



Categorizing Filters

- Surface filtration (example – membrane filters)
 - Particles to be removed are larger than the pore size
 - Clog rapidly and large piezometric head required
 - Depth filtration (not well understood)
 - Particles to be removed may be much smaller than the pore size
 - Require collisions and attachment
 - Can handle more solids before developing excessive head loss
 - Filtration model coming...
- Require more energy
Our focus here

Filtration Outline

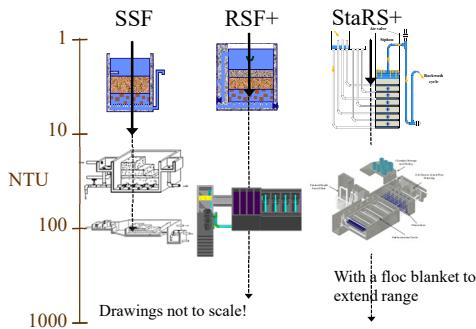
- Sand Filters
 - Slow sand (SSF)
 - Rough Gravel (RGF)
 - Dynamic Gravel (DGF)
 - Rapid (RSF)
 - Stacked Rapid (StaRS)
- Range of applicability
- Net Velocity
- Hydraulics
- Particle Capture theory
 - Transport
 - Dimensional Analysis
 - Model predictions
- Methods to improve performance

Depth Filtration Technologies

- Slow (Filters→English→Slow sand→“Biosand”)
 - First filters used for municipal water treatment
 - Were unable to treat the turbid waters of the Ohio and Mississippi Rivers
 - Can be used after rough gravel filters to extend range
 - Rapid (Mechanical→American→Rapid sand)
 - Used in Conventional Water Treatment Facilities
 - Used after flocculation and sedimentation
 - High flow rates→clog 12 – 90 hr→hydraulic cleaning
 - Stacked Rapid Sand (StaRS)
 - 6 x smaller than rapid sand and self backwashing
- scrape
backwash

“Biosand” is a misnomer based on a misunderstanding of SSF mechanisms. The preferred term is intermittent SSF (ISSF)

Filter range of applicability The “if it is dirty, filter it” Myth



Combining Rapid Sand Filtration and pretreatment

- Conventional: Coagulation – Flocculation – Sedimentation – Filtration
 - Able to treat very high turbidities because filter is the polishing step
 - Effluent from sedimentation should be less than 2 NTU
- Direct: Coagulation - Filtration
 - An option if raw water turbidity is always low
 - less than 5 NTU

Slow Sand Filtration

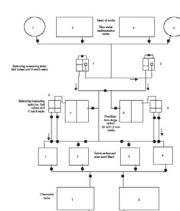


- First filters to be used on a widespread basis
- Fine sand with an effective size of 0.2 mm
- Low flow rates (0.03-0.12 mm/s) Compare with sedimentation
- Schmutzdecke (filter cake) forms on top of the filter
 - causes high head loss
 - must be removed periodically
- Used without coagulation/flocculation!
- Turbidity should always be less than **10 NTU** with a lower average to prevent rapid clogging

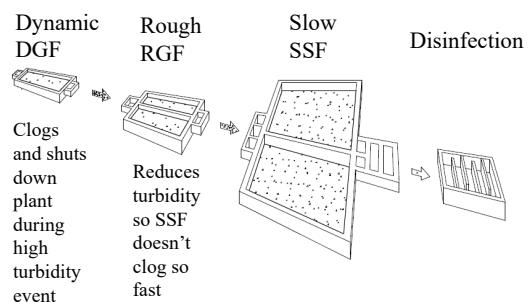


Multistage Filtration

- Another option for communities using surface waters
- Uses no coagulants (if lucky)
- Gravel roughing filters
- Polished with slow sand filters
- Large capital costs for construction
- No coagulant costs
- Labor intensive operation



Multiple Stage Filtration



Assembling Multiple Stage Filters

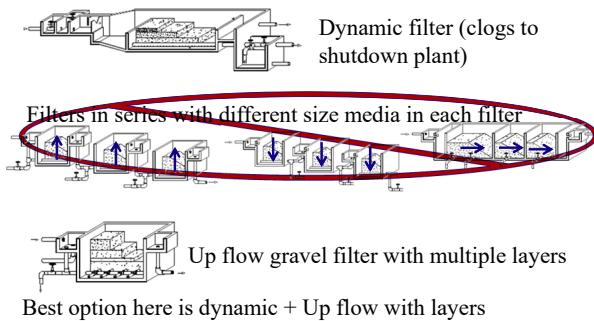




Jesus de Otoro, Honduras: Dynamic Filters

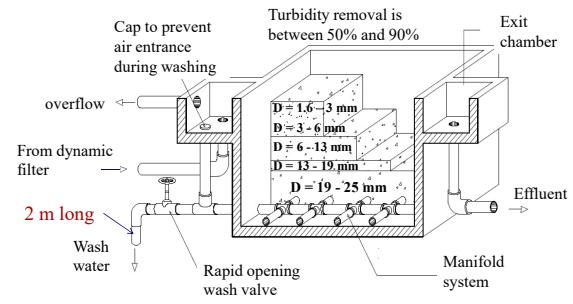


Multiple Stage Filtration Options for pretreatment for SSF



Up flow Roughing filter

0.08 to 0.17 mm/s
50x
5.5 mm/s during wash



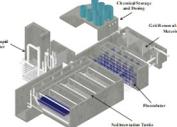
First iteration of covering a roughing filter...



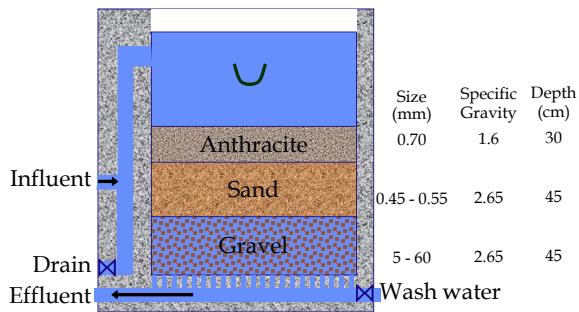
Covered to reduce growth of algae. What is the next logical step?

System Comparison: Multistage vs. Aguacalara water treatment

- Multistage has more expensive hardware (many large valves) and requires 10 to 20 times more land area
- Multistage doesn't use coagulants
- Multistage performs poorly with turbid water
- When might multistage be better than Aguacalara?

