Nanoscopy using a semiconductor heterostructure as the sample stage

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1 Abstract

A special structured illumination microscopy scheme using a two dimensional electron gas as the sample stage is proposed. Terahertz plasma waves generated by a current driven instability illuminate the sample. Meanwhile, a plane wave is used to shift the plasmonic pattern needed to expand the observable range of spatial frequencies. Full coverage of the spatial frequency regime is obtained by tuning the plasma waves through gate voltage control. Hence, it is possible to reconstruct an image with resolution up to two orders of magnitude beyond the diffraction limit. Due to linear nature of the technique, only a weak illumination signal is required, minimizing the chances of radiation damage of sample.

2 Introduction

In conventional wide-field fluorescent microscopy, a sample is uniformly illuminated by a beam of light, and the resulting fluorescence is observed in the far-field through the objective lens. The uniform intensity of the illumination along the sample fundamentally restricts the resolution of the system to half the wavelength of light due to Abbe diffraction limit. With ever growing need to image tiny objects especially in life sciences, modern microscopy techniques such as confocal and linear structured illumination microscopy (SIM) use spatially non-uniform sources of light to illuminate the sample, resulting in resolution extending beyond the diffraction limit by a factor of 2 [1, 2]. Use of pinholes in confocal microscopy makes the technique highly inefficient as a significant portion of light is discarded and it may leave weakly fluorescent objects undetectable. Structured Illumination microscopy is a wide-field technique in which a fine illumination pattern such as a sinusoidal standing wave is used to generate *Moiré fringes* in the observed image. The high frequency content is mathematically reconstructed from a series of images acquired by shifting the pattern. Using a non-linear version of SIM, theoretically unlimited resolution can be achieved [3]. However, high levels of illumination intensity are required, subjecting the sample to significant radiation damage.

Surface waves are electromagnetic waves existing at the boundary of a medium having wavelength and phase velocity much smaller than the homogeneous waves of the same frequency in that given medium. Illuminating a sample by surface waves was first proposed by Nassenstein to realize super-resolution since the sub-diffracted waves contain spatial frequency information of the sample beyond the diffraction limit [4]. Recently, a plasmonic structured illumination microscopy (PSIM) technique was proposed in which surface plasmons existing at a metal-dielectric interface were used to excite a sample at optical frequencies [5]. In another scheme, resolution of up to two orders of magnitude beyond the diffraction limit was achieved through mid-infrared graphene plasmons [6].

In solid-state devices like high electron mobility transistor (HEMT), a two-dimensional electron gas (2DEG) formed at the interface of two epitaxially grown semiconductors, acts as a transistor channel where free electron concentration of metal-like proportions is observed without any doping, along with remarkably high electron mobility. Plasma waves originating in the few atoms thick electron channel of field-effect transistors, discovered more than 40 years ago have lately received interest because of the potential to realize terahertz frequency sources and sensors [7, 8, 9, 10, 11]. For micro-scale lengths, the channel becomes a plasma cavity where the resonant frequency lies in the far-infrared (terahertz) frequency region and remarkably, can be tuned by varying the gate voltage.

In this work, a nanoscale imaging technique is presented in which subwavelength plasma waves, generated in a transistor channel that can be tuned by controlling gate voltage are used as the illumination pattern required for (SIM) that effectively creating a much larger

