



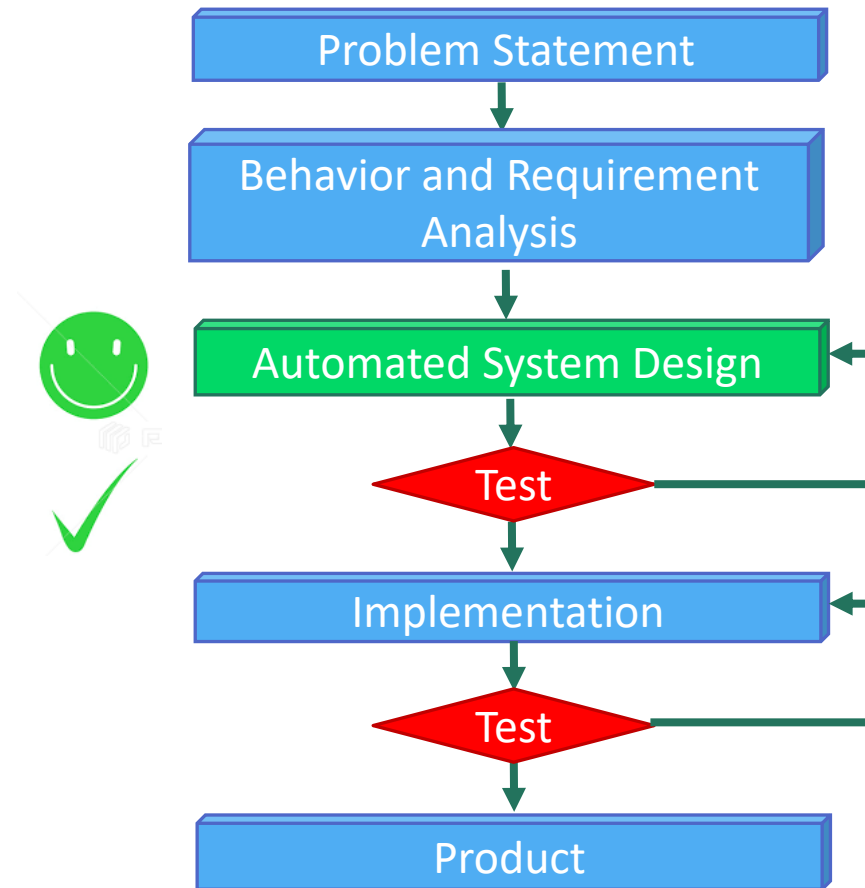
# Digital System Design

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Hajar Falahati

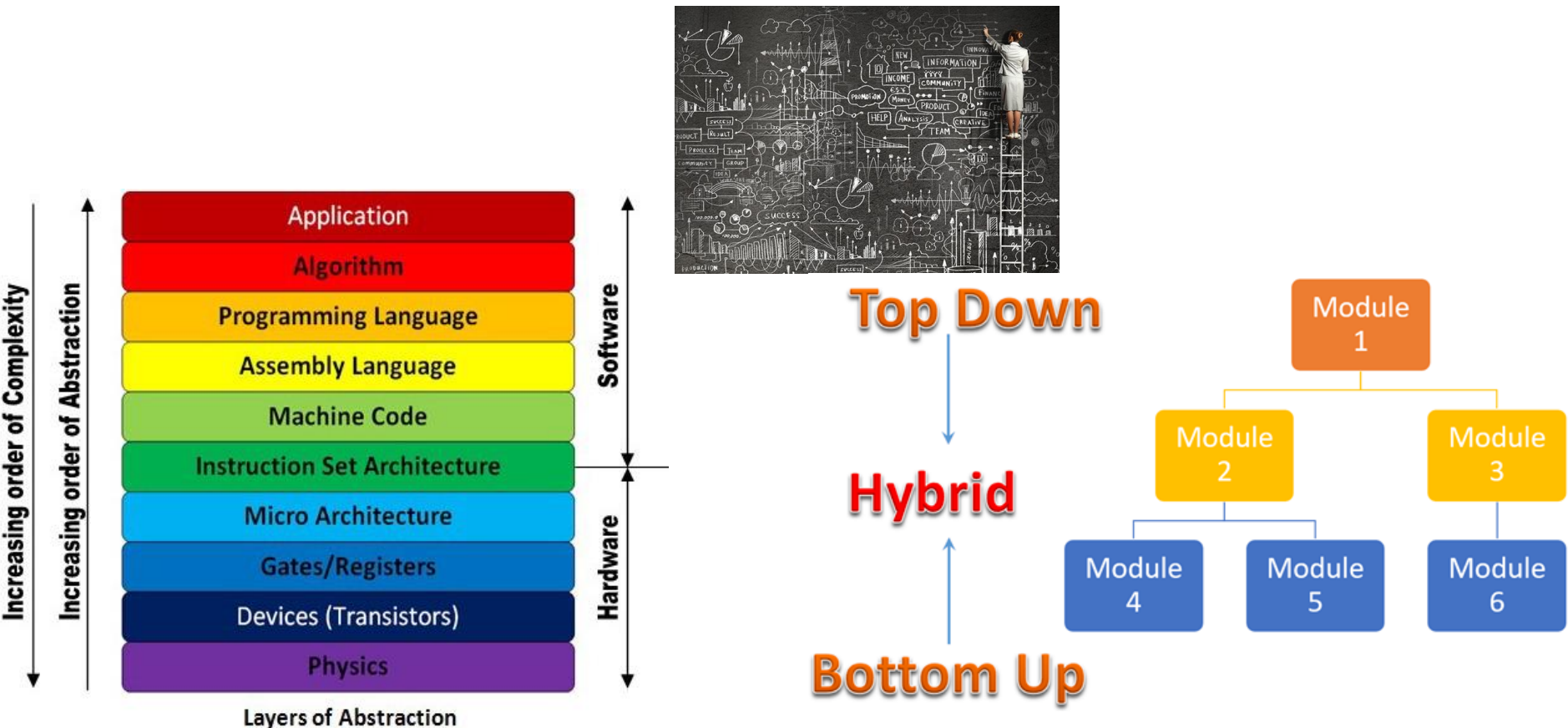
[hfalahati@ipm.ir](mailto:hfalahati@ipm.ir)  
[hfalahati@ce.sharif.edu](mailto:hfalahati@ce.sharif.edu)

# Automated Design Flow



# Design Complexity

- How to handle the **complexity in design and modeling**?



# Outline

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- **Modeling**
  - **Why?**
  - **FSM**
  - **ASM**



# Customers!

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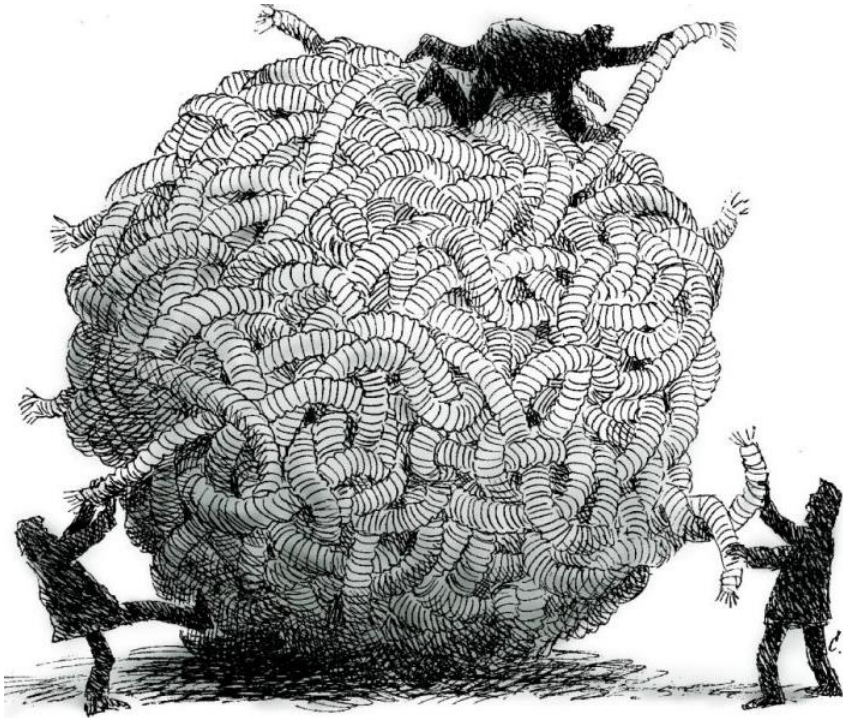
# Help My Friend!

- My friend is a digital designer and recently receives some proposals from different customers.
  - **Handle the traffic problem in a highway and country road intersection**
  - **Monitor the parking space**
  - **Optimize an elevator to reach as fast as possible**
  - **Control forest fire**
  - **Make my home safer**



# What is My Friend's Problem!

- “The customer never knows exactly what he/she wants!”
- “Their descriptions are too high-level”





# Problem Definition

- Identify the **problem**
- What is the **problem**?

GIVEN ONE HOUR TO SAVE THE WORLD, I WOULD SPEND **55 MINUTES** DEFINING THE PROBLEM, AND **5 MINUTES** FINDING THE SOLUTION.

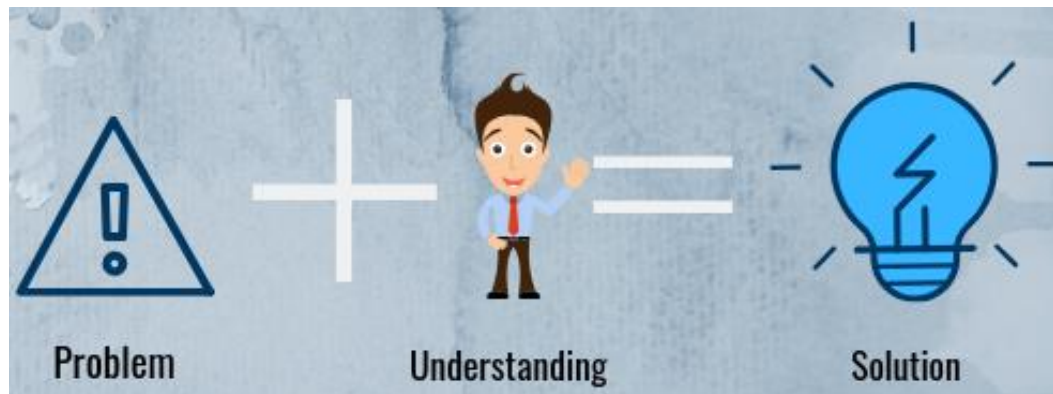
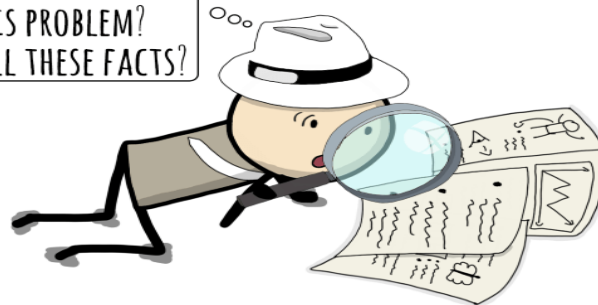




# Problem Understanding

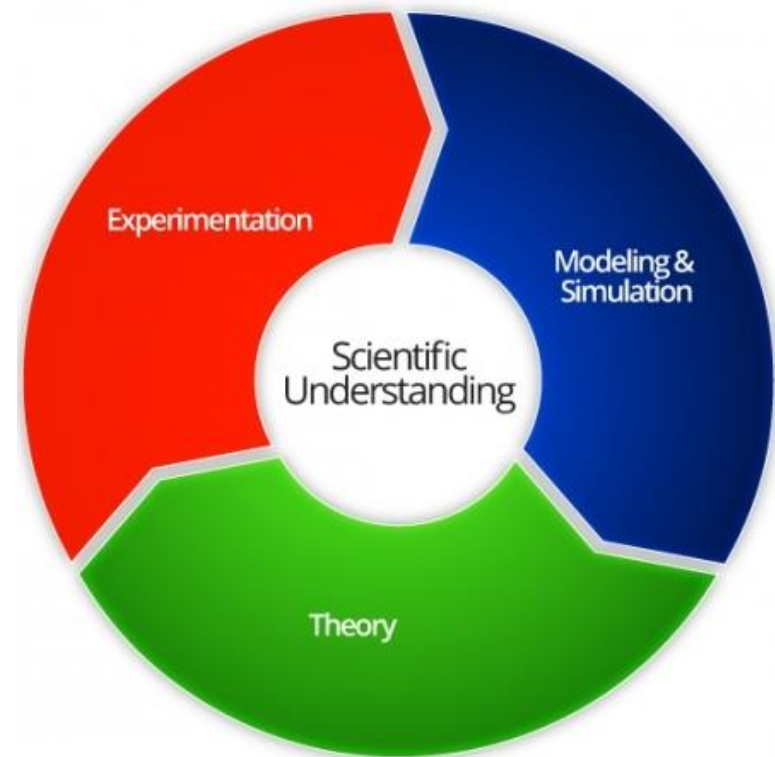
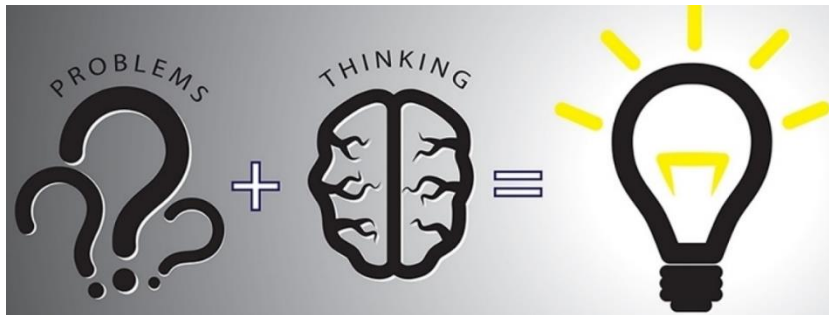
- Carefully explore about the problem
  - Listen to customer

WHAT DO I KNOW ABOUT THIS PROBLEM?  
WHAT DON'T I KNOW ABOUT THIS PROBLEM?  
WHAT'S SIGNIFICANT ABOUT ALL THESE FACTS?