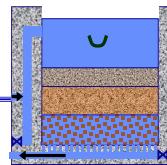


Rapid Sand Filter (Conventional US Treatment)



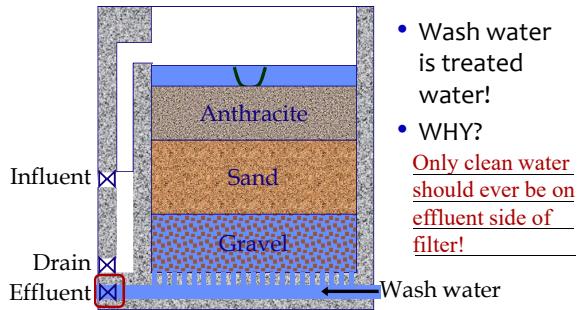
Rapid Sand Filter Design

- Filter media
 - silica sand and anthracite coal
 - non-uniform media will stratify with smaller particles at the top
 - Filtration approach velocities
 - 0.7 – 2.8 mm/s
 - Backwash approach velocities
 - set to obtain a bed porosity, ϕ , of 0.65 to 0.70
 - Fluidized bed
- Similar in size to Aguac Clara Sed (1 mm/s)



Does water speed up or slow down when it enters the sand?

Backwash (Fluidized Bed)



Net Velocity Concept for estimating plant size

$$A_{Dynamic} = \frac{Q}{V_{Dynamic}}$$

$$A_{Total} = \frac{Q}{V_{Dynamic}} + \frac{Q}{V_{Rough}} + \frac{Q}{V_{Slow}}$$

$$V_{Total} = \frac{Q}{A_{Total}}$$

$$V_{Total} = \frac{1}{\frac{1}{V_{Dynamic}} + \frac{1}{V_{Rough}} + \frac{1}{V_{Slow}}}$$

Use this velocity to determine approximate total unit process area. Similar approach for sizing Aguac Clara facility for comparison

Filter type	Velocity (mm/s)	Cleaning	Max (NTU)	pC*	Area (m ²) for 1 L/s
Dynamic	0.4			0	2.5
Roughing ¹	0.17	5.5 mm/s downflow		0.5 ²	5.9
Slow	0.04	Scrape surface	10	0.8 ²	25
Multistage	0.03³		100⁴	1.3	33.4
Rapid	0.7 – 2.8	11 mm/s backwash	5 ⁵	1	0.55
Entrance	8				0.125
Flocculation	4				0.25
Sedimentation	1			2.5	1
Floc Hopper	5				0.2
Stacked Rapid 1.8 x 6	11 mm/s backwash		3 ⁶	1 ⁶	0.093
Aguac Clara	0.6³		1000⁵	3.5	1.67

1 Roughing is up flow in layers

2 Based on table 5.7 https://confluence.cornell.edu/download/attachments/90755680/FiME+CINARA.pdf

3 Net velocity for scaling total plan view area of facility.

4 This assumes a target of 5 NTU in the effluent

5 Assumes a target of 0.3 NTU in the effluent (based on performance of the Atima plant)

6 Data from Tamara.

The 32 L/s plant at San Nicolas has **5.5 m² per L/s** (includes chemical room, bathroom, office, etc.)

Filter Hydraulics

- Porosity
- Sand size definitions
- Clean bed head loss estimates
- Backwash requirements
- Low flow design challenges: the backwash dilemma



Porosity



$$\phi_{FiSand} = \frac{V_{voids}}{V_{total}}$$

volume of voids
total volume

Example: 1 m³ of dry aquifer material weighed 1600 Kg. The aquifer material was then saturated with water and found to weigh 2000 Kg.

What is the porosity of the aquifer material? **0.4**
What is the bulk density of the aquifer material? **1600 kg/m³**
What is the density of the aquifer material? **2700 kg/m³**

Porosity



- Which will have the greatest porosity?
 - A. 1 μm diameter clay
 - B. 0.2 mm diameter sand
 - C. 1 cm diameter gravel
- You have been given 100 kg each of clay, sand and gravel
 - What could you do to create the lowest porosity?

Expanded Bed Porosity

$\Pi_{FiBw} = \frac{H_{FiSandBw}}{H_{FiSand}}$ Filter expansion ratio

Backwash porosity

Pore volume + volume above bed = Expanded bed pore volume

$$\phi_{FiSandBw} = \frac{\phi_{FiSand} H_{FiSand} A_{Fi} + (H_{FiSandBw} - H_{FiSand}) A_{Fi}}{H_{FiSandBw} A_{Fi}}$$

Total expanded volume

Now we can relate porosity and filter expansion ratio

$$\phi_{FiSandBw} = 1 - \frac{1 - \phi_{FiSand}}{\Pi_{FiBw}}$$

$$\Pi_{FiBw} = \frac{\phi_{FiSand}}{\phi_{FiSandBw} - 1}$$

Sand Grading

Effective Size

- D₁₀ - sieve size that 10% by mass of the sand passes through
- Used for filter particle removal modeling
- D₆₀ - sieve size that 60% by mass of the sand passes through
- Used for hydraulic modeling
- Uniformity Coefficient = D₆₀/D₁₀

Clean Bed Head Loss (Karmen Kozeny)

Porosity (0.4 for uniform size media)

Head loss $\frac{h_l}{H_{FiSand}} = 36k \frac{(1 - \phi_{FiSand})^2}{\phi_{FiSand}^3} \frac{\nu V_{Fi}}{g D_{60}^2}$

Flow path length (filter bed depth)

Kinematic viscosity $\frac{h_l}{L} = \frac{32\nu V}{g D^2}$

Approach velocity

Kozeny constant Approximately 5 for most filtration conditions

Assumes laminar flow and is valid up to Re of 6 $Re = \frac{D_{60} V_{Fi}}{\nu}$

Backwash Requirements – Head Loss (force balance)

$P_{Manometer} = \rho_{Water} g (H_{W_1} + H_{W_2}) + \rho_{FB} g H_{FiSand}$
Weight of water/Filter area Weight of sand/Filter area

$$P_{Manometer} = \rho_{Water} g (H_{W_1} + H_{W_2} + \phi_{FiSand} H_{FiSand}) + \rho_{Sand} g (1 - \phi_{FiSand}) H_{FiSand}$$

$||$

$$P_{Manometer} = \rho_{Water} g (H_{W_1} + H_{W_2} + H_{FiSand} + h_{FiBw})$$

$h_{FiBw} = H_{FiSand} (1 - \phi_{FiSand}) \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right)$

For $\phi_{FiSand} = 0.4$ and $\rho_{Sand} = 2650 \text{ kg/m}^3$

$$(1 - \phi_{FiSand}) \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right) = 0.99$$

What does this analysis miss?

