Near-field profile in the Kretschmann configuration with gaussian-beam illumination

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We consider a planar interface, possibly with multiple layers, illuminated under total internal reflection by a gaussian beam from an incident medium (i). We seek to calculate the near-field profile in the outer medium (o).

1 Gaussian beam illumination

Following Novotny, we start with the angular spectrum representation of the incident beam with wavevector $\mathbf{k}_i$. In a frame F_1 attached to the central ray as depicted in Fig. 1, the electric field at a point $\mathbf{r}_1$, is written as a collection of plane waves (Novotny Eq. 3.9 p. 47, Eq. 3.27, p.54),

$$\mathbf{E}_1(\mathbf{r}_1) = \iint a(k_{i1x}, k_{i1y}) \exp\left(i\mathbf{k}_{i1} \cdot \mathbf{r}_1\right) \hat{\mathbf{e}}_1(\mathbf{r}_1, \mathbf{k}_{i1}) \mathrm{d}k_{i1x} \mathrm{d}k_{i1y},$$

where

$$a(\mathbf{k}_1) = \frac{w_0^2}{4\pi} e^{-\frac{w_0^2}{4}(k_{1x}^2 + k_{1y}^2)}$$

describes the gaussian field profile with waist w_0, and $\hat{\mathbf{e}}_1 = (\cos\psi; \sin\psi; 0)^t$ keeps track of the electric field direction (for a focused beam, see Eq. 3 of Burghardt et al.).

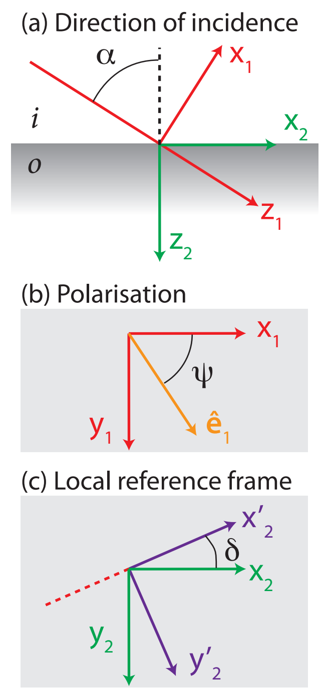

Figure 1: Illustration of the different reference frames used in the derivation. (a) The central ray of the gaussian beam makes an angle α with the normal to the interface. (b) The polarisation is described by the angle Ψ between the electric field and the x_1 axis; values of $\Psi = 0, 90$ correspond to $p-$ and $s-$polarisations, respectively. (c) A rotation of angle δ brings the frame of reference F_2' to coincide with the plane of incidence of a given plane wave.

2 Reference frame

The results should be expressed in a reference frame attached to the planar interface, we thus define a rotation matrix around the axis y_1.

$$R_y(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}. \tag{1}$$

For each individual plane wave in the kernel, a second rotation is performed around the z_2 axis, that brings the new reference frame (x'_2, y'_2, z'_2) to coincide with the plane of incidence of that particular plane wave,

$$R_z(\delta) = \begin{bmatrix} \cos(\delta) & \sin(\delta) & 0 \\ -\sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The angle of rotation δ is given by

$$\delta = \sin^{-1} \frac{s_{2y}}{\sqrt{s_{2x}^2 + s_{2y}^2}}$$

where $\hat{\mathbf{s}}_2 = R_y(\alpha)\hat{\mathbf{s}}_1$ is obtained by rotation of the normalised incident wavevector $\hat{\mathbf{s}}_1 = \mathbf{k}_{i1}/|\mathbf{k}_{i1}|$. Each plane wave, expressed in this dedicated frame of reference, is now written

$$\mathbf{E}_{i2'}(\mathbf{r}_{2'}) = \hat{\mathbf{e}}_{i2'} \exp\left(i\mathbf{k}_{i2'} \cdot \mathbf{r}_{2'}\right).$$

3 Transmission at the interface

We consider an individual plane wave incident on the interface, and express the amplitude in the frame F'_2 of the electric field $\mathbf{E}_{o2'}$ on the outer side using the Fresnel coefficients t^p and t^s (Etchegoin, Le Ru, App. F.3),

$$\mathbf{E}_{o2'}(\mathbf{r}_{2'}) = \begin{bmatrix} \left(\frac{n_i}{n_o}\right)^2 \frac{k_{o2z}}{k_{i2'z}} t^p E_{i2'x} \\ t^s E_{2'y} \\ \left(\frac{n_i}{n_o}\right)^2 t^p E_{i2'z} \end{bmatrix} \exp\left(i\mathbf{k}_{o2'} \cdot \mathbf{r}_{2'}\right).$$

The wave vector of this inhomogeneous plane wave is given by $\mathbf{k}_{o2'} = \left(k_{i2'x}, k_{i2'y}, \sqrt{k_o^2 - (k_{i2'x}^2 + k_{i2'x}^2)}\right)$.

The electric field is finally transformed back into the reference frame F_2 by a rotation of $R_z(-\delta)$, before the integration is performed, in polar coordinates,

$$\mathbf{E}_{o2}(\mathbf{r}_2) = \iint a(\rho, \theta)\mathbf{E}_{o2}(\mathbf{r}_2)\rho \mathrm{d}\rho \mathrm{d}\theta,$$

with

$$\begin{cases} \rho = \sqrt{k_{ix1}^2 + k_{iy1}^2} \\ \theta = \tan^{-1} \dfrac{k_{iy1}}{k_{ix1}} \end{cases}. \tag{2}$$