

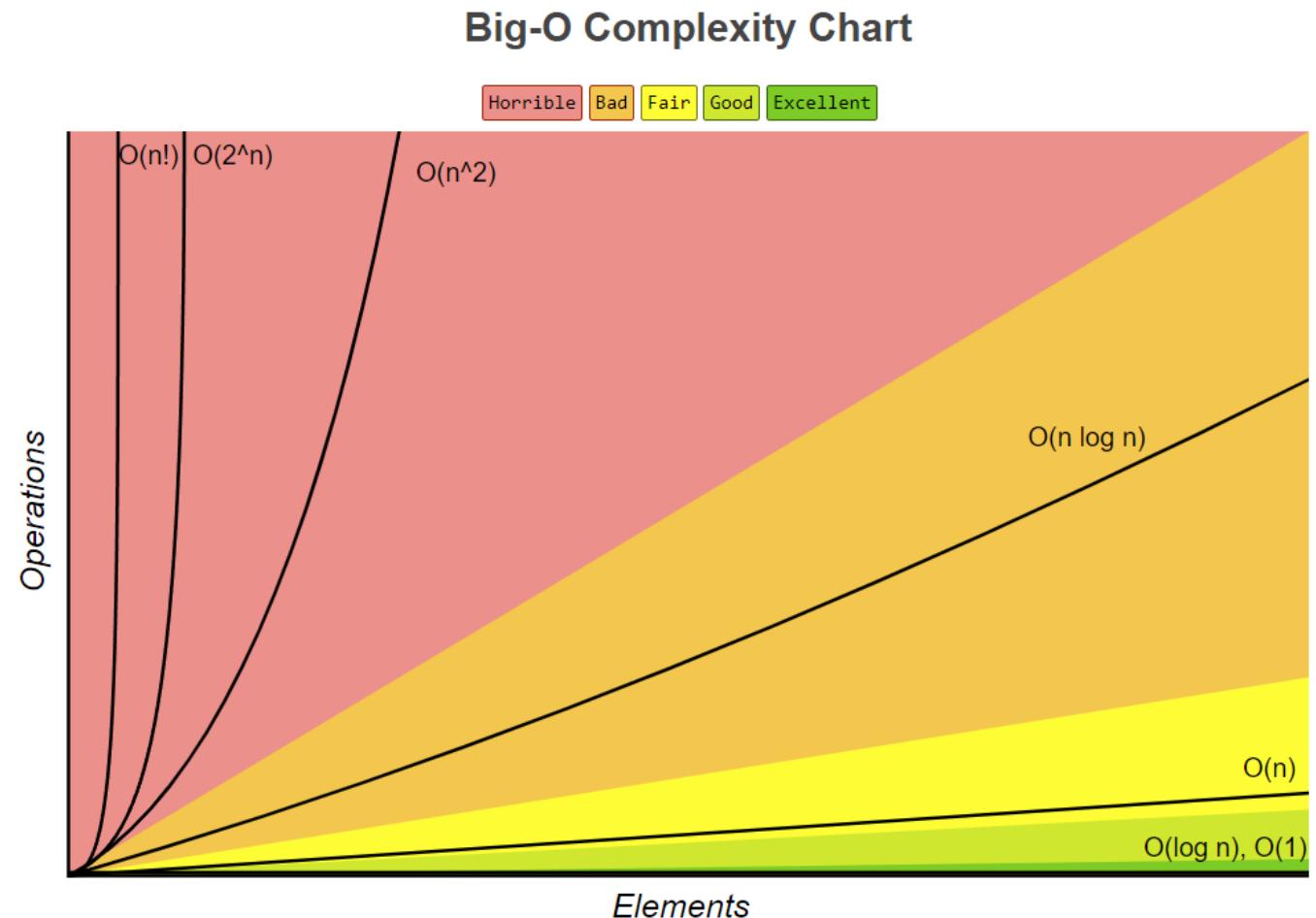
# **Escuela de invierno en computo cuántico**

**Centro de Estudios en  
Computación Avanzada  
CECAV**

**Posgrado en Ciencias Físicas  
Miguel de Jesús González Martínez**

# ¿Para qué necesitamos una computadora cuántica?

- ❖ A groso modo, un algoritmo se dice que es eficiente si corre en un tiempo polinomial, es decir, el tiempo que tarda en correr dicho algoritmo crece con el número de bits de entrada  $n$  de manera polinomial  $n^x$ , con  $x$  algún número entero.
- ❖ Se dice entonces que un problema computacionalmente difícil es aquel cuyo mejor algoritmo clásico que lo resuelve, lo hace de manera exponencial en el número  $n$  de bits de entrada, es decir, como  $a^n$ , con  $a$  alguna constante.



# ¿Qué tipo de problemas se busca resolver con la computación cuántica ?

- Encriptación y seguridad
  - Factorización de números primos en protocolos como el RSA
  - Criptografía cuántica
- Problemas de optimización
  - Buscar la solución más óptima en cierto rango de parámetros
  - Logística, Meteorología, Finanzas
- Desarrollo de materiales
  - Simulación de sistemas cuánticos para determinar los elementos necesarios para obtener determinada característica
  - Química cuántica, Medicina, Biología
- Análisis de datos
  - Se estima que el 90% de toda la información producida por la humanidad, se ha creado en los últimos años. El contar con un gran poder de computo para tareas como ordenar o buscar información, sería de gran ayuda.
- Física básica
  - Simulación de la evolución temporal de ciertos Hamiltonianos
  - Cálculo del espectro de energía

# ¿En qué tipo de problemas se ha estado implementando la computación cuántica ?

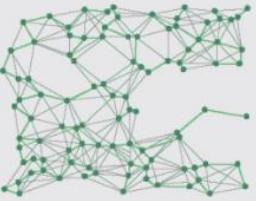
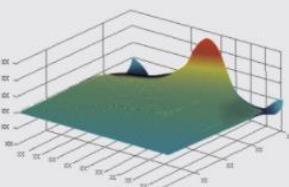
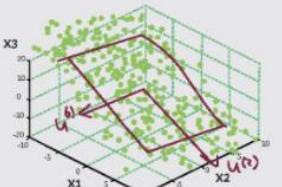
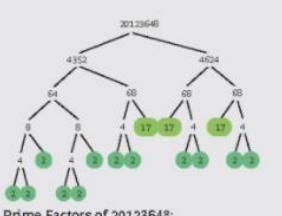
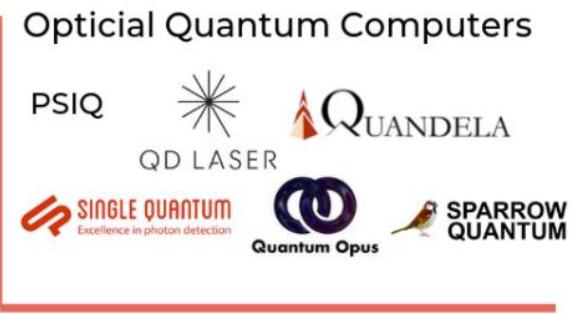
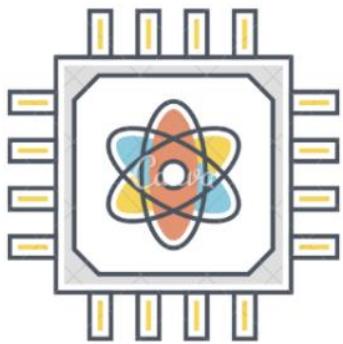
EXHIBIT 1   Quantum-Advantaged Computational Problems			
Type of problem	Useful for...	Industry applications include...	
	<p>Combinatorial optimization</p> <p>Minimizing or maximizing an objective function, such as finding the most efficient allocation of resources or the shortest total distance among a set of points (e.g., the traveling salesman problem)</p>	<ul style="list-style-type: none"><li>Network optimization (e.g., for airlines, taxis)</li><li>Supply chain and logistics optimization</li><li>Portfolio optimization in financial services</li></ul>	
	<p>Differential equations</p> <p>Modeling the behavior of complex systems involving fundamental laws of physics (e.g., Navier Stokes for fluid dynamics and chemistry)</p>	<ul style="list-style-type: none"><li>Fluid dynamics simulations for automotive and aeronautical design and medical devices (e.g., blood flow analysis)</li><li>Molecular simulation for specialty materials design and drug discovery</li></ul>	
	<p>Linear algebra</p> <p>Machine learning tasks involving matrix diagonalization, such as clustering, pattern matching, and principal components analysis, as well as support vector machines, which are ubiquitous in applications across industries</p>	<ul style="list-style-type: none"><li>Risk management in quantitative finance</li><li>DNA sequence classification</li><li>Marketing and customer segmentation</li></ul>	
	<p>Factorization</p> <p>Cryptography and computer security, where the most common protocols today (e.g., RSA) rely on the infeasibility (for classical computers) of factoring the product of two large prime numbers</p>	<ul style="list-style-type: none"><li>Decryption and code breaking (e.g., for governments)</li></ul>	

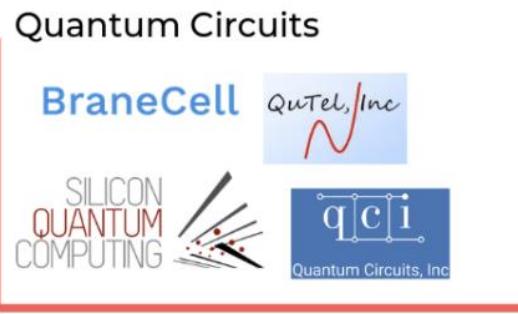
Figura tomada de: [Where Will Quantum Computers Create Value—and When? \(bcg.com\)](http://Where Will Quantum Computers Create Value—and When? (bcg.com))

# ¿Quiénes están invirtiendo?

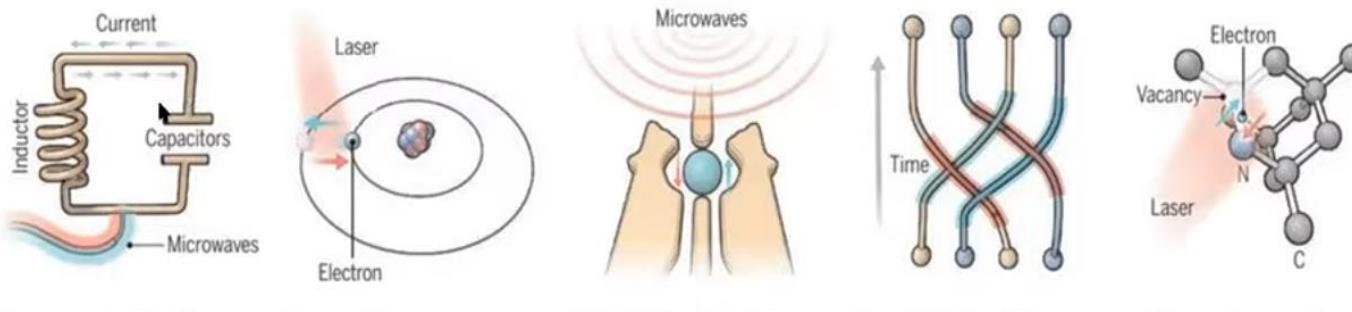
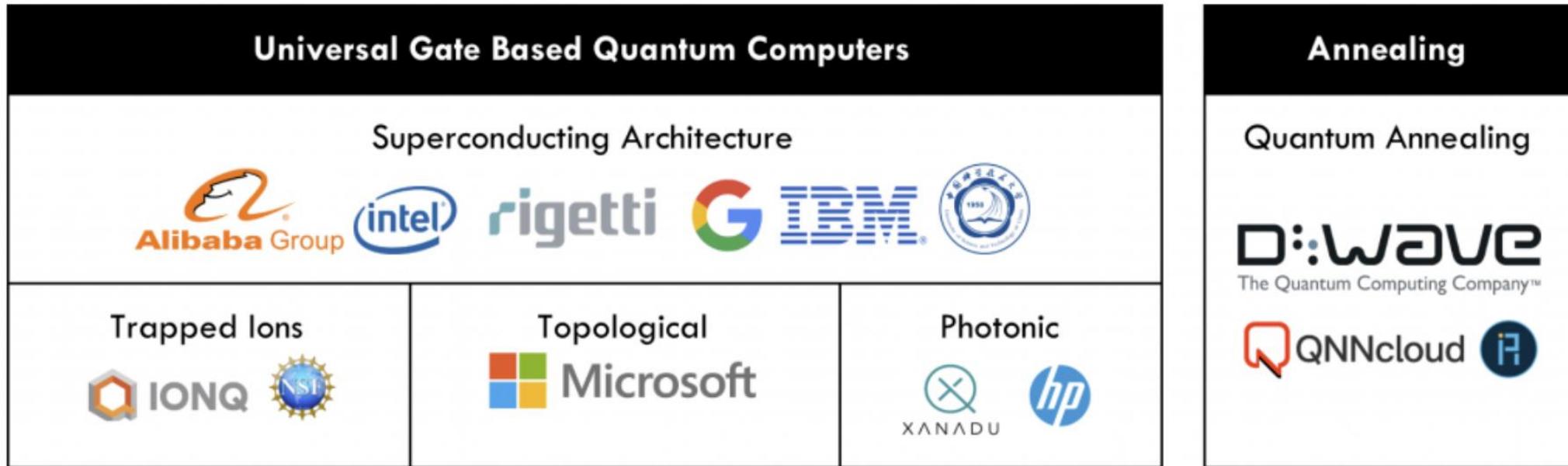
## QUANTUM COMPUTING MARKET MAP



## Tractics



# Implementaciones



Superconducting loops

Company support

Google, IBM, Quantum Circuits

Trapped ions

ionQ

Silicon quantum dots

Intel

Topological qubits

Microsoft,  
Bell Labs

Diamond vacancies

Quantum Diamond  
Technologies

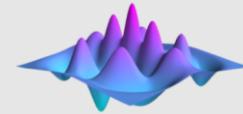
# Lenguajes disponibles



qiskit 0.23.4  
see [release notes](#)

## Open-Source Quantum Development

Qiskit [kiss-kit] is an open source SDK for working with quantum computers at the level of pulses, circuits and application modules.



