Numerical Integration of Sommerfeld Integrals

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The fields of sources in a layered media environment involves computation of Sommerfeld Integrals which are reminiscent of a Hankel transform (a Fourier transform in cylindrical coordinate system):

$$\int_0^\infty \tilde{G}(k_\rho;\mathbf{r}|\mathbf{r}')J_n(k_\rho\rho)k_\rho\,\mathrm{d}k_\rho \tag{1}$$

where $\tilde{G}$ is a spectral domain Green's function of the structure, $\mathbf{r}$ and $\mathbf{r}'$ are the observation and source locations respectively, and J_n is bessel's function of order n. In most cases, integral in (1) cannot be solved analytically. Furthermore, due to singularities and oscillations in the integrand, the numerical evaluation requires careful manipulation and techniques. Using the Cauchy's Integral Theoerem [1, p. 377] by selecting the path of integration along the positive real axis in the complex k_ρ plane as shown in Fig. 1. [2, 3].

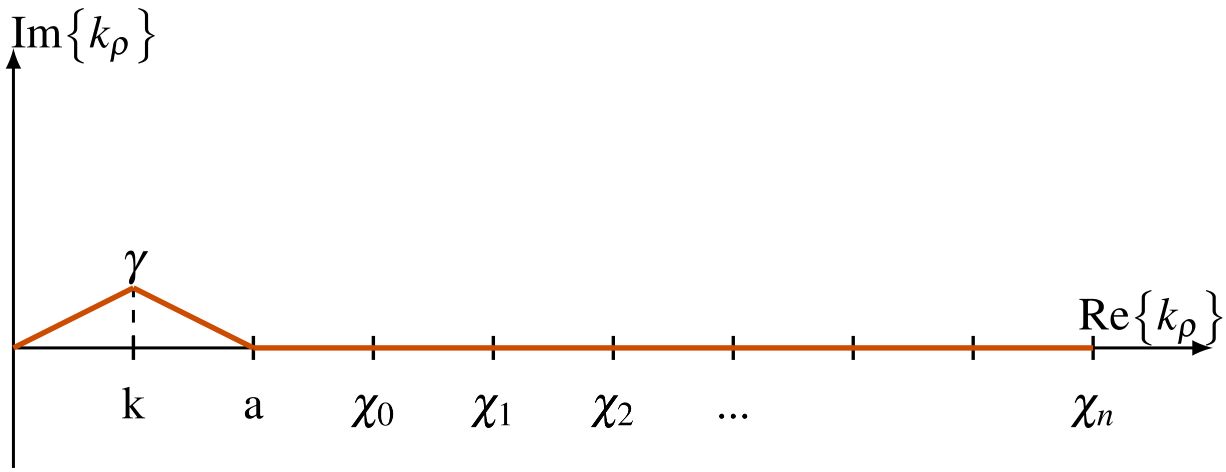

Figure 1: Integration Path along the real axis

The integral in (1) is computed by splitting into three sections:

$$\int_0^\infty \tilde{G}(k_\rho;\mathbf{r}|\mathbf{r}')J_n(k_\rho\rho)k_\rho\,\mathrm{d}k_\rho = \left(\int_0^\gamma + \int_\gamma^a + \int_a^\infty\right)\tilde{G}(k_\rho;\mathbf{r}|\mathbf{r}')J_n(k_\rho\rho)k_\rho\,\mathrm{d}k_\rho \tag{2}$$

where γ represents a slight detour of the integration path from the real axis. This is done to bypass the singularity lying on the real axis. The real part of γ is the same as the singularity location, k whereas the imaginary part should be as small as possible to avoid wild growth of the bessel function illustrated in Fig. 2. Furthermore, a represents a breakpoint, where we switch the numerical integration routine.

*Last Modified: 18:08, Friday 2[nd] September, 2016.

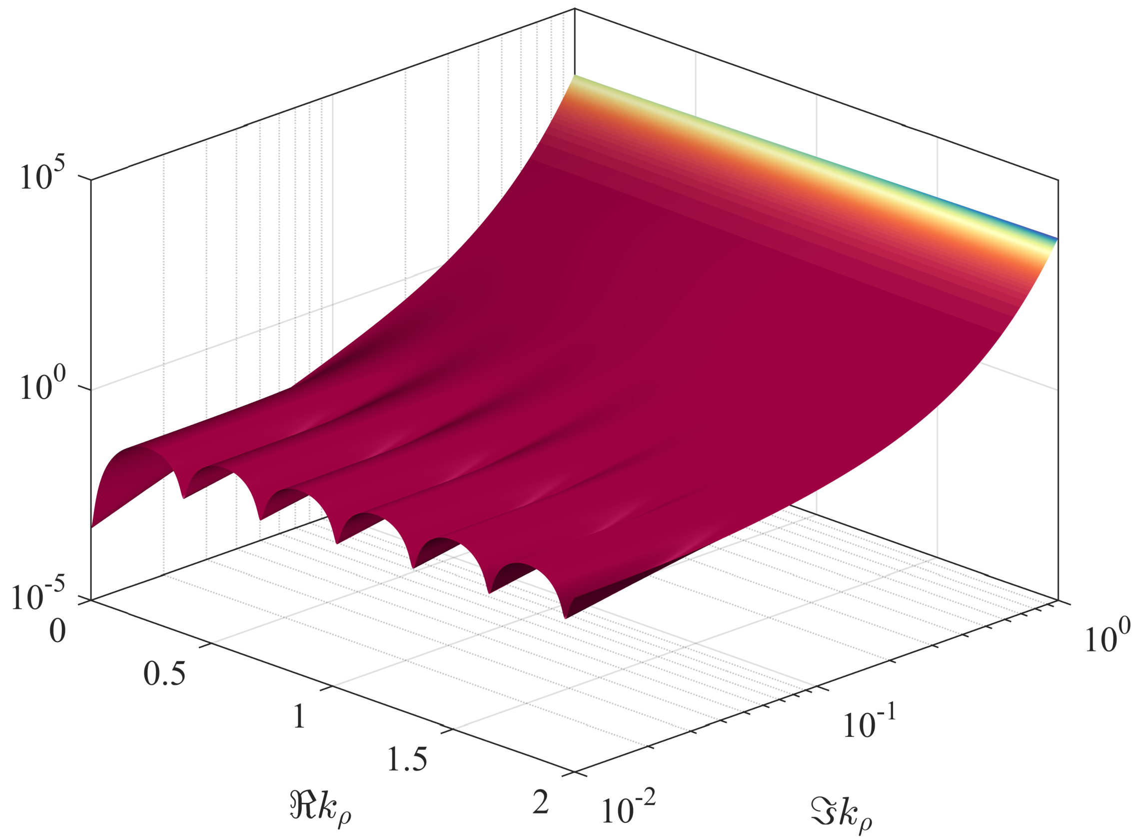

Figure 2: Exponential Growth of Bessel function $J_1(10k_\rho)$ along the imaginary axis

1 Overview of Numerical Integration

References

[1] G. Arfken and H. Weber, *Mathematical Methods for Physicists*. Academic Press, Harcourt/Academic Press, 2001.

[2] R. Golubovic, A. G. Polimeridis, and J. R. Mosig, “Efficient algorithms for computing Sommerfeld integral tails,” *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 5, pp. 2409–2417, 2012.

[3] K. A. Michalski and J. R. Mosig, “Efficient computation of Sommerfeld integral tails–methods and algorithms,” *Journal of Electromagnetic Waves and Applications*, vol. 30, no. 3, pp. 281–317, 2016.