Escuela de invierno en computo cuántico

Centro de Estudios en Computación Avanzada CECAV

Posgrado en Ciencias Físicas

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¿Para qué necesitamos una computadora cuántica?

- A groso modo, un algoritmo se dice que es eficiente si corre en un tiempo polinomial, es decir, el tiempo que tarda en correr dicho algoritmo crece con el número de bits de entrada n de manera polinomial n^x, con x algún número entero.

- Se dice entonces que un problema computacionalmente difícil es aquel cuyo mejor algoritmo clásico que lo resuelve, lo hace de manera exponencial en el número n de bits de entrada, es decir, como a^n, con a alguna constante.

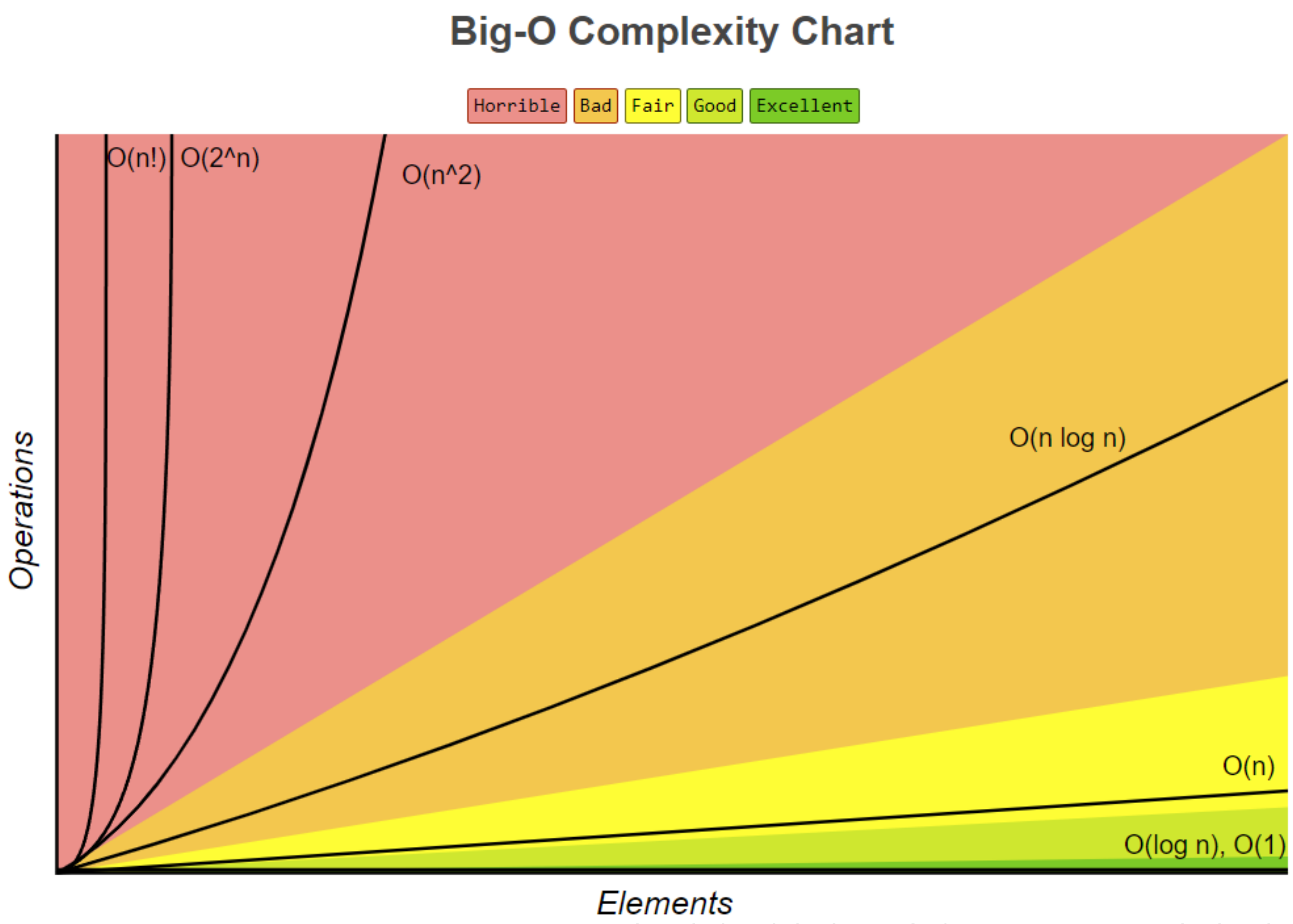


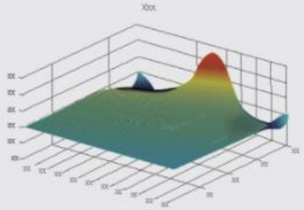
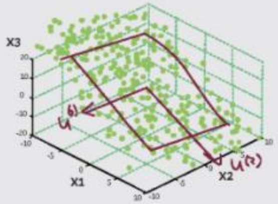
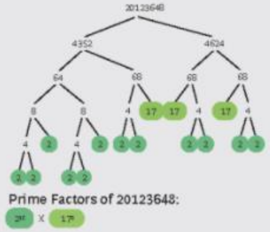
Figura tomada de: Big-O Algorithm Complexity Cheat Sheet (Know Thy Complexities!) @ericdrowell (bigocheatsheet.com)

¿Qué tipo de problemas se busca resolver con la computación cuántica ?

- Encriptación y seguridad
  - Factorización de números primos en protocolos como el RSA
  - Criptografía cuántica
- Problemas de optimización
  - Buscar la solución más óptima en cierto rango de parámetros
  - Logística, Meteorología, Finanzas
- Desarrollo de materiales
  - Simulación de sistemas cuánticos para determinar los elementos necesarios para obtener determinada característica
  - Química cuántica, Medicina, Biología
- Análisis de datos
  - Se estima que el 90% de toda la información producida por la humanidad, se ha creado en los últimos años. El contar con un gran poder de computo para tareas como ordenar o buscar información, sería de gran ayuda.
- Física básica
  - Simulación de la evolución temporal de ciertos Hamiltonianos
  - Cálculo del espectro de energía

¿En qué tipo de problemas se ha estado implementando la computación cuántica ?

EXHIBIT 1 | Quantum-Advantaged Computational Problems

Type of problem		Useful for...	Industry applications include...
	Combinatorial optimization	Minimizing or maximizing an objective function, such as finding the most efficient allocation of resources or the shortest total distance among a set of points (e.g., the traveling salesman problem)	• Network optimization (e.g., for airlines, taxis) • Supply chain and logistics optimization • Portfolio optimization in financial services
	Differential equations	Modeling the behavior of complex systems involving fundamental laws of physics (e.g., Navier Stokes for fluid dynamics and chemistry)	• Fluid dynamics simulations for automotive and aeronautical design and medical devices (e.g., blood flow analysis) • Molecular simulation for specialty materials design and drug discovery
	Linear algebra	Machine learning tasks involving matrix diagonalization, such as clustering, pattern matching, and principal components analysis, as well as support vector machines, which are ubiquitous in applications across industries	• Risk management in quantitative finance • DNA sequence classification • Marketing and customer segmentation
	Factorization	Cryptography and computer security, where the most common protocols today (e.g., RSA) rely on the infeasibility (for classical computers) of factoring the product of two large prime numbers	• Decryption and code breaking (e.g., for governments)

Source: BCG analysis.

Figura tomada de: [Where Will Quantum Computers Create Value—and When? (bcg.com)]

¿Quiénes están invirtiendo?

