Data Structures

Graphs

DataLab

November 19, 2016

Outline

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Outline

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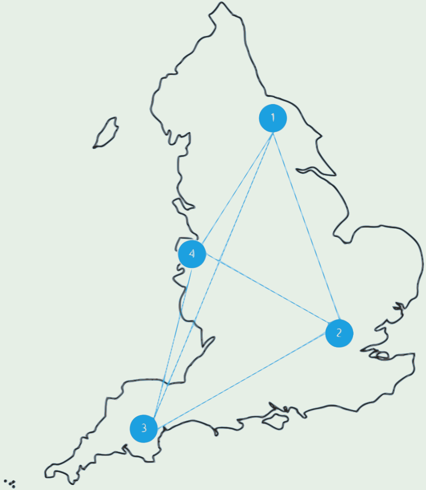
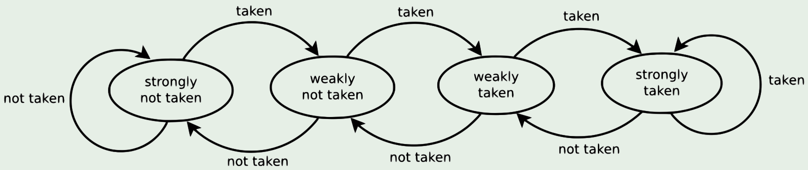
We are full of Graphs

Maps



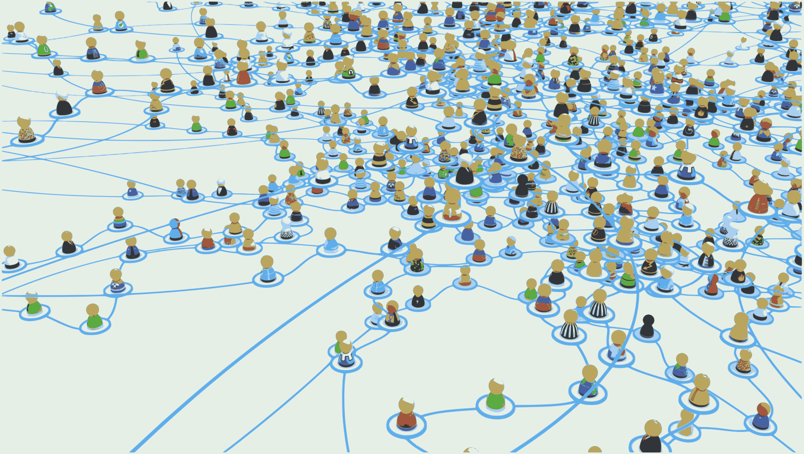
We are full of Graphs

State Machines for Branch Prediction in CPU's



We are full of Graphs

Social Networks



Outline

We need NICE representations

First One

Matrix Representation

Second One

Adjacency Representation

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First One

Matrix Representation

Second One

Adjacency Representation

Outline

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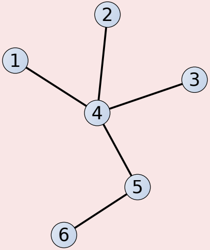
Adjacency Matrix Representation

This is the simplest one

If we number the nodes of an undirected graph:



	1	2	3	4	5	6
1	0	0	0	1	0	0
2	0	0	0	1	0	0
3	0	0	0	1	0	0
4	1	1	1	0	1	0
5	0	0	0	1	0	1
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Adjacency Matrix Representation

In a natural way the edges can be identified by the nodes

For example, the edge between 1 and 4 nodes gets named as (1,4)

Then

How, we use this to represent the graph through a Matrix or and Array of Arrays??!!!

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How, we use this to represent the graph through a Matrix or and Array of Arrays??!!!

What about the following?

How do we indicate that an edge exist given the following matrix

	1	2	3	4	5	6
1	—	—	—	—	—	—
2	—	—	—	—	—	—
3	—	—	—	—	—	—
4	—	—	—	—	—	—
5	—	—	—	—	—	—
6	—	—	—	—	—	—

You say it!!

