

Exercise 3: High Dimensional Data

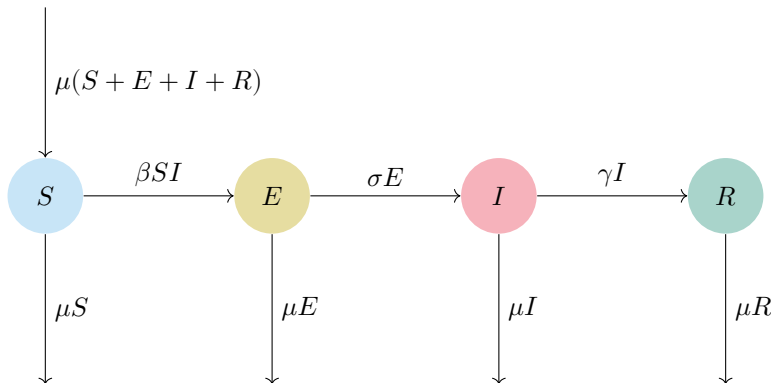
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Complete Exercise 3a and either 3b or 3c.

Exercise 3a

Following is the state diagram for an SEIR model with balanced birth and death. μ is the birth and death rate, indicating zero population change. β is the transmission rate, σ is the progression rate from exposed to infected, and γ is the recovery rate.



\mathcal{R}_0 for this model can be estimated as:

$$\mathcal{R}_0 = \frac{\beta\sigma}{(\mu + \gamma)(\mu + \sigma)},$$

where $\mu \ll \gamma$ and σ is of the order of γ .

Your task is to design a plot to explore variation in \mathcal{R}_0 across a range of γ and σ .

Step 1:

Begin by fixing μ and β and preparing a grid of γ and σ . (You can use `expand.grid()` for this.)

Calculate \mathcal{R}_0 for all values of the grid.

Transform your data into a matrix of \mathcal{R}_0 values, with rows = levels of γ , and columns = levels of μ .

```

library(tidyverse)
# Example with fixed beta, fixed mu, and grid of gamma and sigma
beta <- .2
mu <- .01
gamma <- seq(.15,.3,length.out=10)
sigma <- seq(.1,.2,length.out=10)
results <- expand.grid(gamma = gamma, sigma = sigma)

# Calculate  $R_0$ 
# Note: In tidyverse, use all_of(varname) to refer to an object outside the dataframe.
# Use varname to refer to column of the dataframe.

results <- results %>%
  mutate(R0 = all_of(beta) * sigma / ((all_of(mu) + gamma)*(all_of(mu) + sigma)))

# Prepare a matrix
results.wide <- results %>%
  pivot_wider(names_from = sigma, values_from = R0)
results.matrix <- results.wide %>%
  select(c(-gamma)) %>%
  as.matrix()
dimnames(results.matrix) <- list(gamma = results.wide$gamma,
                                sigma = colnames(results.matrix))

```

Step 2:

Visualize.

A matrix of smoothly changing values can be visualized as a “level plot” (with values mapped to colors), or as a “contour plot” (with contours drawn at set intervals), or both.

A level plot can be made with the `lattice::levelplot()` or `graphics::filled.contour`. Both functions can add contours to the level plot.

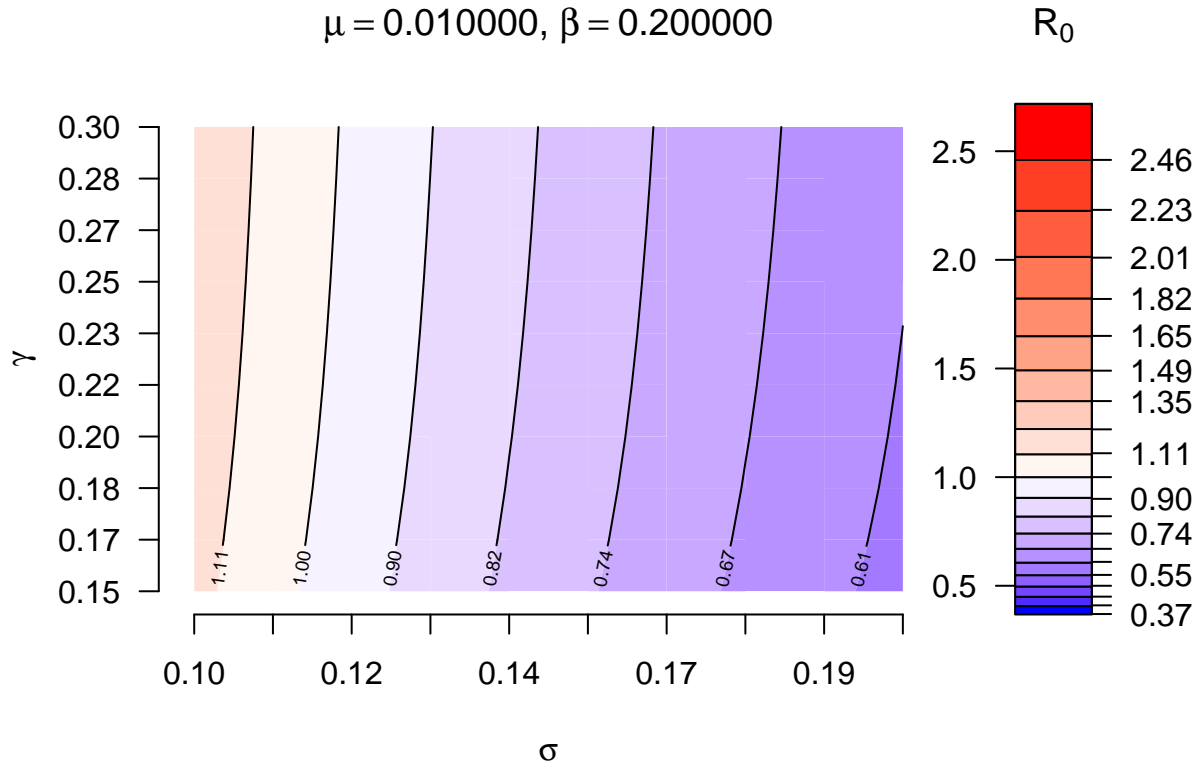
Questions to consider:

