

# Artificial Intelligence

## CE-417, Group 1

### Computer Eng. Department

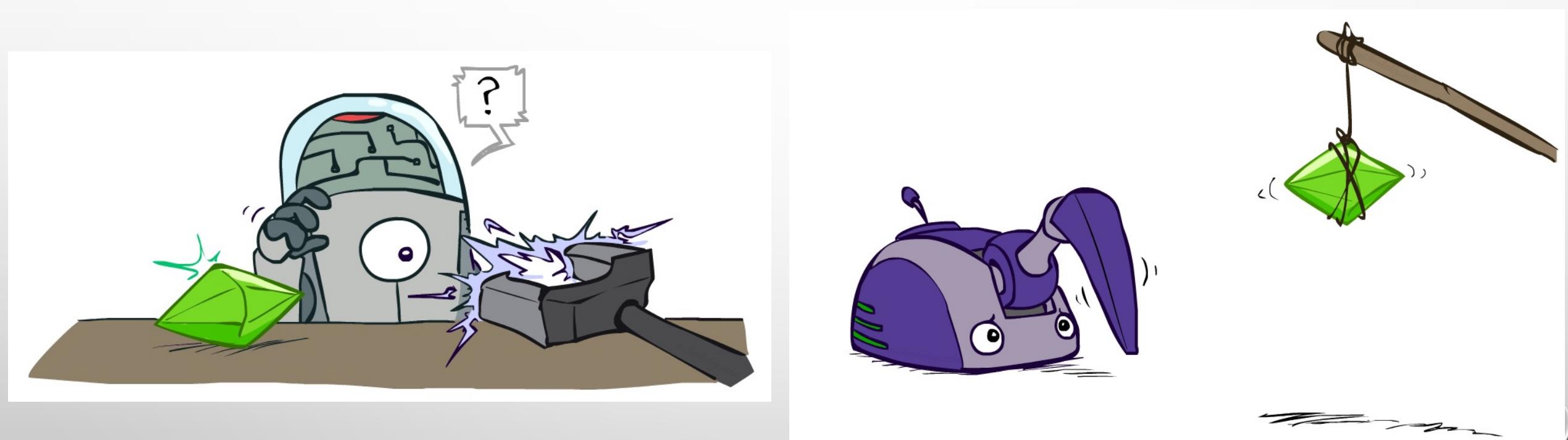
### Sharif University of Technology

Spring 2024

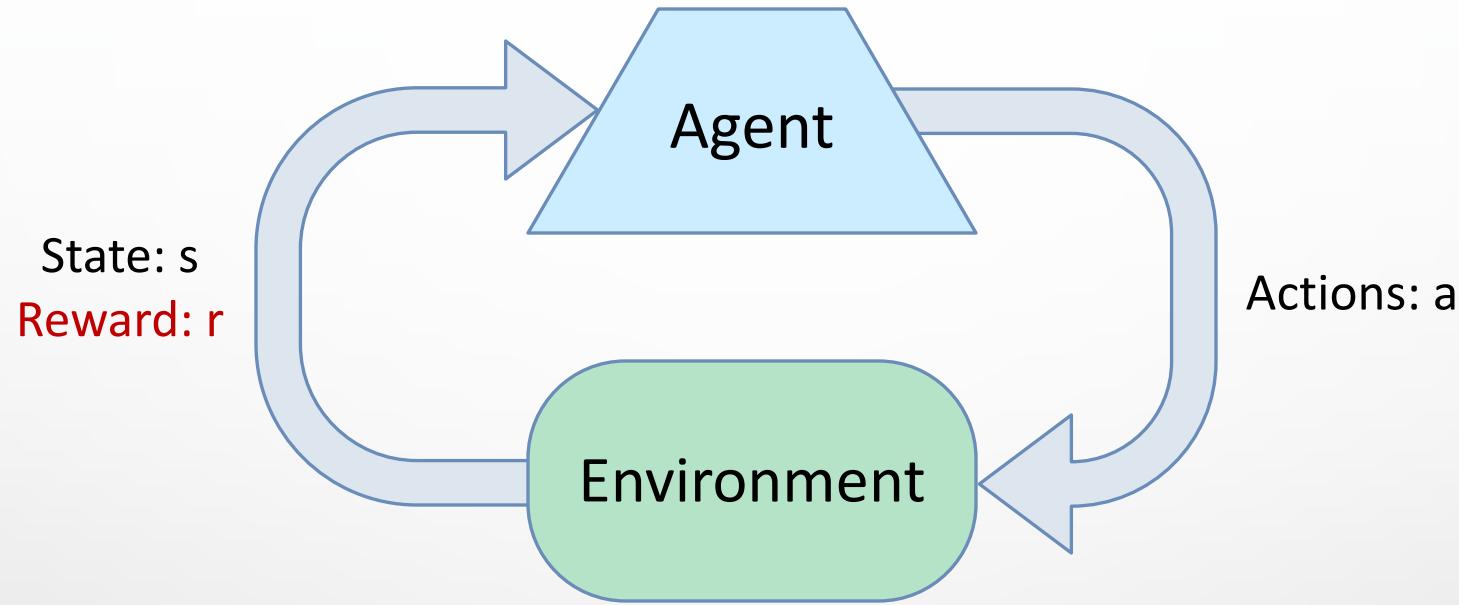
By Mohammad Hossein Rohban, Ph.D.

Courtesy: Most slides are adopted from CSE-573 (Washington U.), original  
slides for the textbook, and CS-188 (UC. Berkeley).

# Reinforcement learning



# Reinforcement Learning



- Basic idea:
  - Receive feedback in the form of **rewards**
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to **maximize expected rewards**
  - All learning is based on observed samples of outcomes!

# Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

# Video of Demo Crawler Bot



# Reinforcement Learning

- Still assume a markov decision process (MDP):

- A **set of states  $s \in S$**
- A **set of actions (per state)  $A$**
- A **model  $T(s,a,s')$**
- A **reward function  $R(s,a,s')$**

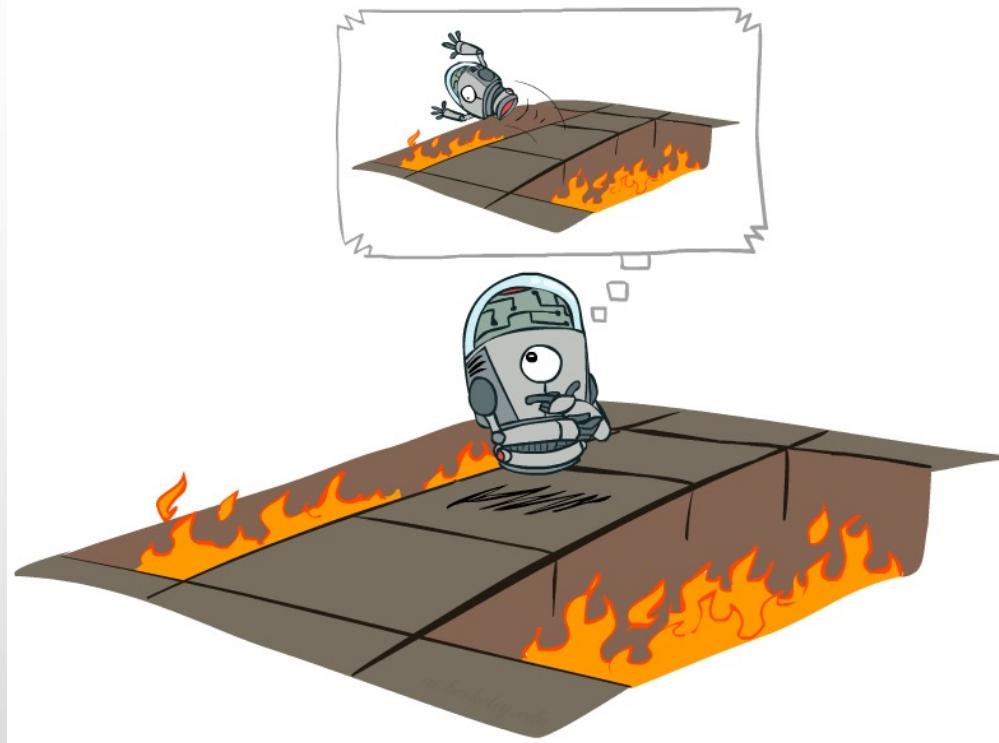
- Still looking for a policy  $\pi(s)$



- New twist: **don't know  $T$  or  $R$**

- i.e. We don't know which states are good or what the actions do
- Must actually try actions and states out to learn