- Use a 0 for no-edge
- Use a 1 for edge

What about the following?

How do we indicate that an edge exist given the following matrix

$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & - & - & - & - & - & - \\
2 & - & - & - & - & - & - \\
3 & - & - & - & - & - & - \\
4 & - & - & - & - & - & - \\
5 & - & - & - & - & - & - \\
6 & - & - & - & - & - & - \\
\end{array}
$$

You say it!!

- Use a 0 for no-edge
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We have then...

Definition

- $0/1$ $N \times N$ matrix with N =Number of nodes or vertices
- $A(i,j) = 1$ iff (i,j) is an edge

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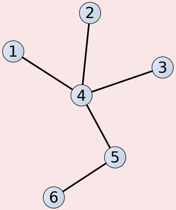
We have then...

For the previous example



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Properties of the Matrix for Undirected Graphs

Property One

Diagonal entries are zero.

Property Two

Adjacency matrix of an undirected graph is symmetric:

$$A(i,j) = A(j,i) \text{ for all } i \text{ and } j$$

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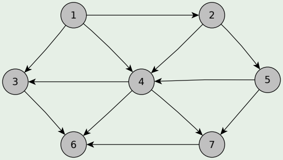
What about direct Graphs!!!

Similar idea

- Use a 0 for no-edge
- Use a 1 for directed edge

Example

We have that



	1	2	3	4	5	6	7
1	0	1	1	1	0	0	0
2	0	0	0	1	1	0	0
3	0	0	0	0	0	1	0
4	0	0	1	0	0	1	1
5	0	0	0	1	0	0	1
6	0	0	0	0	0	0	0
7	0	0	0	0	0	1	0

Outline

What about the code?

Partial Code

We can try to write it!!!

What about Time Complexity?

Operations on a Graph

Most of the basic operations in a graph are:

- Adding an edge – $O(1)$
- Deleting an edge – $O(1)$
- Answering the question "is there an edge between i and j" – $O(1)$
- Finding the successors of a given vertex – $O(N)$
- Finding (if exists) a path between two vertices – $O(N^2)$

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Space Drawbacks of This Representation

We need the following amount of space

If you have N integers of 4 bytes each, we requiere

$$4 \times N \times N = 4N^2$$

space

Which is a killer!!!

If your graph does not have an edge between any two pair of nodes

So

What to do?

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Possible Solutions

If you have an undirected graph

- For an undirected graph, may store only lower or upper triangle (exclude diagonal).
- Space used (Assume Integers): $4 \times \frac{N(N-1)}{2} = 2N(N-1)$

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Use Sparse Matrix Representations

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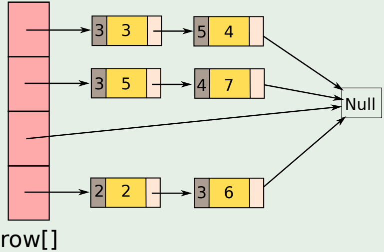
Better

Use Sparse Matrix Representations

We use the sparse Representation of Matrices!!!

Example: Array of Row Chains

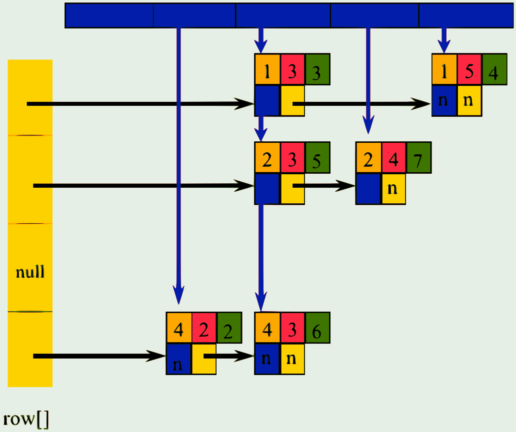
$$
\begin{matrix}
0 & 0 & 3 & 0 & 4 \\
0 & 0 & 5 & 7 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 2 & 6 & 0 & 0
\end{matrix}
$$



We use the sparse Representation of Matrices!!!