QuTiP

Quantum Toolbox in Python



Cirq

An open source framework for programming quantum computers



Quantum Inspire - By QuTech

The multi hardware Quantum Technology platform

Run your own quantum algorithms on one of our simulators or hardware backends and experience the possibilities of quantum computing. Find out more below or get started [here](#).



PRODUCT SOLUTIONS RESEARCH

We build software to get the enterprise quantum-ready™.

We help industry-leading companies understand—and capitalize on—the capabilities of quantum devices in the next 2-5 years and beyond.



The quantum cloud service built for business

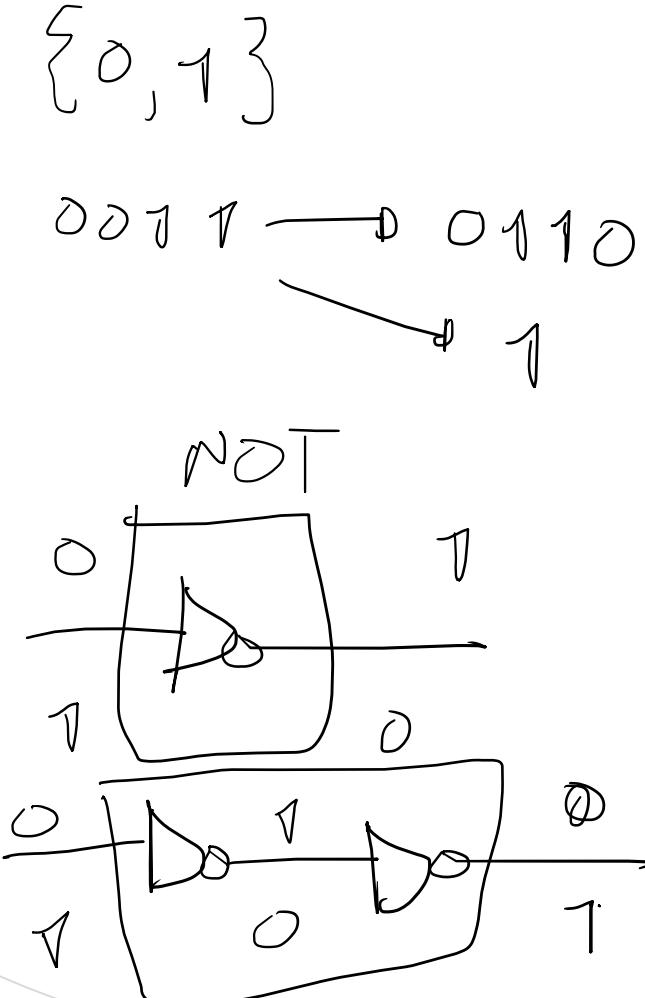
[Sign Up](#)

# **Escuela de invierno en computo cuántico**

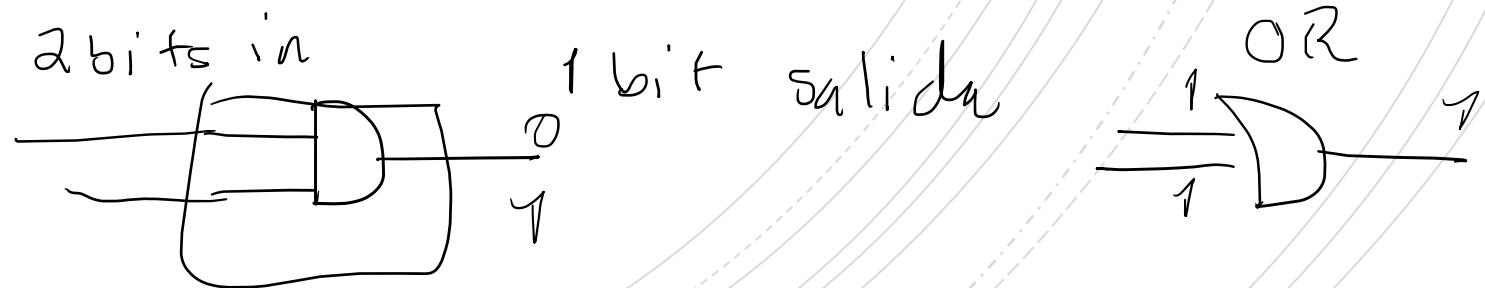
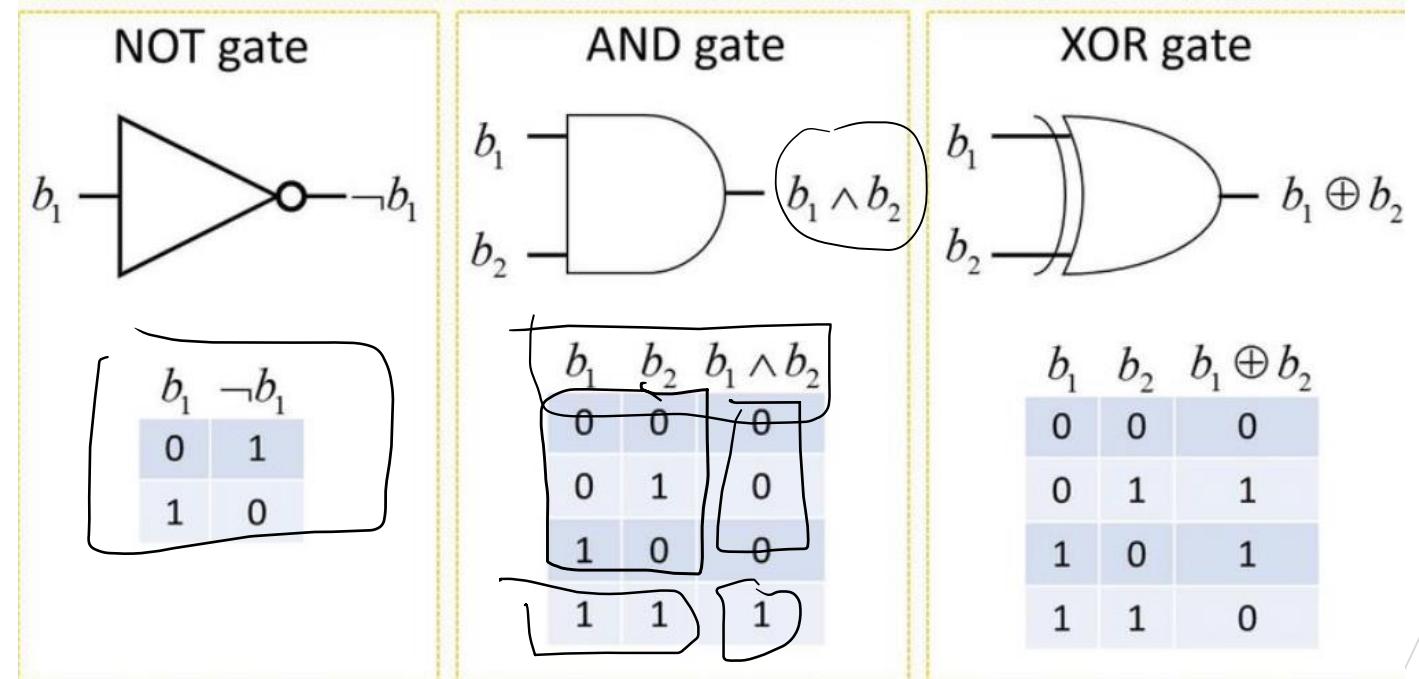
**Iniciamos**

