Multipliers - Carry Save and Wallace Tree

Dr DC Hendry

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Outline I

Higher Performance Multipliers

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- Not all increase the area of the multiplier, rather just reflect clever thinking about the requirements and operation of the circuits.
- Remember that underlying technologies are always changing, the best way to build a circuit changes!

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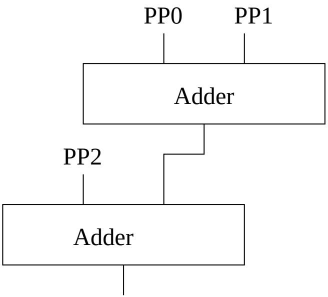

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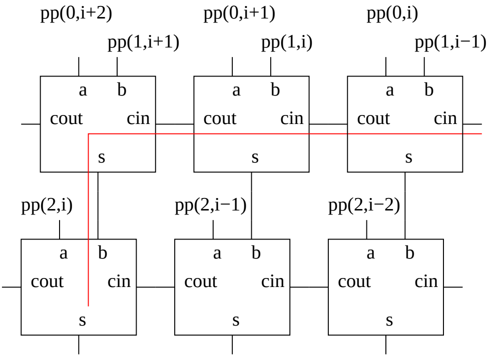

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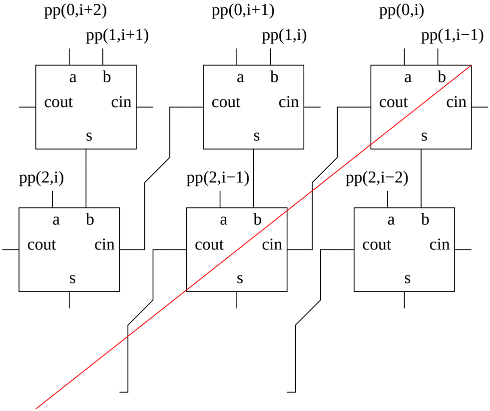
- This leads us to the carry save architecture.


pp(0,i+2)
pp(1,i+1)
pp(0,i+1)
pp(1,i)
pp(0,i)
pp(1,i−1)
a
b
cout
cin
s
pp(2,i)
pp(2,i−1)
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- This is done with a final high speed adder.
- Usually referred to in the literature as the CLA - Carry Lookahead Adder - but can be any fast adder.
- Since signal arrival times are not uniform, a variation on CLA may be used - the synthesis tool may be used for this.

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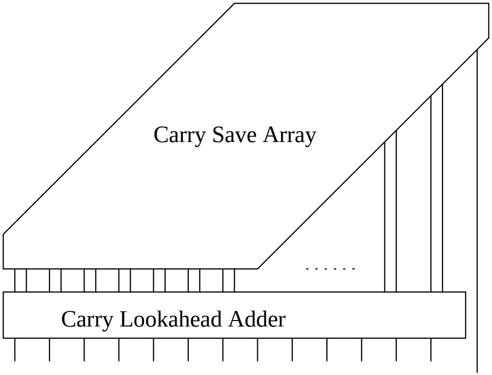

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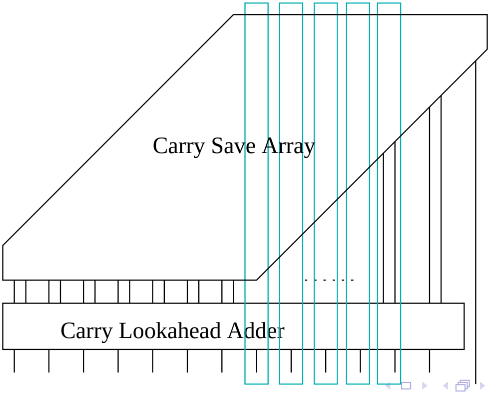

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- the last two sum bits in that column that are passed to the CLA.

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- The carry in comes CSA fashion from the previous column.
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- In comparison to the basic array multiplier, the delay from partial products to final sum bits in a column is $O(ln(n))$ rather than $O(n)$.

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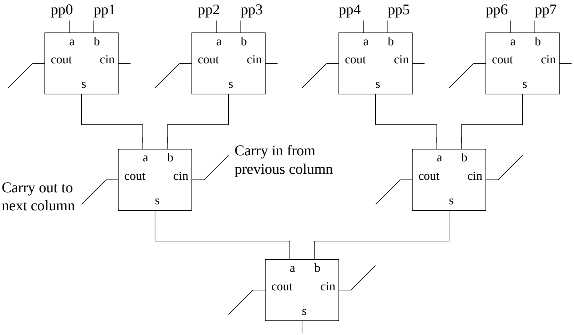

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- The arrival time is when that signal is valid assuming new input data is applied to the circuit at time zero.

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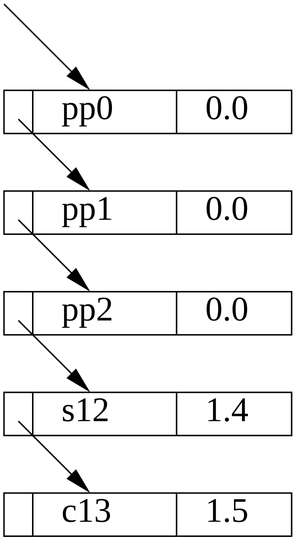

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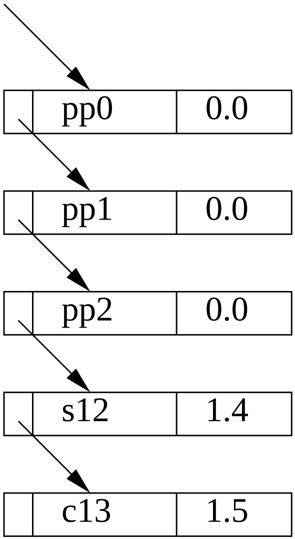

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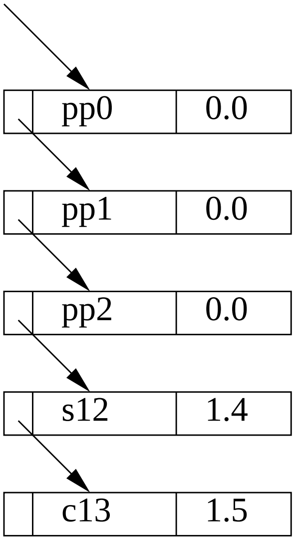

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- When two pass to CLA, when three use half adder and pass to CLA.

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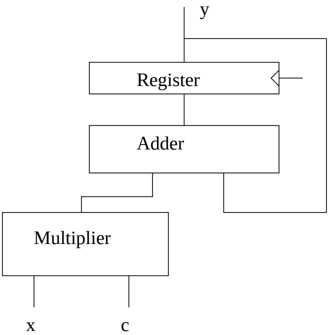

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- But we can still do better.

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- The same idea can be extended to equations such as $y = a * x + b * z$ and so on.
- These are referred to as *fused* multipliers.