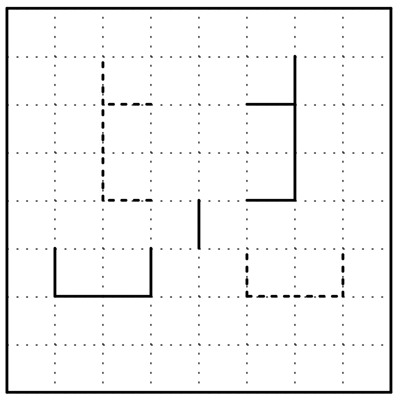
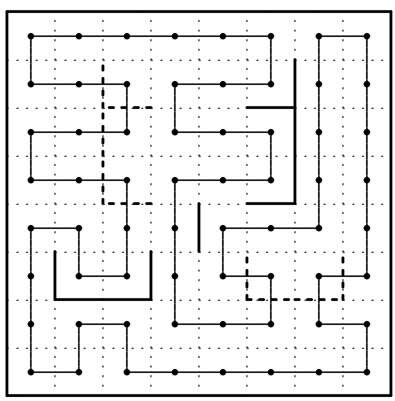
Traffic Signals

Traffic Signals is a puzzle appearing as of February 2, 2025 in the New York Times Magazine under Prasanna Seshadri's byline. The February 2 puzzle, together with its solution, are shown below. The puzzle consists of an 8×8 grid where certain edges are marked green (shown as dashes below) and certain edges are marked red (shown as solid below). You are to find a closed loop horizontally and vertically through the grid that: (i) visits each cell once; (ii) passes through each of the green (dashed) edges; and (iii) avoids each of the red (solid) edges. Here we apply the Metropolis algorithm to generate solutions to these puzzles.

Puzzle



Solution



We number the grid cells 1 through 64 from top to bottom and left to right within each row, so the upper left cell is number 1 and the lower right cell is number 64. Our **state space** S will consist of all permutations of the numbers $\{2, 3, \ldots, 64\}$, so S has $63! \approx 1.98 \times 10^{87}$ elements. Let $(c_2, c_3, \ldots, c_{64})$ denote such a permutation. With $c_1 = c_{65} = 1$, $(c_1, c_2, c_3, \ldots, c_{64}, c_{65})$ will denote a route starting and ending at cell number 1 that visits each cell in the grid. Consecutive cells c_i and c_{i+1} in the route will typically not be adjacent cells, as they must be in the solution.

The energy function, described below, will steer the Markov chain clear of such routes. We will say that route y is a **neighbor** of route x if for some i and j with $2 \le i < j \le 64$ we have

$$\begin{aligned} x &= (1, c_2, \ldots, c_{i-1}, c_i, c_{i+1}, \ldots, c_{j-1}, c_j, c_{j+1}, \ldots, c_{64}, 1), \quad \text{and} \\ y &= (1, c_2, \ldots, c_{i-1}, \overbrace{c_j, c_{j-1}, \ldots, c_{i+1}, c_i}^{\text{reversed}}, c_{j+1}, \ldots, c_{64}, 1). \end{aligned}$$

In this fashion each route has $\binom{63}{2} = 1953$ neighbors. This is identical to the approach taken in §17 (The Traveling Salesman Problem) of "The Metropolis Algorithm: Theory and Examples" (C Douglas Howard, FE Press, 2024).

For any pair of cells a and b, let $d(a,b)$ denote the ℓ^1 distance (sometimes called the Manhattan distance) from a to b. For example, $d(33,41) = d(41,33) = 1$ (they are vertically adjacent cells) and $d(27,46) = d(46,27) = 5$ (46 is down 2, over 3, from 27). The length of a route x, as specified above, will be defined as

$$L(x) = \sum_{i=1}^{64} d(c_i, c_{i+1}).$$

The puzzle's solution will have a length of 64. We also count violations of the traffic signals. Let $R(x)$ denote the number of route steps where $d(c_i, c_{i+1}) = 1$ (i.e, they are adjacent) and their common grid edge is marked red. Let $G(x)$ denote the number of grid edges e marked green which, for no i, are c_i and c_{i+1} adjacent with common edge e. Reflecting both route length and signal violations, our **energy function** is then $E(x) = L(x) + R(x) + G(x)$. The solution will have an energy equal to 64. This is implemented in `TrafficSignals.cpp`, where the `.txt` data input file has the format:

```
9 green signals
10 11
19 11
18 19
26 27
27 35
45 46
47 48
46 54
47 55
10 red signals
14 15
14 22
22 23
30 31
30 38
36 37
41 42
42 50
43 51
43 44
```

This data corresponds to the February 2 puzzle shown above and can be found in `Signals00.txt`. It has 9 green and 10 red traffic signals. Each pair indicates the adjacent cells (horizontally or vertically) whose common edge are colored either green or red. For example, in the figure on the first page the top left green (dashed) edge is generated by the pair `10 11`. The puzzle and solution may be viewed after running `TrafficSignals` by TeXing the file `TS.tex` with LaTeX.

— CDH