Integral Formulation of a Horizontally Oriented Magnetic Dipole in a layered medium

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We model the current distribution in a two-dimensional electron gas (2DEG) at a GaN/AlGaN heterostructure with a horizontal magnetic dipole (HMD) embedded in a layered medium as illustrated in Fig.1.

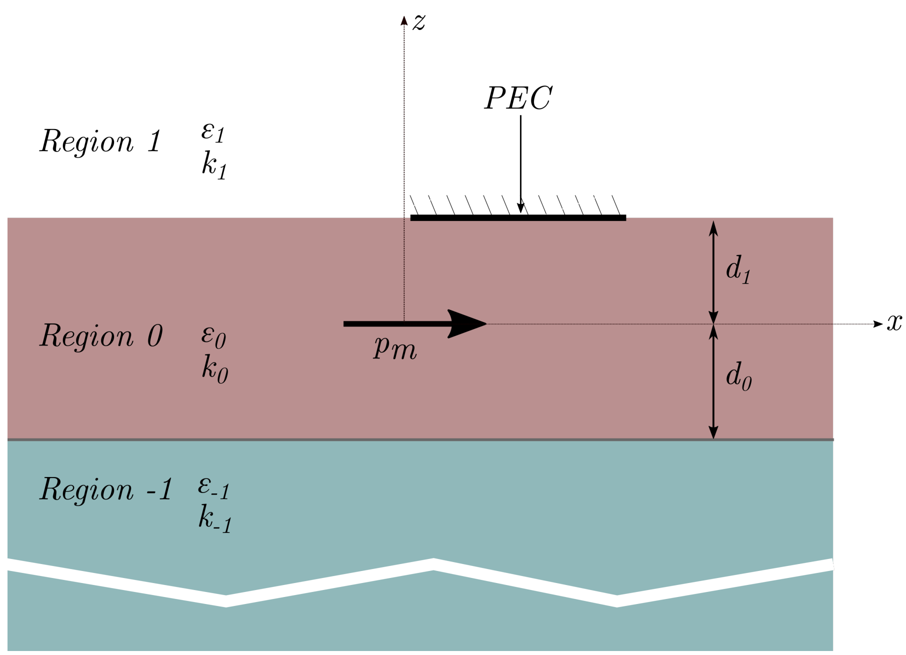

Figure 1: Three layer Structure

1 Green's Function Formulation

We consider a stratified medium with three layers with the dipole placed at the origin and the layered labeled as 0 with dielectric constant ε_0, different from the free-space permittivity. We consider two semi-infinite layers, above (Region 1) at $z = d_1$ and below (Region -1) the source layer at $z = d_0$ that extend to infinity. Assuming a TM case, the fields can be described by the electric field longitudinal component, E_z,

$$E_z = \int_{-\infty}^{\infty} E_z(\rho) dk_\rho \tag{1}$$

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In region i, the E_z field can be written as:

$$E_z^i = \int_{-\infty}^{\infty} \left[A_i e^{jk_{z_i}z} + B_i e^{-jk_{z_i}z}\right] H_n^{(1)}(k_\rho \rho) C_n(\phi) dk_\rho \tag{2}$$

where, $H_n^{(1)}(k_\rho \rho)$ is the nth-order Hankel function of the first kind and $C_n(\phi)$ is ϕ based function depending on the order of Hankel function and the dipole configuration. The remaining electric and magnetic field components can be found by the following Maxwell's equations:

$$E_\rho = \frac{1}{k_\rho^2}\frac{\partial}{\partial \rho}\frac{\partial E_z(k_\rho)}{\partial z} \tag{3a}$$

$$E_\phi = \frac{1}{k_\rho^2}\frac{1}{\rho}\frac{\partial}{\partial \phi}\frac{\partial E_z(k_\rho)}{\partial z} \tag{3b}$$

$$H_\rho = -\frac{j\omega\varepsilon_i}{k_\rho^2}\frac{1}{\rho}\frac{\partial E_z(k_\rho)}{\partial \phi} \tag{4a}$$

$$H_\phi = +\frac{j\omega\varepsilon_i}{k_\rho^2}\frac{1}{\rho}\frac{\partial E_z(k_\rho)}{\partial \rho} \tag{4b}$$