Example: Orthogonal Lists

$$\begin{matrix} 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 & 0 \end{matrix}$$



Outline

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Adjacency List Representation

Definition

Adjacency list for vertex i is a linear list of vertices adjacent from vertex i.

Basically

An array of N adjacency lists.

Thus

Each adjacency list is a chain.

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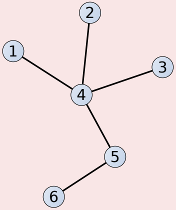

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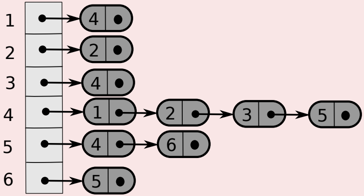

Properties

Space for storage

For undirected or directed graphs $O(V+E)$

Search: Successful or Unsuccessful

$O(1+degree(v))$

- Weight function $w : E \to \mathbb{R}$
- The weight $w(u,v)$ of the edge $(u,v) \in E$ is simply stored with vertex v in u's adjacency list.

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In addition

Adjacency lists can readily be adapted to represent weighted graphs

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Possible Disadvantage

When looking to see if an edge exist

There is no quicker way to determine if a given edge (u,v)

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We can try to write it!!!

Traversing the Graph

Why?

Do you have any examples?

Traversing the Graph

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Do you have any examples?

Yes

- Search for paths satisfying various constraints
  - Shortest Path
- Visit some sets of vertices
  - Tours
- Search for subgraphs
  - Isomorphisms

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Breadth-first search

Definition

Given a graph $G = (V, E)$ and a source vertex s, breadth-first search systematically explores the edges of $G$to "discover" every vertex that is reachable from the vertex s

Something Notable

A vertex is discovered the first time it is encountered during the search

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Defining Some Basic Terms

Color

- Given a node u, we have that
  - $color$ is a field indicating
    - WHITE Never visited node
    - GRAY Node pointer is at the Queue Q
    - BLACK Node has been visited and processed

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Distance d

- Given a node u, we have that
  - d is a field indicating the distance from the node source s so far

Defining Some Basic Terms

Predecesor π

- Given a node u, we have that
  - π is a field indicating who is the immediate predecessor of u in the path from s to u

Breadth-First Search Algorithm

Algorithm

BFS(G, s)

for each vertex $u \in G.V - \{s\}$
  $u.color =$ WHITE
  $u.d = \infty$
  $u.\pi = NIL$
$s.color =$GRAY
$s.d = 0$
$s.\pi =$ NIL
$Q = \emptyset$
Enqueue(Q, s)
while $Q \neq \emptyset$
  $u =$Dequeue(Q)
  for each $v \in G.Adj[u]$
    if $v.color ==$WHITE
      $v.color =$GRAY
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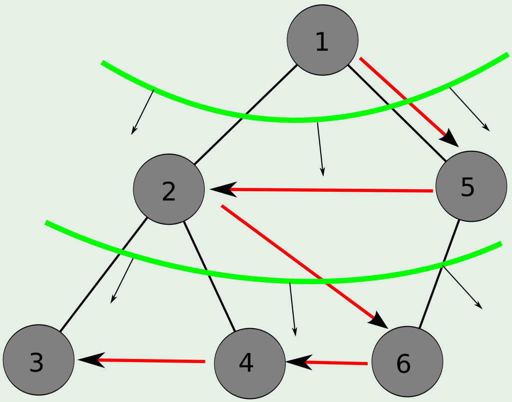
Change the Order of Recursion

It is like a wave going from a node



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Example

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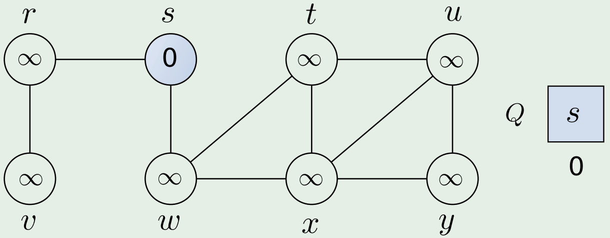

