

## RESULT ANALYSIS OF PAGERANK RANDOM WALK STRATEGY

### STAGE 1 - ORIGINAL NETWORKS:

- *Directed Network:* The original directed network displayed a spectral gap of 0.002572, indicating efficient traversal, with a 90% coverage time of 895.11 units. The network had 482 trapping nodes, potentially affecting navigability.
- *Undirected Network:* Conversely, the undirected network exhibited a significantly smaller spectral gap ( $1.14 \times 10^{-14}$ ), leading to a much longer 90% coverage time (201847617845194 units). The presence of 212 trapping nodes suggests structural constraints affecting network traversal.

### STAGE 2 - INDIVIDUAL LAYERS LINKS INTEGRATION:

- *Directed Network:* Adding individual layer links slightly decreased the spectral gap to 0.002524 and increased the 90% coverage time to 912.13 units, with a reduction in trapping nodes to 479.
- *Undirected Network:* The spectral gap for the undirected network remained extremely small ( $6.05 \times 10^{-15}$ ), with a slightly adjusted 90% coverage time (380547573093462 units), while the count of trapping nodes remained at 212.

### STAGE 3 - TWO-LAYER COMBINATION LINKS INTEGRATION:

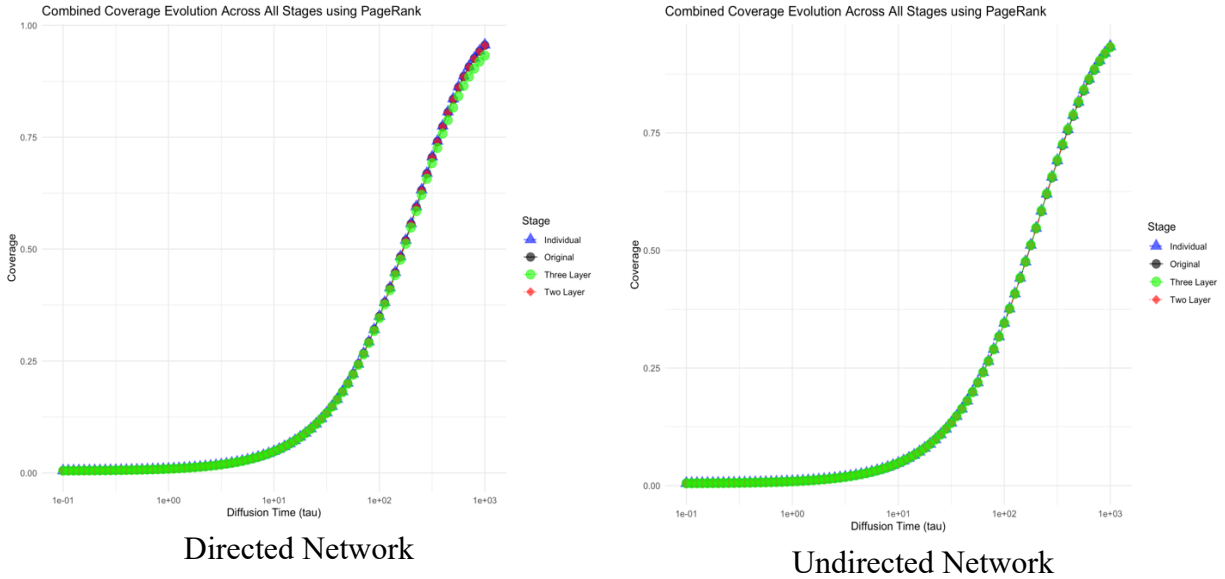
- *Directed Network:* The integration of two-layer combination links slightly increased the spectral gap to 0.002610, leading to a reduced 90% coverage time of 881.98 units. The trapping nodes decreased to 460.
- *Undirected Network:* The spectral gap slightly increased ( $1.35 \times 10^{-14}$ ), with a considerable reduction in the 90% coverage time (170347784259496 units). Trapping nodes were reduced to 183.

### STAGE 4 - THREE-LAYER COMBINATION LINKS INTEGRATION:

- *Directed Network:* With the addition of three-layer combination links, the spectral gap reduced to 0.002268, increasing the 90% coverage time to 1015.11 units. The trapping nodes significantly decreased to 403.
- *Undirected Network:* The spectral gap remained minute ( $3.54 \times 10^{-14}$ ), but the 90% coverage time was reduced to 64964268546887 units. Trapping nodes were notably reduced to 109.