# Offline (MDPs) vs. Online (RL)

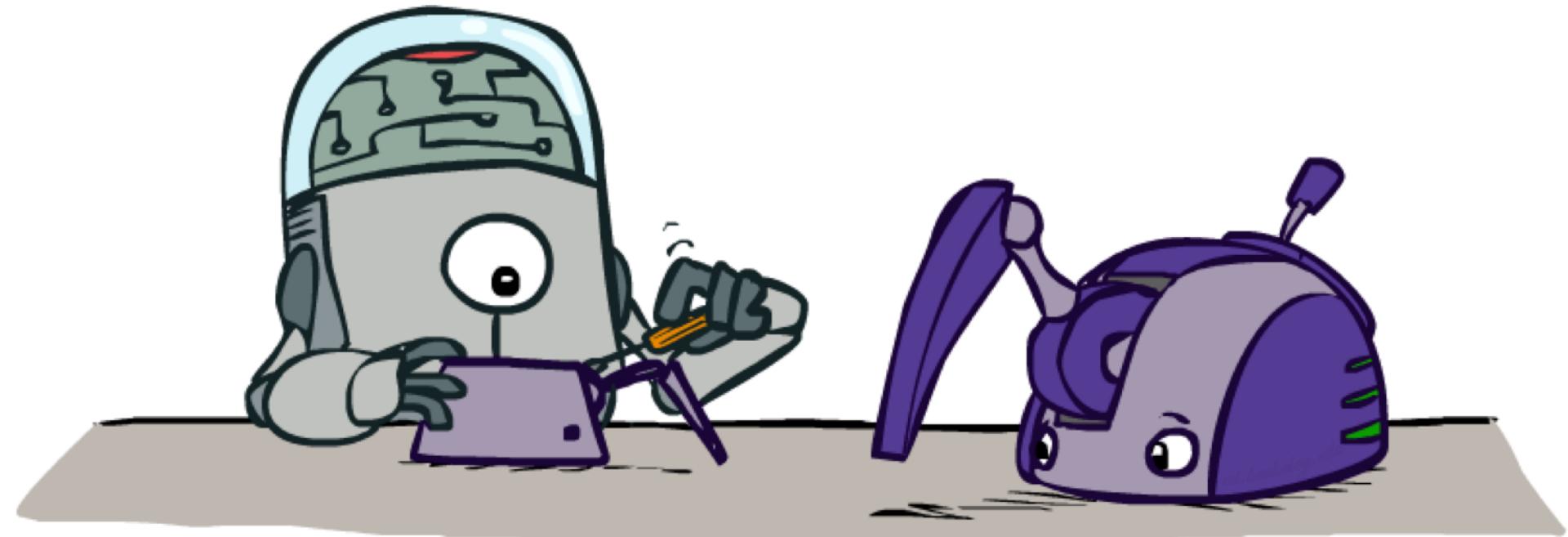


Offline Solution



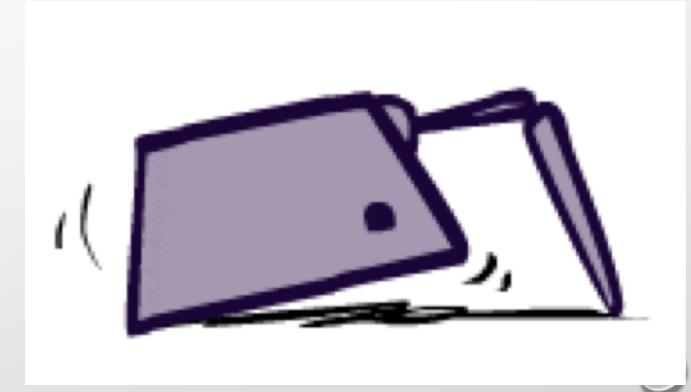
Online Learning

# Model-Based Learning



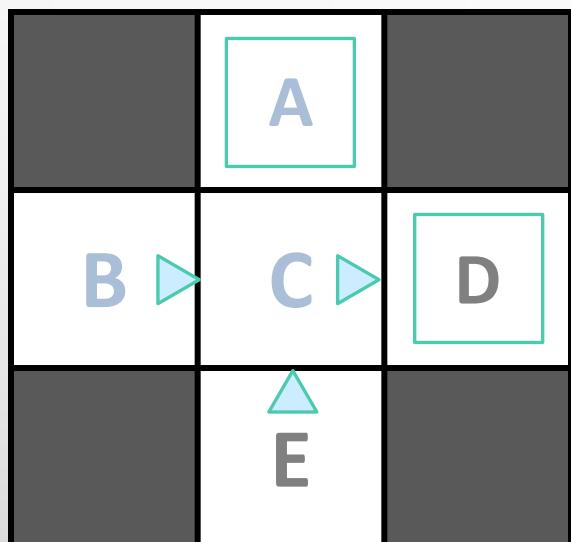
# Model-Based Learning

- Model-based idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- Step 1: learn empirical MDP model
  - Count outcomes  $s'$  for each  $s, a$
  - Normalize to give an estimate of  $\hat{T}(s, a, s')$
  - Discover each  $\hat{R}(s, a, s')$  when we experience  $(s, a, s')$
- Step 2: solve the learned MDP
  - For example, use value iteration, as before



# Example: Model-Based Learning

Input Policy  $\pi$



Assume:  $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

$T(B, \text{east}, C) = 1.00$   
 $T(C, \text{east}, D) = 0.75$   
 $T(C, \text{east}, A) = 0.25$   
...

$\hat{R}(s, a, s')$

$R(B, \text{east}, C) = -1$   
 $R(C, \text{east}, D) = -1$   
 $R(D, \text{exit}, x) = +10$   
...

# Example: Expected Age

Goal: Compute expected age of cs188 students

Known  $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without  $P(A)$ , instead collect samples  $[a_1, a_2, \dots a_N]$

Unknown  $P(A)$ : “Model Based”

Why does this work? Because eventually you learn the right model.

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

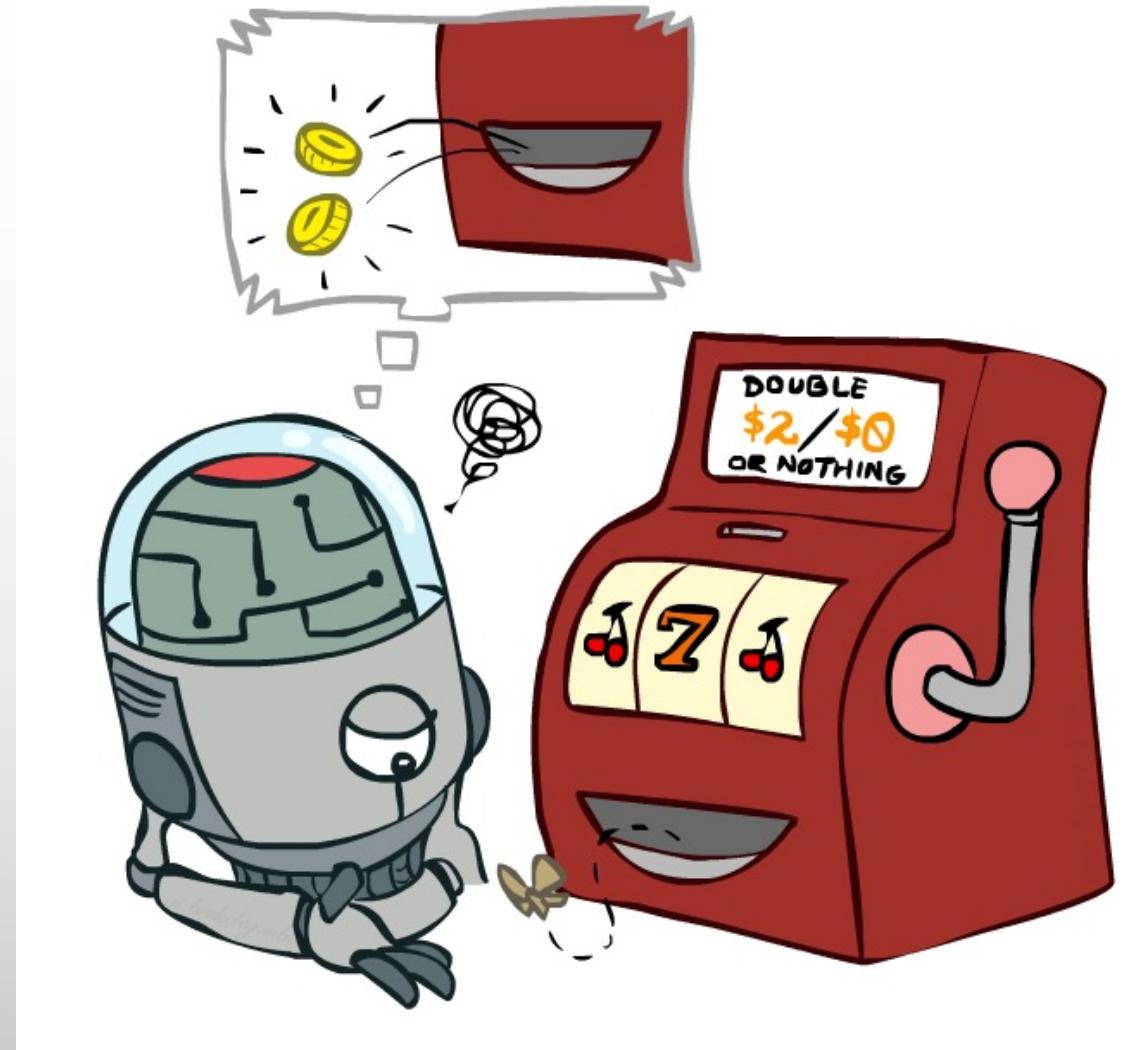
$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown  $P(A)$ : “Model Free”

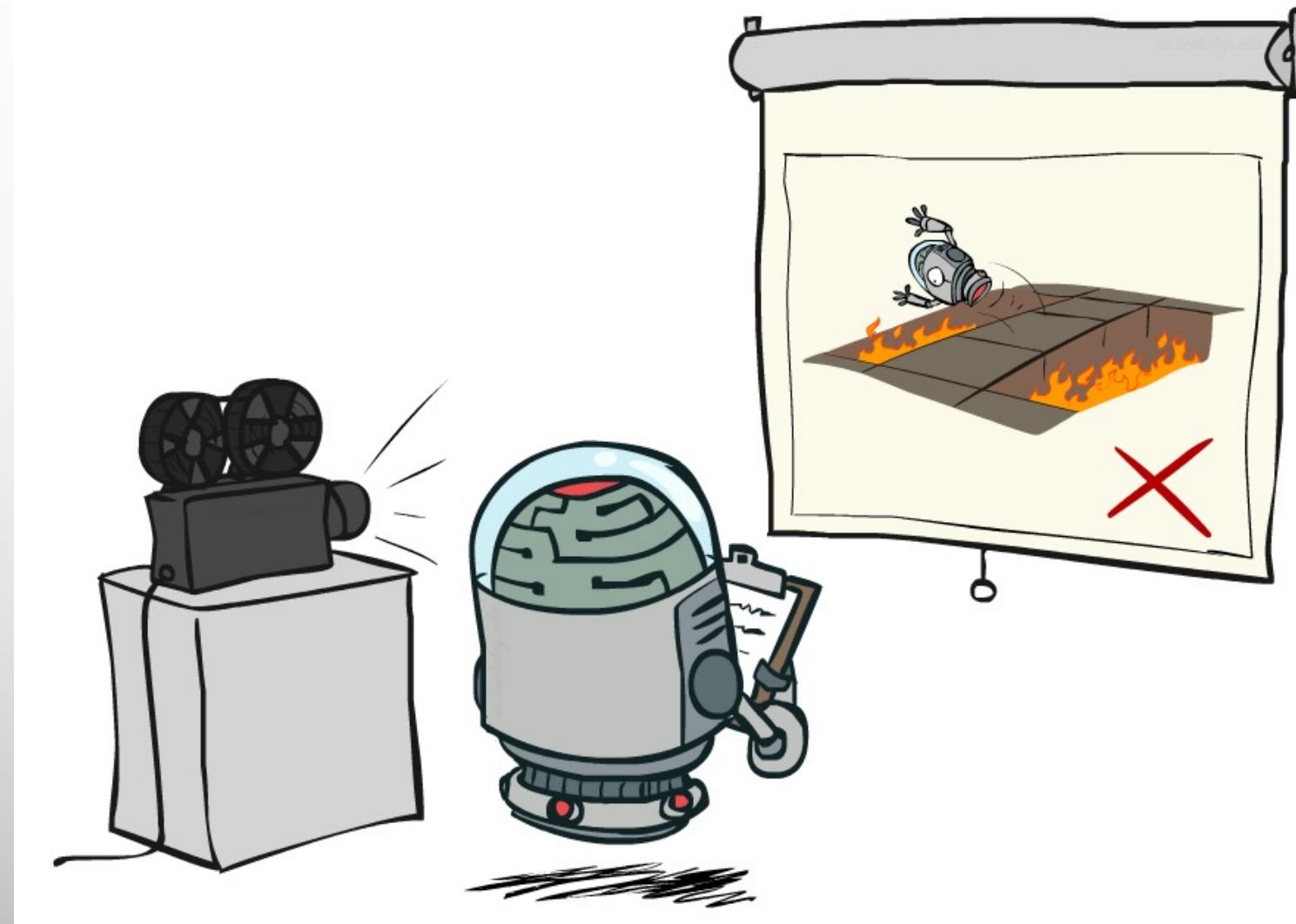
$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