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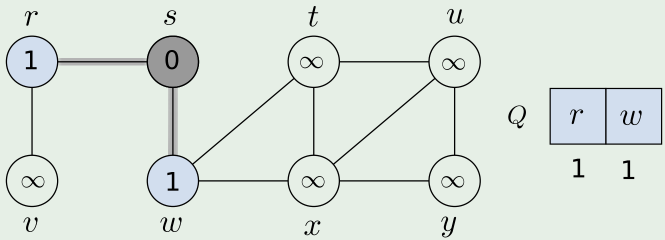

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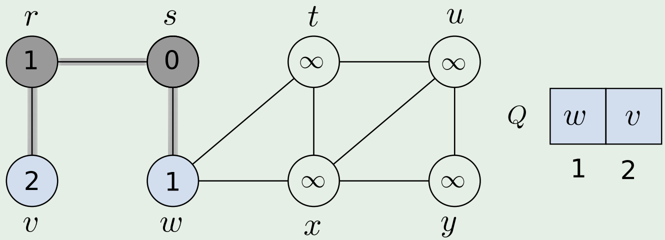

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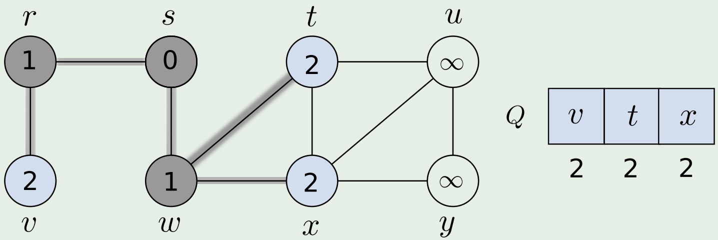

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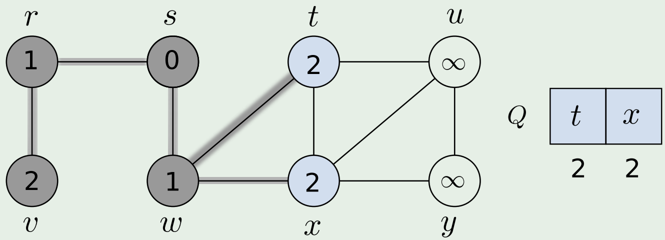

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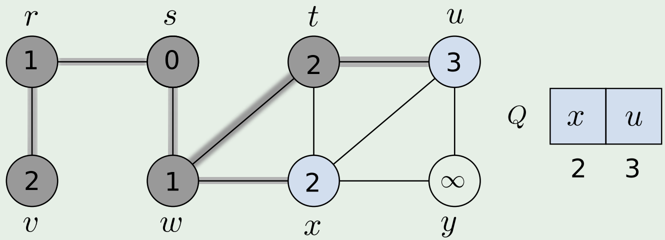

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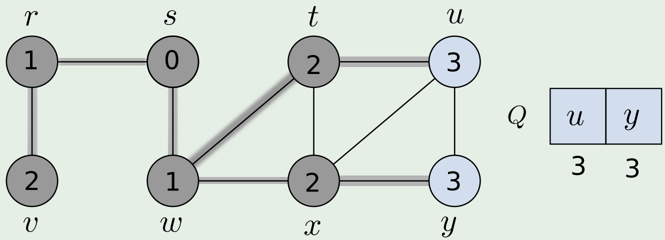

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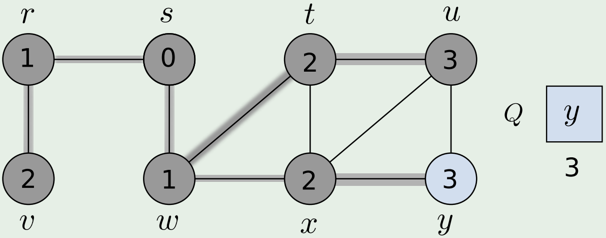