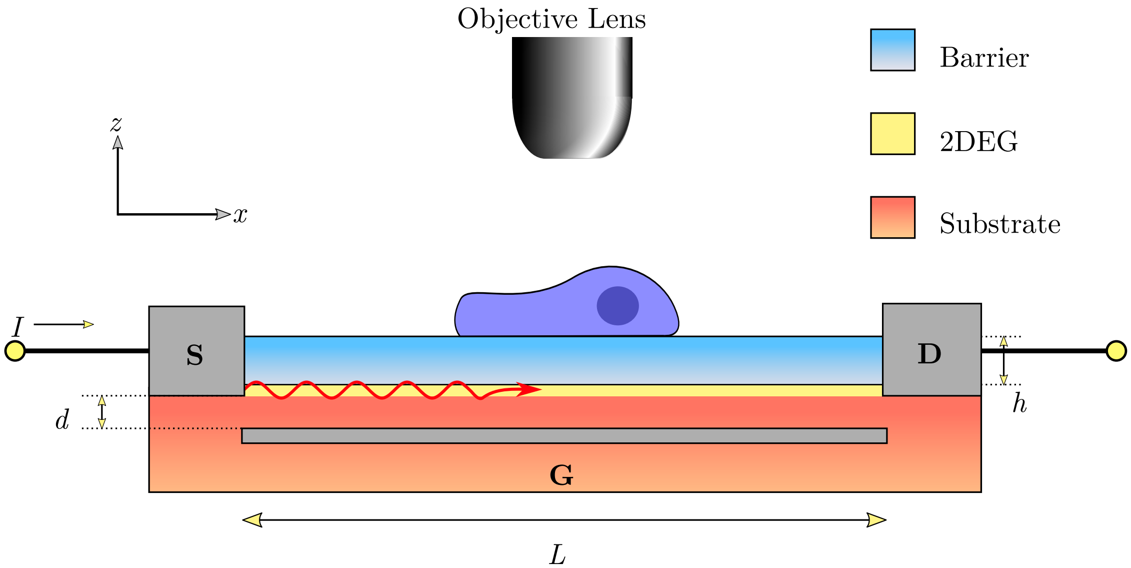

Figure 1: Sample placed on top of HEMT with back gate and excited by 2D plasmons generated by a direct current

observable spatial frequency region as compared to a far-infrared (terahertz) plane wave. Due to the linear nature of the scheme, resolution of up to two orders of magnitude can be obtained with a weak field intensity.

3 Theory

3.1 Dispersion relation

A schematic diagram of the proposed system similar to a transistor is shown in Fig. 1 where a 2DEG that acts as a transistor channel, is formed at the interface of two semiconductor materials of slightly different band-gap energies. Plasma waves are generated in the channel when the source and drain terminals are driven by a current source. Due to reflections from the conducting boundaries, the channel region forms a cavity and the plasma waves exhibit a standing wave pattern. The structure is backed by a gate terminal that spans the length L of the channel and spaced a distance d apart from the 2DEG. The gate capacitively couples with the 2DEG and by varying the voltage, the velocity as well as concentration of electrons in the channel can be controlled. A barrier layer of thickness h separates the sample from the 2DEG.

The dispersion relation that shows a frequency dependent resonance response of plasma waves in the 2D channel is obtained by imposing a transverse resonance condition on an equivalent transmission line (TL) circuit [12, 13]. The 2DEG is modeled as a shunt admittance related to Drude-type surface conductivity,

$$Y_\sigma = \sigma_s = \frac{N_s e^2 \tau}{m^*} \frac{1}{1 - j\omega\tau} \tag{1}$$

where N_s is the surface electron density in the channel, e is the electron charge, m^* is the effective electron mass in the heterostructure, τ is the scattering time of electrons, ω is the angular frequency. The dispersion relation is then written as:

$$Y^{\uparrow}(z_0) + Y^{\downarrow}(z_0) + Y_\sigma = 0. \tag{2}$$

Here $Y^{\uparrow}(z_0)$ and $Y^{\downarrow}(z_0)$ are the upward- and downward-looking TL admittances at $z = 0$,

$$Y^{\uparrow}(z_0) = Y_2 \frac{1 - \Gamma^{\uparrow}(z_0)}{1 + \Gamma^{\uparrow}(z_0)} \tag{3}$$

$$Y^{\downarrow}(z_0) = -jY_1 \cot(k_{z1} d_1) \tag{4}$$

Here, $d_{1,2}$ are the thickness of the first and second layers, respectively, Y_i and k_{zi} where $i = 0, 1, 2$ are the respective TM mode admittances and wavenumbers of the corresponding layer given by:

$$Y_i = \frac{\omega \varepsilon_i \varepsilon_0}{k_{zi}} \qquad\qquad k_{zi} = \pm\sqrt{k_0^2 \varepsilon_i - k_x^2} \tag{5}$$

$$\tag{6}$$

where ε_i is the relative permittivity of i^{th} layer and k_x is the longitudinal propagation constant of the structure. The upward-looking reflection coefficient $\Gamma^{\uparrow}$ in (3) is expressed in terms of the TM mode admittances:

$$\Gamma^{\uparrow}(z_0) = \frac{Y_1 - Y_0}{Y_0 + Y_1} \mathrm{e}^{-2jk_{z2}d_1} \tag{7}$$

An analytical solution of (2) in terms of longitudinal propagation constant k_x is tedious, therefore numerical root-finding techniques such as the Newton method were employed [14]. As an example, the dispersion relation of a Gallium Nitride / Aluminum Gallium Arsenide (GaN/AlGaN) heterostructure is shown in Fig. 2. At 25 THz, the plasma propagation constant is 80 times greater than free-space wavenumber. Such extreme subwavelength phenomenon makes super-resolution possible, extending resolution by two orders of magnitude beyond the diffraction limit. The structure parameters used to compute the dispersion relation using (1)-(7) are briefly discussed. The gate-channel separation is $d = 100$ nm. The channel length L is 2 μm whereas the AlGaN barrier layer is $h = 20$ nm wide. The permittivity of both semiconductor layers is approximated to the static value, i.e., $\varepsilon_1 \approx \varepsilon_2 = 9.5$ [11]. A surface carrier density of $N_s = 7.5 \times 10^{12}$ cm^{-2} and scattering time τ of 114ps corresponding to a temperature of 3K is assumed. Through the gate voltage V_g, the electron