- Sand bed may first lift as a unit and then fall into the fluidized state
- Wall shear (scale effect!!!!) and embedded pipes may prevent bed lift (or require more force than our simple force analysis suggests)
- Sand will stratify if there is even a small range of sizes: Backwash velocity must be sufficient to fluidized the largest sand
- Backwash initiation requires more study

Backwash Requirements – Flow Rate

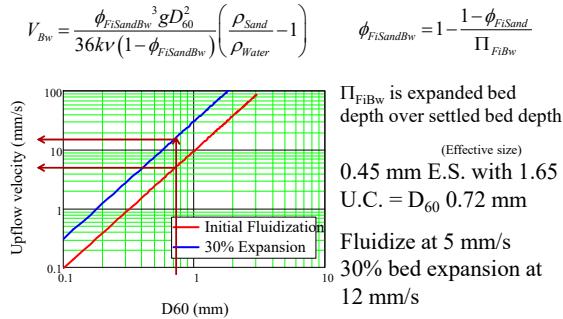
$$\frac{h_l}{H_{FiSand}} = 36k \frac{(1-\phi_{FiSand})^2}{\phi_{FiSand}^3} \frac{\nu V_{Fi}}{g D_{60}^2} \quad \text{Kozeny}$$

$$\frac{h_{FBw}}{H_{FiSand}} = (1-\phi_{FiSand}) \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right) \quad \text{Backwash Head Loss}$$

$$(1-\phi_{FiSand}) \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right) = 36k \frac{(1-\phi_{FiSand})^2}{\phi_{FiSand}^3} \frac{\nu V_{MinFluidization}}{g D_{60}^2} \quad \text{Set them equal}$$

$$V_{MinFluidization} = \frac{\phi_{FiSand}^3 g D_{60}^2}{36k \nu (1-\phi_{FiSand})} \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right) \quad \text{Minimum approach velocity required to fluidize bed}$$

Backwash Velocity



The Backwash Dilemma

- Backwash velocity >> filtration velocity
- Backwash water must be clean water
- Backwash water sources?
 - Pump it from clearwell
 - Set of filters working in parallel to backwash one filter
 - Filtered water stored at adequate elevation

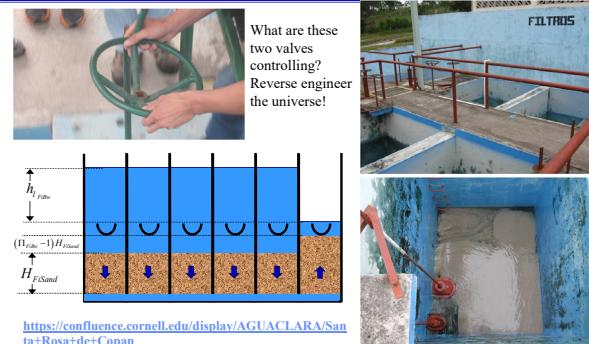


Backwash sand bed stratification

- Suppose the sand stratifies in the filter bed during backwash and suppose that 10% of the bed is smaller than 0.4 mm and 10% of the sand is bigger than 0.8 mm
- What is the ratio of fluidization velocity for the top of the sand bed to the fluidization velocity for the bottom of the sand bed?

$$V_{MinFluidization} = \frac{\phi_{FiSand}^3 g D_{60}^2}{36k \nu (1-\phi_{FiSand})} \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right)$$

Rapid Sand Filtration Battery: Backwash without pumps



The Low Flow Challenge: Filter Set

- Assume plant flow is 6 L/s, Filtration velocity is 1.8 mm/s and backwash velocity is 9 mm/s and that a set of filters will backwash one filter
- How many filters?
- Total filter area?
- What happens if the plant is operating at half capacity (drought conditions)?

$$Q_{Plant} := 6 \frac{\text{L}}{\text{s}}$$

$$V_{Fi} := 1.8 \frac{\text{mm}}{\text{s}}$$

$$V_{Bw} := 9 \frac{\text{mm}}{\text{s}}$$

$$N_{Fi} := \frac{V_{Bw}}{V_{Fi}} + 1 = 6$$

$$A_{FiTotal} := \frac{Q_{Plant}}{V_{Fi}} \cdot \frac{N_{Fi}}{N_{Fi} - 1} = 4 \text{ m}^2$$

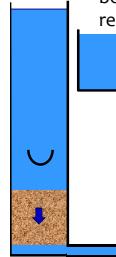
$$A_{Fi} := \frac{A_{FiTotal}}{N_{Fi}} = 0.667 \text{ m}^2$$

$$W_{Fi} := \sqrt{A_{Fi}} = 0.816 \text{ m}$$

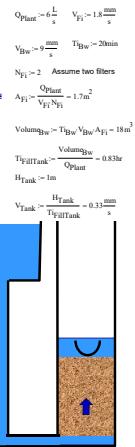
Need filters to be smaller to be able to backwash one!

Store filter water?

- Try two filters and recalculate
- Require a large tank and the tank must be shallow to keep filters from being really deep.



- Filters need to be able to load the tank during filtration and then take water from the tank during backwash. This is accomplished by having the filter water level VERY high during filtration and low during backwash
- Requires more area than sed tanks!



The Low Flow Challenge

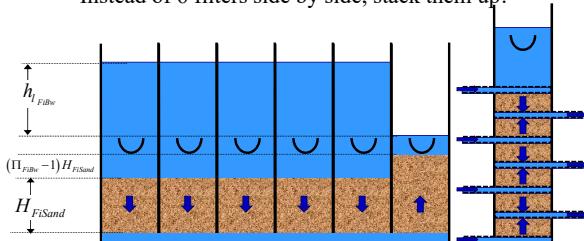
- Requiring 6 filter boxes for a small plant is too costly and difficult to construct and maintain.
- Storing filtered water requires a storage tank with a V_{up} less than the entire AguaClara plant!!!
- If we rule out the pump option, then we have no good options...

Why is this difficult?

- The backwash dilemma is due to the big difference between backwash flow rates and filtration flow rates
- 6 filters are required if backwash velocity is 5 times the filtration velocity
- If we could get those velocities to be similar or equal, then we wouldn't need a set of 6 filters
- Invention time $V_{MinFluidization} = \frac{\phi_{FiSand}^3 g D_{60}^2}{36 k v (1 - \phi_{FiSand})} \left(\frac{\rho_{Sand}}{\rho_{Water}} - 1 \right)$

Geometric Trick: Stacked Rapid Sand Filter (StaRS)

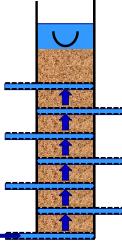
Instead of 6 filters side by side, stack them up!



