

Alexander von Humboldt Stiftung/Foundation



POORMAN'S REVIEW ABOUT: CI - NUDGED ELASTIC BAND

King Chun Lai

lai@fhi-berlin.mpg.de



Microkinetics Modeling

Typical approach studying long term behavior

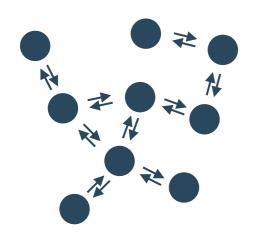
Constructing a Reaction Network



Microkinetic Model



Long Term Behavior



e.g.
Kinetic Monte Carlo (kMC)
Numerical Master Equation

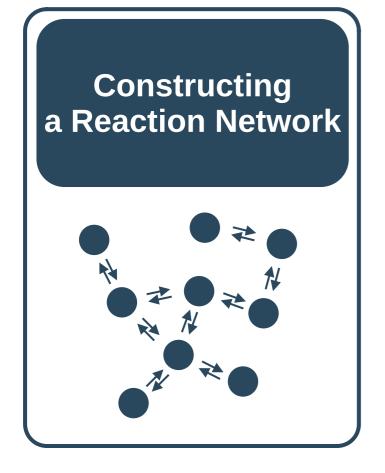
e.g.

Turnover Frequency
Life-time of Nanostructure
Morphological Change



Microkinetics Modeling

Typical approach studying long term behavior



Microkinetic Model

e.g.
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Numerical Master Equation

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Turnover Frequency
Life-time of Nanostructure
Morphological Change

Long Term Behavior



What you have before NEB:

1) A pair of optimized metastable states
Obtained from some sampling
Handwritten



2) A potential energy surface (PES)

Classical force field (EAM)
Machine learned potential energy (MACE, GAP ...)



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What you have before NEB:

1) A pair of optimized metastable states
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Classical force field (EAM)
Machine learned potential energy (MACE, GAP ...)



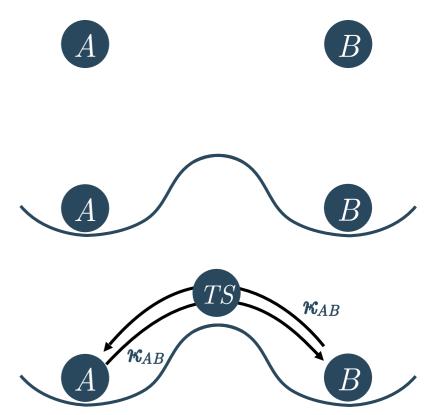
Transition State (TS) on the reaction path linking i,j

What you usually do with that TS:

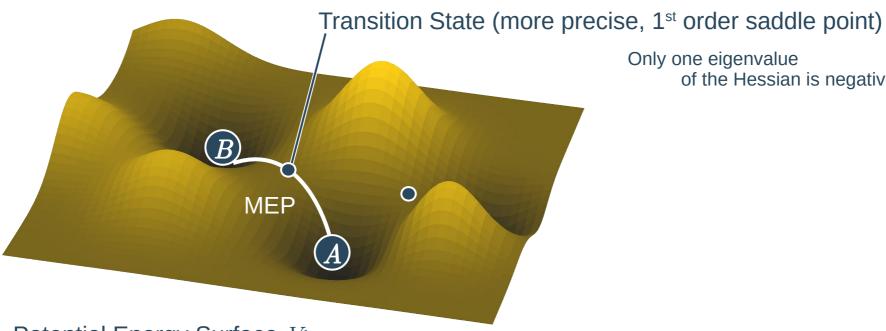
Model the kinetic rate κ_{AB} , κ_{AB} (back-and-forth)

Usually simple Arrhenius form

$$\kappa = \nu \exp\left(-\Delta E_{\rm act}/k_B T\right)$$







Only one eigenvalue of the Hessian is negative

Potential Energy Surface *V*:

Practically 3*N*-Dimension $R_{\theta < m \le 3N}$

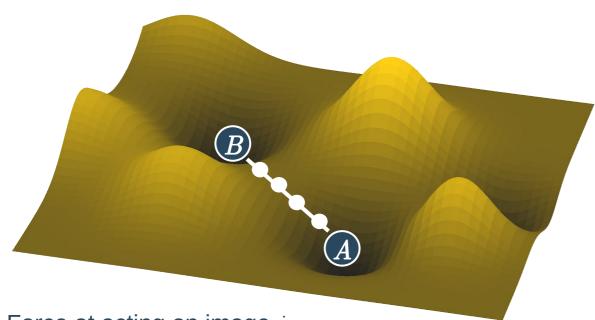
N: Number of atoms free to move

Minimum Energy Path (MEP):

The most energetically favorable path linking A,B

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Force at acting on image i

$$\mathbf{F}_i = -\nabla V(\mathbf{R}) + \mathbf{F}_i^{\mathrm{band}}$$

$$F_i^{\text{band}} = k \left(\mathbf{R}_{i+1} - \mathbf{R}_i \right) + k \left(\mathbf{R}_{i-1} - \mathbf{R}_i \right)$$

To get MEP:

1) Guess a path, typically straight line



2) Put some state-images in between

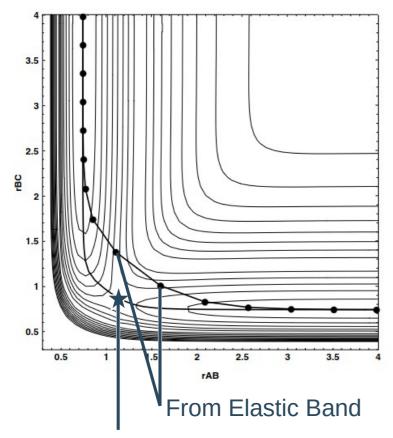


3) Put springs between the 'beads' (Each spring has a spring constant *k*)



4) Optimize image-states on the band





Real Target

- i) The cuts the corner
- ii) It doesn't give the maximum along the band

To get MEP:

...

3) Put springs between the 'beads' (Each spring has a spring constant *k*)

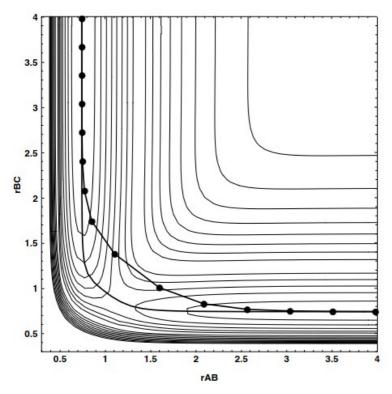


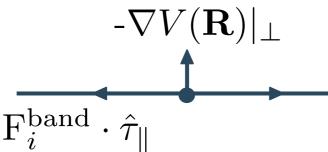
4) Optimize image-states on the band Force at acting on image i

$$\mathbf{F}_i = -\nabla V(\mathbf{R}) + \mathbf{F}_i^{\mathrm{band}}$$

$$F_i^{\text{band}} = k \left(\mathbf{R}_{i+1} - \mathbf{R}_i \right) + k \left(\mathbf{R}_{i-1} - \mathbf{R}_i \right)$$







To get MEP:

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3) Put springs between the 'beads' (Each spring has a spring constant k)

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$$F_i^{\text{band}} = k \left(\mathbf{R}_{i+1} - \mathbf{R}_i \right) + k \left(\mathbf{R}_{i-1} - \mathbf{R}_i \right)$$

