

Data Structures

Stacks

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Outline

- 1 Introduction
 - Insertion and Deletion
 - ADT

- 2 Examples
 - Parentheses Matching

Introduction

Definition of a stack

It is a linear list where:

- One end is called top
- Other end is called bottom

Additionally, adds and removes are at the top end only.

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What is a stack?

First

Stores a set of elements in a particular order.

Second

Stack principle: LAST IN FIRST OUT = LIFO

Meaning

It means: the last element inserted is the first one to be removed

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Example in Real Life

Stack of Coins



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Example

Insert the following items into a stack

List = {A, B, C, D, E}

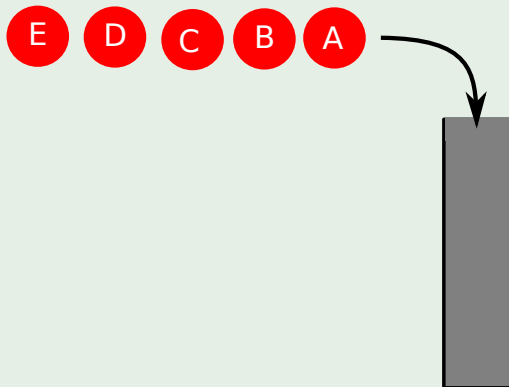
Example

List = {A, B, C, D, E}



Example

List = {A, B, C, D, E}, Push A



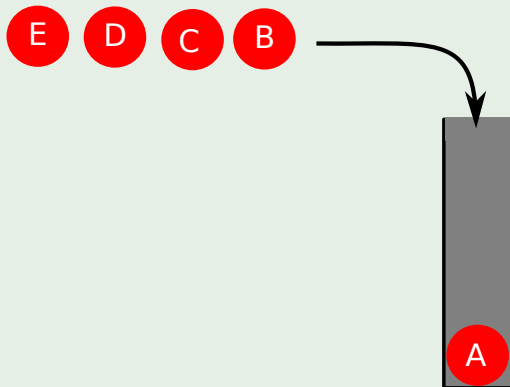
Example

List = {B, C, D, E}



Example

List = {B, C, D, E}, Push B



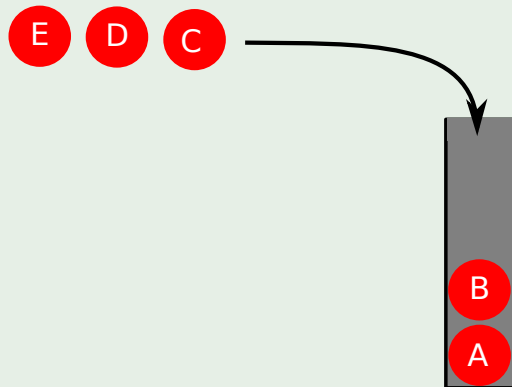
Example

List = {C, D, E}



Example

List = {C, D, E}, Push C



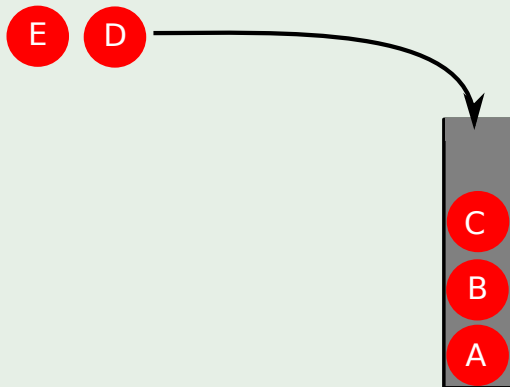