Reduce backwash water volume by factor of number of layers

Water Source for Backwash

- Tradition says Filtered Water
- Why not use Settled water?
- Public Health trade off?
 - Filter to waste after backwash is required in any case due to contamination of manifold tubing
 - Use of settled water significantly simplifies the system
- Same flow rate and water source during filtration and backwash!!!!
- Eliminate empty clearwell failure mode



Water waste

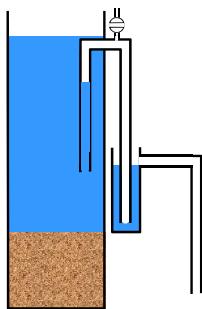


- The amount of water wasted from each unit process is determined by the average solids concentration of wasted water and the solids concentration entering the process.
- The goal is to have the wasted water have as high of a solids concentration as possible
- Backwash water is very dilute compared with the solids concentration from the floc hoppers
- Backwash water can be reprocessed, but that requires a pump (a good option to explore for dry season water saving)

Pipe Gallery (San Nicolas)



Begin Filtration



StaRS Filter Fabrication

- [San Matías construction](#)
- [Backwash operation](#)

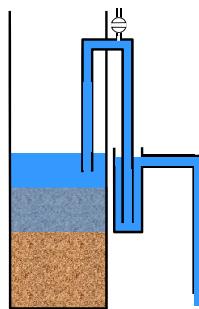


Eliminating 7 big valves: One valve control of a StaRS Filter

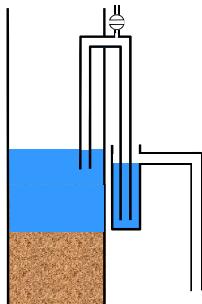
- The following slides illustrate the method of using a siphon tube to initiate backwash
- The filter animation only shows how the siphon will work. It doesn't show how the inlet and outlet manifolds will be controlled
- The Filter Inlet and Outlet control boxes will be designed so that water level changes* during backwash will cause all of the manifold pipes to be hydraulically separated and thus no valves will be required to turn off the inlet or outlet pipes



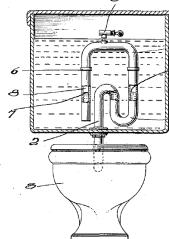
Begin Backwash



End Backwash



The big flush

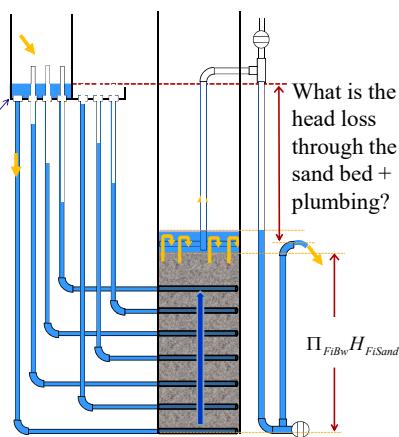


[Patent US2017263 - FLUSHING APPARATUS - Google Patents](https://patents.google.com/patent/US2017263)

End Backwash cycle

Constraints on depth of water in inlet box during backwash?

What is the head loss in the sand bed between an inlet and an outlet?



Head loss between manifolds Assume expansion ratio of 1.3

- Find depth of settled sand in 20 cm of expanded sand

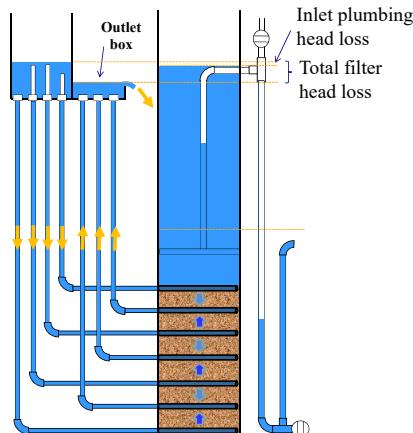
$$\Pi_{FiBw} = \frac{H_{FiSandBw}}{H_{FiSand}}$$

Settled depth Expanded depth (20 cm)

$$H_{FiSand} = \frac{H_{FiSandBw}}{\Pi_{FiBw}} = 15.4\text{cm}$$

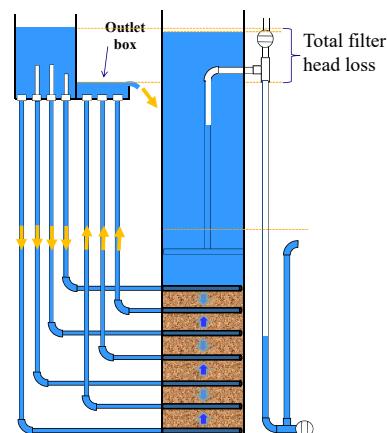
This is head loss per 20 cm of expanded bed!

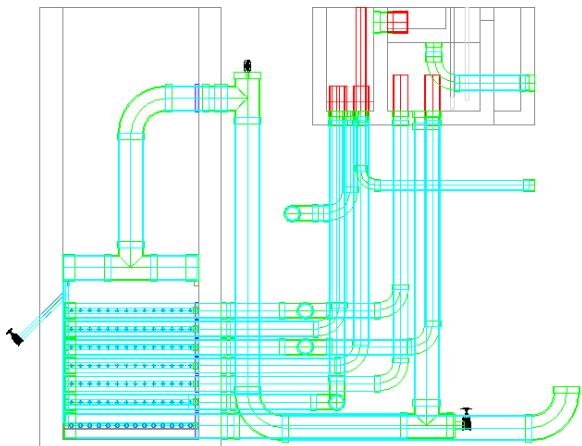
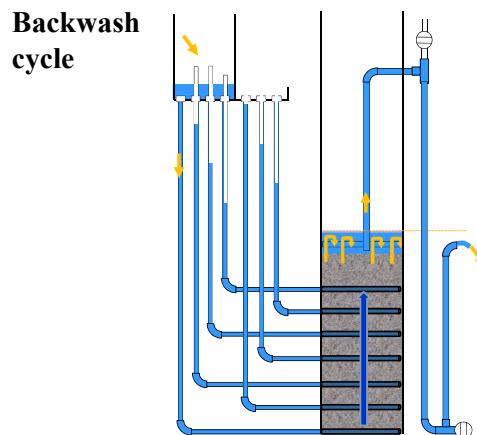
Begin Filtration cycle



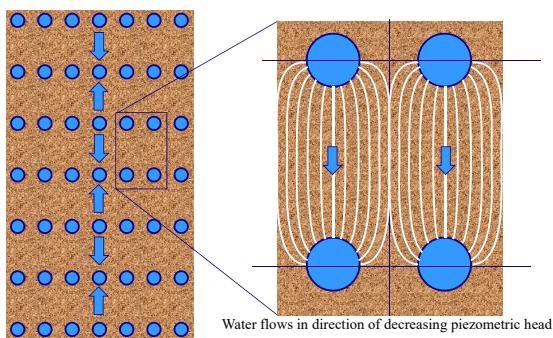
End Filtration cycle

What causes water to flow through the filter bed?
differences in piezometric head





How does water flow through StaRS Filters?