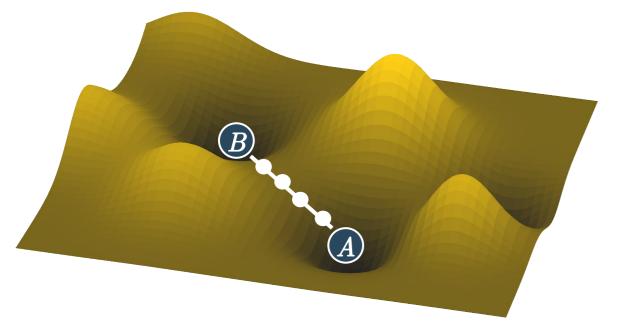
$$\mathbf{F}_{i}^{\mathrm{NEB}} = -\nabla V(\mathbf{R})|_{\perp} + \left(\mathbf{F}_{i}^{\mathrm{band}} \cdot \hat{\tau}_{\parallel}\right) \hat{\tau}_{\parallel}$$

Spring contributes only along the band PES contributes perpendicular to the band



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Climbing Image Nudged Elastic Band Overview



To get MEP:

. . .

3) Put springs between the 'beads' (Each spring has a spring constant *k*)

Force at acting on image i

$$\mathbf{F}_{i}^{\mathrm{NEB}} = -\nabla V(\mathbf{R})|_{\perp} + \left(\mathbf{F}_{i}^{\mathrm{band}} \cdot \hat{\tau}_{\parallel}\right) \hat{\tau}_{\parallel}$$

4) After a few NEB steps:

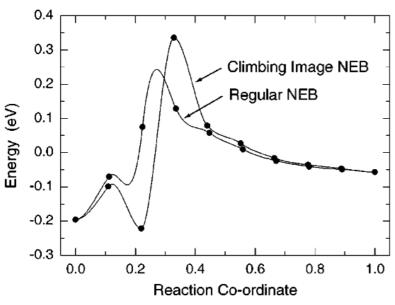
Special treatment for the bead with highest energy

$$F_{i_{\max}}^{\text{ciNEB}} = -\nabla V(\mathbf{R}_{\mathbf{i}_{\max}})|_{\perp} + \nabla V(\mathbf{R}_{\mathbf{i}_{\max}})|_{\parallel}$$

- i) No effect from the band
- ii) Invert the force from PES along the band



Climbing Image Nudged Elastic Band Overview



Henkelman, G., Uberuaga, B. P., & Jónsson, H. J. Chem. Phys. 113.22 (2000): 9901-9904.

To get MEP:

...

3) Put springs between the 'beads' (Each spring has a spring constant *k*)



Force at acting on image i

$$\mathbf{F}_{i}^{\mathrm{NEB}} = -\nabla V(\mathbf{R})|_{\perp} + \left(\mathbf{F}_{i}^{\mathrm{band}} \cdot \hat{\tau}_{\parallel}\right) \hat{\tau}_{\parallel}$$

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Special treatment for the bead with highest energy

$$\mathbf{F}_{i_{\max}}^{\text{ciNEB}} = -\nabla V(\mathbf{R}_{\mathbf{i}_{\max}})|_{\perp} + \nabla V(\mathbf{R}_{\mathbf{i}_{\max}})|_{\parallel}$$

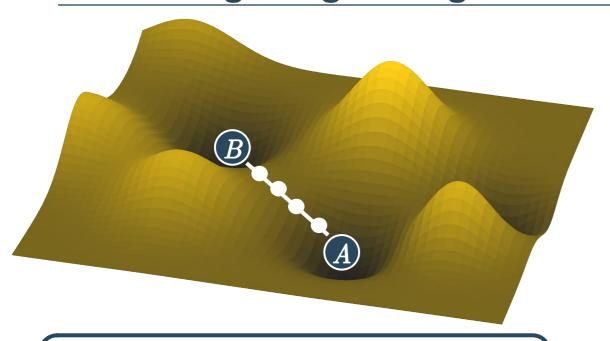
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6



6

Climbing Image Nudged Elastic Band Overview



When generating the initial guess of the band: Besides trivial interpolation of AB, here is a better way Smidstrup, S., Pedersen, A., Stokbro, K., & Jónsson, H. J. Chem. Phys., 140(21) (2014).