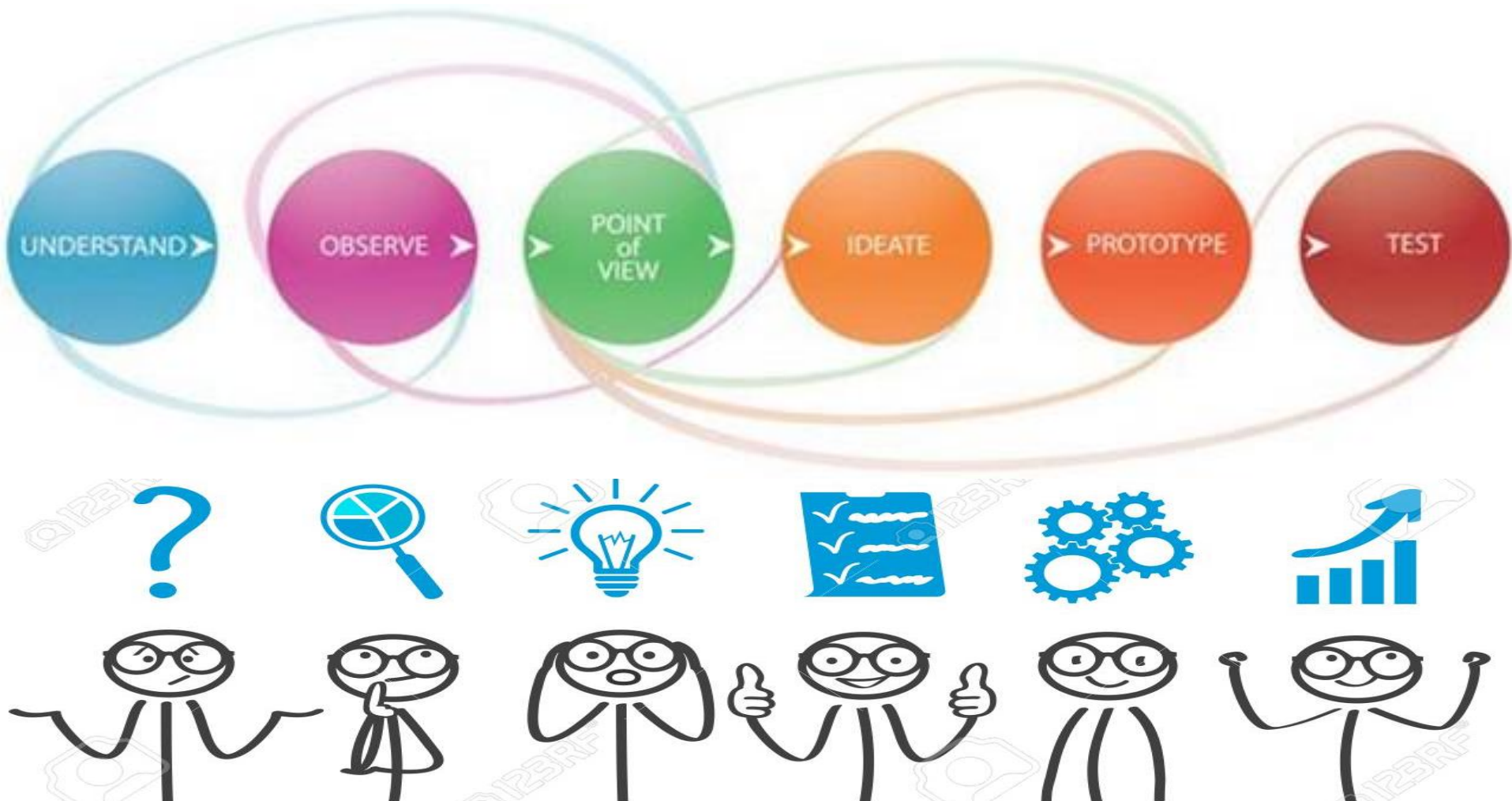


# Problem Modeling

- Formulate **the problem**
  - Involves its **detection, identification, and definition**
- Interpreting **the problem**



# Problem Solving Strategy

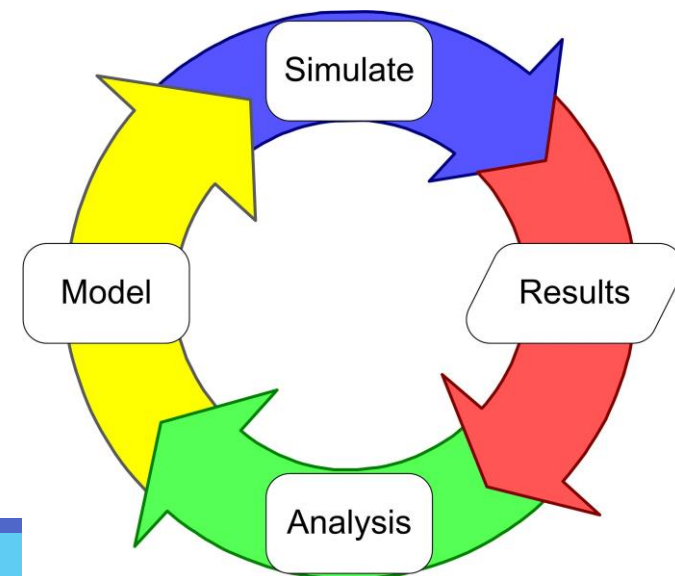


# Modeling

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# Why Modeling?

- Specific language to **describe higher level concepts** and **abstractions**
- Graphical representation helps us in
  - Understanding
  - Documenting
  - Explaining a problem
- Writing down **ideas** in a structured way
  - → Tackle **complexity**



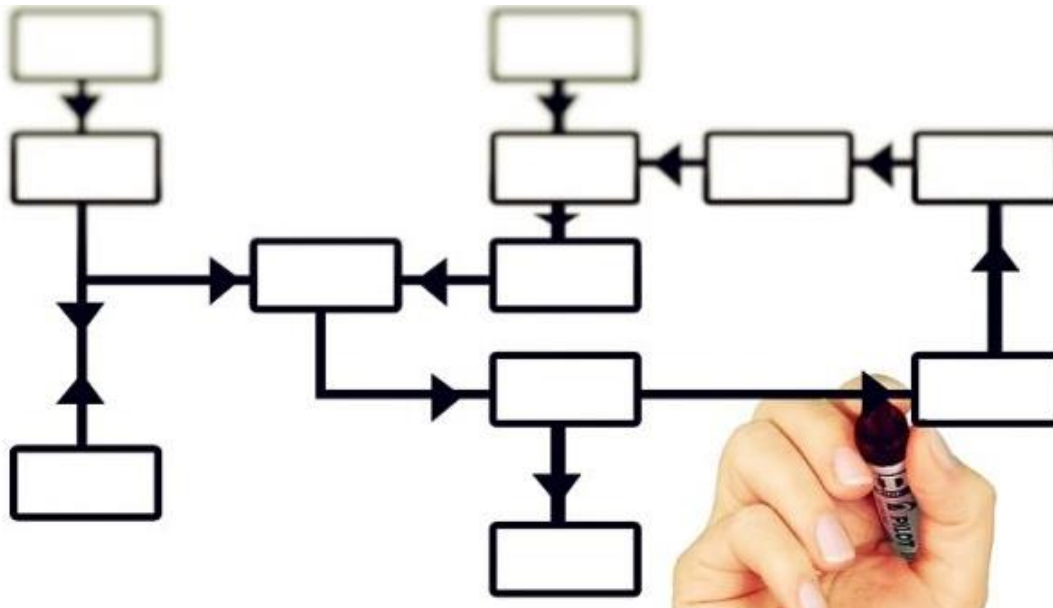
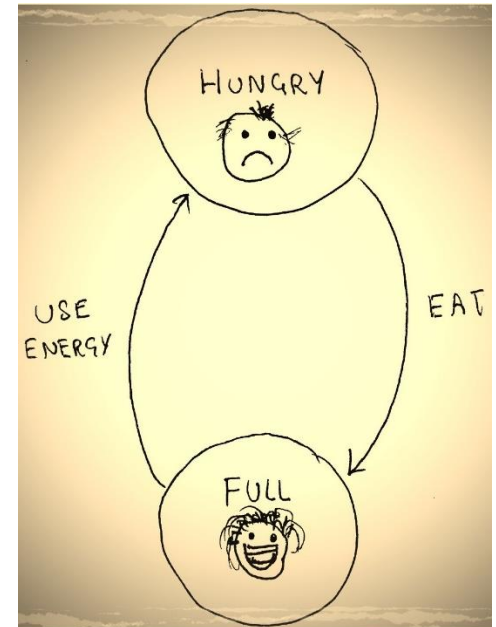
# Modeling is Necessary

- Not All Modeling, Of course !



# Modeling Types

- Finite State Machine
  - FSM
- Algorithmic State Machine
  - ASM



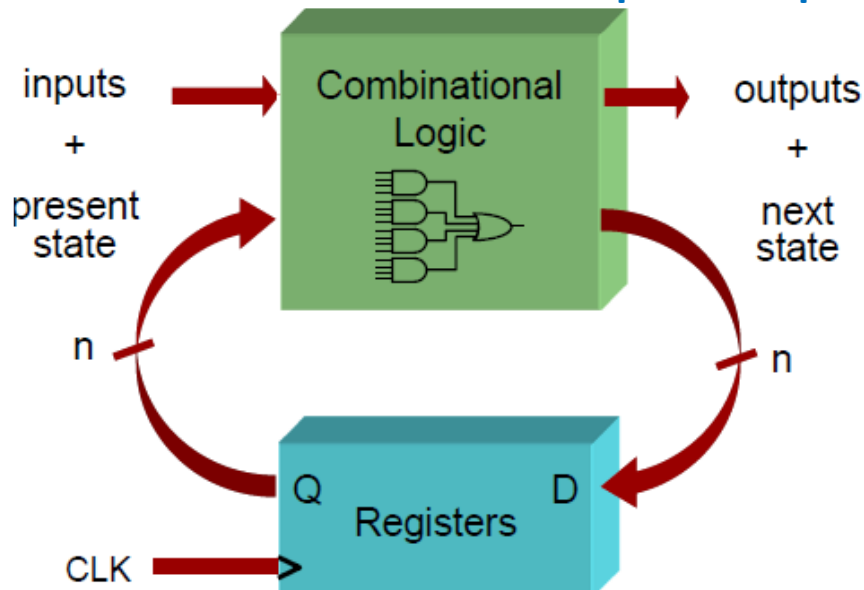


# FSM

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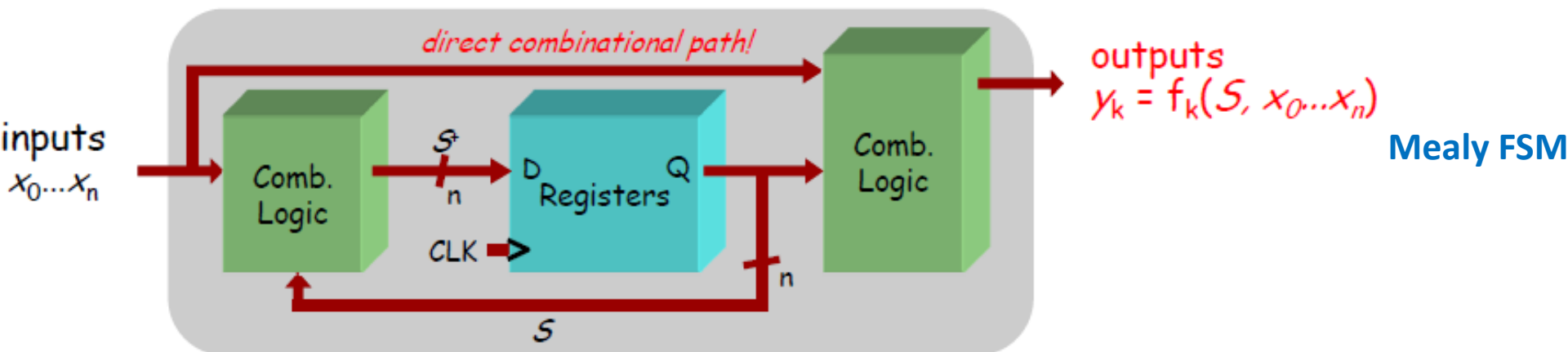
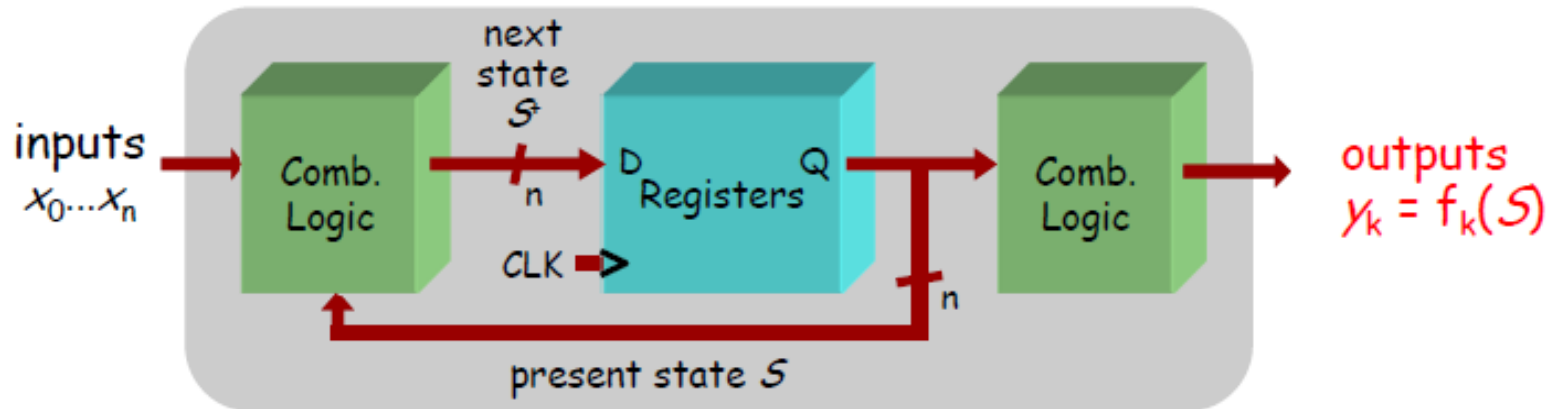
# Finite State Machine (FSM)

- A system that visits a **finite** number of **logically distinct states**
- A useful **abstraction** for **sequential circuits** with **centralized states of operation**
- At each **clock edge**, combinational logic computes
  - **Outputs** and **next state** as a function of **inputs** and **present state**



