Data Structures

Dictionaries

DataLab

November 19, 2016

Outline

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Dictionaries

Definition

The **ADT dictionary**—also called a map, table, or associative array—contains entries that each have two parts:

- A keyword—usually called a search key—such as an English word or a person's name
- A value—such as a definition, an address, or a telephone number—associated with that key

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Example

Dictionary with Duplicates

Pairs are of the form (word, meaning).

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More than a single meaning

- (bolt, a threaded pin)
- (bolt, a crash of thunder)
- (bolt, to shoot forth suddenly)
- (bolt, a gulp)
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- etc.

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Thus

We have possibly in a dictionary

- Sorted keys
- Duplicate keys

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Operation in the ADT dictionary

Common operations with most databases

- **insert** adds a new entry to the dictionary, given a search key and associated value.
- **delete** removes an entry, given its associated search key
- **retrieve** retrieves the value associated with a given search key
- **search** sees whether the dictionary contains a given search key
- **traverse**
  - It traverse all the search keys in the dictionary
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We have the following extra operations

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Specifications: Add

Pseudocode

add(key, value)

Task

It adds the pair (key , value) to the dictionary.

Output : None

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Input and Output

Input: key is an object search key, value is an associated object.

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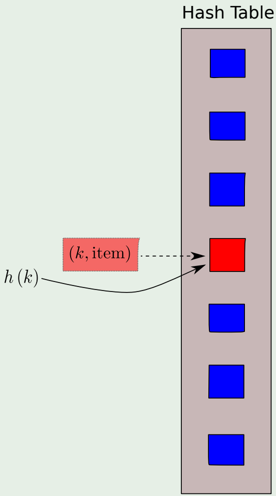
Adding something



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Specifications: Remove

Pseudocode

remove(key)

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It removes from the dictionary the entry that corresponds to a given search key.

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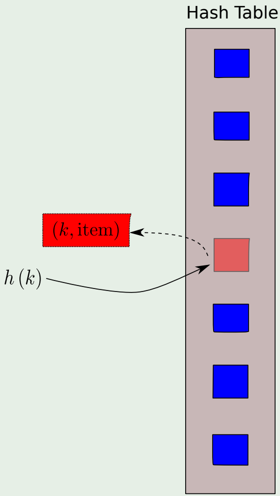

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Other Operations

isEmpty()

It sees whether the dictionary is empty.

getSize()

It gets the size of the dictionary.

clear()

It removes all entries from the dictionary.

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Distinct search keys

Case 1 You can refuse to add another key-value.

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- The methods **remove** and **getValue** must deal with multiple entries that have the same search key.
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Interface

We have the following interface

```
interface DictionaryInterface
{
  add(k, Item);
  remove(k);
  getValue(k);
  contains(k);
  getKeyIterator();
  getValueIterator();
  isEmpty();
  getSize();
  clear();
}
```

Outline

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Where we can use this ADT?

In the phone directory problem

It is a directory that uses a name as the key and adds and returns a phone number

For example

Name	Number
Suzanne Nouveaux	401-555-1234
Andres Mendez-Vazquez	301-123-2345
...	...

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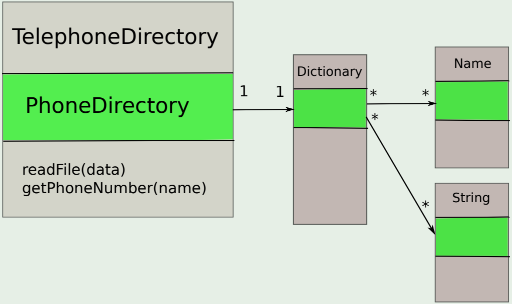
Thus, we have the following diagram

A class Diagram



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Now, the Big Question

It is a big one

How do we implement this data structure?

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Possible ways

- Linear List
- Skip List
- Hash Tables
- ...

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First: Represent It As A Linear List

You have

$L = (e_0, e_1, ..., e_{n-1})$

Where

Each e_i is a pair (key, element).

We can use the following representations

Array or linked representation.