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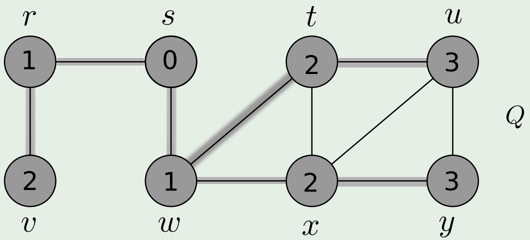

Outline

Complexity

What about the outer loop?

$O(V)$ Enqueue / Dequeue operations – Each adjacency list is processed only once.

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$O(V)$

Then

Total complexity $O(V + E)$

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Properties: Predecessor Graph

Something Notable

Breadth-First Search constructs a **Breadth-First Tree**, initially containing only its root, which is the source vertex s

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We say that u is the predecessor or parent of v in the breadth-first tree.

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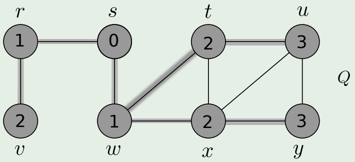

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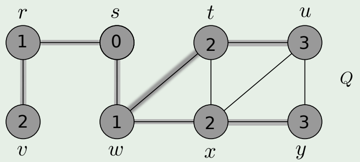

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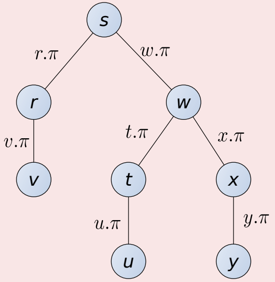

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This is the unweighted version or all weights are equal!!!

We have the following function

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Upon termination of BFS, every vertex $v \in V$ reachable from s has

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Outline

Depth-first search

Given G

- Pick an unvisited vertex v, remember the rest.
  - Recurse on vertices adjacent to v

Outline

The Algorithm

Code for DFS

DFS(G)

for each vertex $u \in G.V$
    $u.color =$ WHITE
    $u.\pi = NIL$
$time = 0$
for each vertex $u \in G.V$
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        DFS-VISIT(G, u)

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Outline

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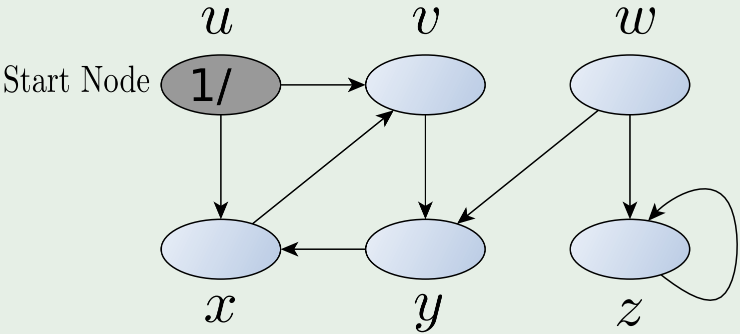

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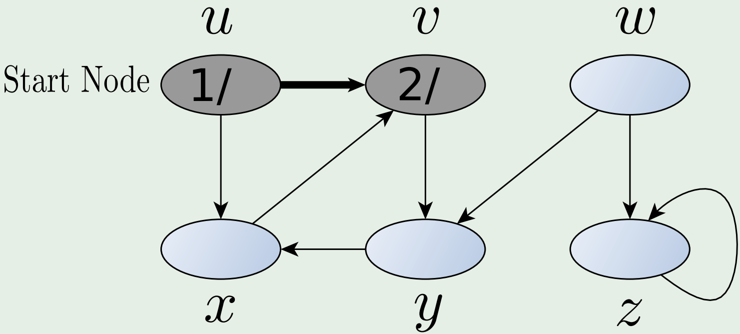