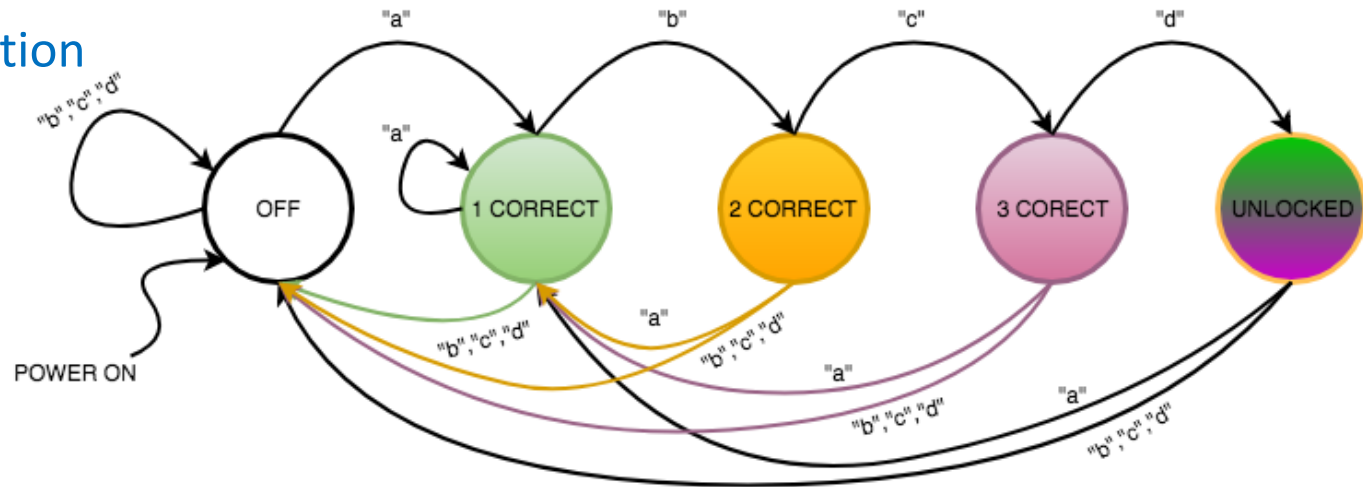
# FSM Types

Moore FSM



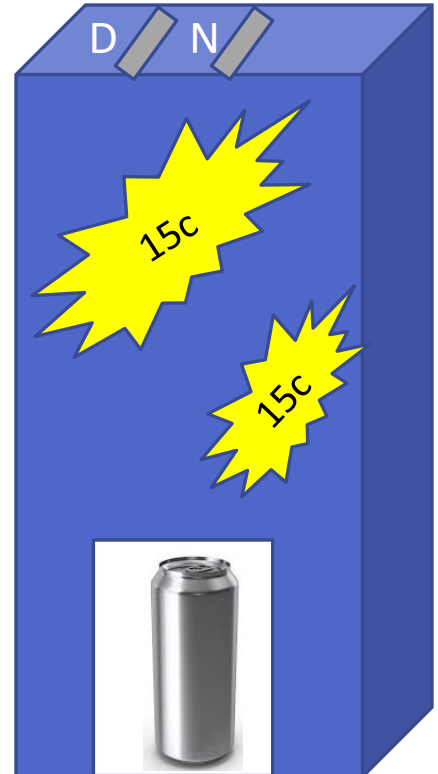
# State Diagram

- A set of states
  - Nodes in a graph
- A set of inputs and outputs
  - Edges in a graph
- A set of state transition function
  - Edges in a graph
- An output function



# Vending Machine

- The computer department need a new soda machine
- We decided to ask computer students to design a controller for the new vending machine



# Vending Machine: Characteristics

- Characteristics

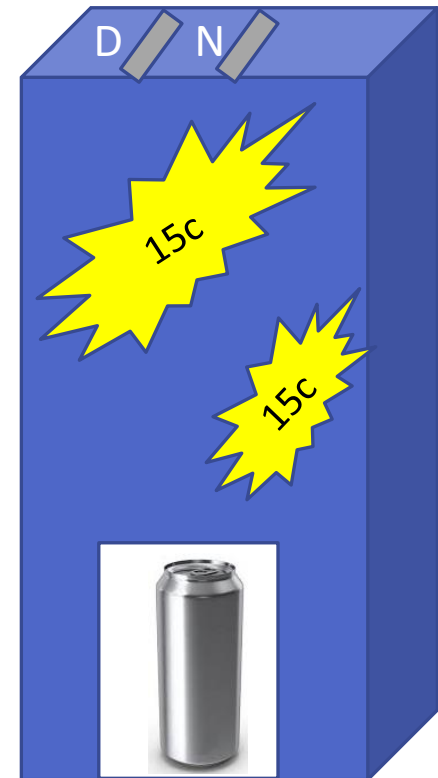
- All selections cost **15** Rial
- Machine does **not return changes!**

- Input

- **D**: dim inserted (10 Rial!)
- **F**: five inserted (5 Rial!)

- Output

- **DC**: dispense can



# Vending Machine: FSM

- **Insert coin**

- Default state
- No money has been inserted

- **5 Rials**

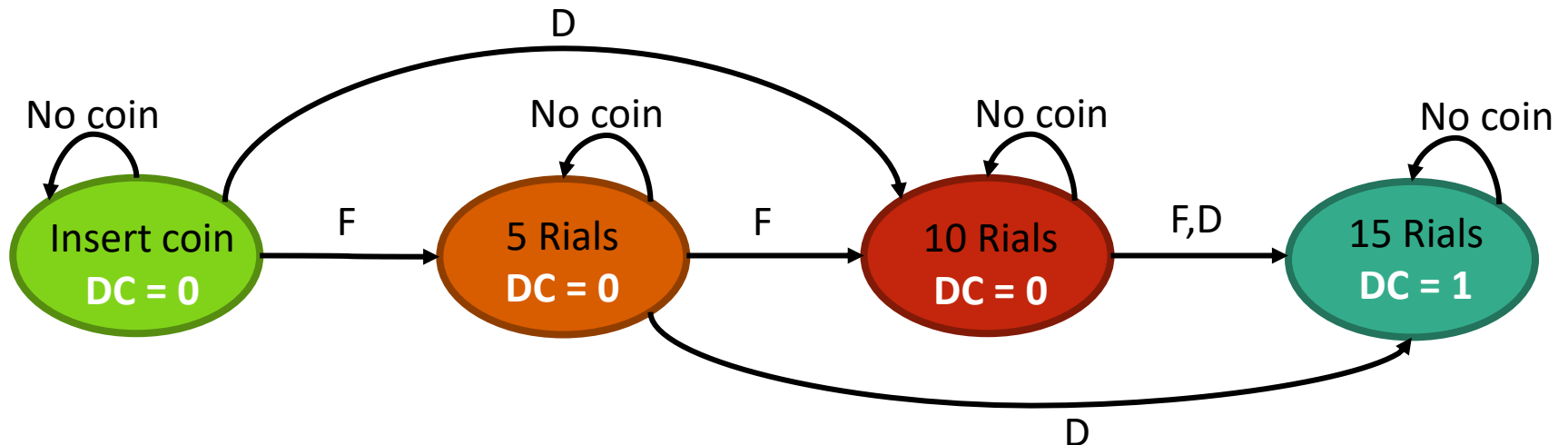
- A 5 Rial coin has been inserted

- **10 Rials**

- A 10 Rial coin has been inserted

- **15 Rials**

- Total money has been reached to 15 Rial
- Done!





# Pattern Recognition

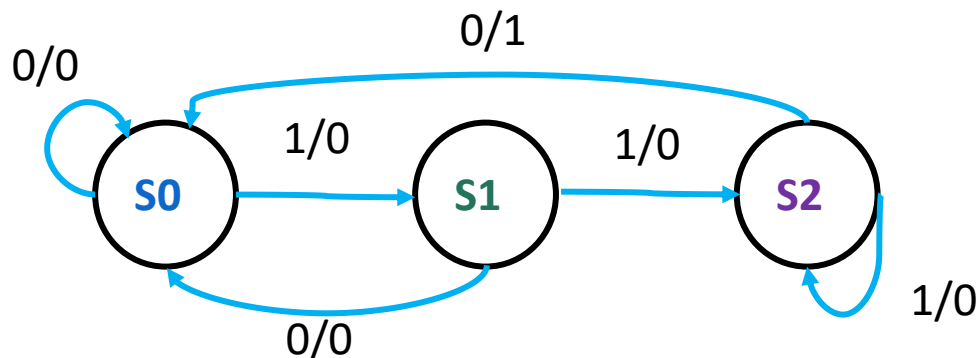
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- Recognize a specific bit pattern (*110*) in a bitstream



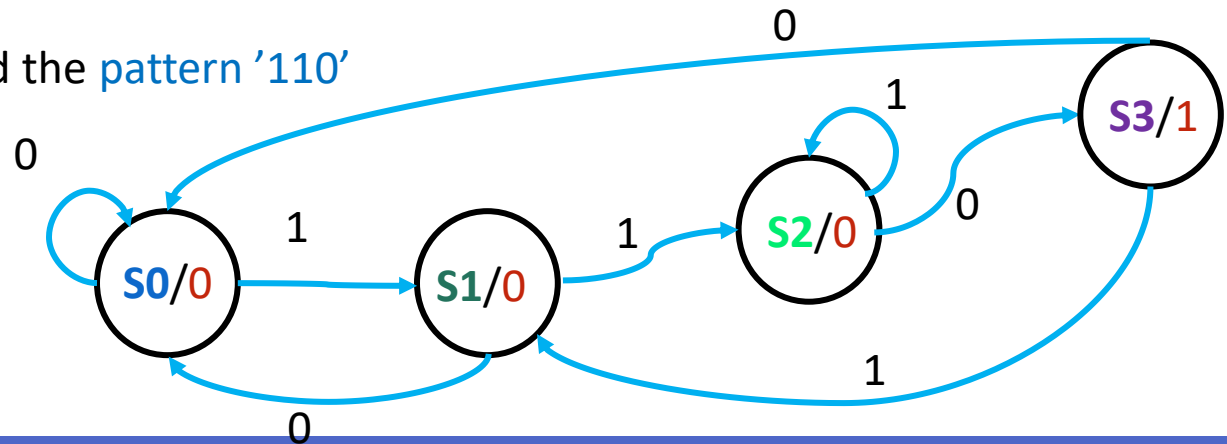
# 110 Mealy Detector

- **State S0**
  - We have **not** recognized **any useful pattern**
- **State S1**
  - We have recognized the **pattern '1'**
- **State S2:**
  - We have recognized the **pattern '11'**
  - **Output:** recognizing an input bit '0' in state S2



# 110 Moore Detector

- State **S0**
  - We have **not** recognized **any** useful pattern
- State **S1**
  - We have recognized the **pattern '1'**
- State **S2**:
  - We have recognized the **pattern '11'**
- State **S3**:
  - We have recognized the **pattern '110'**
  - **Output** becomes 1



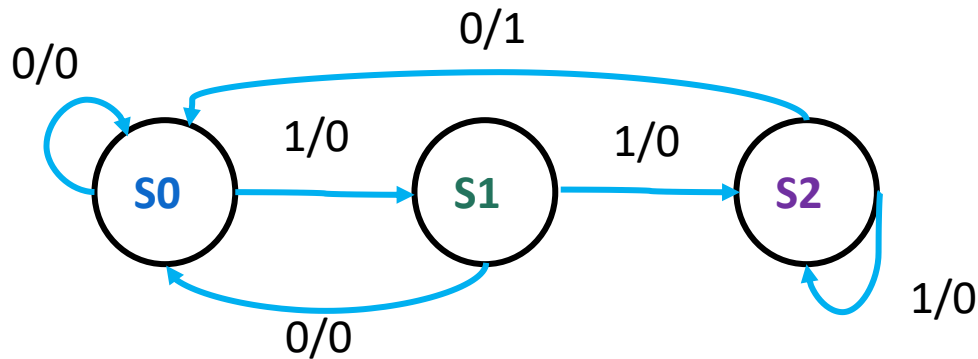
# FSM Type Conversion

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- Mealy → Moore
  - Make a state transition table for the Mealy FSM
  - Next-state is combined with the relevant output
    - State 1 and input '1' → state S2 with output 0 (S2,0)
  - There is one Moore state for each unique (next-state, output) pattern
    - (S2, 0)

# Mealy $\rightarrow$ Moore

- Convert the Mealy 110 detector to Moore 110 detector

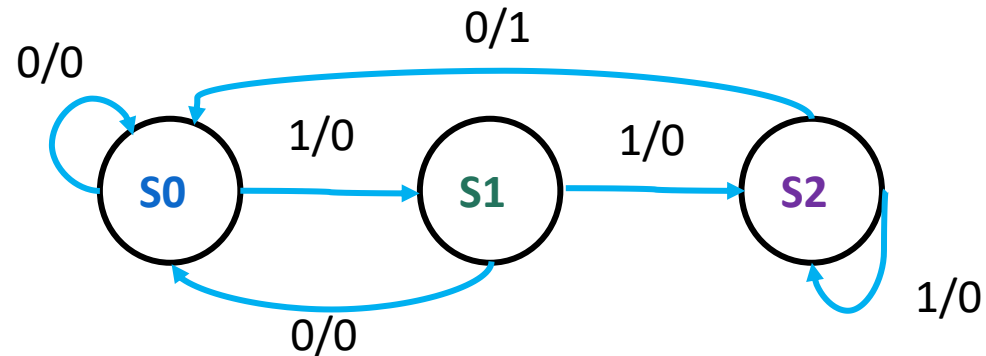


# State Transition Table

- States

- (S0, 0) = SA
- (S0, 1) = SB
- (S1, 0) = SC
- (S2, 0) = SD

Current State	X = 0	X = 1
S0	S0, 0	S1, 0
S1	S0, 0	S2, 0
S2	S0, 1	S2, 0



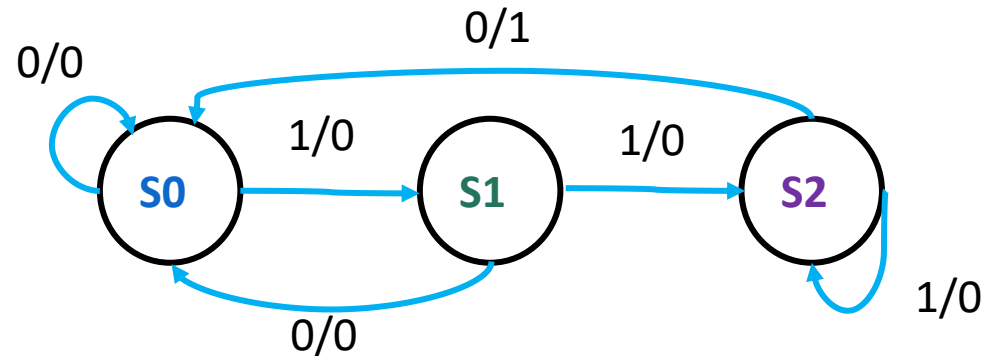
# State Transition Table

- States

- $(S0, 0) = SA$
- $(S0, 1) = SB$
- $(S1, 0) = SC$
- $(S2, 0) = SD$

Current State	$X = 0$	$X = 1$
S0	$S0, 0$	$S1, 0$
S1	$S0, 0$	$S2, 0$
S2	$S0, 1$	$S2, 0$

- There *may* be multiple Moore states for a single Mealy state
  - Mealy state S0 is split into Moore states  $(S0, 0)$  and  $(S0, 1)$





# Make Pair (NextState, Out)

Current State	$X = 0$	$X = 1$
$S_0$	$S_0, 0$	$S_1, 0$
$S_1$	$S_0, 0$	$S_2, 0$
$S_2$	$S_0, 1$	$S_2, 0$

Current State	$X = 0$	$X = 1$	Output
$SA = (S_0, 0)$	$SA$	$SC$	$0$
$SB = (S_0, 1)$	$SA$	$SC$	$1$
$SC = (S_1, 0)$			
$SD = (S_2, 0)$			

# Make Pair (NextState, Out)

Current State	X = 0	X = 1
S0	S0, 0	S1, 0
S1	S0, 0	S2, 0
S2	S0, 1	S2, 0

Current State	X = 0	X = 1	Output
SA = (S0, 0)	SA	SC	0
SB = (S0, 1)	SA	SC	1
SC = (S1, 0)	SA	SD	0
SD = (S2, 0)			0

