جلسه پنجم

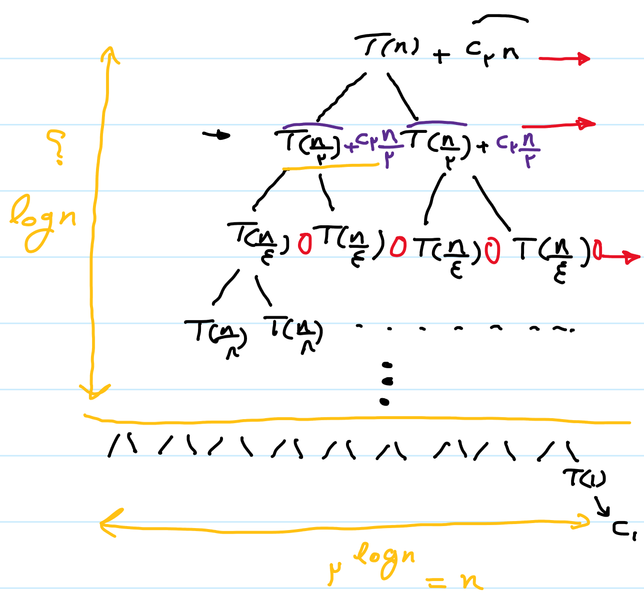
13:15 Sunday, October 3, 2021

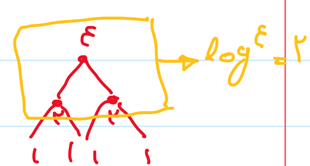
موضوع: روابط بازگشتی : درخت بازگشت - قضیه اصلی.

$$T(1)=O(1) \qquad T(n)=\underbrace{2T(\tfrac{n}{2})}\ +\ \underbrace{O(n)}_{\text{ادغام}} \searrow \Theta(n)$$

$$T(1)\leqslant c_1 \qquad T(n)\leqslant 2T(\tfrac{n}{2})+c_2 n$$

روش درخت بازگشت:

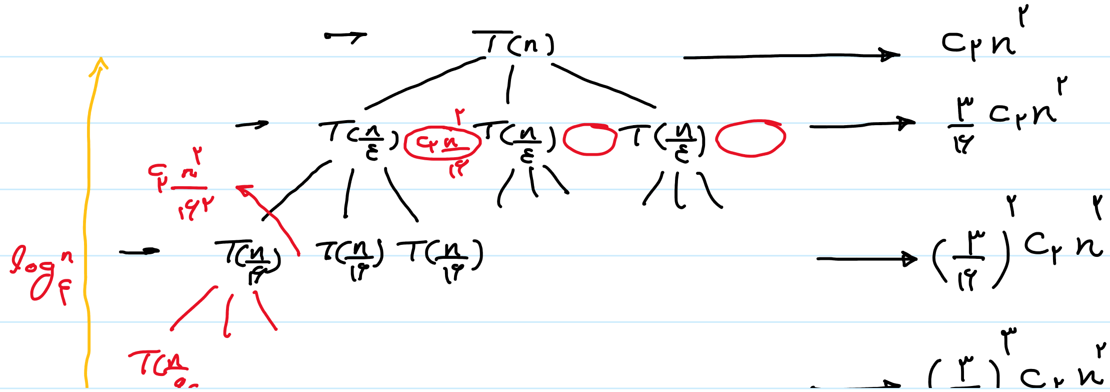


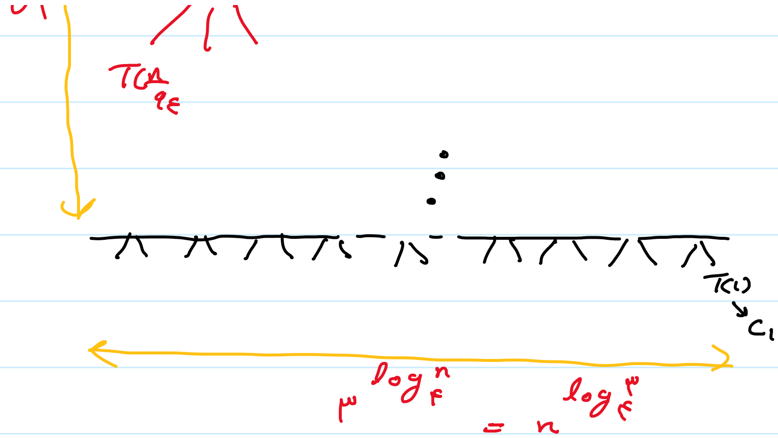


$$T(n)=\underbrace{\log n \times c_2 n}\ +\ \underbrace{n\times c_1}\ =O(n\log n) \searrow \Theta(n\log n)$$

