

# HORIZONTAL ELECTRIC DIPOLE

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The transmission line Greens functions (TLGF) for planar multilayers of a High Electron Mobility Transistor (HEMT) are derived by modeling the radiating 2DEG as a embedded horizontal electric dipole illustrated in Fig. ().

From Transmission Line (TL) theory, the voltage and current in a source-free section can be expressed through the telegrapher's equations:

$$\frac{d}{dz}V(z) = -jk_zZI(z), \quad (1a)$$

$$\frac{d}{dz}I(z) = -jk_zYV(z). \quad (1b)$$

where  $k_z$  is the propagation constant of the TL,  $Z$  and  $Y$  are respectively, the characteristic impedance and admittance of the transmission line. By combining (1a) and (1b), a second-order homogeneous differential equation in either  $V$  or  $I$  can be obtained:

$$\left(\frac{d^2}{dz^2} + k_z^2\right) \begin{Bmatrix} V(z) \\ I(z) \end{Bmatrix} = 0 \quad (2)$$

A travelling wave solution of (2) can be written as:

$$\begin{Bmatrix} V(z) \\ ZI(z) \end{Bmatrix} = V^+e^{-jk_z(z-z')} + V^-e^{jk_z(z-z')} \quad (3)$$

where the two terms represent forward and backward propagating waves with amplitudes  $V^+$  and  $V^-$  respectively. The exponentials can be phased referenced to any point; the current choice will become evident in the treatment of a multilayer case.

For a transmission line of infinite extent excited by a current source of unit amplitude located at  $z = z'$ , the voltage and current can be expressed as:

$$V(z) = \frac{Z}{2}e^{-jk_z|z-z'|}, \quad (4a)$$

$$I(z) = \pm \frac{1}{2}e^{-jk_z|z-z'|}. \quad (4b)$$

The  $+$  sign in (4b) is for  $z > z'$  and vice versa. The discontinuity due to the presence of the source at  $z = z'$  should also be observed due to change of signs at  $z = z'$ .

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# 1 Multi-section Transmission Line

We now consider a multi-section transmission line excited by a unit strength current source located in the  $n^{th}$  section as illustrated in Fig. ().

The voltage and current expressions in any section now consist of a particular solution due to source and a homogeneous solution accounting for boundaries of the section. Due to a current source at  $z = z'$ , the solutions at any point  $z$  in the  $n^{th}$  section with propagation constant  $k_{zn}$  become:

$$V_i(z, z') = \frac{Z}{2} e^{-jk_z|z-z'|} + A_n e^{-jk_{zn}z} + B_n e^{jk_{zn}z}, \quad (5a)$$

$$I_i(z, z') = \pm \frac{1}{2} e^{-jk_z|z-z'|} + A_n e^{-jk_{zn}z} - B_n e^{jk_{zn}z}. \quad (5b)$$

In order to find the unknowns  $A_n$  and  $B_n$ , we invoke the boundary conditions that ensure continuity of the characteristic impedance along the transmission line. At the left edge  $z_n$  and the right edge  $z_{n+1}$ , we write:

$$\frac{V_i(z_n, z')}{I_i(z_n, z')} = -\overleftarrow{Z}_n, \quad z = z_n \quad (6a)$$

$$\frac{V_i(z_{n+1}, z')}{I_i(z_{n+1}, z')} = \overrightarrow{Z}_n, \quad z = z_{n+1}. \quad (6b)$$

From (5) and (6), we obtain:

$$\left( Z_n - \overleftarrow{Z}_n \right) e^{-jk_{zn}(z'-z_n)} + A_n \left( Z_n + \overleftarrow{Z}_n \right) e^{-jk_{zn}z_n} + B_n \left( Z_n - \overleftarrow{Z}_n \right) e^{+jk_{zn}z_n} = 0 \quad (7a)$$

$$\left( Z_n - \overrightarrow{Z}_n \right) e^{-jk_{zn}(z_{n+1}-z')} + A_n \left( Z_n + \overrightarrow{Z}_n \right) e^{-jk_{zn}z_{n+1}} + B_n \left( Z_n - \overrightarrow{Z}_n \right) e^{+jk_{zn}z_{n+1}} = 0 \quad (7b)$$

The solution of the system of two linear equations (7) can be written in general form as:

$$A_n = \frac{ce - bf}{bd - ae}, \quad B_n = \frac{af - cd}{bd - ae} \quad (8)$$

where,

$$a = \left( Z_n + \overleftarrow{Z}_n \right) e^{-jk_{zn}z_n}, \quad (9a)$$

$$b = \left( Z_n - \overleftarrow{Z}_n \right) e^{+jk_{zn}z_n}, \quad (9b)$$

$$c = \left( Z_n - \overleftarrow{Z}_n \right) e^{-jk_{zn}(z'-z_n)}, \quad (9c)$$

$$d = \left( Z_n + \overrightarrow{Z}_n \right) e^{-jk_{zn}z_{n+1}}, \quad (9d)$$

$$e = \left( Z_n - \overrightarrow{Z}_n \right) e^{+jk_{zn}z_{n+1}}, \quad (9e)$$

$$f = \left( Z_n - \overrightarrow{Z}_n \right) e^{-jk_{zn}(z_{n+1}-z')}. \quad (9f)$$

The solution for unknowns, therefore is [1, p. 1178]:

$$A_n = \frac{\overleftarrow{\Gamma}_n e^{jk_{zn}2z_n}}{1 - \overleftarrow{\Gamma}_n \overrightarrow{\Gamma}_n e^{-2jk_{zn}d_n}} \left[ e^{-jk_{zn}z'} + \overrightarrow{\Gamma}_n e^{-jk_{zn}(2z_{n+1}-z')} \right], \quad (10a)$$

$$B_n = \frac{\overrightarrow{\Gamma}_n e^{-jk_{zn}2z_{n+1}}}{1 - \overleftarrow{\Gamma}_n \overrightarrow{\Gamma}_n e^{-2jk_{zn}d_n}} \left[ e^{+jk_{zn}z'} + \overleftarrow{\Gamma}_n e^{+jk_{zn}(2z_n-z')} \right] \quad (10b)$$

where  $\overleftarrow{\Gamma}_n$  and  $\overrightarrow{\Gamma}_n$  are the left and right looking reflection coefficients:

$$\overleftarrow{\Gamma}_n = \frac{\overleftarrow{Z}_n - Z_n}{\overleftarrow{Z}_n + Z_n} \quad (11a)$$

$$\overrightarrow{\Gamma}_n = \frac{\overrightarrow{Z}_n - Z_n}{\overrightarrow{Z}_n + Z_n} \quad (11b)$$

and  $d_n = z_{n+1} - z_n$  is the thickness of the  $n^{th}$  section of the transmission line as illustrated in Fig.()

## References

- [1] K. A. Michalski, “Electromagnetic field computation in planar multilayers,” *Encyclopedia of RF and microwave engineering*, 2005.