Data Structures

Dictionaries

DataLab

November 12, 2016

Outline

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Outline

Dictionaries

Definition

The **ADT dictionary**—also called a map, table, or associative array—contains entries that each have two parts:

- A keyword—usually called a search key—such as an English word or a person's name
- A value—such as a definition, an address, or a telephone number—associated with that key

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Example

Dictionary with Duplicates

Pairs are of the form (word, meaning).

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More than a single meaning

- (bolt, a threaded pin)
- (bolt, a crash of thunder)
- (bolt, to shoot forth suddenly)
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Thus

We have possibly in a dictionary

- Sorted keys
- Duplicate keys

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Operation in the ADT dictionary

Common operations with most databases

- **insert** adds a new entry to the dictionary, given a search key and associated value.
- **delete** removes an entry, given its associated search key
- **retrieve** retrieves the value associated with a given search key
- **search** sees whether the dictionary contains a given search key
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  - It traverse all the search keys in the dictionary
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Specifications: Add

Pseudocode

add(key, value)

Task

It adds the pair (key , value) to the dictionary.

Output : None

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Input: key is an object search key, value is an associated object.

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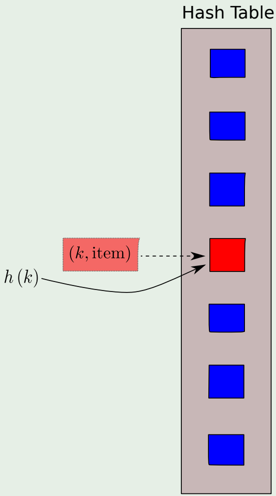

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Specifications: Remove

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It removes from the dictionary the entry that corresponds to a given search key.

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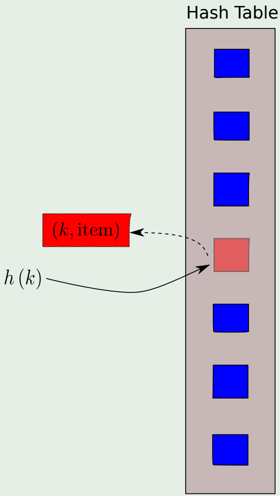

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Other Operations

isEmpty()

It sees whether the dictionary is empty.

getSize()

It gets the size of the dictionary.

clear()

It removes all entries from the dictionary.

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Distinct search keys

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Interface

We have the following interface

```
interface DictionaryInterface
{
  add(k, Item);
  remove(k);
  getValue(k);
  contains(k);
  getKeyIterator();
  getValueIterator();
  isEmpty();
  getSize();
  clear();
}
```

Outline

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Where we can use this ADT?

In the phone directory problem

It is a directory that uses a name as the key and adds and returns a phone number

For example

Name	Number
Suzanne Nouveaux	401-555-1234
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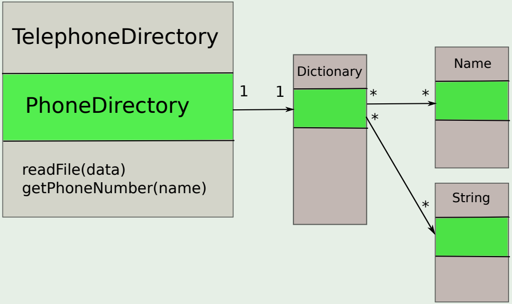
Thus, we have the following diagram

A class Diagram



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Now, the Big Question

It is a big one

How do we implement this data structure?

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Possible ways

- Linear List
- Skip List
- Hash Tables
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First: Represent It As A Linear List

You have

$L = (e_0, e_1, ..., e_{n-1})$

Where

Each e_i is a pair (key, element).

We can use the following representations

Array or linked representation.

