Karnaugh Map(K-MAP)

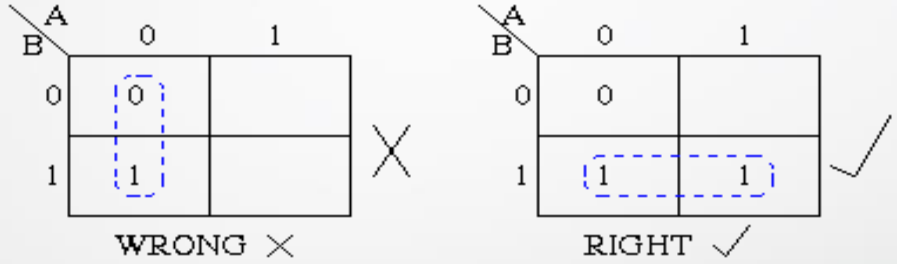
Presented by Nabanita Das

Karnaugh Map

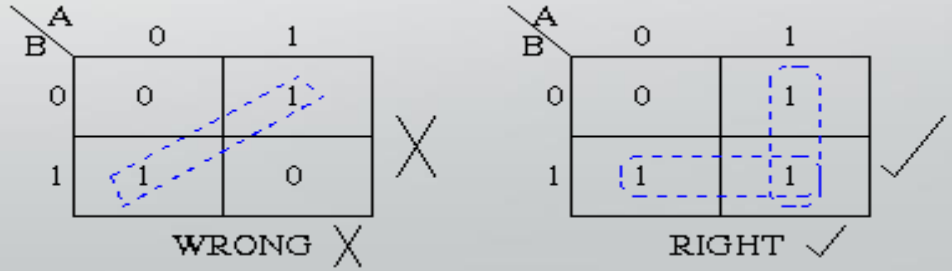
- Karnaugh maps provide an alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map.
- A K-map is a matrix consisting of rows and columns that represent the output values of a Boolean function.
- The output values placed in each cell are derived from the minterms *or* maxterms of a Boolean function.
- The arrangement of 0's and 1's within the map helps you to visualize the logic relationships between the variables and leads directly to a simplified Boolean statement.

The Karnaugh map uses the following rules for the simplification of expressions by grouping together adjacent cells containing *ones*.

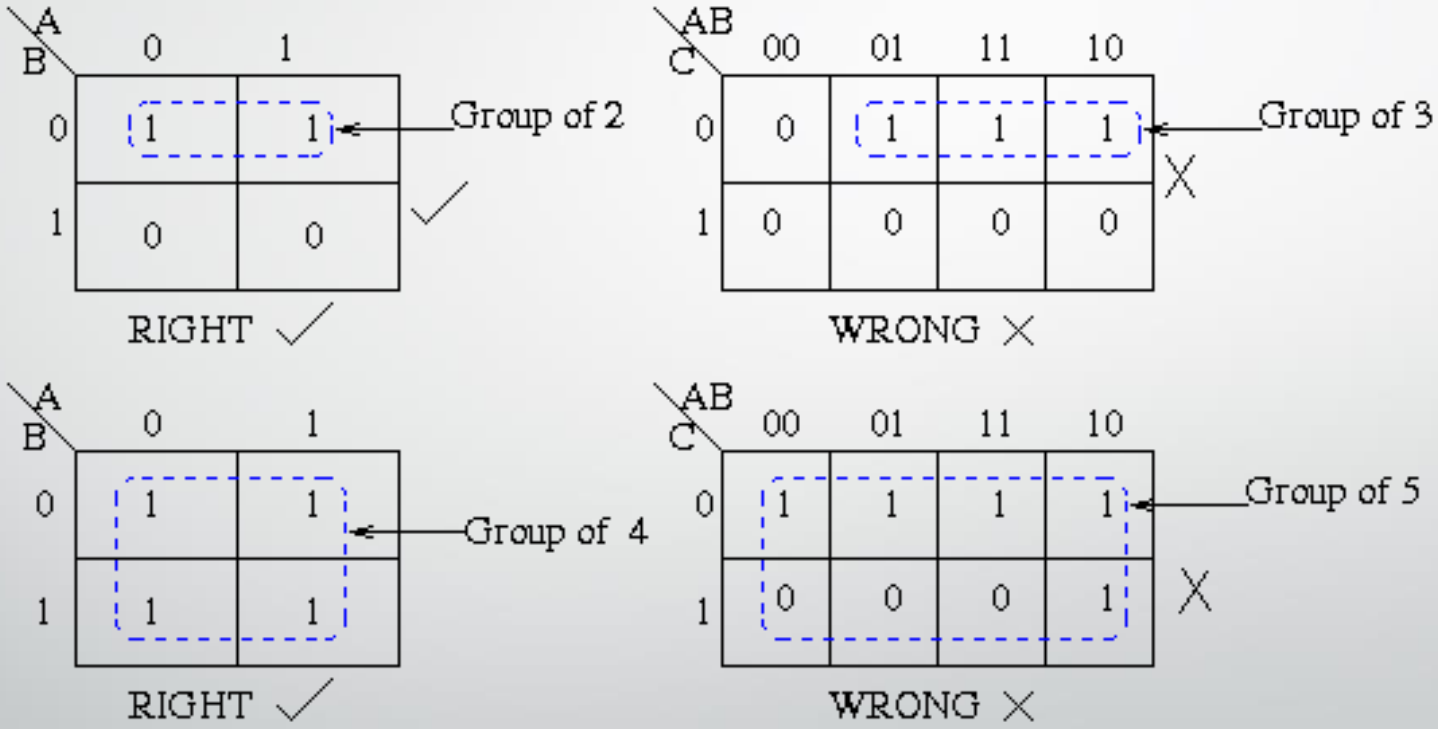
1.Groups may not include any cell containing a zero.