# Algoritmo clásico

## Elementos básicos



## Compuertas (operadores)



# Algoritmo clásico

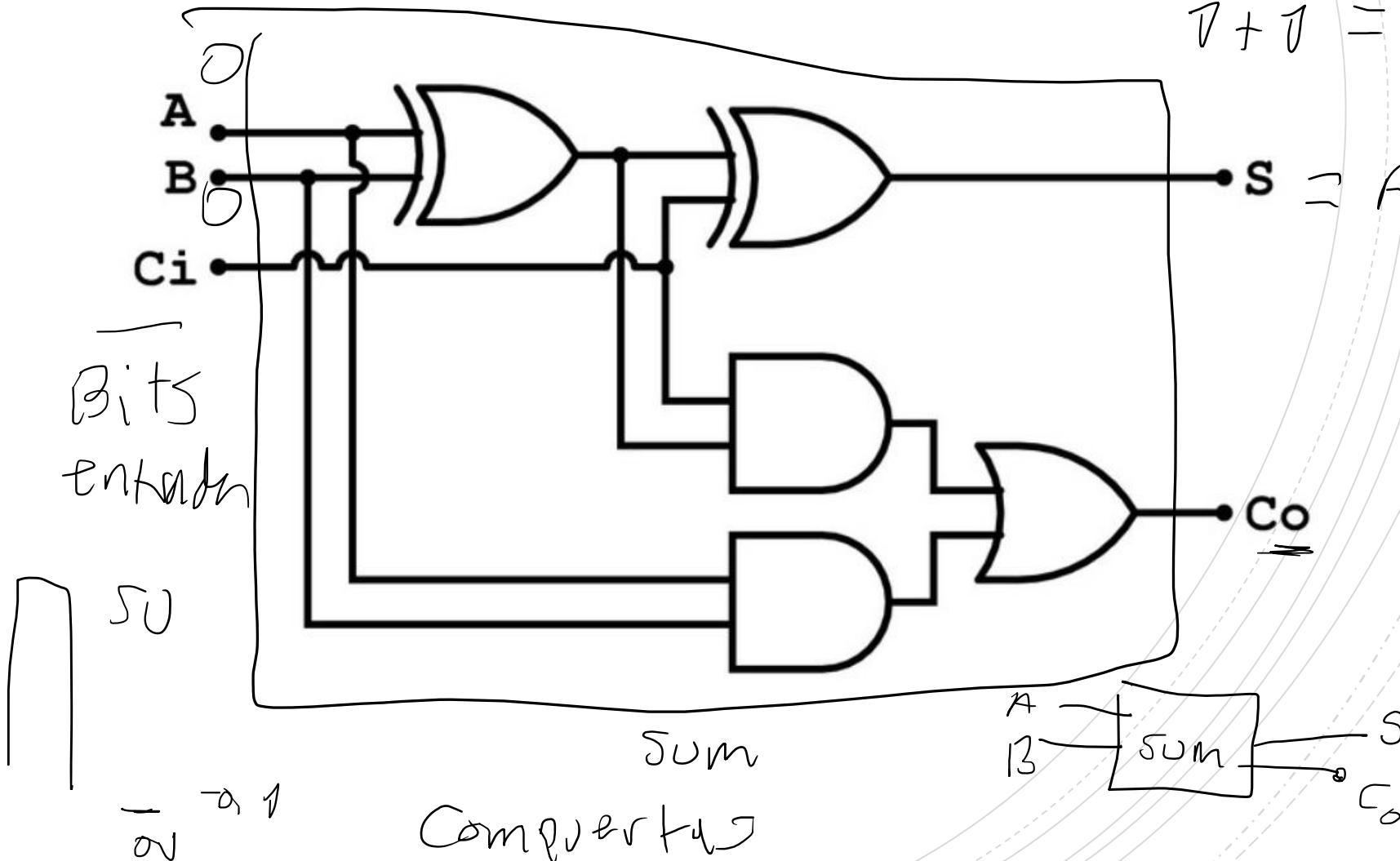
## Circuito sumador de un bit

$$A, B \in \{0, 1\}$$

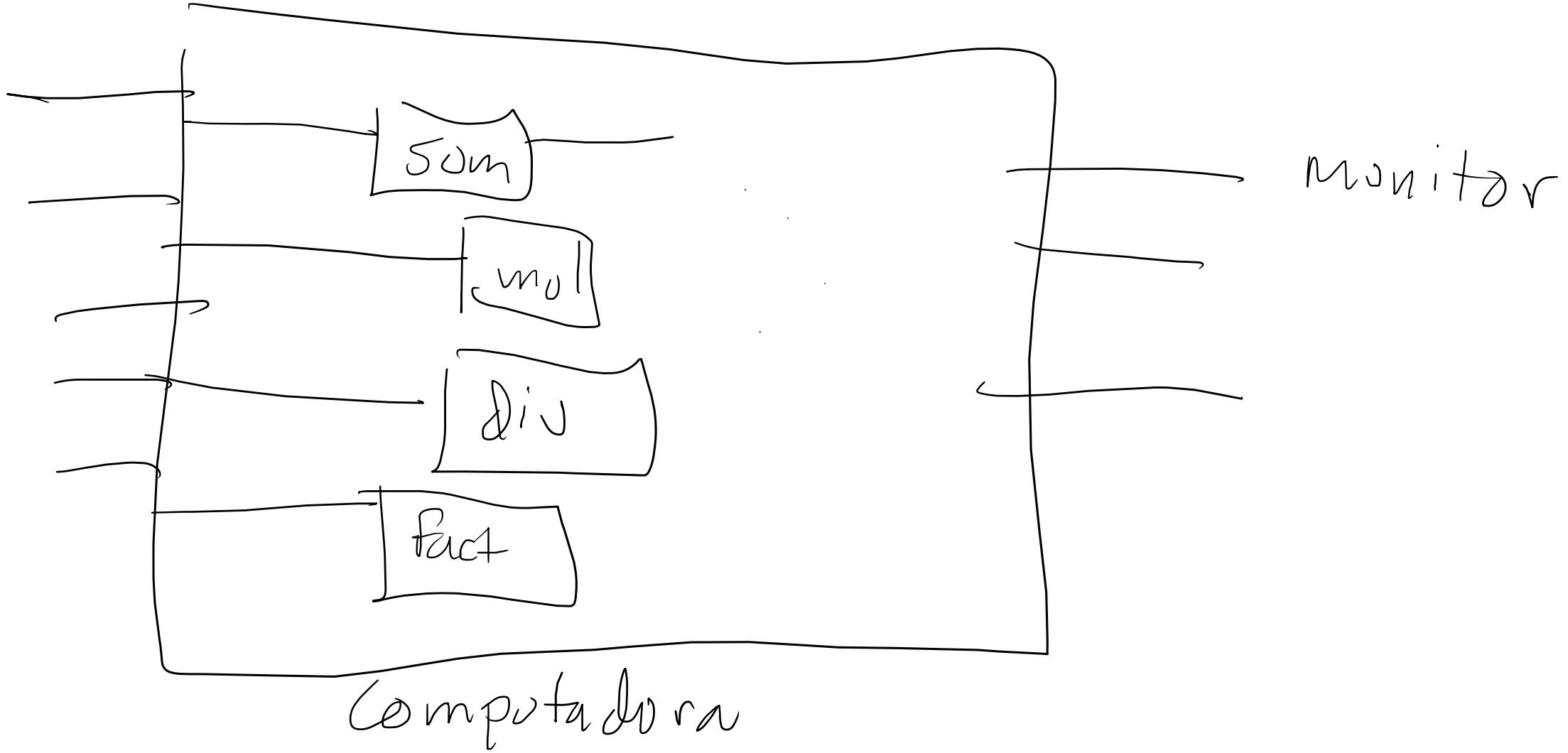
$$\begin{array}{l} 0+0 = 0 \\ 1+0 = 1 \\ 0+1 = 1 \\ 1+1 = 10 \end{array}$$