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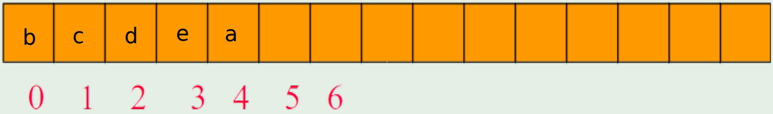

We have then

Operation in Array Representation	Complexity
getValue(theKey)	O(size)
add(theKey, theItem)	O(size) to find duplicate
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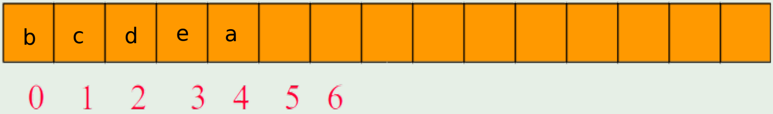

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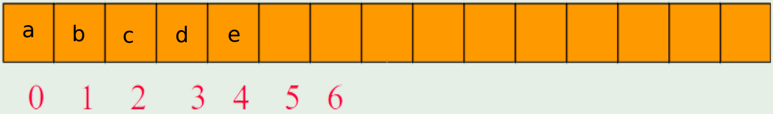

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Unsorted Chain

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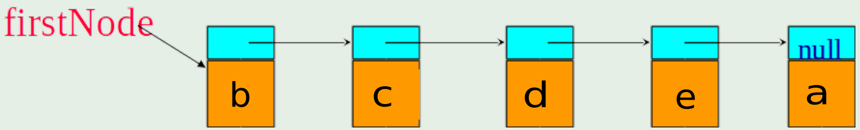

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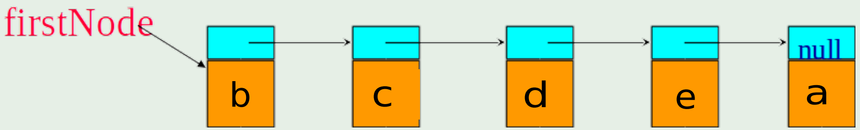

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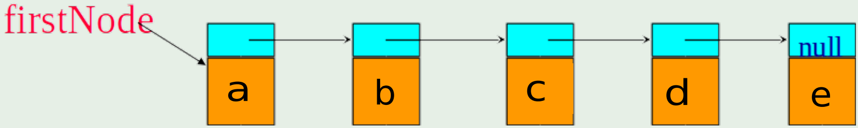

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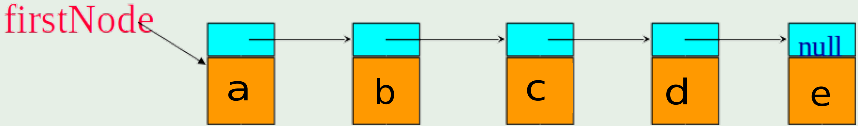

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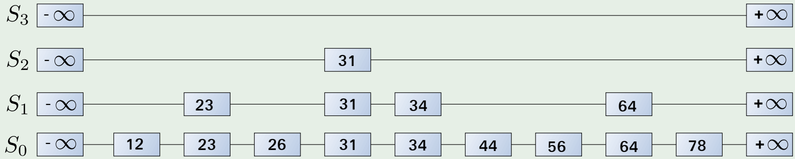

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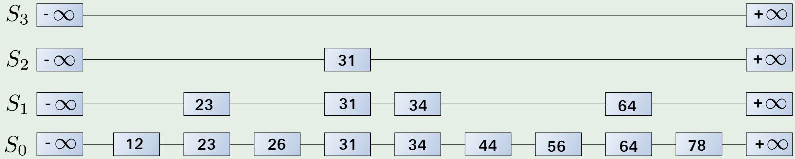

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- A hash table or hash map T is a data structure, most commonly an array, that uses a hash function to efficiently map certain identifiers of keys (e.g. person names) to associated values.

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You have two cases for this data structure

First

Small universe of keys.

Second

Large number of keys

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- Key values are direct addresses in the array.
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Example

Imagine that you have

A 1D array (or table) table$[0 : m-1]$.

Thus

$h(k)$is the home bucket for key k.