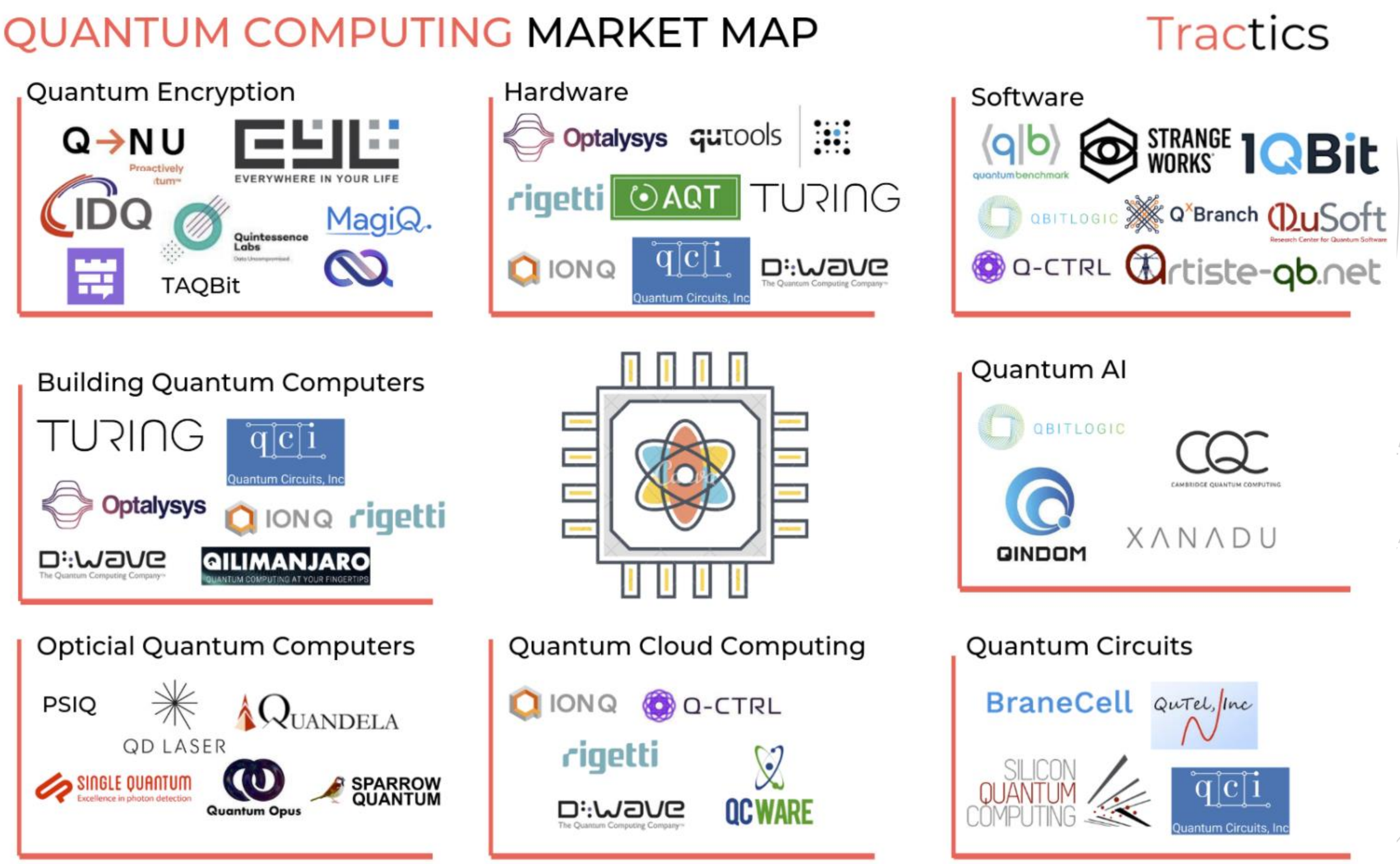
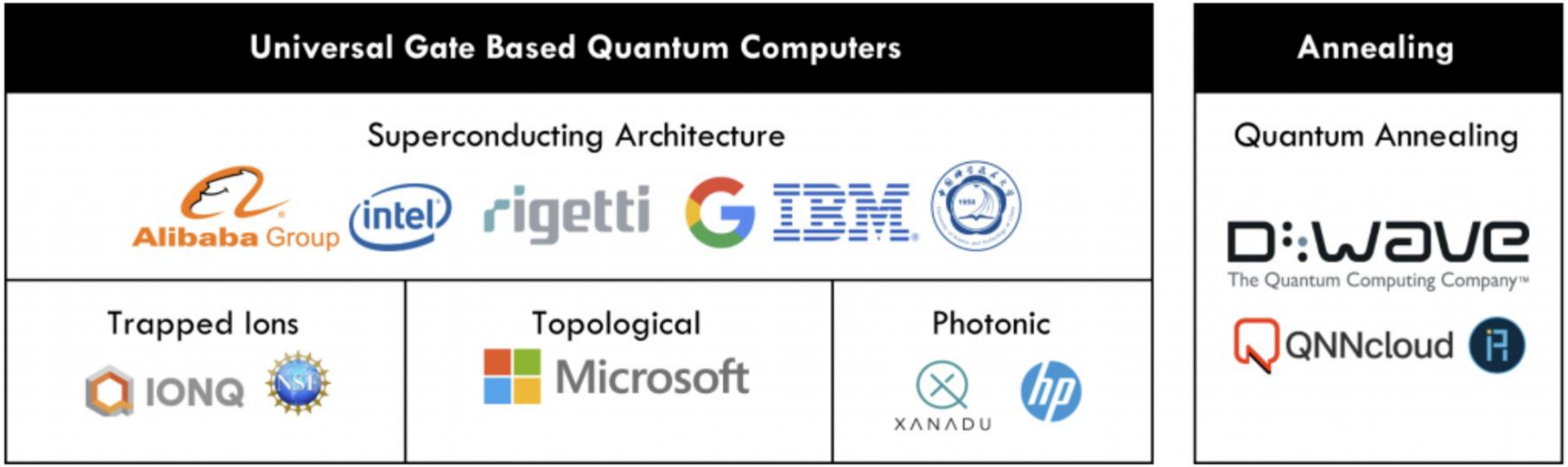


Figura tomada de:[QC companies of 2022: Guide based on 4 ecosystem maps (aimultiple.com)]

Implementaciones



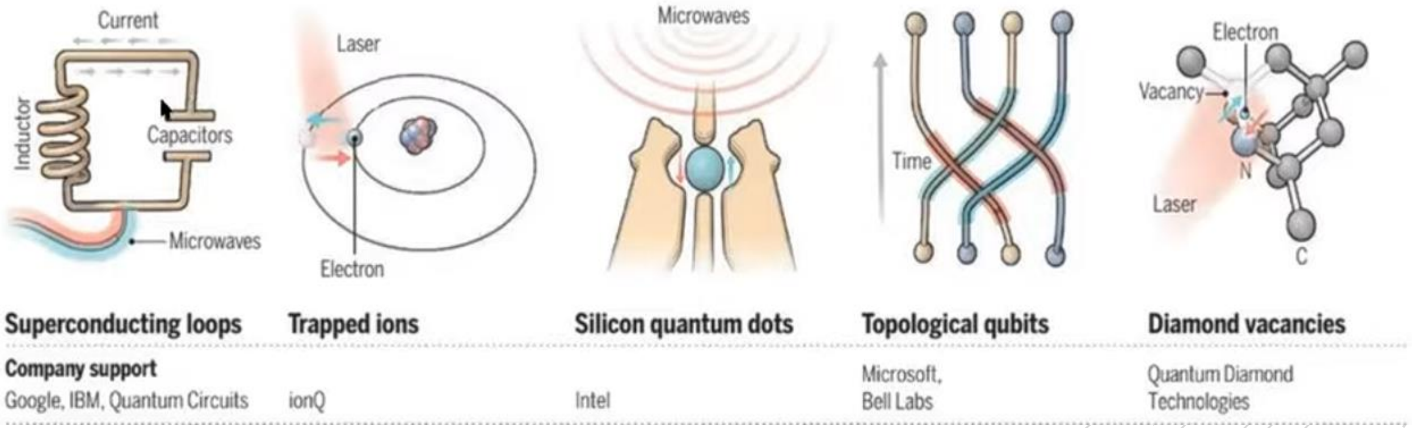


Figura tomada de:[QC companies of 2022: Guide based on 4 ecosystem maps (aimultiple.com)]

Lenguajes disponibles

qiskit 0.23.4
see release notes

Open-Source Quantum Development

Qiskit [kiss-kit] is an open source SDK for working with quantum computers at the level of pulses, circuits and application modules.

QuTiP
Quantum Toolbox in Python

ProjectQ
Powerful open source software for quantum computing

Cirq
An open source framework for programming quantum computers

XANADU
PENNYLANE
A cross-platform Python library for differentiable programming of quantum computers. Train a quantum computer the same way as a neural network.

TensorFlow Quantum

Visual Studio | Marketplace
Visual Studio > Tools > Microsoft Quantum Development Kit
Microsoft Quantum Development Kit
Microsoft DevLabs | 65.801 installs | ★★★★★ (35)
Microsoft Quantum Development Kit provides support for developing quantum algorithms in the Q# programming language.
Download

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Escuela de invierno en computo cuántico

Iniciamos

Algoritmo clásico

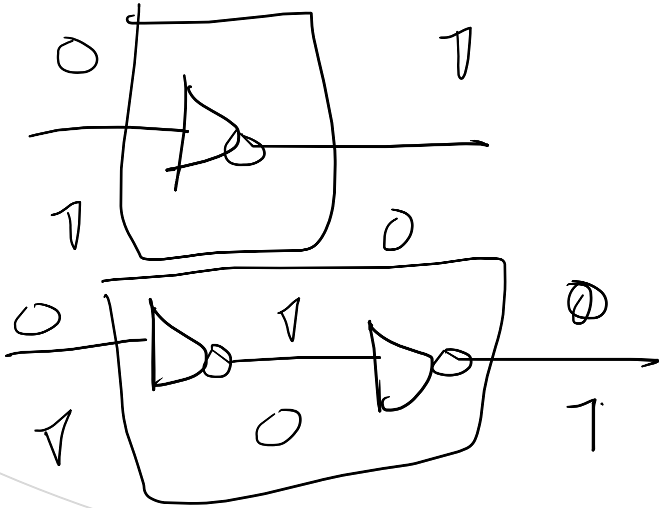
Elementos básicos

{0, 1}

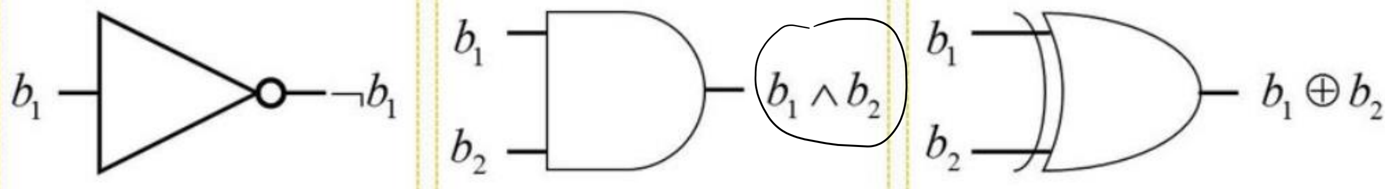
0011 → 0110

→ 1

NOT



Compuertas (operadores)



b_1	$\neg b_1$
0	1
1	0

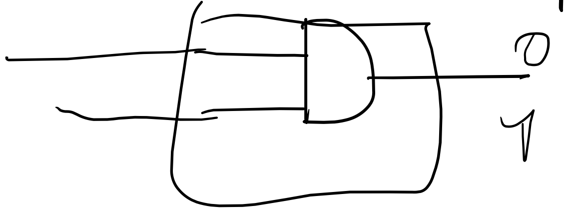
b_1	b_2	$b_1 \wedge b_2$
0	0	0
0	1	0
1	0	0
1	1	1