# Model-Free Learning

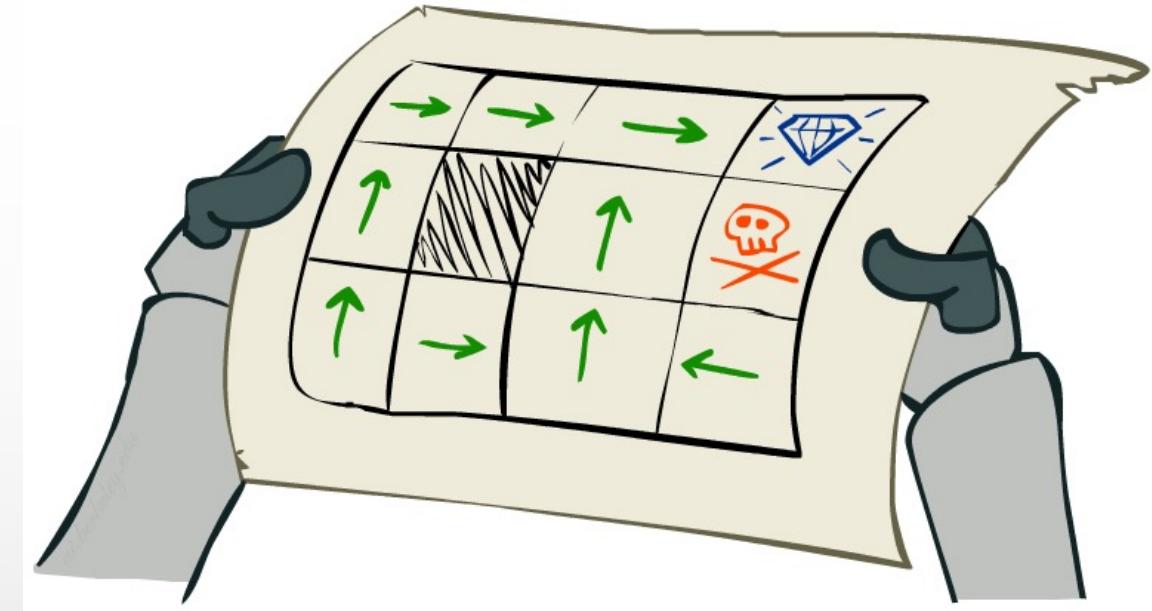


# Passive Reinforcement Learning



# Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy  $\pi(s)$
  - You don't know the transitions  $T(s,a,s')$
  - You don't know the rewards  $R(s,a,s')$
  - **Goal: learn the state values**
- In this case:
  - Learner is “along for the ride”
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world.



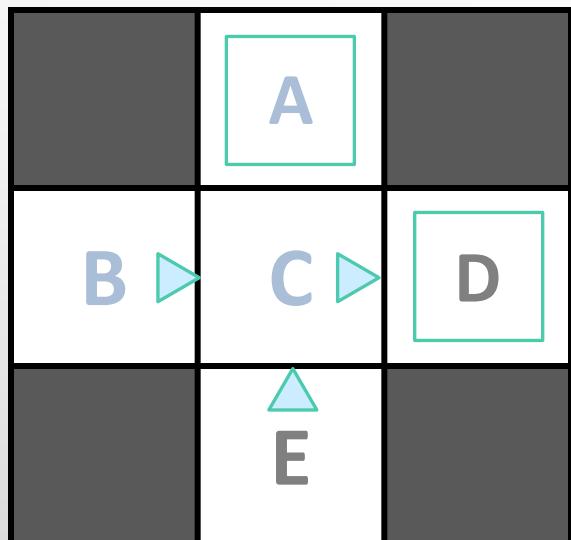
# Direct Evaluation

- Goal: compute values for each state under  $\pi$
- Idea: average together observed sample values
  - Act according to  $\pi$
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples
- This is called direct evaluation



# Example: Direct Evaluation

Input Policy  $\pi$



Assume:  $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Output Values

		-10
	A	
+8		+4
B	C	D
		-2
	E	

# Problems with Direct Evaluation

- What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of  $T$ ,  $R$
- It eventually computes the correct average values, using just sample transitions

- What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

## Output Values

		-10 A	
	+8 B	+4 C	+10 D
		-2 E	

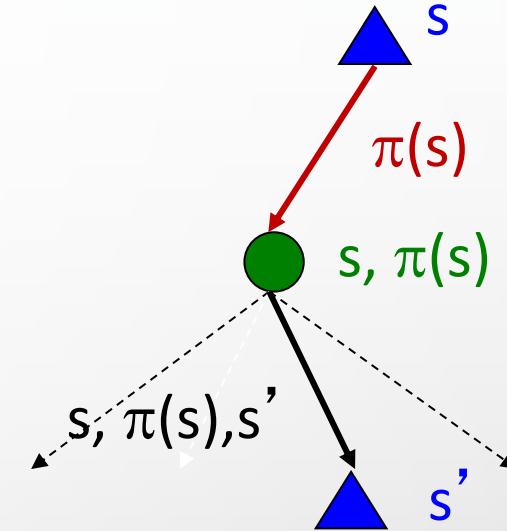
*If B and E both go to C under this policy, how can their values be different?*

# Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate  $V$  for a fixed policy:
  - Each round, replace  $V$  with a one-step-look-ahead layer over  $V$

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need  $T$  and  $R$  to do it!
- Key question: how can we do this update to  $V$  without knowing  $T$  and  $R$ ?
  - In other words, how to we take a weighted average without knowing the weights?

# Sample-Based Policy Evaluation?

- We want to improve our estimate of  $V$  by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: take samples of outcomes  $s'$  (by doing the action!) and average

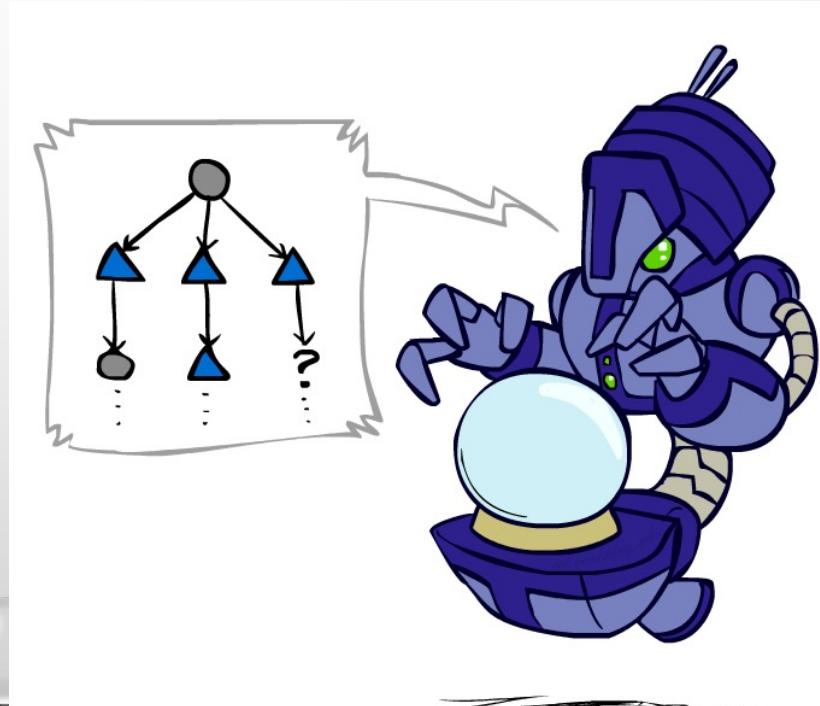
$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

...

