NSE 654 - Computational Particle Transport

Homework # 2
Slab Geometry Transport
Due: Friday, May 13, 11:59 pm

The goal of this assignment is for you to develop, test and observe the resulting solution behavior from a code to solve the Discrete Ordinates, mono-energetic, steady-state, isotropic scattering/isotropic source, slab geometry transport equation. Each of you has been assigned a different spatial discretization approach, and one of our goals will be compare results amongst the various options that we have implemented.

Write the code to allow general spatial heterogeneity in cross-sections (σ, σ_s), boundary conditions (incident isotropic/anisotropic angular flux, reflecting), spatial mesh (each cell can have its own cell size Δx, source and angular quadrature.

We'll be solving this equation:

$$\mu_m \frac{\partial \psi_m(x)}{\partial x} + \sigma(x)\psi_m(x) = \frac{\sigma_s(x)}{2}\sum_{n=1}^{N} w_n \psi_n(x) + \frac{Q(x)}{2}, \qquad m = 1 \cdots N, \tag{1}$$

employing Source Iteration. In Source Iteration, we do the following:

$$\mu_m \frac{\partial \psi_m^{(l+1)}(x)}{\partial x} + \sigma(x)\psi_m^{(l+1)}(x) = \frac{\sigma_s(x)\phi^{(l)}(x)}{2} + \frac{Q(x)}{2}, \qquad m = 1 \cdots N, \tag{2}$$

$$\phi^{(l+1)} = \sum_{n=1}^{N} w_n \psi_n^{(l+1)}(x). \tag{3}$$

After you have written this code, demonstrate its use to solve several problems for which the exact solution is known:

1. Uniform infinite medium

2. Source free pure absorber

3. Source free half-space (use the provided spreadsheet to compare with your code...)

Pay attention to the rate of convergence of Source Iteration as a function of the largest scattering ratio in any cell of the problem, the maximum optical thickness of any cell in the problem, and the boundary conditions.

If you are convinced that your code is working properly, then use it to solve the following two problems:

1. Optically thick, diffusive problem:

   $\sigma = 100$ cm^{-1},

   $\sigma_s = 99.5$ cm^{-1},

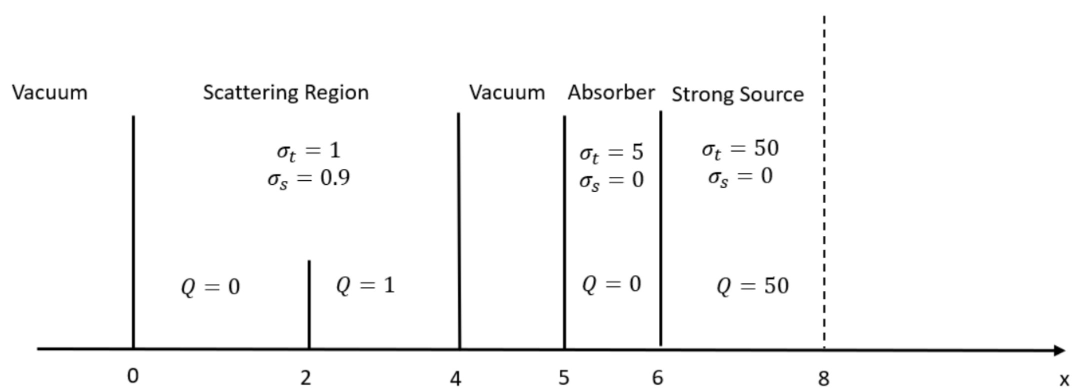
   $Q = 5.0$ particles per cm^3,

   $\psi_{inc,L} = 10\ \delta(\mu - 1)$

   reflecting boundary on the right side of the slab (at $x = 10$ cm),

   spatial meshes - 10 uniform cells and 100 uniform cells.

2. Reed's problem:



   Reflecting boundary on the right, and use a spatial grid that is uniform in each region with each cell being 0.1 mfp thick. [In the interior of the problem "vacuum" means void.]

As you prepare the document for submission, please plot scalar flux and current for each problem, and do your best to make plots that are consistent with your specific discretization.

Those of you writing in Python will likely be able to call a function to generate even-order symmetric Gauss-Legendre quadrature sets very easily (see `https://numpy.org/doc/stable/reference/generated/numpy.polynomial.legendre.leggauss.html`).