Lab 01: Coordinate systems and their transformations

CE334 Modern Methods in Geoinformatics	2025-26 II
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In this lab, you will work on coordinate transformations, including geodetic to Cartesian, spherical to geodetic, and topocentric to geocentric conversions. The theoretical background and formulas for these transformations are provided in the appendix section.

Coordinate Transformations

The geodetic coordinates of Kanpur are based on the WGS84 ellipsoid and the required parameters of WGS84 are given.

1. Latitude of Kanpur : $26°26'59.7228''$ N
   Longitude of Kanpur : $80°19'54.7356''$ E
   Elevation of Kanpur : $126.630\,\mathrm{m}$
2. **World Geodetic System 1984**
   Earth's center of mass being defined for the whole Earth including oceans and atmosphere
   Semi-major axis $a = 6378137.0\,\mathrm{m}$ and flattening Factor of the Earth, $\frac{1}{f} = 298.257223563$.

Based on the above information compute the following:

1. Write a function to transform geodetic coordinates into cartesian coordinates and use the function to find out the cartesian coordinates of your hometown. Also, write a separate function for the inverse transformation.
2. Write a program to transform spherical coordinates into cartesian coordinates, and *vice versa*.
3. Write a function to compute spherical coordinates from the given geodetic coordinates. The user should provide the reference values for the ellipsoid and the sphere. For the case of the sphere, the default radius of the reference sphere can be taken to be the same as the semi-major axis of the reference ellipsoid.
4. Find out the topocentric coordinates of a satellite that is flying exactly overhead of Kanpur at an altitude of $800\,\mathrm{km}$ from the earth's surface at Kanpur. Assume the topocentre to be at your hometown. Write a function to transform geodetic coordinates into topocentric coordinates.

Bonus

What are the geodetic coordinates of an aircraft flying at an altitude of $6\,\mathrm{km}$ above your hometown with its east and north topocentric coordinates reading $-800\,\mathrm{m}$ and $900\,\mathrm{m}$? The topocentre will be your hometown for which the coordinates have been given previously.

Appendix

A Geodetic and spherical coordinates

A.1 Spherical coordinates

Points on the Earth are usually specified in the spherical coordinate system.

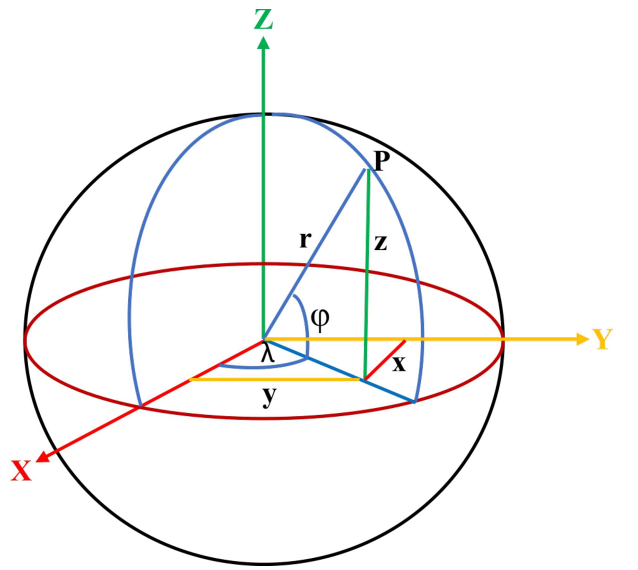

Figure 1: Depiction of spherical coordinates

The spherical coordinates of an arbitrary cartesian point $P(x, y, z)$ are:

1. The latitude φ which is the angle between position vector and the equator.
2. The longitude λ which is the angle between the prime meridian and the meridian containing point P.
3. The radius r which is the distance of P from the origin.

Since the radius is constant, the two angles (φ, λ) are sufficient to specify a point on the Earth:

$$\begin{aligned} -90^\circ \leq \quad & \varphi \quad \leq 90^\circ \text{ (Latitude along meridian)} \\ -180^\circ < \quad & \lambda \quad \leq 180^\circ \text{ (Longitude along equator)} \end{aligned}$$

The longitude at the poles is arbitrary and is usually set to 0° by convention.
We can now compute the cartesian coordinates from the spherical coordinates:

$$\begin{aligned} x &= r\cos\varphi\cos\lambda \\ y &= r\cos\varphi\sin\lambda \\ z &= r\sin\varphi \end{aligned}$$

Solving these equations for (φ, λ, r) yields the opposite relationship:

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \varphi &= \arcsin(\frac{z}{r}) \\ \lambda &= \arctan(\frac{y}{x}) \end{aligned}$$

A.2 Geodetic coordinates (Jekeli 2016)

1. The Cartesian coordinates of a point (x, y, z) can be obtained from the geodetic coordinates (φ, λ, h) by the expressions :

$$\begin{aligned} x &= (N+h)\cos\phi\cos\lambda \\ y &= (N+h)\cos\phi\sin\lambda \\ z &= \big((1-e^2)N+h\big)\sin\phi \end{aligned}$$

   where N is the radius of curvature in the prime vertical.