The unknowns A_i and B_i can be found by applying boundary conditions at the two interfaces at $z = d_1$ and $z = d_0$ that require the continuity of tangential components of the electric and magnetic fields. From (2), (3a) and (4a), we obtain:

$$k_{z_i}\left(A_i e^{jk_{z_i}d_i} - B_i e^{-jk_{z_i}d_i}\right) = k_{z_{i-1}}\left(A_{i-1} e^{jk_{z_{i-1}}d_i} - B_{i-1} e^{-jk_{z_{i-1}}d_i}\right) \tag{5}$$

$$\varepsilon_i\left(A_i e^{jk_{z_i}d_i} - B_i e^{-jk_{z_i}d_i}\right) = \varepsilon_{i-1}\left(A_{i-1} e^{jk_{z_{i-1}}d_i} - B_{i-1} e^{-jk_{z_{i-1}}d_i}\right) \tag{6}$$

In the region 0, the position of the observation point above or below the source leads to different values of unknowns A_0 and B_0. To make it distinct, for $z > 0$,the unknowns are written as $A^>$ and $B^>$ and for $z < 0$, $A^<$ and $B^<$ respectively. For HMD, the following conditions are used.[1]

$$A_0^> = A_{hmd} + E_{hmd} \tag{7a}$$

$$A_0^< = A_{hmd} \tag{7b}$$

$$B_0^> = B_{hmd} \tag{7c}$$

$$B_0^< = B_{hmd} + E_{hmd} \tag{7d}$$

where,

$$E_{hmd} = \frac{p_m \omega \mu_0 k_\rho^2}{8\pi k_{0z}} \tag{8}$$

$$H_{hmd} = -\frac{p_m k_\rho^2}{8\pi} \tag{9}$$

where p_m is the dipole moment of the current source. The longitudinal propagation constant in region i is given by:

$$k_z^i = \sqrt{k_i^2 - k_\rho^2} \tag{10}$$

Now using (5) and (6), at $z = d_1$, we have:

$$k_{z_1}\left(A_1 e^{jk_{z_1}d_1} - B_1 e^{-jk_{z_1}d_1}\right) = k_{z_0}\left(A_0^{>} e^{jk_{z_0}d_1} - B_0^{>} e^{-jk_{z_0}d_1}\right) \tag{11}$$

$$\varepsilon_1\left(A_1 e^{jk_{z_1}d_1} - B_1 e^{-jk_{z_1}d_1}\right) = \varepsilon_0\left(A_0^{>} e^{jk_{z_0}d_1} - B_0^{>} e^{-jk_{z_0}d_1}\right) \tag{12}$$

By adding (11) and (12) we get:

$$(k_{z_1} + \varepsilon_1) A_1 e^{jk_{z_1}d_1} - (k_{z_1} - \varepsilon_1) B_1 e^{-jk_{z_1}d_1} = (k_{z_0} + \varepsilon_0) A_0^{>} e^{jk_{z_0}d_1} - (k_{z_0} - \varepsilon_0) B_0^{>} e^{-jk_{z_0}d_1} \tag{13}$$

The unknowns A_1 and B_1 can be expressed in terms of $A_0^{>}$ and $B_0^{>}$ as [1]:

$$A_1 e^{jk_{z_1}d_1} = \frac{1}{2}\left[\frac{\varepsilon_0}{\varepsilon_1} + \frac{k_{z_0}}{k_{z_1}}\right] \times \left[A_0^{>} e^{jk_{z_0}d_1} + R^{\uparrow} B_0^{>} e^{-jk_{z_0}d_1}\right] \tag{14}$$

$$B_1 e^{-jk_{z_1}d_1} = \frac{1}{2}\left[\frac{\varepsilon_0}{\varepsilon_1} + \frac{k_{z_0}}{k_{z_1}}\right] \times \left[R^{\uparrow} A_0^{>} e^{jk_{z_0}d_1} + B_0^{>} e^{-jk_{z_0}d_1}\right] \tag{15}$$

where, $R^{\uparrow}$ is the TM Fresnel reflection coefficient at $z = d_1$ as seen from region 0,

$$R^{\uparrow} = \frac{-\varepsilon_0 k_{z_1} + \varepsilon_1 k_{z_0}}{\varepsilon^0 k_{z_1} + \varepsilon^1 k_{z_0}} \tag{16}$$

