data input file has the format:

```

9
1 4 0
2 6 1
3 2 0
4 4 0
4 8 0
5 2 0
5 5 1
5 8 1
6 4 0

```

The initial “9” indicates that this puzzle has 9 clues. The next lines indicate clue data. For example, “1 4 0” means that in row 1, column 4, the clue circle is white (1 means black). The above data corresponds to the 9/22/2024 puzzle shown above and can be found in `9-22-2024.txt`. The puzzle and solution may be viewed by TeXing the file `NA.tex` with Plain TeX (don't use Latex).