Example

List = {D, E}



Example

List = {D, E}, Push D



Example

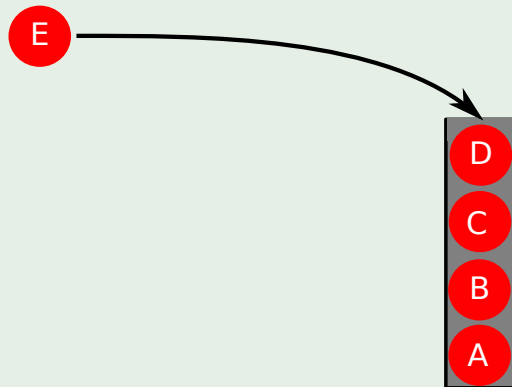
List = {E}

E

D
C
B
A

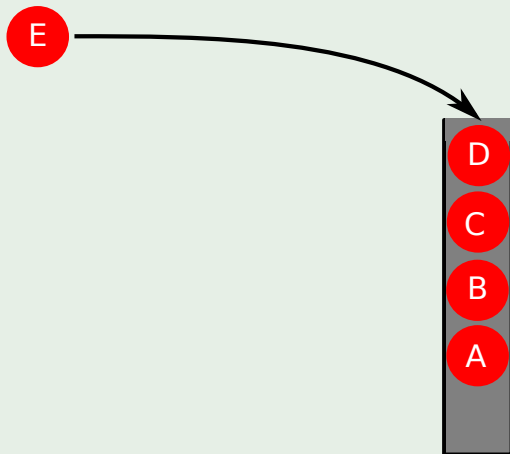
Example

List = {E}, Push E



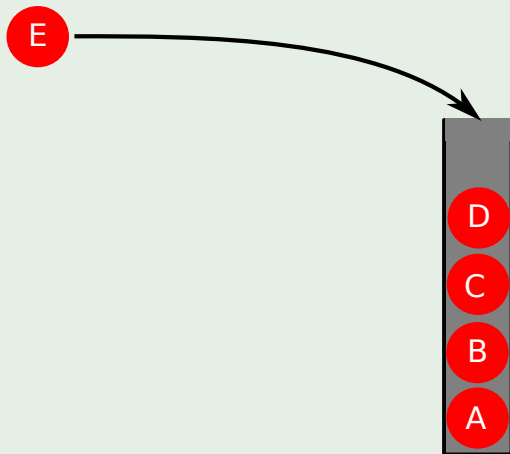
Example

List = {E}, No space!!! Make Space



Example

List = {E}, Push E



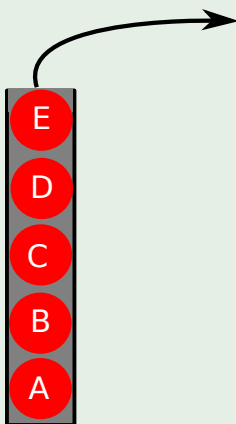
Example

List = {}



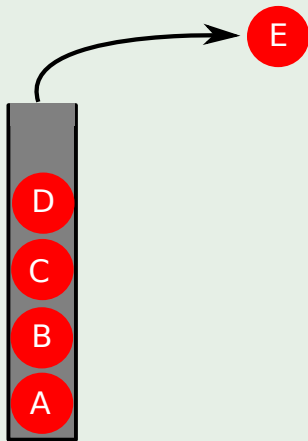
Example

List = {}, Pop



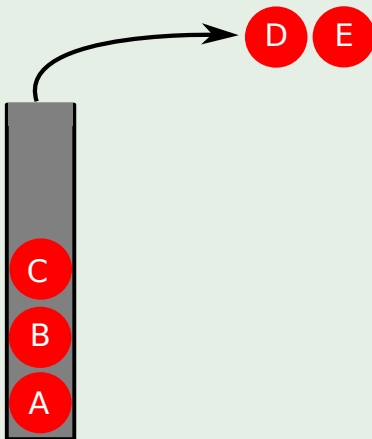
Example

List = {E}, Pop



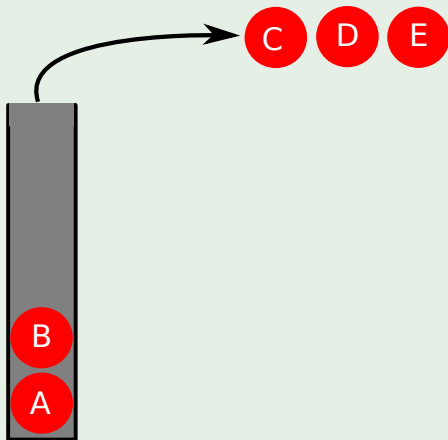
Example

List = {E,D}, Pop



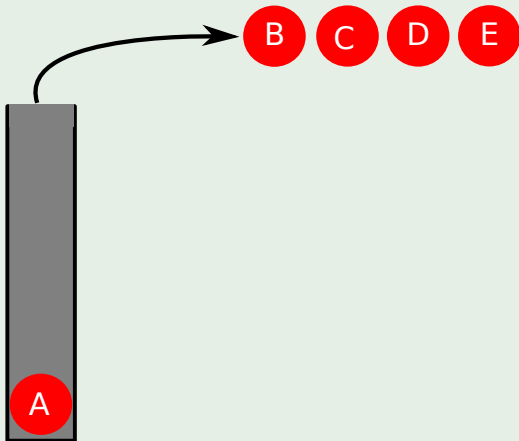
Example

List = {E,D,C}, Pop



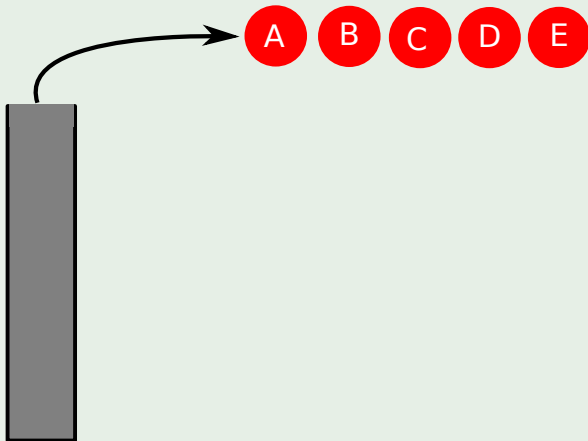
Example

List = {E,D,C,B}, Pop



Example

List = {E,D,C,B,A}



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- ADT

2

Examples

- Parentheses Matching

Stacks ADT

Interface

```
interface Stack
{
    public boolean empty();
    public Item peek();
    public void push(Object);
    public Item pop();
}
```

Explanation of the ADT I

`peek()`

This method allows to look at the top of the stack without removing it!!!

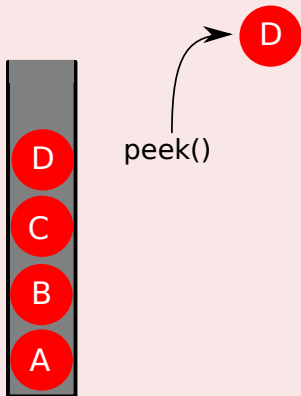
For Example

