LINE SOURCE ABOVE A DIELECTRIC HALF-SPACE

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1 Green's Function Derivation

We consider the problem of a horizontally oriented magnetic line source located above a lossy dielectric half-space in air and follow the Stinson's approach [**?**, **?**]. We assume the source to be time-harmonic and z-directed as shown in **??**. The lossy dielectric characterized by a complex dielctric constant (ε_b) exists in the region $y < 0$. For simplicity, we assume that the source lies at a height $y = h$ above the interface, $y = 0$ and is expressed as:

$$\overrightarrow{M} = \mathscr{I}_m \delta(x)\delta(y-h)\widehat{z} \tag{1}$$

where $\mathscr{I}_m$ is the amplitude of the source.

For the two-dimensional problem at hand, we write the scalar Helmholtz equations for the respective media.

$$\left(\nabla_t^2 + k_a^2\right) F_z^a = -\varepsilon_0 \mathscr{I}_m \delta(x)\delta(y-h), y > 0 \tag{2a}$$

$$\left(\nabla_t^2 + k_b^2\right) F_z^b = 0, y <= 0. \tag{2b}$$

where ∇_t^2 is the Laplacian in the transverse direction to the source (xy-plane). In terms of the magnetic vector potential, the electric and magnetic fields are given by:

$$\overrightarrow{E} = \frac{-1}{\varepsilon} \nabla \times \overrightarrow{F} \tag{3a}$$

$$\overrightarrow{H} = \frac{-j\omega}{k^2} \left(k^2 + \nabla\nabla\cdot\right) \overrightarrow{F} \tag{3b}$$

*Last Modified: 13:41, Wednesday 25th May, 2016.

The boundary conditions extracted from the continuity of the tangential fields at the interface $y = 0$ are:

$$\widetilde{F}_z^a = \widetilde{F}_z^b \tag{4a}$$

$$1/\varepsilon_0 \frac{\partial \widetilde{F}_z^a}{\partial y} = 1/\varepsilon_b \frac{\partial \widetilde{F}_z^b}{\partial y} \tag{4b}$$

The $\sim$ in the preceding equations indicates that the magnetic potential has been Fourier transformed in one dimension from x to k_x. Eqs. (2a) and (5b) are transformed to:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + (k_a^2 - k_y^2)\right) \widetilde{F}_z^a = -\varepsilon_0 \widetilde{\mathscr{I}}_m \delta(y-h), y > 0 \tag{5a}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + (k_b^2 - k_y^2)\right) \widetilde{F}_z^b = 0, y <= 0 \tag{5b}$$

Where,

$$\widetilde{\mathscr{I}}_m = \frac{\mathscr{I}_m}{\sqrt{2\pi}} \tag{6}$$

The above particular solutions of the equations above can be written as:

$$\widetilde{F}_z^a = Ae^{(-j\beta_a(y-h))}, y \geq h \tag{7a}$$

$$\widetilde{F}_z^a = Be^{(j\beta_a(y-h))} + Ce^{(-j\beta_a(y+h))}, 0 \leq y \leq h \tag{7b}$$

$$\widetilde{F}_z^b = De^{(j\beta_b y)}, y \leq 0 \tag{7c}$$

The four unknown constants can be found by using the boundary conditions (4) and source strength (6). They are:

$$A = \frac{\mathscr{I}_m}{j2\beta_a\sqrt{2\pi}}, \tag{8a}$$

$$B = \mathscr{R}A, \tag{8b}$$

$$C = A\left(1 + Re^{-2j\beta_a h}\right), \tag{8c}$$

$$D = \frac{2\beta_a}{\beta_b + \varepsilon_b\beta_a}A. \tag{8d}$$

Where,

$$\mathscr{R} = 1 - \frac{-2\beta_b}{\beta_b + \varepsilon_b\beta_a} \tag{9}$$

and

$$\beta_{a,b} = \sqrt{k_{a,b}^2 - k_y^2} \tag{10}$$

The general solutions for the mangetic potentials in the x-space can, therefore be constructed as:

$$F_z^a = \frac{\mathscr{I}_m}{4\pi j} \int_{-\infty}^{\infty} \frac{1}{\beta_a} \left[e^{-j\beta_a |y-h|} + \mathscr{R} e^{-j\beta_a (y+h)} \right] dk_x \tag{11a}$$

$$F_z^b = \frac{\mathscr{I}_m}{2\pi j} \int_{-\infty}^{\infty} \frac{1}{\beta_b + \varepsilon_b \beta_a} \left[e^{-j\beta_b y} \right] dk_x \tag{11b}$$

2 Field Computation