LWE parameters for Brakerski/Fan-Vercauteren scheme implementation in Cingulata

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This document contains information about parameter sets in the database. They are adapted for the Brakerski/Fan-Vercauteren implementation in Cingulata. Security is estimated using the LWE Estimator (Commit ID = a2296b8). The security expressed in the filename is an approximated value. Estimated security is indicated in the corresponding file.

We avoid the notation σ due to different usages in the literature, in the same context.

$$\begin{aligned} & \text{Gaussian_width} = 2\sqrt{n} \\ & \text{noise_rate} = \frac{\text{Gaussian_width}}{q} = \frac{2\sqrt{n}}{q} \\ & \text{std_dev} = \frac{\text{Gaussian_width}}{\sqrt{2\pi}} = \sqrt{\frac{2n}{\pi}} \end{aligned}$$

Remark: Note that some parameters generated with multiplicative depth 1 are inconsistent (lower parameter with bigger estimated security with the same reduction cost model).

| Filename | Reference |
|--------------|---------------------------|
| BKZ Enum | [CheNgu12] |
| BKZ Sieve | [BDGL16] |
| Core Sieve | [ADPS16] (mode classical) |
| Q-Core Sieve | [ADPS16] (mode quantum) |

Table 1: Four BKZ reduction cost models considered in CinguParam.

References

- [CheNgu12] Yuanmi Chen and Phong Q. Nguyen. BKZ 2.0: Better lattice security estimates (Full Version). http://www.di.ens.fr/~ychen/research/Full_BKZ.pdf
- [BDGL16] Becker, A., Ducas, L., Gama, N., Laarhoven, T. New directions in nearest neighbor searching with applications to lattice sieving. SODA 2016
- [ADPS16] Edem Alkim, Léo Ducas, Thomas Pöppelmann, & Peter Schwabe Post-quantum key exchange A New Hope. USENIX Security 16 (pp. 327–343).

 $Syntax: \ \underline{multiplicative-depth_reduction-cost-model_desired-security-level}$

| Filename | n | $\log_2(q)$ | std dev |
|---|------|-------------|---------|
| 1_q_core_sieve_192 | | 02(1) | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | 54 | 51 |
| 1 core sieve 256 | | | |
| 2_bkz_sieve_192 | | 76 | |
| $\begin{bmatrix} 2_okz_sieve_132 \\ 2_core_sieve_128 \end{bmatrix}$ | | | |
| 2_q_core_sieve_128 | | | |
| $\begin{bmatrix} 2 & q & core & sieve & 128 \\ 2 & q & core & sieve & 128 \end{bmatrix}$ | | | |
| $\begin{bmatrix} 2 & q & core & sieve & 128 \\ 2 & q & core & sieve & 128 \end{bmatrix}$ | | | |
| 3_core_sieve_80 | | 101 | |
| $\begin{bmatrix} 3_core_sieve_60 \\ 3_bkz_enum_256 \end{bmatrix}$ | | | |
| $\begin{bmatrix} 3_bkz_emam_250 \\ 3_bkz_sieve_128 \end{bmatrix}$ | | | |
| $3_q_core_sieve_80$ | | | |
| $\frac{3_{q}core_sieve_50}{1 \ bkz \ sieve \ 80}$ | 4096 | | |
| 1_core_sieve_80 | | 117 | |
| $\begin{bmatrix} 1_corc_siece_co\\ 1_bkz \ enum \ 192 \end{bmatrix}$ | | | |
| $\begin{bmatrix} 1_bkz_enum_192 \\ 2_bkz_enum_192 \end{bmatrix}$ | | | |
| $\begin{bmatrix} 2_bkz_sieve_80 \end{bmatrix}$ | | | |
| $\begin{bmatrix} 2 & core \\ 2 & core \\ \end{bmatrix}$ sieve $\begin{bmatrix} 80 \\ \end{bmatrix}$ | | | |
| $\begin{bmatrix} 2_core_sieve_co\\ 3_bkz_sieve_80 \end{bmatrix}$ | | | |
| $3_bkz_enum_192$ | | | |
| $\frac{0_bkz_enum_102}{4\ bkz\ enum\ 128}$ | | | |
| $4_bkz_sieve_80$ | | 126 | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | 120 | |
| $\frac{1-bkz-enum-50}{5-bkz-enum-128}$ | | 151 | |
| $\begin{bmatrix} 5 & bkz & sieve & 80 \end{bmatrix}$ | | | |
| $6 \ bkz \ enum \ 80$ | | 176 | |
| 5_bkz_enum_80 | | 181 | |