Through a similar procedure by applying boundary conditions at $z = d_{-1}$, coefficients A_{-1} and B_{-1} are obtained:

$$A_{-1} e^{jk_{z_{-1}}d_0} = \frac{1}{2}\left[\frac{\varepsilon_0}{\varepsilon_{-1}} + \frac{k_{z_0}}{k_{z_{-1}}}\right] \times \left[A_0^{<} e^{jk_z z_{-1} d_0} + R^{\downarrow} B_0^{<} e^{-jk_{z_{-1}}d_0}\right] \tag{17}$$

$$B_{-1} e^{-jk_{z_{-1}}d_0} = \frac{1}{2}\left[\frac{\varepsilon_0}{\varepsilon_{-1}} + \frac{k_{z_0}}{k_{z_{-1}}}\right] \times \left[R^{\downarrow} A_0^{<} e^{jk_{z_{-1}}d_0} + B_0^{<} e^{-jk_{z_{-1}}d_0}\right] \tag{18}$$

where, $R^{\downarrow}$ is the TM Fresnel reflection coefficient at $z = d_0$ as seen from region 0,

$$R^{\downarrow} = \frac{-\varepsilon_0 k_{z_{-1}} + \varepsilon_{-1} k_{z_0}}{\varepsilon_0 k_{z_{-1}} + \varepsilon_{-1} k_{z_0}} \tag{19}$$

For a structure having three regions, it is observed that B_1 and A_{-1} equals zero due to the open nature of region 1 and -1 respectively. By manipulating (11) and (12), we obtain the reflection coefficient in region 0 at $z = d_1$:

$$\Gamma_0^{\uparrow} = \frac{B_0^{>}}{A_0^{>}} = +\frac{e^{j2k_{z_0}d_1}}{R^{\uparrow}} + \frac{\left[1 - (1/{R^{\uparrow}}^2)\right] e^{j2(k_{z_0}+k_{z_1})d_1}}{(1/R^{\uparrow}) e^{j2k_{z_1}d_1}} \tag{20}$$

Similarly, the reflection coefficient in region 0 at $z = d_0$ is given by:

$$\Gamma_0^{\downarrow} = \frac{A_0^{<}}{B_0^{<}} = +\frac{e^{-j2k_{z_0}d_0}}{R^{\downarrow}} + \frac{\left[1 - (1/{R^{\downarrow}}^2)\right] e^{-j2(k_{z_0}+k_{z_{-1}})d_0}}{(1/R^{\downarrow}) e^{-j2k_{z_{-1}}d_0}} \tag{21}$$

From (9),(20) and (22), we obtain:

$$\Gamma_0^{\downarrow} = \frac{A_0^{<}}{B_0^{<}} = +\frac{e^{-j2k_{z_0}d_0}}{R^{\downarrow}} + \frac{\left[1-(1/R^{\downarrow 2})\right]e^{-j2(k_{z_0}+k_{z_{-1}})d_0}}{(1/R^{\downarrow})e^{-j2k_{z_{-1}}d_0}} \tag{22}$$

$$\Gamma_0^{\uparrow} = \frac{B_0^{>}}{A_0^{>}} = \frac{B_{hmd}}{A_{hmd}+E_{hmd}} \tag{23a}$$

$$\Gamma_0^{\downarrow} = \frac{A_0^{<}}{B_0^{<}} = \frac{A_{hmd}}{B_{hmd}+E_{hmd}} \tag{23b}$$

The coefficients in region 0, therefore can be written as:

$$A_0^{>} = \frac{1+\Gamma_0^{\downarrow}}{1-\Gamma_0^{\uparrow}\Gamma_0^{\downarrow}}E_{hmd} \tag{24a}$$

$$B_0^{>} = \frac{\Gamma_0^{\uparrow}(1+\Gamma_0^{\downarrow})}{1-\Gamma_0^{\uparrow}\Gamma_0^{\downarrow}}E_{hmd} \tag{24b}$$

$$A_0^{<} = \frac{\Gamma_0^{\uparrow}(1+\Gamma_0^{\uparrow})}{1-\Gamma_0^{\uparrow}\Gamma_0^{\downarrow}}E_{hmd} \tag{24c}$$

$$B_0^{>} = \frac{1+\Gamma_0^{\uparrow}}{1-\Gamma_0^{\uparrow}\Gamma_0^{\downarrow}}E_{hmd} \tag{24d}$$

(24e)

Once the electric field in region 0 is found using (2) and (24), fields in other regions can be found using Eqs. (14)-(18).

1.1 Gated Region

A perfect electric conductor (PEC) is placed at the interface of region 0 and 1 at $z = d_1$. The Fresnel reflection coefficient (19) reduces to -1 in this case.

References

[1] J. Kong, *Electromagnetic Wave Theory*. A Wiley-interscience publication, Wiley, 1990.