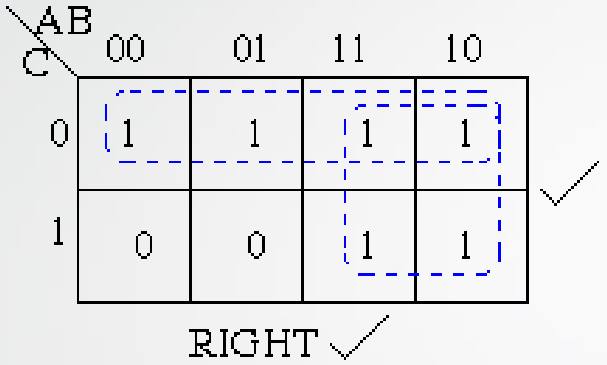
2.Groups may be horizontal or vertical, but not diagonal.

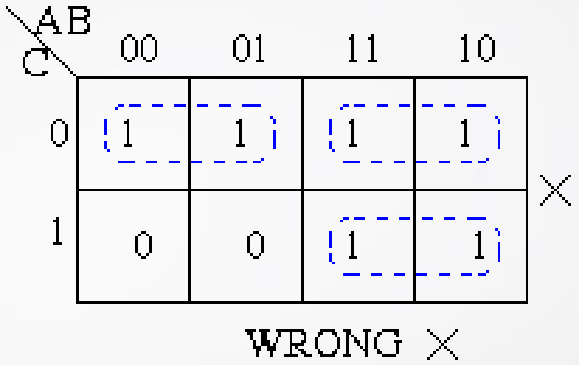


3. In an n- variable K-map, there are 2^n cells. That is if n = 1, a group will contain two 1's since $2^1 = 2$. If n = 2, a group will contain four 1's since $2^2 = 4$.