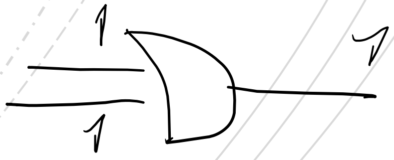
b_1	b_2	$b_1 \oplus b_2$
0	0	0
0	1	1
1	0	1
1	1	0

2 bits in

1 bit salida

OR



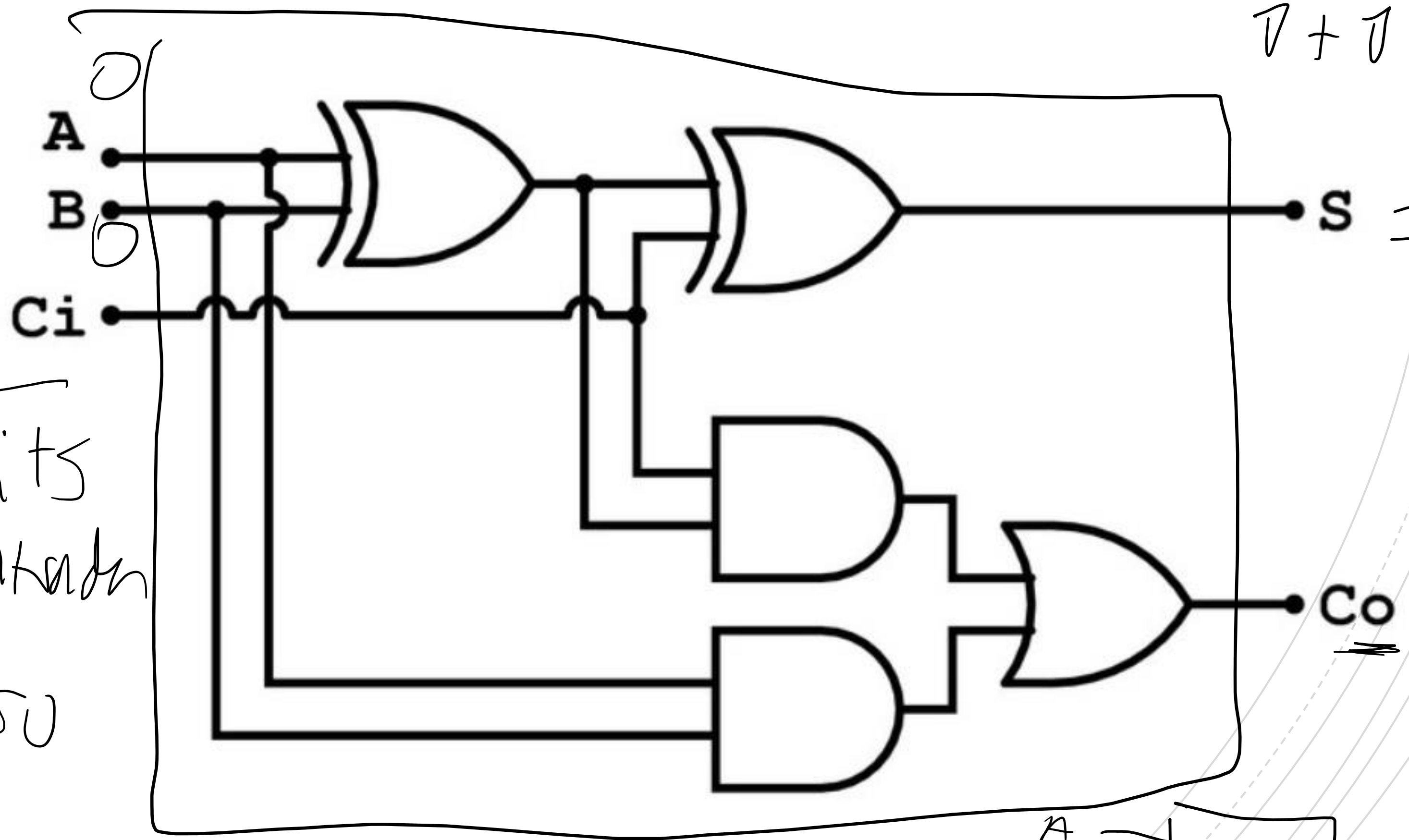


Algoritmo clásico

Circuito sumador de un bit

A, B {0, 1}

0+0 = 00
1+0 = 01
0+1 = 01
1+1 = 10



0 0

S = A + B = 0

1

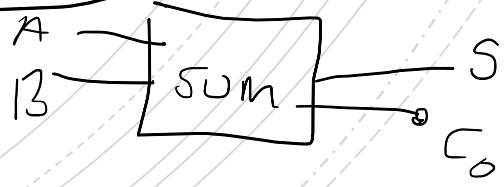
