پاسخ سوال 1: با توجه به مدار داریم: $i_x = -i_4$ (I) $i_2 - i_4 = 6A$ $\boxed{i_3 = 6A}$

(II): $+4 - 5(i_1 - i_4) - 4(-i_4) = 0$

(III): $-8i_4 - (i_4 - i_3)\times 4 - (i_2 - i_3)\times 5 + 4(-i_4) - 5\times(i_4 - i_1) = 0$

$i_4 = \frac{1}{9}[5i_1 - 4]$ (*) $\xrightarrow{(I)}$ $i_2 = 6 + \frac{1}{9}[5i_1 - 4] = \frac{50}{9} + \frac{5}{9}i_1$

$-21\left[\frac{1}{9}[5i_1 - 4]\right] + 54 - 5i_2 + 5i_1 = 0$

$\frac{28}{3} - \frac{35}{3}i_1 + 54 + 5i_1 - 5\left[\frac{50}{9} + \frac{5}{9}i_1\right] = i_1\left[-\frac{35}{3} + 5 - \frac{25}{9}\right] + \left(\frac{28}{3} + 54 - \frac{250}{9}\right) = 0$

$\Rightarrow i_1\left(\frac{-85}{9}\right) = \frac{250 - 3\times 28 - 9\times 54}{9} = \frac{-320}{9} \Rightarrow \boxed{i_1 = \frac{64}{17}}$

(*) $\Rightarrow i_4 = \frac{5}{9}\times\frac{64}{17} - \frac{4}{9} = \frac{5\times 64 - 4\times 17}{9\times 17} = \frac{+252}{9\times 17} = +\frac{28}{17} \Rightarrow \boxed{i_4 = +\frac{28}{17}}$

(I) $\rightarrow$ $i_2 = 6 + \left(\frac{+28}{17}\right) = \frac{130}{17} \Rightarrow \boxed{i_2 = \frac{130}{17}}$

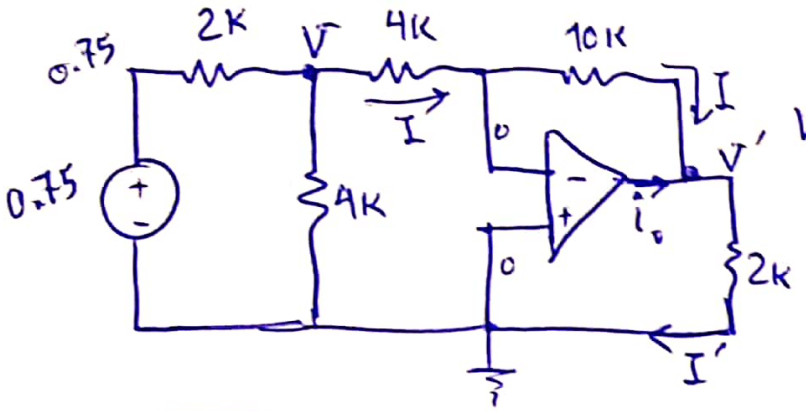
توان تحویل داده شده به مقاومت 5 اهمی چپ

$$= (i_4 - i_1)^2 \times 5 = \left(+\frac{28}{17} - \frac{64}{17}\right)^2 \times 5 = \frac{36^2}{17^2} \times 5 \simeq 22.12\,w$$

توان تحویل داده شده به منبع وابسته

$$= (i_1 - i_2) \times 4(-i_4) = \left(\frac{64-130}{17}\right) \times 4 \times \left(\frac{-28}{17}\right) \simeq +25.58\,w$$

پاسخ سوال 2:



$$\bar{V}: \frac{\bar{V}-0.75}{2K} + \frac{\bar{V}-0}{4K} + \frac{\bar{V}-0}{4K} = 0$$