Explanation of the ADT I

`peek()`

This method allows to look at the top of the stack without removing it!!!

For Example



Explanation of the ADT II

`pop()`

This method allows to pop stuff from the top of the stack!!!

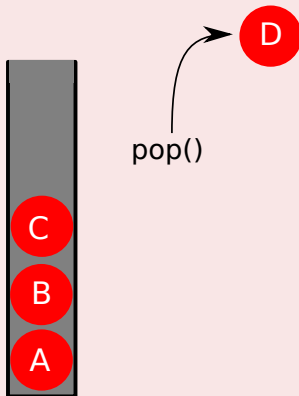
For Example

Explanation of the ADT II

`pop()`

This method allows to pop stuff from the top of the stack!!!

For Example



Explanation of the ADT III

`push()`

This method allows to push stuff to the top of the stack!!!

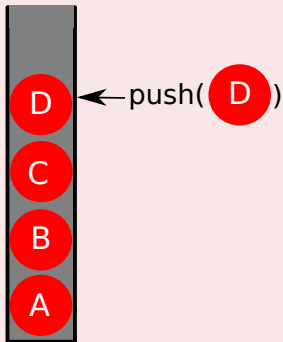
For Example

Explanation of the ADT III

`push()`

This method allows to push stuff to the top of the stack!!!

For Example



Explanation of the ADT III

`empty()`

This method allows to know if the stack is empty!!!

Stack Applications

Real life

- Pile of books
- Plate trays

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More applications related to computer science

- Program execution stack (You will know about this in OS or CA)
- Evaluating expressions

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What do we do now?

First

Instead of going toward the implementations!!!

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Why not examples?