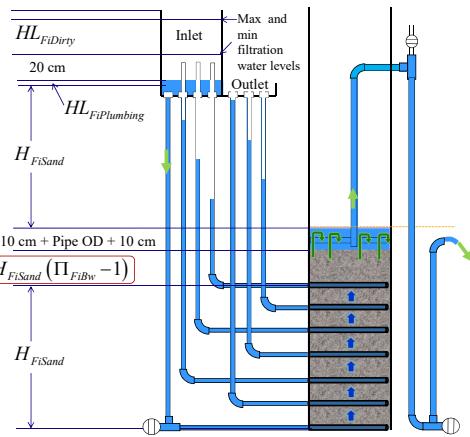


Filter Design Constraints

- Spacing between inlet pipes (or outlet pipes) must be small compared with layer depth
- Filter should be able to be backwashed at the beginning of filtration cycle if needed
- All inlets and outlets (except bottom inlet) must be high and dry during backwash
- Kinetic energy in trunk lines and inlet/outlet pipes must be less than 10% of the clean bed head loss to achieve uniform flow distribution in sand bed*



* Edge of knowledge



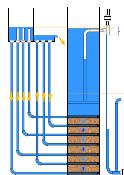
EStaRS and OStaRS Enclosed vs. Open

- Fabrication becomes difficult with filter dimensions less than $1\text{ m} \times 1\text{ m}$
- Minimum OStaRS flow rate is about 8 L/s
- EStaRS doesn't require excavation because filter is operated under vacuum for backwash



StaRS Backwash Advantage

- Where does the water come from that fills the filter box after the siphon is broken at the end of backwash?
- Where does the water come from that slowly fills up the filter box during a filtration run?
- Unlike RSF the backwash water above the StaRS bed does not pass through the sand during filtration

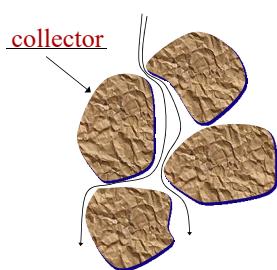


StaRS Challenges

- Inlet slotted pipes clog
 - 2015 replaced with [image]
- Water level changes above sand result in unnecessary water waste
- Could we replace Outlet slotted pipes?



Particle Removal Mechanisms in Filters



Transport to a surface

Molecular diffusion

Inertia

Gravity

Interception

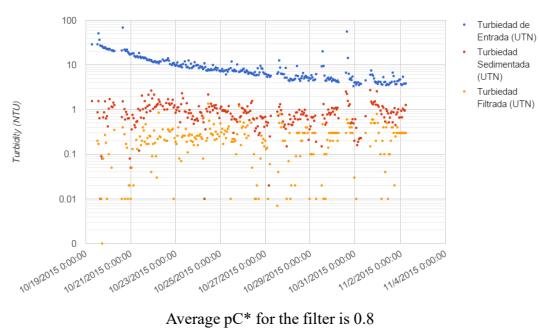
Attachment

Straining

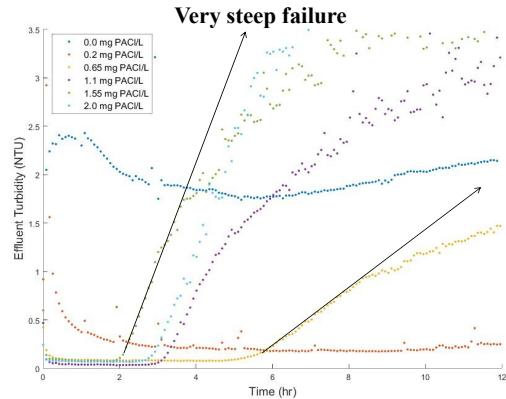
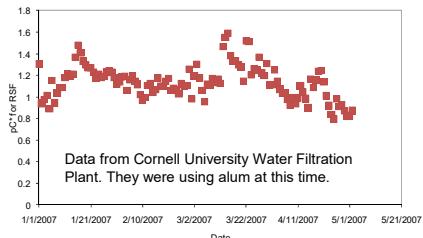
Polar coagulant

Gravity

San Matías Performance

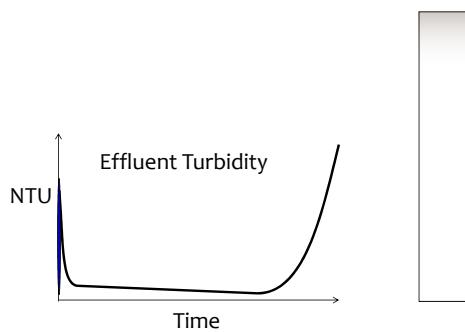


RSF Performance

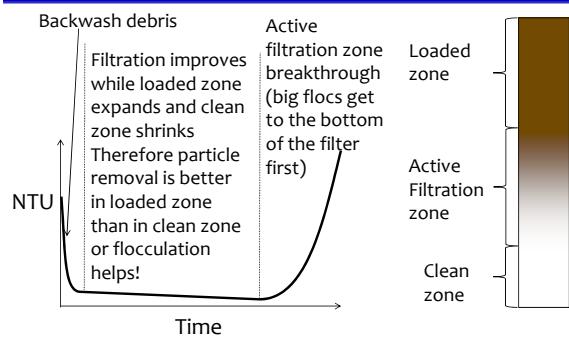


Lucinda Li, Theresa Chu, Jonathan Harris, Mythri Krishnamoorthysujatha, Rose Linchan, William H. Pennock

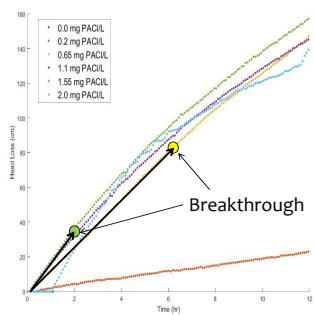
Active filtration zone slowly moves down through the filter



Active Filtration zone



Head loss in Filters is close to linear before breakthrough



Head loss and fraction loaded are increasing linearly together

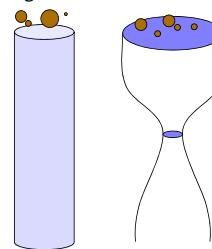
- Head loss continues to increase after the active zone reaches the filter effluent
- Head loss increase becomes nonlinear with time after the active zone reaches the filter effluent

What do we know?