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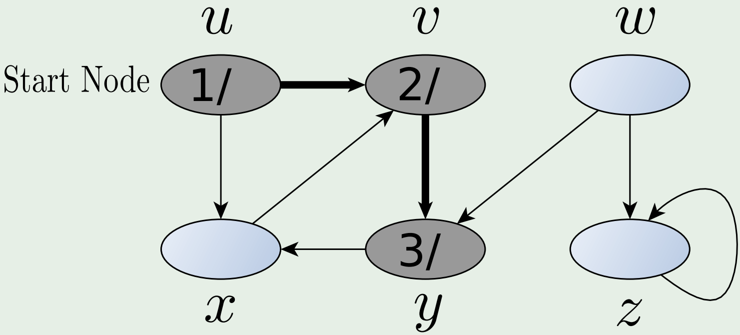

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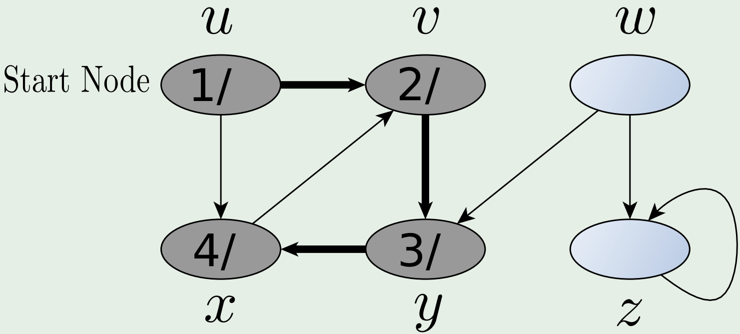

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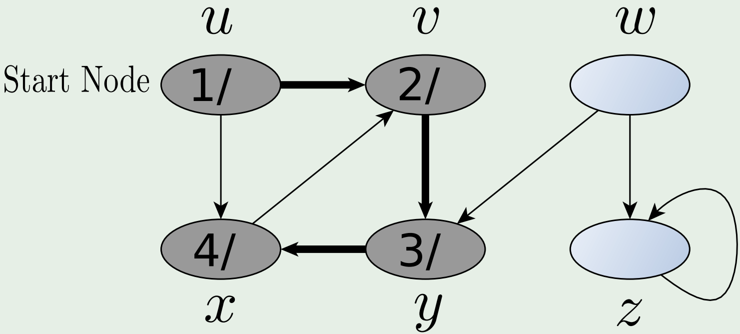

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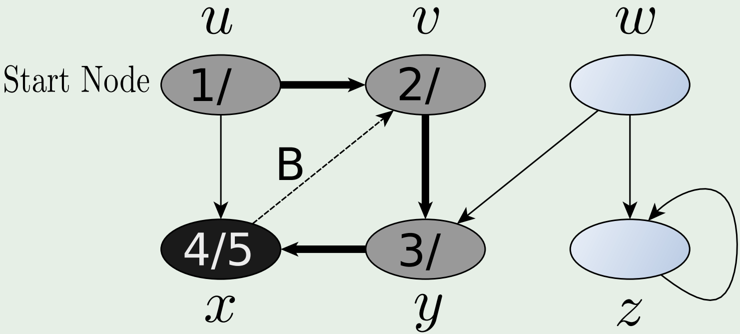

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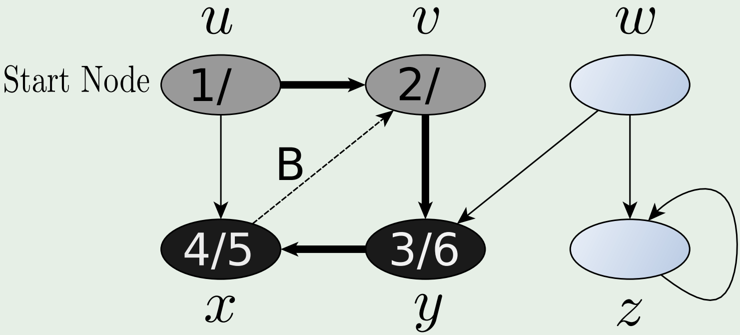