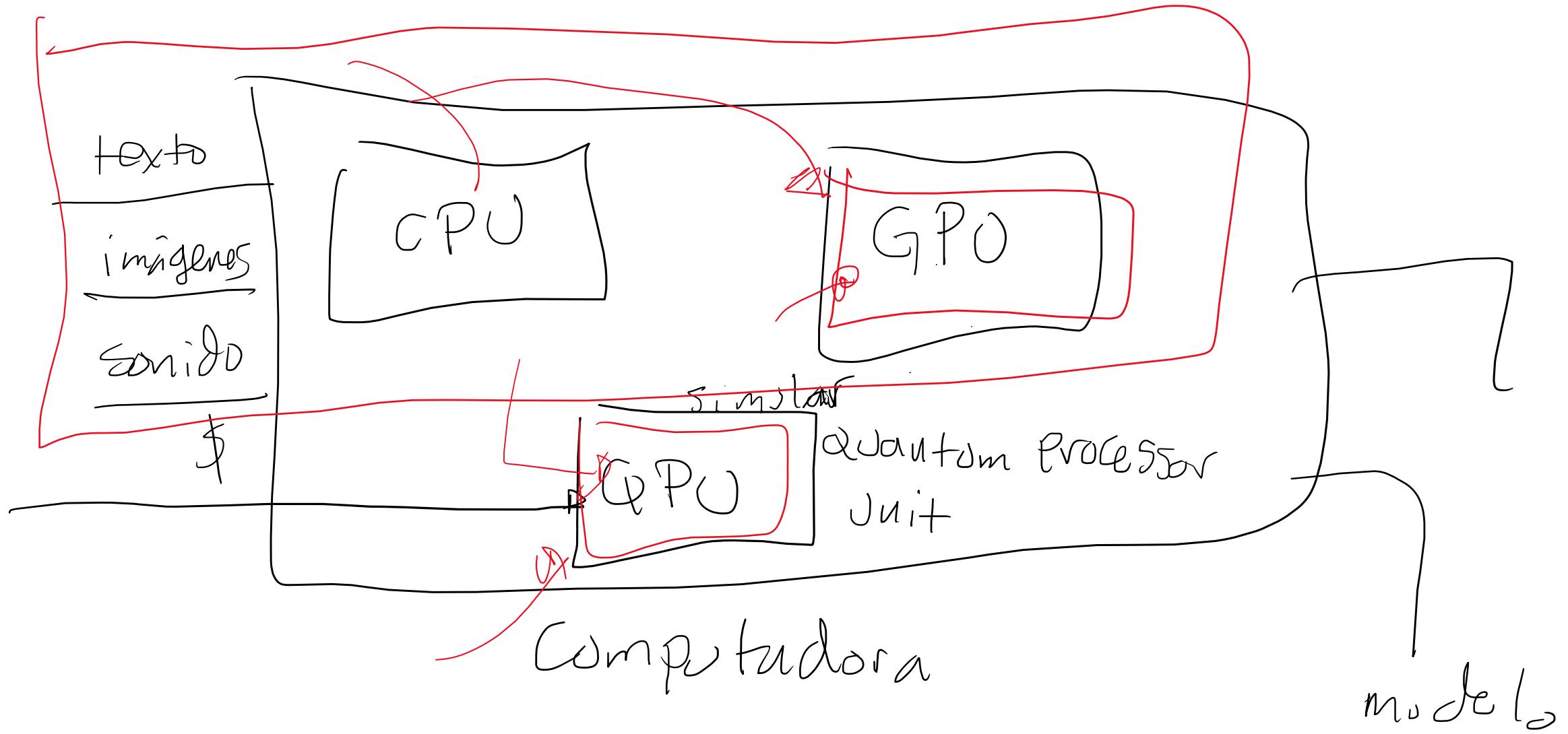
$$= A + B = 0 \\ 1$$

Bits  
salida



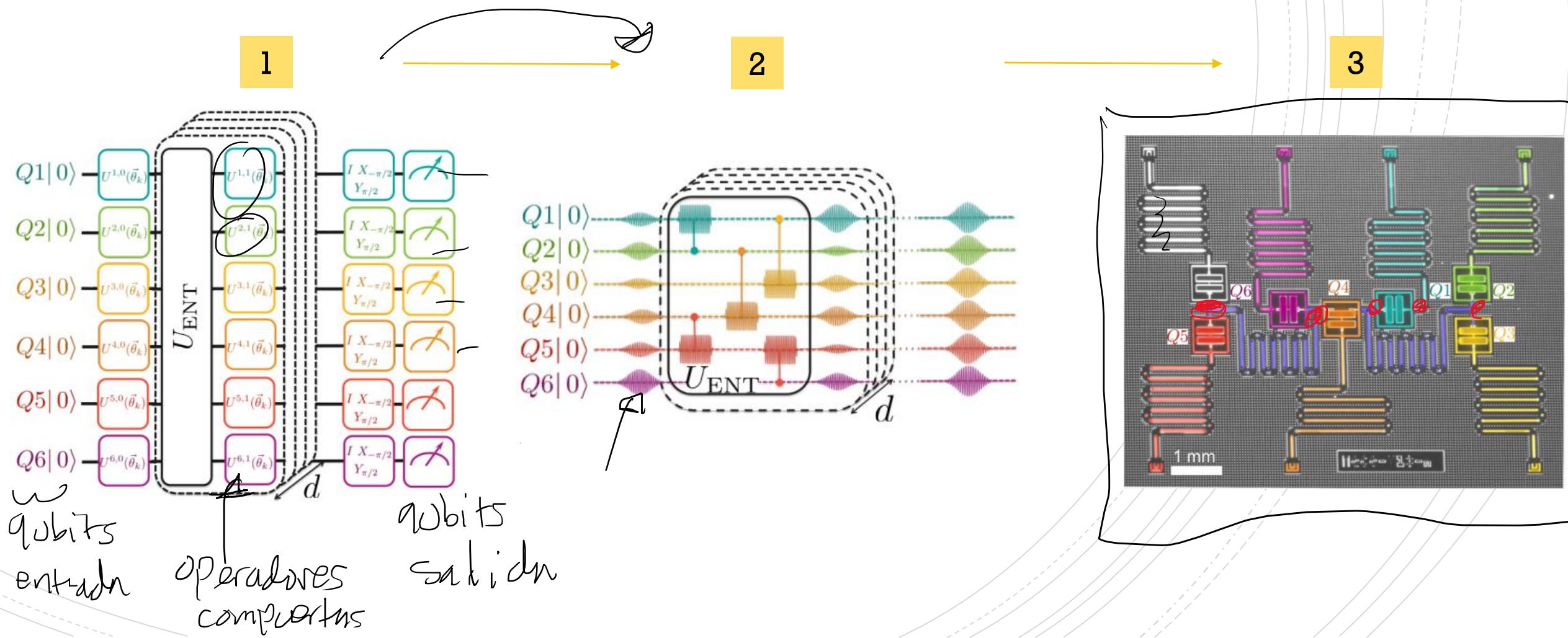
I/O



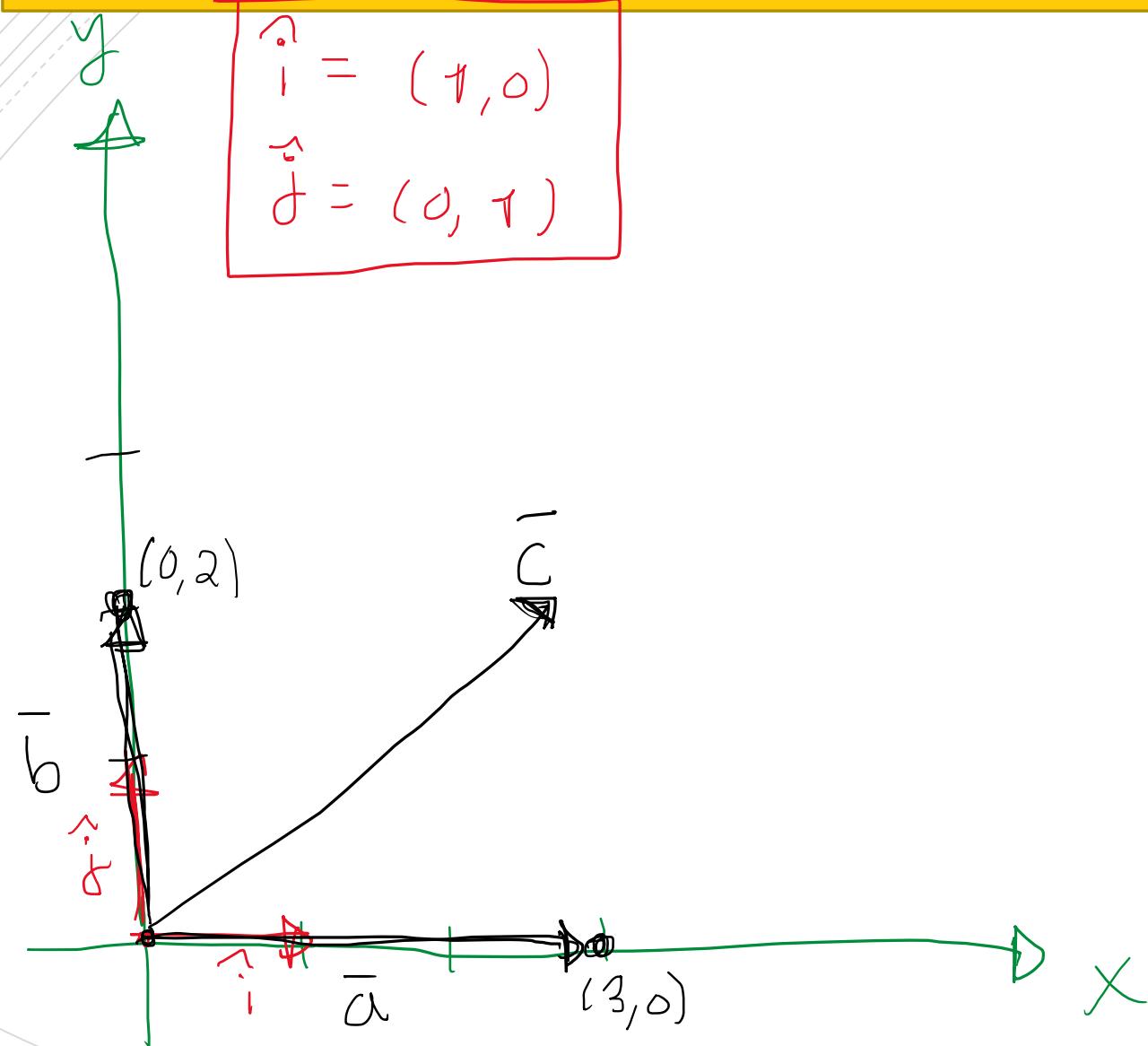


# Computadora cuántica

## Niveles de abstracción



## Vectores en $R^2$ y “notación de Dirac”



$$(3, 0), (0, 2)$$

$$\bar{a} = 3 \hat{i}$$

$$\bar{b} = 2 \hat{j}$$

$$\bar{c} = \bar{a} + \bar{b}$$

$$\bar{c} = 3 \hat{i} + 2 \hat{j}$$

cuantico

Ket

$$|c\rangle = 3 |\hat{i}\rangle + 2 |\hat{j}\rangle$$

$$|c\rangle = 3 |0\rangle + 2 |1\rangle$$

# Elementos básicos de un algoritmo cuántico

Estados cuánticos

$|\psi\rangle :=$  Estado cuántico (Ket)

$$\{ |0\rangle, |1\rangle \}$$

Base computacional  
Para 1 qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\alpha, \beta \in \mathbb{C}$

$$|\psi_1\rangle = |0\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$|\psi_2\rangle = |1\rangle$$

Condición de  
normalización

$\alpha, \beta :=$  amplitudes de probabilidad

$|\alpha|^2 :=$  Probabilidad del estado  $|\psi\rangle$  de estar en  $|0\rangle$

# Espacio de Hilbert (pre-Hilbert)

$$\{|0\rangle, |1\rangle\} \quad \text{2 = 2}$$

① un qubit

Base computacional

$$|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

(vector, vector de estado, qubit)

• Base Computacion  $\{|0\rangle, |1\rangle\}$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|a\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix} = \boxed{\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}$$

$$|a\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

Ket  $|a\rangle$

Bra  $\langle a|$

$$|a\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \Rightarrow$$

Conjugada

$$\langle a| = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}^{\star T}$$

Vector columna

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T = (1, 0)$$

vector renglón

$$\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T = (0, 1)$$

$$|a\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \quad \text{• Ket} \quad \text{• } \cancel{\text{ret }} |a\rangle$$

$$\langle a | = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} \langle 0 | + \frac{1}{\sqrt{2}} \langle 1 | \quad \text{• Bra}$$

$$|b\rangle = \frac{i}{\sqrt{3}} |0\rangle - i\sqrt{2/3} |1\rangle$$

$$\langle b | = \left( \frac{i}{\sqrt{3}} \langle 0 | \right)^* - \left( i\sqrt{2/3} \langle 1 | \right)^*$$

$$|b\rangle = \frac{i}{\sqrt{3}} |0\rangle - i\sqrt{2/3} |1\rangle$$

$$\langle b| = \left( \frac{i}{\sqrt{3}} \langle 0| \right)^{*T} - \left( i\sqrt{2/3} \langle 1| \right)^{*T}$$

Let  $|a\rangle$

$$z = \alpha + \beta i$$

$$\bar{z}^* = \alpha - \beta i$$

$$\alpha, \beta \in \mathbb{R}$$

$$(|0\rangle)^{*T} = \langle 0|$$

$$(|1\rangle)^{*T} = \langle 1|$$

$$\langle b| = \frac{-i}{\sqrt{3}} \langle 0| - i\sqrt{2/3} \langle 1|$$

Bra

$$\langle b| = \frac{-i}{\sqrt{3}} \langle 0| + i\sqrt{2/3} \langle 1|$$

• Normal  $|\psi\rangle$

$$|\psi| = \sqrt{\langle\psi|\psi\rangle}$$

$$\langle\psi|\psi\rangle =$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\psi| = ?$$

•  $|\alpha\rangle$  y  $|\psi\rangle$  son diferentes?

$$|\psi\rangle = \sqrt{\langle\psi|\psi\rangle}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} \end{pmatrix}$$

$$\langle\psi| = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}^T = (1/\sqrt{2}, -i/\sqrt{2})$$

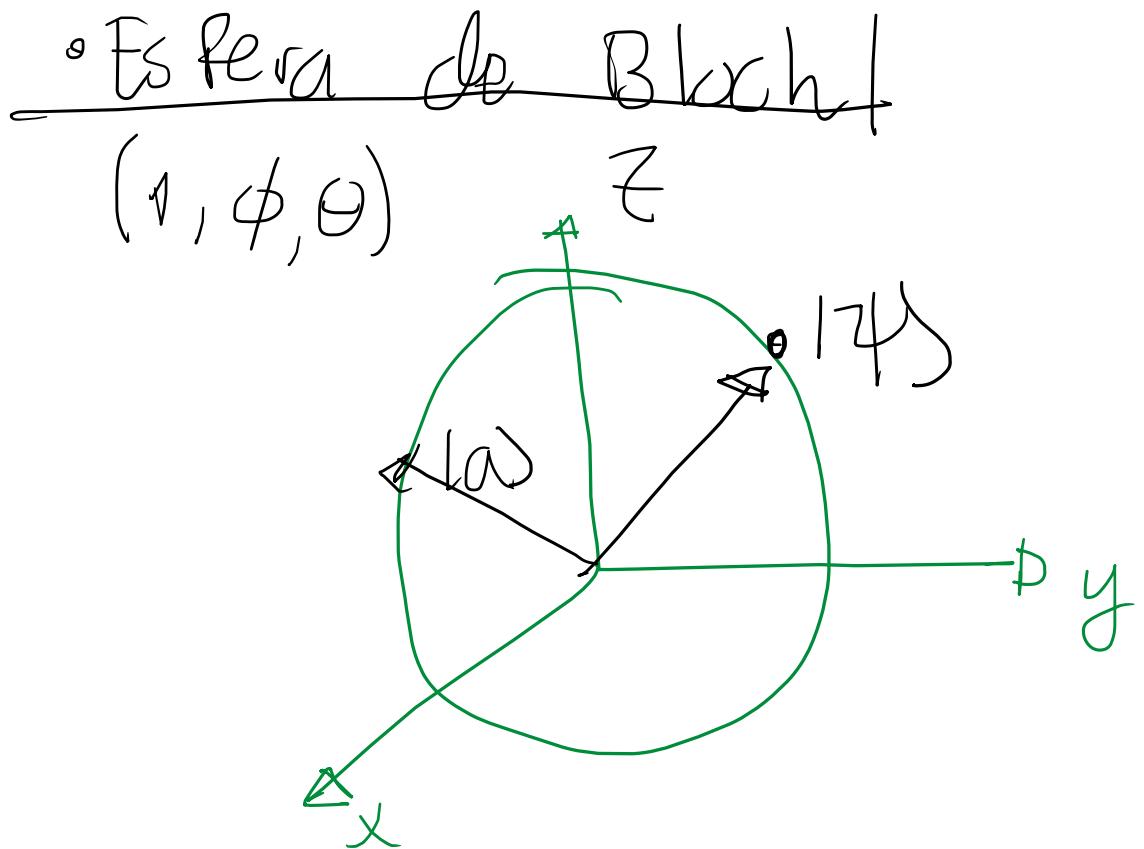
$$\langle\psi|\psi\rangle = (1/\sqrt{2}, -i/\sqrt{2}) \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix} = (1/\sqrt{2})(1/\sqrt{2}) + (-i/\sqrt{2})(-i/\sqrt{2})$$

$$|\psi\rangle = \sqrt{\langle\psi|\psi\rangle} = \sqrt{1} = 1 \checkmark$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$



$$|\psi\rangle = \underline{\alpha}|0\rangle + \underline{\beta}|1\rangle$$

$$|\psi\rangle = \underline{\cos\theta}|0\rangle + e^{\frac{i\phi}{2}} \underline{\sin\theta}|1\rangle$$

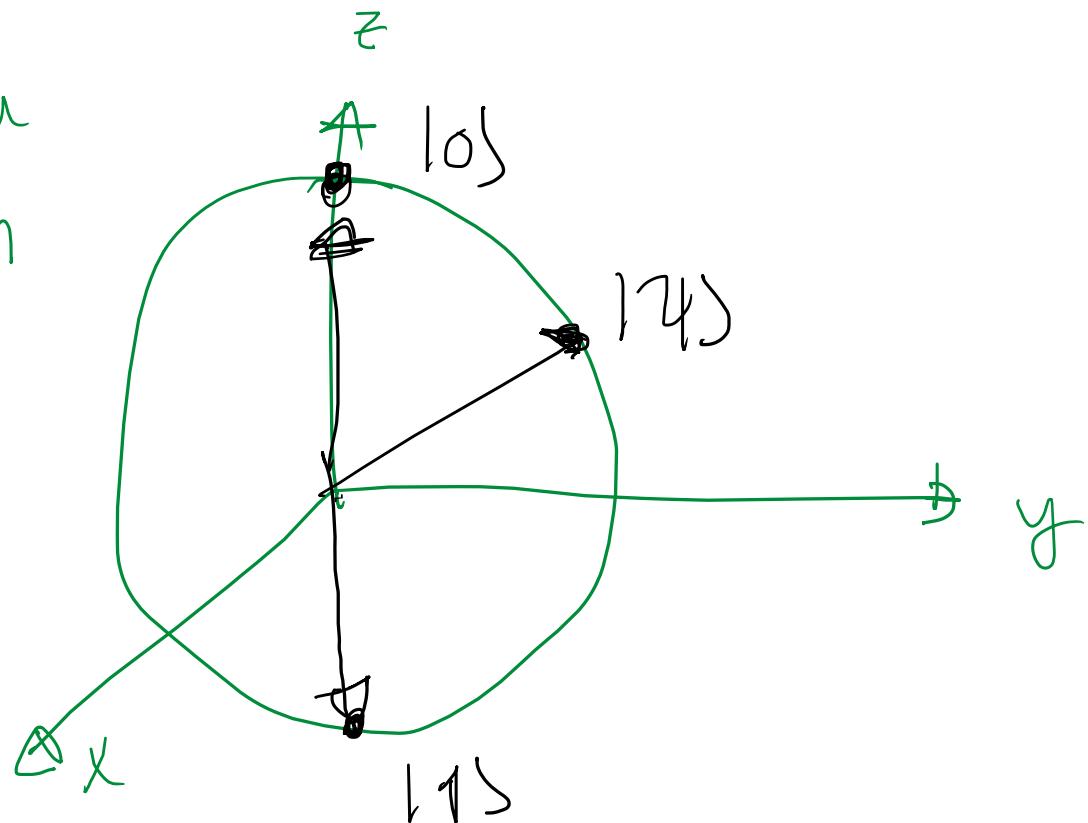
$$\alpha, \beta \sim \theta, \phi$$

$$|OS\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow |\sim OS\rangle + |\sim IS\rangle$$

$$|IS\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\Psi\rangle = \cos\theta |OS\rangle + e^{i\phi} \sin\theta |IS\rangle$$