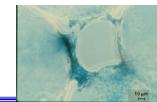
- Flocs occupy a few percent of the void volume in a loaded filter
 - Insufficient floc volume to uniformly decrease void volume and increase fluid velocity, shear, and major losses
- Head loss is close to linear with time
- Clean bed filtration models
 - Removal is primarily due to interception
 - Sedimentation may be important too

Interception occurs where streamlines converge

No removal by interception in straight tube

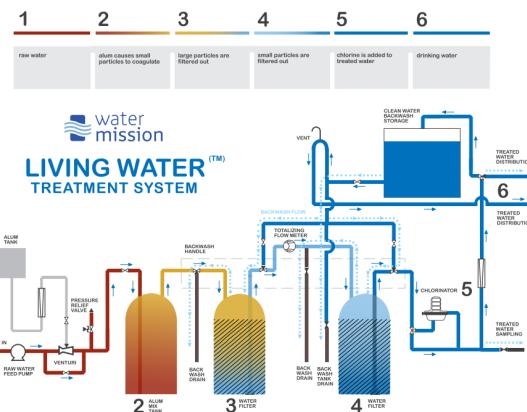


- Particles are deposited at the constriction
- The biggest part of the filter pore remains empty!
- Big particles can't attach to partially filled restrictions



StaRS Optimization

- Sand Diameter – smaller diameter is more efficient at particle capture, requires a lower backwash velocity and thus a lower filtration velocity could be used
- Layer height – filter box rapidly becomes very deep to accommodate deep layers
- Slot width – wider slots would take longer to clog
- More research required to improve the design



Conclusions...

- Many different filtration technologies are available
- Filters are well suited for taking clean water and making it cleaner. They are not a good choice for treating very turbid surface waters
- Pretreat using flocculation/floc blanket/plate (or tube) settlers

...Conclusions...

- Filters could remove particles more efficiently if the attachment efficiency were increased
- StaRS 3 main advantages over RSF
 - Self backwashing (no pumps)
 - Smaller filter area required
 - Single valve control
- RSF (and StaRS) require pretreatment with coagulant

...Conclusions

- RSF have pC^* of 1 to 1.5 under normal conditions (max influent turbidity is about 5 NTU)
- StaRS have pC^* of 1 (preliminary data)
- SSF have pC^* of 1 to 1.5, but they perform poorly if the water contains much colloidal clay
- Sand Filters CAN NOT treat turbid water

References

- Tufenkji, N. and M. Elimelech (2004). "Correlation equation for predicting single-collector efficiency in physicochemical filtration in saturated porous media." *Environmental Science and Technology* 38(2): 529-536.
- Cushing, R. S. and D. F. Lawler (1998). "Depth Filtration: Fundamental Investigation through Three-Dimensional Trajectory Analysis." *Environmental Science and Technology* 32(23): 3793-3801.
- Tobiason, J. E. and C. R. O'Melia (1988). "Physicochemical Aspects of Particle Removal in Depth Filtration." *Journal American Water Works Association* 80(12): 54-64.
- Yao, K.-M., M. T. Habibian, et al. (1971). "Water and Waste Water Filtration: Concepts and Applications." *Environmental Science and Technology* 5(1): 1105.
- M.A. Elliott*, C.E. Stauber, F. Koksal, K.R. Liang, D.K. Huslage, F.A. DiGiano, M.D. Sobsey, (2006) The operation, flow conditions and microbial reductions of an intermittently operated, household-scale slow sand filter
- Enhanced Filter Performance by Fluidized-Bed Pretreatment with Al(OH)₃(am): Observations and Model Simulation
Po-Hsun Lin, Leonard W. Lion, and Monroe L. Weber-Shirk.
Journal of Environmental Engineering 1, 389 (2011).

Filtration Performance: Dimensional Analysis

Performance = f(forces, geometry)

- What is the parameter we are interested in measuring? Effluent concentration
- How could we make performance dimensionless? C/C_0 or pC^* $pC^* = -\log\left(\frac{C}{C_0}\right)$
- What are the important forces?

transport

Inertia Thermal

Viscous Gravitational

attachment

Sticky? (α – attachment efficiency)

Need to create dimensionless force ratios!

What is the Reynolds number for filtration flow?

- What are the possible length scales?
 - Void size (collector size) 0.7 mm in RSF
 - Particle size
- Velocities
 - V_{fi} varies between 0.03 mm/s (SSF) and 2.8 mm/s (RSF)
- Take the largest length scale and highest velocity to find max Re

$$Re = \frac{Vl}{\nu} \quad Re = \frac{\left(2.8 \times 10^{-3} \frac{m}{s}\right)(0.7 \times 10^{-3} m)}{\left(10^{-6} \frac{m^2}{s}\right)} = 2$$
- Flow through SSF, RSF, and StaRS is laminar during filtration

Choose viscosity!

- In Fluid Mechanics inertia is a significant "force" for most problems
- In porous media filtration viscosity is more important than inertia.
- We will use viscosity as the repeating parameter and get a different set of dimensionless force ratios

$$\Pi_g \rightarrow \frac{\text{Gravitational}}{\text{Viscous}}$$

$$\Pi_{Br} \rightarrow \frac{\text{Thermal}}{\text{Viscous}}$$

Dimensionless Force Ratios $f_i = \rho \frac{V^2}{l}$

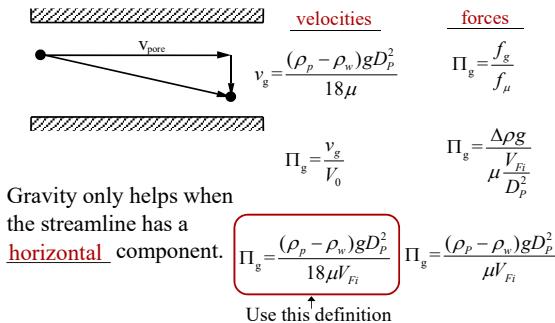
- Reynolds Number $Re = \frac{\rho V l}{\mu}$ $f_u = \mu \frac{V}{l^2}$
- Froude Number $Fr = \frac{V}{\sqrt{gl}}$ $f_g = \rho g$
- Weber Number $W = \frac{V^2 l \rho}{\sigma}$ $f_\sigma = \frac{\sigma}{l^2}$
- Mach Number $M = \frac{V}{c}$ $f_{E_v} = \frac{\rho c^2}{l}$
- Pressure/Drag Coefficients $(\Delta p + \rho g \Delta z)$
 - (dependent parameters that we measure experimentally)

$$C_p = \frac{-2(\Delta p)}{\rho V^2} \quad C_d = \frac{2 \text{Drag}}{\rho V^2 A}$$

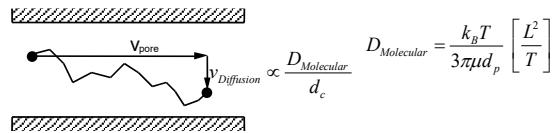
Gravity

$$f_u = \mu \frac{V}{l^2}$$

$$f_g = \rho g$$



Diffusion (Brownian Motion)



Diffusion velocity is high when the particle diameter is small.

