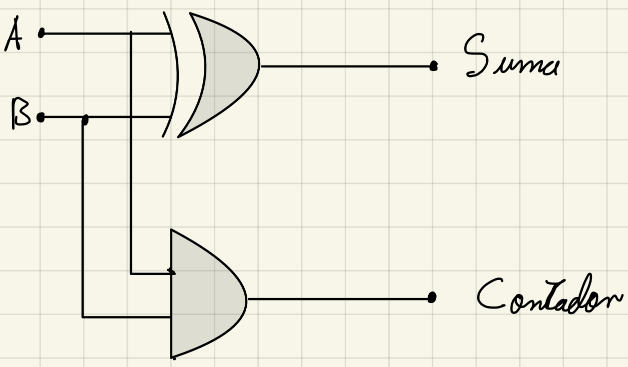
Sumador Cuantico

Ejemplo:

$$\begin{array}{r} 10111 \\ +\ 1010 \\ \hline 1 \end{array} \qquad \begin{array}{r} {\scriptstyle 1}\ \ \\ 10111 \\ +\ 1010 \\ \hline 01 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1}\ \ \\ 10111 \\ +\ 1010 \\ \hline 001 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1}\ \ \\ 10111 \\ +\ 1010 \\ \hline 0001 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1\,1}\ \ \\ 10111 \\ +\ 1010 \\ \hline 00001 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1\,1}\ \ \\ 10111 \\ +\ 1010 \\ \hline 100001 \end{array}$$

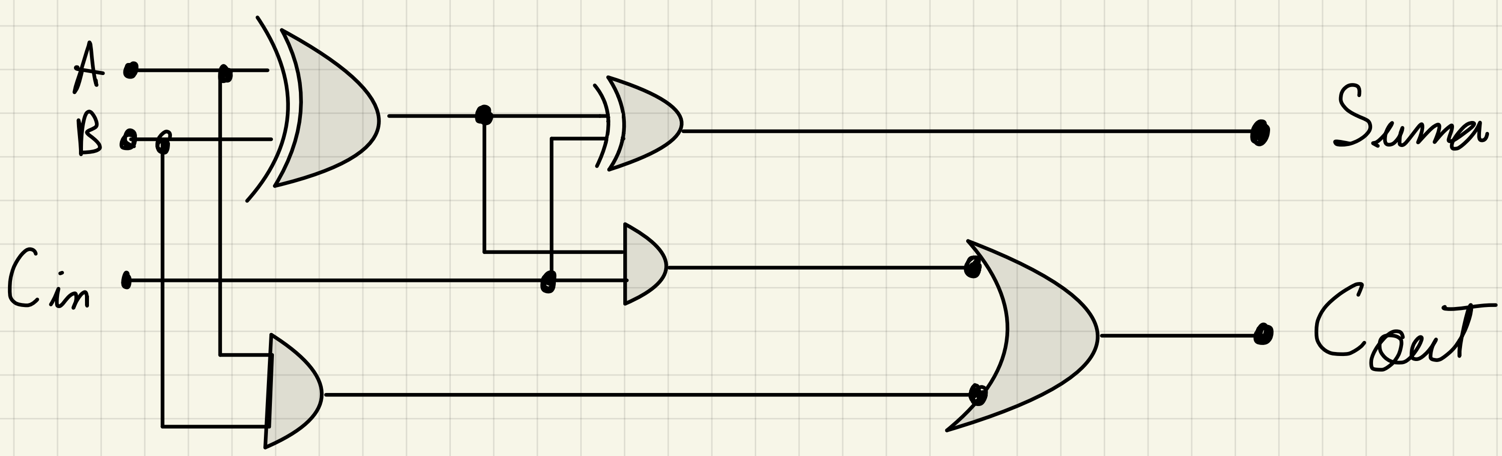
El sumador parcial clasico



A	B	A(XOR)B
0	0	0
0	1	1
1	0	1
1	1	0

A	B	A(AND)B
0	0	0
0	1	0
1	0	0
1	1	1

El sumador total clasico



Suma

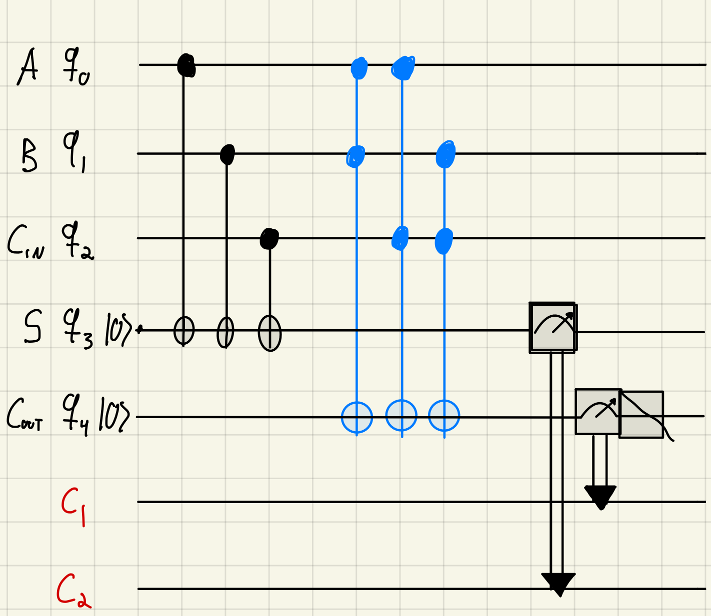
Cin	B	A	Cin	A(XOR)B	Cin (XOR) [A(XOR)B]
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	1	0	1