$$\bar{V} = \frac{0.75/2}{1/2 + 1/4 + 1/4} = \frac{0.75}{1+1} = \frac{0.75}{2}$$

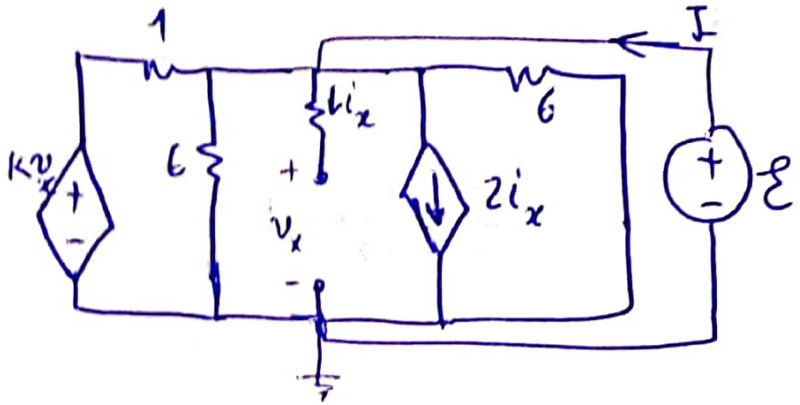
(I) $\boxed{I = \frac{\bar{V}}{4K}} \Rightarrow V' = 0 - 10K \times I = 0 - \frac{10}{4}\bar{V} = -\frac{10}{4} \times \frac{0.75}{2} = -\frac{5}{4} \times 0.75$

$$I' = \frac{V'-0}{2K} = \frac{-\frac{5}{4} \times 0.75}{2K} = \boxed{-\frac{5}{8} \times 0.75 \text{ mA}} \quad \text{(II)}$$

از طرفی $I' = i_o + I \xrightarrow[\text{II}]{\text{I}}$ $i_o = I' - I = -\frac{5}{8} \times 0.75 - \frac{0.75}{8} = -\frac{3}{4} \times 0.75$

$$\Rightarrow \boxed{i_o = -\frac{2.25}{4}}$$

پاسخ سوال 3 : (الف) منابع مستقل را خاموش می کنیم و به ab یک منبع مستقل ولتاژ اضافه می کنیم، داریم :



$$i_x = 0 \Rightarrow v_x = \mathcal{E}$$

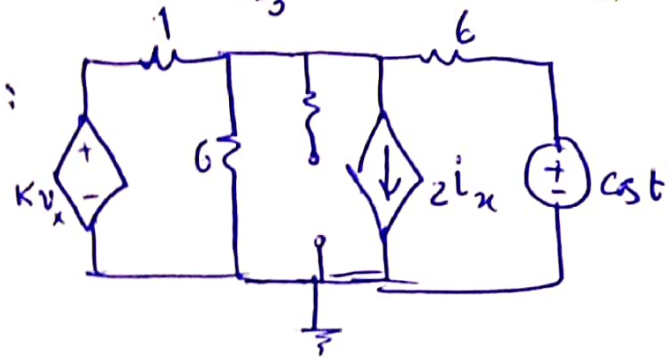
KCL:
$$\frac{\mathcal{E}-0}{6} + \frac{\mathcal{E}-0}{6} + \frac{\mathcal{E}-K\mathcal{E}}{1} = I$$

$$\mathcal{E}\left[\frac{1}{3}+1-K\right] = I \rightarrow R_{th} = \frac{\mathcal{E}}{I} = \frac{1}{\frac{4}{3}-K}$$

$$R_{th} = 2 = \frac{1}{4/3-K} \Rightarrow K = \frac{4}{3} - \frac{1}{2} = \boxed{\frac{5}{6}} \rightarrow \text{پاسخ الف}$$

(ب)

منابع ولتاژ روشن :

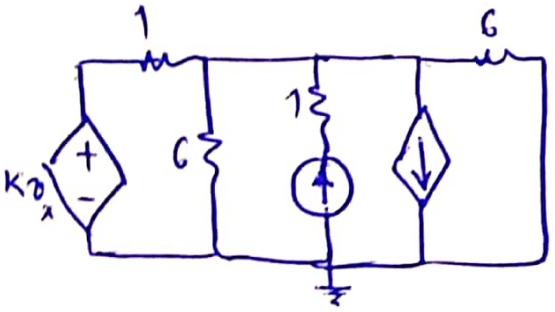


$$i_x = 0 \qquad v_x = v_{ab_1}$$

$$\frac{v_{ab_1} - \cos t}{6} + \frac{v_{ab_1} - 0}{6} + \frac{v_{ab_1} - \left(\frac{5}{6}\right)(v_{ab_1})}{1} = 0$$

$$v_{ab_1}\left[\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right] = \frac{\cos t}{6} \rightarrow \boxed{v_{ab_1} = \frac{\cos t}{3}} \quad (I)$$