# Make Pair (NextState, Out)

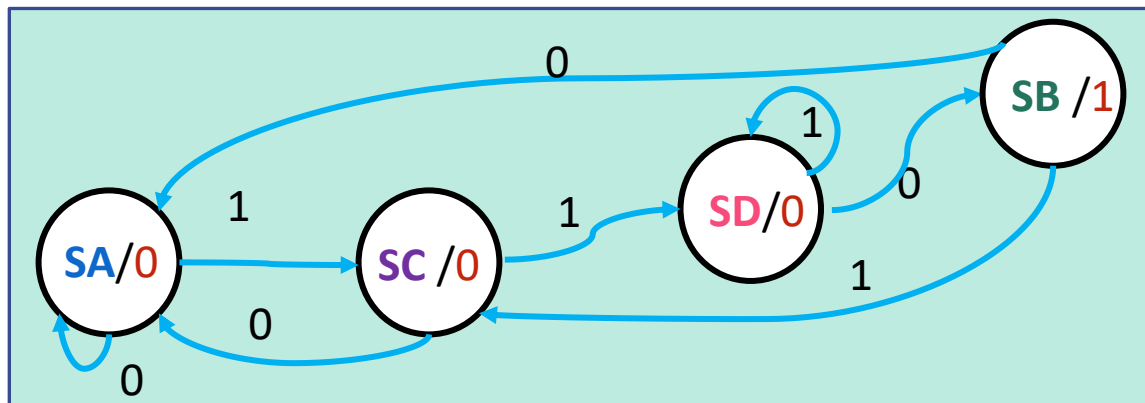
Current State	X = 0	X = 1
S0	S0, 0	S1, 0
S1	S0, 0	S2, 0
S2	S0, 1	S2, 0

Current State	X = 0	X = 1	Output
SA = (S0, 0)	SA	SC	0
SB = (S0, 1)	SA	SC	1
SC = (S1, 0)	SA	SD	0
SD = (S2, 0)	SB	SD	0

# Make Pair (NextState, Out)

Current State	X = 0	X = 1
S0	S0, 0	S1, 0
S1	S0, 0	S2, 0
S2	S0, 1	S2, 0

Current State	X = 0	X = 1	Output
SA = (S0, 0)	SA	SC	0
SB = (S0, 1)	SA	SC	1
SC = (S1, 0)	SA	SD	0
SD = (S2, 0)	SB	SD	0

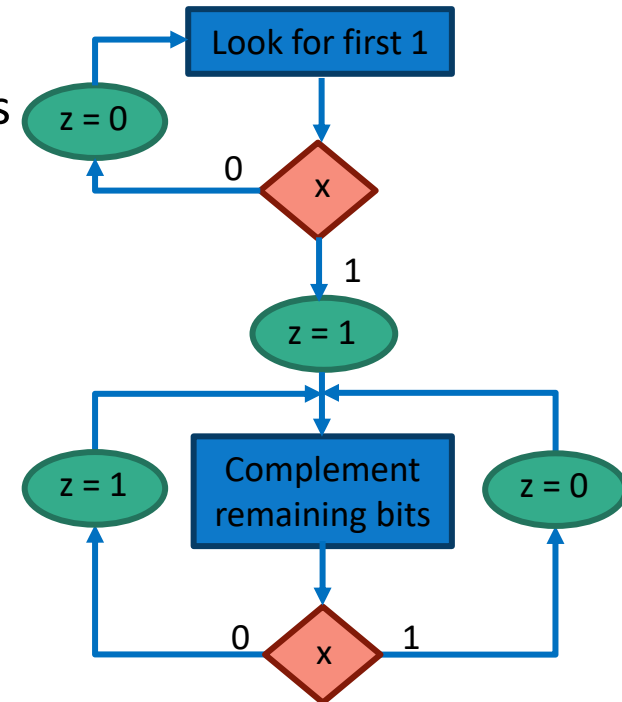


# ASM

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# Algorithmic State Machine (ASM)

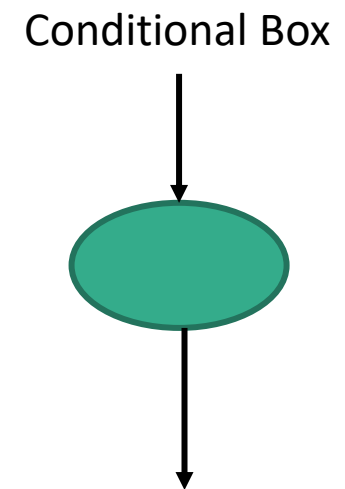
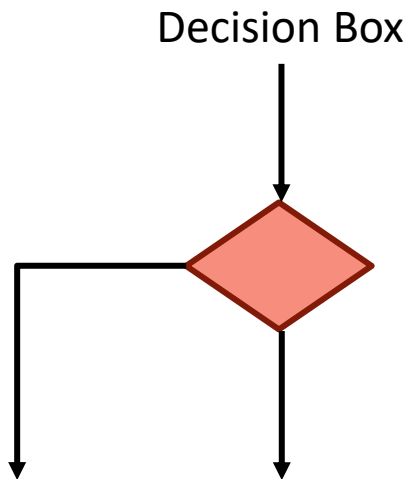
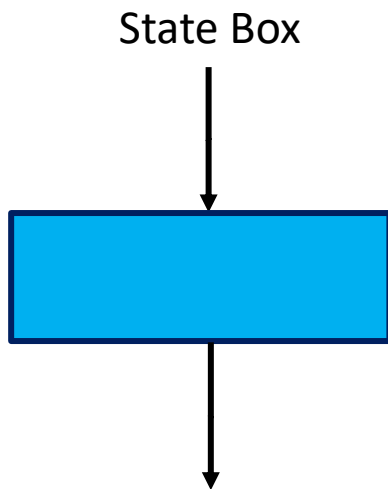
- A **systematic** way to **design complex** digital systems
  - **Complex** digital systems
  - **Large** number of **inputs** and **outputs**
- **Describes** the **sequence of events**
- **Describes timing relationship** between the states
- **Behavioral model**
- **Basic Idea**
  - Flowchart



# ASM chart Elements

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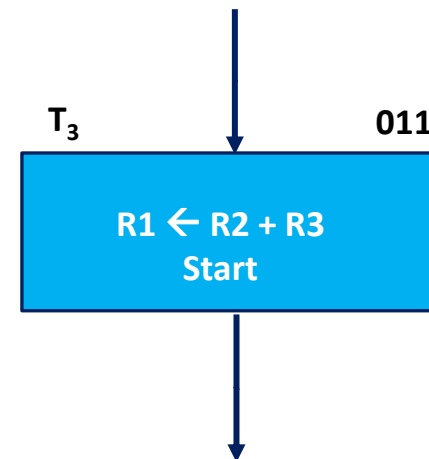
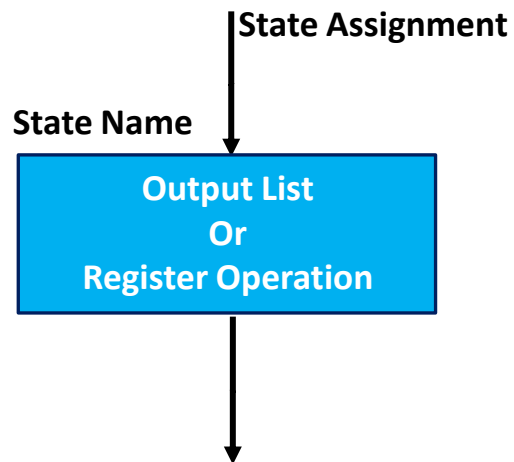
- ASM chart elements
  - State box
  - Decision box
  - Conditional box





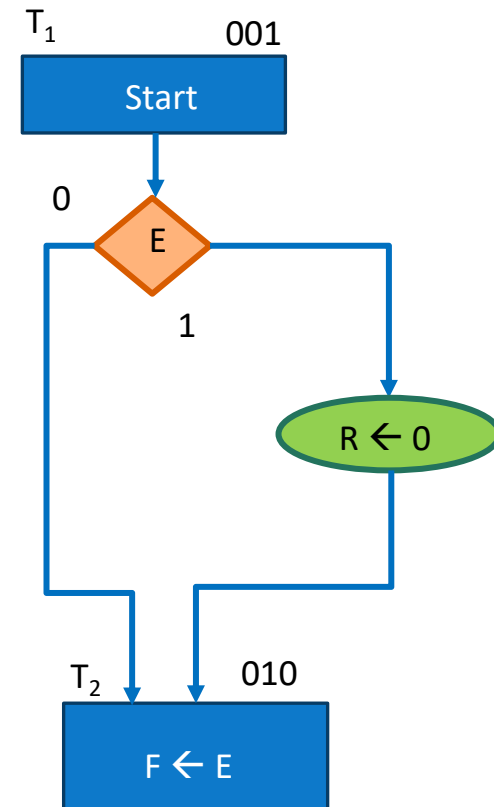
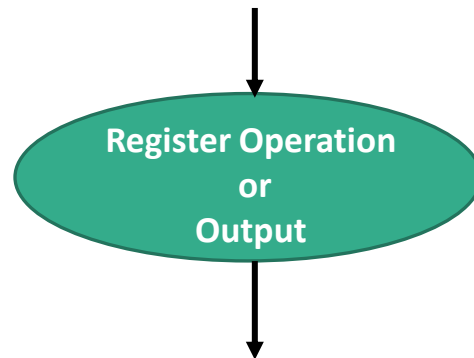
# State Box

- Represents the **state** of the system
- Register Transfers
- Takes **one cycle** to be executed
  - $R1 \leftarrow R2 + R3$



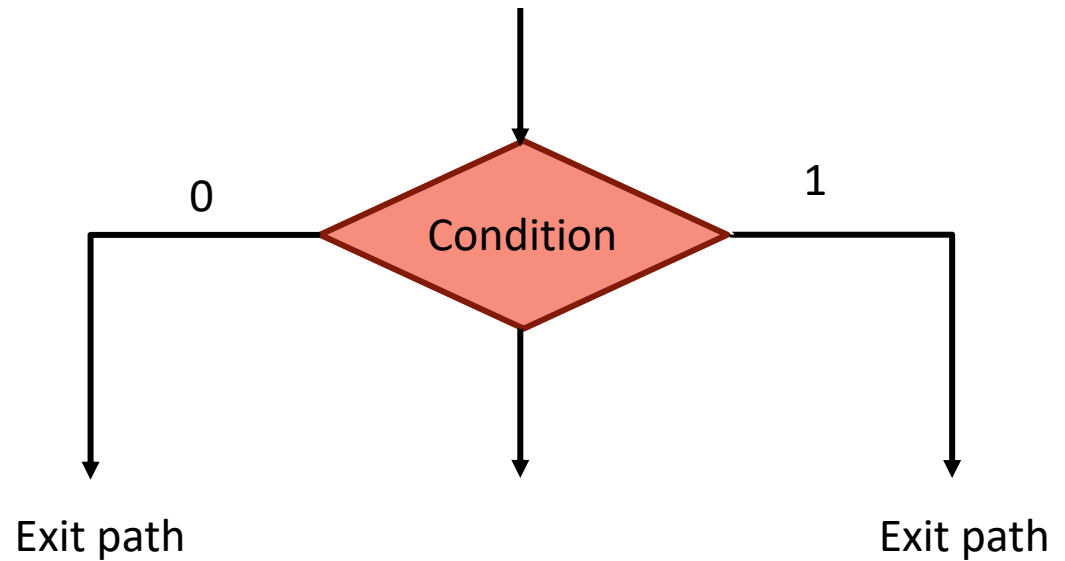
# Conditional Box

- Register Transfers
- Contain conditional output list
  - Depends on both the **state** of the system and the **inputs**
  - A.k.a., **mealy** output
  - A condition output must follow a decision box
- Takes one cycle to be executed
  - $R1 \leftarrow R2 + R3$



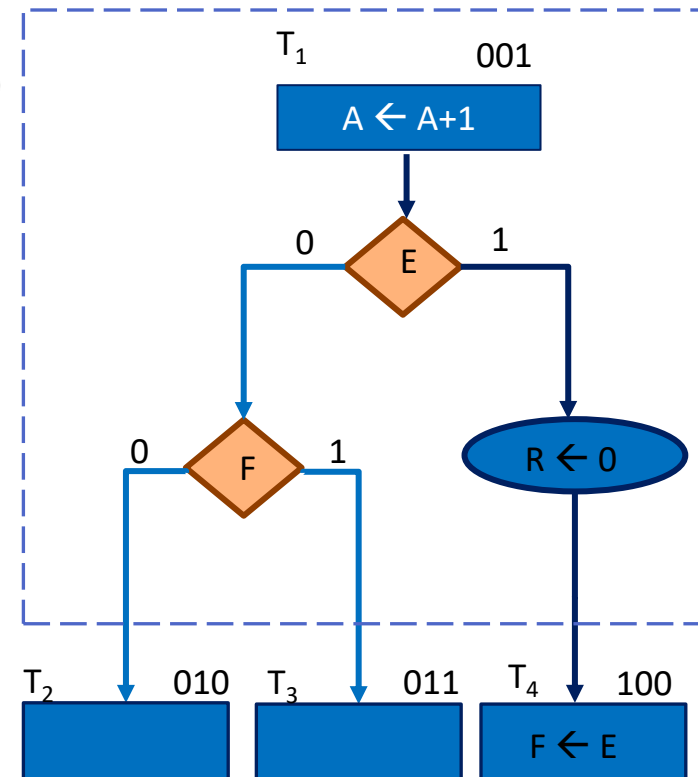
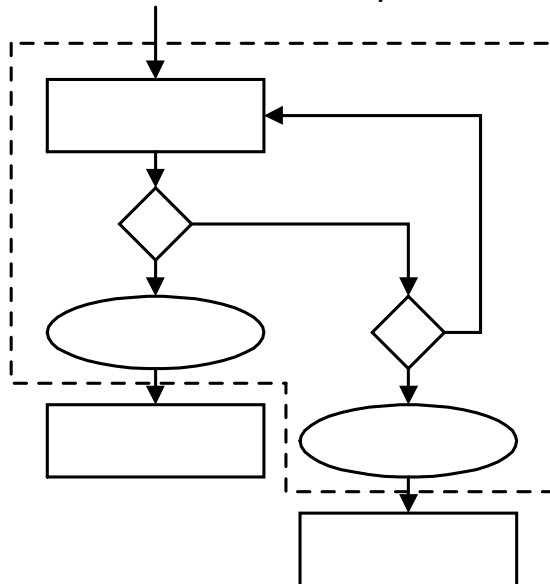
# Decision Box

- Binary expressions
  - $\sim R1[7]$



# ASM Block

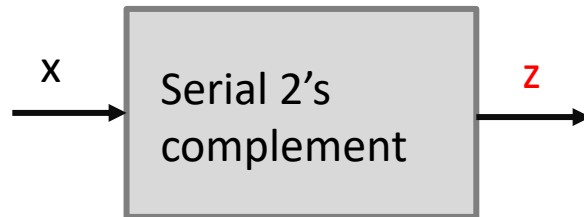
- Represents **what happens** in the system during **one** clock cycle
- Includes
  - **Only one state box**
  - All other boxes (**decision** and **conditional** boxes)
    - Connected to the exit path of the **state box**