Bits entrada

Bits salida

5V

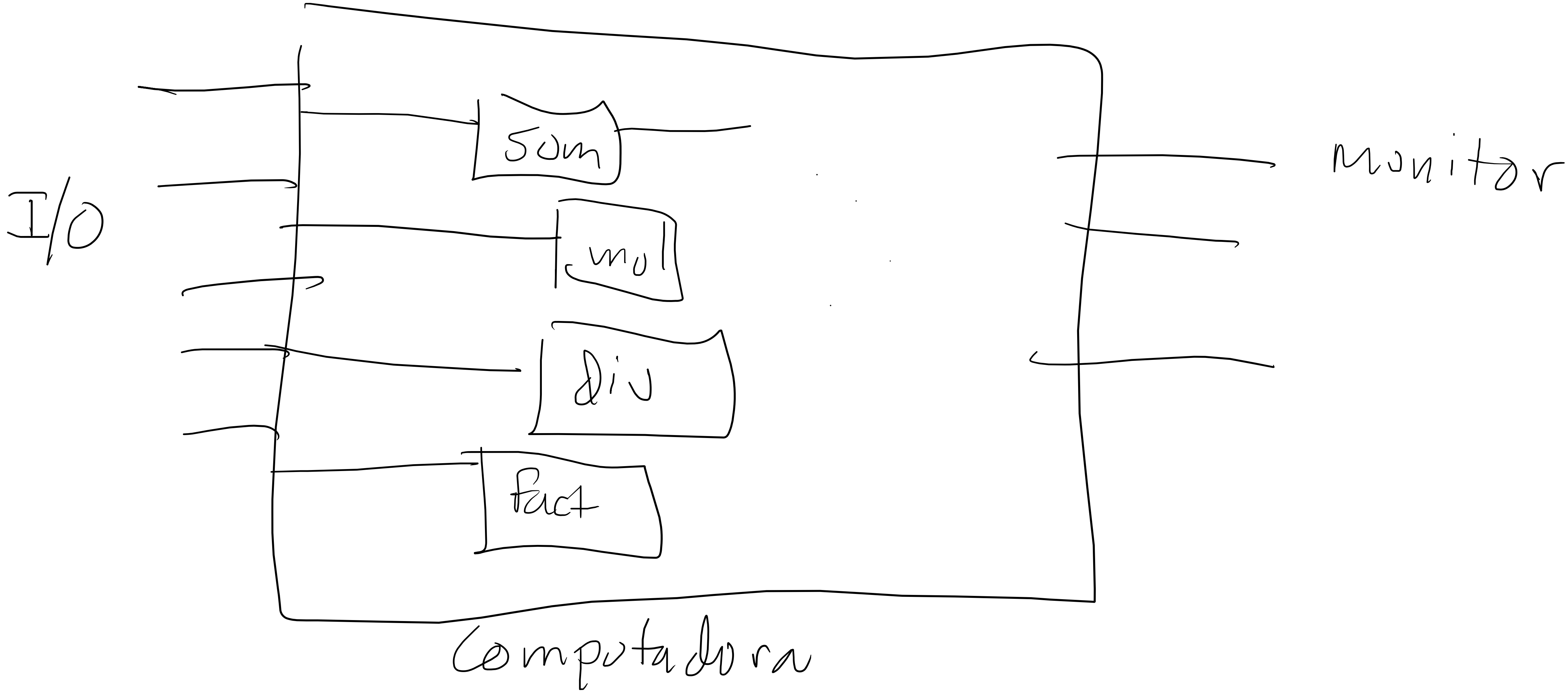
Sum

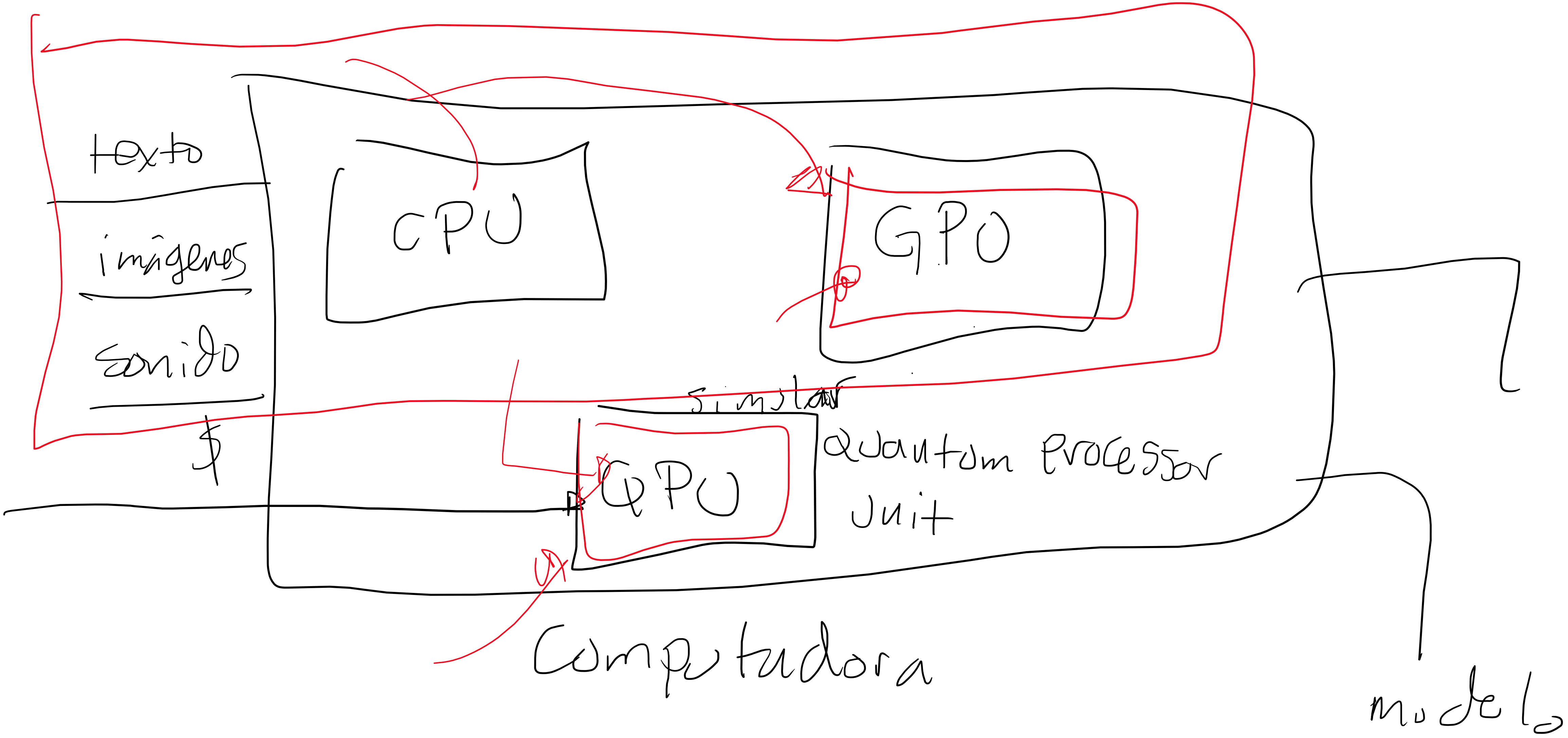
Compuertas



0 0V

0V → 1

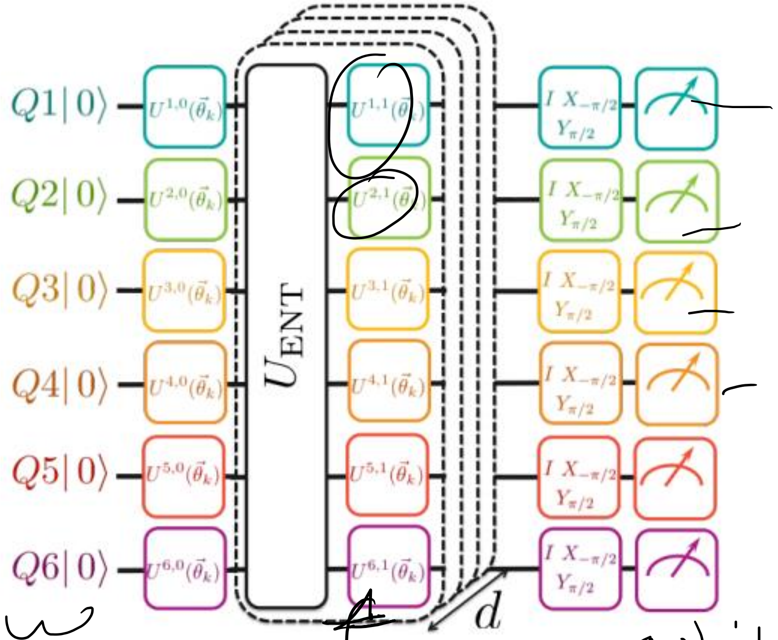

I/O
som
mul
div
fact
monitor
Computadora


texto
imágenes
sonido
$
CPU
GPU
similar
QPU
Quantum processor unit
Computadora
modelo