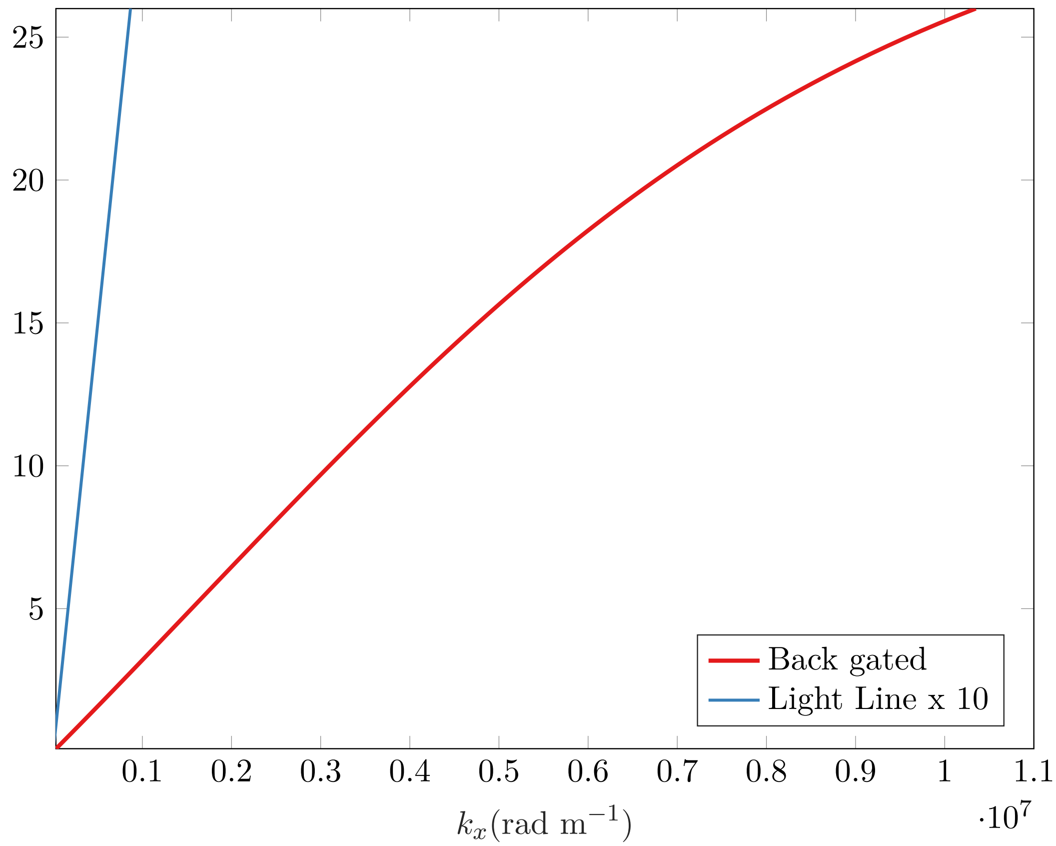

Figure 2: Dispersion curve of back gated plasmons at zero gate bias

density N_s of the channel can be varied using the relation:

$$N_s = N_0 \times (1 - \frac{V_g}{V_T}) \tag{8}$$

where N_0 is the zero-bias density and V_T is the threshold voltage of the transistor. The plasma frequency ω_p of the channel can therefore be tuned by the expression [15]:

$$\omega_p = \sqrt{\frac{N_s e^2 d}{m_* \varepsilon}} \frac{\pi}{L} \tag{9}$$

For a threshold voltage of -0.764V, the effect of gate bias on the plasma frequency is shown in Fig. 3. where V_{th} is the gate threshold voltage. To derive the dispersion relation, the