# Sample Design 1

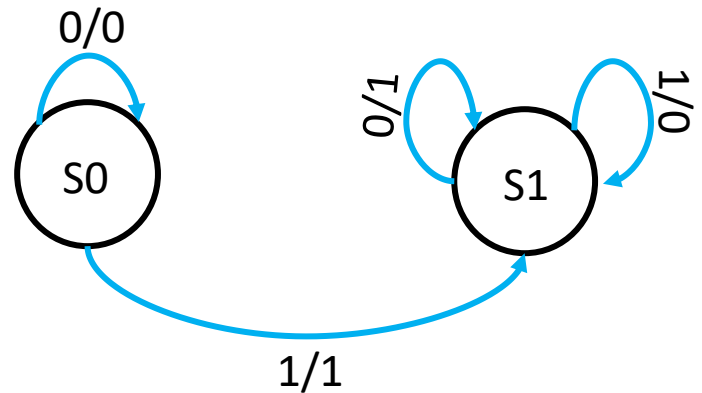
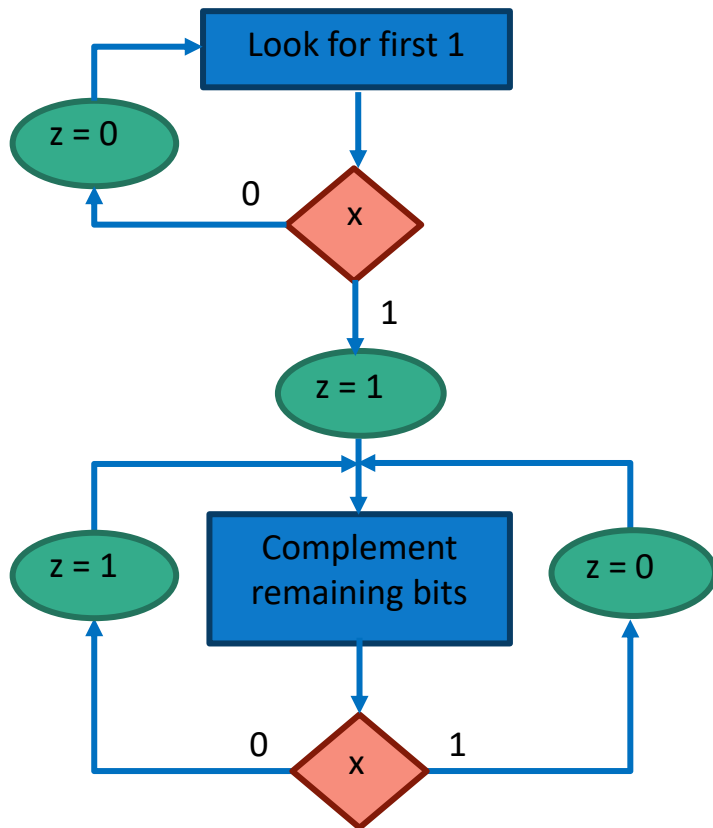
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- Design a control unit for a serial 2's complement

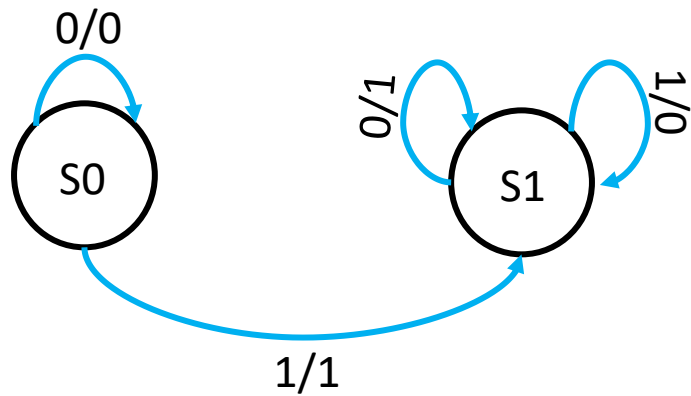


<b>t</b>	<b>x</b>	<b>z</b>
0	0	0
1	0	0
2	1	1
3	1	0

# Sample Design 1: Modeling



# Sample Design 1: Excitation Table



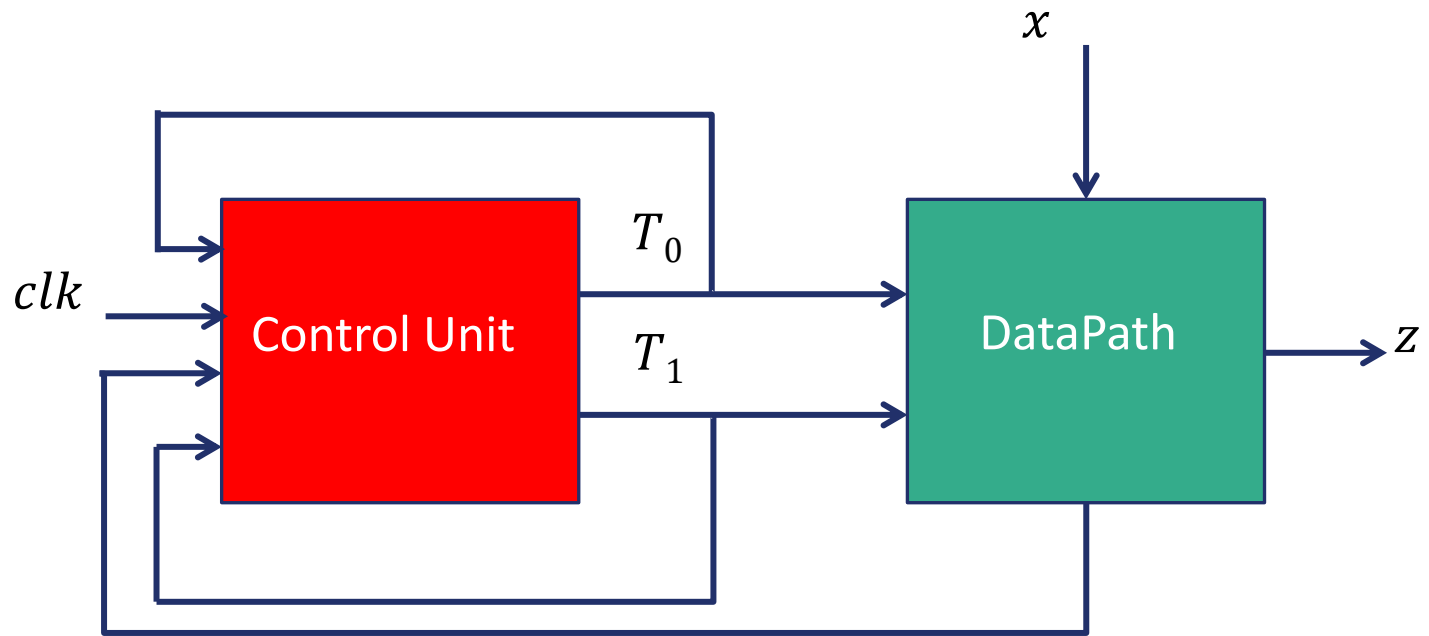
State	Input (x)	Next State	Output (z)	D
A(0)	0	A	0	0
A	1	B	1	1
B(1)	0	B	1	1
B	1	B	0	1

$$z = Ax + Bx'$$

$$D = Ax + B$$

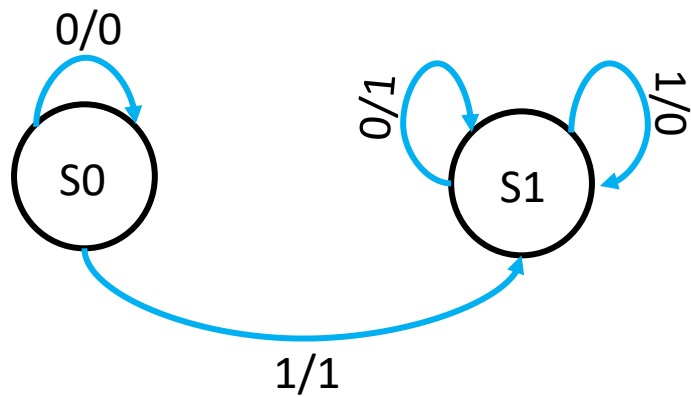
# Sample Design 1: Block Diagram

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# Sample Design 1: Design



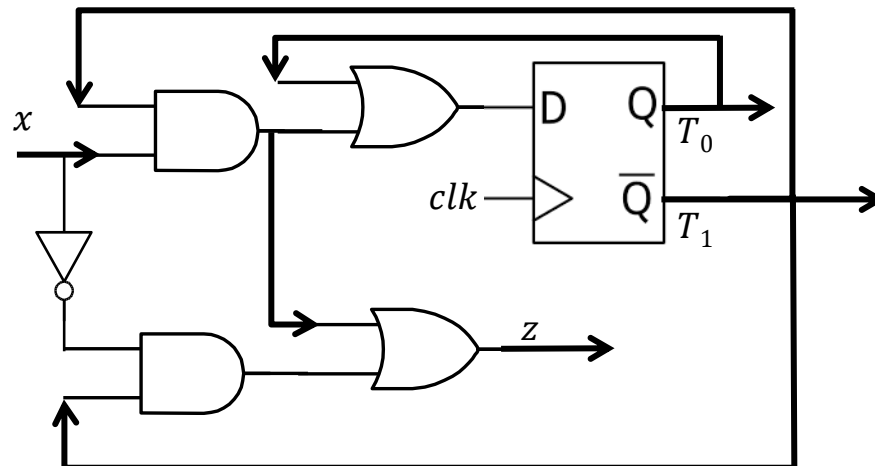
State	Input (x)	Next State	Output (z)	D
A(0)	0	A	0	0
A	1	B	1	1
B(1)	0	B	1	1
B	1	B	0	1

$$z = Ax + Bx'$$

$$z = Q'x + Qx'$$

$$D = Ax + B$$

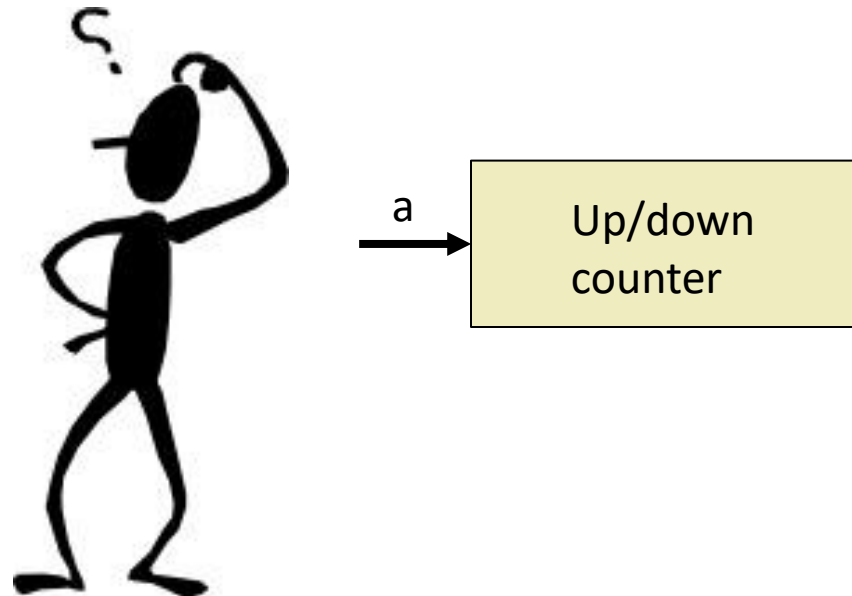
$$D = Q'x + Q$$



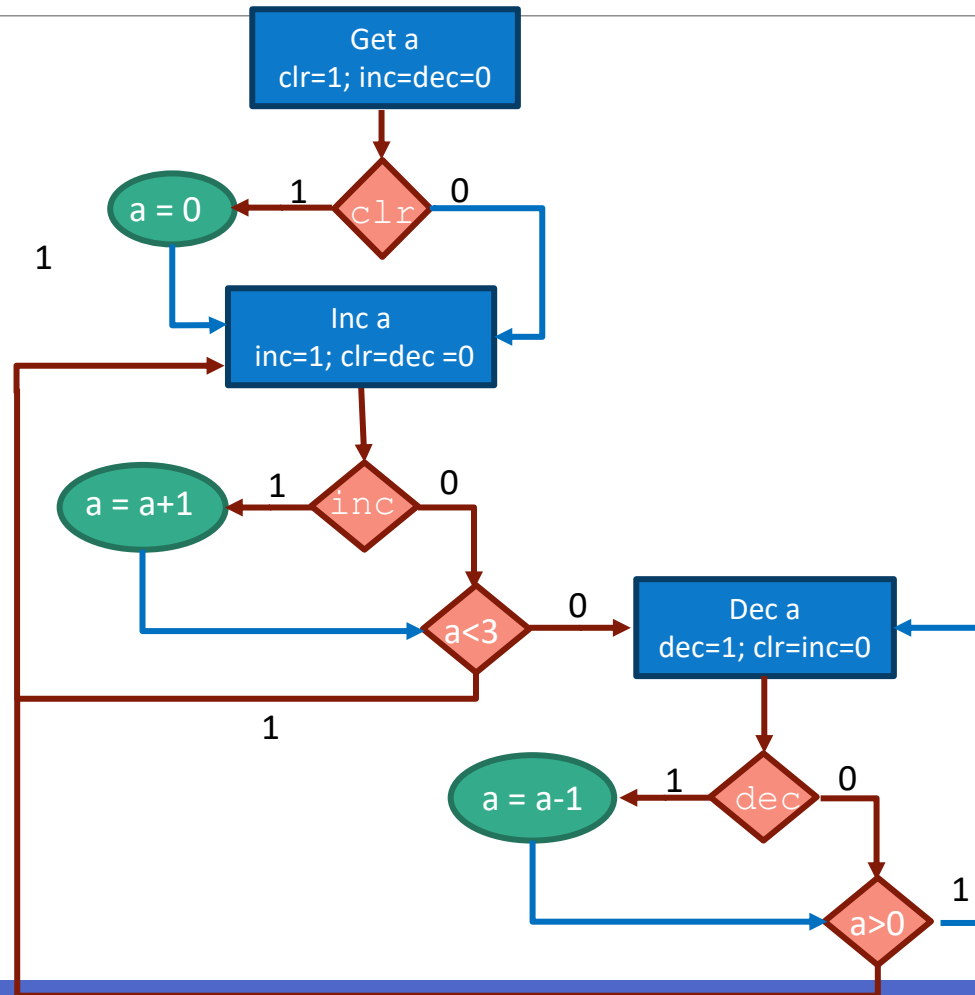
# Sample 3

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- Design an up/down counter



# Up/Down Counter ASM



# FSMD

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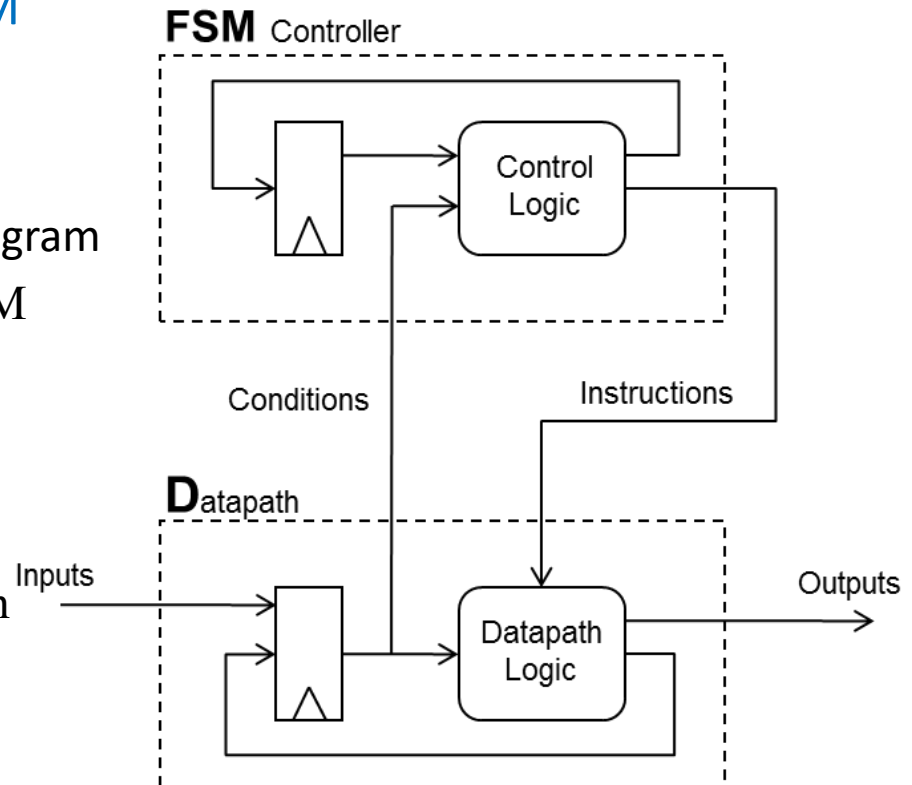
# FSMD

