

# The Data Science Cycle

## Unsupervised Learning - Clustering

DataLab

September 21, 2016

# Outline

## 1 Supervised Learning vs. Unsupervised Learning

- Supervised vs Unsupervised
- Clustering
- Pattern Recognition
- Aspect of Clustering

## 2 K-Means Clustering

- K-Means Clustering
- Convergence Criterion
- The Distance Function
- Example
- Properties of K-Means

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# Supervised Learning vs. Unsupervised Learning

## Supervised learning:

- Discover patterns in the data that relate data attributes with a target (class) attribute.

## Unsupervised learning:

The data have no target attribute.

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Search for structure in data

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## Elements of Numerical Pattern Recognition

### 1 Process Description

- ▶ Feature Nomination, Test Data, Design Data

### 2 Feature Analysis

- ▶ Preprocessing, Extraction, Selection, ...

### 3 Cluster Analysis

- ▶ Labeling, Validity, ...

### 4 Classifier Design

- ▶ Classification, Estimation, Prediction, Control, ...

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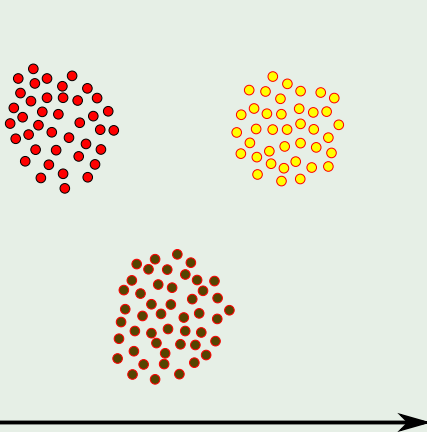
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# An illustration

The data set has three natural groups of data points, i.e., 3 natural clusters.



# Examples

## Example 1

Groups people of similar sizes together to make “small”, “medium” and “large” T-Shirts.

## Example 2

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For this, we use the following concept

**Clustering!!!**

Basically,

We want to “reveal” the organization of patterns into “sensible” clusters (groups).

Intuitively,

Clustering is one of the most primitive mental activities of humans, used to handle the huge amount of information they receive every day.



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# Aspects of clustering

## A clustering algorithm - They are Many!!!

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## Clustering quality

- Inter-clusters distance  $\rightarrow$  maximized.
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# K-Means - Stuart Lloyd (Circa 1957)

## History

Invented by Stuart Lloyd in Bell Labs to obtain the best quantization in a signal data set.

## Something Notable

The paper was published until 1982

Especially given  $N$  vectors  $x_1, \dots, x_N \in \mathbb{R}^d$

It tries to find  $k$  points  $\mu_1, \dots, \mu_k \in \mathbb{R}^d$  that minimize the expression (i.e. a partition  $S$  of the vector points):

$$\sum_{k=1}^N \sum_{i: x_i \in C_k} \|x_i - \mu_k\|^2 = \sum_{k=1}^N \sum_{i: x_i \in C_k} (x_i - \mu_k)^T (x_i - \mu_k)$$

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### Definition

Let the set of data points (or instances)  $D$  be  $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  where  $\mathbf{x}_i = (x_{i1}, \dots, x_{ir})^T$ :

- The  $K$ -means algorithm partitions the given data into  $K$  clusters.
- Each cluster has a cluster center, called centroid.
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# $K$ -means algorithm

The  $K$ -means algorithm works as follows

Given  $k$  as the possible number of cluster:

- 1 Randomly choose  $K$  data points (seeds) to be the initial **centroids**, cluster centers,

- ▶  $\{v_1, \dots, v_k\}$

- 2 Assign each data point to the closest **centroid**

- ▶  $c_i = \arg \min_j \{dist(x_i - v_j)\}$

- 3 Re-compute the **centroids** using the current cluster memberships.

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$$v_j = \frac{\sum_{i=1}^n I(c_i = j) x_i}{\sum_{i=1}^n I(c_i = j)}$$

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It is trying to find a partition  $S$

$K$ -means tries to find a partition  $S$  such that it minimizes the cost function:

$$\min_S \sum_{k=1}^N \sum_{i: x_i \in C_k} (x_i - \mu_k)^T (x_i - \mu_k) \quad (1)$$

Where  $\mu_k$  is the centroid for cluster  $C_k$ .