Cout

Cin	B	A	A(AND)B	Cin	A(XOR)B	Cin (AND) [A(XOR)B]	[A(AND)B] (OR) [Cin (AND) [A(XOR)B]]
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	1	0	0
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	0
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	0	0	1

Cin	B	A	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

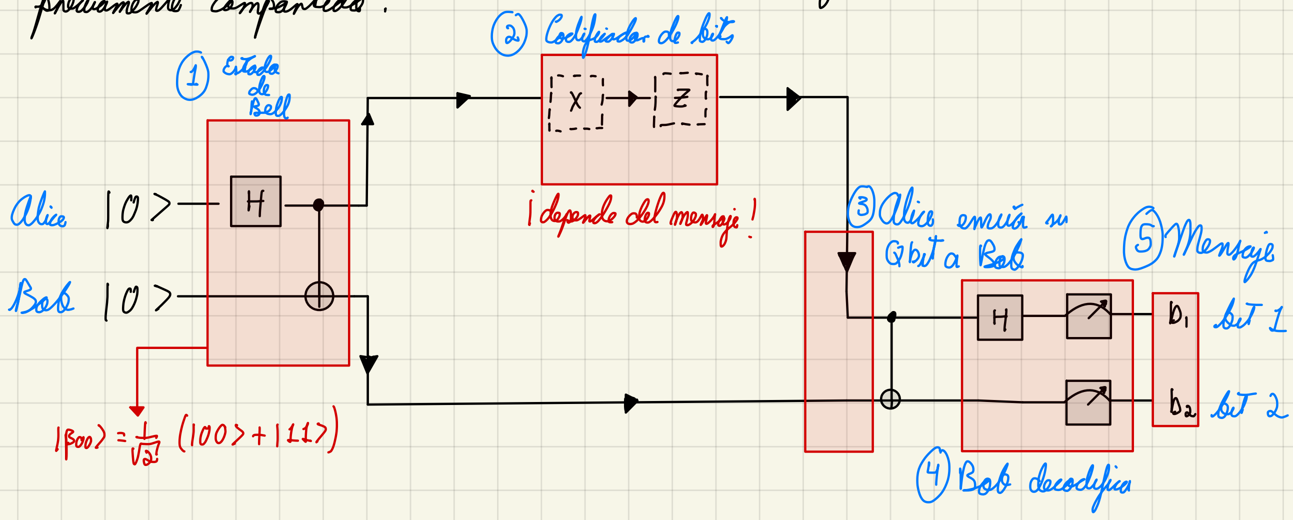
¿Como Podemos expesarlo en computo cuántico?



C_{IN}	B	A	C_{OUT}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Superdense Coding (1992)

El superdense coding es un protocolo mediante el cual se transmiten dos bits clasicos de información utilizando solo un qbit y un estado entrelazado previamente compartido.



1.- Se inicializa con un estado entrelazado compartido entre el

Emisor (Alice)

Receptor (Bob)

$$|0\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$|\psi\rangle = CNOT\left(\hat{H}|0\rangle \otimes \hat{I}|0\rangle\right) = \overbrace{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\1&0&0&0\end{pmatrix}}^{CNOT} \left[\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\1&0&0&0\end{pmatrix}\left[\begin{pmatrix}1\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = |\beta_{00}\rangle$$

2.- Alice Codifica   3.- Decodificar

Alice quiere enviar un mensaje a Bob (00, 01, 10, 11).

