Horizontal Electric Dipole

*

The transmission line Greens functions (TLGF) for planar multilayers of a High Electron Mobility Transistor (HEMT) are derived by modeling the radiating 2DEG as a embedded horizontal electric dipole illustrated in Fig. 1.

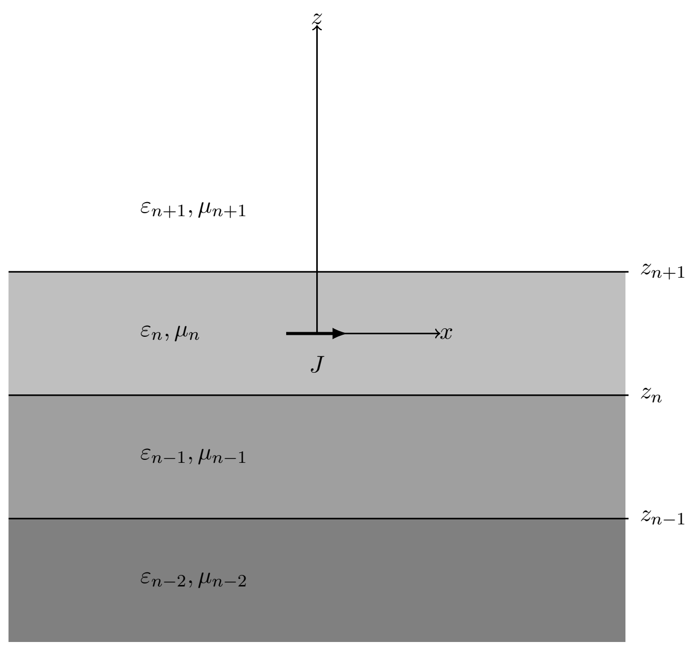

Figure 1: Current Source in a multi-section transmission line

From Transmission Line (TL) theory, the voltage and current in a source-free section can be expressed through the telegrapher's equations:

$$\frac{\mathrm{d}}{\mathrm{d}z}V(z) = -jk_z Z I(z), \tag{1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}I(z) = -jk_z Y V(z). \tag{1b}$$

where k_z is the propagation constant of the TL, Z and Y are respectively, the characteristic impedance and admittance of the transmission line. By combining (1a) and (1b), a second-order homogoneous differential equation in either V or I can be obtained:

*Last Modified: 16:54, Monday 19$^{\text{th}}$ September, 2016.

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k_z^2\right) \begin{cases} V(z) \\ I(z) \end{cases} = 0 \tag{2}$$

A travelling wave solution of (2) can be written as:

$$\left.\begin{matrix} V(z) \\ ZI(z) \end{matrix}\right\} = V^+ e^{-jk_z(z-z')} + V^- e^{jk_z(z-z')} \tag{3}$$

where the two terms represent forward and backward propagating waves with amplitudes V^+ and V^- respectively. The exponentials can be phased referenced to any point; the current choice will become evident in the treatment of a multilayer case.

For a transmission line of infinite extent excited by a current source of unit amplitude located at $z = z'$, the voltage and current can be expressed as:

$$V(z) = \frac{Z}{2} e^{-jk_z|z-z'|}, \tag{4a}$$

$$I(z) = \pm \frac{1}{2} e^{-jk_z|z-z'|}. \tag{4b}$$

The + sign in (4b) is for $z > z'$ and vice versa. The discontinuity due to the presence of the source at $z = z'$ should also be observed due to change of signs at $z = z'$.

1 Multi-section Transmission Line

We now consider a multi-section transmission line excited by a unit strength current source located in the n^{th} section as illustrated in Fig. 2.

The voltage and current expressions in any section now consist of a particular solution due to source and a homogeneous solution accounting for boundaries of the section. Due to a current source at $z = z'$, the solutions at any point z in the n^{th} section with propagation constant k_{zn} become:

$$V_i(z, z') = \frac{Z}{2} e^{-jk_z|z-z'|} + A_n e^{-jk_{zn}z} + B_n e^{jk_{zn}z}, \tag{5a}$$

$$I_i(z, z') = \pm \frac{1}{2} e^{-jk_z|z-z'|} + A_n e^{-jk_{zn}z} - B_n e^{jk_{zn}z}. \tag{5b}$$

In order to find the unknowns A_n and B_n, we invoke the boundary conditions that ensure continuity of the characteristic impedance along the transmission line. At the left edge z_n and the right edge z_{n+1}, we write:

$$\frac{V_i(z_n, z')}{I_i(z_n, z')} = -\overleftarrow{Z_n}, \quad z = z_n \tag{6a}$$

$$\frac{V_i(z_{n+1}, z')}{I_i(z_{n+1}, z')} = \overrightarrow{Z_n}, \quad z = z_{n+1}. \tag{6b}$$