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Minimum decrease in the sum of squared error (SSE),

- $C_k$  is cluster  $k$ .
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$$SSE = \sum_{k=1}^K \sum_{\mathbf{x} \in C_k} \text{dist}(\mathbf{x}, \mathbf{v}_k)^2$$

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# The distance function

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$$d_{\text{Euc}}(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{(\mathbf{x} - \mathbf{y})^T (\mathbf{x} - \mathbf{y})}$$

## Manhattan

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## Mahalanobis

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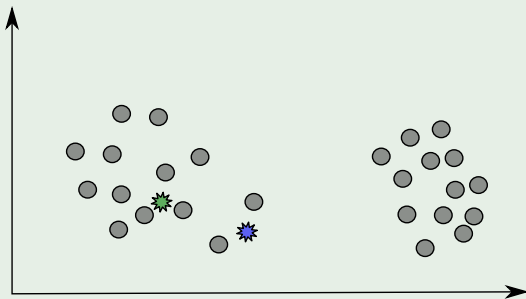
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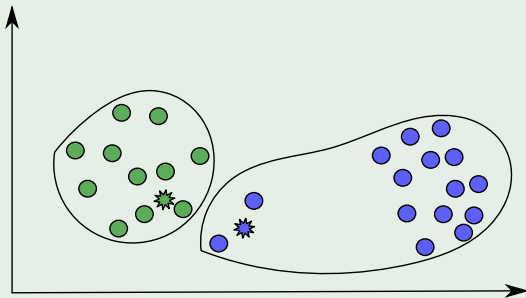
## An example

### Dropping two possible centroids



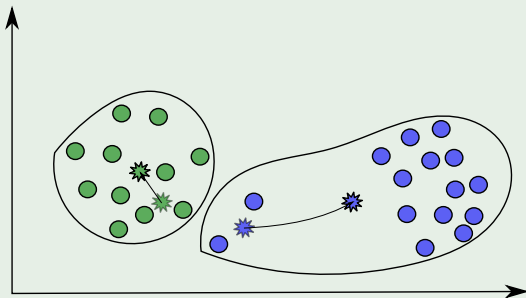
# An example

Calculate the memberships



# An example

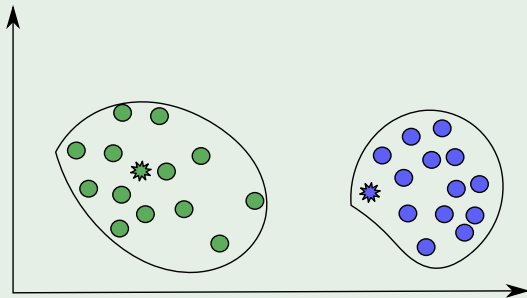
We re-calculate centroids





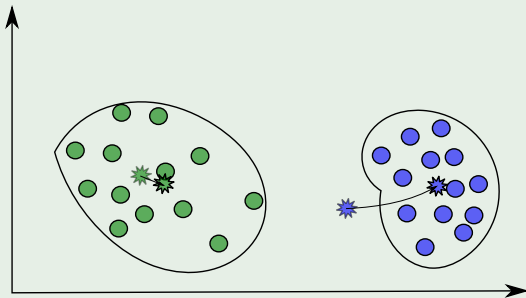
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## An example

We re-calculate centroids and keep going



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# Strengths of $K$ -means

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- Simple: easy to understand and to implement
- Efficient: Time complexity:  $O(tKN)$ , where  $N$  is the number of data points,  $K$  is the number of clusters, and  $t$  is the number of iterations.
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## Popularity

$K$ -means is the most popular clustering algorithm.