Computadora cuántica

Niveles de abstracción

1

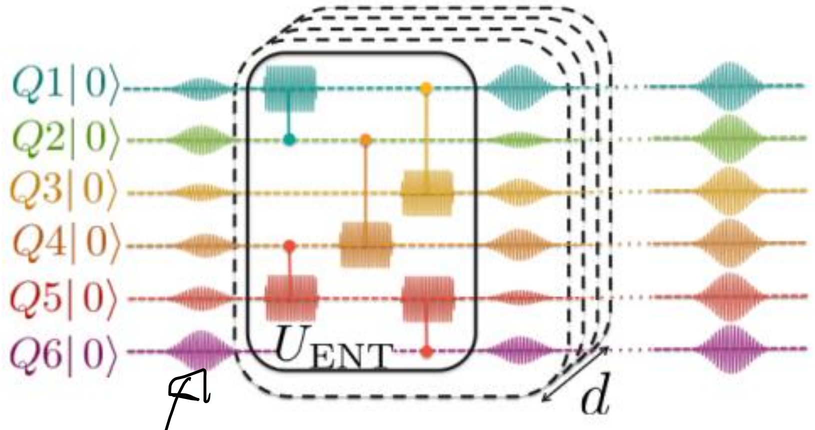


qubits entrada

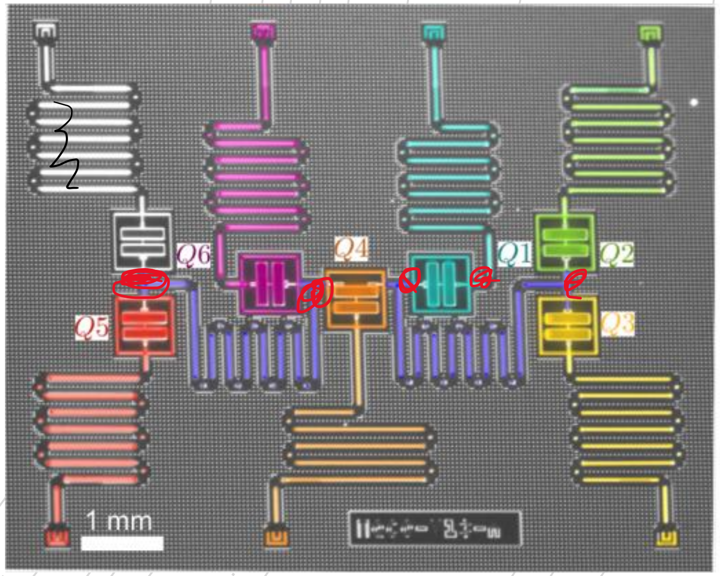
operadores compuertas

qubits salida

2



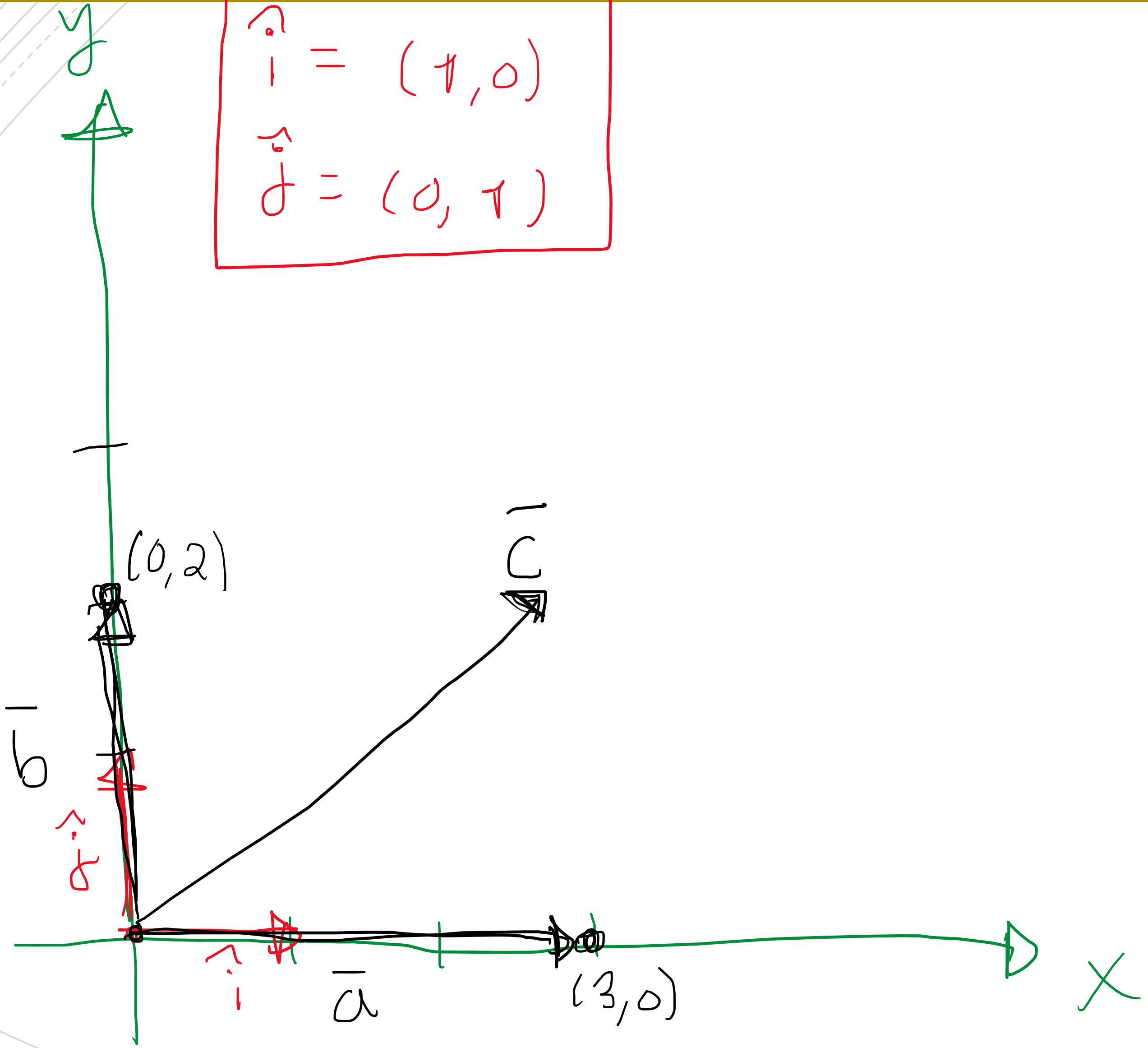
3



Vectores en R^2 y "notación de Dirac"

$\hat{i} = (1,0)$

$\hat{j} = (0,1)$



$(3,0)$, $(0,2)$

$\bar{a} = 3\hat{i}$ $\quad$ $\bar{b} = 2\hat{j}$

$\bar{c} = \bar{a} + \bar{b}$

$\bar{c} = 3\hat{i} + 2\hat{j}$

cuántico

ket

$|c\rangle = 3|i\rangle + 2|j\rangle$

$|c\rangle = 3|0\rangle + 2|1\rangle$

Elementos básicos de un algoritmo cuántico

Estados cuánticos

$|\psi\rangle$:= Estado cuántico (Ket)

psi

$\{|0\rangle, |1\rangle\}$

Base computacional para 1 qubit

$$\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle} \quad \boxed{\alpha, \beta \in \mathbb{C}}$$

$|\psi_1\rangle = |0\rangle$

$|\psi_2\rangle = |1\rangle$

$$\boxed{|\alpha|^2 + |\beta|^2 = 1}$$

condición de normalización

α, β := amplitudes de probabilidad

$|\alpha|^2$:= Probabilidad del estado $|\psi\rangle$ de estar en $|0\rangle$

Espacio de Hilbert (pre-Hilbert)

$$\{|0\rangle, |1\rangle\} \qquad 2^{1} = 2$$

un qubit

Base computacional

$$\boxed{|a\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle}$$

(vector, vector de estado, qubit)

- Base computacional $\{|0\rangle, |1\rangle\}$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|a\rangle = 1/\sqrt{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1/\sqrt{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix} = \boxed{\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}$$

$|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

- Ket $|a\rangle$

- Bra $\langle a|$

conjugada

traspuesta

$$|a\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \Longrightarrow \langle a| = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}^{*T}$$

- vector columna

$$= \left(1/\sqrt{2} \,,\, 1/\sqrt{2} \right)$$

$$\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^{*T} = (1, 0) \qquad \langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^{*T} = (0, 1)$$

vector renglón

$|a\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$ • Ket

• ~~Ket~~ $|a\rangle$

$\langle a| = (1/\sqrt{2},\ 1/\sqrt{2}) = 1/\sqrt{2}\,\langle 0| + 1/\sqrt{2}\,\langle 1|$ • Bra

---

$$|b\rangle = \frac{i}{\sqrt{3}}|0\rangle - i\sqrt{2/3}\,|1\rangle$$

$$\langle b| = \left(\frac{i}{\sqrt{3}}|0\rangle\right)^{*T} - \left(i\sqrt{2/3}\,|1\rangle\right)^{*T}$$

$$|b\rangle = \frac{i}{\sqrt{3}}|0\rangle - i\sqrt{2/3}\,|1\rangle$$

set $|a\rangle$

$$z = \alpha + \beta i$$
$$z^* = \alpha - \beta i$$
$$\alpha, \beta \in \mathbb{R}$$

$$\langle b| = \left(\frac{i}{\sqrt{3}}|0\rangle\right)^{*T} - \left(i\sqrt{2/3}\,|1\rangle\right)^{*T}$$

$$(|0\rangle)^{*T} = \langle 0|$$
$$(|1\rangle)^{*T} = \langle 1|$$

$$\langle b| = \frac{-i}{\sqrt{3}}\langle 0| - -i\sqrt{2/3}\,\langle 1|$$

Bra

$$\langle b| = \frac{-i}{\sqrt{3}}\langle 0| + i\sqrt{2/3}\,\langle 1|$$

- Norma $|\psi\rangle$

$$\|\psi\| = \sqrt{\langle\psi|\psi\rangle}$$

$$\langle\psi|\psi\rangle =$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\|\psi\| = ?$$

¿ $|a\rangle$ y $|\psi\rangle$ son diferentes?

$$|\psi| = \sqrt{\langle\psi|\psi\rangle}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$$

$$\langle\psi| = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}^{*T} = \left(1/\sqrt{2}, \quad i/\sqrt{2} \right)$$

$$\langle\psi|\psi\rangle = \left(1/\sqrt{2}, \quad i/\sqrt{2} \right) \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix} = (1/\sqrt{2})(1/\sqrt{2}) + \left(\frac{i}{\sqrt{2}}\right)\left(\frac{-i}{\sqrt{2}}\right)$$

$$= 1/2 + \frac{1}{2} = 1 \checkmark$$

$$|\psi| = \sqrt{\langle\psi|\psi\rangle} = \sqrt{1} = 1$$

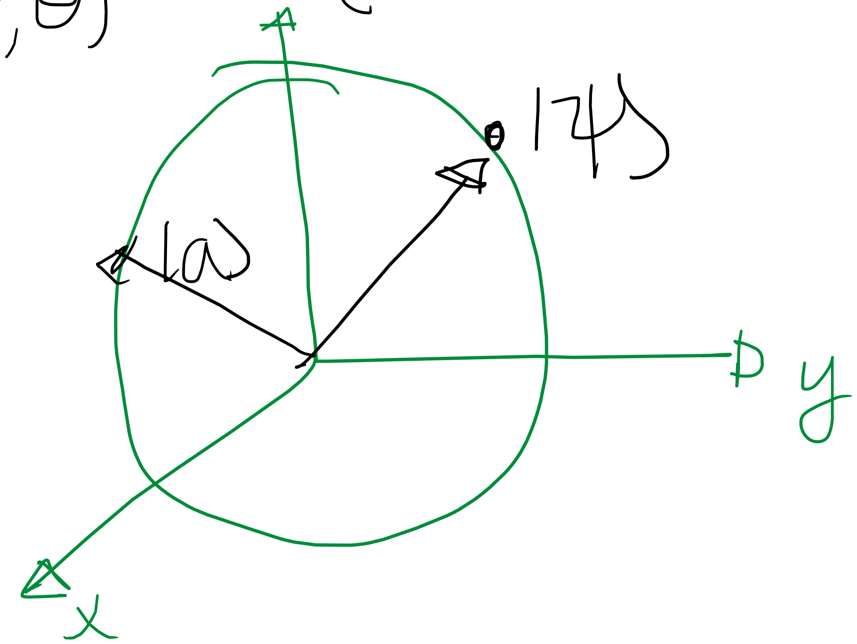
$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\boxed{|\alpha|^2 + |\beta|^2 = 1}$$