Then

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First Issue

The choice of the possible hash function.

Second

The collision handling method

Third

The size (number of buckets) at the hash table

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Hash Functions

They have two parts

1. The conversion of the key into an integer in the case the key is not an integer.
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Analysis of hashing: Which hash function?

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Good hash functions should maintain the property of simple uniform hashing!

- The keys have the same probability $1/m$ to be hashed to any bucket!!!
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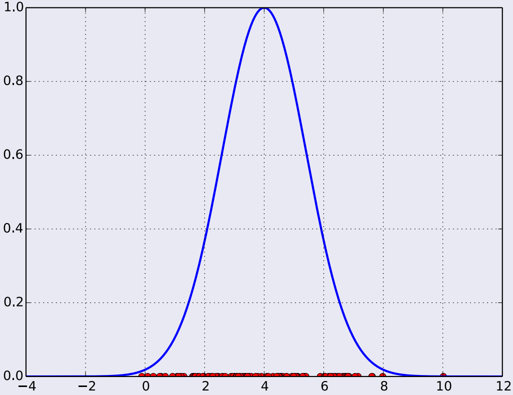
Question:

What about something with keys in a normal distribution?



Hashing By Division

Universe of keys

keySpace = all integers.

Thus, we have that

For every m, the number of integers that get mapped (hashed) into bucket i is approximately $2^{32}/m$.

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Odd number and m an even number

Odd integers hash into odd home buckets

- $15\%14 = 1$, $3\%14 = 3$, $23\%14 = 9$

and m an even number

Even integers into even home buckets.

- $20\%14 = 6$, $30\%14 = 2$, $8\%14 = 8$

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The effect of each prime divisor p of m decreases as p gets larger.

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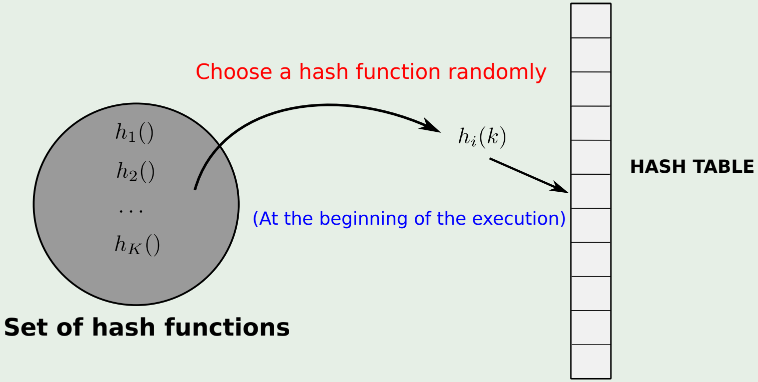
Universal Hashing

Example



Example of Universal Hash

Proceed as follows:

- Choose a primer number p large enough so that every possible key k is in the range $[0, ..., p-1]$

$$\mathbb{Z}_p = \{0, 1, ..., p-1\} \text{and } \mathbb{Z}_p^* = \{1, ..., p-1\}$$

- Define the following hash function:

$$h_{a,b}(k) = ((ak+b) \mod p) \mod m, \forall a \in Z_p^* \text{and } b \in Z_p$$

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Example: Universal hash functions

Example

- $p = 977$, $m = 50$, a and b random numbers
  - $h_{a,b}(k) = ((ak + b) \mod p) \mod m$