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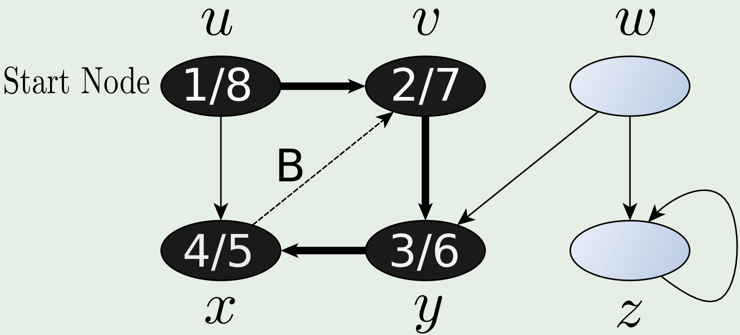

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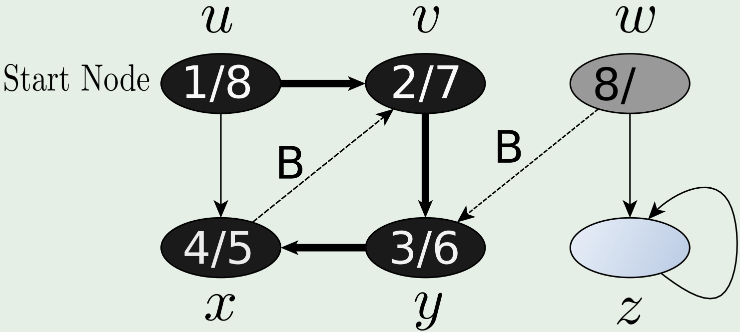

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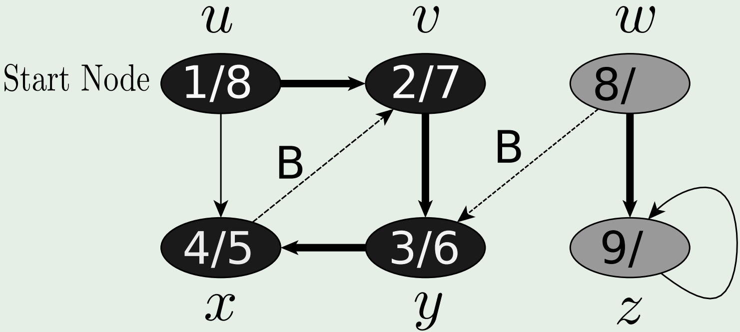

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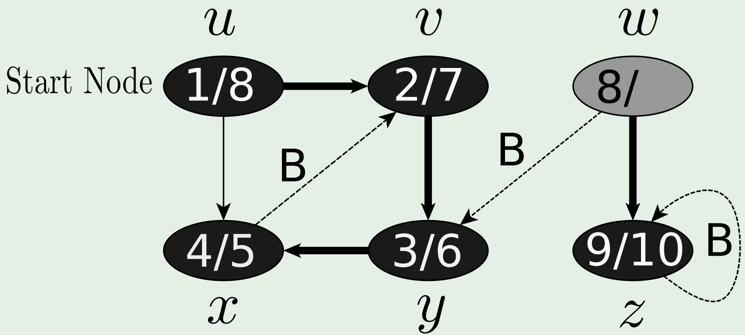

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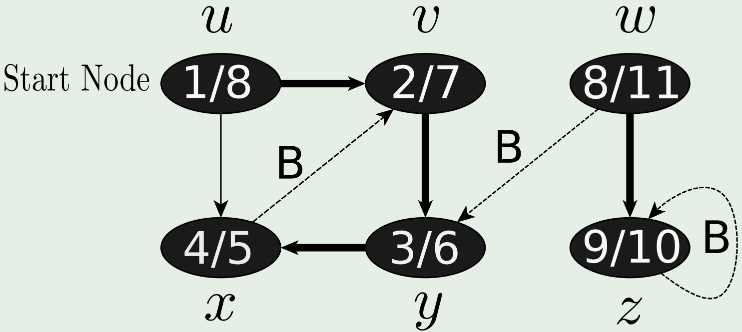