- Esfera de Bloch

$(1, \phi, \theta)$



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \cos\theta|0\rangle + e^{i\phi}\,\mathrm{sen}\,\theta\,|1\rangle$$

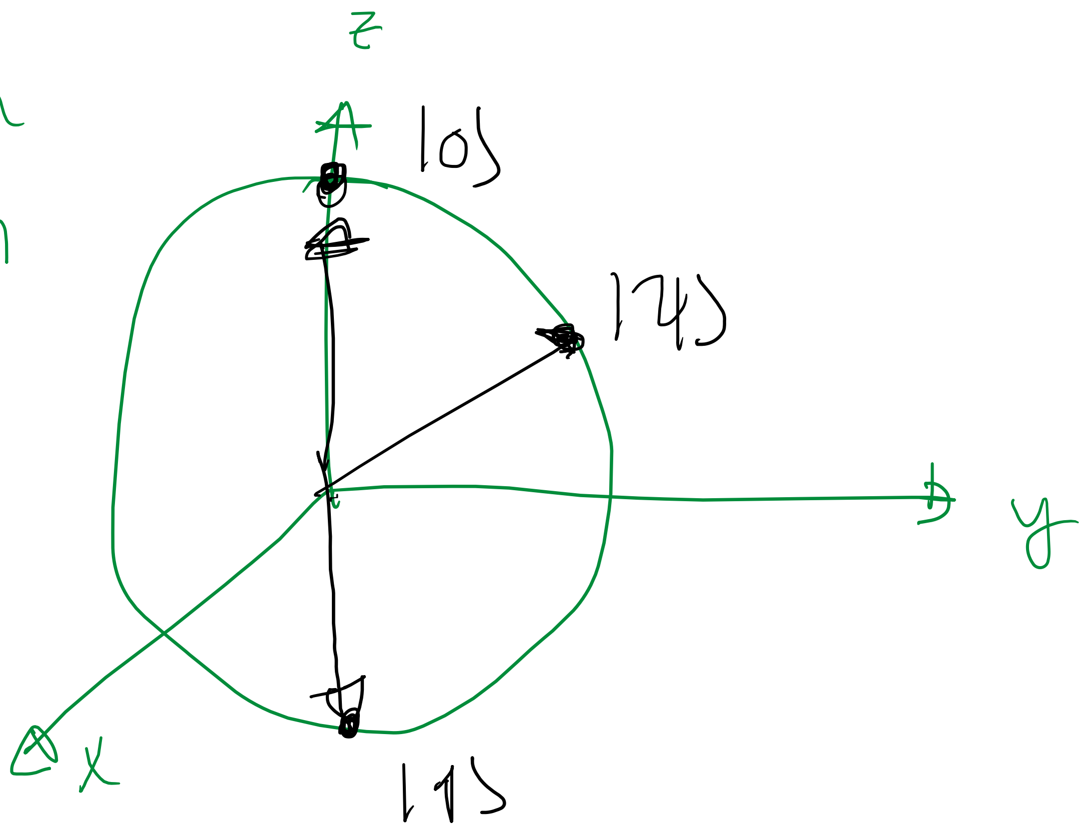
$\alpha, \beta \sim \theta, \phi$

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \sim |0\rangle + \sim |1\rangle$$

$$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\psi\rangle = \cos\theta\,|0\rangle + e^{i\phi}\,\mathrm{sen}\,\theta\,|1\rangle$$

Esfera Bloch



Herramientas matemáticas

Compuertas cuánticas

$\hat{\sigma}_z = \hat{Z}$:= Matriz de Pauli Z

$\langle 0|0\rangle = (1\ 0)\begin{pmatrix}1\\0\end{pmatrix} = 1$

$\langle 0|1\rangle = 0$

$\langle 1|0\rangle = 0$

$\langle 1|1\rangle = 1$

$\hat{\sigma}_z|0\rangle = |0\rangle$

$\Longleftrightarrow \quad \hat{\sigma}_z = \begin{pmatrix} 1_1 & 0_2 \\ 0_3 & -1_4 \end{pmatrix}$

$\hat{\sigma}_z|1\rangle = -|1\rangle$

$\langle 0|\hat{\sigma}_z|0\rangle = 1_0 \quad \langle 0|\hat{\sigma}_z|1\rangle = 2_0$

$\langle 1|\hat{\sigma}_z|0\rangle = 3_0 \quad \langle 1|\hat{\sigma}_z|1\rangle = 4_0$

$$\hat{\sigma}_z |0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} (1)(1) + (0)(0) \\ (0)(1) + (-1)(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\hat{\sigma}_z |1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 + 0 \\ 0 + -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= -1 |1\rangle$$

$$= -|1\rangle$$

- Cómo actúan los operadores $\quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$\hat{A}|\psi\rangle = \hat{A}\left(\alpha|0\rangle + \beta|1\rangle\right)$$

$$= \alpha\hat{A}|0\rangle + \beta\hat{A}|1\rangle$$

$$=$$

- Modelo de circuito

$$|\psi\rangle \xrightarrow{\boxed{\hat{\sigma}_z}} |\psi'\rangle$$

$$|0\rangle \qquad |0\rangle$$

$$|1\rangle \qquad -|1\rangle$$

$$|a\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$$

$$\hat{\sigma}_z|a\rangle = 1/\sqrt{2}\,\hat{\sigma}_z|0\rangle + 1/\sqrt{2}\,\hat{\sigma}_z|1\rangle$$

$$|a\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$$

$$\boxed{\begin{aligned}\hat{\sigma}_z|0\rangle &= |0\rangle \\ \hat{\sigma}_z|1\rangle &= -|1\rangle\end{aligned}}$$

$$\hat{\sigma}_z|a\rangle = 1/\sqrt{2}\,\hat{\sigma}_z|0\rangle + 1/\sqrt{2}\,\hat{\sigma}_z|1\rangle$$

$$= 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,(-|1\rangle)$$

$$= 1/\sqrt{2}\,|0\rangle - 1/\sqrt{2}\,|1\rangle$$

$$|a\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_z |a\rangle = \hat{\sigma}_z \left(\begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix} \right)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = 1/\sqrt{2}\,|0\rangle - 1/\sqrt{2}\,|1\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = |b\rangle$$

* Matrices de Pauli $(\hat{x}, \hat{y}, \hat{z})$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{\sigma}_x |0\rangle =$$

$$\hat{\sigma}_x |1\rangle =$$

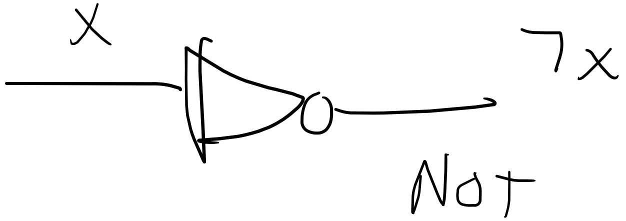
$$\hat{\sigma}_y |0\rangle =$$

$$\hat{\sigma}_y |1\rangle =$$

$$\hat{\sigma}_x |0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; ; \; \hat{\sigma}_x |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{\hat{\sigma}_x |0\rangle = |1\rangle \; ; \; \hat{\sigma}_x |1\rangle = |0\rangle}$$

análogo cuántico a



{ AND, NOT, COPY }

XOR, OR, NAND

$$\hat{\sigma}_y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\begin{pmatrix} 0 \\ 1 \end{pmatrix} = i|1\rangle$$

$$\hat{\sigma}_y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -i|0\rangle$$

$$\boxed{\begin{aligned} \hat{\sigma}_y|0\rangle &= i|1\rangle \\ \hat{\sigma}_y|1\rangle &= -i|0\rangle \end{aligned}}$$

- Identidad

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{I}|0\rangle = |0\rangle$$

$$\hat{I}|1\rangle = |1\rangle$$

Herramientas matemáticas

Esfera de Bloch

Herramientas matemáticas

Medición de los estados cuánticos

$$\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle}$$

$\alpha, \beta :=$ amplitudes de probabilidad (son complejas en general)

$|\alpha|^2 =:$ Probabilidad del estado $|\psi\rangle$ de estar en $|0\rangle$

$|\beta|^2 =:$,, ,, ,, ,, ,, ,, ,, $|1\rangle$

$$\boxed{P_{|\psi\rangle}(|0\rangle) = |\langle 0|\psi\rangle|^2}$$

¿cuál es la probabilidad del estado $|\psi\rangle$ de estar en $|0\rangle$?

