

INTEGRAL FORMULATION OF A HORIZONTALLY ORIENTED MAGNETIC DIPOLE IN A LAYERED MEDIUM

*

We model the current distribution in a two-dimensional electron gas (2DEG) at a GaN/AlGaIn heterostructure with a horizontal magnetic dipole (HMD) embedded in a layered medium as illustrated in ??.

1 Green's Function Formulation

We consider a stratified medium with three layers with the dipole placed at the origin and the layered labeled as 0 with dielectric constant ϵ_0 , different from the free-space permittivity. We consider two semi-infinite layers, above (Region 1) at $z = d_1$ and below (Region -1) the source layer at $z = d_0$ that extend to infinity. Assuming a TM case, the fields can be described by the electric field longitudinal component, E_z ,

$$E_z = \int_{-\infty}^{\infty} E_z(\rho) dk_\rho \quad (1)$$

In region i , the E_z field can be written as:

$$E_z^i = \int_{-\infty}^{\infty} \left[A_i e^{jk_{zi}z} + B_i e^{-jk_{zi}z} \right] H_n^{(1)}(k_\rho \rho) C_n(\phi) dk_\rho \quad (2)$$

where, $H_n^{(1)}(k_\rho \rho)$ is the n th-order Hankel function of the first kind and $C_n(\phi)$ is ϕ based function depending on the order of Hankel function and the dipole configuration. The remaining electric and magnetic field components can be found by the following Maxwell's equations:

$$E_\rho = \frac{1}{k_\rho^2} \frac{\partial}{\partial \rho} \frac{\partial E_z(k_\rho)}{\partial z} \quad (3a)$$

$$E_\phi = \frac{1}{k_\rho^2} \frac{1}{\rho} \frac{\partial}{\partial \phi} \frac{\partial E_z(k_\rho)}{\partial z} \quad (3b)$$

*Last Modified: 18:51, Thursday 7th July, 2016.

$$H_\rho = -\frac{j\omega\varepsilon_i}{k_\rho^2} \frac{1}{\rho} \frac{\partial E_z(k_\rho)}{\partial \phi} \quad (4a)$$

$$H_\phi = +\frac{j\omega\varepsilon_i}{k_\rho^2} \frac{1}{\rho} \frac{\partial E_z(k_\rho)}{\partial \rho} \quad (4b)$$

The unknowns A_i and B_i can be found by applying boundary conditions at the two interfaces at $z = d_1$ and $z = d_0$ that require the continuity of tangential components of the electric and magnetic fields. From (2), (3a) and (4a), we obtain:

$$k_{z_i} \left(A_i e^{jk_{z_i} d_i} - B_i e^{-jk_{z_i} d_i} \right) = k_{z_{i-1}} \left(A_{i-1} e^{jk_{z_{i-1}} d_i} - B_{i-1} e^{-jk_{z_{i-1}} d_i} \right) \quad (5)$$

$$\varepsilon_i \left(A_i e^{jk_{z_i} d_i} - B_i e^{-jk_{z_i} d_i} \right) = \varepsilon_{i-1} \left(A_{i-1} e^{jk_{z_{i-1}} d_i} - B_{i-1} e^{-jk_{z_{i-1}} d_i} \right) \quad (6)$$

In the region 0, the position of the observation point above or below the source leads to different values of unknowns A_0 and B_0 . To make it distinct, for $z > 0$, the unknowns are written as $A^>$ and $B^>$ and for $z < 0$, $A^<$ and $B^<$ respectively. For HMD, the following conditions are used [[?]].

$$A_0^> = A_{hmd} + E_{hmd} \quad (7a)$$

$$A_0^< = A_{hmd} \quad (7b)$$

$$B_0^> = B_{hmd} \quad (7c)$$

$$B_0^< = B_{hmd} + E_{hmd} \quad (7d)$$

where,

$$E_{hmd} = \frac{p_m \omega \mu_0 k_\rho^2}{8\pi k_{0z}} \quad (8)$$

$$H_{hmd} = -\frac{p_m k_\rho^2}{8\pi} \quad (9)$$

where p_m is the dipole moment of the current source. The longitudinal propagation constant in region i is given by:

$$k_z^i = \sqrt{k_i^2 - k_\rho^2} \quad (10)$$

Now using (5) and (6), at $z = d_1$, we have:

$$k_{z_1} \left(A_1 e^{jk_{z_1} d_1} - B_1 e^{-jk_{z_1} d_1} \right) = k_{z_0} \left(A_0^> e^{jk_{z_0} d_1} - B_0^> e^{-jk_{z_0} d_1} \right) \quad (11)$$

$$\varepsilon_1 \left(A_1 e^{jk_{z_1} d_1} - B_1 e^{-jk_{z_1} d_1} \right) = \varepsilon_0 \left(A_0^> e^{jk_{z_0} d_1} - B_0^> e^{-jk_{z_0} d_1} \right) \quad (12)$$

By adding (11) and (12) we get:

$$(k_{z_1} + \varepsilon_1)A_1 e^{jk_{z_1}d_1} - (k_{z_1} - \varepsilon_1)B_1 e^{-jk_{z_1}d_1} = (k_{z_0} + \varepsilon_0)A_0^> e^{jk_{z_0}d_1} - (k_{z_0} - \varepsilon_0)B_0^> e^{-jk_{z_0}d_1} \quad (13)$$