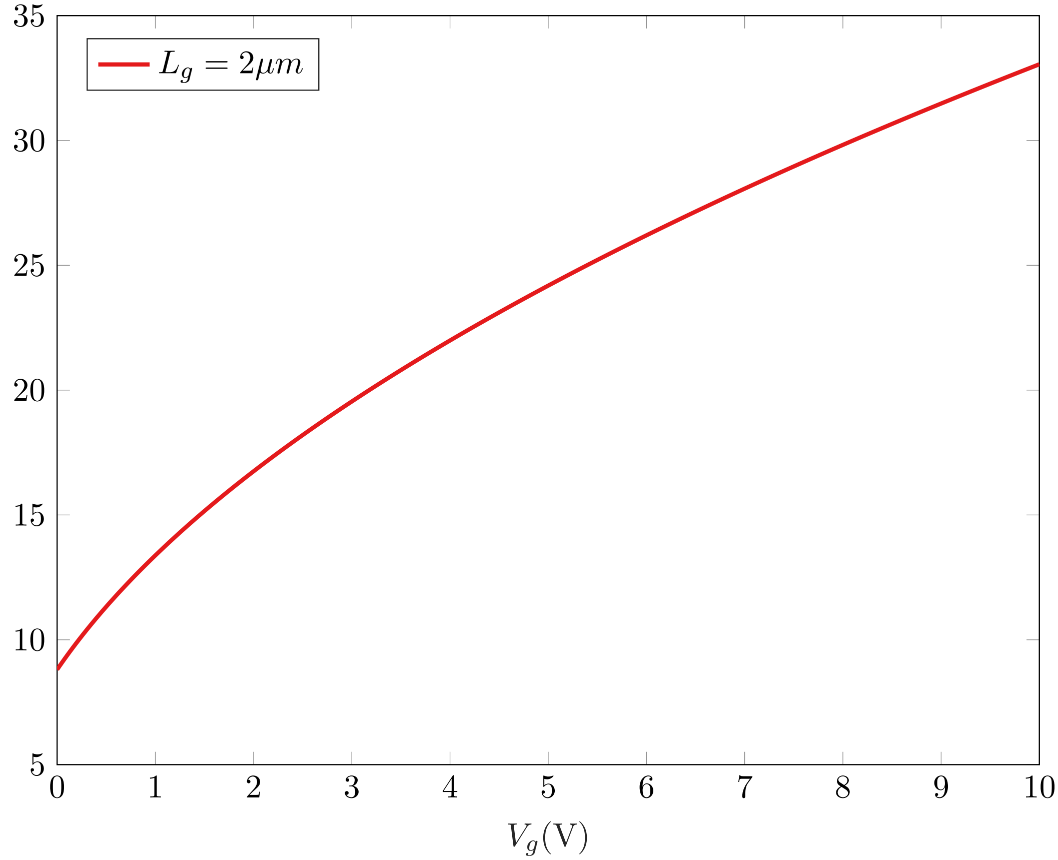

Figure 3: Plasma frequency tunable using gate bias with $V_T = -0.76$V

2DEG is described by a surface conductivity function:

3.2 Image Reconstruction

Consider $I(\mathbf{r})$ as the sinusoidal illumination intensity:

$$I(\mathbf{r}) = 1 + \cos\big(\mathbf{k}_\rho \cdot \mathbf{r} + \phi\big) \tag{10}$$

where $\mathbf{k}_\rho = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$ is the spatial frequency wavevector, $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ is the two-dimensional positional vector and ϕ is the pattern phase. The image of a sample $F(\mathbf{r})$ observed through a microscope can be expressed as:

$$M(\mathbf{r}) = \big[F(\mathbf{r}) \cdot I(\mathbf{r})\big] \otimes H(\mathbf{r}) \tag{11}$$