| Filename | n | $\log_2(q)$ | std_dev |
|---|------|-------------|---------|
| 3_q_core_sieve_192 3 bkz sieve 256 | | 108 | |
| $3_core_sieve_256$ | | | |
| 1_core_sieve_192 | | | |
| 2_q_core_sieve_192 2 bkz sieve 256 | | 117 | |
| 2_core_sieve_192 | | 111 | |
| 3_core_sieve_192 | | | |
| $4_q_core_sieve_128$ | | | |
| 4_bkz_sieve_192 4_core_sieve_192 | | 135 | |
| 5 bkz sieve 128 | | | |
| 5_core_sieve_128 | | 162 | |
| 5_q_core_sieve_128 | | | |
| 4_q_core_sieve_80 | | | |
| $\begin{array}{c} 4_bkz_enum_256 \\ 4_bkz_sieve_128 \end{array}$ | | | |
| 4_core_sieve_128 | 8192 | 181 | 72 |
| 5_q_core_sieve_80 | | | |
| 5_bkz_enum_256 | | | |
| 6_q_core_sieve_80 6_core_sieve_80 | | | |
| $\begin{bmatrix} 6_core_sieve_so \\ 6_bkz_enum_256 \end{bmatrix}$ | | 189 | |
| $6_bkz_sieve_128$ | | | |
| 7_core_sieve_80 | | | |
| 7_q_core_sieve_80 7_bkz_enum_192 | | 216 | |
| 7_ <i>bkz_sieve_</i> 128 | | | |
| 8_bkz_enum_128 | | | |
| $8_bkz_sieve_80$ | | 243 | |
| 8_core_sieve_80 | | | |
| 6_bkz_enum_128 6_bkz_sieve_80 | | | |
| 7_bkz_enum_128 | | 245 | |
| 7_bkz_sieve_80 | | | |
| 9_bkz_enum_128 | | 270 | |
| 9_bkz_sieve_80 10_bkz_sieve_80 | | | |
| 10_0kz_steve_80 10_bkz_enum_128 | | 297 | |
| 9_ <i>bkz_enum_</i> 80 | | 309 | - |
| 10_bkz_enum_80 | | 303 | |
| 11_bkz_enum_80 11_bkz_sieve_80 | | 324 | |
| 11_0kz_steve_80 12_bkz_enum_80 | | 351 | |
| 13_bkz_enum_80 | | 378 | |