The unknowns A_1 and B_1 can be expressed in terms of $A_0^>$ and $B_0^>$ as [[?]]:

$$A_1 e^{jk_{z_1}d_1} = \frac{1}{2} \left[\frac{\varepsilon_0}{\varepsilon_1} + \frac{k_{z_0}}{k_{z_1}} \right] \times \left[A_0^> e^{jk_{z_0}d_1} + R^\uparrow B_0^> e^{-jk_{z_0}d_1} \right] \quad (14)$$

$$B_1 e^{-jk_{z_1}d_1} = \frac{1}{2} \left[\frac{\varepsilon_0}{\varepsilon_1} + \frac{k_{z_0}}{k_{z_1}} \right] \times \left[R^\uparrow A_0^> e^{jk_{z_0}d_1} + B_0^> e^{-jk_{z_0}d_1} \right] \quad (15)$$

where, R^\uparrow is the TM Fresnel reflection coefficient at $z = d_1$ as seen from region 0,

$$R^\uparrow = \frac{-\varepsilon_0 k_{z_1} + \varepsilon_1 k_{z_0}}{\varepsilon^0 k_{z_1} + \varepsilon^1 k_{z_0}} \quad (16)$$

Through a similar procedure by applying boundary conditions at $z = d_{-1}$, coefficients A_{-1} and B_{-1} are obtained:

$$A_{-1} e^{jk_{z_{-1}}d_0} = \frac{1}{2} \left[\frac{\varepsilon_0}{\varepsilon_{-1}} + \frac{k_{z_0}}{k_{z_{-1}}} \right] \times \left[A_0^< e^{jk_{z_{-1}}d_0} + R^\downarrow B_0^< e^{-jk_{z_{-1}}d_0} \right] \quad (17)$$

$$B_{-1} e^{-jk_{z_{-1}}d_0} = \frac{1}{2} \left[\frac{\varepsilon_0}{\varepsilon_{-1}} + \frac{k_{z_0}}{k_{z_{-1}}} \right] \times \left[R^\downarrow A_0^< e^{jk_{z_{-1}}d_0} + B_0^< e^{-jk_{z_{-1}}d_0} \right] \quad (18)$$

where, R^\downarrow is the TM Fresnel reflection coefficient at $z = d_0$ as seen from region 0,

$$R^\downarrow = \frac{-\varepsilon_0 k_{z_{-1}} + \varepsilon_{-1} k_{z_0}}{\varepsilon_0 k_{z_{-1}} + \varepsilon_{-1} k_{z_0}} \quad (19)$$

For a structure having three regions, it is observed that B_1 and A_{-1} equals zero due to the open nature of region 1 and -1 respectively. By manipulating (11) and (12), we obtain the reflection coefficient in region 0 at $z = d_1$:

$$\Gamma_0^\uparrow = \frac{B_0^>}{A_0^>} = + \frac{e^{j2k_{z_0}d_1}}{R^\uparrow} + \frac{\left[1 - (1/R^\uparrow)^2 \right] e^{j2(k_{z_0} + k_{z_1})d_1}}{(1/R^\uparrow) e^{j2k_{z_1}d_1}} \quad (20)$$

Similarly, the reflection coefficient in region 0 at $z = d_0$ is given by:

$$\Gamma_0^\downarrow = \frac{A_0^<}{B_0^<} = + \frac{e^{-j2k_{z_0}d_0}}{R^\downarrow} + \frac{\left[1 - (1/R^\downarrow)^2 \right] e^{-j2(k_{z_0} + k_{z_{-1}})d_0}}{(1/R^\downarrow) e^{-j2k_{z_{-1}}d_0}} \quad (21)$$

From (9),(20) and (22), we obtain:

$$\Gamma_0^\downarrow = \frac{A_0^<}{B_0^<} = + \frac{e^{-j2k_{z_0}d_0}}{R^\downarrow} + \frac{\left[1 - (1/R^\downarrow)^2 \right] e^{-j2(k_{z_0} + k_{z_{-1}})d_0}}{(1/R^\downarrow) e^{-j2k_{z_{-1}}d_0}} \quad (22)$$

$$\Gamma_0^\uparrow = \frac{B_0^>}{A_0^>} = \frac{B_{hmd}}{A_{hmd} + E_{hmd}} \quad (23a)$$

$$\Gamma_0^\downarrow = \frac{A_0^<}{B_0^<} = \frac{A_{hmd}}{B_{hmd} + E_{hmd}} \quad (23b)$$

The coefficients in region 0, therefore can be written as:

$$A_0^> = \frac{1 + \Gamma_0^\downarrow}{1 - \Gamma_0^\uparrow \Gamma_0^\downarrow} E_{hmd} \quad (24a)$$

$$B_0^> = \frac{\Gamma_0^\uparrow (1 + \Gamma_0^\downarrow)}{1 - \Gamma_0^\uparrow \Gamma_0^\downarrow} E_{hmd} \quad (24b)$$

$$A_0^< = \frac{\Gamma_0^\uparrow (1 + \Gamma_0^\downarrow)}{1 - \Gamma_0^\uparrow \Gamma_0^\downarrow} E_{hmd} \quad (24c)$$

$$B_0^> = \frac{1 + \Gamma_0^\uparrow}{1 - \Gamma_0^\uparrow \Gamma_0^\downarrow} E_{hmd} \quad (24d)$$

$$(24e)$$

Once the electric field is found in region 0 using (2) and (24), fields in other regions can be found using Eqs. (14)-(18).

1.1 Ungated Region

For the ungated region of the three-layer structre, region 1 is characterized by free-space.

References

- [1] J. Kong, *Electromagnetic Wave Theory*. A Wiley-interscience publication, Wiley, 1990.