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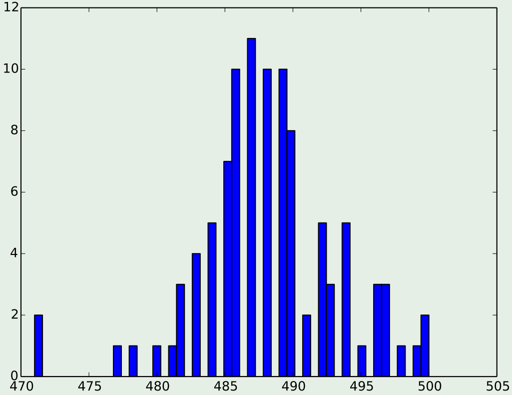
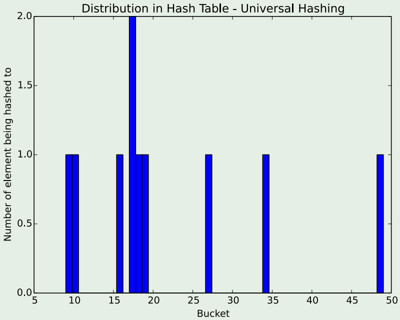
Example of key distribution

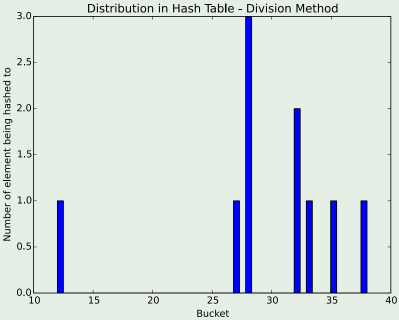
Example, mean $=$ 488.5 and dispersion $=$ 5



Example with 10 keys

Universal Hashing Vs Division Method





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Example with 50 keys

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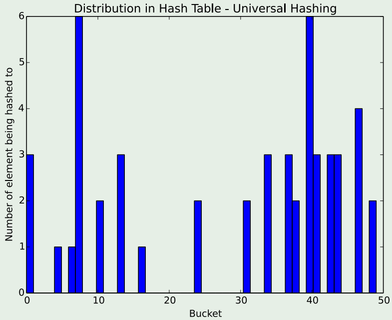

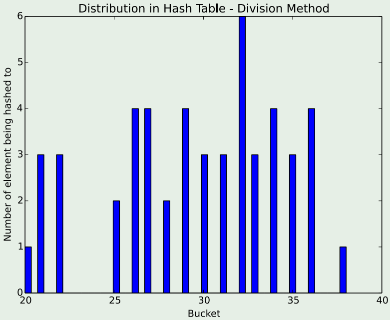

Example with 100 keys

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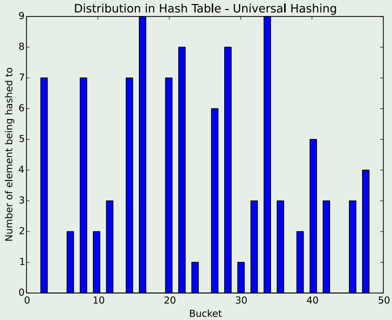

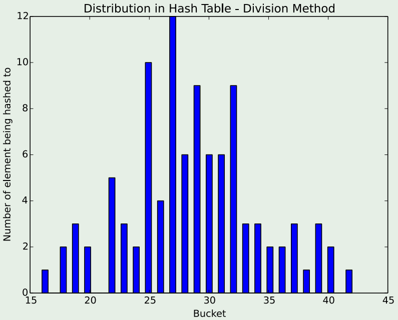

Example with 200 keys

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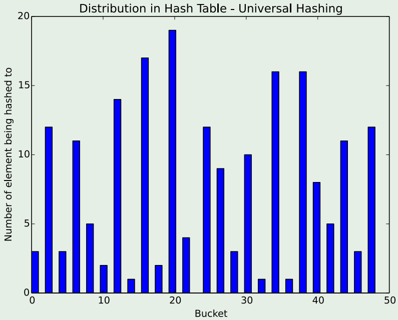

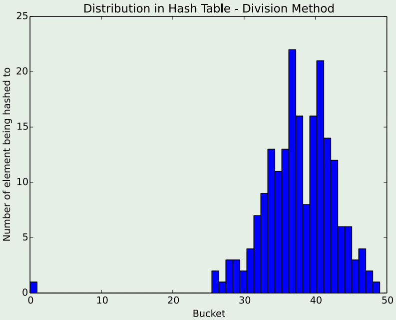

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An overflow occurs when the home bucket for a new pair (key, element) is full.