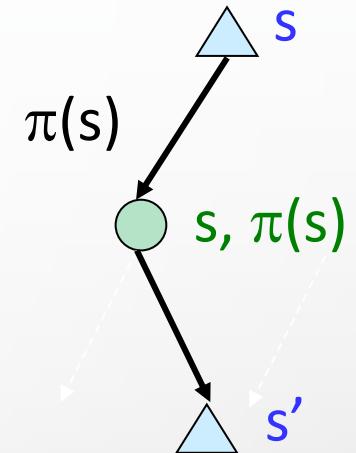
$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i sample_i$$



# Temporal Difference Learning

- Big idea: learn from every experience!
  - Update  $V(s)$  each time we experience a transition  $(s, a, s', r)$
  - Likely outcomes  $s'$  will contribute updates more often
- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average



Sample of  $V(s)$ :  $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to  $V(s)$ :  $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update:  $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$

# Exponential Moving Average

- Exponential moving average

- The running interpolation update:  $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

# Example: Temporal Difference Learning

States

	A	
B	C	D
	E	

Observed Transitions

B, east, C, -2

	0	
0	0	8
	0	

C, east, D, -2

	0	
-1	0	8
	0	

	0	
-1	3	8
	0	

Assume:  $\gamma = 1, \alpha = 1/2$

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

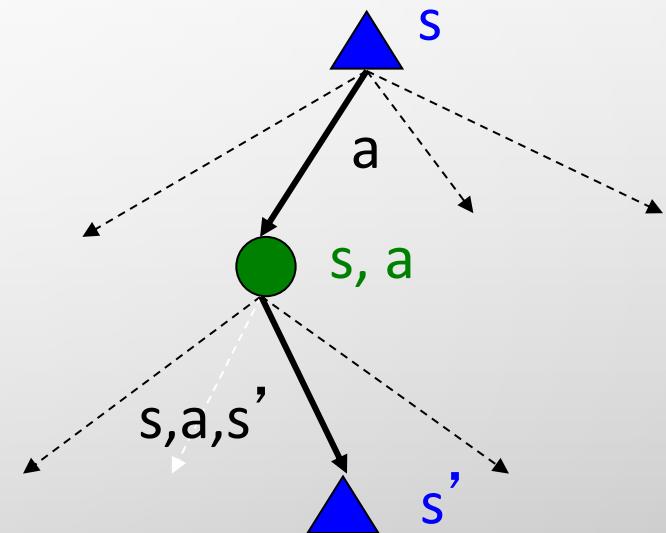
# Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!



# Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with  $V_0(s) = 0$ , which we know is right
  - Given  $V_k$ , calculate the depth  $k+1$  values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead
  - Start with  $Q_0(s, a) = 0$ , which we know is right
  - Given  $Q_k$ , calculate the depth  $k+1$  Q-values for all Q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

# Q-Learning

- Q-learning: sample-based q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

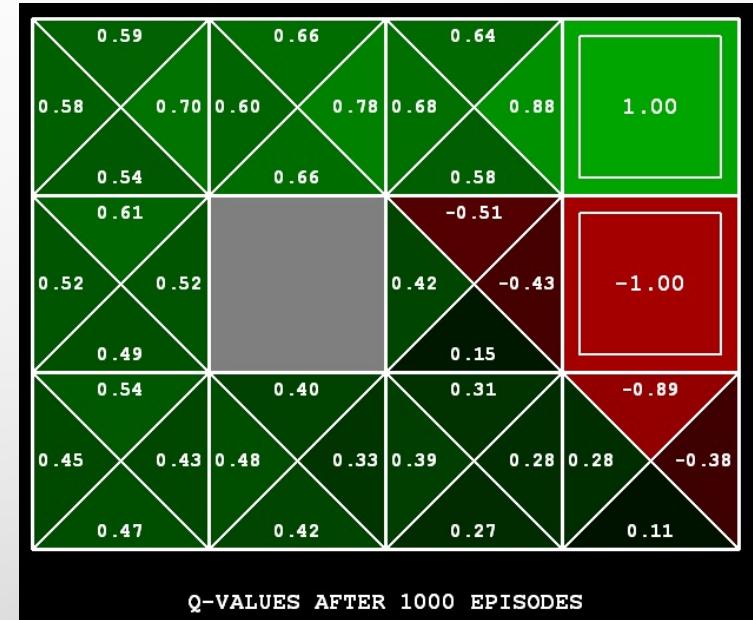
- Learn Q(s,a) values as you go

- Receive a sample  $(s, a, s', r)$
- Consider your old estimate:  $Q(s, a)$
- Consider your new sample estimate:

$$\text{sample} = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [\text{sample}]$$



# Video of Demo Q-Learning -- Gridworld

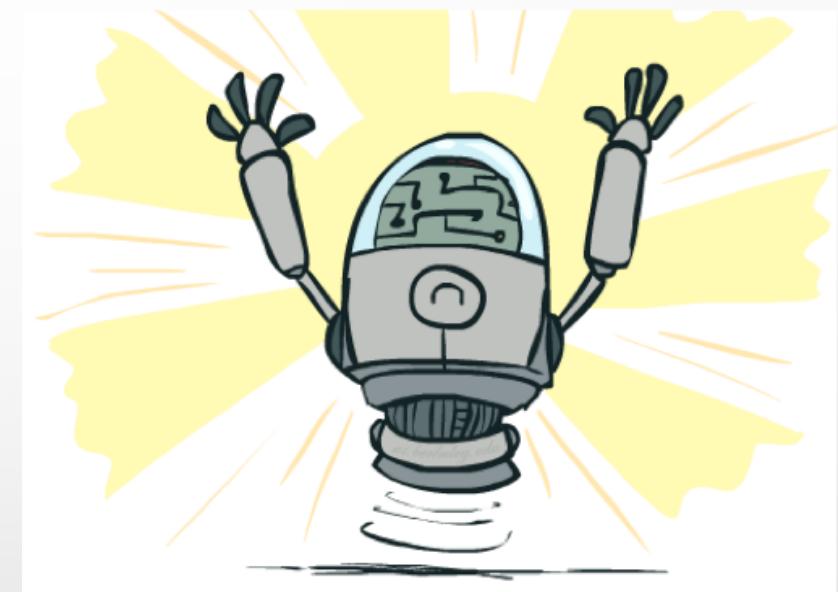


# Video of Demo Q-Learning -- Crawler

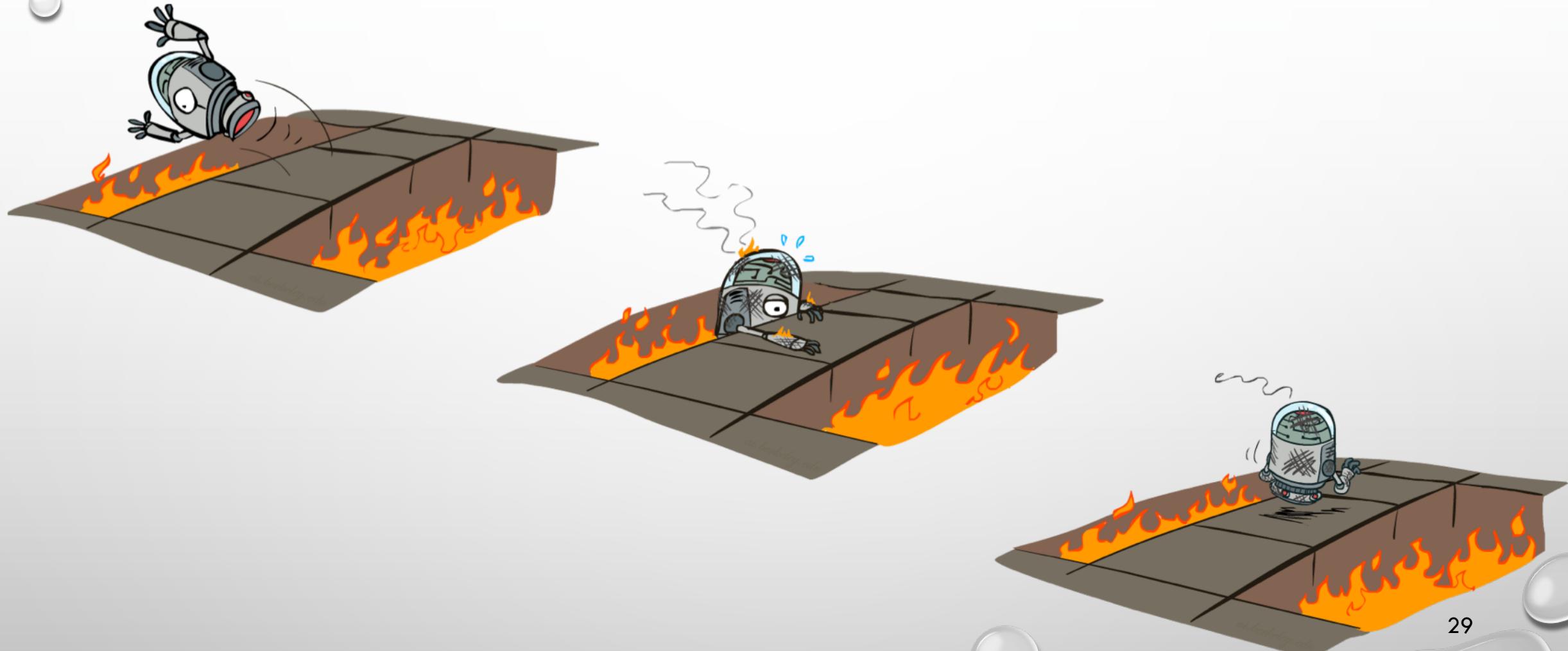


# Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting sub-optimally!
- This is called **off-policy learning**
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)

