MAGIC SQUARES

A **Magic Square** of order n is an $n \times n$ grid containing the numbers 1 through n^2 arranged so that all the rows and columns and both diagonals sum to the "magic number", which is $n(n^2+1)/2$. Below is an example of an order eight magic square with magic number 260.

16	52	29	12	43	26	34	48
53	4	3	31	46	55	49	19
21	63	61	38	40	22	10	5
35	64	7	23	27	56	9	39
11	1	51	36	60	6	54	41
44	33	20	24	14	62	45	18
50	28	47	37	13	25	2	58
30	15	42	59	17	8	57	32

The code `MagicSquare.cpp` uses the Metropolis algorithm to generate magic squares. To implement Metropolis, our **state space** S consists of all ways to allocate the digits $\{1, 2, 3, \ldots, n^2\}$ to the n^2 grid sites, so S has $n^2!$ elements. When $n = 8$, $|S| \approx 1.27 \times 10^{89}$. (Of course, there are symmetries — rotations and reflection — producing eight equivalent arrangements. Also, if each number in a magic square is subtracted from n^2+1 the result is a "complementary" magic square. We ignore these considerations.) Two states x and y are **neighbors** if y can be obtained from x by swapping x's digits at two sites, so each state has $\binom{n^2}{2}$ neighbors. As for the **energy** $E(x)$ of a state x, for each row, column, and diagonal, form the sum, call it s, of the corresponding n numbers. Each such sum s contributes $|s - m|$ to $E(x)$, where m is the magic number. A magic square will have an energy equal to 0.

`MagicSquare.cpp` prompts the user to seed the random number generator (with an integer) and input the desired order. It also prompts the user for the temperature parameter. The optimal temperature varies with the order of the square. For example, for squares of order 5, 10, 15, and 20, the optimal temperatures are approximately 0.72, 0.42, .35, and 0.31, respectively. It reports a resulting magic square to the screen and creates the file `MS.tex` which can be processed with LaTeX to produce a figure like the one above.

The program also produces an output file `MSout.txt` containing the magic square data. This file is read in by the program `VerifyMS.cpp`, which verifies that the square is, indeed, magic. I have obtained an order 60 magic square, which you may verify by copying the file `Order60.txt` into `MSout.txt` and running the veri-

fication code. This order 60 magic square is generated by `MagicSquare.cpp` after 7.7 billion steps of the Markov chain with an RNG seed of 1261727 (as randomly generated by my fingers on the keyboard!) and a temperature of 0.285.

— CDH