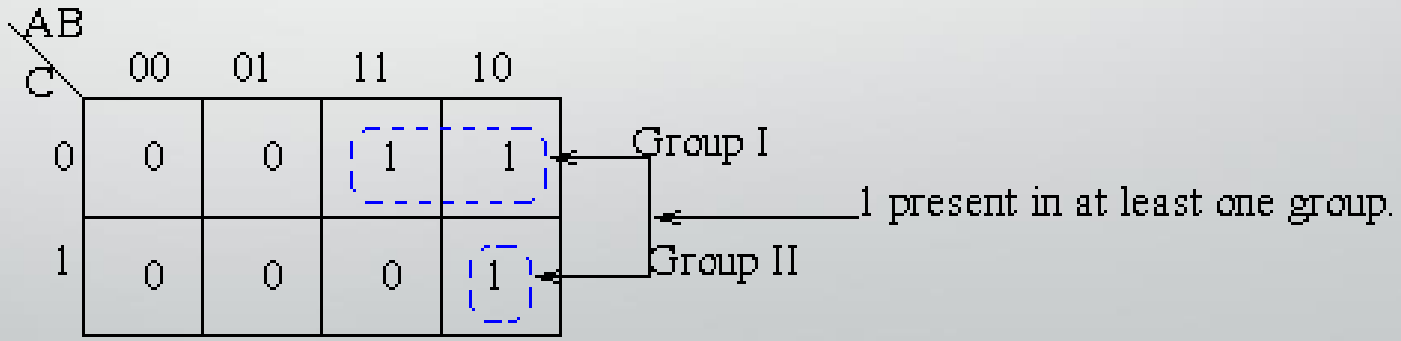
4. Each group should be as large aspossible.