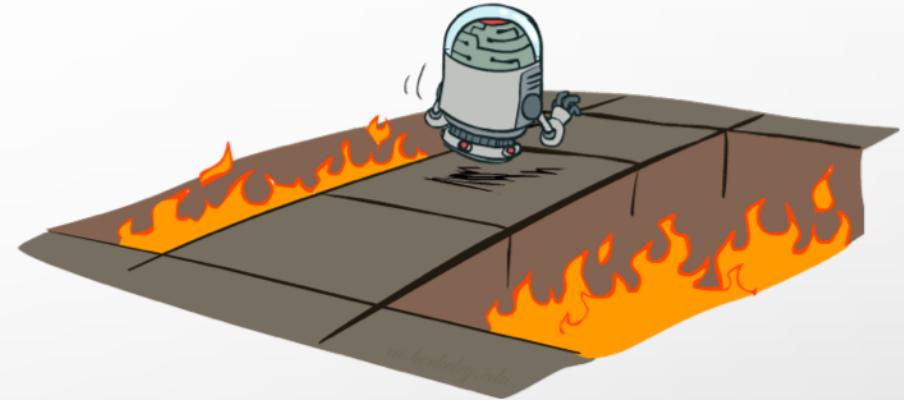


# Active Reinforcement Learning

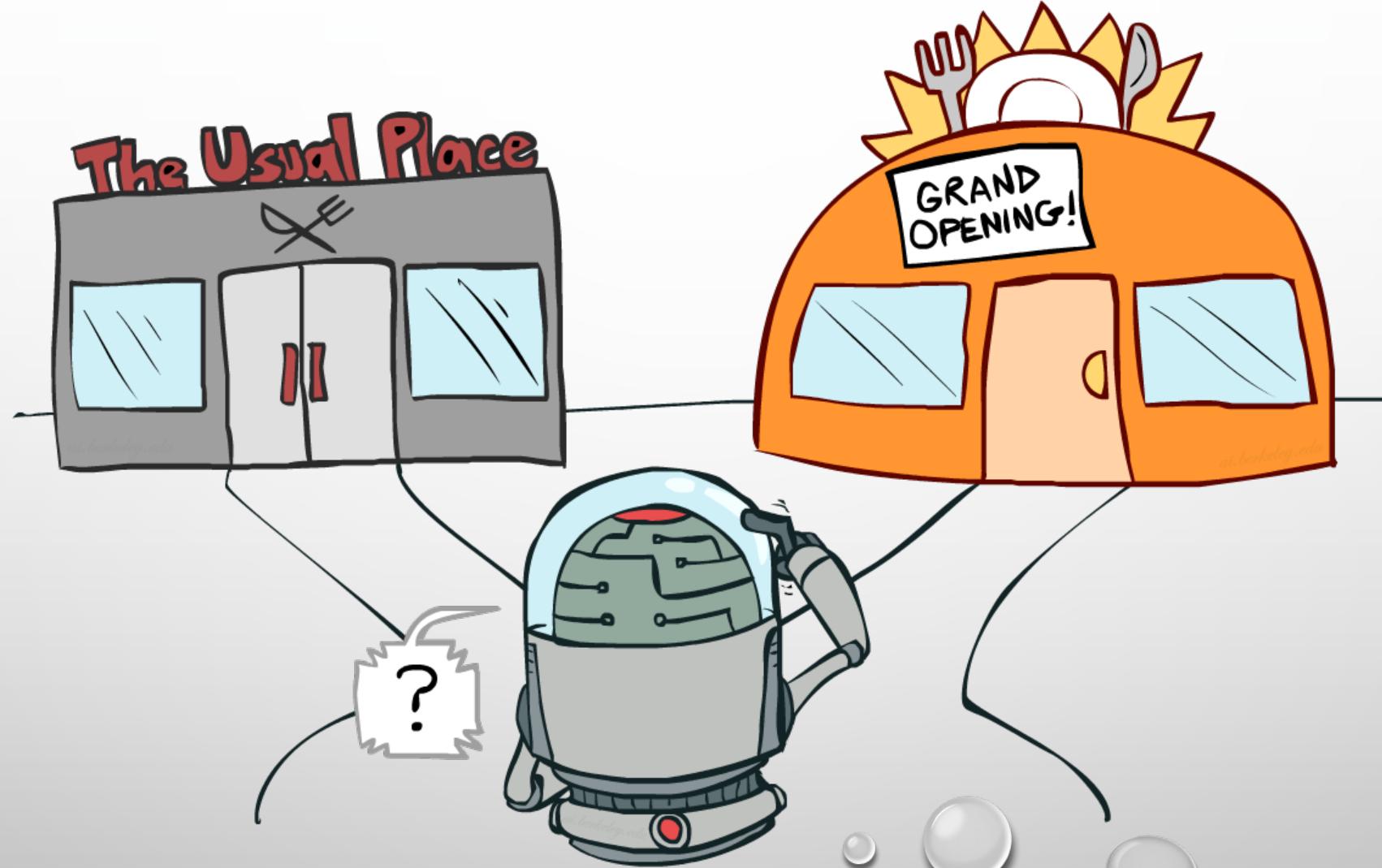


# Active Reinforcement Learning

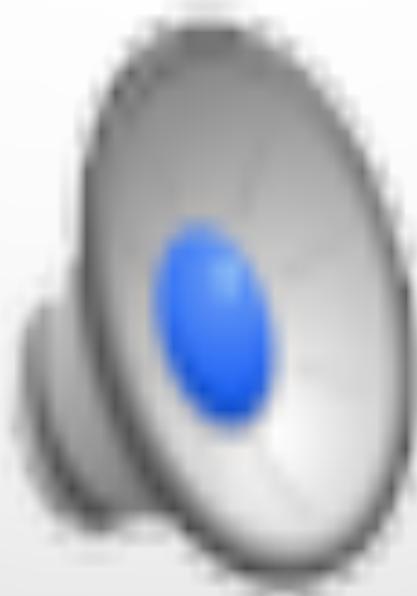
- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions  $T(s,a,s')$
  - You don't know the rewards  $R(s,a,s')$
  - You choose the actions now
  - **Goal: learn the optimal policy / values**
- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens...



# Exploration vs. Exploitation



# Video of Demo Q-learning – Manual Exploration – Bridge Grid



# How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions ( $\varepsilon$ -greedy)
    - Every time step, flip a coin
    - With (small) probability  $\varepsilon$ , act randomly
    - With (large) probability  $1-\varepsilon$ , act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower  $\varepsilon$  over time
    - Another solution: exploration functions



# Video of Demo Q-learning – Epsilon-Greedy – Crawler

# Exploration Functions

- When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- Exploration function

- Takes a value estimate  $u$  and a visit count  $n$ , and returns an optimistic utility, e.g.

$$f(u, n) = u + k/n$$



Regular Q-update: 
$$Q(s, a) \leftarrow \alpha R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

- Note: this propagates the “bonus” back to states that lead to unknown states as well!

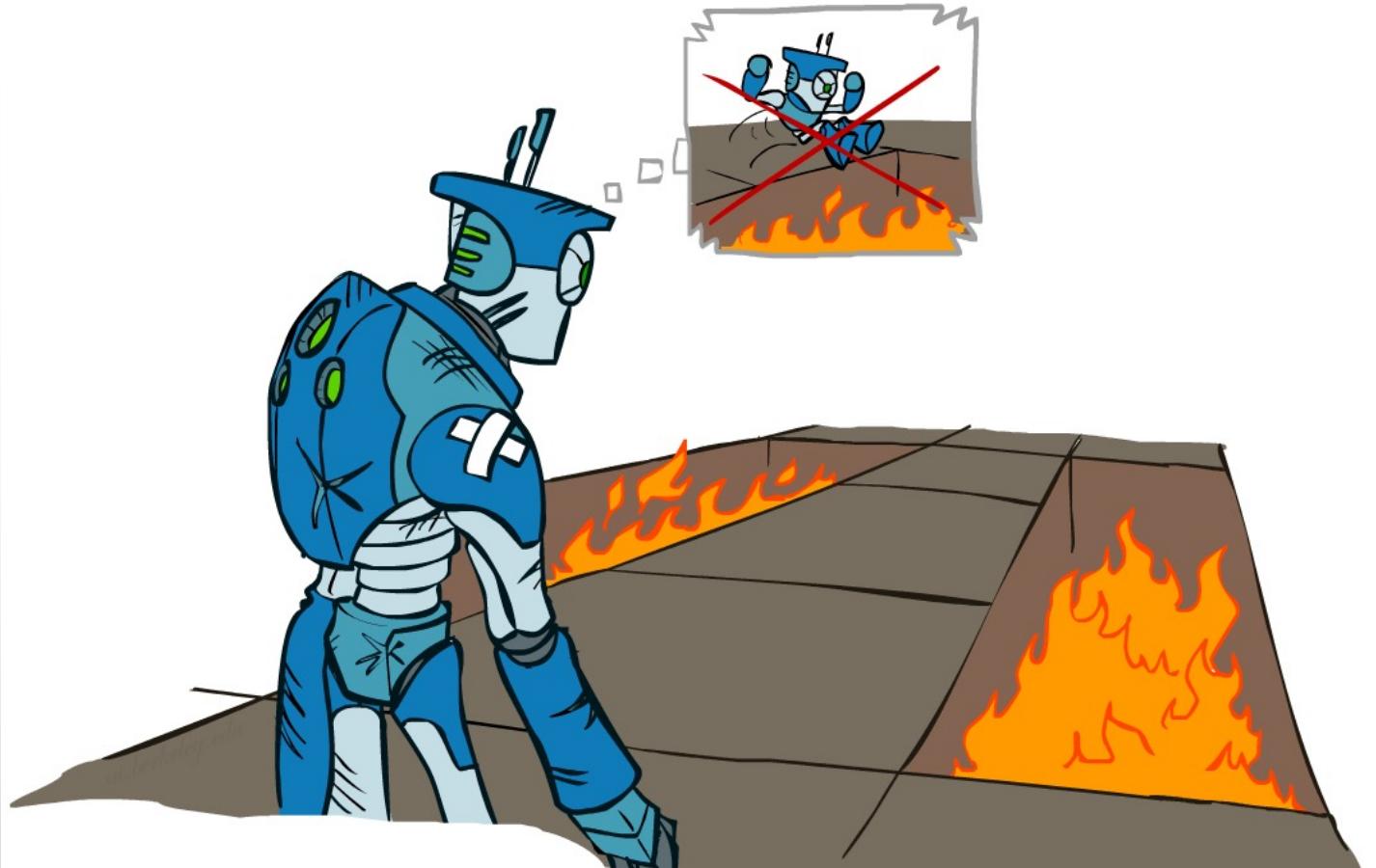
Modified Q-update: 
$$Q(s, a) \leftarrow \alpha R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$$

