Vertical Electric Dipole Above a Dielectric Half-space

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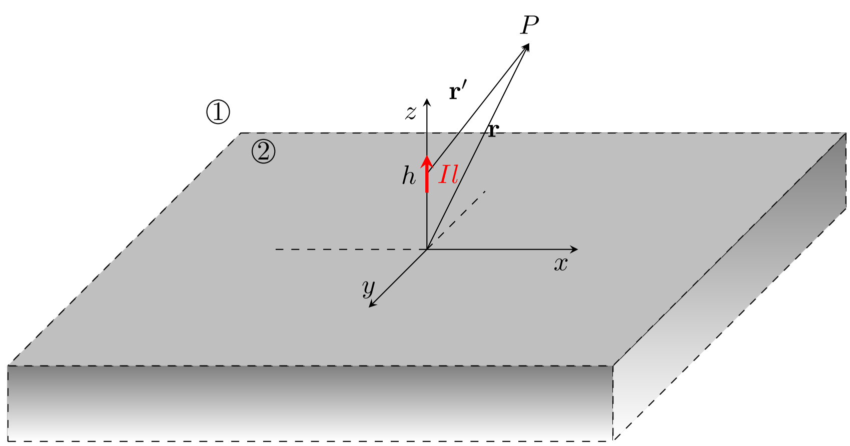

Figure 1: A vertically oriented Hertzian dipole above a dielectric half-space

An infinitesimally small electric dipole is vertically oriented in free space (medium 1) at $z = h$ and lies above a dielectric half space (medium 2). The media interface is at $z = 0$. An observation point P is chosen as illustrated in Fig. 1. The current on the dipole is expressed as,

$$\mathbf{J} = Il\delta(x)\delta(y)\delta(z-h)\hat{\mathbf{z}} \tag{1}$$

where the product of current I and length l gives the dipole moment.

Before formulating the problem, we express the propagation constant in each medium,

$$\begin{aligned} k_1 &= \omega\mu_1\varepsilon_1 \\ &= \omega\mu_0\varepsilon_0, \end{aligned} \tag{2a}$$

$$\begin{aligned} k_2 &= \omega\mu_2\varepsilon_2 \\ &= \omega\mu_0\varepsilon_0\left(\varepsilon' - j\varepsilon''\right). \end{aligned} \tag{2b}$$

Here we consider the dielectric to be non-magnetic and lossy which is manifested by a complex dielectric constant. In order to find the solution of the radiation problem at hand, we first look at the Maxwell's equations,

$$\nabla \cdot \mathbf{D} = \rho, \tag{3a}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{3b}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \tag{3c}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}. \tag{3d}$$

It is convenient to introduce magnetic vector potential, $\mathbf{A}$ at this point and set up the corresponding Helmholtz equation,

$$\begin{aligned} \left(\nabla^2 + k^2\right) \mathbf{A} &= -\mu \mathbf{J} \\ &= -\mu I l \delta(x)\delta(y)\delta(z-h)\hat{\mathbf{z}}, \mathbf{z} > \mathbf{0} \end{aligned} \tag{4a}$$

$$\left(\nabla^2 + k^2\right) \mathbf{A} = 0, z < 0. \tag{4b}$$

The electric and magnetic fields can be expressed in term of the potential function, $\mathbf{A}$ as:

$$\mathbf{E} = -\frac{j\omega}{k^2} \left(k^2 + \nabla\nabla\cdot\right) \mathbf{A}, \tag{5a}$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A}. \tag{5b}$$

In order to solve (4), continuity of tangential components of electric and magnetic fields is considered. The electric and magnetic fields can be resolved into Cartesian components as:

$$\begin{aligned} E_x &= -\frac{j\omega}{k^2} \frac{\partial^2 A_z}{\partial z^2} & H_x &= \frac{1}{\mu} \frac{\partial A_z}{\partial y} \\ E_y &= -\frac{j\omega}{k^2} \frac{\partial^2 A_z}{\partial y \partial z} & H_y &= -\frac{1}{\mu} \frac{\partial A_z}{\partial x} \\ E_z &= -\frac{j\omega}{k^2} \left(k^2 + \frac{\partial^2 A_z}{\partial z^2}\right) & H_z &= 0. \end{aligned}$$