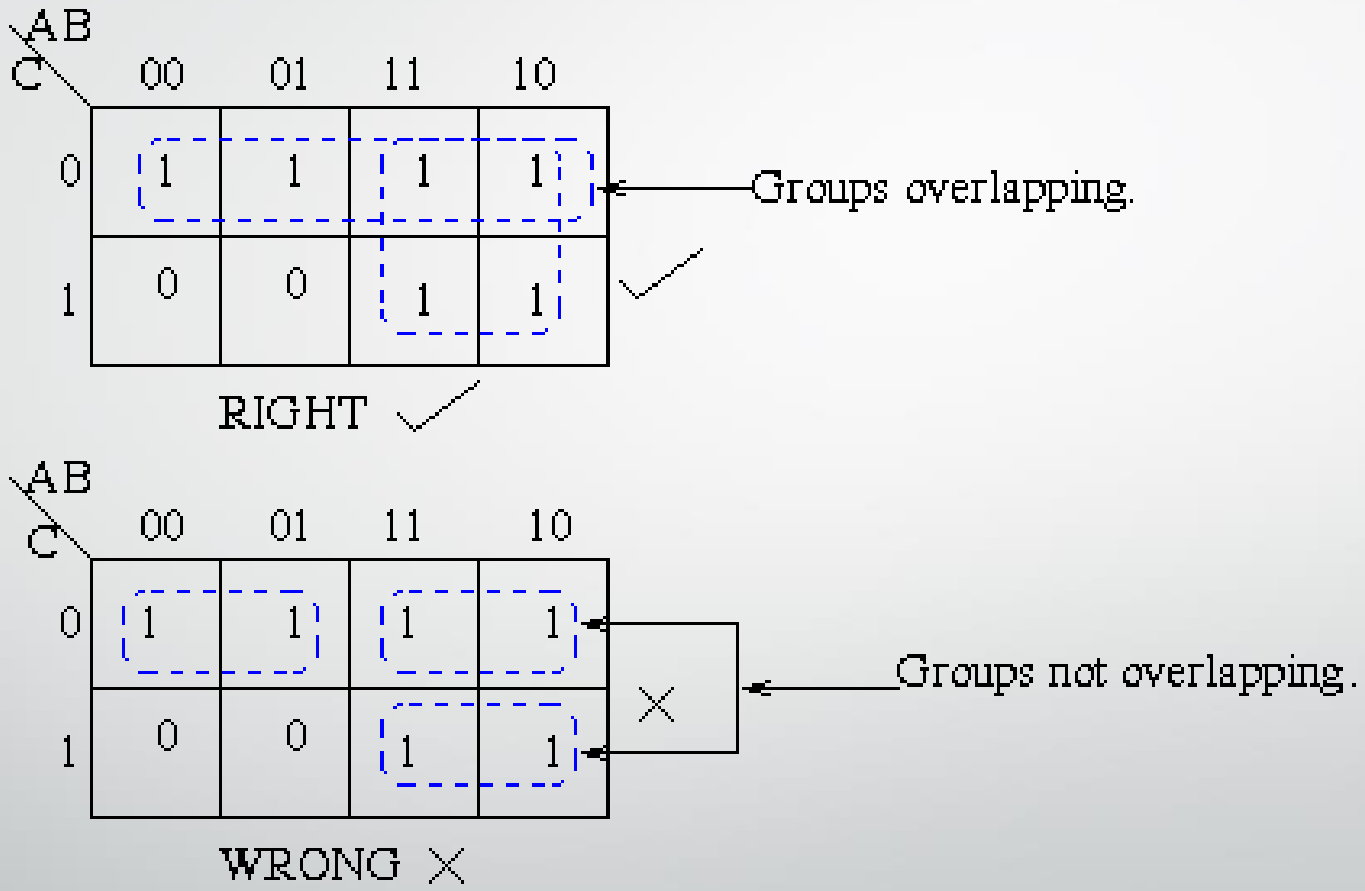


(Note that no Boolean laws broken,
but not sufficiently minimal)

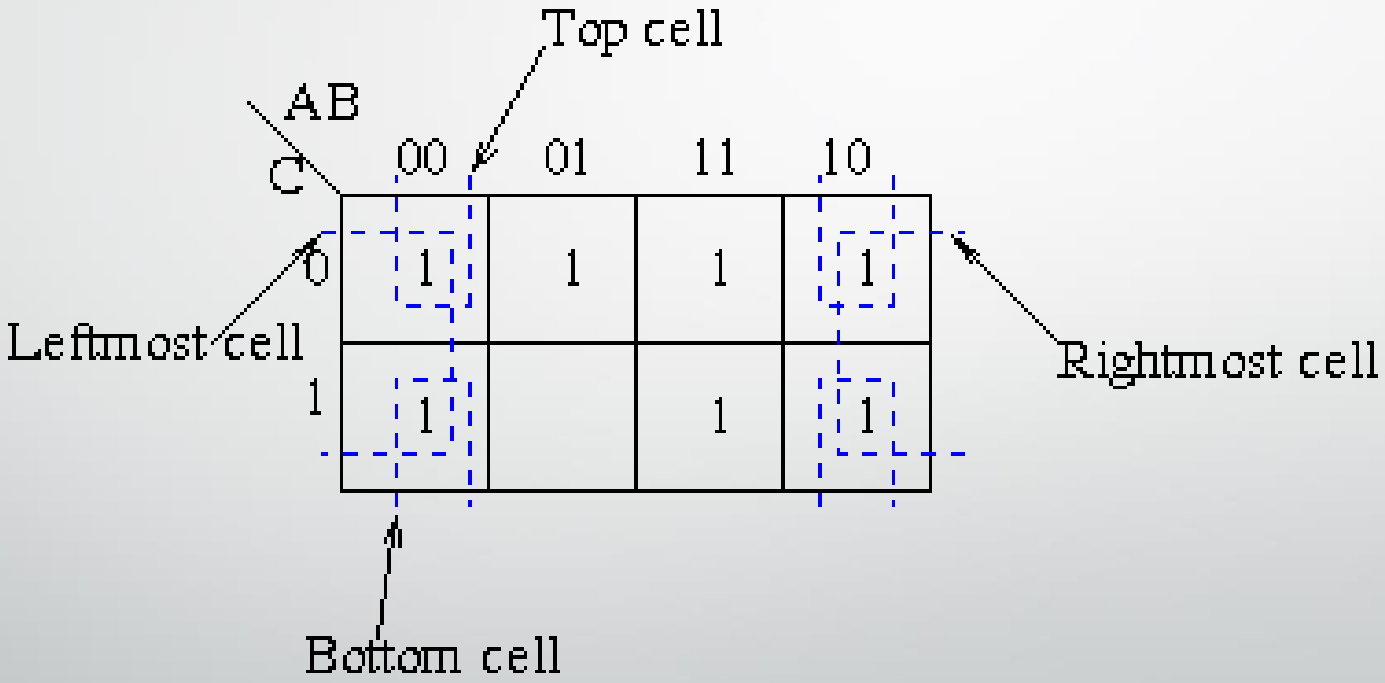
5.Each cell containing a one must be in at least one group.