# Video of Demo Q-learning – Exploration Function – Crawler

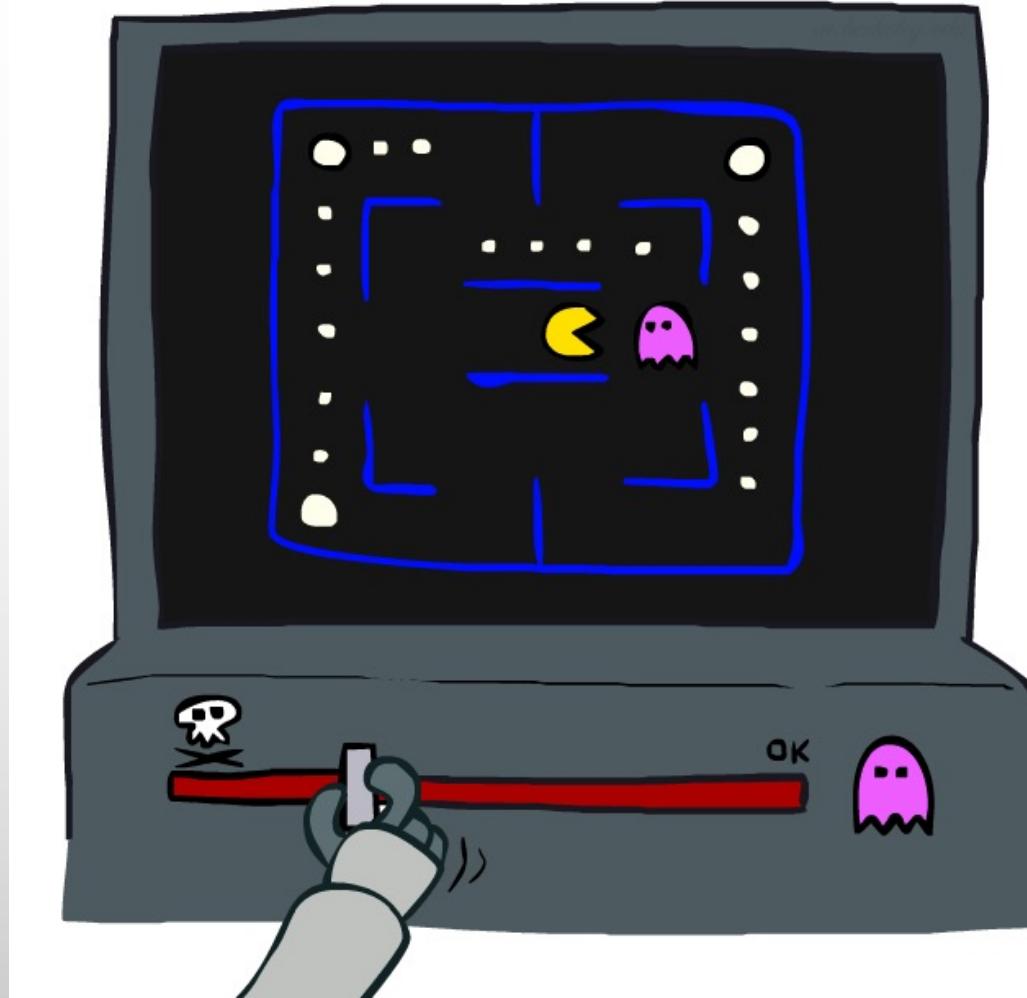


# Regret

- Even if you learn the optimal policy, you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

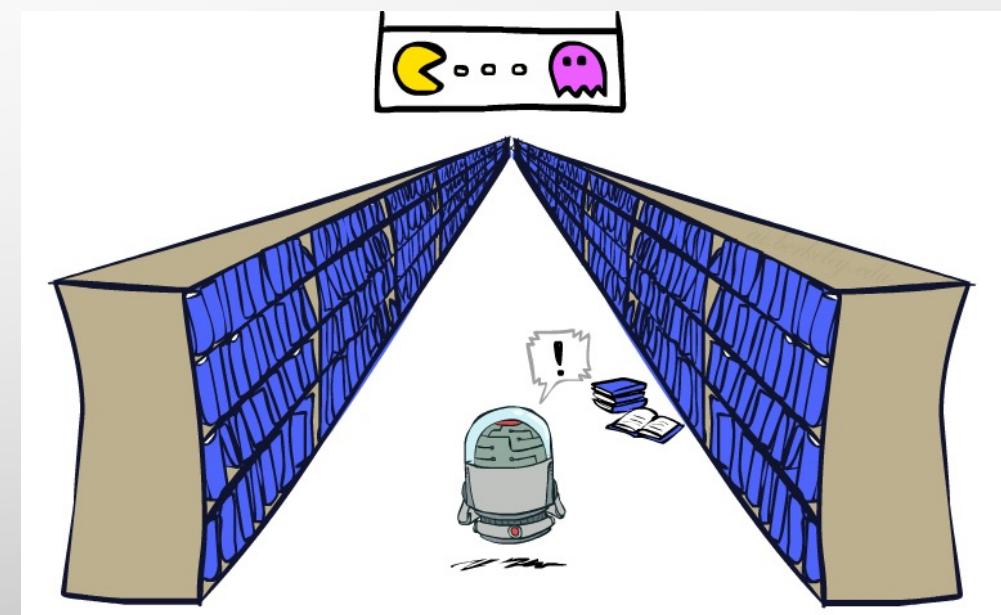
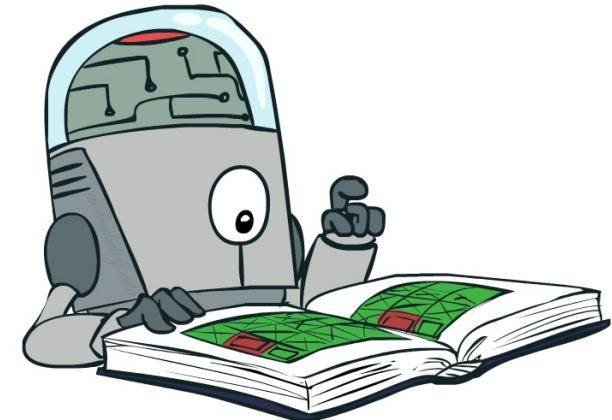


# Approximate Q-Learning



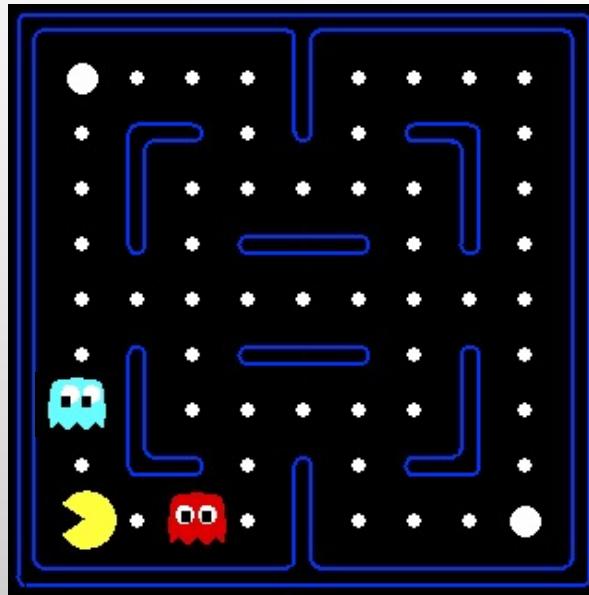
# Generalizing Across States

- Basic Q-learning keeps a table of all Q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

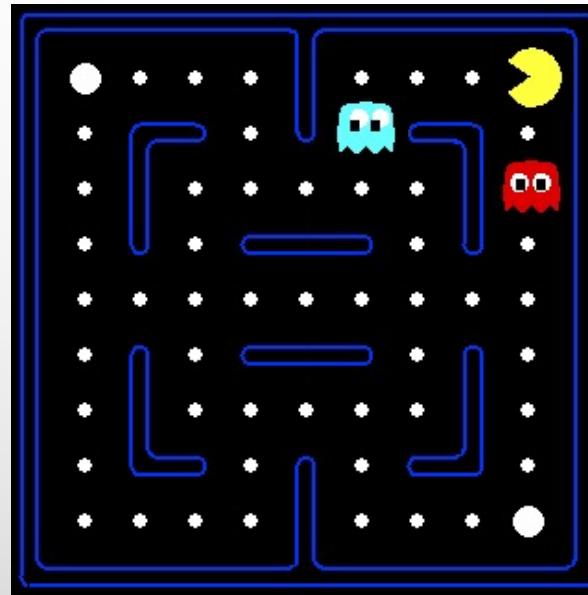


## Example: Pacman

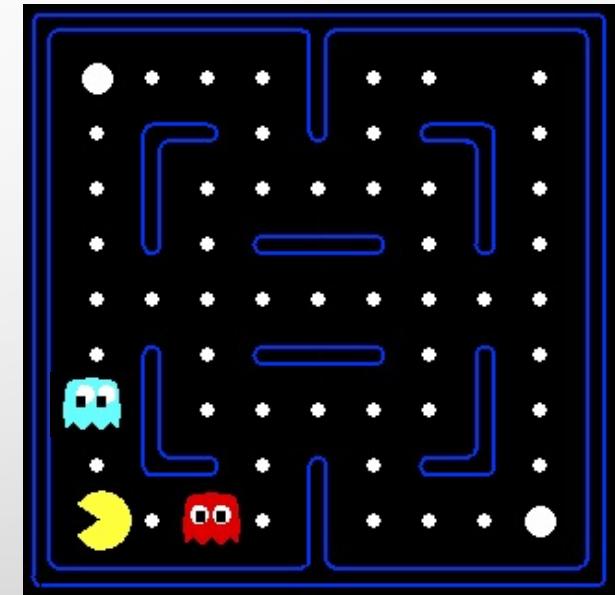
Let's say we discover through experience that this state is bad:



In naïve Q-learning, we know nothing about this state:



Or even this one!