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- Combines a hardware control model (an FSM) with a datapath
- FSMD datapath uses conditionally executed **always blocks**
  - Instructions will be executed ONLY when a controller invokes them
- FSMDs are useful because they capture control-flow as well as data-flow in hardware
  - C programs are a combination of control-flow and data-flow which implicitly 'connects' hardware FSMD models and C programs

# FSMD (cont'd)

- An FSMD contains two stacked FSM
- **Controller**
  - Top FSM
  - Specified using a state transition diagram
  - Sends instructions to the bottom FSM
- **Datapath**
  - Bottom FSM
  - Specified using expressions
  - Receives status information in return



# Sample 4

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Greatest Common Divisor (GCD) using Euclid's algorithm.



# GCD

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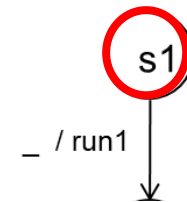
```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:       else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```



# GCD Control Unit

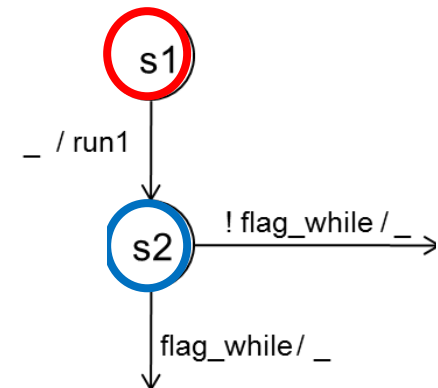
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```



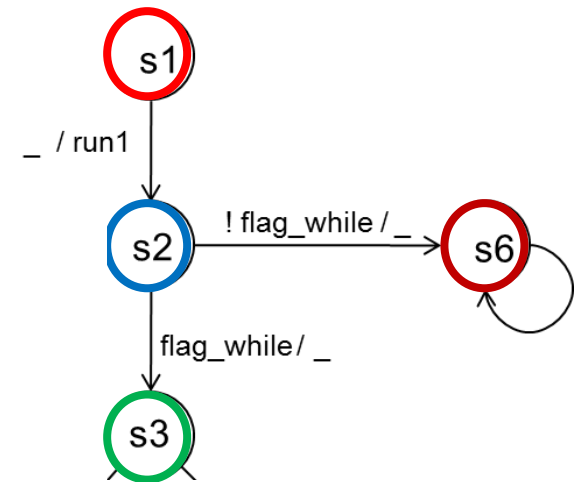
# GCD Control Unit

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   }  
7:   return a;  
}
```



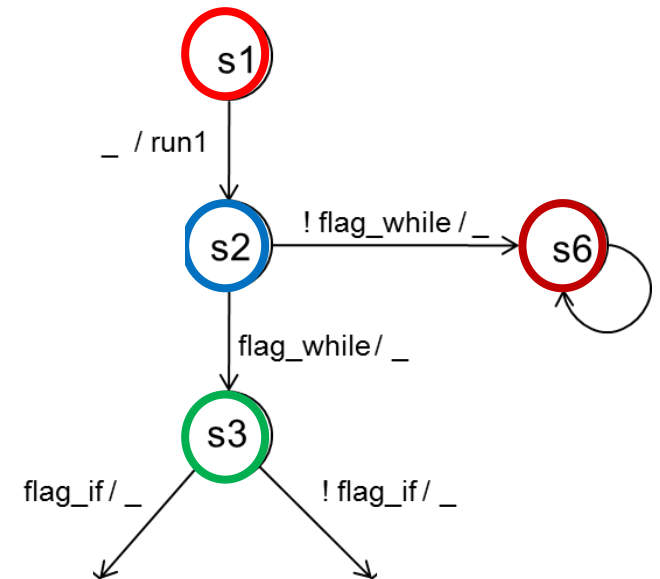
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```



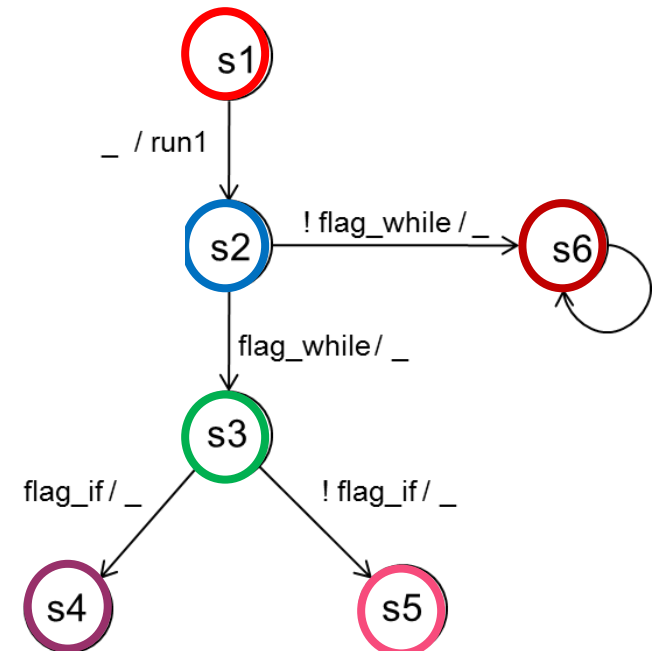
# GCD Control Unit

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```



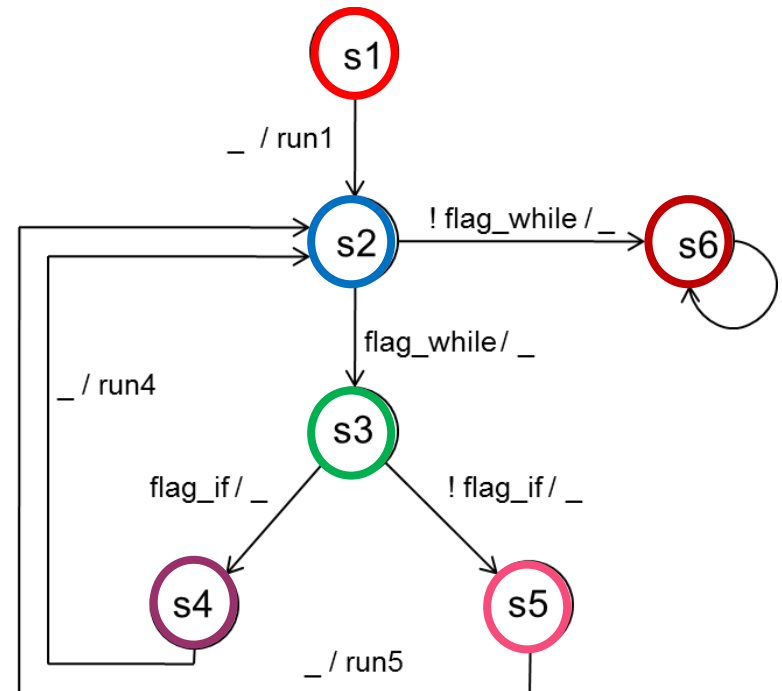
# GCD Control Unit

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1: int gcd(int a, int b) {  
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5:       b = b - a;  
6:   }  
   return a;  
}
```



# GCD Control Unit

```
1: int gcd(int a, int b) {  
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3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
   }  
   return a;  
}
```



# GCD Datapath

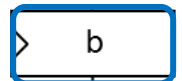
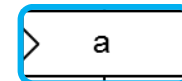
---

```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:       else  
6:       b = b - a;  
7:     }  
8:   return a;  
9: }
```

# Variables → Registers

---

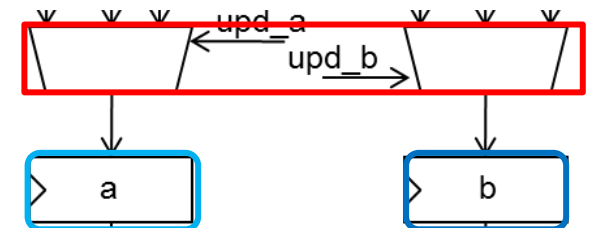
```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:       else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```





# Who Feeds the Registers?

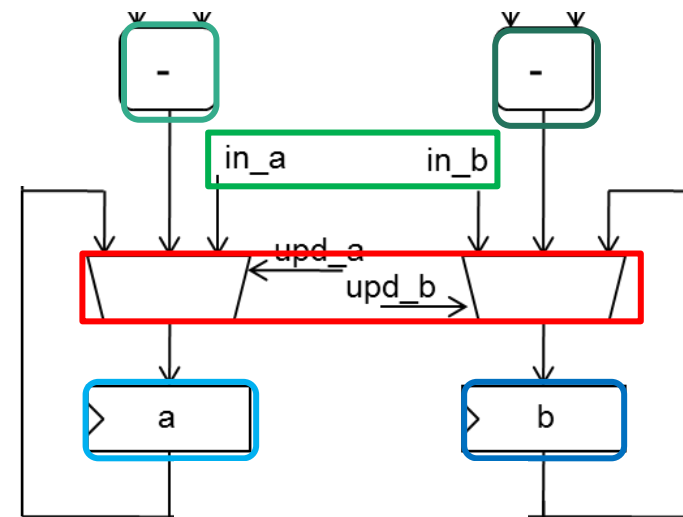
```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
   }  
7: return a;  
}
```



More than 1!

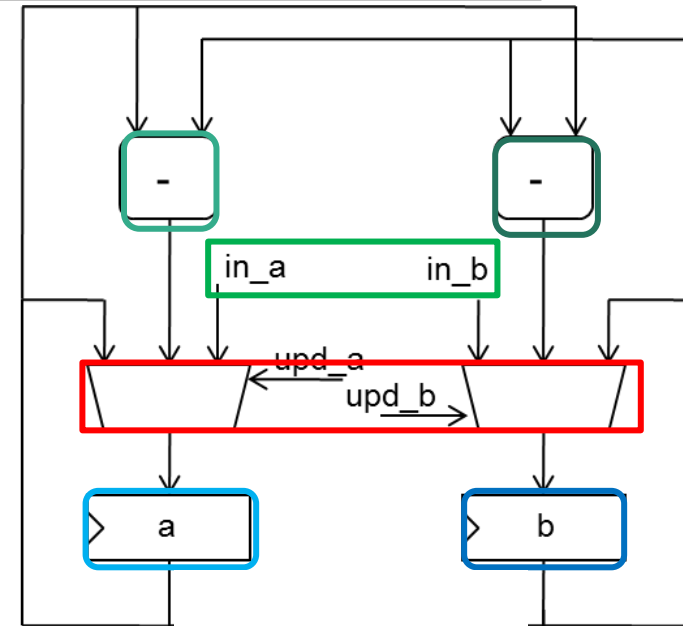
# Sources!

```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```



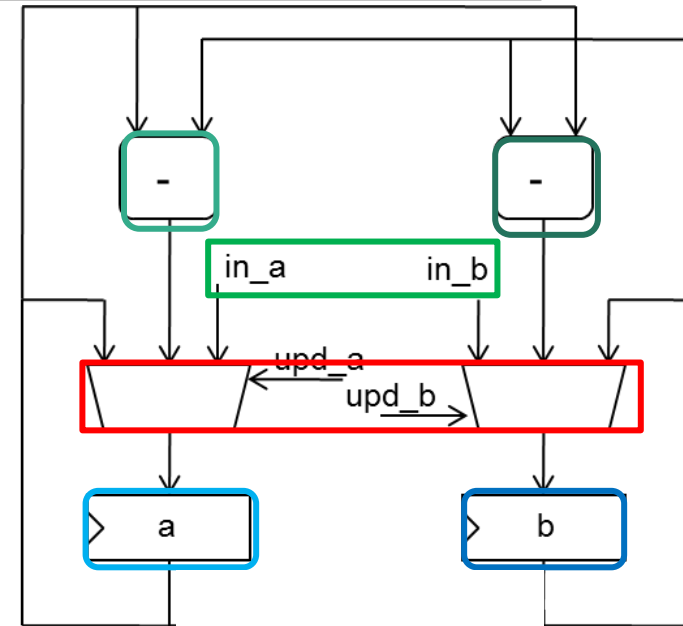
# Functional Units