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Array Representation

Our classic array

b	c	d	e	a										
0	1	2	3	4	5	6								

We have then

Operation in Array Representation	Complexity
getValue(theKey)	O(size)
add(theKey, theItem)	O(size) to find duplicate
	O(1) to add at right end
remove(theKey)	O(size)

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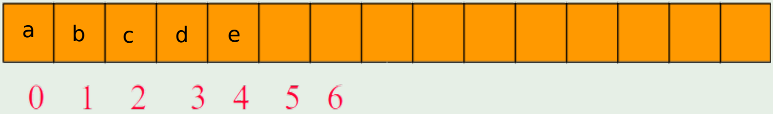
What if we sort the array?

Sorted Keys



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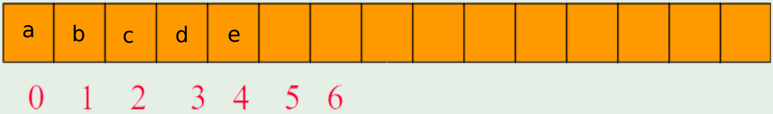

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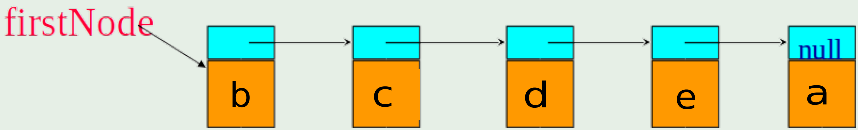
Unsorted Chain

Our Structure



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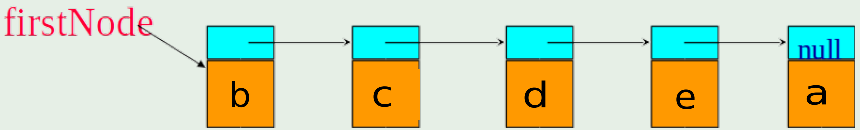

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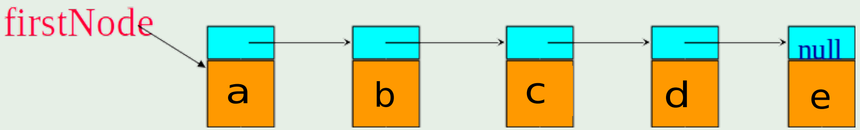

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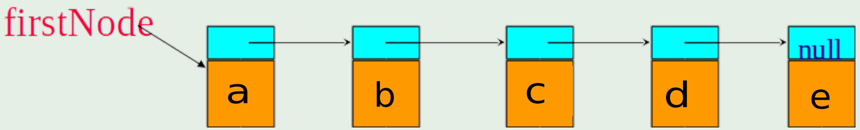

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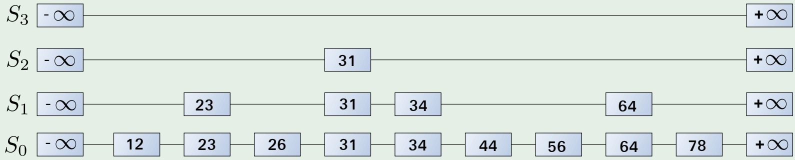

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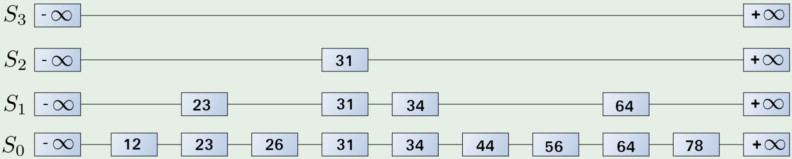

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We will concentrate our efforts in the Hash Tables

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- A hash table or hash map T is a data structure, most commonly an array, that uses a hash function to efficiently map certain identifiers of keys (e.g. person names) to associated values.

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You have two cases for this data structure

First

Small universe of keys.

Second

Large number of keys

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We can do the following

- Key values are direct addresses in the array.
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Then, it is impractical to store a table of the size of $|U|$.

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$$h : U \rightarrow \{0, 1, ..., m-1\} \tag{1}$$

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Example

Imagine that you have

A 1D array (or table) table$[0 : m-1]$.