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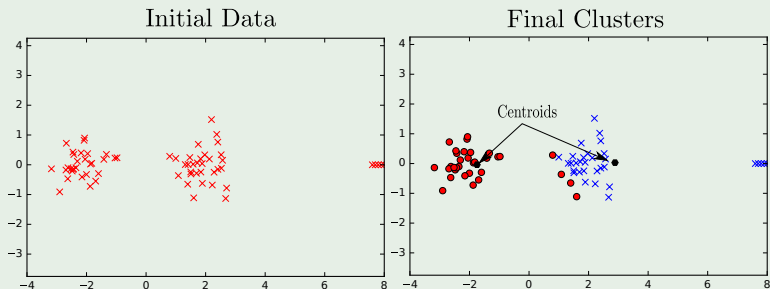
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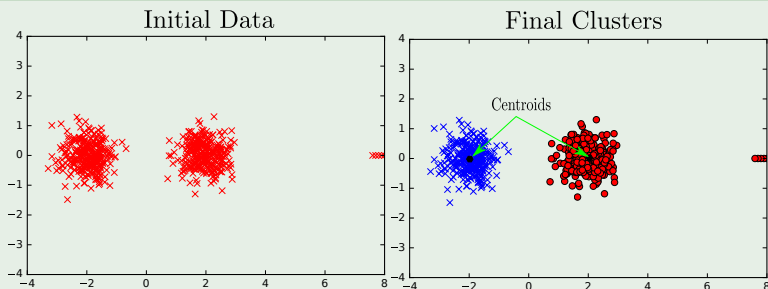
# Weaknesses of $K$ -means: Problems with outliers

## A series of outliers



# Weaknesses of $K$ -means: Problems with outliers

Nevertheless, if you have more dense clusters





## Weaknesses of $K$ -means: How to deal with outliers

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To perform random sampling.

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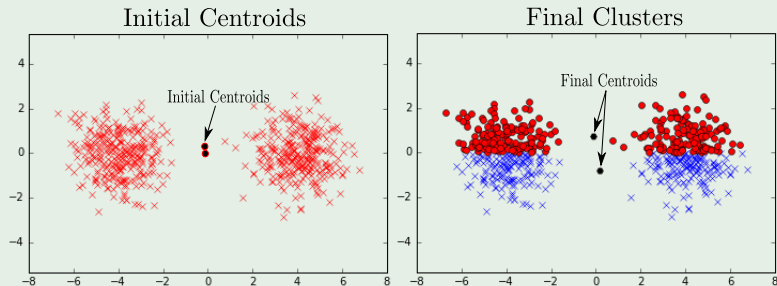
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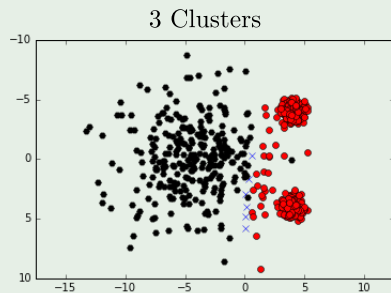
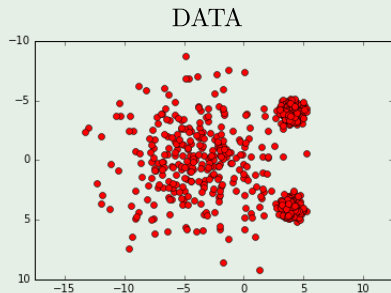
# Weaknesses of $K$ -means (cont...)

The algorithm is sensitive to **initial seeds**



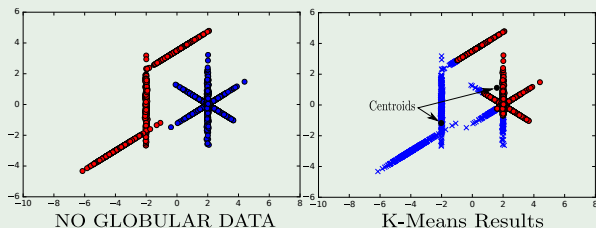
# Weaknesses of $K$ -means : Different Densities

We have three cluster nevertheless



# Weaknesses of $K$ -means: Non-globular Shapes

Here, we notice that  $K$ -means may only detect globular shapes





# Weaknesses of $K$ -means: Non-globular Shapes

However, it sometimes work better than expected