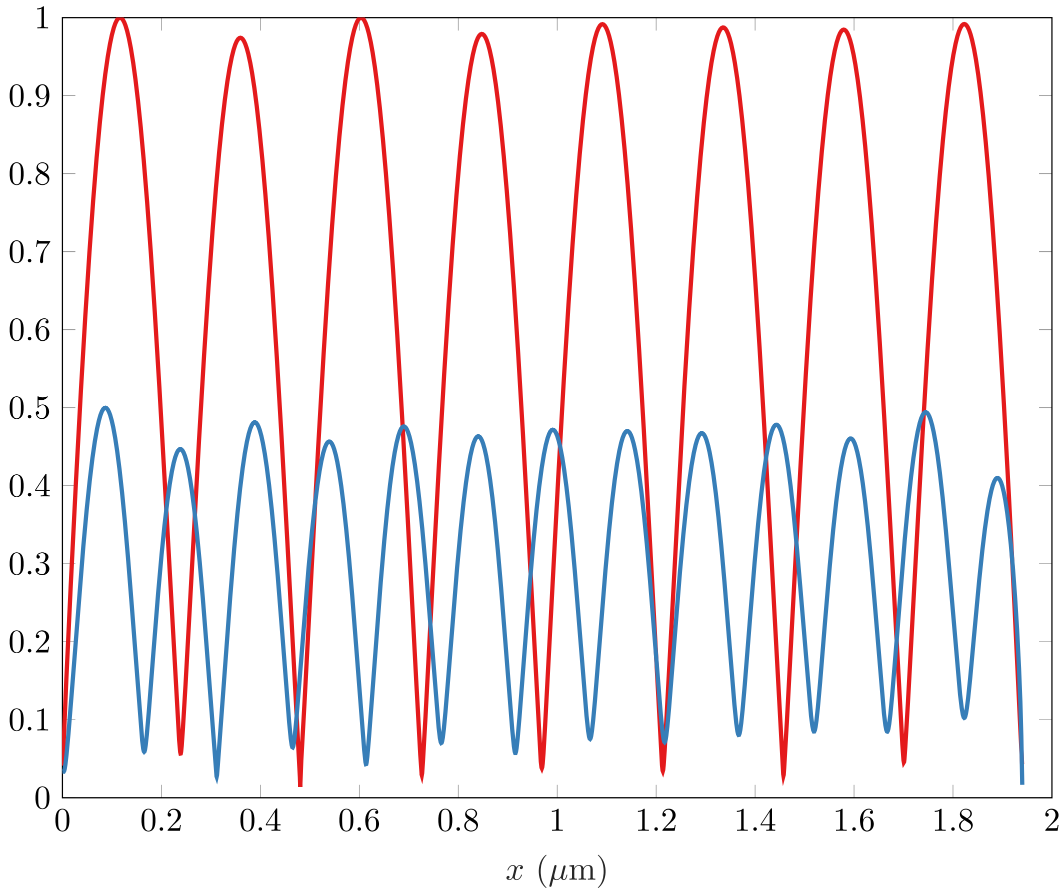

Figure 4: Different frequency

where $H(\mathbf{r})$ is the point spread function (PSF) of the microscope, and $\cdot, \otimes$ denote multiplication and convolution operations in the spatial domain respectively. A frequency domain representation of the image by taking the Fourier transform is expressed as:

$$\begin{aligned} \tilde{M}(\mathbf{k}) &= \left[\tilde{F}(\mathbf{k}) \otimes \tilde{I}(\mathbf{k})\right] \cdot \tilde{H}(\mathbf{k}) \\ &= \frac{1}{2}\left[2\tilde{F}(\mathbf{k}) + \tilde{F}(\mathbf{k} - \mathbf{k}_\rho)\mathrm{e}^{-j\phi} + \tilde{F}(\mathbf{k} + \mathbf{k}_\rho)\mathrm{e}^{j\phi}\right] \cdot \tilde{H}(\mathbf{k}) \end{aligned} \tag{12}$$

where $\sim$ over the letters indicates a frequency domain term and $\tilde{H}(k)$ is the optical transfer function (OTF) of the microscope. As evident in (12), a sinusoidal illumination pattern has three frequency components which generates an image which is linear combination of the sample along with two shifted versions as shown in Fig. 5(b). To reconstruct the sample, three different images need to be captured with different phase term ϕ. The