Thus

$h(k)$is the home bucket for key k.

Then

Every dictionary pair (key, Item) is stored in its home bucket table$[h[key]]$.

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Every dictionary pair (key, Item) is stored in its home bucket table$[h[key]]$.

Example

Imagine that you have

A 1D array (or table) table$[0 : m - 1]$.

Thus

$h(k)$is the home bucket for key k.

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Every dictionary pair (key, Item) is stored in its home bucket table$[h[key]]$.

Example

Push the following pairs in a hash table of size $m = 8$

(22,a), (33,c), (3,d), (73,e), (85,f).

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First Issue

The choice of the possible hash function.

Second

The collision handling method

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The size (number of buckets) at the hash table

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Outline

Hash Functions

They have two parts

1. The conversion of the key into an integer in the case the key is not an integer.
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Analysis of hashing: Which hash function?

Consider that:

Good hash functions should maintain the property of simple uniform hashing!

- The keys have the same probability $1/m$ to be hashed to any bucket!!!
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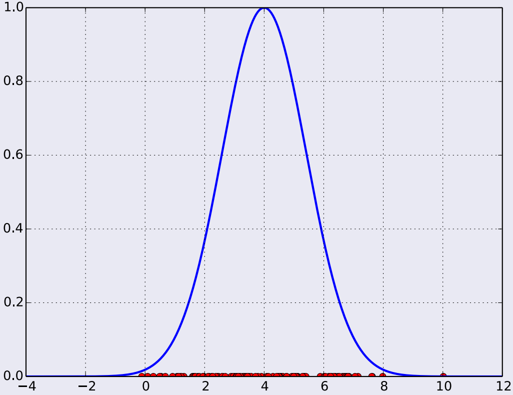
What if...?

Question:

What about something with keys in a normal distribution?



Hashing By Division

Universe of keys

keySpace = all integers.

Thus, we have that

For every m, the number of integers that get mapped (hashed) into bucket i is approximately $2^{32}/m$.

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Odd number and m an even number

Odd integers hash into odd home buckets

- $15\%14 = 1$, $3\%14 = 3$, $23\%14 = 9$

and m an even number

Even integers into even home buckets.

- $20\%14 = 6$, $30\%14 = 2$, $8\%14 = 8$

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Another Problem

Similar biased distribution of home buckets is seen, in practice, when the divisor is a multiple of prime numbers such as 3, 5, 7, . . .

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The effect of each prime divisor p of m decreases as p gets larger.

Rules of Choosing m

- Ideally, choose m so that it is a prime number.
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Universal Hashing

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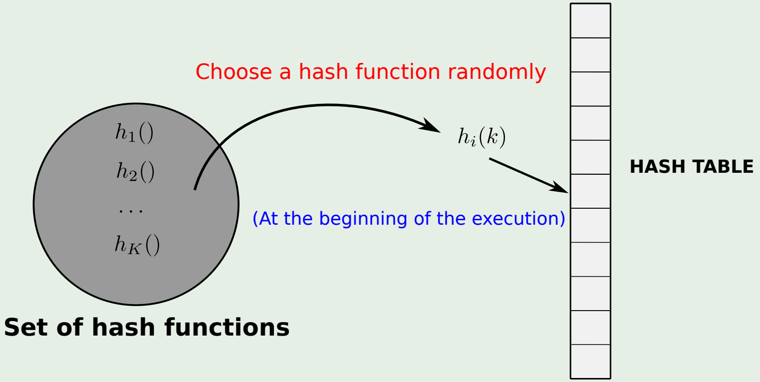

Example of Universal Hash

Proceed as follows:

- Choose a primer number p large enough so that every possible key k is in the range $[0, ..., p-1]$

$$\mathbb{Z}_p = \{0, 1, ..., p-1\} \text{and } \mathbb{Z}_p^* = \{1, ..., p-1\}$$

- Define the following hash function:

$$h_{a,b}(k) = ((ak+b) \mod p) \mod m, \forall a \in Z_p^* \text{and } b \in Z_p$$

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Example: Universal hash functions