The band is a set of straight lines linking the beads! How to estimate "tangential"/"perpendicular" direction G. Henkelman and H. Jonsson,

J. Chem. Phys. 113, 9978 (2000)

To get MEP:

...

3) Put springs between the 'beads' (Each spring has a spring constant *k*)

Force at acting on image i

$$\mathbf{F}_{i}^{\mathrm{NEB}} = -\nabla V(\mathbf{R})|_{\perp} + \left(\mathbf{F}_{i}^{\mathrm{band}} \cdot \hat{\tau}_{\parallel}\right) \hat{\tau}_{\parallel}$$

4) After a few NEB steps:

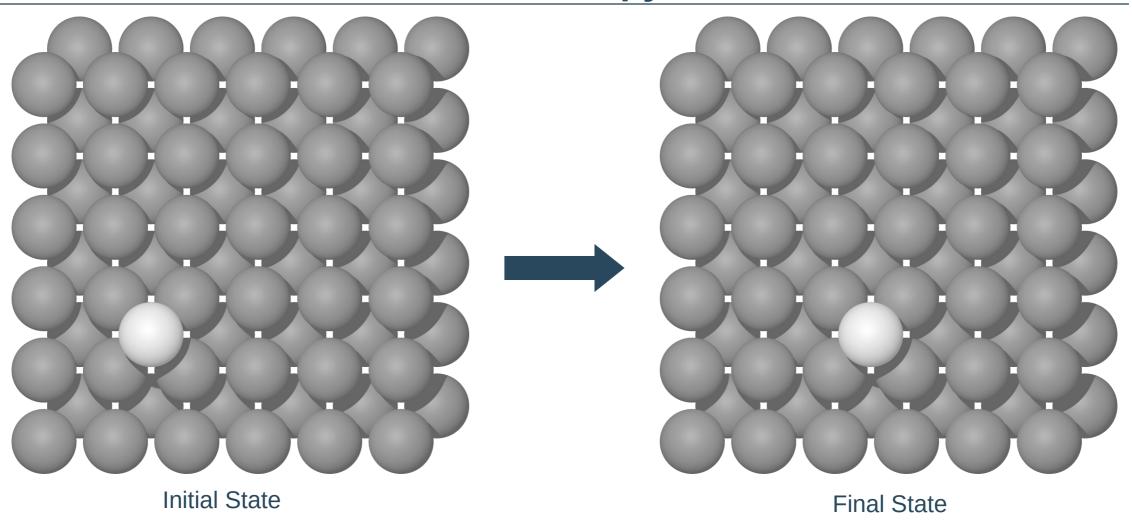
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$$F_{i_{\text{max}}}^{\text{ciNEB}} = -\nabla V(\mathbf{R}_{\mathbf{i}_{\text{max}}})|_{\perp} + \nabla V(\mathbf{R}_{\mathbf{i}_{\text{max}}})|_{\parallel}$$

- i) No effect from the band
- ii) Invert the force from PES along the band



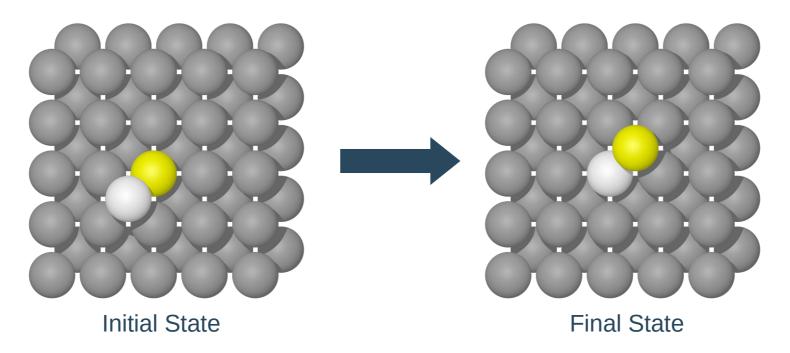
Demo Time Please Look at the Jupyter Notebook



