A Variance Deconvolution Approach to Uncertainty Quantification for Monte Carlo Radiation Transport – Selected Results

Kayla Clements[a], Gianluca Geraci[b], Aaron Olson[b]

Center for Exascale Monte-Carlo Neutron Transport (CEMeNT), [a]Oregon State University, [b]Sandia National Lab

Objective

Compare new variance deconvolution method for Monte Carlo radiation transport solvers, EVADE, with existing $\text{EVADE}_{\text{old},[1]}$ method.

Theory

- Deconvolve parametric uncertainty ξ from MC transport solver uncertainty η in code response
- Law of total variance:

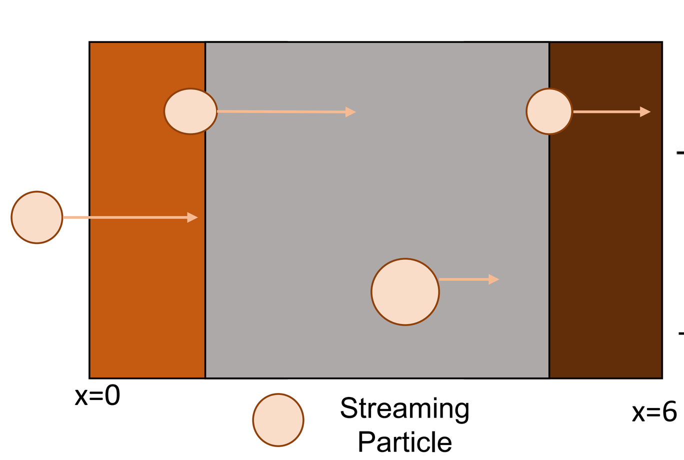
$$\mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)] = \mathbb{V}ar_\xi\left[\mathbb{E}_\eta[\tilde{T}(\xi,\eta)]\right] + \mathbb{E}_\xi\left[\mathbb{V}ar_\eta[\tilde{T}(\xi,\eta)]\right]$$

- Simplified[2]:

$$\mathbb{V}ar_\xi[T] = \mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)] - \frac{1}{N_\eta}\mathbb{E}_\xi[\sigma_\eta^2]$$

1D Attenuation Problem

$$\mu\frac{\partial\psi(x,\mu)}{\partial x} + \Sigma_t(x)\psi(x,\mu) = 0$$



	m = 1	m = 2	m = 3
x_R	2.0	5.0	6.0
$\bar{\Sigma}_{t,m}$	0.90	0.15	0.60
$\Sigma^{\Delta}_{t,m}$	0.70	0.12	0.50

Table 1: 1D attenuation problem parameters.

Algorithm Comparison

EVADE

Determine:
- number of repetitions N_{real}
- number of realizations N_ξ
- number of histories N_η

For $q = 1 : N_{real}$

For $i = 1 : N_\xi$

Sample ξ and calculate $\Sigma_t(\xi)$

Run N_η particle simulations to compute $\mathrm{T}(\xi_i,\eta_j)$

Calculate mean transmittance:

$$\tilde{T}(\xi_i) = \frac{1}{N_\eta}\sum_{j=1}^{N_\eta} T(\xi_i,\eta_j)$$

Calculate MC variance over histories:

$$\sigma_\eta^2(\xi_i) \approx \frac{1}{N_\eta - 1}\sum_{j=1}^{N_\eta}\left(T(\xi_i,\eta_j) - \tilde{T}(\xi_i)\right)^2$$

Calculate average MC solver noise over parameter space:

$$\mathbb{E}[\sigma_\eta^2] \approx \frac{1}{N_\xi}\sum_{i=1}^{N_\xi}\sigma_\eta^2(\xi_i)$$

Calculate total variance over parameter space:

$$\mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)] \approx \frac{1}{N_\xi - 1}\sum_{j=1}^{N_\xi}\left(\tilde{T}(\xi_i) - \tilde{T}_{avg}(\xi)\right)^2$$

Subtract MC noise from total variance to solve for true variance over parameter space:

$$\mathbb{V}ar_\xi[T] = \mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)] - \frac{1}{N_\eta}\mathbb{E}_\xi[\sigma_\eta^2]$$

Average variances over N_{real}

$\text{EVADE}_{\text{old}}$

Number of histories N_η = 2

Determine:
- number of repetitions N_{real}
- number of realizations N_ξ

For $q = 1 : N_{real}$

For $i = 1 : N_\xi$

Sample ξ and calculate $\Sigma_t(\xi)$

Run 2 particle simulations to compute $\mathrm{T}(\xi_i,\eta_j)$
Store first history's tally in $\mathrm{T}(\xi_i,1)$