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

   In the above expression, e is the eccentricity and it is related to the semi-major axis a, the semi-minor axis b and the flattening factor $f = \left(1 - \frac{b}{a}\right)$ by

$$e^2 = \frac{a^2-b^2}{a^2} = 2f - f^2$$

2. The ellipsoidal coordinates of a point (φ, λ, h) can be obtained from the Cartesian coordinates (x, y, z) as follows :
   The longitude λ is given by

$$\lambda = \arctan\left(\frac{y}{x}\right)$$

   The latitude is computed by an iterative procedure.

   i. The initial value is given by

$$\varphi_{(0)} = \arctan\left(\frac{z/p}{1-e^2}\right)$$

   with $p = \sqrt{x^2+y^2}$

   ii. Improved values of φ, as well as the height h, are computed by iterating the equations

$$N_{(i)} = \left[\frac{a}{\sqrt{1-e^2\sin^2\phi_{(i-1)}}}\right]$$

$$h_{(i)} = \left[\frac{p}{\cos\phi_{(i-1)}} - N_{(i)}\right]$$

$$\varphi_{(i)} = \left[\arctan\frac{\frac{z}{p}}{1-\frac{N_{(i)}}{N_{(i)}+h_{(i)}}e^2}\right]$$

   The iterations are repeated until the change between two successive values of $\varphi_{(i)}$ is smaller than the precision required.

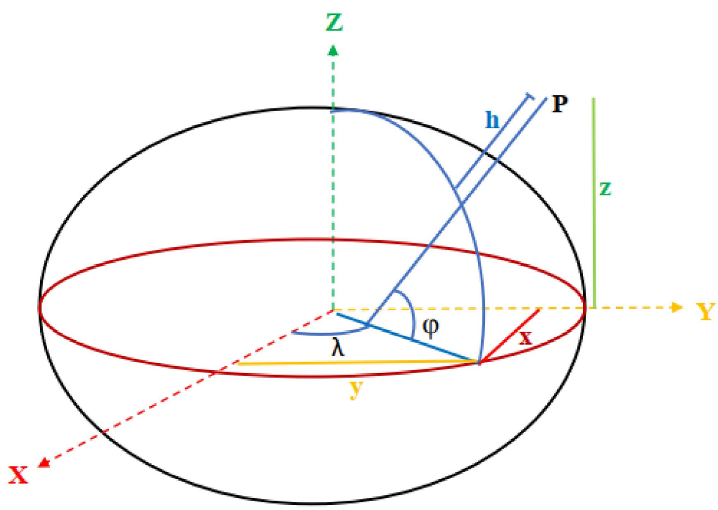

Figure 2: Depiction of geodetic coordinates

A.3 Geodetic to spherical coordinates

The conversion of geodetic coordinates to spherical ones or *vice versa* is a two-step process. We first convert the given geodetic coordinates to cartesian coordinates and then change them to spherical coordinates.

1. Transformation of geodetic coordinates (ϕ, λ, h) to cartesian coordinates (x, y, z).
2. Transformation of cartesian coordinates (x, y, z) to spherical coordinates (φ, λ, r).

B Geocentric and topocentric coordinates

Let (x, y, z) be the earth-centered-earth-fixed (ECEF) cartesian coordinates of a given point P and (φ, λ) its associated latitude and longitude. Let $\mathbf{\Delta r} = (\Delta x,\ \Delta y,\ \Delta z)$ be a displacement vector from that point. These vector coordinates can be transformed from the ECEF to the local system East-North-Up (Δe, Δn, Δu coordinates, and vice-versa, by two rotations.

B.1 Geocentric to topocentric coordinates

1. By using the properties of the rotation matrices $R_i(\alpha)$, that is $R_i^{-1}(\alpha) = R_i(-\alpha) = R_i^{\mathrm{T}}(\alpha)$.