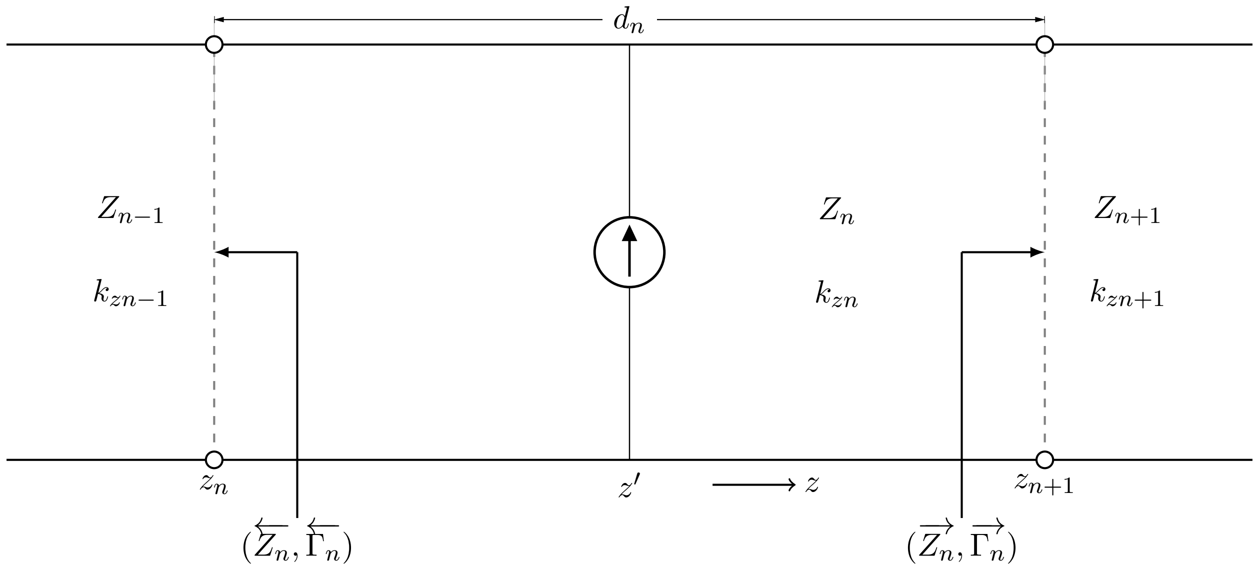

Figure 2: Current Source in a multi-section transmission line

From (5) and (6), we obtain:

$$\left(Z_n - \overleftarrow{Z_n}\right) e^{-jk_{zn}(z'-z_n)} + A_n \left(Z_n + \overleftarrow{Z_n}\right) e^{-jk_{zn}z_n} + B_n \left(Z_n - \overleftarrow{Z_n}\right) e^{+jk_{zn}z_n} = 0 \tag{7a}$$

$$\left(Z_n - \overrightarrow{Z_n}\right) e^{-jk_{zn}(z_{n+1}-z')} + A_n \left(Z_n - \overrightarrow{Z_n}\right) e^{-jk_{zn}z_{n+1}} + B_n \left(Z_n - \overrightarrow{Z_n}\right) e^{+jk_{zn}z_{n+1}} = 0 \tag{7b}$$

The solution of the system of two linear equations (7) can be written in general form as:

$$A_n = \frac{ce - bf}{bd - ae} \quad , \quad B_n = \frac{af - cd}{bd - ae} \tag{8}$$

where,

$$a = \left(Z_n + \overleftarrow{Z_n}\right) e^{-jk_{zn}z_n}, \tag{9a}$$

$$b = \left(Z_n - \overleftarrow{Z_n}\right) e^{+jk_{zn}z_n}, \tag{9b}$$

$$c = \left(Z_n - \overleftarrow{Z_n}\right) e^{-jk_{zn}(z'-z_n)}, \tag{9c}$$

$$d = \left(Z_n - \overrightarrow{Z_n}\right) e^{-jk_{zn}z_{n+1}}, \tag{9d}$$

$$e = \left(Z_n - \overrightarrow{Z_n}\right) e^{+jk_{zn}z_{n+1}}, \tag{9e}$$

$$f = \left(Z_n - \overrightarrow{Z_n}\right) e^{-jk_{zn}(z_{n+1}-z')}. \tag{9f}$$

The solution for unknowns, therefore is [1, p. 1178]:

$$A_n = \frac{\overleftarrow{\Gamma}_n e^{jk_{zn}2z_n}}{1 - \overleftarrow{\Gamma}_n \overrightarrow{\Gamma}_n e^{-2jk_{zn}d_n}} \left[e^{-jk_{zn}z'} + \overrightarrow{\Gamma}_n e^{-jk_{zn}(2z_{n+1}-z')} \right], \tag{10a}$$

$$B_n = \frac{\overrightarrow{\Gamma}_n e^{-jk_{zn}2z_{n+1}}}{1 - \overleftarrow{\Gamma}_n \overrightarrow{\Gamma}_n e^{-2jk_{zn}d_n}} \left[e^{+jk_{zn}z'} + \overleftarrow{\Gamma}_n e^{+jk_{zn}(2z_n-z')} \right] \tag{10b}$$

where $\overleftarrow{\Gamma}_n$ and $\overrightarrow{\Gamma}_n$ are the left and right looking reflection coefficients:

$$\overleftarrow{\Gamma}_n = \frac{\overleftarrow{Z}_n - Z_n}{\overleftarrow{Z}_n + Z_n} \tag{11a}$$

$$\overrightarrow{\Gamma}_n = \frac{\overrightarrow{Z}_n - Z_n}{\overrightarrow{Z}_n + Z_n} \tag{11b}$$

and $d_n = z_{n+1} - z_n$ is the thickness of the n^{th} section of the transmission line as illustrated in Fig. 2.

References

[1] K. A. Michalski, "Electromagnetic field computation in planar multilayers," *Encyclopedia of RF and microwave engineering*, 2005.