Outline

Complexity

Analysis

1. The loops on lines 1–3 and lines 5–7 of DFS take $\Theta(V)$.
2. The procedure DFS-VISIT is called exactly once for each vertex $v \in V$.
3. During an execution of DFS-VISIT(G, v) the loop on lines 4–7 executes $|Adj(v)|$ times.
4. But $\sum_{v \in V} |Adj(v)| = \Theta(E)$ we have that the cost of executing g lines 4–7 of DFS-VISIT is $\Theta(E)$.

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We have several

- Finding a path between nodes
- Strongly Connected Components
- Spanning Trees
- Topological Sort - The Program (or Project) Evaluation and Review (PERT)
- Computer Vision Algorithms
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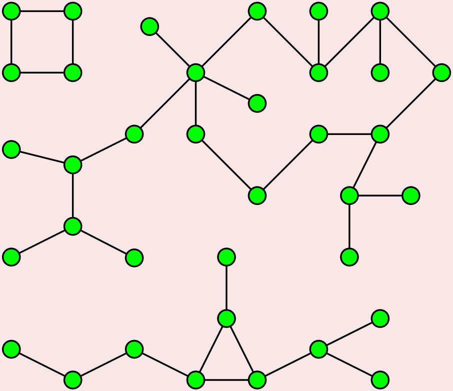

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Thus

Newly visited vertices (plus edges between them) define a component.

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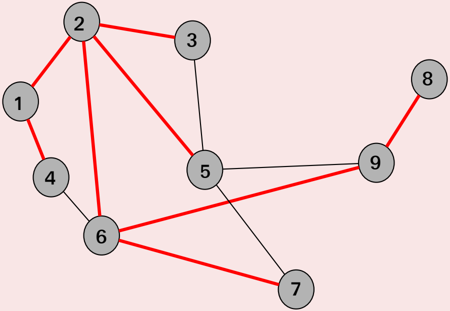

Procedure

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If graph is connected, the $n-1$ edges used to get to **unvisited vertices** define a spanning tree (**Breadth-First Spanning Tree**).

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Topological Sorting

Definitions

A topological sort (sometimes abbreviated topsort or toposort) or topological ordering of a directed graph is a linear ordering of its vertices such that for every directed edge (u, v) from vertex u to vertex y, u comes before v in the ordering.

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We have that

The jobs are represented by vertices, and there is an edge from x to y if job x must be completed before job y can be started.

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When washing clothes, the washing machine must finish before we put the clothes to dry.

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Algorithm

TOPOLOGICAL-SORT

1. Call DFS(G) to compute finishing times $v.f$ for each vertex v.
2. As each vertex is finished, insert it onto the front of a linked list
3. Return the linked list of vertices

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TOPOLOGICAL-SORT

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2. As each vertex is finished, insert it onto the front of a linked list
3. Return the linked list of vertices

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Algorithm

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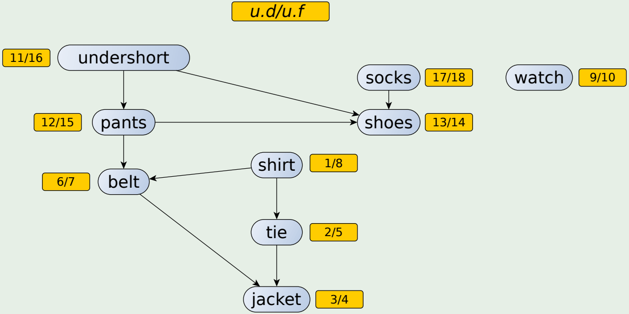
Example

Dressing



Thus

Using the $u.f$

As each vertex is finished, insert it onto the front of a linked list

Example

After Sorting