Espera  
Bloch



# Herramientas matemáticas

## Compuertas cuánticas

$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  := Matriz de Pauli  $z$

$$\langle 0|0\rangle = \langle 1|0\rangle \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$$\langle 0|1\rangle = 0$$

$$\langle 1|0\rangle = 0$$

$$\langle 1|1\rangle = 1$$

$$\hat{\sigma}_z |0\rangle = |0\rangle \quad \iff$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_z |1\rangle = -|1\rangle$$

$$\langle 0|\hat{\sigma}_z|0\rangle = 1_0 \quad \langle 0|\hat{\sigma}_z|1\rangle = 2_0$$

$$\langle 1|\hat{\sigma}_z|0\rangle = 3_0 \quad \langle 1|\hat{\sigma}_z|1\rangle = 4_0$$

$$\hat{\sigma}_z |0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} (1)(1) + (0)(0) \\ (0)(1) + (-1)(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underline{|0\rangle}$$

$$\hat{\sigma}_z |1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 + 0 \\ 0 + -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= -1 |1\rangle$$

$$= \underline{-|1\rangle}$$

① Cómo actúan los operadores

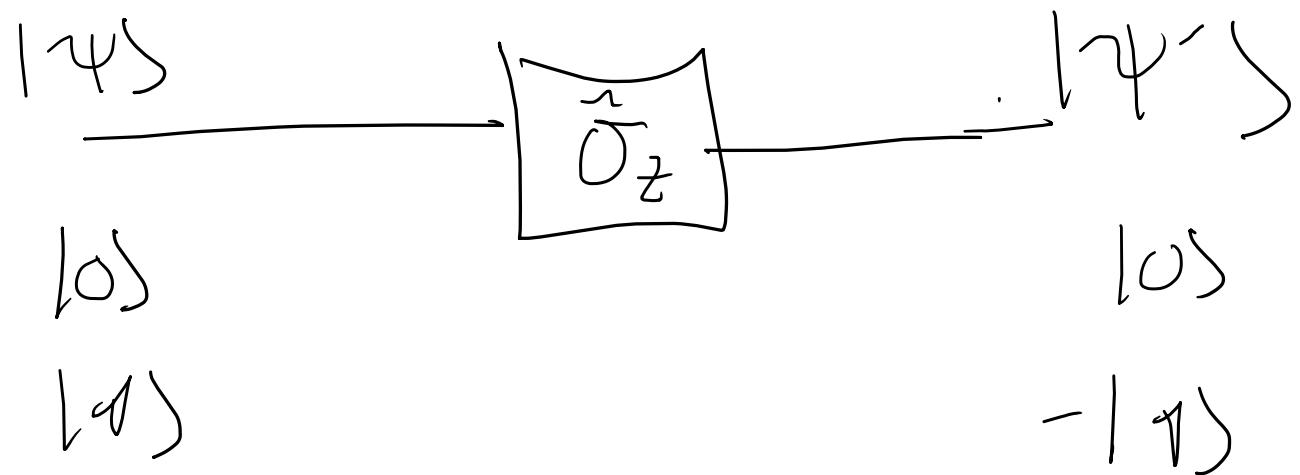
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\hat{A}|\psi\rangle = \hat{A}(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha\hat{A}|0\rangle + \beta\hat{A}|1\rangle$$

||

• Modelos de circuito



$$|a\rangle = \sqrt{2} |\phi\rangle + \sqrt{2} |\gamma\rangle$$

$$\hat{O}_z |a\rangle = \sqrt{2} \hat{O}_z |\phi\rangle + \sqrt{2} \hat{O}_z |\gamma\rangle$$

$$|\alpha\rangle = \sqrt{2} |\psi\rangle + \sqrt{2} |\tau\rangle$$

$$\hat{\sigma}_z |\alpha\rangle = \sqrt{2} \hat{\sigma}_z |\psi\rangle + \sqrt{2} \hat{\sigma}_z |\tau\rangle$$

$$= \sqrt{2} |\psi\rangle + \sqrt{2} (-|\tau\rangle)$$

$$= \sqrt{2} |\psi\rangle - \sqrt{2} |\tau\rangle$$

$$\hat{\sigma}_z |\psi\rangle = |\psi\rangle$$

$$\hat{\sigma}_z |\tau\rangle = -|\tau\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\phi\rangle + \frac{1}{\sqrt{2}} |\psi\rangle$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_z |\psi\rangle = \hat{\sigma}_z \left( \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix} \right)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} |\phi\rangle - \frac{1}{\sqrt{2}} |\psi\rangle$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = |\phi\rangle - |\psi\rangle$$

\* Matrices de Pauli ( $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$ )

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{\sigma}_x |s\rangle =$$

$$\hat{\sigma}_x |y\rangle =$$

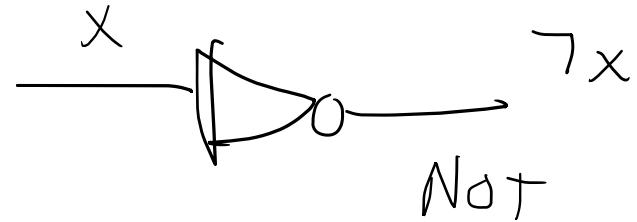
$$\hat{\sigma}_y |s\rangle =$$

$$\hat{\sigma}_y |y\rangle =$$

$$\hat{\sigma}_x |0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad \hat{\sigma}_x |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\hat{\sigma}_x |0\rangle = |1\rangle ; \quad \hat{\sigma}_x |1\rangle = |0\rangle$

análogo cuántico a



{ AND, NOT, COPY }

XOR, OR, NAND

$$\hat{\sigma}_y |0\rangle = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i \begin{pmatrix} 0 \\ 1 \end{pmatrix} = i |1\rangle$$

$$\hat{\sigma}_y |1\rangle = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix} = -i \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -i |0\rangle$$

$$\begin{aligned}\hat{\sigma}_y |0\rangle &= i |1\rangle \\ \hat{\sigma}_y |1\rangle &= -i |0\rangle\end{aligned}$$

◦ Identidad

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{I}|0\rangle = |0\rangle$$

$$\hat{I}|1\rangle = |1\rangle$$

# Herramientas matemáticas

## Esfera de Bloch

# Herramientas matemáticas

## Medición de los estados cuánticos

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

$\alpha, \beta$  := amplitudes de probabilidad  
(son complejas en general)

$|\alpha|^2$  = Probabilidad del estado  $|\psi\rangle$  de estar en  $|0\rangle$

$|\beta|^2$  = , , , , , , , , ,  $|1\rangle$

$$P(|0\rangle) = |\langle 0 | \psi \rangle|^2$$

¿Cuál es la probabilidad del estado  $|\psi\rangle$  de estar en  $|0\rangle$ ?  
Regla de Born

$$P_{|\psi\rangle}(|0\rangle) = |\langle 0 | \psi \rangle|^2 = \left| \langle 0 | (\underbrace{\alpha |0\rangle + \beta |1\rangle}_{\text{Red wavy line}}) \right|^2$$

$$= |\alpha \langle 0 | 0 \rangle + \beta \langle 0 | 1 \rangle|^2$$

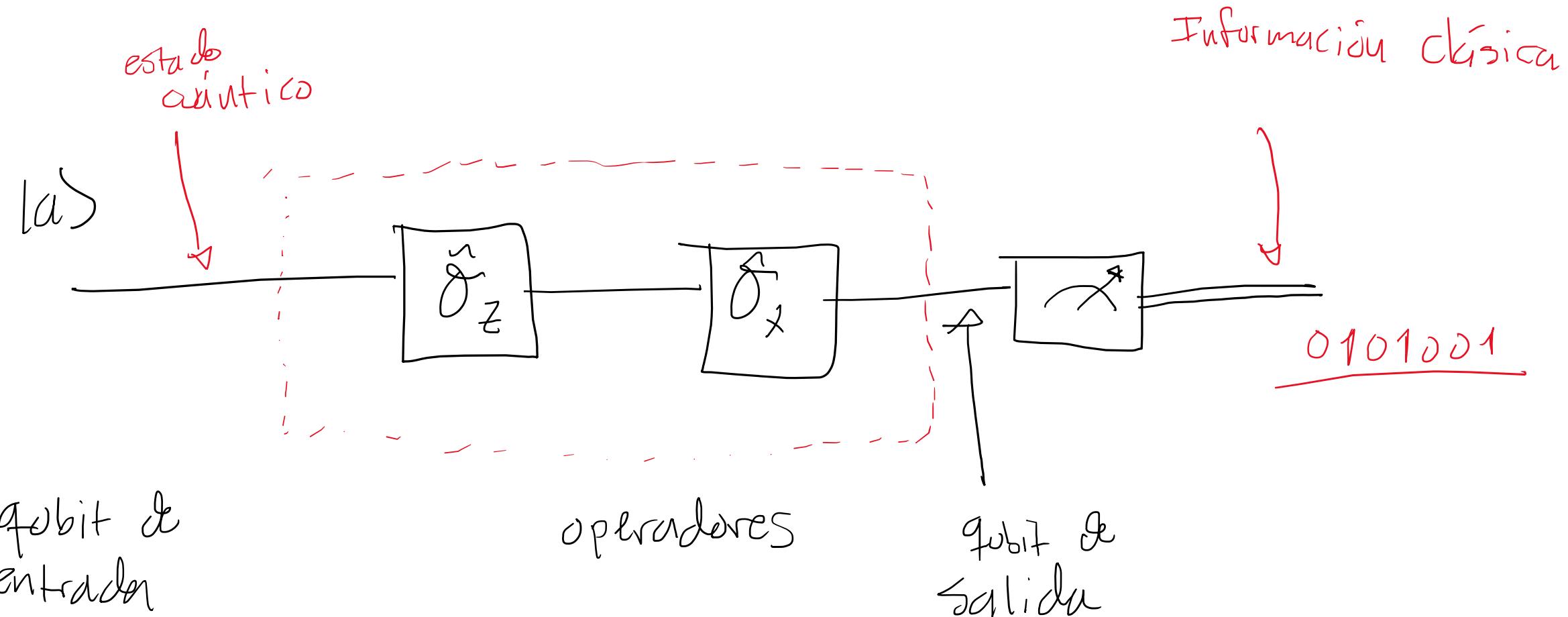
$$= |\alpha(1) + \beta(0)|^2$$

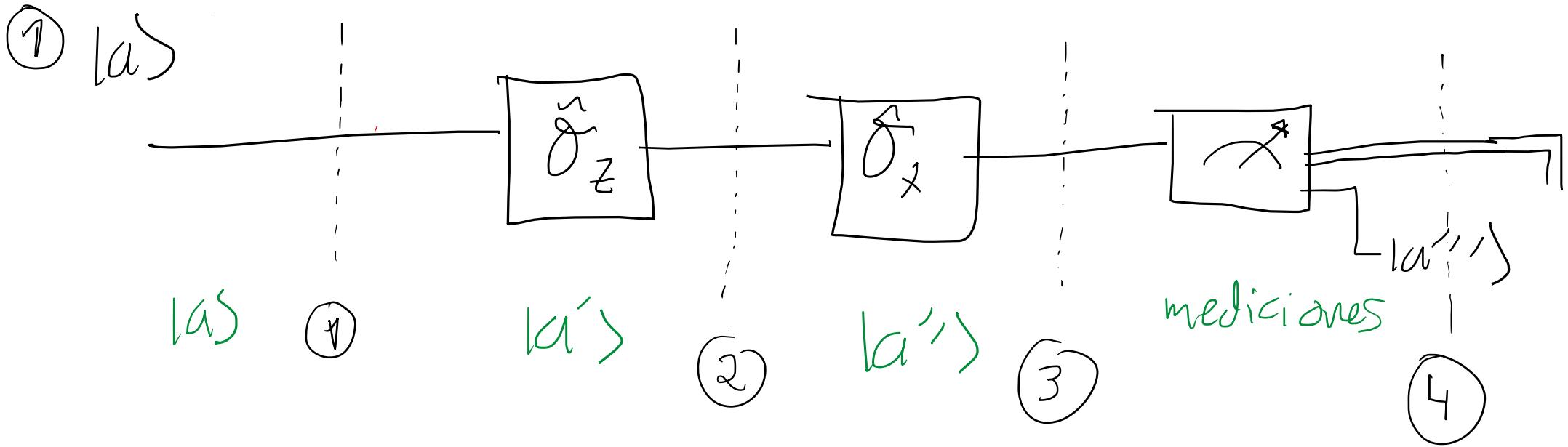
$$= \underline{|\alpha|^2}$$

$$P_{|\psi\rangle}(|1\rangle) = |\langle 1 | \psi \rangle|^2 = \underline{|\beta|^2}$$