Figure 4 illustrates the coverage evolution of our energy network using the PageRank random walk strategy across different stages of link addition. Despite structural modifications—integrating

individual, two-layer, and three-layer links—the coverage plots for both directed and undirected networks display remarkable congruence, suggesting minimal impact from these augmentations. This implies that the PageRank algorithm, characterized by its inherent damping factor, maintains a consistent performance in terms of network coverage regardless of the incremental topological changes. The plots reaffirm that while the spectral gap and trapping nodes vary, these structural adjustments do not significantly alter the PageRank traversal dynamics, leading to a steady coverage profile as visualized.



**Figure 4. Coverage Evolution in Directed and Undirected Networks using PageRank.**

## RESULT ANALYSIS OF DIFFUSIVE RANDOM WALK STRATEGY

For the Diffusive Random Walk analysis, the network dynamics varied considerably between the directed and undirected networks across all stages:

### STAGE 1: ORIGINAL NETWORK

- Directed: A spectral gap of 0.0003109 suggests a slower coverage, with a 90% coverage time of approximately 7,405.
- Undirected: A much smaller spectral gap of  $1.50e-06$  indicates an even slower process, with a 90% coverage time exceeding 1,535,852 units.

### STAGE 2: INDIVIDUAL LAYERS LINKS INTEGRATION

- Directed: The spectral gap decreases slightly to 0.0002393, increasing the coverage time to around 9,621 units.

- Undirected: The spectral gap marginally diminishes to  $1.12\text{e-}06$ , with the coverage time rising significantly to over 2,047,858 units.

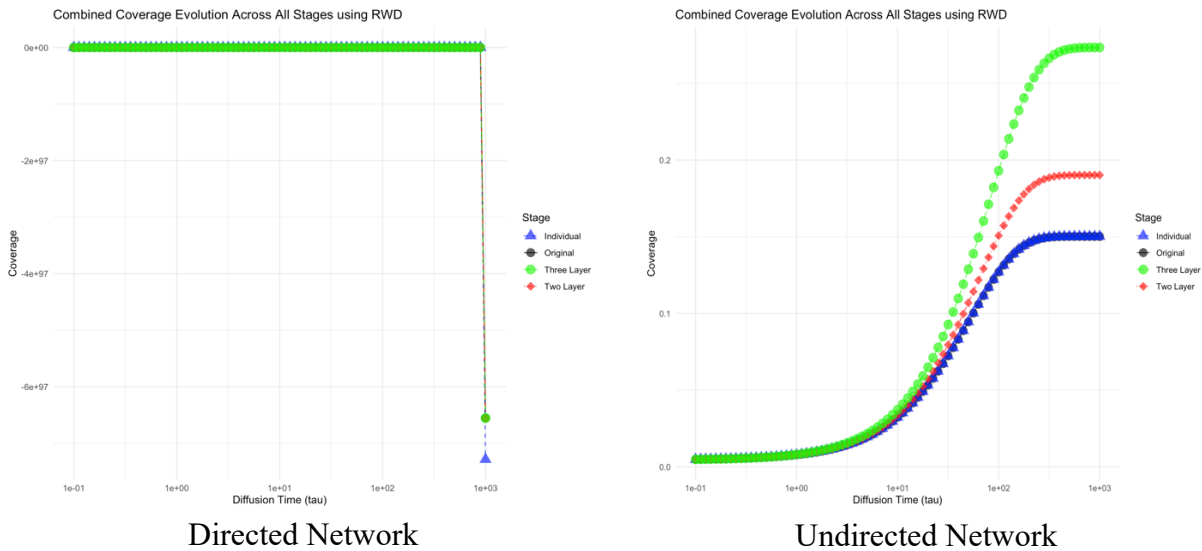
### STAGE 3: TWO-LAYERS COMBINATION LINKS INTEGRATION

- Directed: The gap lessens to 0.0002309, with the coverage time edging up to 9,973 units.
- Undirected: Here, the spectral gap increases slightly to  $4.09\text{e-}06$ , yet the coverage time shortens to 562,561 units.

### STAGE 4: THREE-LAYER COMBINATION LINKS INTEGRATION

- Directed: The spectral gap further contracts to  $4.45\text{e-}05$ , leading to a substantial coverage time of 51,725 units.
- Undirected: The spectral gap expands to  $3.41\text{e-}06$ , with the coverage time increasing to 676,034 units.

These observations indicate that the directed network has a quicker response to link additions, enhancing traversal efficiency. In contrast, the undirected network maintains long coverage times despite link modifications. The coverage plots in Figure 5 vividly illustrate these dynamics, underscoring the contrasting behaviors between the two network types.



**Figure 4. Coverage Evolution in Directed and Undirected Networks using Diffusive RW.**