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- Quadratic probing.
- Random probing.

- Array linear list.
- Chain.

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Linear List Of Synonyms

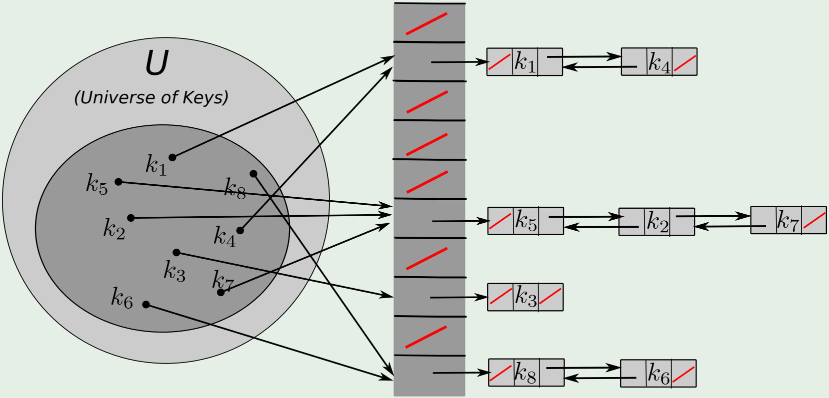
Thus

- Each bucket keeps a linear list of all pairs for which it is the home bucket.
- The linear list may or may not be sorted by key.
- The linear list may be an array linear list or a chain.

Collision Handling: Chaining

A Possible Solution

Insert the elements that hash to the same slot into a linked list.