$$|\alpha\rangle = \frac{1}{\sqrt{3}}|0\rangle - i\sqrt{\frac{2}{3}}|1\rangle$$

$$|\alpha|=1$$





②  $\hat{\sigma}_z |a\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{2/3}|1\rangle = |a'\rangle$

③  $\hat{\sigma}_x |a'\rangle = \boxed{\frac{1}{\sqrt{3}}|1\rangle + i\sqrt{2/3}|0\rangle = |a''\rangle}$

④  $P_{|a''\rangle}(|0\rangle), P_{|a''\rangle}(|1\rangle)$

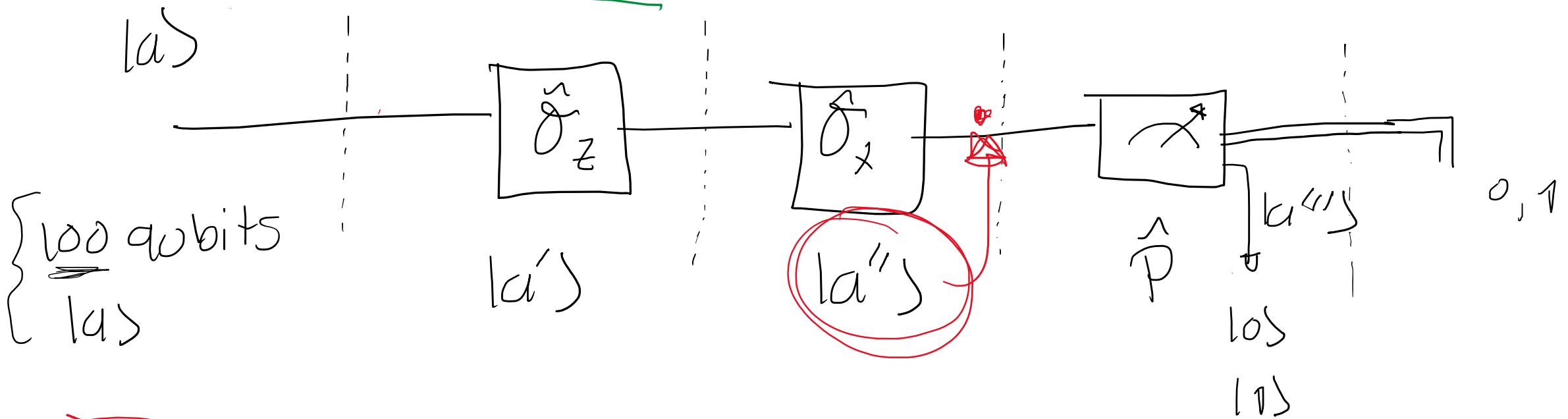
$$P_{|a''\rangle}(|0\rangle) = |\langle 0 | a'' \rangle|^2 = |\langle 0 | \left( \frac{1}{\sqrt{3}} |1\rangle + i\sqrt{\frac{2}{3}} |0\rangle \right) |^2$$

$$= \left| i\sqrt{\frac{2}{3}} \right|^2 = \left| i \right| \left| \sqrt{\frac{2}{3}} \right|^2 = \textcircled{2/3} =$$

$$P_{|a''\rangle}(|1\rangle) = |\langle 1 | a'' \rangle|^2 = |\langle 1 | \left( \frac{1}{\sqrt{3}} |1\rangle + i\sqrt{\frac{2}{3}} |0\rangle \right) |^2$$

$$= \left| \frac{1}{\sqrt{3}} \right|^2 = \textcircled{1/3}$$

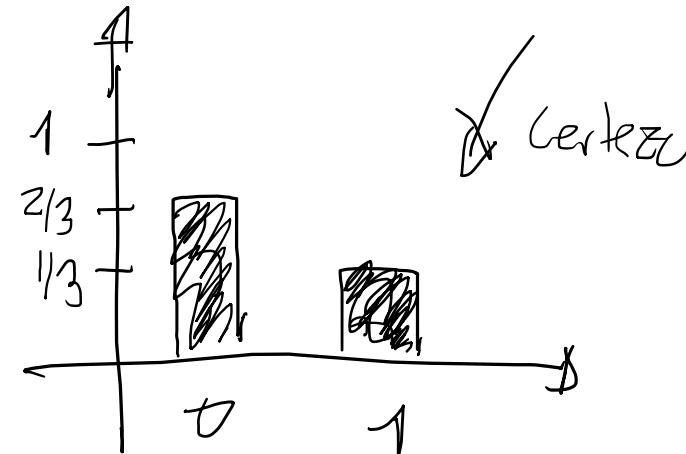
• "Colapso del estado después de medir"



~~Aleatorio~~

$$\hat{P}|a''\rangle = |0\rangle \quad ?$$

$$\hat{P}|a''\rangle = |1\rangle$$



# Herramientas matemáticas

Más compuertas de 1 qubit

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\hat{P} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$\begin{aligned}\hat{H}|0\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \\ &= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle\end{aligned}$$

apliquen 2 veces  $\hat{S}$

$$\hat{S} = \sqrt{\hat{\sigma}_z} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$
$$\hat{T} = \sqrt{\hat{S}} = \sqrt[4]{\hat{\sigma}_z} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

apliquen  
4 veces  $\hat{T}$

$$|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$$

$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

① Base computacional  $\{|0\rangle, |1\rangle\}$   $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

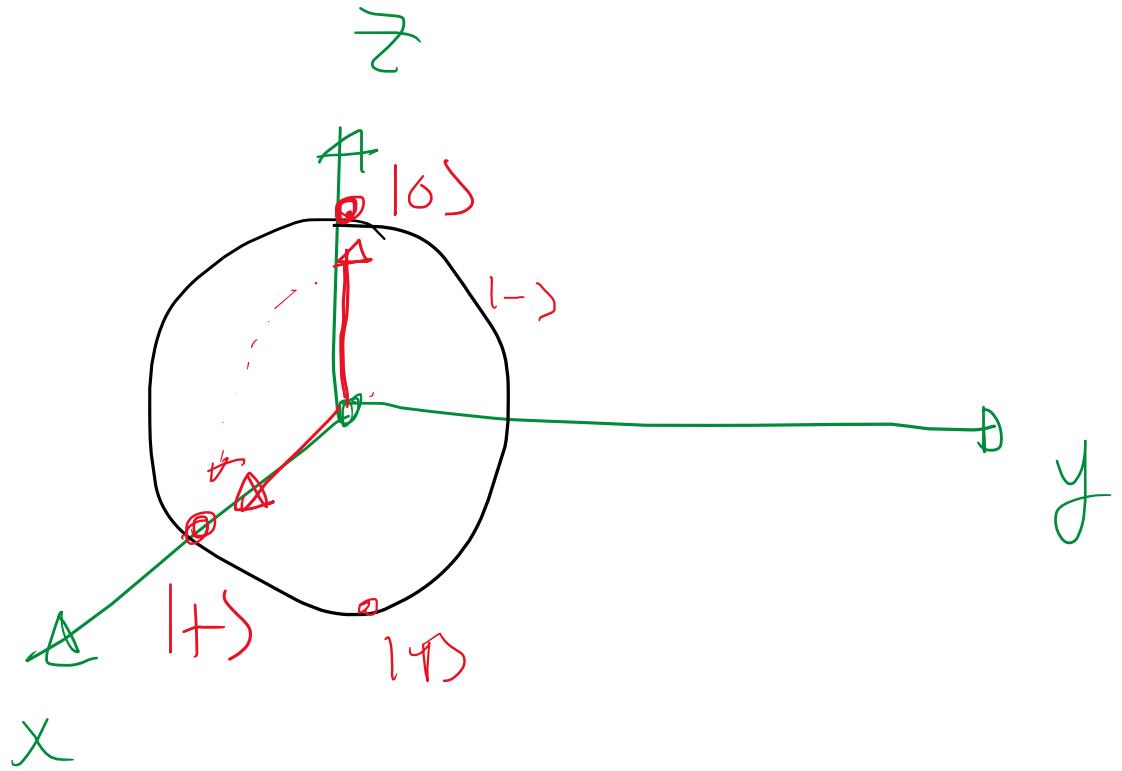
② Base  $\hat{\sigma}_x$   $\{|\+\rangle, |\-\rangle\}$   $|\psi\rangle = \gamma |\+\rangle + \delta |\-\rangle$

$$\hat{P} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}; \quad |\psi\rangle = \underbrace{\alpha|0\rangle + \beta|1\rangle}_{+}$$

$$\hat{P}|\psi\rangle = \hat{P}(\alpha|0\rangle + \beta|1\rangle) = \alpha \hat{P}|0\rangle + \beta \hat{P}|1\rangle$$

$$= \alpha \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{P}|\psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ e^{i\theta} \end{pmatrix} = \alpha|0\rangle + \beta e^{i\theta} \underline{|1\rangle}$$



$$\hat{H}|0\rangle = |+\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

# Herramientas matemáticas

## Operación daga

º Bra, Ket

$$\langle 0 | = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^* {}^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^+ = | 0 \rangle^+$$

$$| 1 \rangle^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^* {}^T = (0 \ 1) = \langle 1 |$$

$$\tilde{\sigma}_z^+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{*T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tilde{\sigma}_z^-$$

$$\tilde{\sigma}_y^+ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}^{*T} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \tilde{\sigma}_y^-$$

$$\tilde{H}^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{*T} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \tilde{H}^-$$

$$\tilde{P}^+ = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}^{*T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

# Herramientas matemáticas

## Producto tensorial y compuertas de 2 qubits

### • Producto tensorial

$$|a\rangle \otimes |b\rangle = |a\rangle |b\rangle = \underbrace{|ab\rangle}_{\text{Estado de 2 qubits}}$$

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \hat{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\hat{I} \otimes \hat{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2 \times 2} \otimes \begin{pmatrix} ab \\ cd \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 1(ab) & 0(ab) \\ 0(ab) & 1(ab) \end{pmatrix}_{4 \times 4}$$

$$\frac{1}{2} \hat{\sigma}_x \hat{\sigma}_z = \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{pmatrix}$$

$$\hat{\sigma}_z \hat{\sigma}_x = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}_{4 \times 4}$$

$$\hat{\sigma}_x \hat{\sigma}_z$$

$$\hat{\sigma}_z \hat{\sigma}_y$$

---


$$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{2 \times 1} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{2 \times 1} = \begin{pmatrix} 1(1) \\ 0(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{4 \times 1}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\underline{|\psi\rangle} \otimes |\gamma\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |\gamma\rangle$$

$$= \alpha|0\rangle \otimes |\gamma\rangle + \beta|1\rangle \otimes |\gamma\rangle$$

$$= \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle |\gamma\rangle = \alpha \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \alpha \underline{|01\rangle} + \beta \underline{|10\rangle}$$

• Base Computacional de 2 qubits  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$   $d=4$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