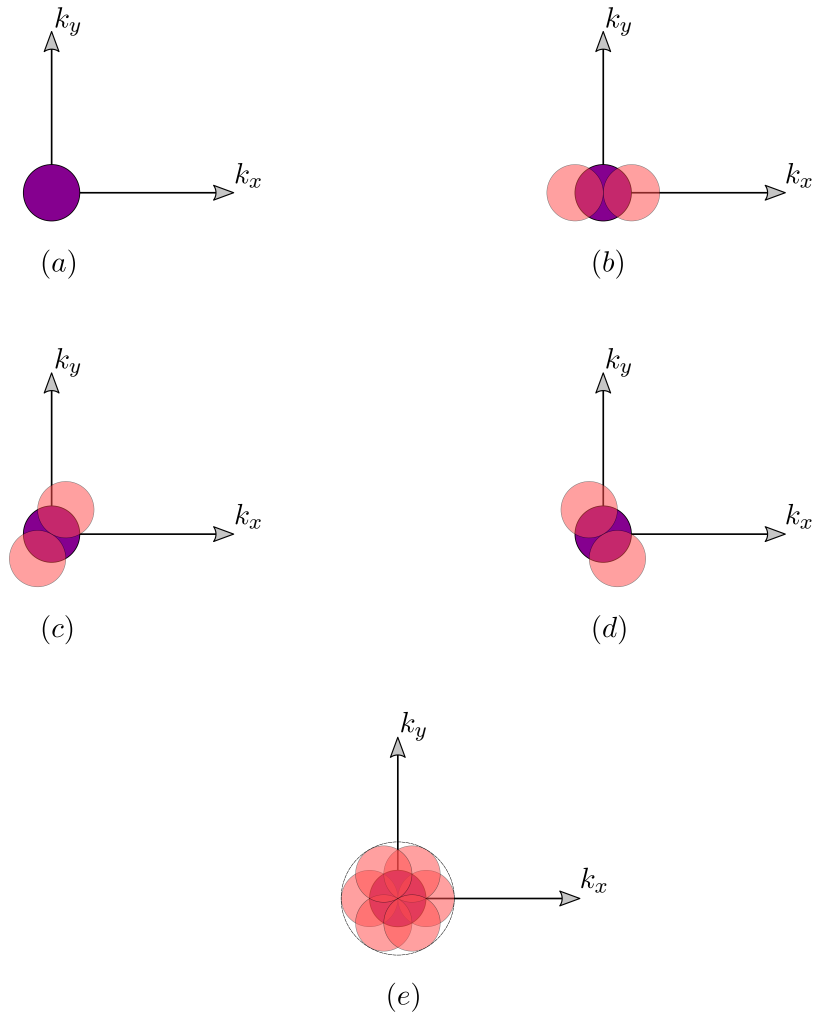

Figure 5: Resolution enhancement through SIM: (a) Diffraction limited observable region in frequency domain. Moiré effect using a sinusoidal illumination pattern bringing high frequency content under the observable region. The sample is rotated: (b) 0°, (c) 60°, (d) 120°. (e) Doubling of lateral resolution with effective coverage area twice the size of (a)

process can be expressed as a system of linear equations,

$$\tilde{H}(\mathbf{k}) \cdot \begin{bmatrix} \tilde{F}(\mathbf{k}) \\ \tilde{F}(\mathbf{k}-\mathbf{k}_\rho) \\ \tilde{F}(\mathbf{k}+\mathbf{k}_\rho) \end{bmatrix} = \begin{bmatrix} 2 & \mathrm{e}^{-j\phi_1} & \mathrm{e}^{j\phi_1} \\ 2 & \mathrm{e}^{-j\phi_2} & \mathrm{e}^{j\phi_2} \\ 2 & \mathrm{e}^{-j\phi_3} & \mathrm{e}^{j\phi_3} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{M}_1(\mathbf{k}) \\ \tilde{M}_2(\mathbf{k}) \\ \tilde{M}_3(\mathbf{k}) \end{bmatrix} \tag{13}$$

The phase shifts in (13) are known beforehand. Frequency content of the sample up to k_ρ can, therefore be observed due the Moiré effect which transports the high frequency information in to the observation region. To achieve two-dimensional enhancement in resolution, the sample has to be rotated about the optical axis of the microscope or the angular distribution of the illumination needs to be varied. In this work, an external plane wave is used to shift the plasma wave pattern laterally in the sample stage. The electric field of a TM polarized plane wave is expressed as, $\mathbf{E}_{ext} = a\hat{\mathbf{x}} + b\hat{\mathbf{z}}$. The total field is the sum of the plasmonic field and external plane wave. The total intensity is then expressed as:

$$\begin{aligned} |E|^2 &= (a + \cos k_\rho x)^2 + (b + \sin k_\rho x)^2 \\ &= a^2 + b^2 + 1 + 2\chi \cos\bigl(k_\rho x + \psi\bigr) \end{aligned} \tag{14}$$

where k_ρ is the spatial frequency of plasma wave, $\chi = \sqrt{a^2 + b^2}$ and $\psi = \mathrm{atan}\bigl(b/a\bigr)$. Using a commercial full-wave electromagnetic simulation tool [16], the pattern shifting is shown in An example of phased shift is shown in Fig. 6.