| Filename | n | $\log_2(q)$ | std_dev |
|---|-------|-------------|---------|
| $5_q_core_sieve_256$ | | 174 | |
| $4_q_core_sieve_256$ | | 181 | |
| 6_core_sieve_256 | | | |
| 6_bkz_sieve_256 | | 203 | |
| 6_q_core_sieve_256 | | | |
| 7_core_sieve_192 7_q_core_sieve_192 | | 232 | |
| 7_q_core_sieve_192 7_bkz_sieve_256 | | 232 | |
| 6_q_core_sieve_192 | | | |
| 6 core sieve 192 | | 245 | |
| 8_ <i>bkz_sieve_</i> 192 | | | |
| $8_core_sieve_192$ | | 261 | |
| $8_q_core_sieve_128$ | | | |
| $9_bkz_sieve_192$ | | | |
| 9_core_sieve_128 | | 290 | |
| 9_q_core_sieve_128 | | 200 | |
| 8_core_sieve_128 10_q_core_sieve_128 | | 309 | |
| 10_q_core_sieve_128 | | 319 | |
| 10_core_sieve_128 | | 013 | |
| 11 bkz sieve 128 | | | |
| 11_core_sieve_128 | 16384 | 348 | 102 |
| 11_q_core_sieve_80 | | | |
| 10_core_sieve_80 | | | |
| 10_q_core_sieve_80 | | 070 | |
| 10_bkz_enum_256 | | 373 | |
| 11_bkz_enum_256 11 core sieve 80 | | | |
| 12_bkz_enum_256 | | | |
| 12 bkz sieve 128 | | | |
| 12_q_core_sieve_80 | | 377 | |
| 12_core_sieve_80 | | | |
| 13_bkz_enum_192 | | | |
| 13_core_sieve_80 | | 406 | |
| 13_q_core_sieve_80 | | 100 | |
| 13_bkz_sieve_128 | | | |
| 14_bkz_enum_192 14_core_sieve_80 | | | |
| 14_ <i>q_core_sieve_</i> 80 | | 435 | |
| 14 bkz sieve 128 | | | |
| $12_bkz_enum_192$ | | 437 | |
| $15_bkz_enum_192$ | | | |
| 15_bkz_sieve_128 | | 464 | |
| 15_core_sieve_80 | | | |
| 16_bkz_enum_128 | | 493 | |
| 16_bkz_sieve_80 15_bkz_sieve_80 | | | |
| 15 bkz enum 128 | | 501 | |
| 17_bkz_enum_128 | | F00 | |
| 17_bkz_sieve_80 | | 522 | |
| 18_bkz_enum_128 | | 551 | |
| 18_bkz_sieve_80 | | | |
| 19_bkz_enum_128 19_bkz_sieve_80 | | 580 | |
| 19_bkz_sieve_80 20 bkz enum 80 | | | |
| 20_bkz_enam_80 20_bkz_sieve_80 | | 609 | |
| 19_bkz_enum_80 | | 629 | |
| | | | |

| Filename | n | $\log_2(q)$ | std_dev |
|---|-------|-------------|---------|
| 11_q_core_sieve_256 | | | |
| 11_core_sieve_256 | | 371 | |
| $10_q_core_sieve_256$ | | 373 | |
| $10_core_sieve_256$ | | 313 | |
| $12_q_core_sieve_256$ | | 402 | |
| $_12_core_sieve_256$ | | 402 | |
| $13_bkz_sieve_256$ | | | |
| 13_core_sieve_256 | | 433 | |
| 13_q_core_sieve_192 | | | |
| 12_q_core_sieve_192 | | 437 | |
| 12_bkz_sieve_256 | | | |
| 14_bkz_sieve_256 | 22-22 | 101 | 4.4 |
| 14_core_sieve_192 | 32768 | 464 | 144 |
| 14_q_core_sieve_192 | | | |
| 15_core_sieve_192 | | 405 | |
| 15_bkz_sieve_192 | | 495 | |
| 15_q_core_sieve_192 14_bkz_sieve_192 | | 501 | |
| 14_0kz_sieve_192 16_bkz_sieve_192 | | 301 | |
| 16_0kz_sieve_192 16_core_sieve_192 | | 526 | |
| 16_q_core_sieve_128 | | 520 | |
| 17 bkz sieve 192 | | | |
| 17_core_sieve_128 | | 557 | |
| 17_q_core_sieve_128 | | 551 | |
| 16_core_sieve_128 | | 565 | |
| 18_bkz_sieve_192 | | | |
| 18_core_sieve_128 | | 588 | |
| 18 <i>q core sieve</i> 128 | | | |
| $19 \ bkz \ sieve \ 192$ | | | |
| 19_core_sieve_128 | | 619 | |
| 19_ <i>q_core_sieve</i> _128 | | | |
| $20_bkz_sieve_128$ | | | |
| 20_core_sieve_128 | | 650 | |
| 20_ <i>q_core_sieve</i> _128 | | | |
| $20_q_core_sieve_80$ | | 693 | |
| $20_q_core_sieve_256$ | 65536 | 692 | 204 |
| $\boxed{19_q_core_sieve_256}$ | 00000 | 693 | 204 |