Regla de Born

$$P_{|\psi\rangle}(|0\rangle) = |\langle 0|\psi\rangle|^2 = |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2$$

$$= |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2$$

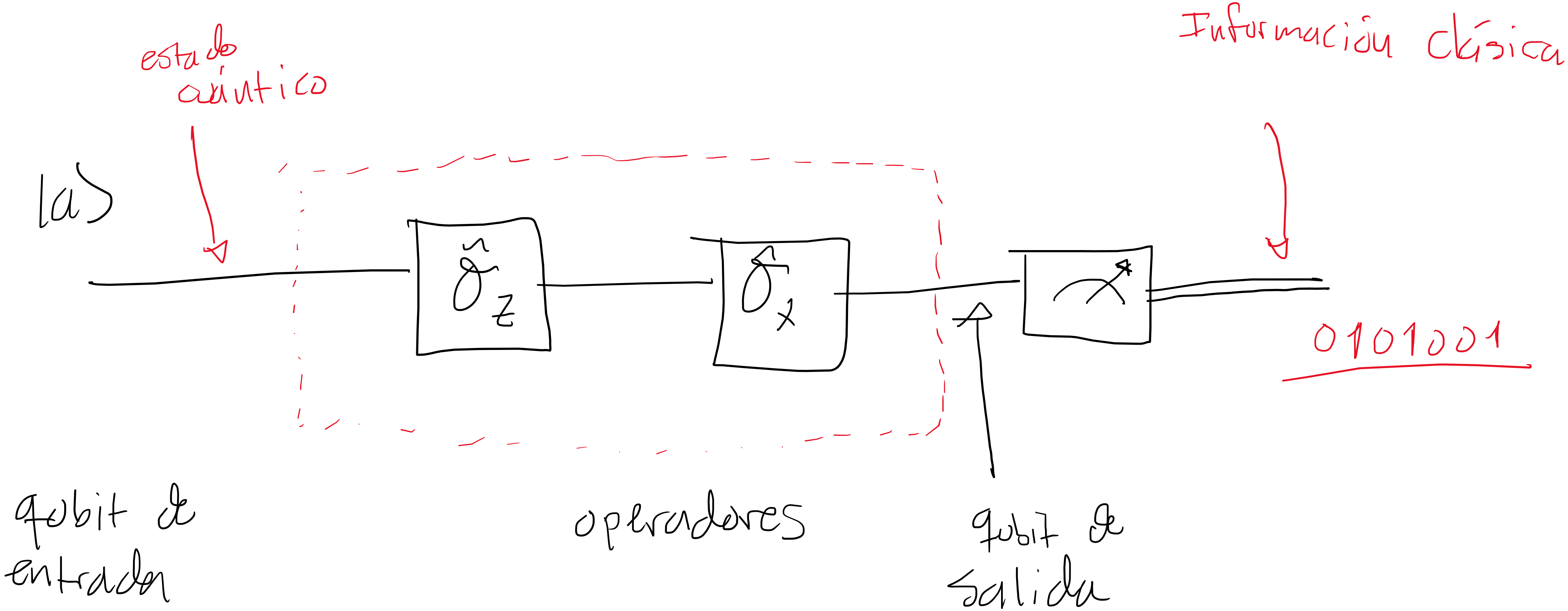
$$= |\alpha(1) + \cancel{\beta(0)}|^2$$

$$= \boxed{|\alpha|^2}$$

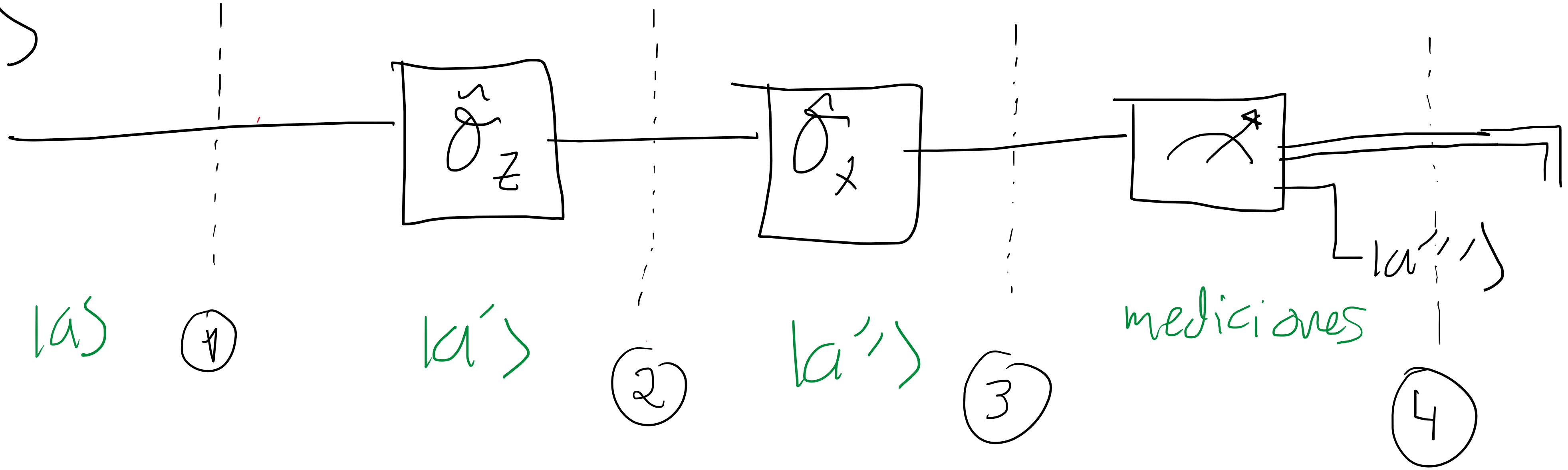
$$P_{|\psi\rangle}(|1\rangle) = |\langle 1|\psi\rangle|^2 = \boxed{|\beta|^2}$$

$$|a\rangle = \frac{1}{\sqrt{3}}|0\rangle - i\sqrt{\frac{2}{3}}\,|1\rangle$$

$|a| = 1$



1 $|a\rangle$



2 $\hat{\sigma}_z|a\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{2/3}\,|1\rangle = |a'\rangle$

3 $\hat{\sigma}_x|a'\rangle = \boxed{\frac{1}{\sqrt{3}}|1\rangle + i\sqrt{2/3}\,|0\rangle = |a''\rangle}$

4 $P_{|a''\rangle}(|0\rangle)$, $P_{|a''\rangle}(|1\rangle)$

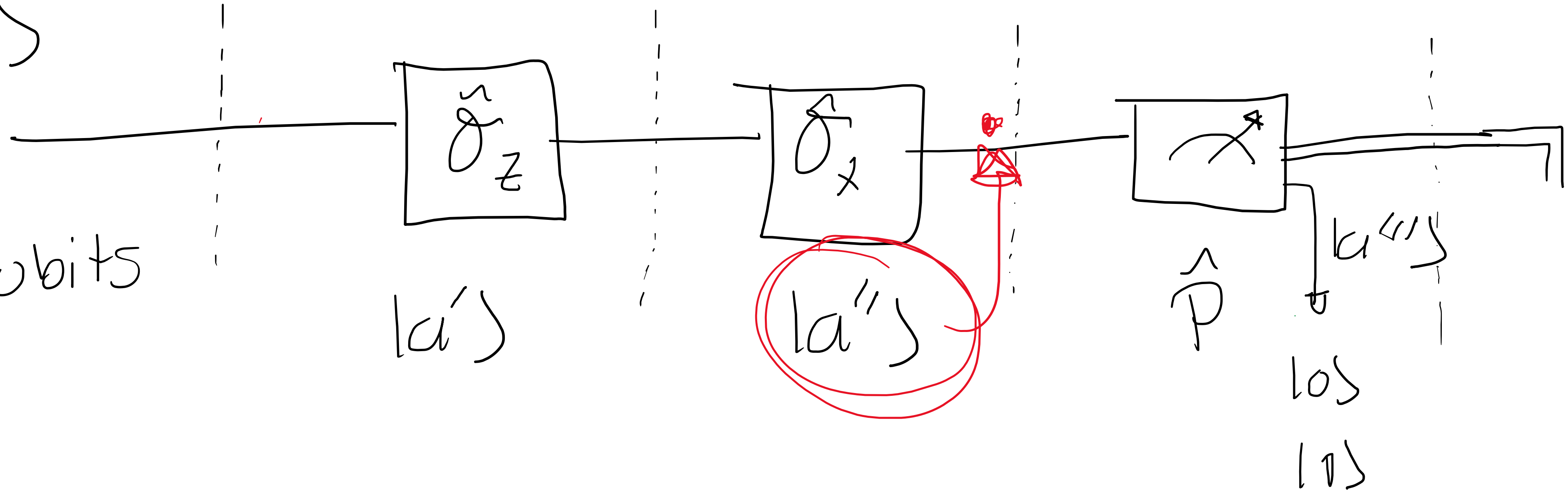
$$P_{|a''\rangle}(|0\rangle) = |\langle 0|a''\rangle|^2 = \left|\langle 0|\left(\tfrac{1}{\sqrt{3}}|1\rangle + i\sqrt{2/3}\,|0\rangle\right)\right|^2$$

$$= \left|i\sqrt{2/3}\right|^2 = |i|\,\left|\sqrt{2/3}\right|^2 = \underline{2/3} =$$

$$P_{|a''\rangle}(|1\rangle) = |\langle 1|a''\rangle|^2 = \left|\langle 1|\left(\tfrac{1}{\sqrt{3}}|1\rangle + i\sqrt{2/3}\,|0\rangle\right)\right|^2$$

$$= \left|1/\sqrt{3}\right|^2 = 1/3$$

- "colapso del estado después de medir"