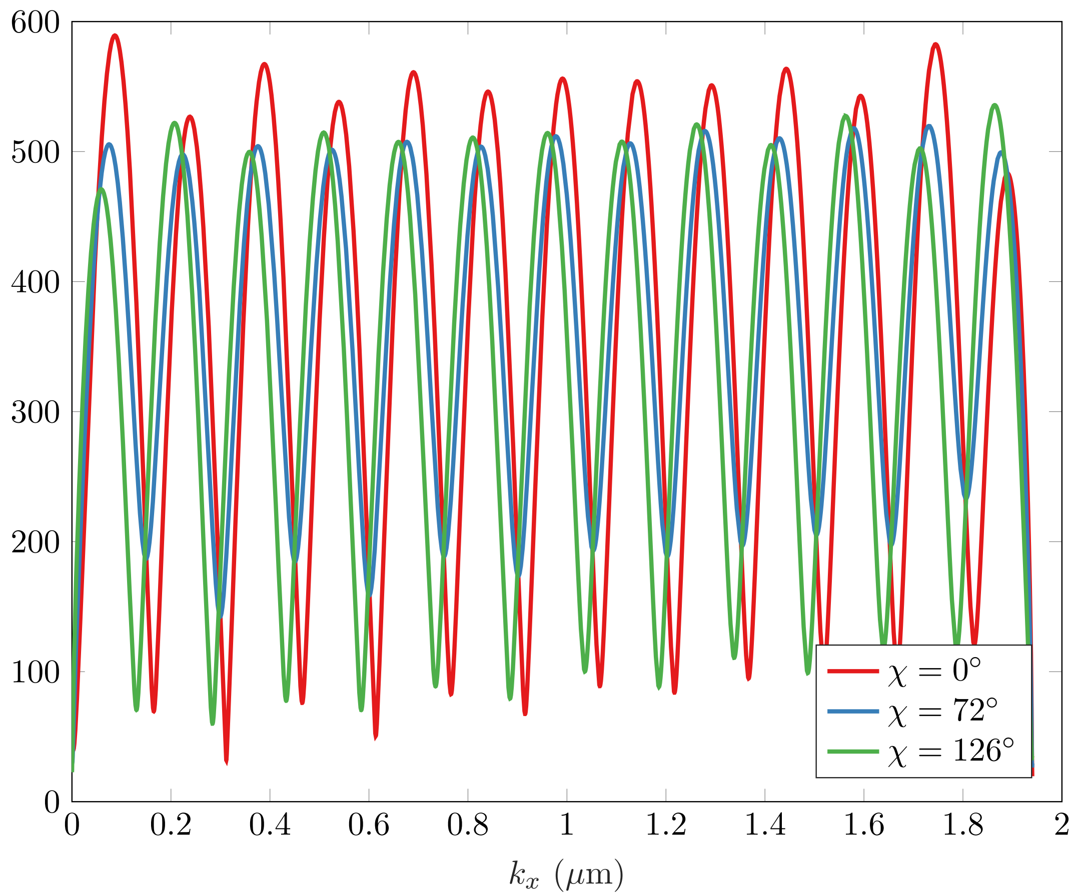

Figure 6: Lateral shifting of standing wave pattern using external wave

4 Results

We consider a 500 nm square sample with nano-sized fluorescent beads randomly dispersed illustrated in Fig. 7. To reconstruct the image from the spatial frequency information, the inverse Fourier transform of (12) should be ideally be performed with an infinite bandwidth. However, due to diffraction limits, the high-frequency information is lost leaving fine details blurred or lost.