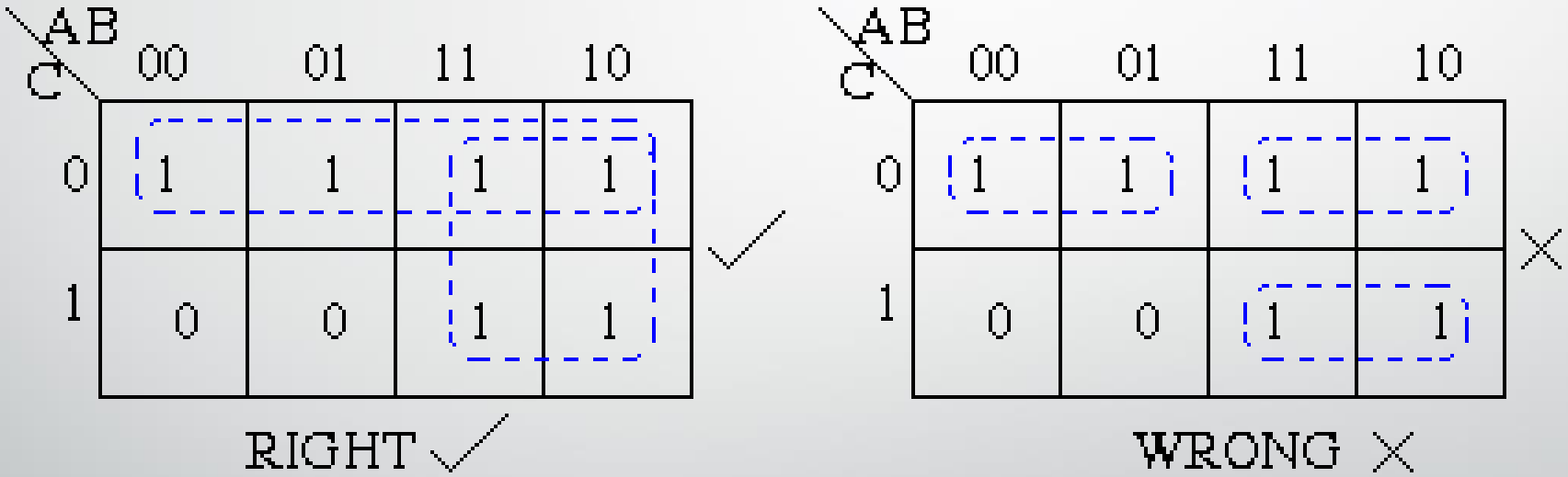
6.Groups may overlap.



7.Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.



8. There should be as few groups as possible, as long as this does not contradict any of the previous rules



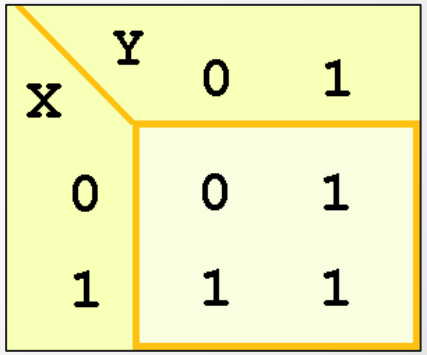
Summmary:

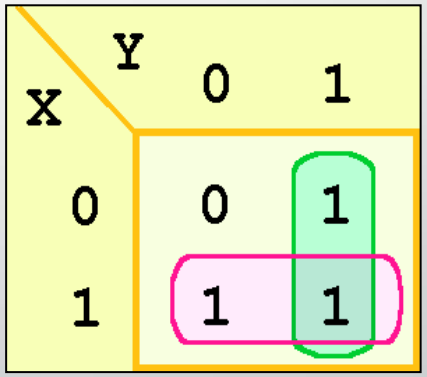
1. No zeros allowed.
2. No diagonals.
3. Only power of 2 number of cells in each group.
4. Groups should beas large as possible.
5. Every one must be in at least one group.
6. Overlapping allowed.
7. Wrap around allowed.
8. Fewest number of groups possible.

K-map Simplification for Two Variables

- In our example, we have two such groups.

- We see that both groups are powers of two and that the groups overlap.





K-map Simplification for Three Variables

- Consider the function:

$$F(X,Y) = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}Z + XYZ$$

- Its K-map is given below.
  - What is the largest group of 1s that is a power of 2?

X \ YZ	00	01	11	10
0	0^0	1^1	1^3	0^2
1	0^4	1^5	1^7	0^6

This means that the function, reduces to $F(x) = z$

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

You could verify this reduction with identities or a truth table.