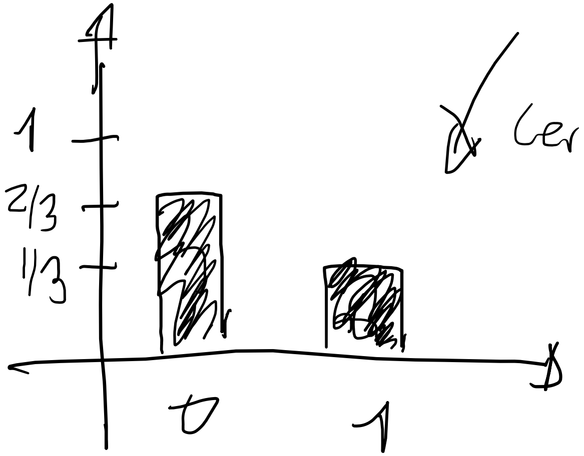


100 qubits
|a⟩

Aleatorio

$$\hat{P}|a''\rangle = |0\rangle \quad ¿?$$

$$\hat{P}|a''\rangle = |1\rangle$$



✗ certeza

Herramientas matemáticas

Producto tensorial y compuertas de 2 qubits

- Producto tensorial

$$|a\rangle \otimes |b\rangle = |a\rangle|b\rangle = \underline{|ab\rangle}$$ Estado de 2 qubits

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \hat{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\hat{I} \otimes \hat{A} = \underset{2\times 2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \otimes \underset{2\times 2}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \begin{pmatrix} 1\begin{pmatrix} a & b \\ c & d \end{pmatrix} & 0\begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ 0\begin{pmatrix} a & b \\ c & d \end{pmatrix} & 1\begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{pmatrix}_{4\times 4}$$

$$\hat{I} \otimes \hat{A} = \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{pmatrix}$$

$$\hat{\sigma}_z \otimes \hat{\sigma}_x = \left(\ddots \right)_{4\times 4}$$

$$\hat{\sigma}_x \otimes \hat{\sigma}_z$$

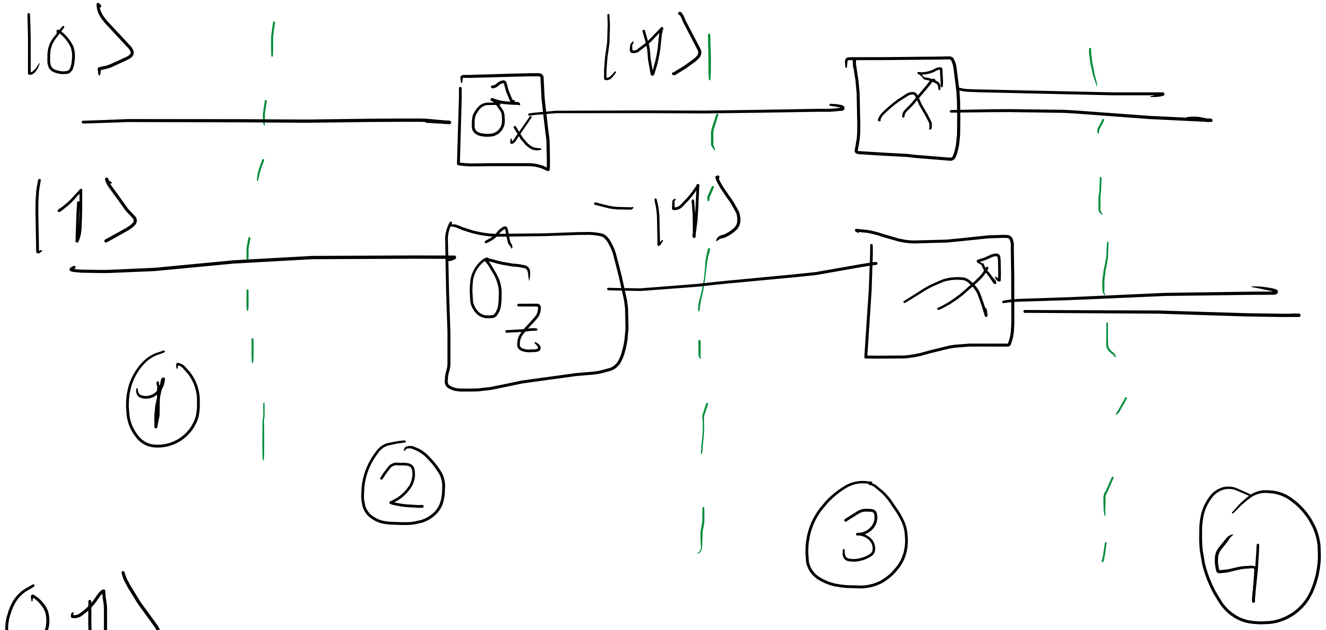
$$\hat{\sigma}_z \otimes \hat{\sigma}_y$$

Herramientas matemáticas

Entrelazamiento cuántico

Base de 2 qubits $\left(2^2 = 4\right)$

$$\underbrace{\{|0,0\rangle,\ |0,1\rangle,\ |1,0\rangle,\ |1,1\rangle\}}_{\text{Base de 4 elementos}}$$



1. $|01\rangle$

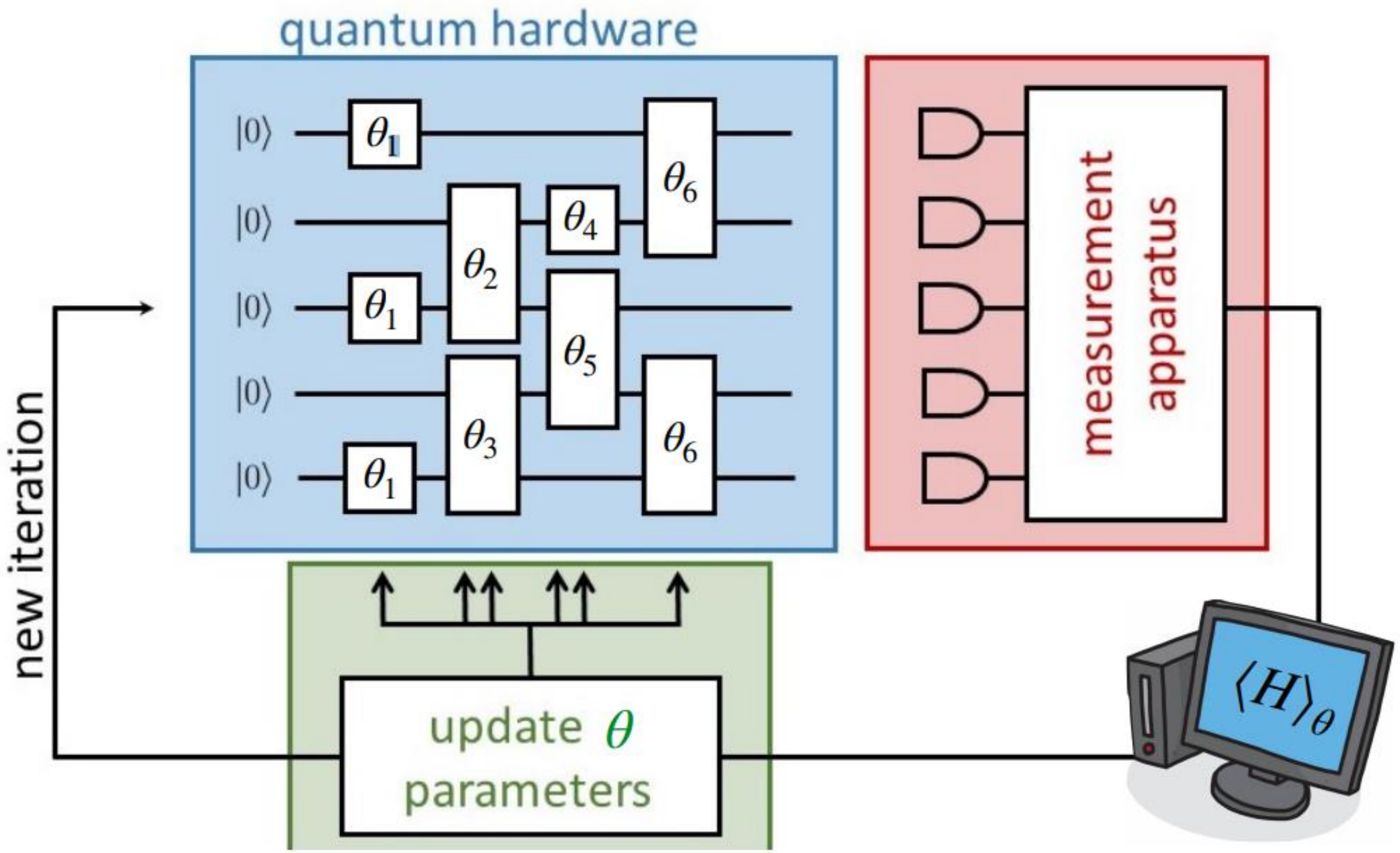
2. $-|11\rangle$

3. Mediciones

Algoritmo de teleportación cuántica

Aplicaciones (variational quantum eigensolver)

Peruzzo et al, Nat. Comm. '13



Quantum circuit as a variational ansatz