Example

- $p = 977$, $m = 50$, a and b random numbers
  - $h_{a,b}(k) = ((ak + b) \mod p) \mod m$

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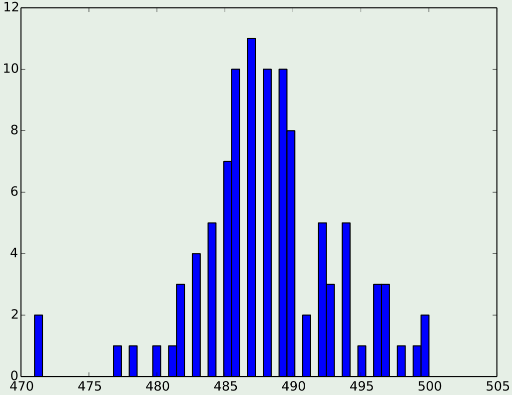
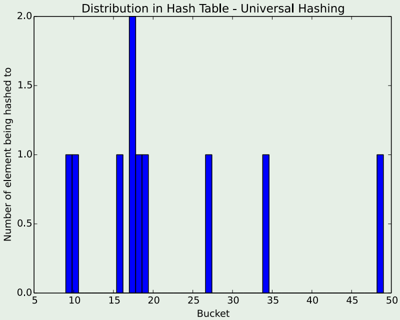
Example of key distribution

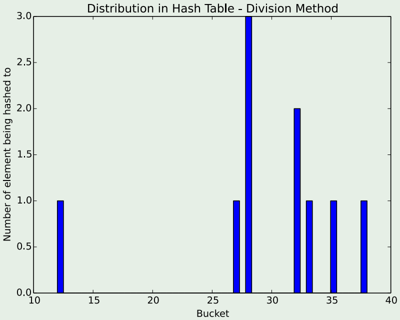
Example, mean = 488.5 and dispersion = 5



Example with 10 keys

Universal Hashing Vs Division Method





Example with 50 keys

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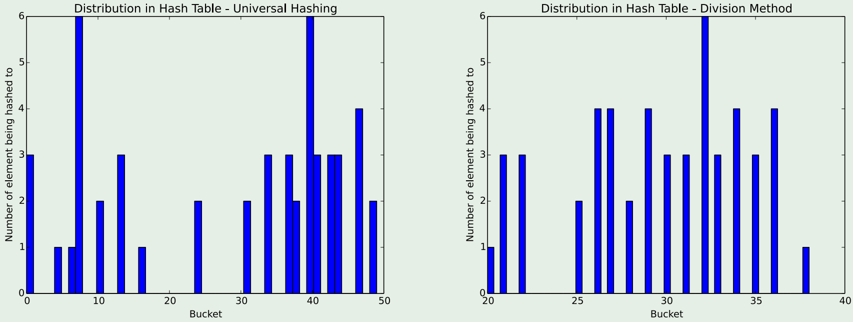

Example with 100 keys

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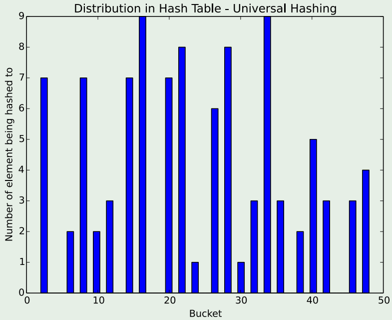

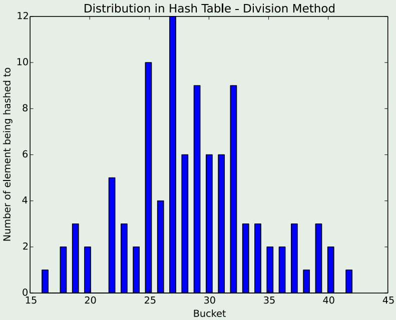

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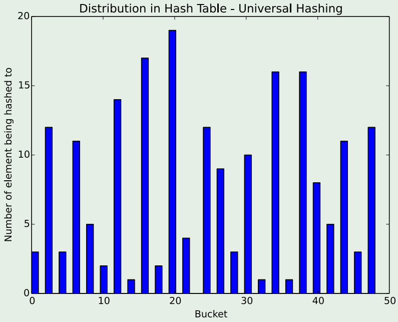