$$\Pi_u = \mu \frac{V}{l^2}$$

$k_B = 1.38 \times 10^{-23} \text{ J/K}$
T = absolute temperature
 d_c is diameter of the collector

$$\Pi_{Br} = \frac{k_B T}{3\pi\mu d_p V_{F_i} d_c}$$

Geometric Parameters

- What are the length scales that are related to particle capture by a filter?
 - Filter depth (z)
 - Collector diameter (media size) (d_c)
 - Particle diameter (d_p)
 - Porosity (void volume/filter volume) (ϕ)
- Create dimensionless groups
 - Choose the repeating length (d_c)

Number of collectors!

$$\Pi_z(z, d_c) := \frac{3(1 - \phi p_{cr})}{24\pi(10)} \left(\frac{z}{d_c} \right)^3$$

Definition used in model

Write the functional relationship

Length ratios Force ratios

$$pC^* = \alpha f \left(\Pi_R, \Pi_z, \phi, \Pi_g, \Pi_{Br} \right)$$

$\alpha = \text{Attachment efficiency}$

If we double depth of filter what does pC^* do? doubles

$$pC^* = \alpha \Pi_z f \left(\Pi_R, \phi, \Pi_g, \Pi_{Br} \right)$$

How do we get more detail on this functional relationship?

Empirical measurements

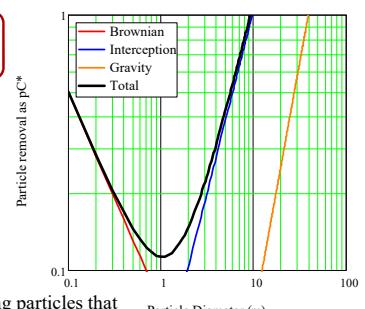
Numerical models

Filtration Model Limitations

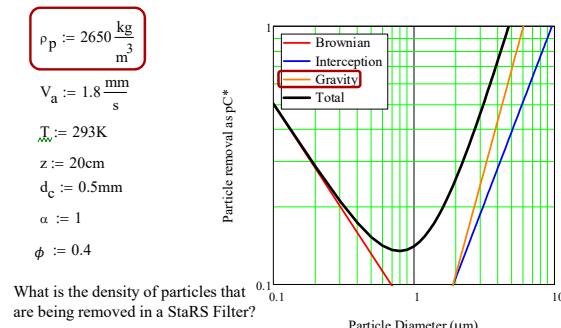
- Filtration model is really a “clean bed” model
 - Likely not able to explain performance during a filter run
 - Doesn’t account for pore filling
- pC^* is proportional to depth (not consistent with observations of operating filters)
- pC^* is independent of influent particle concentration (not consistent with observations of operating filters)
- Influent particle size distribution is likely very important and may control pC^* for real filters

Stacked Rapid Sand Filter predicted performance for biological particle

$$\begin{aligned} \rho_p &:= 1040 \frac{\text{kg}}{\text{m}^3} \\ V_a &:= 1.8 \frac{\text{mm}}{\text{s}} \\ T &:= 293 \text{K} \\ z &:= 20 \text{cm} \\ d_c &:= 0.5 \text{mm} \\ \alpha &:= 1 \\ \phi &:= 0.4 \end{aligned}$$

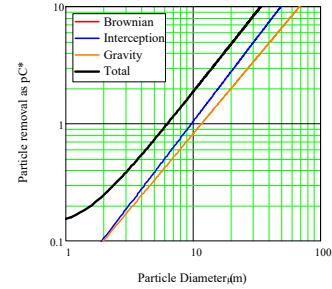


Stacked Rapid Sand Filter predicted performance for inorganic particle



StaRS and Fractal Flocs predicted performance

The filtration model combined with the fractal flocculation model suggests that filtration is dominated by interception for all particles sizes for approach velocities above about 1 mm/s.



Filter as Flocculator?

- Head loss of 40 cm $G\theta = \sqrt{\frac{gh_e}{\nu} \theta}$
- Residence time of 44 s
- $G\theta$ of 13,000!
- Filter provides one last chance for flocculation especially after head loss increases

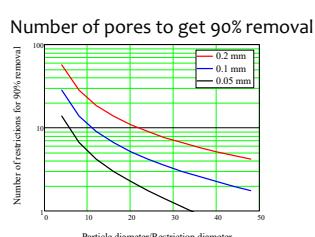
A series of restrictions takes a toll on even the small particles

If we assume complete mixing in each pore then the probability of slipping through N pores is

$$C_{Filter}^* = \left(\frac{D_c - D_p}{D_c} \right)^{2N_{Pores}}$$

Number of pores to get a target fraction remaining

$$N_{Pores} = \frac{\ln C_{Filter}^*}{2 \ln \left(\frac{D_c - D_p}{D_c} \right)}$$

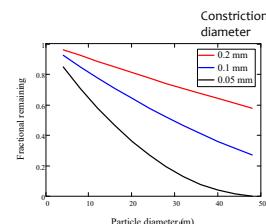


How much mixing happens in each pore? Is the outer ring of fluid refreshed?

Small particles slip through a small restriction

$$C_{Pore}^* = \left(\frac{D_c - D_p}{D_c} \right)^2$$

The vast majority of small particles slide right through a constriction

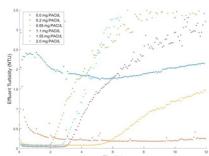


Tiny (or slippery) bad guys sneak through all the security

- Some small particles escape flocculation, floc blanket, plate settlers, and filter
 - Because they are small (not likely because effluent turbidity stayed the same when flow rate was decreased in a filter)
 - Or because a few particles aren't coated with nanoparticles (they are slippery)
 - Because they never got enough nanoparticles of coagulant
 - Or the nanoparticles were sheared off during
 - Failed attachments
 - Breakup
- How could we determine if it is small size or lack of nanoparticles or something else?

What could cause slippery particles (with too few nanoparticles)?