How fine of a grid do you need? What should the bounds be?

What kind of color scale would be appropriate to represent \mathcal{R}_0 ? (Think carefully about what \mathcal{R}_0 represents, the critical value(s), boundaries, etc.)

Does a log scale make sense for the independent variables?

Does a log scale make sense for the dependent variable (\mathcal{R}_0)? If so, how would you construct a log color scale?



Exercise 3b

\mathcal{R}_0 for a model with both environmental and direct transmission can be estimated as

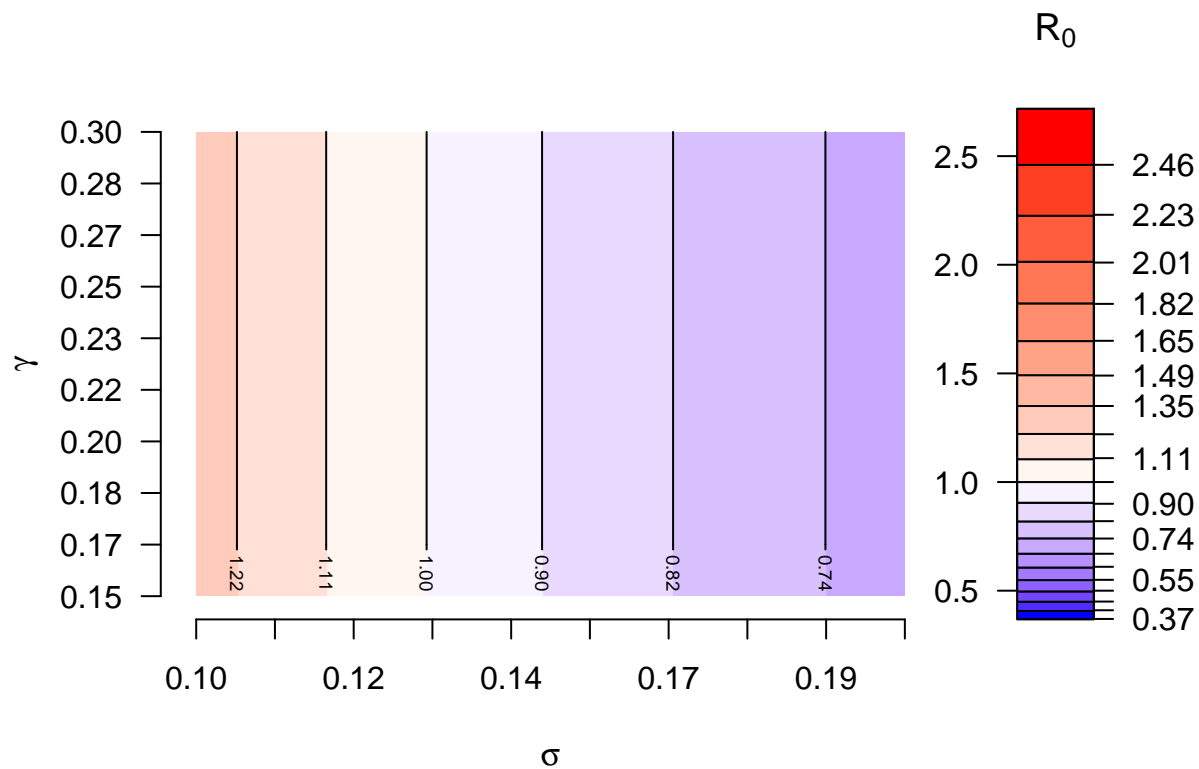
$$\mathcal{R}_0 = \frac{\beta}{(\mu + \gamma)} + \frac{\beta_v \omega}{\rho(\mu + \gamma)},$$

where β is the basic transmission rate, β_v is the environmental transmission rate, ω is ? , and ρ is ? .

How can you use the level plot approach to explore \mathcal{R}_0 across this parameter space?

Can you imagine alternate ways to visualize \mathcal{R}_0 across this parameter space? Sketch your ideas on paper.

$\mu = 0.01, \beta = 0.2, \beta_v = 0.2, \rho = 0.1, \omega = 0.1$



Excercise 3c

For a simple COVID model with asymptomatic transmission

$$\mathcal{R}_0 = \frac{\kappa\beta}{\gamma_A} + \frac{(1-\kappa)\beta}{\gamma}$$