

INSTRUCTIONS

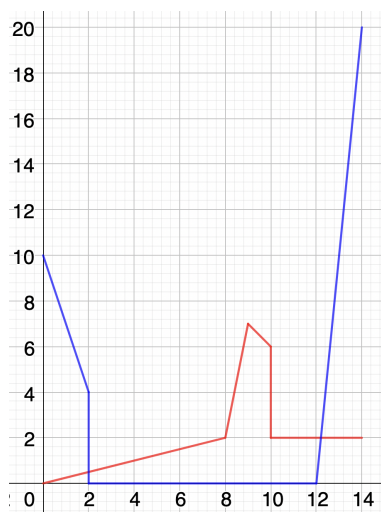
1. DO NOT SCROLL DOWN TO THE PROBLEMS UNTIL YOU ARE READY.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil if you would like to get one from [here](#). Check the blackened circles for accuracy and erase errors and stray marks completely. However, only answers on the MIMC Google Form found on the AoPS community page or here will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, rulers, compass, and erasers. No calculators, smart-watches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When you are ready to start the test, you can begin working on the problems. You will have 75 minutes to complete the test.
8. When you finish the exam, fill in and submit the Google Form.
9. Enjoy the problems!

The MIMC Committee reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

The Committee will publish Distinction and Distinguished Honor Roll cutoffs after the contest ends.

1. Find the value of $1 + 2 + 3 + 4 + 5 - 4 - 3 - 2 - 1$.
(A) 1 (B) 3 (C) 5 (D) 7 (E) 9
2. Find the number of three digit positive integers $\underline{1BC}$ that is divisible by 36.
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
3. How many integers $a \neq 0$ are there such that $\frac{36}{a}$ is an integer?
(A) 6 (B) 9 (C) 12 (D) 18 (E) 24
4. Joshua is racing with Andrew on a straight track. Joshua starts $40m$ in front of Andrew, and Joshua's running speed is $3m/s$ while Andrew's running speed is $4m/s$. How long does it take for Andrew to reach Joshua assume that they never stop and they both runs in constant speed.
(A) 10 (B) 20 (C) 40 (D) 80 (E) Andrew will never reach Joshua
5. Find n if the number n satisfies $n - \frac{11}{12} = \frac{n}{3}$
(A) $\frac{11}{2}$ (B) $\frac{11}{4}$ (C) $\frac{11}{8}$ (D) $\frac{11}{24}$ (E) $\frac{11}{36}$
6. If $a \heartsuit b = a^2 + 2ab + b^2$, find the value of $9 \heartsuit (1 \heartsuit 3)$.
(A) 13 (B) 16 (C) 25 (D) 256 (E) 625
7. Find the value of $1 + 2 - 2 + 3 - 3 + 4 - 4 + 5 - 5 + \dots + 99 - 99 + 100 - 100$.
(A) -99 (B) 0 (C) 1 (D) 101 (E) 5050
8. The mean of $1, 2, 2, 3, 4, x, y, 8$ is 8 such that $4 \leq x \leq y \leq 8$. What is the value of $x + y$?
(A) 14 (B) 24 (C) 34 (D) 44 (E) x and y don't exist.

9. The following figure shows the velocity vs. time graph of two people, Amma and Belle, chasing each other. The horizontal axis represents time, in minutes and the vertical axis represents velocity, in kilometers per hour. If they started on the same position and ran in one-dimensional space, what is the positive difference of their running distance, in kilometers, after 14 minutes? (Every vertex point has integer coordinates.)



- (A) 1 (B) 2 (C) 4 (D) 7 (E) 10
10. A right cone has a diameter x and height h . A circle parallel to the to the base is cut resulting a smaller cone with diameter $\frac{x}{3}$. What is the volume of the truncated cone after the smaller cone was cut?
- (A) $\frac{13}{162}\pi x^2 h$ (B) $\frac{13}{54}\pi x^2 h$ (C) $\frac{4}{9}\pi x^2 h$ (D) $\frac{13}{27}\pi x^2 h$ (E) $\frac{1}{2}\pi x^2 h$
11. The graph of $x^2 + 2^x + 1 = y$ passes through which of the following point?
- (A) (0, 1) (B) (0, 2) (C) (1, 2) (D) (1, 3) (E) (2, 8)
12. In a right triangle ABC with right angle B , two sides of $\triangle ABC$ are $\sqrt{5}$ and 1, respectively. What is the sum of the two possible third sides?
- (A) $2 + \sqrt{6}$ (B) 5 (C) $3 + \sqrt{5}$ (D) $3 + \sqrt{6}$ (E) 10
13. Let $f(a, b)$ be a function that outputs the positive difference of the arithmetic mean of a, b and their geometric mean. Find $f(4, 9)$.
- (A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$ (E) $\frac{9}{2}$