\* Productos punto en 2 qubits

$$|A\rangle = \alpha|01\rangle + \beta|11\rangle, \quad |B\rangle = \gamma|01\rangle$$

$$\langle A| = |A|^+ = (\alpha|01\rangle + \beta|11\rangle)^*{}^T = (\alpha^* \langle 01| + \beta^* \langle 11|)$$

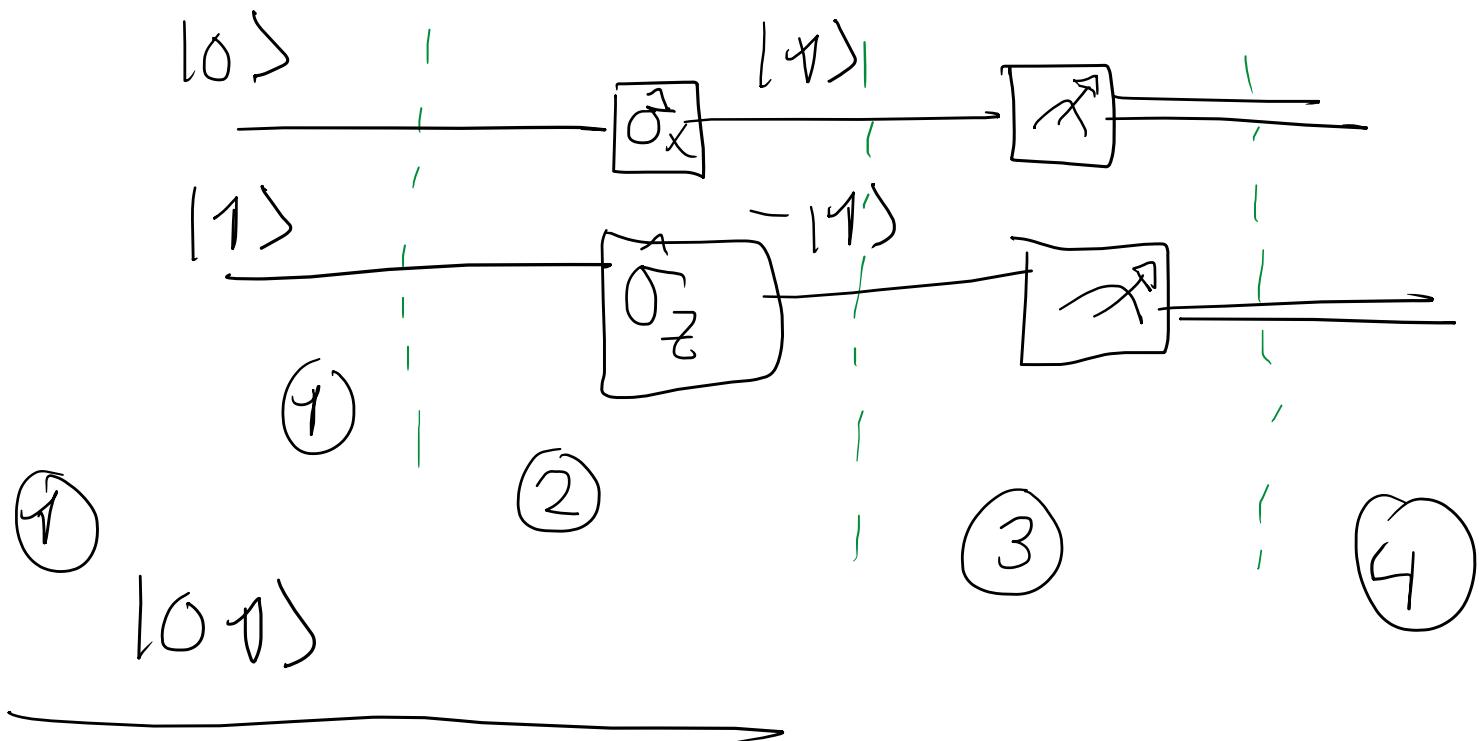
$$\langle A|B\rangle = (\alpha^* \langle 01| + \beta^* \langle 11|)(\gamma|01\rangle)$$

$$\langle A | B \rangle = (\alpha^* \underbrace{\langle 01 |}_{\text{red}} + \beta^* \langle 11 |}_{\text{red}}) \langle S | 01 \rangle$$

$$= \alpha^* \delta \langle 01 | 01 \rangle + \beta^* \delta \langle 11 | 01 \rangle$$

$$= \alpha^* \delta(1) + \cancel{\beta^* \delta(0)}$$

$$= \underline{\alpha^* \delta}$$



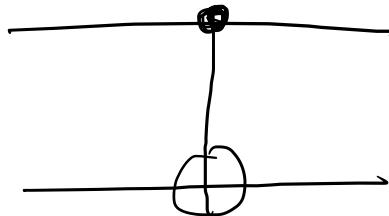
② -  $|11\rangle$

③) Mediciones

# Herramientas matemáticas

## Compuertas de 2 o más qubits

$$C-NOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$$(CNOT |00\rangle) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$



$$CNOT |01\rangle = \begin{pmatrix} 1000 \\ 0100 \\ 0001 \\ 0010 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |01\rangle$$

$$CNOT |10\rangle = \begin{pmatrix} 1000 \\ 0100 \\ 0001 \\ 0010 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle$$

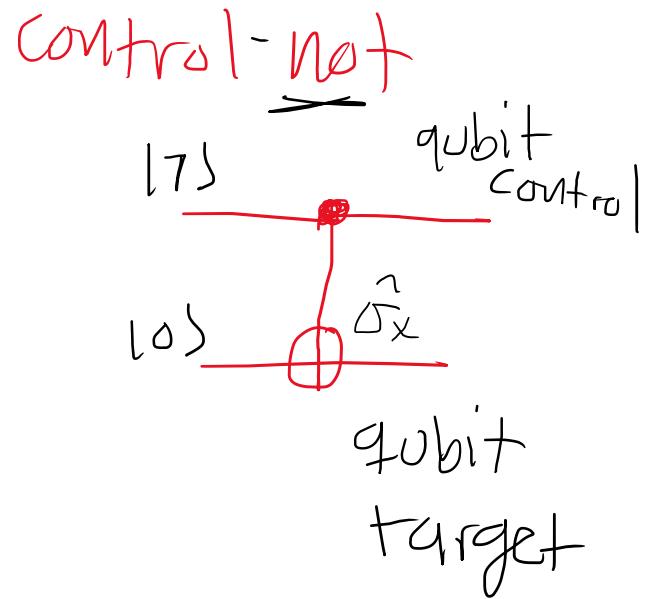
$$CNOT |11\rangle = \begin{pmatrix} 1000 \\ 0100 \\ 0001 \\ 0010 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |10\rangle$$

$$CNOT |00\rangle = |00\rangle$$

$$CNOT |01\rangle = |01\rangle$$

$$CNOT |10\rangle = |11\rangle$$

$$CNOT |11\rangle = |10\rangle$$



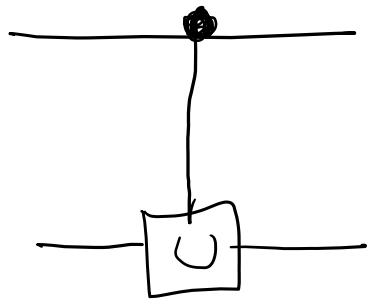
$$C-P = \begin{pmatrix} 10 & 00 \\ 01 & 00 \\ 00 & 10 \\ 00 & 0 e^{i\theta} \end{pmatrix}$$

$$C-P |00\rangle = |00\rangle$$

$$C-P |10\rangle = |10\rangle$$

$$CP |11\rangle = e^{i\theta} |11\rangle$$

C-U



$$C_U |00\rangle = |00\rangle$$

$$C_U |01\rangle = |01\rangle$$

$$C_U |10\rangle = |1\rangle \otimes \underline{U} |0\rangle$$

$$C_U |11\rangle = |1\rangle \otimes \underline{G} |1\rangle$$

# Enredamiento cuántico

## Estados productos y estados enredados

$$|00\rangle = |0\rangle \otimes |0\rangle$$

estado  
de  
2 qubits

↗  
estado producto de  
dos estados de 1 qubit

$$|\Psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$$

$$|\Psi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \otimes \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \alpha_2 \\ \alpha_1 \beta_2 \\ \beta_1 \alpha_2 \\ \beta_1 \beta_2 \end{pmatrix}$$

$$|\phi\rangle_2 = C_0 |00\rangle + C_1 |01\rangle + C_2 |10\rangle + C_3 |11\rangle$$

$$C_0 C_3 = C_1 C_2$$

Estados producto

$$|C_0|^2 + |C_1|^2 + |C_2|^2 + |C_3|^2 = 1$$

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|\underline{00}\rangle + |\underline{11}\rangle) \quad \checkmark$$

$$= \frac{1}{\sqrt{2}} |00\rangle + 0 |01\rangle + 0 |10\rangle + \frac{1}{\sqrt{2}} |11\rangle$$

$$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = (0)(0) \quad \begin{matrix} \checkmark \\ 0 \end{matrix} \text{ contradicción}$$

∴  $|\beta_{00}\rangle$  es un estado enredado

$$|\beta_{00}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$$

Sistema ①

$$|\psi_1\rangle$$

Sistema ②

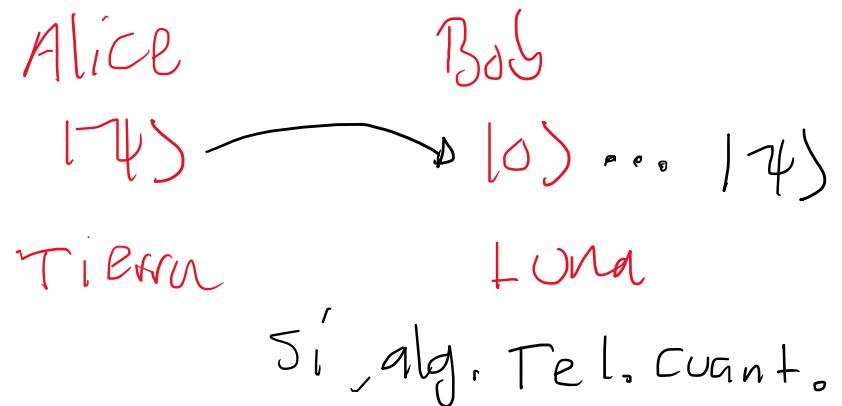
$$|\psi_2\rangle$$

Correlaciones en  
las propiedades del  
sistema global

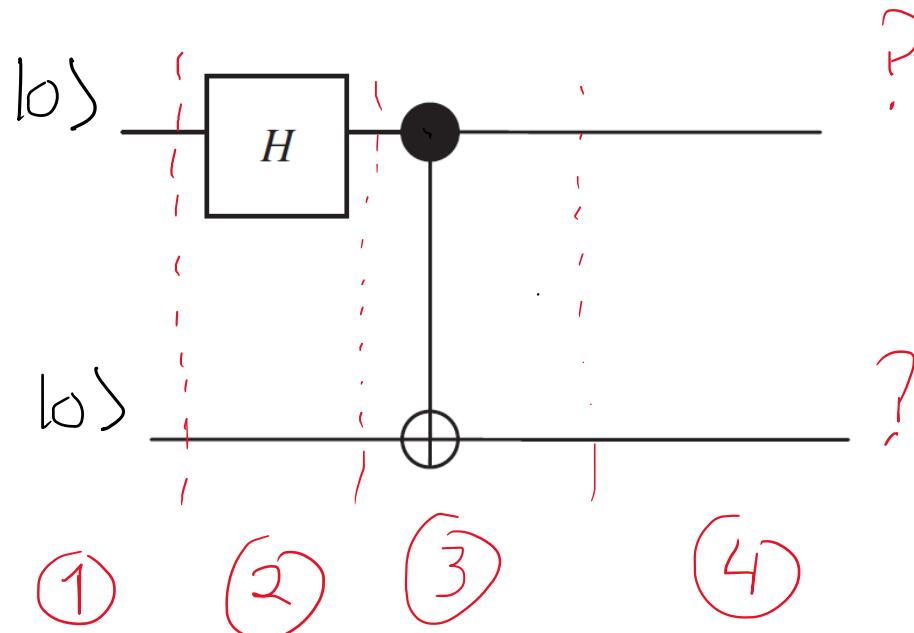
¿Cómo es el estado global de ambos sistemas?