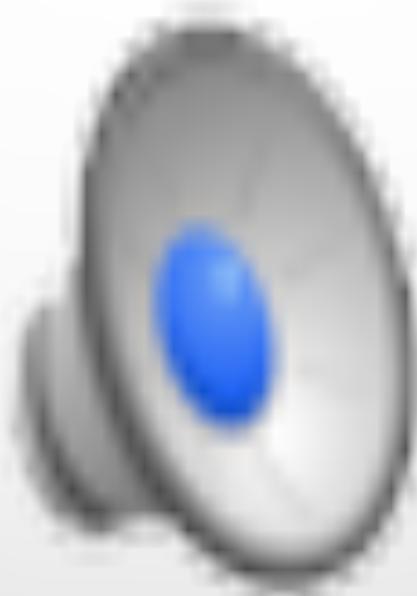


[Demo: Q-learning – pacman – tiny – watch all (L11D5)]  
[Demo: Q-learning – pacman – tiny – silent train (L11D6)]  
[Demo: Q-learning – pacman – tricky – watch all (L11D7)]

# Video of Demo Q-Learning Pacman – Tiny – Watch All



# Video of demo Q-learning Pacman – tiny – silent train

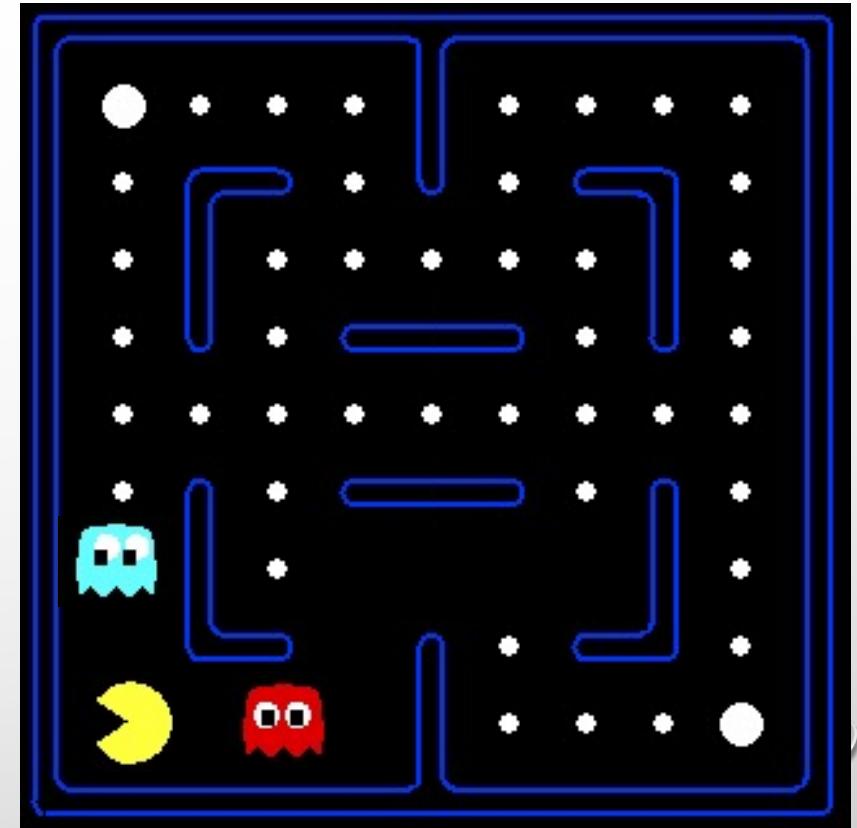


# Video of Demo Q-Learning Pacman – Tricky – Watch All



# Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1 / (\text{dist. to dot})^2$
    - Is Pacman in a tunnel? (0/1)
    - ..... Etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state  $(s, a)$  with features (e.g. Action moves closer to food)



# Linear Value Functions

- Using a feature representation, we can write a Q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

# Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear q-functions:

transition =  $(s, a, r, s')$

difference =  $[r + \gamma \max_{a'} Q(s', a')] - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]}$

Exact Q's

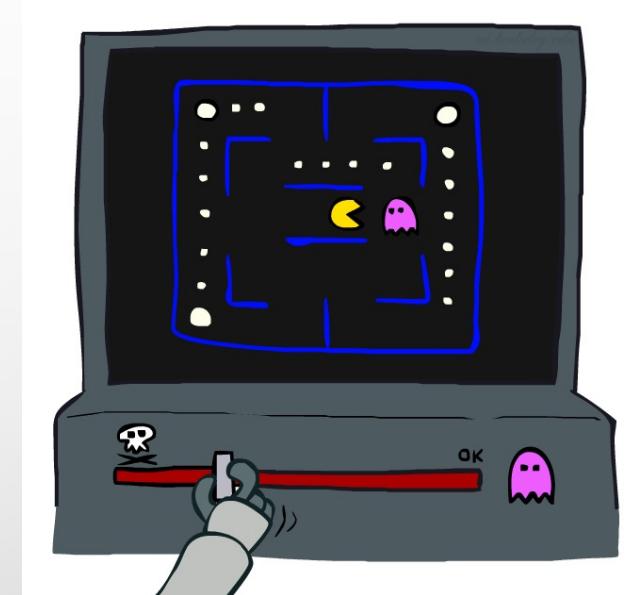
$w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a)$

Approximate Q's

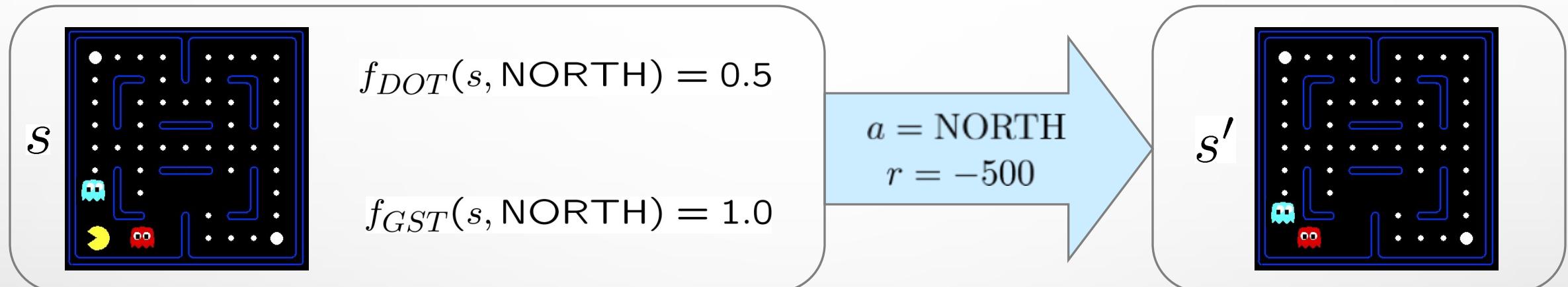
- Intuitive interpretation:

- Adjust weights of active features
- e.g., If something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

- Formal justification: online least squares



$$Q(s, a) = 4.0f_{DOT}(s, a) - 1.0f_{GST}(s, a)$$



difference =  $-501$

$$\begin{aligned} w_{DOT} &\leftarrow 4.0 + \alpha [-501] 0.5 \\ w_{GST} &\leftarrow -1.0 + \alpha [-501] 1.0 \end{aligned}$$

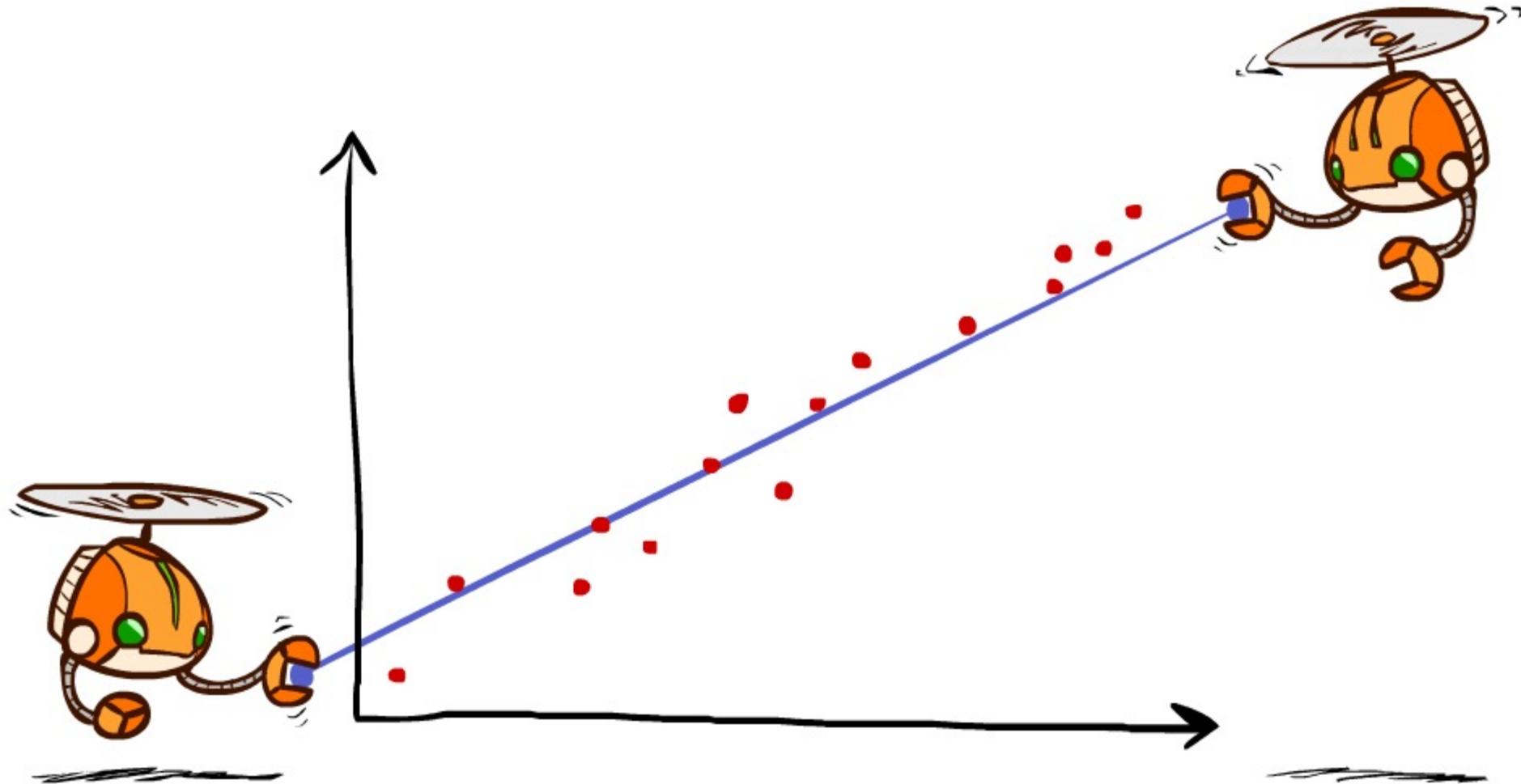
$$Q(s, a) = 3.0f_{DOT}(s, a) - 3.0f_{GST}(s, a)$$

