Monte Carlo Moment Matching: Make Your Own Luck

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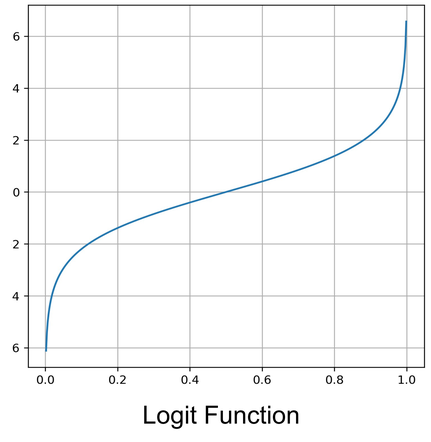
Objective

By applying a small shift to all samples in a random uniform set we can match the first moment of the set with the first moment, or expectation, of the function. This technique was applied to a Monte Carlo integration of a complex integral and an implicit Monte Carlo radiation transport code [1].

Moment Matching

- To avoid shifting samples outside of the desired domain, we can transform the samples to be on $-\infty$ to ∞ using the logit function.
- The next step is to solve for a shift in this new domain using a root finding method then transform back to the original domain using the inverse logit function.

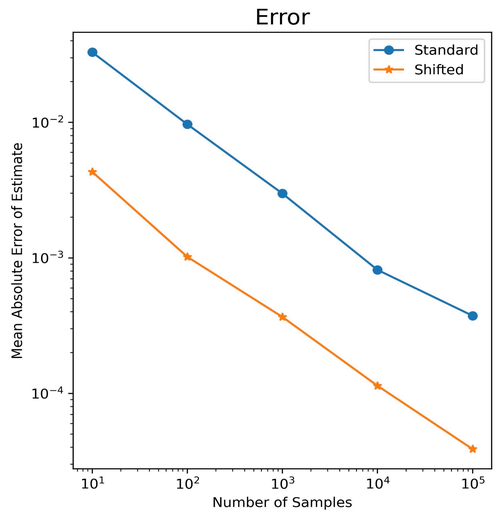


Implementation

Algorithm Outline

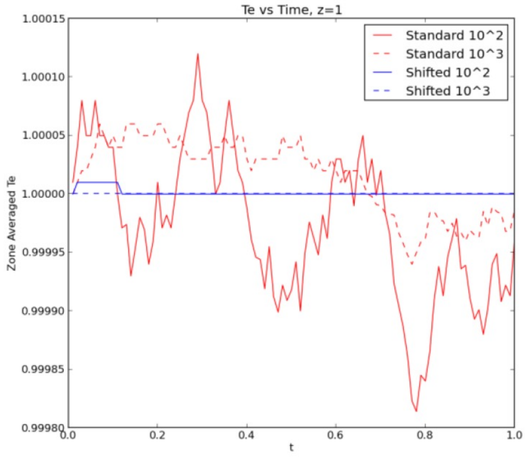
1. Generate Samples x_n
2. Pass samples through logit function
   - $y_n = g(x_n) = \log(\frac{x_n}{1-x_n})$
3. Find δ such that:
   - $\frac{1}{N}\sum_{n=1}^{N} g^{-1}(y_n + \delta) = E$
4. Shift samples
   - $Y_n = y_n + \delta$
5. Transform samples back
   - $x_n = g^{-1}(Y_n)$

Integral Test

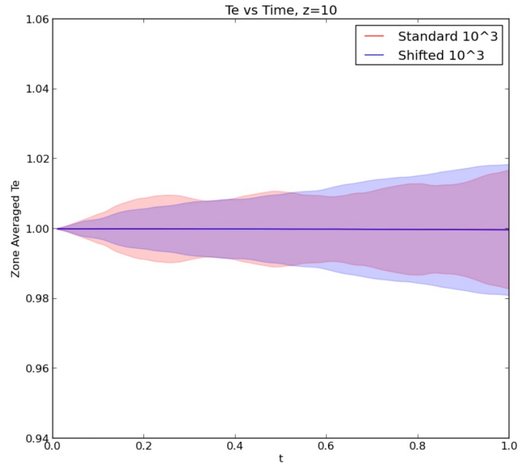


$$\int_0^1 \frac{tan^{-1}(\sqrt{x^2+2})}{(x^2+1)\sqrt{x^2+2}}dx$$

One-Zone Test



Ten-Zone Test



Conclusions

The moment matching technique worked well to increase accuracy and decrease standard deviation in one-zone and low particle count radiation transport problems. As the number of particles and zones increase the benefits of the method diminish.

References

1. J.A. Fleck Jr, J.D. Cummings Jr, "An implicit Monte Carlo scheme for calculating time and frequency dependent nonlinear radiation transport", Journal of Computational Physics, vol. 8, pp 313-342, 1971.

Acknowledgements

This work was supported by the Center for Exascale Monte-Carlo Neutron Transport (CEMeNT) a PSAAP-III project funded by the Department of Energy, grant number DE-NA003967 and performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC, LLNL-PRES-826203.