a)  $|\psi_1\rangle \otimes |\psi_2\rangle$

b)  $\boxed{|\beta_{00}\rangle} \neq |\psi_1\rangle \otimes |\psi_2\rangle$



# Enredamiento cuántico



$$\begin{aligned}\hat{H}|0\rangle &= |+\rangle \\ &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\end{aligned}$$

①  $|0\rangle \otimes |0\rangle = |00\rangle$  Estados iniciales

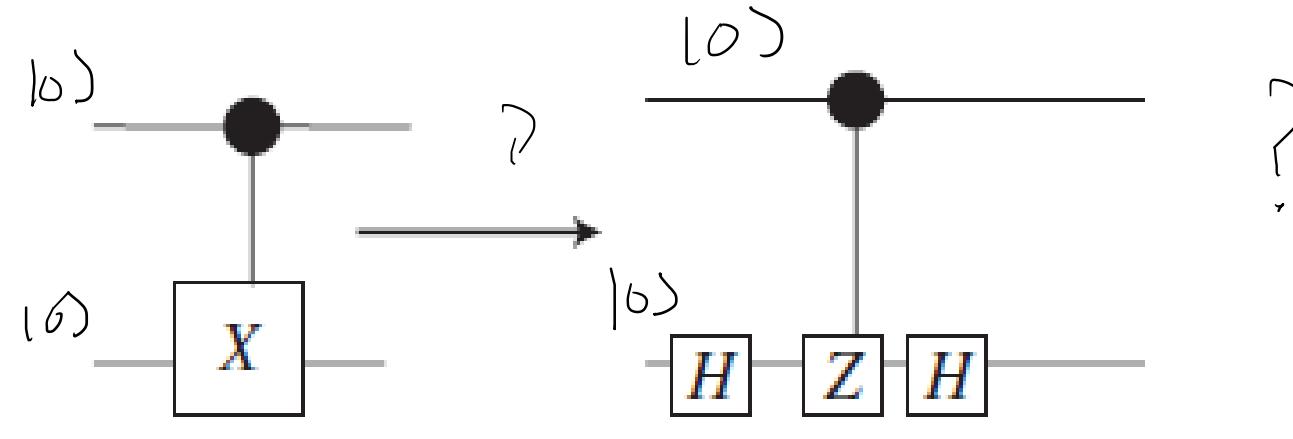
②  $\underbrace{\hat{H}_1}_{\text{Preparación}} \otimes \underbrace{\hat{I}_2}_{\text{del estado}} |00\rangle = \cancel{\hat{H}_1} \otimes \cancel{\hat{I}_2} |0\rangle \otimes |0\rangle = \underbrace{\hat{H}_1}_{\text{Preparación}} \otimes \underbrace{\hat{I}_2}_{\text{del estado}} |0\rangle$

$$\begin{aligned}
 \textcircled{2} \quad & \hat{H} \otimes \hat{\mathbb{I}} |00\rangle = \cancel{\hat{H}} \otimes \cancel{\mathbb{I}} |00\rangle = \cancel{\hat{H}} |00\rangle \otimes \cancel{\mathbb{I}} |0\rangle \\
 & = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |0\rangle + |1\rangle \otimes |0\rangle) \\
 & \qquad \qquad \text{of control} \qquad \qquad \text{of target} \\
 & = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad & \text{CNOT } \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}} (\text{CNOT } |00\rangle + \text{CNOT } |10\rangle) \quad \text{Compoto Cinti} \\
 & = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle \quad \text{Estado de Bell}
 \end{aligned}$$

# Herramientas matemáticas

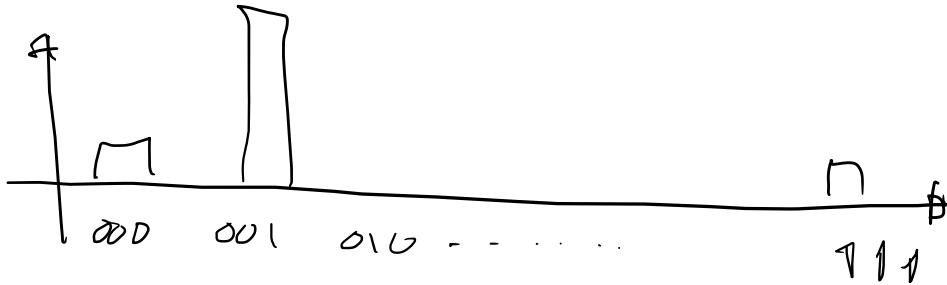
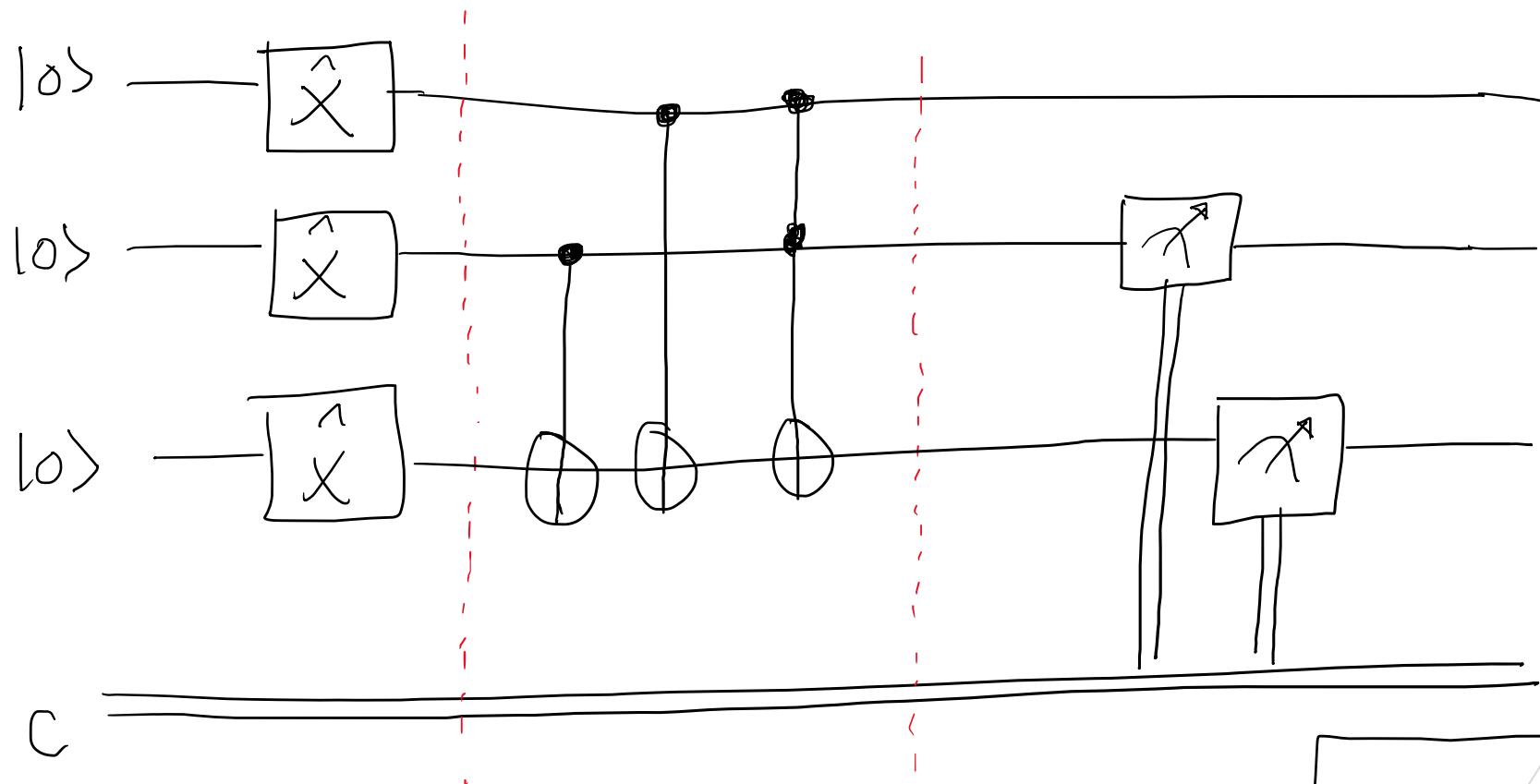
## Descomposición de compuertas



CNOT

$$\hat{H} - C - \hat{Z} - \hat{H}$$

# Algoritmo de suma de 3 bits

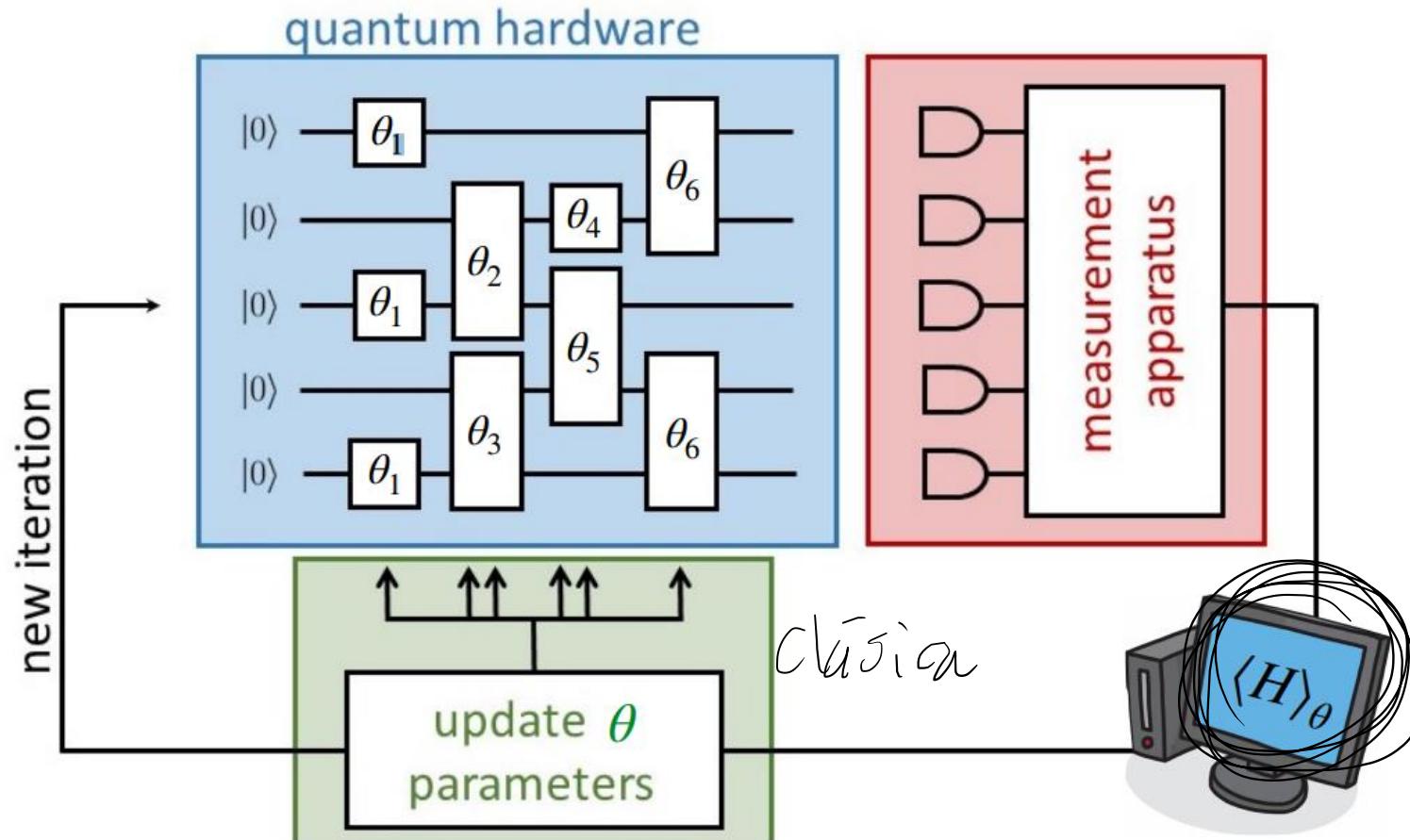


Toffoli, C-C-Not  
compuerta de 3  
qubits

$$2^3 = 8$$

# Aplicaciones (variational quantum eigensolver)

Peruzzo et al, Nat. Comm. '13



Quantum circuit as a variational ansatz