[Demo: approximate Q-learning pacman (L11D10)]

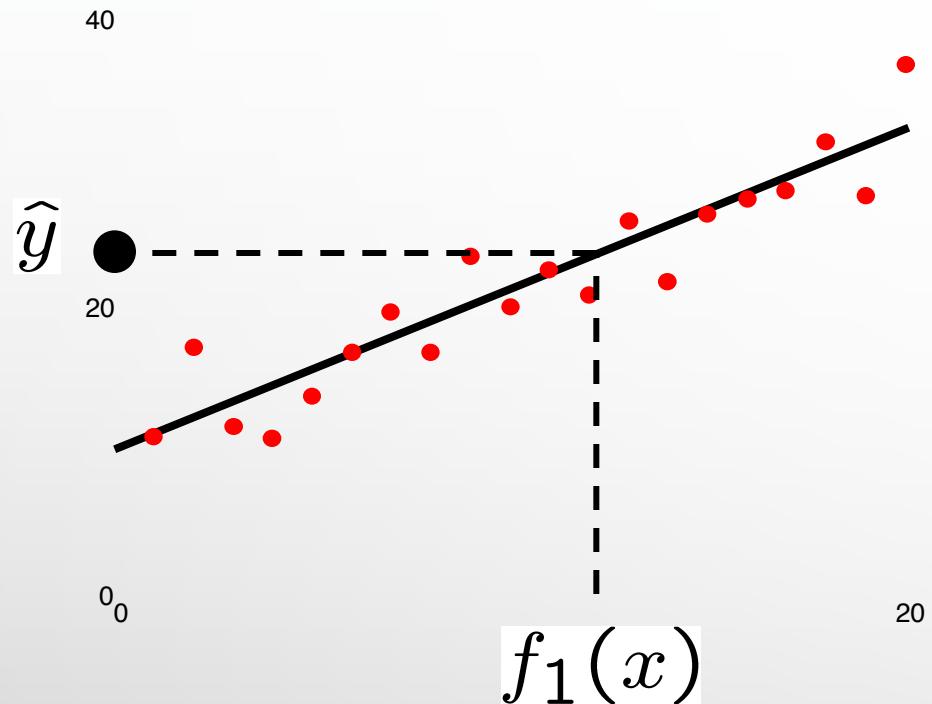
# Video of Demo Approximate Q-Learning -- Pacman



# Q-Learning and Least Squares

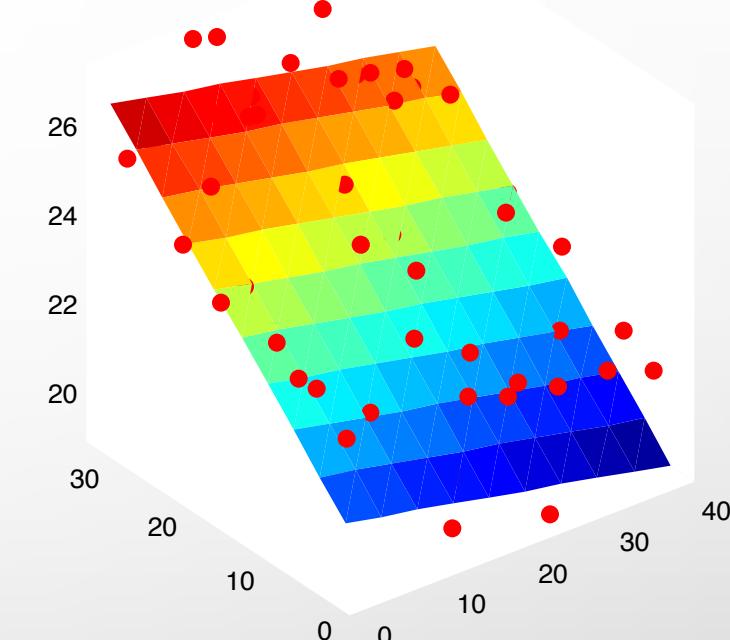


# Linear Approximation: Regression\*



Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

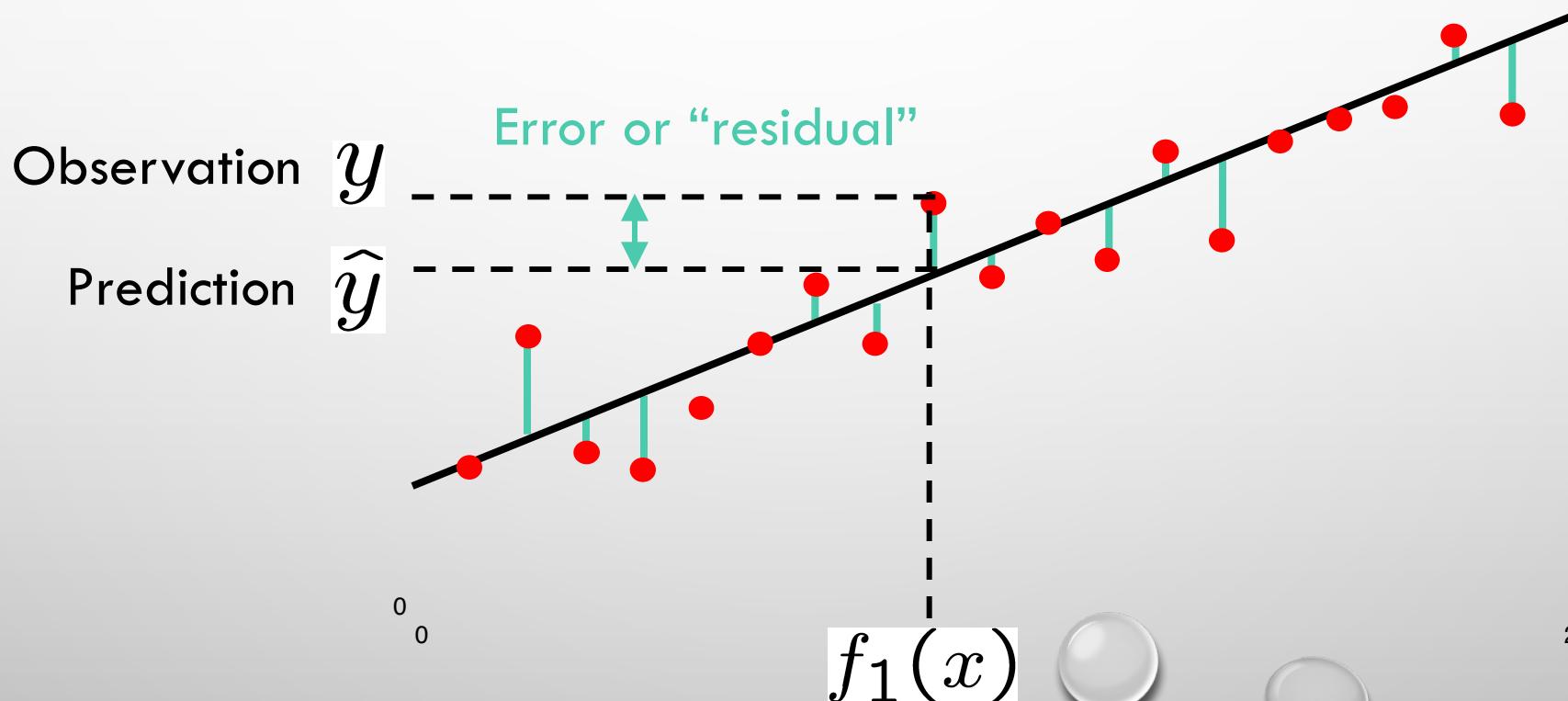


Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

# Optimization: Least Squares\*

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left( y_i - \sum_k w_k f_k(x_i) \right)^2$$



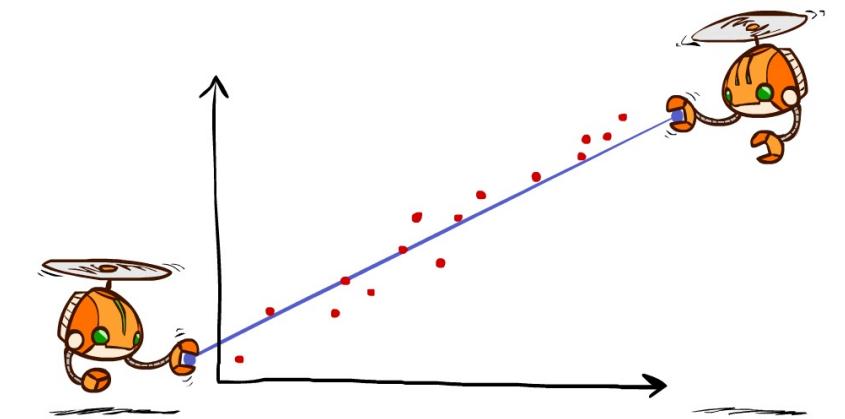
# Minimizing Error\*

Imagine we had only one point  $x$ , with features  $f(x)$ , target value  $y$ , and weights  $w$ :

$$\text{error}(w) = \frac{1}{2} \left( y - \sum_k w_k f_k(x) \right)^2$$

$$\frac{\partial \text{error}(w)}{\partial w_m} = - \left( y - \sum_k w_k f_k(x) \right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left( y - \sum_k w_k f_k(x) \right) f_m(x)$$



Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[ r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$

“target”

“prediction”

# Conclusion

- We're done with part I: search and planning!
- We've seen how AI methods can solve problems in:
  - Search
  - Constraint satisfaction problems
  - Games
  - Markov decision problems
  - Reinforcement learning