```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```



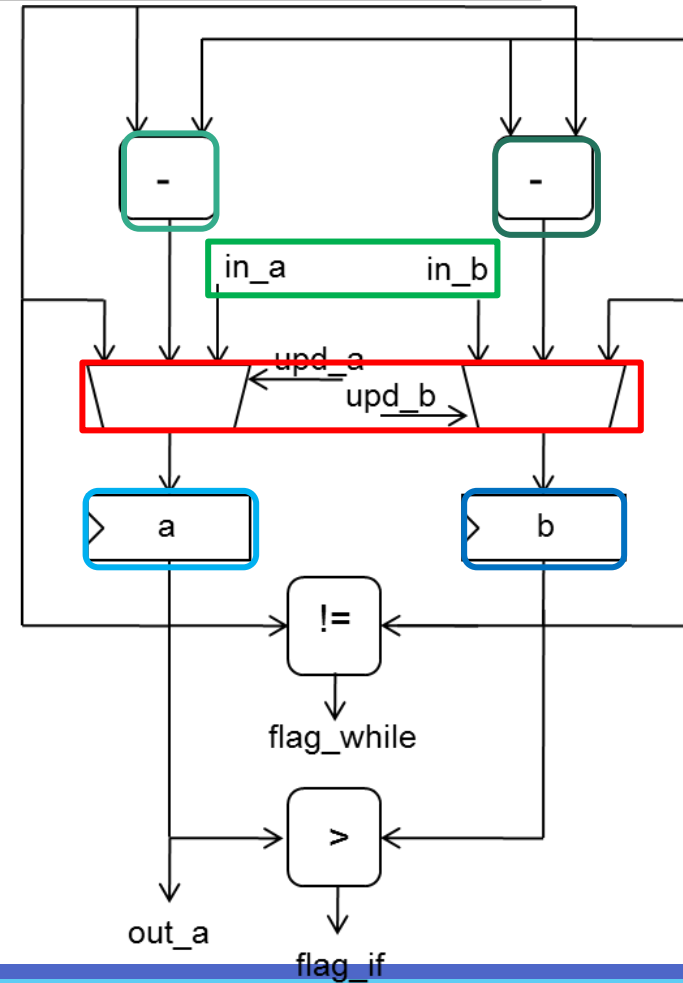
# Generate Control Signals

```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```



# Generate Control Signals