K-map Simplification for Three Variables

- Now we take a example for a more complicated K-map. Consider the function:

$$F(X,Y,Z) = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}YZ + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$$

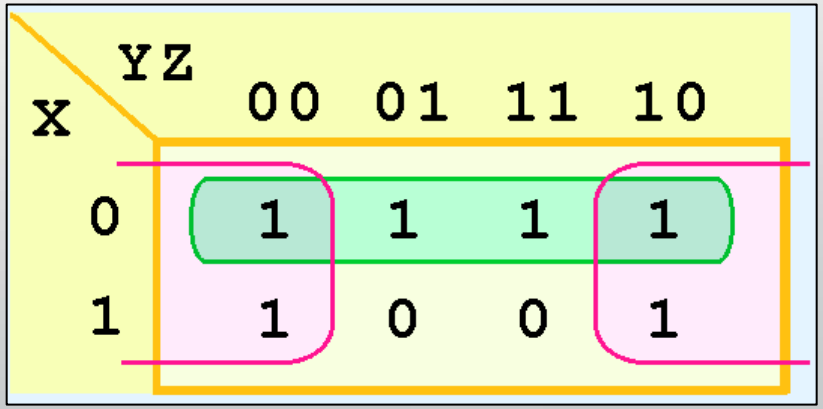
- Its K-map is shown below. There are (only) two groupings of 1s.

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

K-map Simplification for Three Variables

- The green group in the top row tells us that only the value of *x* is significant in that group.
- We see that it is complemented in that row, so the other term of the reduced function is $\overline{x}$
- Our reduced function is: $F(X,Y,Z) = \overline{X} + \overline{Z}$

Recall that we had six minterms in our original function!



K-map Simplification for Four Variables

- Our model can be extended to accommodate the 16 minterms that are produced by a four-input function.
- This is the format for a 16-minterm K-map.

WX \ YZ	00	01	11	10
00	$\bar{W}\bar{X}\bar{Y}\bar{Z}$	$\bar{W}\bar{X}\bar{Y}Z$	$\bar{W}\bar{X}YZ$	$\bar{W}\bar{X}Y\bar{Z}$
01	$\bar{W}X\bar{Y}\bar{Z}$	$\bar{W}X\bar{Y}Z$	$\bar{W}XYZ$	$\bar{W}XY\bar{Z}$
11	$WX\bar{Y}\bar{Z}$	$WX\bar{Y}Z$	$WXYZ$	$WXY\bar{Z}$
10	$W\bar{X}\bar{Y}\bar{Z}$	$W\bar{X}\bar{Y}Z$	$W\bar{X}YZ$	$W\bar{X}Y\bar{Z}$

4 variables K-map for Sum of Products

WX \ YZ	[00] Y'Z'	[01] Y'Z	[11] YZ	[10] YZ'
[00] W'X'	0	1	3	2
[01] W'X	4	5	7	6
[11] WX	12	13	15	14
[10] WX'	8	9	11	10

A	B	F	Minterm
0	0	0	$A'B' = m_0$
0	1	1	$A'B = m_1$
1	0	1	$AB' = m_2$
1	1	0	$AB = m_3$

4 variables K-map for Products of Sum

WX \ YZ	[00] Y+Z	[01] Y+Z'	[11] Y'+Z'	[10] Y'+Z
[00] W+X	0	1	3	2
[01] W+X'	4	5	7	6
[11] W'+X'	12	13	15	14
[10] W'+X	8	9	11	10

A	B	F	Maxterm
0	0	0	$A+B = M_0$
0	1	1	$A+B' = M_1$
1	0	1	$A'+B = M_2$
1	1	0	$A'+B' = M_3$

K-map Simplification for Four Variables

- We have plotted the K-map which has shown below with the non zero minterms from the function:

$$F(W,X,Y,Z) = \bar{W}\bar{X}\bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Y}Z + \bar{W}\bar{X}Y\bar{Z} + \bar{W}X\bar{Y}\bar{Z} + W\bar{X}\bar{Y}\bar{Z} + W\bar{X}\bar{Y}Z + W\bar{X}Y\bar{Z}$$