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = R_1\left(\frac{\pi}{2} - \phi\right) R_3\left(\frac{\pi}{2} + \lambda\right) \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

where the transformation matrix of the above equation is the transpose of the matrix given below

$$R_1\left[\frac{\pi}{2} - \phi\right] R_3\left[\frac{\pi}{2} + \lambda\right] = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\varphi & -\sin\lambda\sin\varphi & \cos\varphi \\ \cos\lambda\cos\varphi & \sin\lambda\cos\varphi & \sin\varphi \end{bmatrix}$$

The unit vectors are in the ECEF $\hat{x}, \hat{y}$ and $\hat{z}$ directions, as expressed in ENU coordinates, are given by,

$$
\begin{aligned}
\hat{x} &= (-\sin\lambda, -\cos\lambda\sin\varphi, \cos\lambda cos\varphi) \\
\hat{y} &= (\cos\lambda, -\sin\lambda\sin\varphi, \sin\lambda\cos\varphi) \\
\hat{z} &= (0, \cos\varphi, \sin\varphi)
\end{aligned}
$$

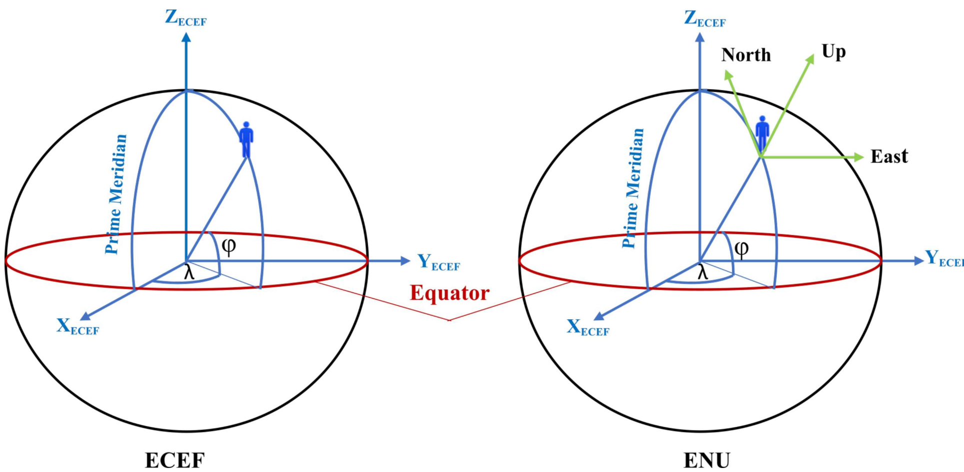

Figure 3: Depiction of ECEF and topocentric (ENU) coordinates.

B.2 Topocentric to geocentric Coordinates

1. A clockwise rotation over the z-axis by an angle of 90+λ to align the East-axis with the x-axis.

$$R_3\left[-\left(\frac{\pi}{2}+\lambda\right)\right]$$

2. A clockwise rotation over the East-axis by an angle of 90-φ to align the up-axis with the z-axis.

$$R_1\left[-\left(\frac{\pi}{2}-\phi\right)\right]$$

3.

$$\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} = R_3[-(\pi/2-\phi)]R_1[-(\pi/2+\lambda)]\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta u\end{bmatrix}$$

where

$$R_3[-(\pi/2-\phi)]R_1[-(\pi/2+\lambda)] = \begin{bmatrix}-\sin\lambda & -\cos\lambda\sin\varphi & \cos\lambda\cos\varphi\\ \cos\lambda & -\sin\lambda\sin\varphi & \sin\lambda\cos\varphi\\ 0 & \cos\varphi & \sin\varphi\end{bmatrix}$$

The unit vectors in the local east, north and up directions as expressed in ECEF Cartesian coordinates are given by,

$$
\begin{aligned}
\hat{e} &= (-\sin\lambda,\ \cos\lambda,\ 0) \\
\hat{n} &= (-\cos\lambda\sin\varphi,\ -\sin\lambda\sin\varphi,\ \cos\varphi) \\
\hat{u} &= (\cos\lambda\cos\varphi,\ \sin\lambda\cos\varphi,\ \sin\varphi)
\end{aligned}
$$

Note If (φ, λ) are ellipsoidal coordinates, then the vector $\hat{u}$ is orthogonal to the tangent plane to the ellipsoid, which is defined by $(\hat{e},\hat{n})$. If (φ, λ) are taken as the spherical latitude and longitude, then the vector $\hat{u}$ is in the radial direction and $(\hat{e},\hat{n})$ defines the tangent plane to the sphere.

References

Jekeli, Christopher (2016). Geometric Reference Systems in Geodesy. Ohio State University, Division of Geodetic Science, School of Earth Sciences.