منابع جریان روشن



$$i_x = -2$$

$$v_{ab_2} = v_x - 2$$

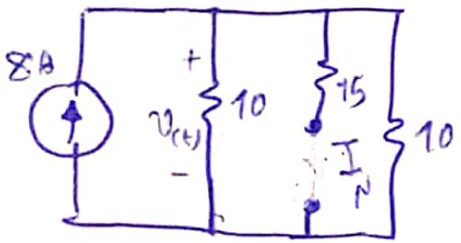
$$\frac{v_{ab_2} - 0}{6} + 2(-2) - 2 + \frac{v_{ab_2} - 0}{6} + \frac{v_{ab_2} - \left(\frac{5}{6}\right)(v_{ab_2}+2)}{1} = 0$$

$$\Rightarrow v_{ab_2}\left[\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right]=6+\frac{5}{3}\longrightarrow v_{ab_2}=12+\frac{10}{3}=\boxed{\frac{46}{3}}$$

$$\Rightarrow v_{ab}=v_{ab_1}+v_{ab_2}=\frac{cost}{3}+\frac{46}{3}\Rightarrow \boxed{v_{ab}=\frac{1}{3}(cost+46)}$$

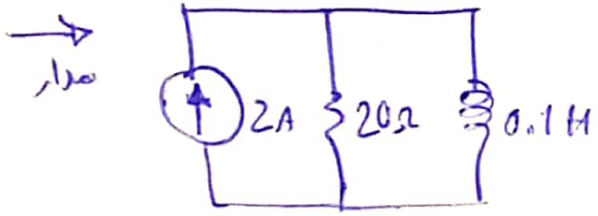
باسمه تعالی

پاسخ سوال 4: ابتدا سرسلف مدار را معادل نورتن می‌گیریم:



$$I_N = 8 \times \frac{5}{5+15} = 2A \quad , \quad R_N = 15 + \frac{10 \times 10}{10+10} = 20\,\Omega$$

(با خاموش کردن منبع مستقل)



مدار

$$i(t) = i_{(t=0)}\, e^{-\frac{t}{\tau}} + i_{(t=\infty)} \left(1 - e^{-\frac{t}{\tau}}\right)$$

$i_{(t=0)} = 0$ چون در $t=0^-$ نیز 0 بوده است.

$i_{(t=\infty)} = 2A$

$$\tau = L/R = \frac{0.1}{20} = \frac{1}{200}$$

$$\Rightarrow \boxed{i(t) = 2\left(1 - e^{-200t}\right)}$$

از طرفین با توجه به مدار سوال

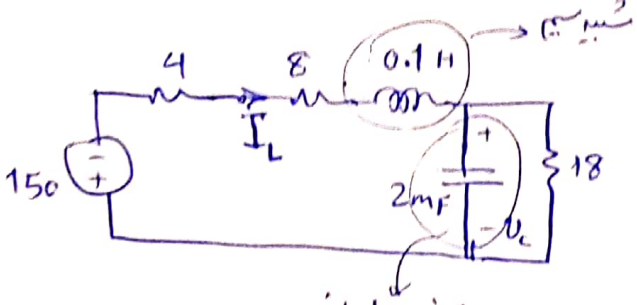
$$v(t) = L\frac{di}{dt} + 15\,i$$

$$\Rightarrow v(t) = 0.1\left[2 \times 200\, e^{-200t}\right] + 30\left(1 - e^{-200t}\right)$$

$$\boxed{v(t) = 30 + 10\, e^{-200t}}$$

پاسخ سوال 5 :

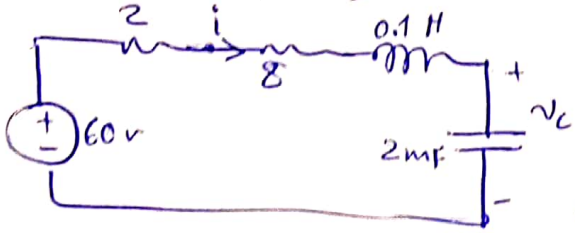
$t=0^-$ ،،



$$I_L(t=0^-) = \frac{150}{30} = 5A$$

$$v_c(t=0^-) = -5 \times 18 = -90v$$

عین مدار باز



$t \geq 0^+$ ،،

$$i = C\frac{dv_c}{dt} = 2m\frac{dv_c}{dt}$$

$$+60 - 10i - L\frac{di}{dt} - v_c = 0$$

$$60 - 10\left[2\times10^{-3}\frac{dv_c}{dt}\right] - 0.1\left[2\times10^{-3}\frac{d^2v_c}{dt^2}\right] - v_c = 0$$