Para esto, Alice realiza una tranformación a su Qbit según el mensaje que quiera enviar

a) Para enviar 00, Alice aplica $\hat{I}$. i.e. no hace nada

$$\hat{I}|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

decodificamos

$$(\hat{H}\otimes\hat{I})\, CNOT\, |\beta_{00}\rangle = \frac{1}{2}(\hat{H}\otimes\hat{I})\left(|00\rangle + |10\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\0\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} = |00\rangle$$

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b) Para enviar 01, Alice aplica $\hat{X}$. i.e. no hace nada

$$\hat{X}\otimes\hat{I}|\beta_{00}\rangle = \left[\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

decodificamos

$$\frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\, CNOT\left(|01\rangle + |10\rangle\right) = \frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\, CNOT\left(|01\rangle + |10\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\left(|01\rangle + |11\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0\\2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = |01\rangle$$

c) Para enviar 10, Alice aplica $\hat{Z}$.

$$\hat{Z}\otimes\hat{I}\,|B_{00}\rangle = \left[\begin{pmatrix}1&0\\0&-1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|00\rangle-|11\rangle\right)$$

decodificamos

$$\frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\,CNot\left(|00\rangle-|11\rangle\right) = \frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\,CNot\left(|00\rangle-|10\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\left(|00\rangle-|10\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0\\0\\2\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \underline{|10\rangle}$$

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d) Para enviar 11, Alice aplica $\hat{Z}\hat{X}$. i.e. no hace nada

$$(\hat{Z}\otimes\hat{I})(\hat{X}\otimes\hat{I})\,|B_{00}\rangle = \left[\begin{pmatrix}1&0\\0&-1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]\left[\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|01\rangle-|10\rangle\right)$$

decodificamos

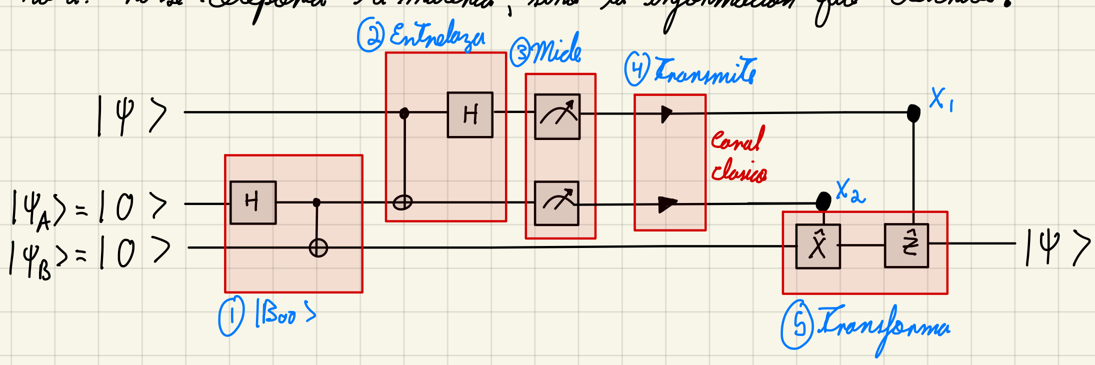
$$\frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\,CNot\left(|01\rangle-|10\rangle\right) = \frac{1}{\sqrt{2}}(\hat{H}\otimes\hat{I})\,CNot\left(|01\rangle-|10\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\left(|01\rangle-|11\rangle\right) = \frac{1}{2}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0\\0\\0\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \underline{|11\rangle}$$

Teleportación

El algoritmo de teleportación permite transferir el estado cuántico de una partícula ("mensaje") a otra partícula previamente entrelazada.

Nota: No se teleporta la materia, sino la información que describe.



1.- Preparar Estado de Bell entre Alice y Bob

2.- Alice entrelaza su partícula del estado de Bell con otra partícula en estado $|\psi\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\theta\rangle = |\psi\rangle|\beta_{00}\rangle = |\psi\rangle \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = (\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

Aplicamos CNoT con QBit (3) como control y QBit (2) como target

$$CNoT \otimes \hat{I}\,|\theta\rangle = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

Aplicamos Hadamard al QBit 3

Obs 1.- $\hat{H} = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)$

Obs 2.- $\left.\begin{matrix}\langle 0|0\rangle = 1\\ \langle 1|1\rangle = 1\end{matrix}\right\}$ escalares $\quad \left.\begin{matrix}\langle 0|1\rangle = 0\\ \langle 1|0\rangle = 0\end{matrix}\right\}$ escalares

$$\hat{H}\otimes\hat{I}\otimes\hat{I} = \left[\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)\otimes\hat{I}\otimes\hat{I}\right]\frac{1}{\sqrt{2}}\left[\alpha|0\rangle|00\rangle + \alpha|0\rangle|11\rangle + \beta|1\rangle|10\rangle + \beta|1\rangle|01\rangle\right]$$

$$= \frac{1}{2}\alpha\left[|0\rangle\underbrace{\langle 0|0\rangle}_{1}|00\rangle + |0\rangle\cancelto{0}{\langle 1|0\rangle}|00\rangle + |1\rangle\underbrace{\langle 0|0\rangle}_{1}|00\rangle - |1\rangle\cancelto{0}{\langle 1|0\rangle}|00\rangle\right]$$