مثال ۲)

$$T(1)=c_1 \qquad T(n)=3T(\tfrac{n}{4})+c_2 n^2$$





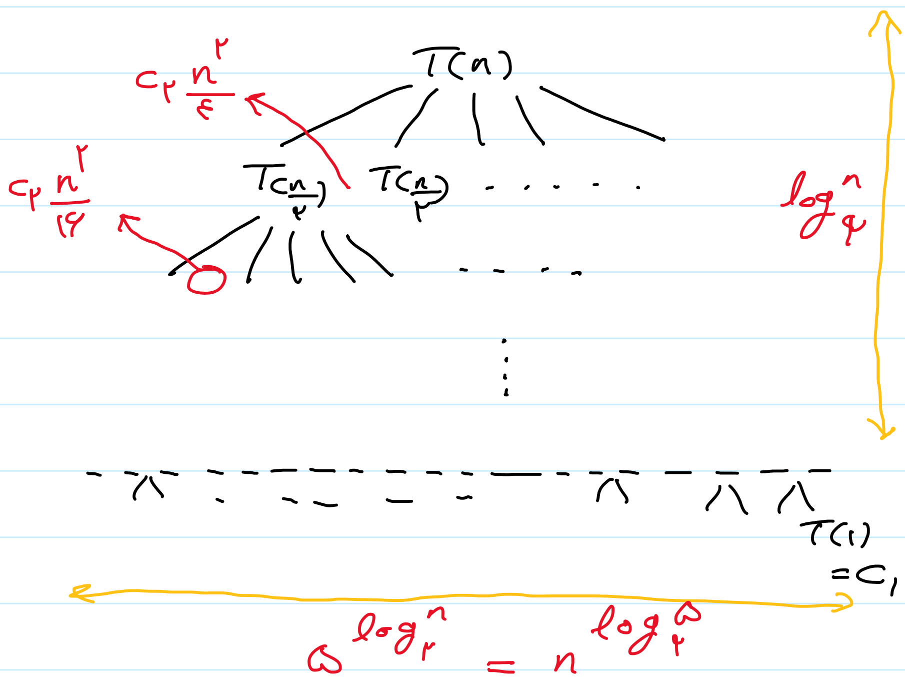
$\longrightarrow (\frac{3}{16})^3 c_2 n^2$

$$T(n) = \underline{c_1 n^{\log_4 3}} + \underline{c_2 n^2 \left(1 + \frac{3}{16} + (\frac{3}{16})^2 + \cdots + (\frac{3}{16})^{\log_4 n}\right)}$$

حتی اگر تا ∞ هم می‌رفت باز هم مقدار ثابت می‌شد.

$$= O(n^2) \to \Theta(n^2)$$

مثال ۳) $T(n) = 5T(\frac{n}{2}) + c_2 n^2$ $\qquad T_1 = c_1$



$+ c_2 n^2$

$+ \frac{5}{4} c_2 n^2$

$+ (\frac{5}{4})^2 c_2 n^2$

$+ (\frac{5}{4})^3 c_2 n^2$

$$T(n) = c_1 \times \underbrace{n^{(\log_2 5)}} + \underbrace{c_2 n^2}_{} \overbrace{\left[1 + \frac{5}{4} + (\frac{5}{4})^2 + \cdots + (\frac{5}{4})^{\log_2 n}\right.}^{\alpha}$$

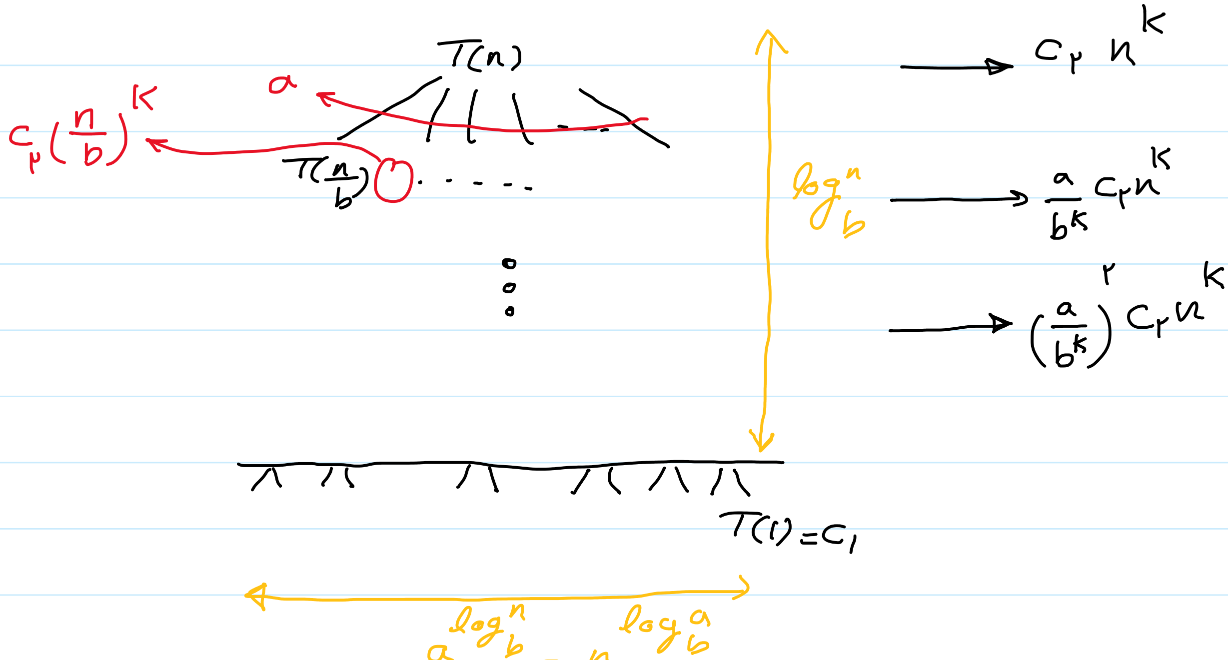
$$T(n) = C_1 \times n^{\log_2 ۵} + C_2 n^{۲} \left[1 + \frac{۵}{۴} + \left(\frac{۵}{۴}\right)^{۲} + \cdots + \left(\frac{۵}{۴}\right)^{\log_2 n} \right]$$