```
1: int gcd(int a, int b) {  
2:   while (a != b) {  
3:     if (a > b)  
4:       a = a - b;  
5:     else  
6:       b = b - a;  
7:   }  
8:   return a;  
9: }
```



# Samples

---

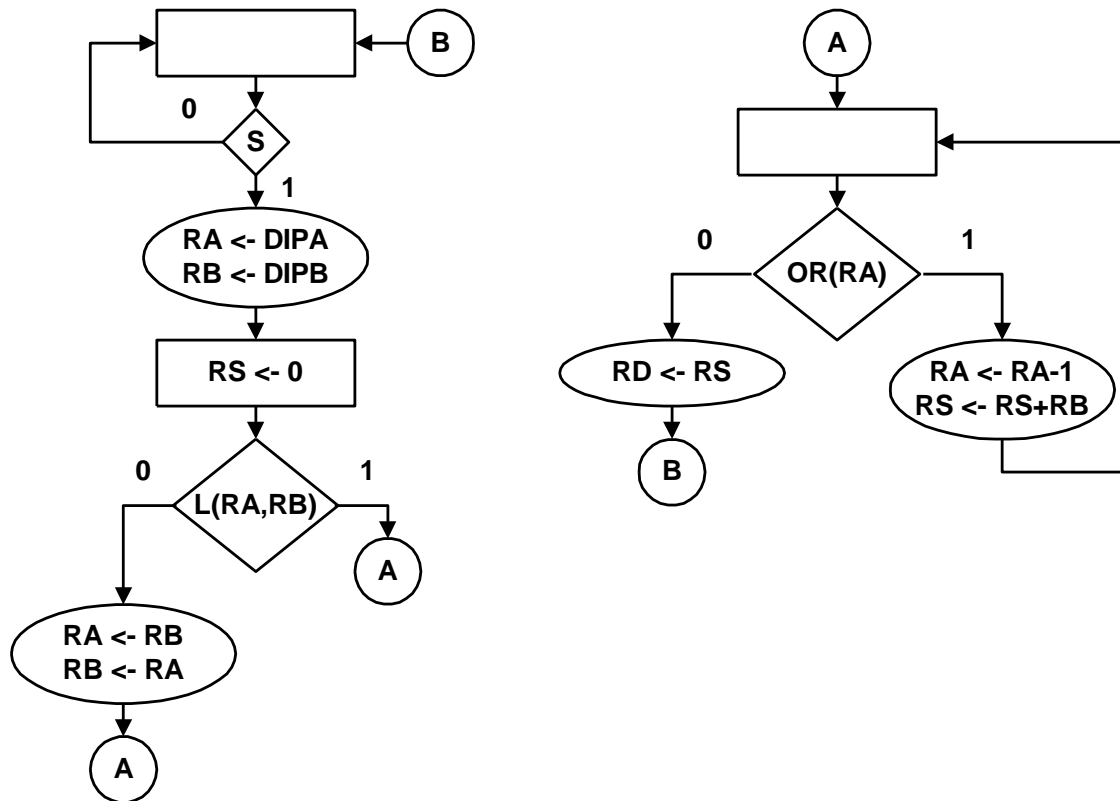
# Sample 5

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- ASM for multiplier?

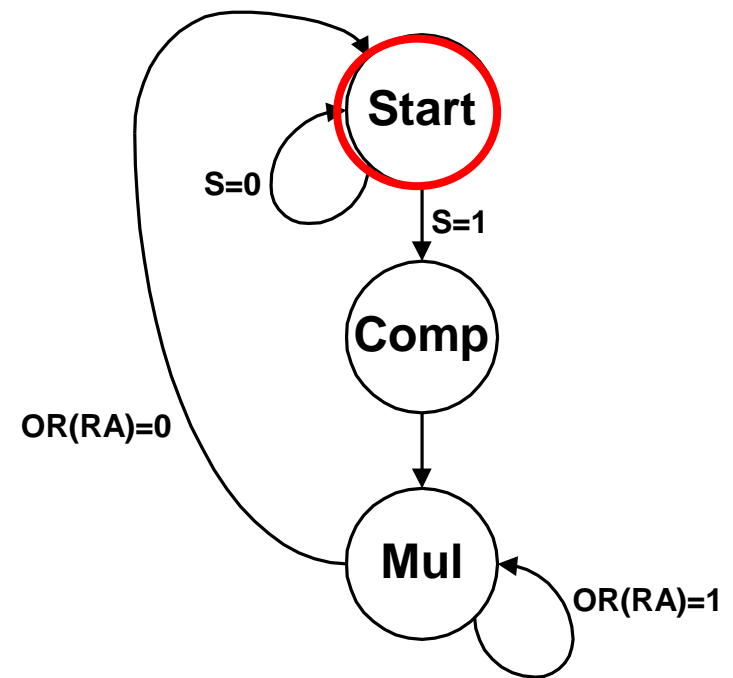
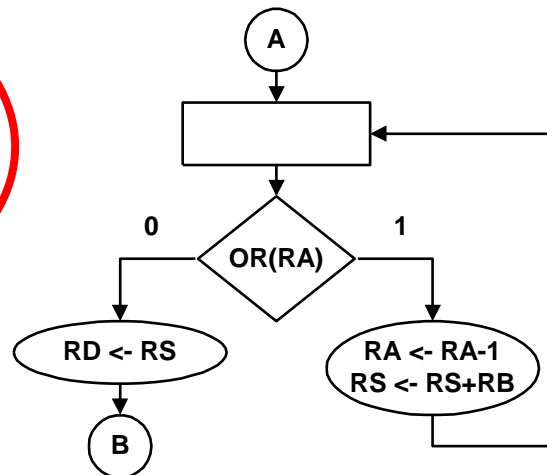
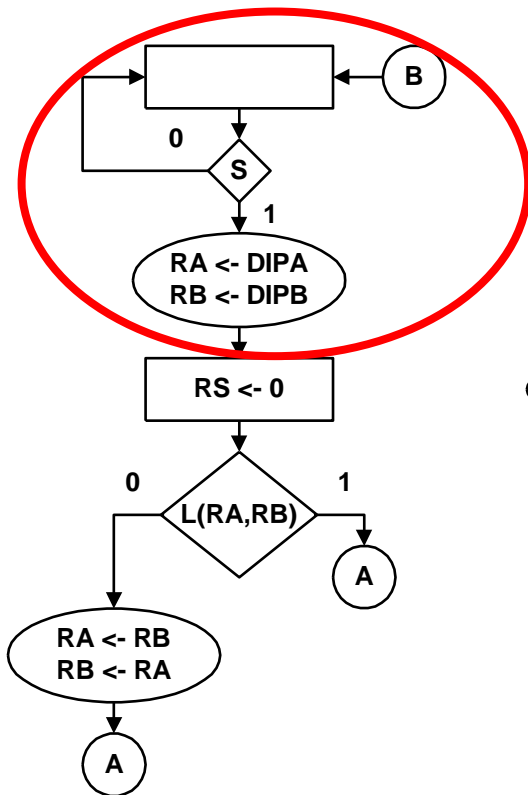


# Multiplier ASM

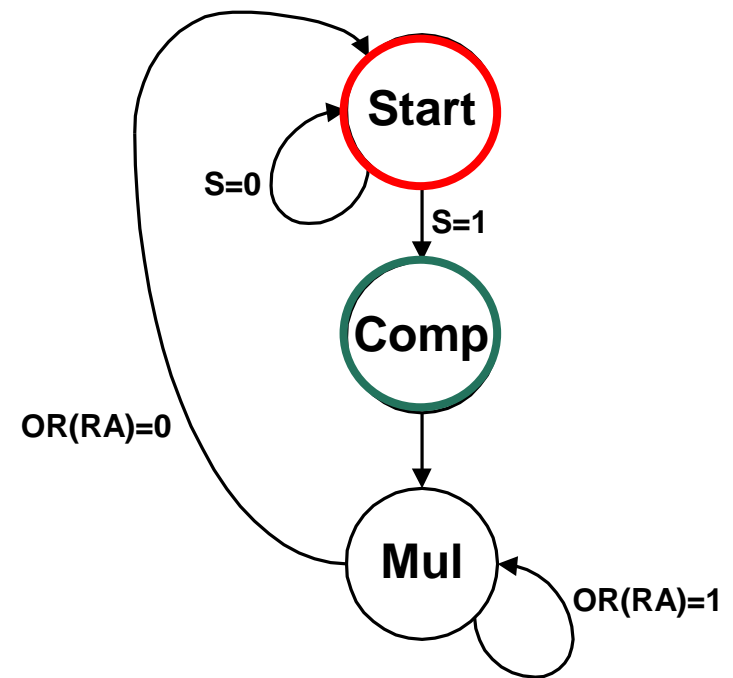
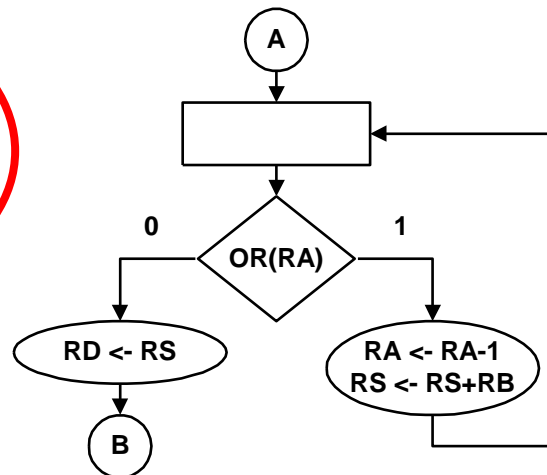
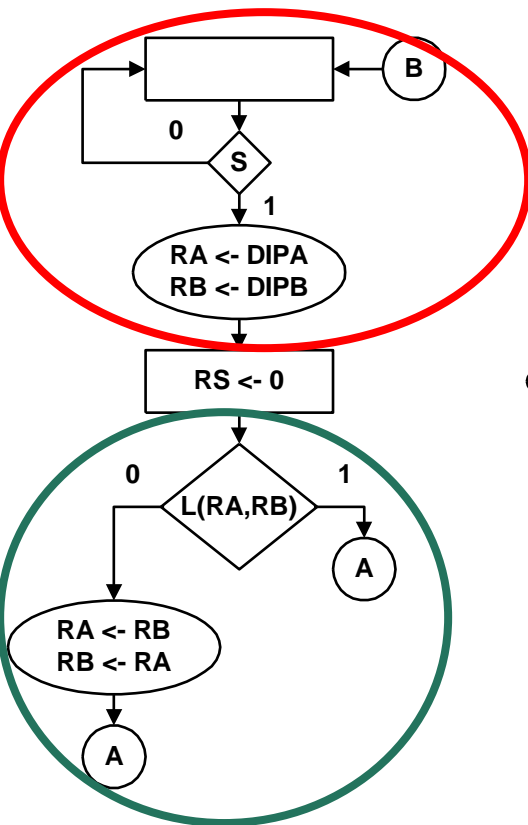




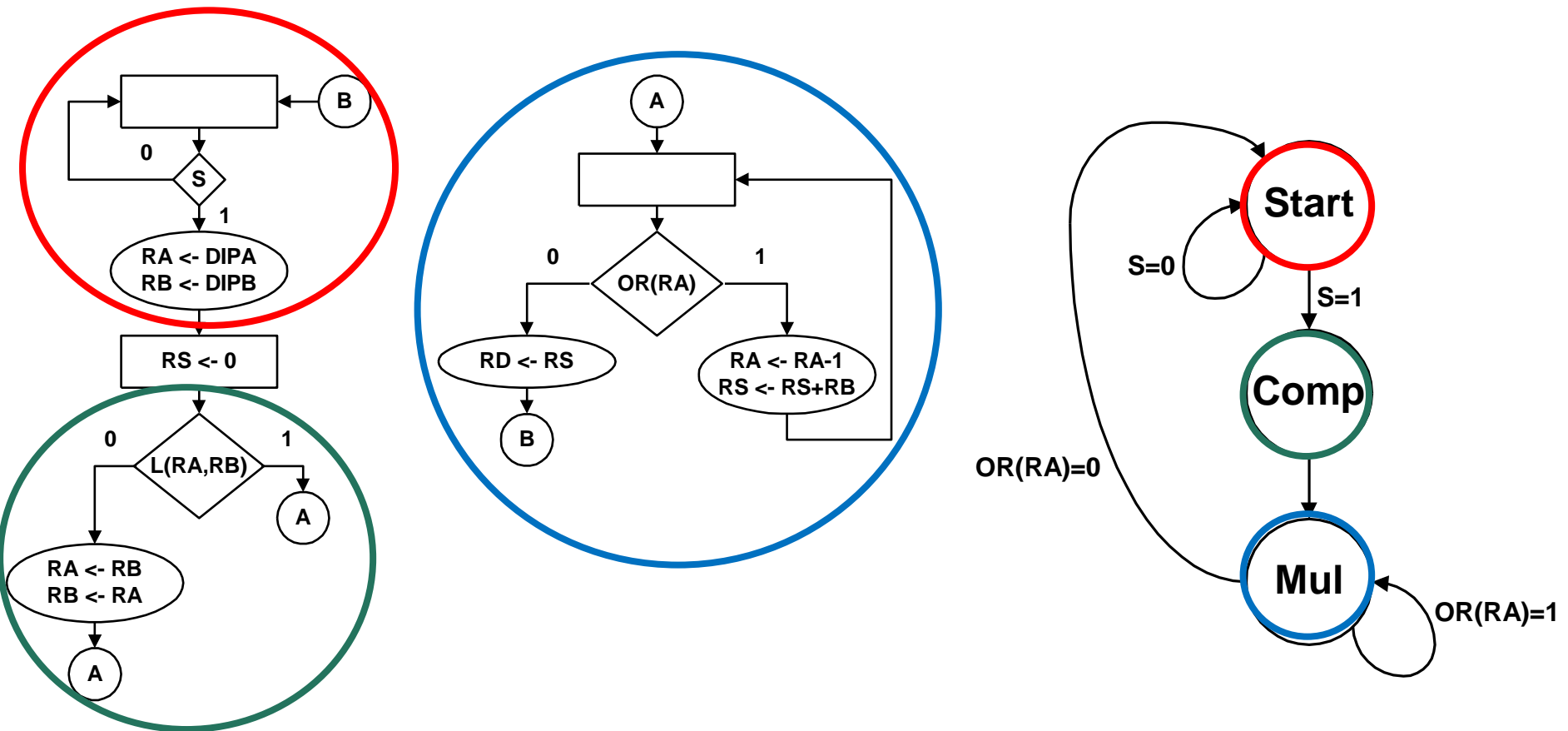
# Multiplier: Control Unit



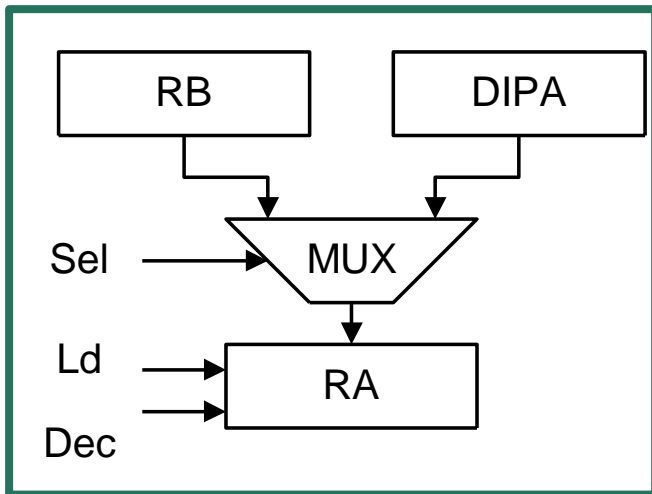
# Multiplier: Control Unit



# Multiplier: Control Unit



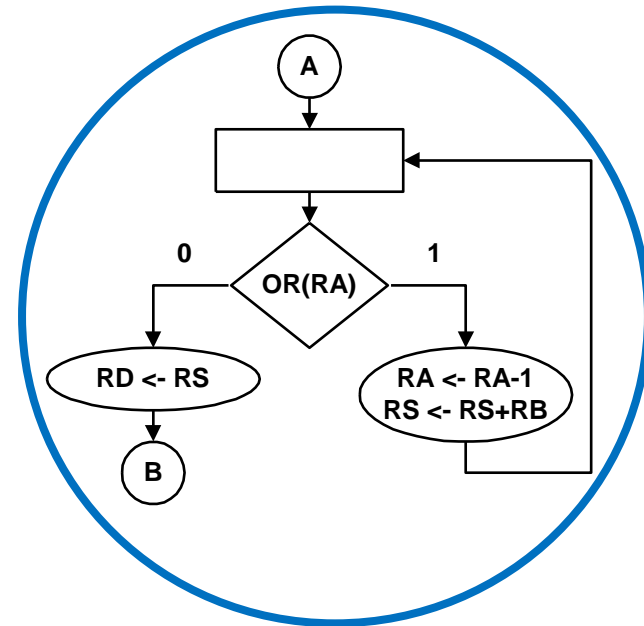
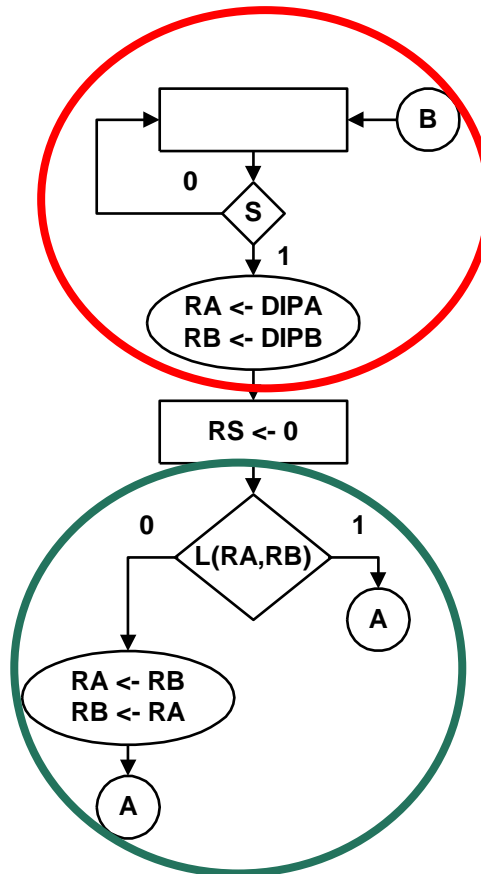
# Multiplier Datapath



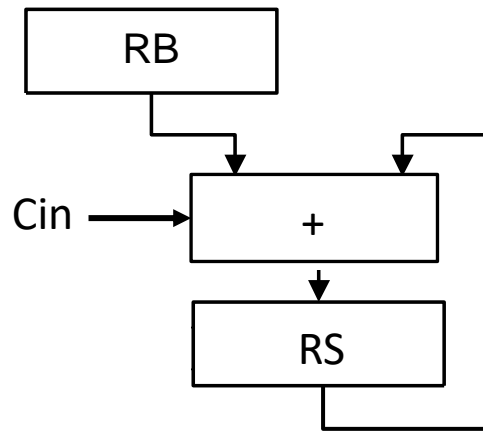
$RA \leftarrow DIPA$

$RA \leftarrow RB$

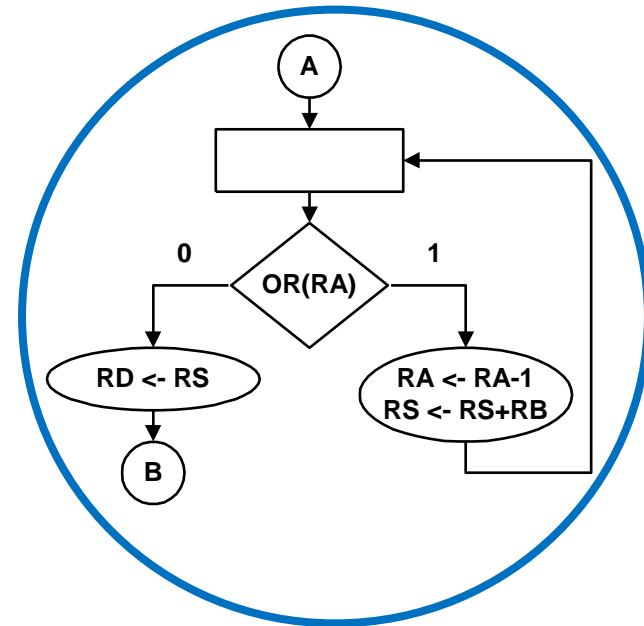
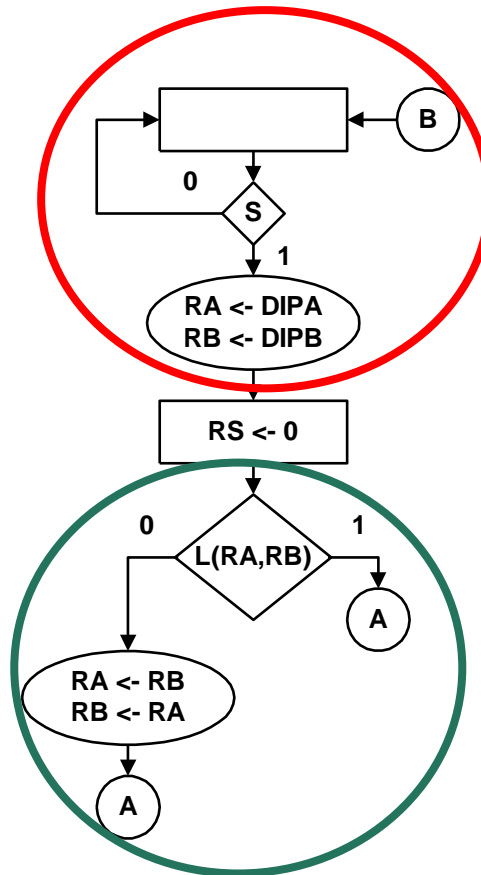
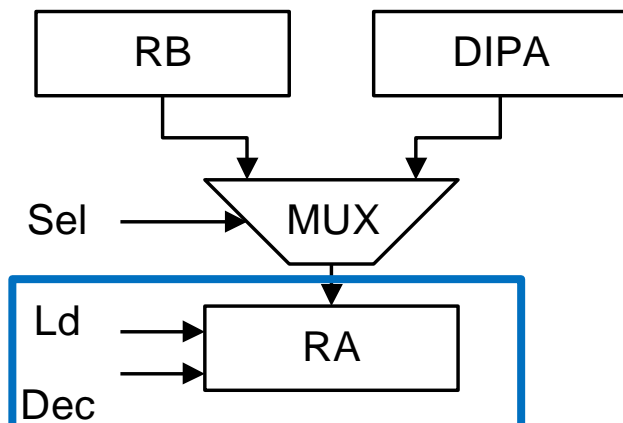
$RA \leftarrow RA - 1$



# Multiplier Datapath

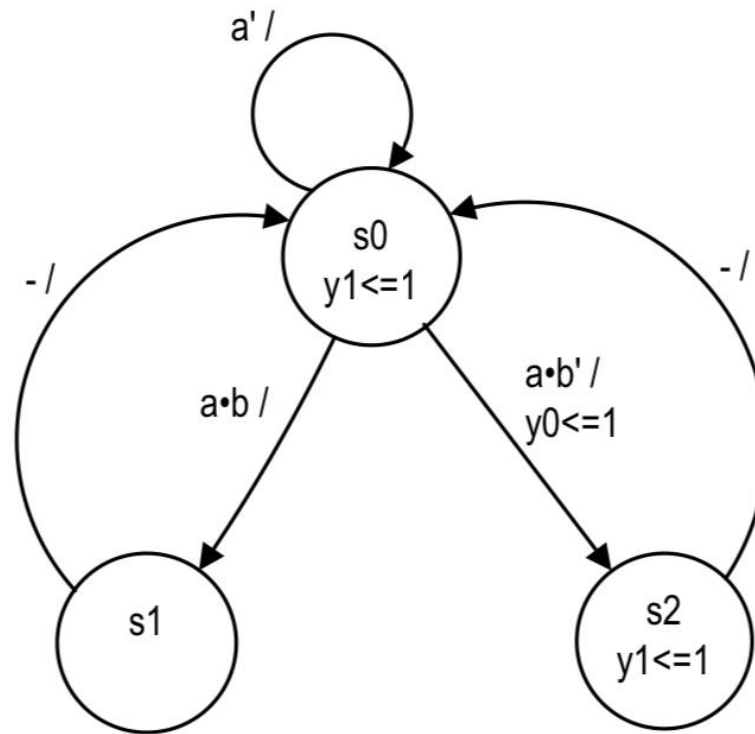


$RS \leftarrow RS + RB$



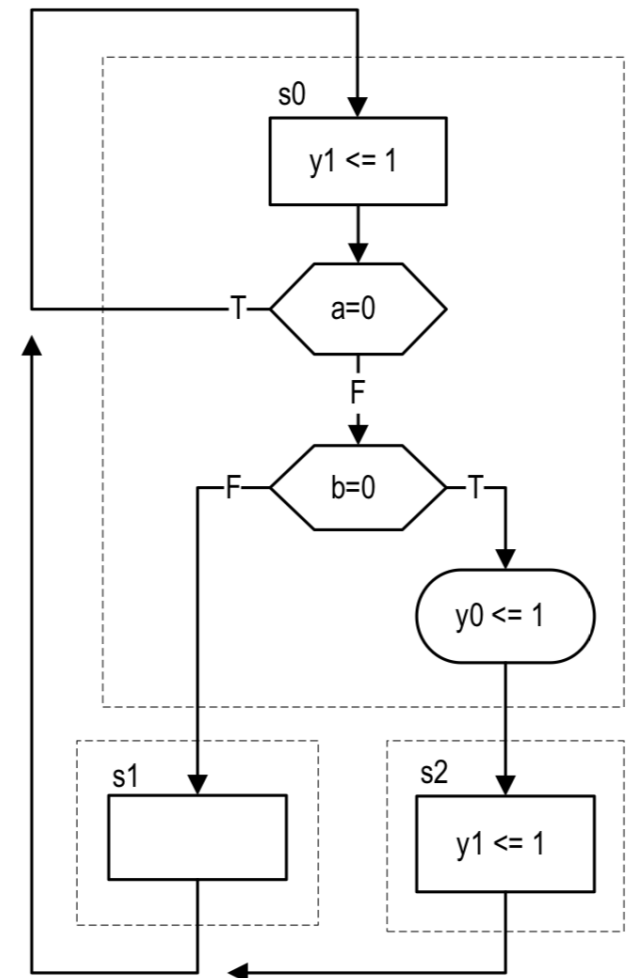
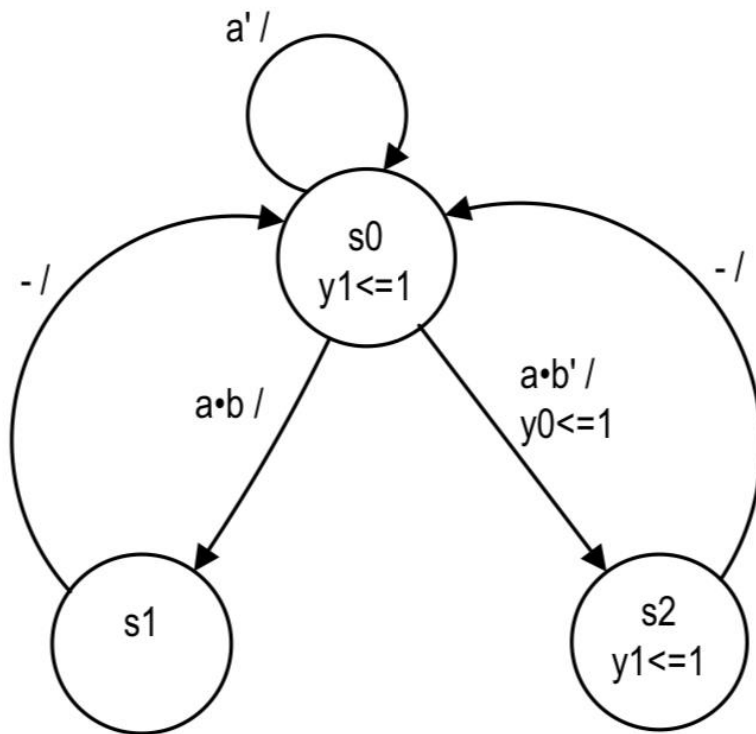
# Sample 6

ASM chart for the below FSM?



# ASM

ASM chart for the below FSM?



# Thank You

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