Example Sorted Chains

Add to a hash table with $m = 11$

Put in pairs whose keys are 6, 17, 12, 23, 28, 5, 16, 3, 8

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The Table

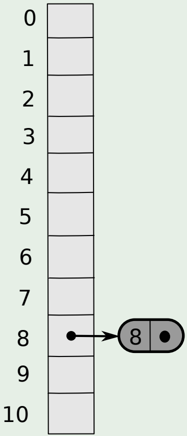
6, 17, 12, 23, 28, 5, 16, 3, 8

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Example

The Table

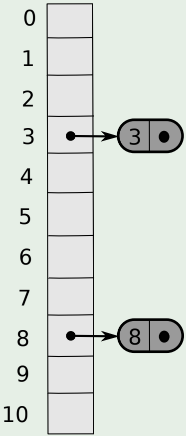
6, 17, 12, 23, 28, 5, 16, 3



Example

The Table

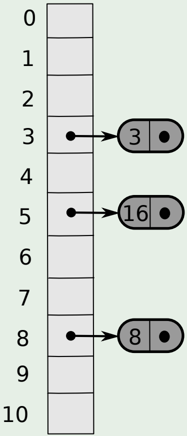
6, 17, 12, 23, 28, 5, 16



Example

The Table

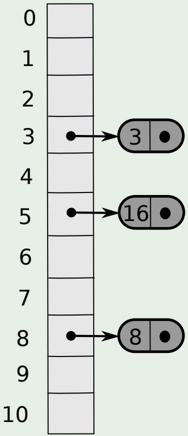
6, 17, 12, 23, 28, 5



Example

The Table

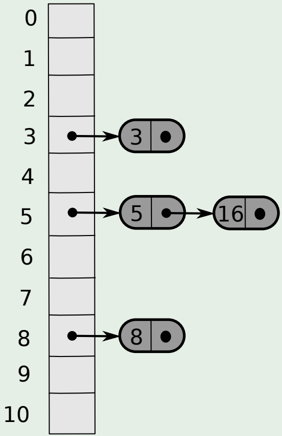
6, 17, 12, 23, 28, 5



Example

The Table

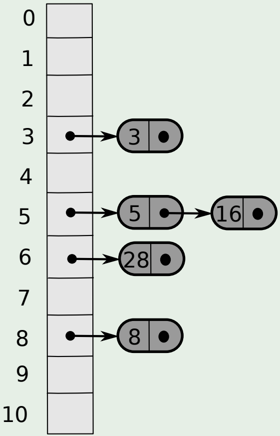
6, 17, 12, 23, 28



Example

The Table

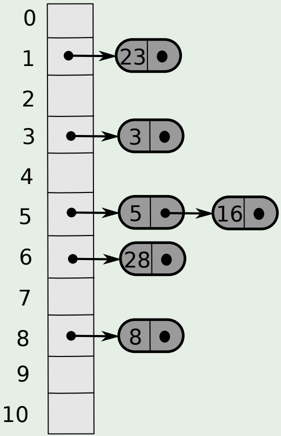
6, 17, 12, 23



Example

The Table

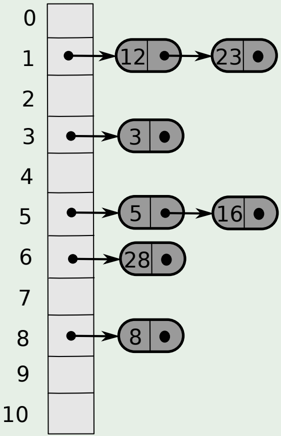
6, 17, 12



Example

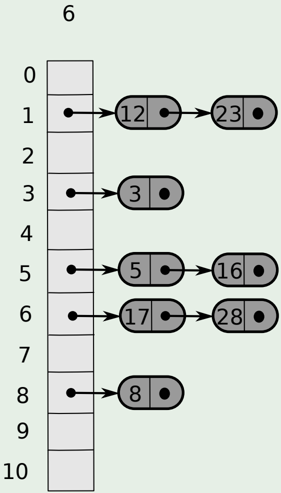
The Table

6, 17



Example

The Table



Example

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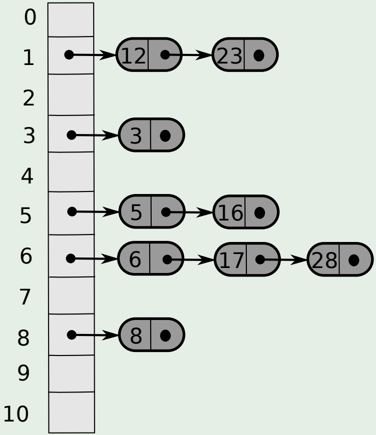
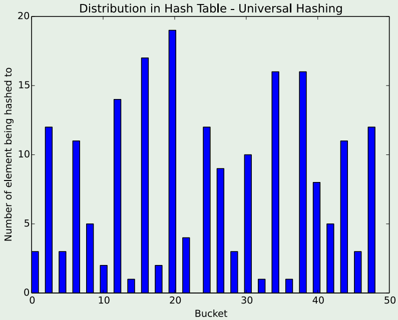
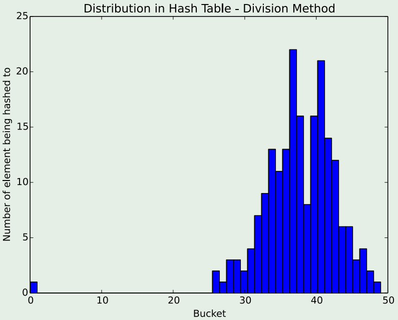


Do You Remember This?

Universal Hashing Vs Division Method





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Expected Complexity of Hash Table under Chaining

We have for unsuccessful search

$$U_n = O\left(1 + \alpha\right) \tag{2}$$

We have for successful search

$$S_n = O\left(1 + \alpha\right) \tag{3}$$

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Now, Back to the real world

java.util.Hashtable

- It uses unsorted chains.
- It uses a default initial m = divisor = 101
- It uses a default $\alpha \leq 0.75$
- When loading density exceeds a max permissible threshold, It rehash with new $m = 2m+1$.

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