- Pulsing peristaltic pumps
- Statistical variability given small number of nanoparticles per clay
- Nanoparticles removed during unsuccessful collisions (this could occur during flocculation and/or during filtration)



Flocs on their way to the filter

- We use weirs and free fall to ensure that the flow is divided equally between filters and is not affected by the head loss of the filter
- The elevation drop into the entrance box is approximately 0.4 m.
- Velocity is 2.8 m/s
- If we assume a 1 cm thick jet of water then the energy dissipation rate is 140 W/kg
- The max diameter of a floc is thus 14 μm

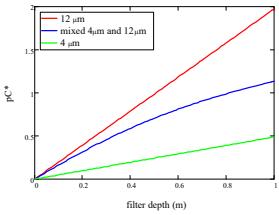
Filtration of mixed particle sizes

$$pC_i^* = \alpha \Pi_z f(d_i) \quad \text{Filtration model}$$

$C_i = C_{0,i} 10^{-pC_i^*}$ Definition of pC^* where i represents the bins containing different size fractions

$C_i = C_{0,i} 10^{-\alpha \Pi_z f(d_i)}$ Effluent concentration of each size class

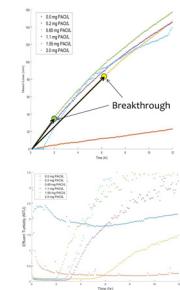
$$pC^* = -\log \left(\frac{\sum_{i=0}^n C_i}{\sum_{i=0}^n C_{0,i}} \right)$$



Mixtures produce nonlinear relationship between pC^* and filter bed depth

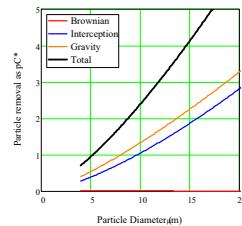
Head loss in filters is close to linear before breakthrough

- Why does increased coagulant dose result in earlier failure?
- Higher coagulant dose results in bigger flocs and fewer small flocs
- Thus failure must be from big flocs
- Big flocs can't deposit in partially formed restrictions
- Why is head loss accumulation faster with more coagulant?
- More coagulant = bigger flocs and bigger flocs are less dense/occupy more volume



Floc removal efficiency increases rapidly with floc size

Perhaps filter performance is largely controlled by the size distribution of influent flocs. We need particle size distribution entering and exiting the filter

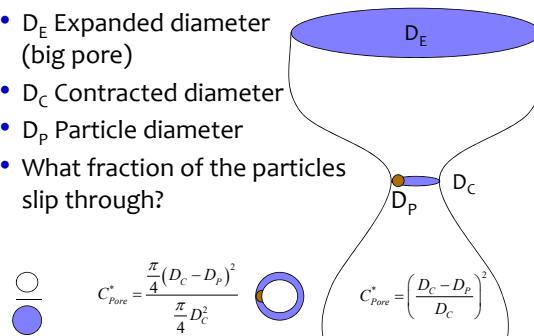


Individual clay particles would be hard to capture

20 cm bed of 0.5 mm sand

Interception

- D_E Expanded diameter (big pore)
- D_C Contracted diameter
- D_P Particle diameter
- What fraction of the particles slip through?



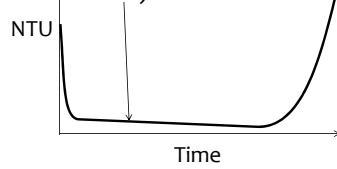
Methods to improve Rapid Sand Filter Performance

- Increase attachment efficiency
- Flocculate particles to make them larger
- Add coagulant to the filter influent to make the filter media and previously captured particles stickier (50 µg/L of Al has been shown to be effective at Cornell Water Treatment Plant)

Po-Hsun Lin 2011

Flocculation may be a significant mechanism inside a filter

- The equations are simplified major losses in a clean filter bed
- Flocculation increases as head loss increases (another reason for this)



$$h_f = \frac{32\mu\theta V^2}{\rho g D^2}$$

$$\bar{z} = \frac{gh_f}{\theta}$$

$$\bar{G} = \sqrt{\frac{\bar{z}}{V}}$$

$$\theta = \frac{L}{V}$$

$$\bar{G}\theta = 4\sqrt{2} \frac{V}{D}$$

$$\frac{20\text{cm}}{0.2\text{mm}} = 4\sqrt{2} \frac{V}{D} = 5657$$

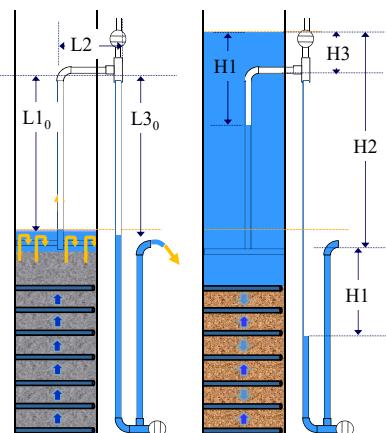
StaRS Design Guidelines

- Settled water is divided evenly between filters that are in filtration mode
 - Weir into entrance box
 - need to eliminate air entrainment problem
- Backwash filter gets design flow
- Each filter has an overflow box
- Filtered water flows over a weir into a box taking water to the distribution tank

Siphon Geometry for Air trap

What determines how much air we trap?
What happens as head loss increases?

Success requires two things
1. Start backwash whenever desired.
2. Prevent backwash from starting.



Siphon Geometry for Air trap

- What must be true for the air in the siphon air trap?
 - Mass of air must be conserved
 - Volume of air is NOT conserved
 - Pressure in the air is independent of elevation
 - Depth of "submergence" (hydrostatic pressure) must be the same at both ends of the siphon!
- What relationship can we use to find volume of air given initial volume of air?
 - $PV=nRT$



Siphon Geometry for Air trap

$$\mathcal{V}_0 = (L1_0 + L2 + L3_0) A_{Siphon}$$

$$\mathcal{V}_0 = (H_2 - H_3 - HL_{Siphon} + L2 + H_2 - H_3) A_{Siphon}$$

$$\mathcal{V}_0 = (2H_2 - 2H_3 - HL_{Siphon} + L2) A_{Siphon}$$

$$\mathcal{V}_1 = (H_1 - H_3 + L2 + H_1 + H_2 - H_3) A_{Siphon}$$

$$\mathcal{V}_1 = (2H_1 - 2H_3 + L2 + H_2) A_{Siphon}$$

$$\frac{\mathcal{V}_1}{\mathcal{V}_0} = \frac{P_0 \mathcal{V}_0}{P_1 \mathcal{V}_1}$$

$$P_0 = 1atm$$

$$P_1 = P_{atm} + \rho g H_1$$

$$(2H_1 - 2H_3 + L2 + H_2) = \frac{P_{atm} (2H_2 - 2H_3 - HL_{Siphon} + L2)}{P_{atm} + \rho g H_1}$$

Maximum water level that can be held by siphon

$$(2H_1 - 2H_3 + L2 + H_2) = \frac{P_{atm} (2H_2 - 2H_3 - HL_{Siphon} + L2)}{P_{atm} + \rho g H_1}$$

Failure mode is when $H_1 = H_3$
Set $H_1 = H_3$, then solve for H_3

$$(L2 + H_2) = \frac{P_{atm} (2H_2 - 2H_3 - HL_{Siphon} + L2)}{P_{atm} + \rho g H_3}$$

$H_3 = \frac{(H_2 - HL_{Siphon}) P_{atm}}{\rho g (L2 + H_2) + 2P_{atm}}$ This is the maximum value of H_3