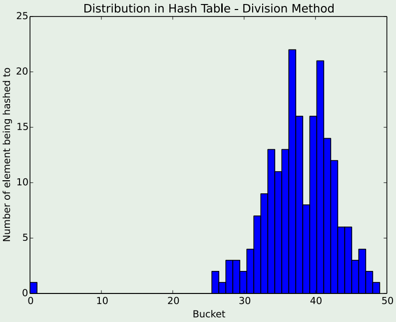

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Outline

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Overflow

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- Quadratic probing.
- Random probing.

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Linear List Of Synonyms

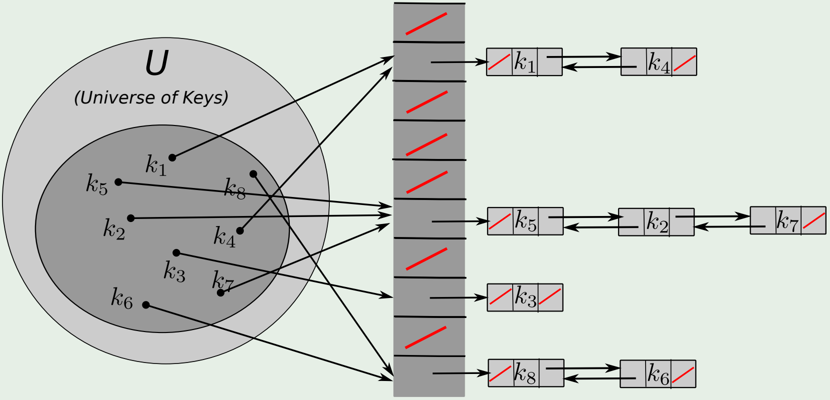
Thus

- Each bucket keeps a linear list of all pairs for which it is the home bucket.
- The linear list may or may not be sorted by key.
- The linear list may be an array linear list or a chain.

Collision Handling: Chaining

A Possible Solution

Insert the elements that hash to the same slot into a linked list.



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Example Sorted Chains

Add to a hash table with $m = 11$

Put in pairs whose keys are 6, 17, 12, 23, 28, 5, 16, 3, 8

So, we have

Home bucket = key % 11.

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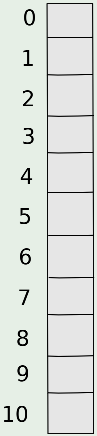

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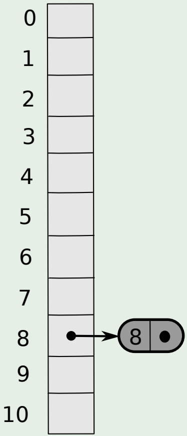

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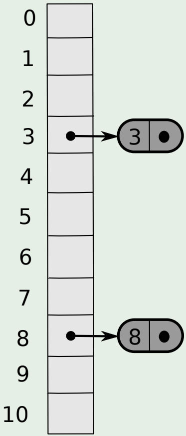
The Table