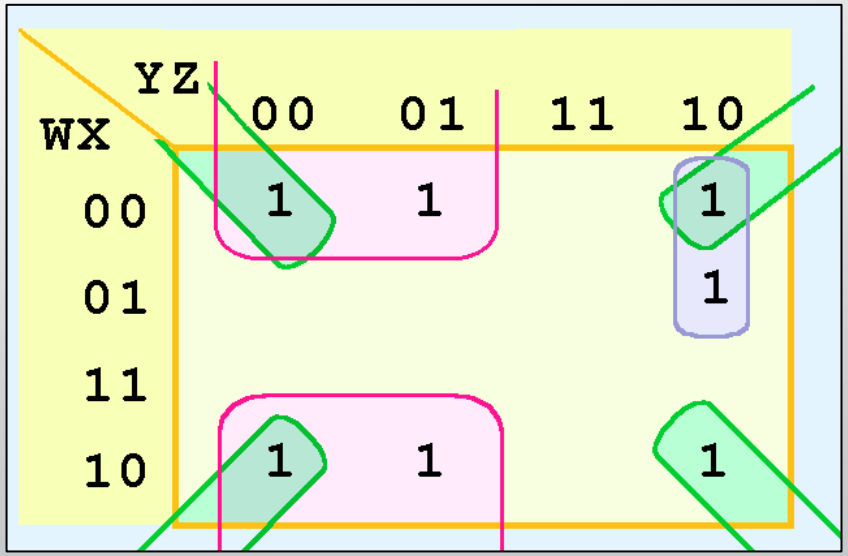
Recall that groups can overlap.

WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

K-map Simplification for Four Variables

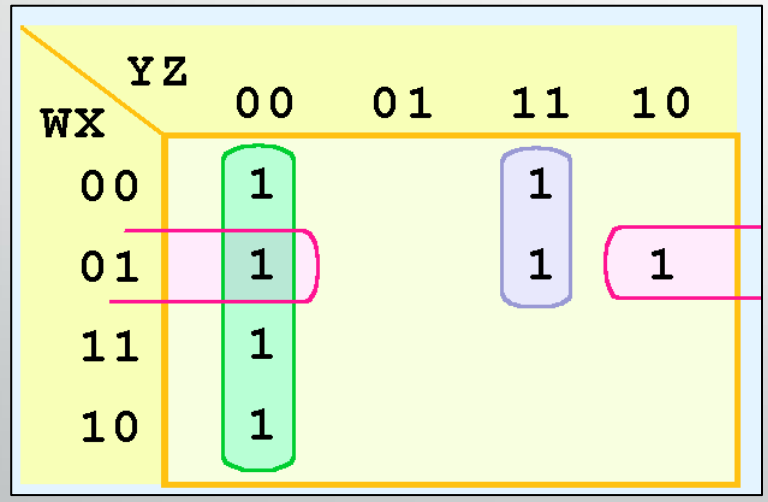
- Three groups consist of:
  - A purple group entirely within the K-map at the right.
  - A pink group that wraps the top and bottom.
  - A green group that spans the corners.
- Thus we have three terms in our final function:

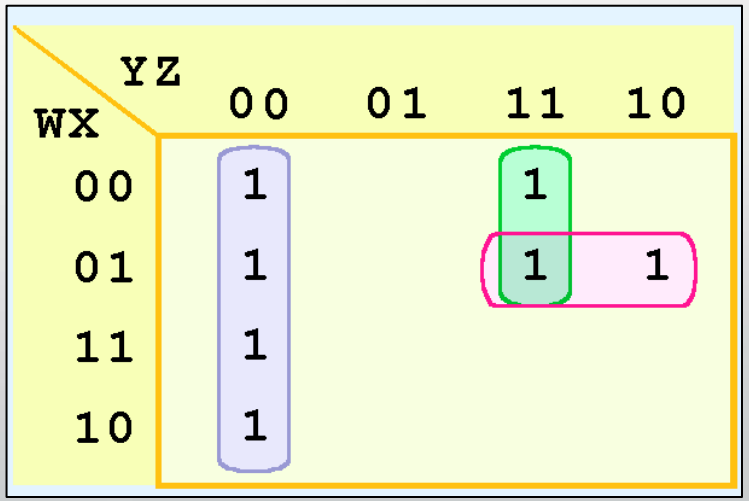
$$F(W,X,Y,Z) = W'YZ' + X'Y' + X'Z'$$



K-map Simplification for Four Variables

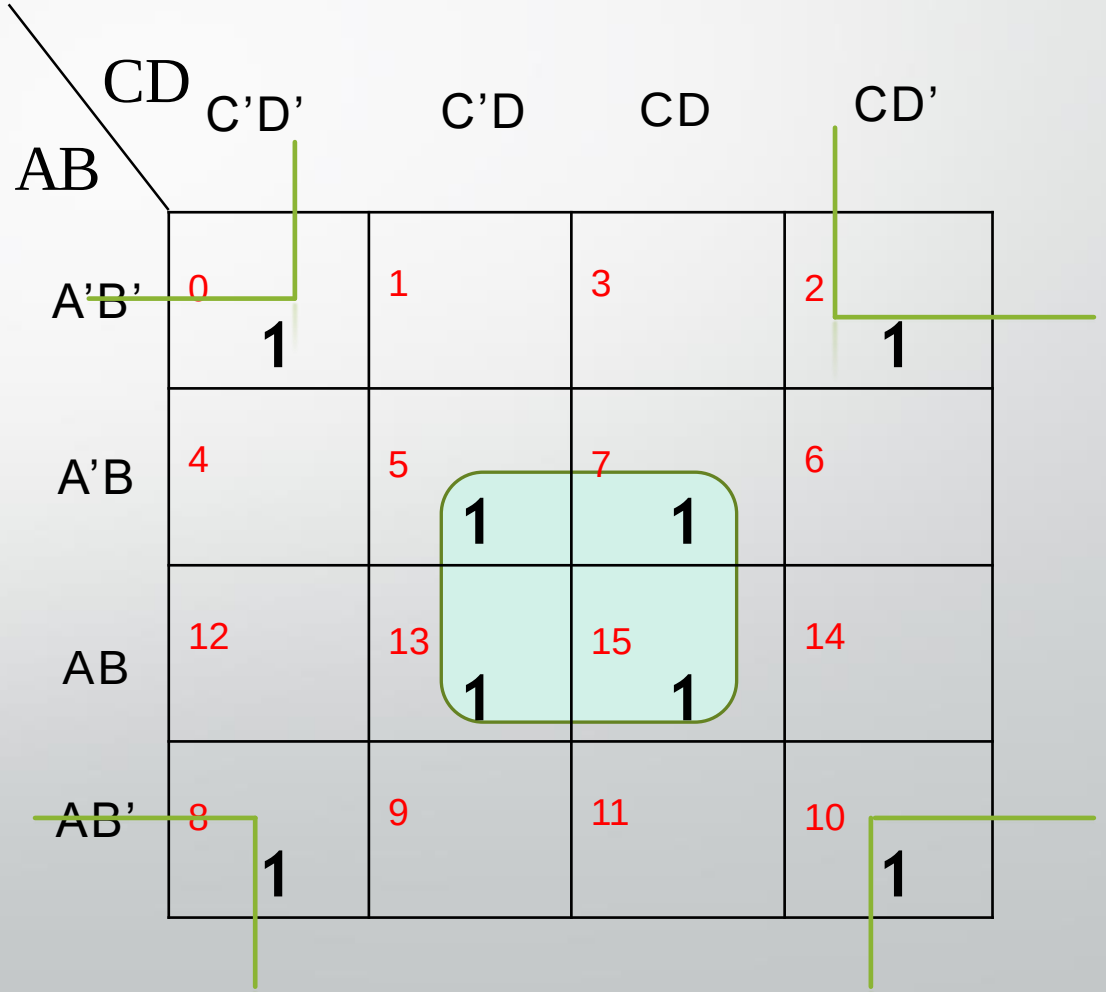
- It is possible to have a choice as to how to pick groups within a K-map, while keeping the groups as large as possible.
- The (different) functions that result from the groupings below are logically equivalent.





K-map Simplification for Four Variables by SOP and POS Form

- F(A,B,C,D)=∑(0,2,5,7,8,10,13,15)
- **Final expression (BD+B’D’)**



K-map Simplification for Three Variables by POS Form

- F(A,B,C)=π(0,3,6,7)
- **Final expression (A+B+C)(B’+C’)(A’+B’+C)**