$$\times \frac{-1}{2}\times10^4$$

$$\frac{d^2v_c}{dt^2} + 100\frac{dv_c}{dt} + \frac{1}{2}\times10^4 v_c = 30\times10^4$$

$$s^2 + 100s + 5\times10^3 = 0 \rightsquigarrow s_{1,2} = \frac{-100 \pm \sqrt{10^4 - 20\times10^3}}{2} = \frac{-100 \pm 100i}{2}$$

$$s_{1,2} = -50 \pm 50i = -50(1 \pm i)$$

$$\Rightarrow v_c = e^{-50t}(A\sin(50t) + B\cos(50t)) + 60$$

$$v_c(t=0^+) = v_c(t=0^-) = -90 = B + 60 \Rightarrow \boxed{B = -150}$$

$$i_L(t=0^+) = i_L(t=0^-) = 2\times10^{-3}\left(\frac{dv_c}{dt}\right)_{t=0^+} \Rightarrow \left.\frac{dv_c}{dt}\right|_{t=0^+} = -2500$$

($i_L(t=0^-) = -5A$)

$$\frac{dv_c}{dt} = -50e^{-50t}(A\sin(50t) - 150\cos(50t)) + e^{-50t}(50A\cos(50t) + 50\times150\sin(50t))\Big|_{t=0^+} = -50(-150) + 50A = -2500$$

$$\boxed{A = -200}$$

$$\Rightarrow \boxed{v_c = e^{-50t}(-200\sin(50t) - 150\cos(50t))}$$

پاسخ سؤال 6:

الف) می‌دانیم $\cos(3t) = \frac{e^{3it} + e^{-3it}}{2}$

حال:

$$\cos^3(3t) = \cos 3t \left(\frac{1+\cos 6t}{2}\right) = \frac{\cos 3t}{2} + \frac{\cos 3t \cos 6t}{2}$$

$$= \frac{\cos 3t}{2} + \frac{1}{2}\left(\frac{e^{3it}+e^{-3it}}{2}\right)\left(\frac{e^{6it}+e^{-6it}}{2}\right)$$

$$= \frac{\cos 3t}{2} + \frac{1}{8}\left(e^{9it} + e^{-3it} + e^{3it} + e^{-9it}\right)$$

$$= \frac{\cos 3t}{2} + \frac{1}{8}\left(2\cos 9t + 2\cos 3t\right) = \frac{3}{4}\cos 3t + \frac{1}{4}\cos 9t$$

$$\Rightarrow \boxed{\mathcal{L}(\cos^3(3t)) = \frac{3}{4}\left(\frac{s}{s^2+9}\right) + \frac{1}{4}\left(\frac{s}{s^2+81}\right)}$$

ب)

$$f(t) = te^{2t}\sin(3t)$$

$$\mathcal{L}(t\sin 3t) = -1\frac{d}{ds}\left(\frac{3}{s^2+3^2}\right) = +3\frac{2s}{(s^2+9)^2}$$

$$\mathcal{L}(e^{2t}t\sin(3t)) = \frac{6(s-2)}{((s-2)^2+9)^2}$$

حاسع سؤال 7 :

ا) $F(s) = \frac{2s+3}{s^2+4s+13} = \frac{2s+3}{(s+2)^2+9} = \frac{2(s+1)}{(s+2)^2+9} - \frac{1}{(s+2)^2+9}$

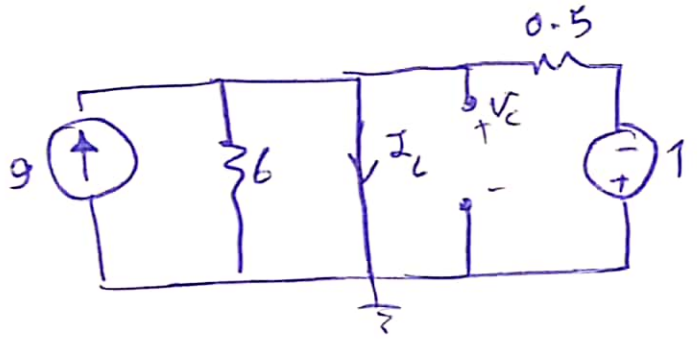
$= \frac{2(s+2)}{(s+2)^2+3^2} + \frac{-1}{(s+2)^2+9}$

$\Rightarrow \boxed{\mathcal{L}^{-1}(F(s)) = 2e^{-2t}\cos(3t) - \frac{1}{3}e^{-2t}\sin(3t)}$