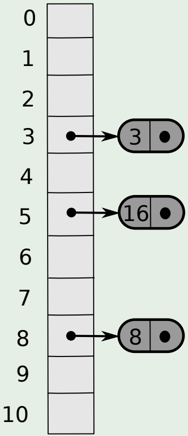
6, 17, 12, 23, 28, 5, 16



Example

The Table

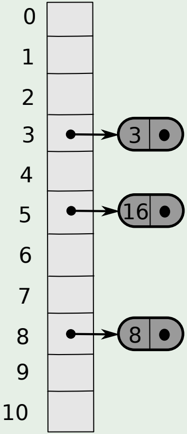
6, 17, 12, 23, 28, 5



Example

The Table

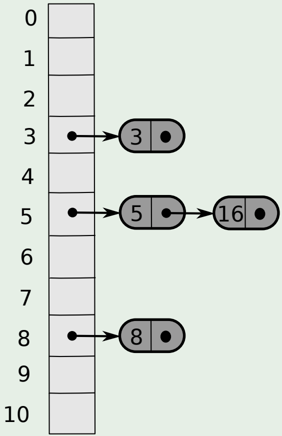
6, 17, 12, 23, 28, 5



Example

The Table

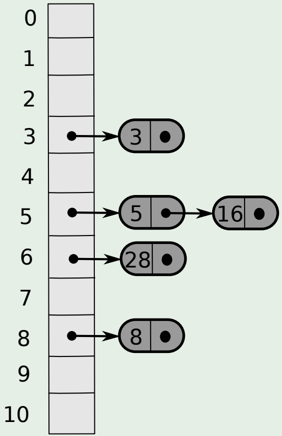
6, 17, 12, 23, 28



Example

The Table

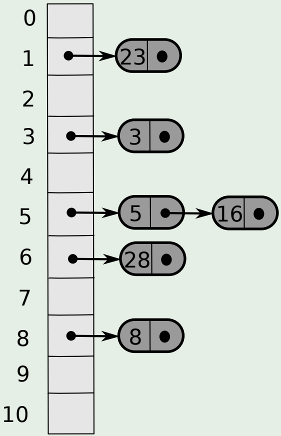
6, 17, 12, 23



Example

The Table

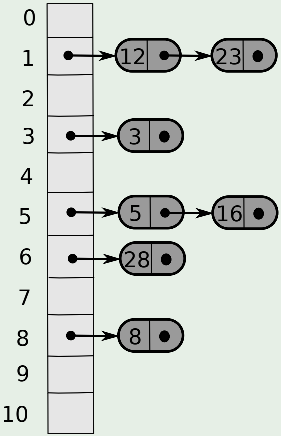
6, 17, 12



Example

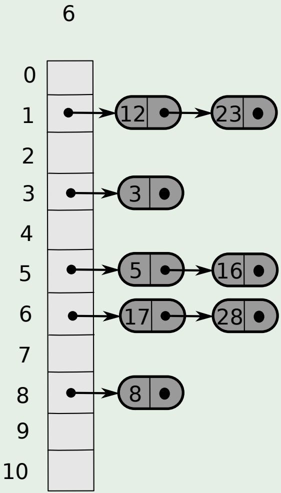
The Table

6, 17



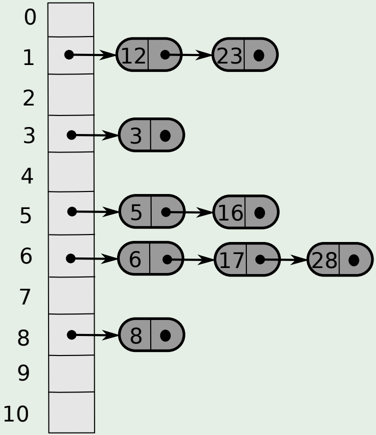
Example

The Table



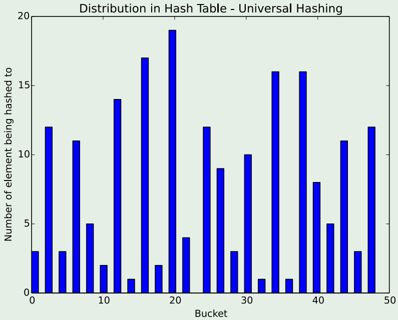
Example

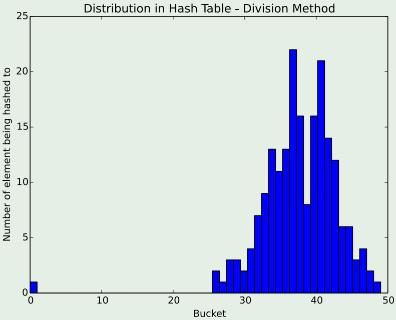
The Table



Do You Remember This?

Universal Hashing Vs Division Method





DataLab
Data Science Community

Expected Complexity of Hash Table under Chaining

We have for unsuccessful search

$$U_n = O\left(1 + \alpha\right) \tag{2}$$

We have for successful search

$$S_n = O\left(1 + \alpha\right) \tag{3}$$

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