INTEGRAL FORMULATION OF A HORIZONTALLY ORIENTED MAGNETIC DIPOLE IN A LAYERED MEDIUM

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We model the current distribution in a two-dimensional electron gas (2DEG) at a GaN/AlGaN heterostructure with a horizontal magnetic dipole (HMD) embedded in a layered medium as illustrated in **??**.

1 Green's Function Formulation

We consider a stratified medium with three layers with the dipole placed at the origin and the layered labeled as 0 with dielectric constant ε_0, different from the free-space permittivity. We consider two semi-infinite layers, above (Region 1) at $z = d_1$ and below (Region -1) the source layer at $z = d_{-1}$ that extend to infinity. Assuming a TM case, the fields can be described by the electric field longitudinal component, E_z,

$$E_z = \int_{-\infty}^{\infty} E_z(\rho) dk_\rho \tag{1}$$

In region i, the E_z field can be written as:

$$E_z^i = \int_{-\infty}^{\infty} \left[A_i e^{jk_{iz}z} + B_i e^{-jk_{iz}z} \right] H_n^{(1)}(k_\rho \rho) C_n(\phi) dk_\rho \tag{2}$$

where, $H_n^{(1)}(k_\rho \rho)$ is the nth-order Hankel function of the first kind and $C_n(\phi)$ is ϕ based function depending on the order of Hankel function and the dipole configuration. The remaining electric and magnetic field components can be found by the following Maxwell's equations:

$$E_\rho = \frac{1}{k_\rho^2} \frac{\partial}{\partial \rho} \frac{\partial E_z(k_\rho)}{\partial z} \tag{3a}$$

$$E_\phi = \frac{1}{k_\rho^2} \frac{1}{\rho} \frac{\partial}{\partial \phi} \frac{\partial E_z(k_\rho)}{\partial z} \tag{3b}$$

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$$H_\rho = -\frac{j\omega\varepsilon_i}{k_\rho^2}\frac{1}{\rho}\frac{\partial E_z(k_\rho)}{\partial \phi} \tag{4a}$$

$$H_\phi = +\frac{j\omega\varepsilon_i}{k_\rho^2}\frac{1}{\rho}\frac{\partial E_z(k_\rho)}{\partial \rho} \tag{4b}$$

The unknowns A_i and B_i can be found by applying boundary conditions at the two interfaces at $z = d$ and $z = -d$ that require the continuity of tangential components of the electric and magnetic fields. From (2) and (3b), we obtain:

$$k_z^i \left(A_i e^{jk_z^i d_i} - B_i e^{-jk_z^i d_i}\right) = k_z^{i-1}\left(A_{i-1} e^{jk_z^{i-1} d_i} - B_{i-1} e^{-jk_z^{i-1} d_i}\right) \tag{5}$$

$$\varepsilon^i \left(A_i e^{jk_z^i d_i} - B_i e^{-jk_z^i d_i}\right) = \varepsilon^{i-1}\left(A_{i-1} e^{jk_z^{i-1} d_i} - B_{i-1} e^{-jk_z^{i-1} d_i}\right) \tag{6}$$

In the region 0, the position of the observation point above or below the source leads to different values of unknowns A_0 and B_0. To make it distinct, for $z > 0$,the unknowns are written as $A^>$ and $B^>$ and for $z < 0$, $A^<$ and $B^<$ respectively. For HMD, the following conditions are used [[**?**]].

$$A_0^> = A_{HMD} + E_{HMD} \tag{7a}$$

$$A_0^< = A_{HMD} \tag{7b}$$

$$B_0^> = B_{HMD} \tag{7c}$$

$$B_0^< = B_{HMD} + E_{HMD} \tag{7d}$$

where,

$$E_{HMD} = \frac{p_m \omega \mu_0 k_\rho^2}{8\pi k_{0z}} \tag{8}$$

$$H_{HMD} = -\frac{p_m k_\rho^2}{8\pi} \tag{9}$$

The longitudinal propagation constant in region i is given by:

$$k_z^i = \sqrt{k_i^2 - k_\rho^2} \tag{10}$$

Now using (5) and (6), at $z = d_1$, we have:

$$k_z^1 \left(A_1 e^{jk_z^1 d_1} - B_1 e^{-jk_z^1 d_1}\right) = k_z^0\left(A_0^> e^{jk_z^0 d_1} - B_0^> e^{-jk_z^0 d_1}\right) \tag{11}$$

$$\varepsilon^1 \left(A_1 e^{jk_z^1 d_1} - B_1 e^{-jk_z^1 d_1}\right) = \varepsilon^0\left(A_0^> e^{jk_z^0 d_1} - B_0^> e^{-jk_z^0 d_1}\right) \tag{12}$$

After adding (11) and (12), we get,