$$+ \frac{1}{2}\alpha\left[|0\rangle\underbrace{\langle 0|0\rangle}_{1}|11\rangle + |0\rangle\cancelto{0}{\langle 1|0\rangle}|11\rangle + |1\rangle\underbrace{\langle 0|0\rangle}_{1}|11\rangle - |1\rangle\cancelto{0}{\langle 1|0\rangle}|00\rangle\right]$$

$$+ \frac{1}{2}\beta\left[|0\rangle\cancelto{0}{\langle 0|1\rangle}|10\rangle + |0\rangle\underbrace{\langle 1|1\rangle}_{1}|10\rangle + |1\rangle\cancelto{0}{\langle 0|1\rangle}|10\rangle - |1\rangle\underbrace{\langle 1|1\rangle}_{1}|10\rangle\right]$$

$$+ \frac{1}{2}\beta\left[|0\rangle\cancelto{0}{\langle 0|1\rangle}|01\rangle + |0\rangle\underbrace{\langle 1|1\rangle}_{1}|01\rangle + |1\rangle\cancelto{0}{\langle 0|1\rangle}|01\rangle - |1\rangle\underbrace{\langle 1|1\rangle}_{1}|01\rangle\right]$$

$$= \frac{1}{2}\left[\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle\alpha|0\rangle + |10\rangle\alpha|0\rangle + |01\rangle\alpha|1\rangle + |11\rangle\alpha|1\rangle + |01\rangle\beta|0\rangle - |11\rangle\beta|0\rangle + |00\rangle\beta|1\rangle - |10\rangle\beta|1\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\right]$$

$$= \frac{1}{2}\Big[\underbrace{|00\rangle(\alpha|0\rangle+\beta|1\rangle)}_{|\phi_1\rangle} + \underbrace{|10\rangle(\alpha|0\rangle-\beta|1\rangle)}_{|\phi_2\rangle} + \underbrace{|01\rangle(\alpha|1\rangle+\beta|0\rangle)}_{|\phi_3\rangle} + \underbrace{|11\rangle(\alpha|1\rangle-\beta|0\rangle)}_{|\phi_4\rangle}\Big]$$

3.- Alice mide sus dos partículas y comparte por un canal clasico la información a Bob

a) Alice: de $|\psi\rangle$ mide $X_1 = 0$
$|\psi_A\rangle$ mide $X_2 = 0$

Entonces Bob no aplica ninguna operación y el único estado posible es $|\phi_1\rangle$

$$\hat{I}|\psi_B\rangle = \hat{I}\big(\alpha|0\rangle+\beta|1\rangle\big) = \alpha|0\rangle+\beta|1\rangle = |\psi\rangle$$

b) Alice: de $|\psi\rangle$ mide $X_1 = 1$
$|\psi_A\rangle$ mide $X_2 = 0$

Entonces Bob aplica $\hat{Z}$ al único estado posible que es $|\phi_2\rangle$

$$\hat{Z}|\psi_B\rangle = \hat{Z}\big(\alpha|0\rangle-\beta|1\rangle\big) = \big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)\big(\alpha|0\rangle-\beta|1\rangle\big) = \alpha|0\rangle\underbrace{\langle 0|0\rangle}_{1} - \beta|0\rangle\cancel{\langle 0|1\rangle}^{0} - \alpha|1\rangle\cancel{\langle 1|0\rangle}^{0} + \beta|1\rangle\underbrace{\langle 1|1\rangle}_{1}$$
$$= \alpha|0\rangle + \beta|1\rangle = |\psi\rangle$$

c) Alice: de $|\psi\rangle$ mide $X_1 = 0$
$|\psi_A\rangle$ mide $X_2 = 1$

Entonces Bob aplica $\hat{X}$ al único estado posible que es $|\phi_3\rangle$

$$\hat{X}|\psi_B\rangle = \hat{X}\big(\alpha|1\rangle+\beta|0\rangle\big) = \alpha\hat{X}|1\rangle + \beta\hat{X}|0\rangle = \alpha|0\rangle+\beta|1\rangle = |\psi\rangle$$

d) Alice: de $|\psi\rangle$ mide $X_1 = 1$
$|\psi_A\rangle$ mide $X_2 = 1$

Entonces Bob aplica $\hat{Z}\hat{X}$ al único estado posible que es $|\phi_4\rangle$

$$\hat{Z}\hat{X}|\psi_B\rangle = \hat{Z}\hat{X}\big(\alpha|1\rangle-\beta|0\rangle\big) = \hat{Z}\big(\alpha\hat{X}|1\rangle-\beta\hat{X}|0\rangle\big) = \hat{Z}\big(\alpha|0\rangle-\beta|1\rangle\big) = \alpha\hat{Z}|0\rangle - \beta\hat{Z}|1\rangle$$
$$= \alpha|0\rangle - \beta(-1)|1\rangle = \alpha|0\rangle+\beta|1\rangle = |\psi\rangle$$