$$\log_2 n$$

$$T(n) = O\left(n^{\log_2 ۵}\right) = O\left(n^{۲٫۳۲}\right)$$

مثال ۴) فرم کلی درخت بازگشت :

$$T(1) = C_1 \qquad T(n) = aT\left(\frac{n}{b}\right) + C_2 n^k$$



$$T(n) = c_1 n^{\log_b a} + C_2 n^k \left[1 + \frac{a}{b^k} + \left(\frac{a}{b^k}\right)^2 + \cdots + \left(\frac{a}{b^k}\right)^{\log_b n} \right]$$

$$k > \log_b a \longrightarrow b^k > a \longrightarrow O(n^k)$$

$$k = \log_b a \longrightarrow b^k = a \longrightarrow O(n^k \log n)$$

$$\begin{cases} \to K = \log_b a \longrightarrow b^k = a \longrightarrow O(n \log n) \\ K < \log_b a \longrightarrow b^k < a \longrightarrow O(n^{\log_b a}) \end{cases}$$

$$\boxed{n^{\varepsilon}} \qquad \log n \qquad \log n = O(n^{\varepsilon})$$

① ۲

$n^{0/000000\cdots01}$ $\underbrace{\qquad}_{۴۵0\cdots \times 10^9}$

$T(n) = aT(\frac{n}{b}) + \underbrace{f(n)}$ $T(1) = O(1)$: قضیه اصلی

$$\underline{f(n)} \qquad \underline{n^{\log_b a}}$$

$T(n) = \Theta(n^{\log_b a}) \Leftarrow f(n) = O(n^{\log_b a - \varepsilon})$: case 1

$T(n) = \Theta(n^{\log_b a} \log n) \Leftarrow f(n) = \Theta(n^{\log_b a})$: case 2

$\to T(n) = \underbrace{\Theta(f(n))} \Leftarrow \underbrace{f(n)} = \Omega(n^{\log_b a + \varepsilon})$: case 3

مثال:

$$T(n) = \underbrace{\overset{a}{۳}T(\frac{n}{۲})}_{b} + \underbrace{n^۲}_{f(n)}$$

$\to n^{\log_۲ ۳}$ $\qquad \underline{f(n) = n^۲}$

$$T(n) = \Theta(n^۲)$$

$$T(n) = 3T\left(\frac{n}{2}\right) + \underline{O(n^2)}$$

$$\rightarrow T(n) = O(n^2)$$

$$T(n) = 3T\left(\frac{n}{2}\right) + \Omega(n^2)$$

$$\rightarrow T(n) = \Omega(n^2)$$

$$\Theta(n^2)$$

$$\Theta(n^2)$$

مثال

$$T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$$

$$n^{\log_b^a} = n^{\log_3^3} = n \overset{O}{>} n^{1/2}$$

$$T(n) = \Theta(n)$$

$$\sqrt{n} \rightarrow O(\sqrt{n}) \quad \Theta(n)$$

$$\sqrt{n} \rightarrow \Omega(\sqrt{n}) \quad \Omega(n)$$

* مثال

$$T(n) = 16T\left(\frac{n}{4}\right) + n!$$

$$n^{\log_b^a} = n^{\log_4^{16}} = n^2 < f(n) = n!$$

$$\rightarrow T(n) = \Theta(n!)$$

* مثال:

$$T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$$

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$$T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$$

$$n^{\log_b^a} = \underline{n} \qquad \underline{f(n) = \frac{n}{2}}$$

$$n = \Theta\left(\frac{n}{2}\right)$$

$$T(n) = \Theta(n \log n) \quad : \text{case } \underline{II}$$

مثال آخر

$$T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{\log n}$$

$$n^{\log_3^3} = n \qquad f(n) = \frac{n}{\log n}$$

$$n \quad ? \quad \frac{n}{\log n}$$

$$\times \qquad \underline{\frac{n}{\log n}} = O(n^{1-\varepsilon}) \qquad ?$$

$$\times \qquad \frac{n}{\log n} = \Omega(n^{1+\varepsilon})$$

$$\times \qquad \frac{n}{\log n} = \Theta(n)$$

← این مثال را با استفاده از قضیه اصلی نمی توان حل کرد.