- ① Parentheses Matching
- ② Towers Of Hanoi/Brahma
- ③ Switch Box Routing
- ④ Try-Throw-Catch in Java
- ⑤ Rat In A Maze

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Parentheses Matching

Example - Input

$((a+b)*c+d-e)/(f+g)-(h+j))$

(((<i>a</i>	+	<i>b</i>)	*	<i>c</i>	+	<i>d</i>	-	<i>e</i>)	/	...
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
(<i>f</i>	+	<i>g</i>)	-	(<i>h</i>	+	<i>j</i>))				
15	16	17	18	19	20	21	22	23	24	25	26				

Output

Output pairs (u,v) such that the left parenthesis at position u is matched with the right parenthesis at v .

Or

$(2,6)$ $(1,13)$ $(15,19)$ $(21,25)$ $(0,26)$

Parentheses Matching

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$((a+b)*c+d-e)/(f+g)-(h+j))$

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Wrong Matching

Input

$(a+b)*((c+d)$

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$(a+b))*((c+d)$

Output

❶ (0,4)

❷ WRONG!!! Right parenthesis at 5 has no matching left parenthesis

❸ (8,12)

❹ WRONG!!! Left parenthesis at 7 has no matching right parenthesis

Wrong Matching

Input

$(a+b))*((c+d)$

Output

- 1 (0,4)
- 2 **WRONG!!!** Right parenthesis at 5 has no matching left parenthesis
- 3 (8,12)
- 4 **WRONG!!!** Left parenthesis at 7 has no matching right parenthesis

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Developing a Recursive Solution I

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What do we do? Ideas

Look at this

What if we have $(a+b)$?

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What if we have $(a+b)$?

Initial Idea

```
boolean Rec-Paren(Chain List)
```

```
1 if (List.get(0)=='(')
```

```
    List.remove(0)
```

```
    return Rec-Paren(List)
```

Initial Idea

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```
1 if (List.get(0)=='(')
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```
1 List.remove(0)
```

```
2 return Rec-Paren(List)
```

Now

What else?

Initial Idea

boolean Rec-Paren(Chain List)

- 1 if (List.get(0)=='(')
 - 1 List.remove(0)
 - 2 return Rec-Paren(List)

Now

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boolean Rec-Paren(Chain List)

- 1 if (List.get(0)=='(')
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Now

What else?

Next Case

Cont...

```
2. else if (List.get(0)=='[0-9]|[+-]')
```

Next Case

Cont...

```
2. else if (List.get(0)=='[0-9][+-]')
```

```
    1 List.remove(0)
```

```
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```

Last Step

```
3. else if (List.get(0)=='')
```

```
    1 List.remove(0)
```

```
    2 return true
```

```
3. else return false
```

Next Case

Cont...

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 - ① List.remove(0)
 - ② return Rec-Paren(List)

Last Step

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- 3. else return false

Developing a Recursive Solution II

What if you have?

What if we have $(a+b)$?

What about

What if we have $a+b$)?

Developing a Recursive Solution II

What if you have?

What if we have $(a+b)$?

What about

What if we have $a+b$)?

This solution fails!!!

So, we need to send something down the recursion

What do we do? Ideas

What about...?

what if

We send down a flag!!

Thus

We need a flag to send down the recursion!!!

To tell the logic if we saw a left parenthesis

Ok

For simple problems fine!!! However...

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To tell the logic if we saw a left parenthesis

Ok

For simple problems fine!!! However...

Then

For problems like these ones

$((a+b)$

Nope

It does not work

Then

For problems like these ones

$((a+b))$

Nope

It does not work

We need something more complex

A counter!!!

To see how many “(” we have seen down the recursion!!!

Recursive Solution - You assume a list of characters

```
def Balanced(ChainLinearList List , int Counter):  
  
    if (len(List)==0): // Check empty  
    if (Counter == 0): // Check Counter  
        return True  
    else:  
        return False  
    if (List[0]=='('): // Case (  
        Counter++;  
        List.pop(0)  
        return Balanced(List , Counter)  
    elif (List.get(0)==')'): // Case )  
        if (Counter > 0):  
            Counter—  
            List.pop(0)  
            return Balanced(List , Counter)  
        else  
            return False:  
    else: // Case Number or -+  
        List.pop(0)  
        return Balanced(List , Counter)
```

Can we simplify our code?

Yes

Using this memory container the **STACK!!!**