The Simplest Example: Direct Hop of Adatom on FCC(100) Pd With Classical Force Field (EAM)



Your Turn! Exercise 1: Exchange Terrace Diffusion



Consider the exchange terrace diffusion on Pd(100) surface (all atoms shown are Pd)

- 1) Perform ciNEB and find the energy barrier and prefactors in both directions
- 2) Plot the energy on the minimal energy pathway from ciNEB

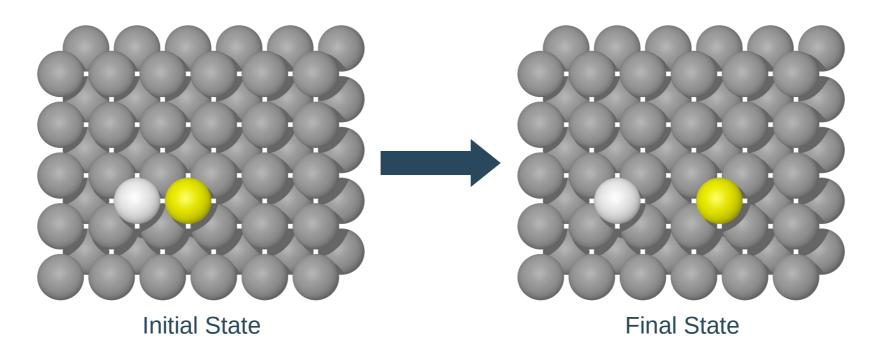
CAUTION:

It is an exchange process, white atom is squeezing out the yellow atom. NOT the white atom jumping to the diagonal site!

When you calculate the vibrational modes, which atoms should be included???



Your Turn! Exercise 2: Bond Breaking Direct Hop?



Consider the bond breaking process on Pd(100) surface (all atoms shown are Pd)

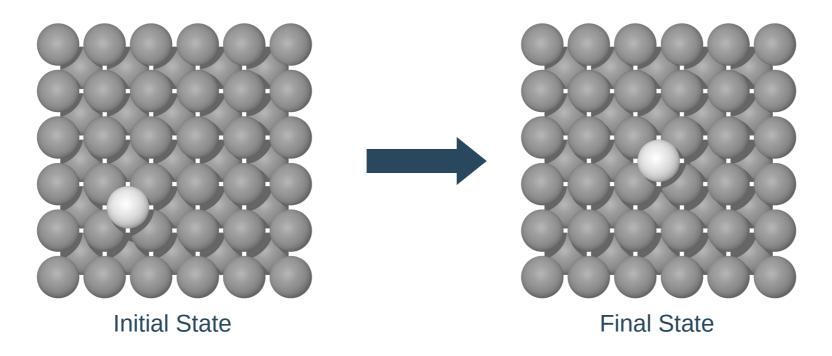
- 1) Perform ciNEB and find the energy barrier and prefactors in both directions
- 2) Plot the energy on the minimal energy pathway from ciNEB

CAUTION:

Take a closer look at the MEP plot.



Your Turn! Exercise 3: Hmmm



What about this process, if you perform ciNEB, take a closer look at the results. Why is it not physical? How would you model this instead???

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Footnote

Think about it again!

In general,

You want to calculate energy (construct PES) with a different force field /potential What to do?

You may identify which atoms to include in the vibrational mode calculation Do you want to do this identification by-hand every time?

Here the initial and final states are given, no matter they are physical or not What structures / systems you are going to consider is up to you. How you organize the work flow to generate these?

If you are choosing other tools instead of ASE, the idea of doing it is basically the same. How you substitute each step with the tool(s) you choose?