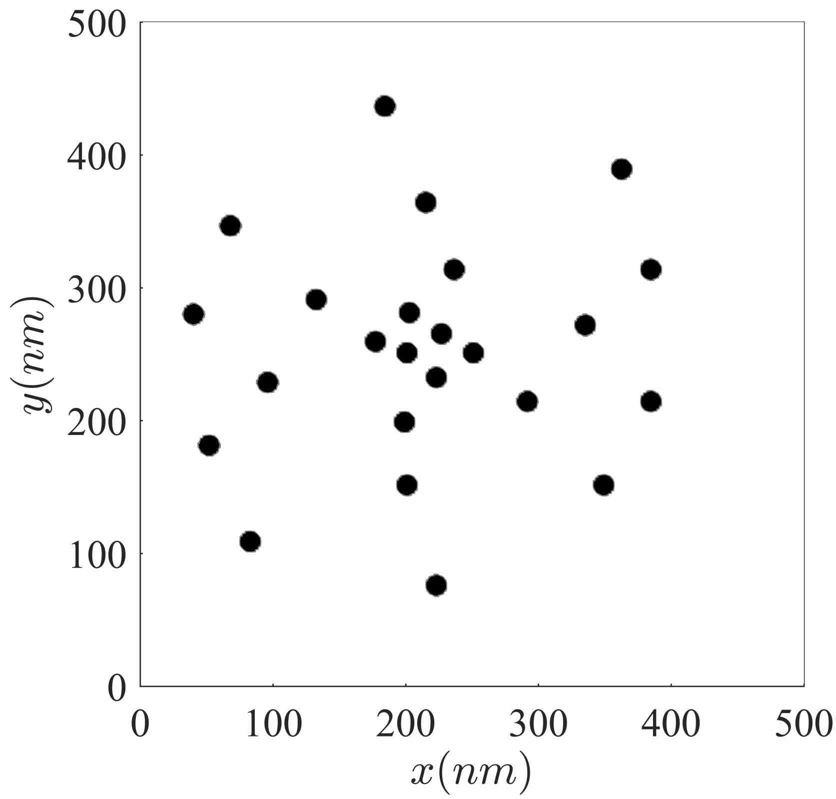

Figure 7: Test image with 15 nm fluorescent beads in a 500 nm square space

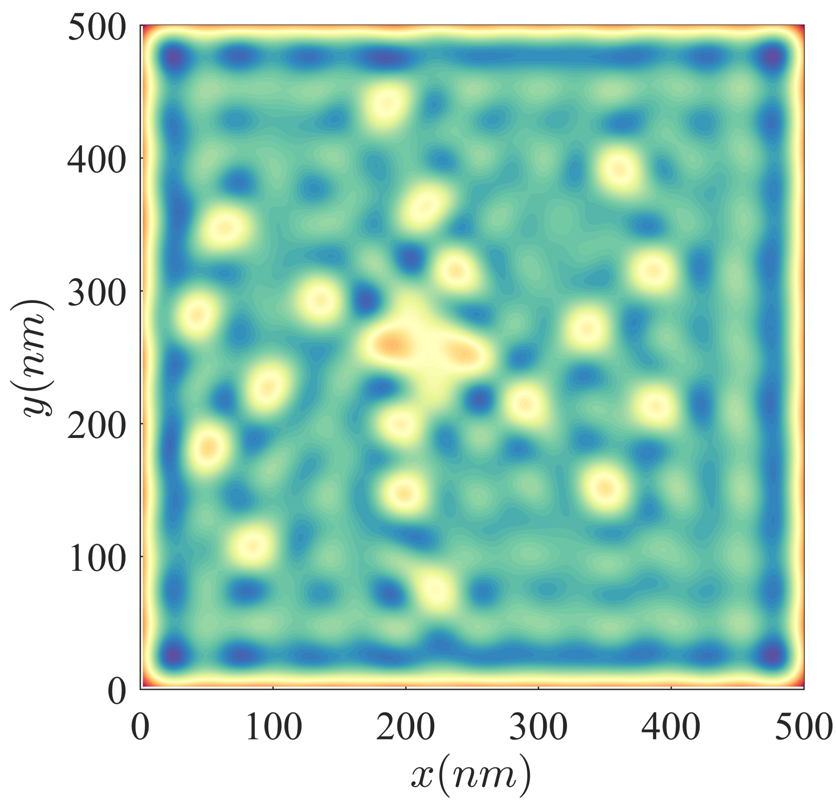

Figure 8: Simulation with GaN/AlGaN 2DEG at 20 THz corresponding to $\mathrm{Re}(k_p) = 39k_0$

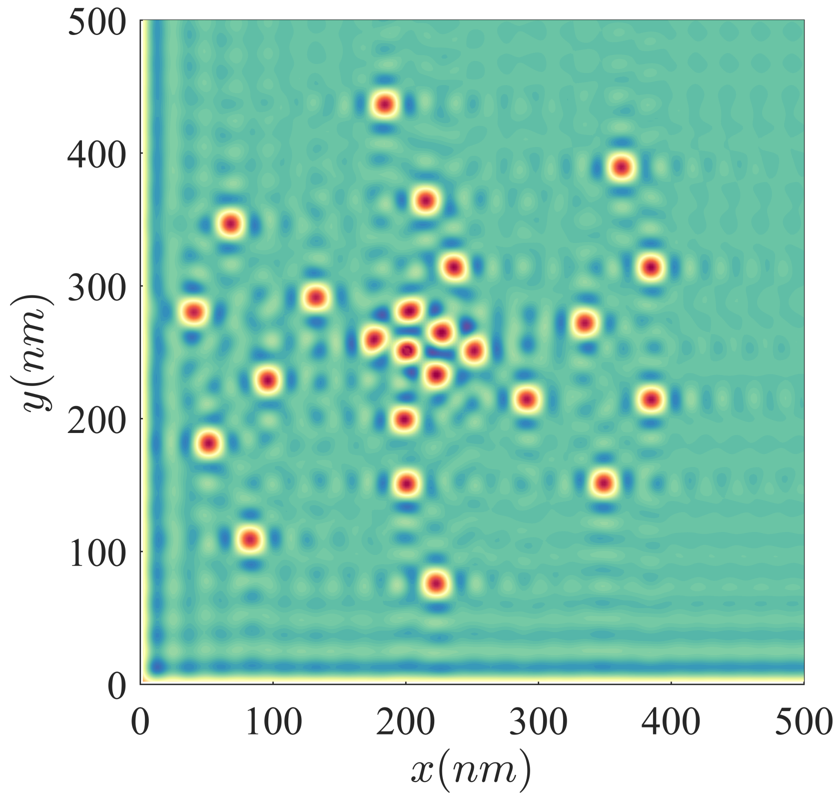

Figure 9: Simulation with GaN/AlGaN 2DEG at 25 THz corresponding to $\mathrm{Re}(k_p) = 80k_0$

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