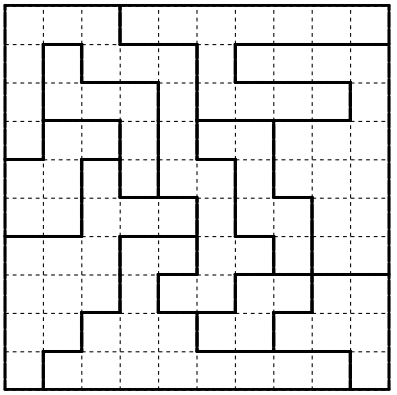
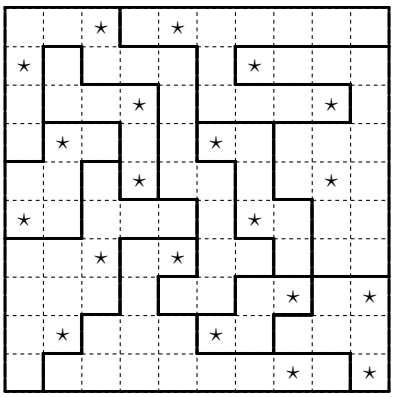
Two Not Touch

Two Not Touch is a popular puzzle appearing in the Arts section of the New York Times under a `www.krazydad.com` copyright. Here we apply the Metropolis algorithm to generate solutions to these puzzles. The puzzle consists of a 10×10 grid with 10 highlighted regions. You are to place 20 stars in 20 different locations on the grid in such a way that: (i) each row, column, and region contains exactly two stars; and (ii) no two stars touch, not even diagonally. One of the two puzzles appearing on 7/8/2024, together with its solution, is shown below.

Puzzle



Solution



Our **state space** S will consist of all possible arrangements of the 20 stars where each row contains exactly two stars. There are $\binom{10}{2}^{10} \approx 3.405 \times 10^{16}$ such configurations in S. Two configurations x and y will be **neighbors** if y can be obtained from x by moving exactly one star to a different location *in the same row*. This is easily seen to adhere to the good neighbor rules of Chapter 3 of

"The Metropolis Algorithm: Theory and Examples" (C Douglas Howard, FE Press, 2024). In particular, each configuration has $10 \times 2 \times 8 = 160$ neighbors. One randomly chooses a configuration's neighbor by: (i) randomly choosing a row; (ii) randomly choosing one of the two stars in that row; and (iii) randomly moving that star to one of the 8 possible new locations in that row. The **energy function** $E(x)$ will count deviations in configuration x of the rule-of-two-stars for the columns and regions (the rows automatically all have two stars). For example, a column containing five stars will contribute $|5-2| = 3$ to $E(x)$; similarly, a region containing only one star will contribute $|1-2| = 1$. $E(x)$ is then further augmented by the number of pairs of stars in x that violate the no-touching rule. The solution will have an energy equal to 0. This is implemented in `TwoNotTouch.cpp`, where the `.txt` data input file has the format:

```
AAABBBBBBB
ACAAABDDDD
ACCCABBBBD
AEECAFFDDD
EEGCAAFDDD
EEGGGAFFDD
GGGHHAAFDD
GGGHAAIIJJ
GGHHHIIJJJ
GHHHHHHHHJ
```

The ten letters `A` through `J` show the regions of the grid. The above data corresponds to the 7/8/2024 puzzle shown above and can be found in `7-8-2024.txt`. The puzzle and solution may be viewed by TeXing `TNT.tex` with Plain TeX (don't use Latex).