Calculate mean transmittance:

$$\tilde{T}(\xi_i) = \frac{1}{2}\sum_{j=1}^{N_\eta} T(\xi_i,\eta_j)$$

Calculate MC variance over histories:

$$\sigma_\eta^2(\xi_i) \approx \sum_{j=1}^{N_\eta}\left(T(\xi_i,\eta_j) - \tilde{T}(\xi_i)\right)^2$$

Calculate average MC solver noise over parameter space:

$$\mathbb{E}[\sigma_\eta^2] \approx \frac{1}{N_\xi}\sum_{i=1}^{N_\xi}\sigma_\eta^2(\xi_i)$$

Calculate total variance over parameter space: $\mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)]$

$$\approx \frac{1}{N_\xi - 1}\sum_{j=1}^{N_\xi}\left(\tilde{T}(\xi_i,1) - \tilde{T}_{avg}(\xi,1)\right)^2$$

Subtract MC noise from total variance to solve for true variance over parameter space:

$$\mathbb{V}ar_\xi[T] = \mathbb{V}ar_\xi[\tilde{T}(\xi,\eta)] - \mathbb{E}_\xi[\sigma_\eta^2]$$

Average variances over N_{real}

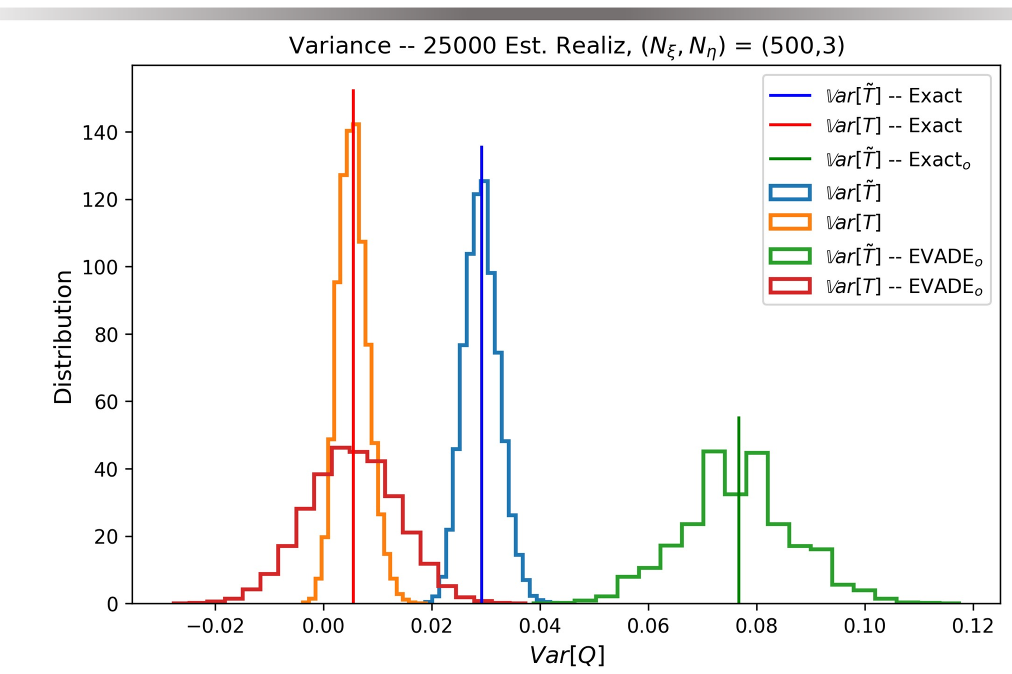
Differences

- Same number of histories to calculate all variances vs. one history for total variance, two histories for MC solver variance
- $\text{EVADE}_{\text{old}}$: MC variance averaged over one history used for total variance

Conclusion

New method has lower variance in estimate of parametric variance.

Results



- EVADE has tighter distribution around analytic solution than $\text{EVADE}_{\text{old}}$
- EVADE has lower variance in estimates of $\mathbb{V}ar_\xi[T]$ over 25,000 repetitions

References

1. A. J. Olson, "Calculation of parametric variance using variance deconvolution," Transactions of ANS, vol. 120, 2019.
2. A. J. Olson and G. Geraci, "Impact of sampling strategies in the polynomial chaos surrogate construction for Monte Carlo transport applications," 2021.

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