14. Let $a = 2^{2022}$. How many positive divisors of a leave a remainder of 1 when divided by 7?
(A) 337 (B) 338 (C) 674 (D) 675 (E) 676
15. How many ways are there to arrange the letters $AAABCCCCDE$ such that B, D, E must be together? (Identical letters are indistinguishable, but different letters are distinguishable.)
(A) 210 (B) 420 (C) 840 (D) 1680 (E) 3360
16. Let O be a circle such that the numerical value of its area is $\sqrt{2}$ times the numerical value of its circumference in centimeters. Find the diameter of O in centimeters.
(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) 8
17. In the diagram, there are four concentric circles with diameters 2, 4, 6, 10, respectively. In each circle, an inscribed square was shaded, but excluding the previous circle. The ratio between the shaded and non-shaded region can be expressed as

$$\frac{a - b\pi}{c\pi - d}$$

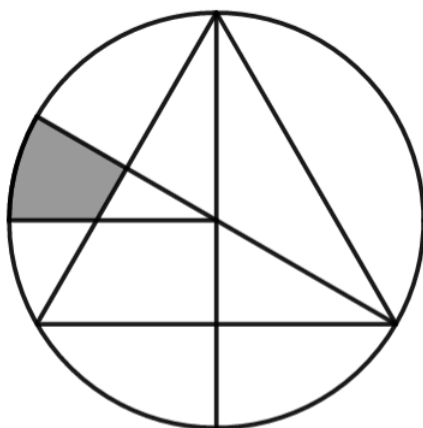
such that $\gcd(a, b, c, d) = 1$. Find $a + b + c + d$.



- (A) 201 (B) 202 (C) 203 (D) 204 (E) 205

18. Let a positive integer be *addy* if it can be expressed as $a^2 - b^2$ such that both a, b are non-integers. Which of the following answer choice is not *addy*?
(A) 4 (B) 12 (C) 24 (D) 26 (E) 45
19. Call a five digit positive integer *Semi-Palindrome* if the number can be written in the form $ABABA$ which A and B denotes two different number digits. How many *Semi-Palindrome* s are divisible by 12?
(A) 4 (B) 6 (C) 8 (D) 12 (E) 16
20. Given that a, b, c forms a geometric progression, in that order, such that $a + b + c = 15$ and $abc = 15$. What is b ?
(A) 2 (B) $\sqrt{5}$ (C) $\sqrt[3]{15}$ (D) $\sqrt{15}$ (E) 5
21. Let $f(x)$ be a function such that $af(a) + bf(b) = af(b) + bf(a) = f(ab)$. Given that $f(18) = 18$, what is the value of $f(3) + f(6)$?
(A) 2 (B) 4 (C) 9 (D) 12 (E) 18
22. Given that a and b are positive real numbers such that $(a + 2b)a = b^2$. What is the value of $\frac{a}{b}$?
(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{1}{4}$ (C) $\sqrt{2} - 1$ (D) $\frac{1}{2}$ (E) $\sqrt{2} + 1$
23. Given that x is a real number greater than 1 satisfying $x + \frac{1}{x} = 3$. What is the value of $x^4 - \frac{1}{x^4}$?
(A) $21\sqrt{5}$ (B) $28\sqrt{5}$ (C) $49\sqrt{5}$ (D) $140\sqrt{5}$ (E) $147\sqrt{5}$

24. An equilateral triangle was inscribed inside a circle with area π . The shaded region was formed by the perpendicular bisector on the side and the perpendicular bisector of the diameter drawn from one of the other vertices, as shown. Find the area of the shaded region.



- (A) $\frac{\pi}{12} - \frac{\sqrt{3}}{24}$ (B) $\frac{\pi}{12} - \frac{\sqrt{3}}{12}$ (C) $\frac{\pi}{6} - \frac{\sqrt{3}}{36}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{24}$ (E) $\frac{\pi}{6} - \frac{\sqrt{3}}{12}$
25. Find the number of positive divisors of

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 + 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 + \cdots + 96 \cdot 97 \cdot 98 \cdot 99 \cdot 100.$$

- (A) 1296 (B) 1400 (C) 1600 (D) 1728 (E) 2187

ADDITIONAL INFORMATION

1. The Committee on the Michael595 & Interstigation Math Contest (MIMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The MIMC also reserves the right to disqualify score from a test taker if it is determined that the required security procedures were not followed.
2. The publication, reproduction or communication of the problems or solutions of the MIMC 10 will result in disqualification. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules except the private discussion form.

Sincerely, the MIMC mock contest cannot come true without the contributions from the following testsolvers, problem writers and advisors:

Michael595 (Problem Writer)

Interstigation (Problem Writer)