ب) $F(s) = \frac{1+e^{-2s}}{s^2+6} = \frac{1}{s^2+6} + \left(\frac{1}{s^2+6}\right)e^{-2s}$

$\Rightarrow \boxed{\mathcal{L}^{-1}(F(s)) = \frac{1}{\sqrt{6}}\sin(\sqrt{6}\,t) + \frac{1}{\sqrt{6}}\sin(\sqrt{6}\,(t-2))\,u(t-2)}$

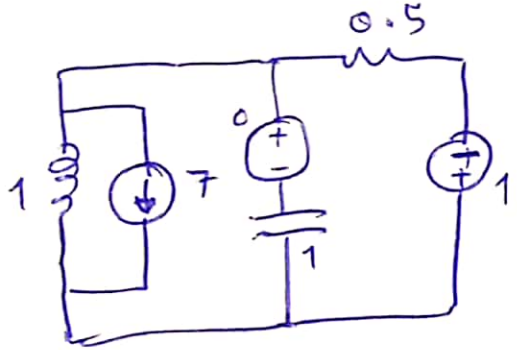
پاسخ سوال 8 ؛ "$t \le 0^-$"



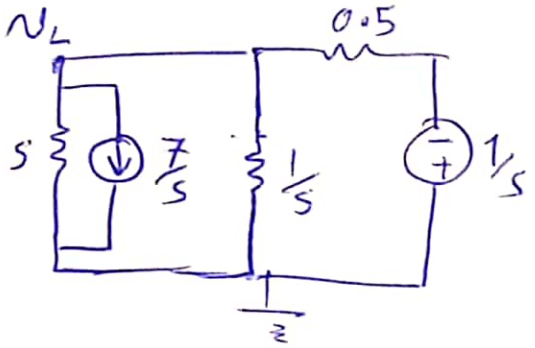
$$\frac{0-(-1)}{0.5} + I_L + \frac{0-0}{6} - 9 = 0$$

$$I_L = 9 - 2 = 7\,A \qquad v_c(t=0^-) = 0$$

بعد از باز شدن



لاپلاس



$$\frac{v_L-(-\frac{1}{s})}{0.5} + \frac{v_L-0}{\frac{1}{s}} + \frac{7}{s} + \frac{v_L-0}{s} = 0 \longrightarrow v_L\left[2+s+\frac{1}{s}\right] = -\frac{2}{s} - \frac{7}{s}$$

$$\Rightarrow v_L = \frac{-9/s}{2+s+1/s} = \frac{-9}{s^2+2s+1} = \frac{-9}{(s+1)^2} \xrightarrow{L^{-1}} \boxed{v_L = -9te^{-t}}$$

پاسخ سوال ۹)

$$V = (Ae^{-3t} + Bte^{-3t})u(t) + C\delta(t)$$

$$\frac{dV}{dt} = (-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t})u(t) + (Ae^{-3t} + Bte^{-3t})\delta(t) + C\delta'(t)$$

$$\frac{d^2V}{dt^2} = (9Ae^{-3t} - 3Be^{-3t} - 3Be^{-3t} + 9Bte^{-3t})u(t) + (-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t})\delta(t) + (-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t})\delta(t) + (Ae^{-3t} + Bte^{-3t})\delta'(t) + C\delta''(t)$$

ضرایب $u(t)$:
$$9Ae^{-3t} - 6Be^{-3t} + 9Bte^{-3t} + 6(-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t}) + 9(Ae^{-3t} + Bte^{-3t}) = 0$$

ضرایب $\delta(t)$:
$$-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t} + (-3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t}) + 6(Ae^{-3t} + Bte^{-3t}) + 9C = \frac{1}{3} \xrightarrow{t=0} \boxed{2B + 9C = \frac{1}{3}}$$

ضرایب $\delta'(t)$:
$$Ae^{-3t} + Bte^{-3t} + 6C = \frac{1}{2} \xrightarrow{t=0} \boxed{A + 6C = \frac{1}{2}}$$

ضرایب $\delta''(t)$:
$$\boxed{C = 1} \Longrightarrow \boxed{B = -\frac{13}{3}}, \boxed{A = -\frac{11}{2}}$$

$$\Rightarrow \boxed{V = \left(-\frac{11}{2}e^{-3t} - \frac{13}{3}te^{-3t}\right)u(t) + \delta(t)}$$