BINOMIAL HEAP

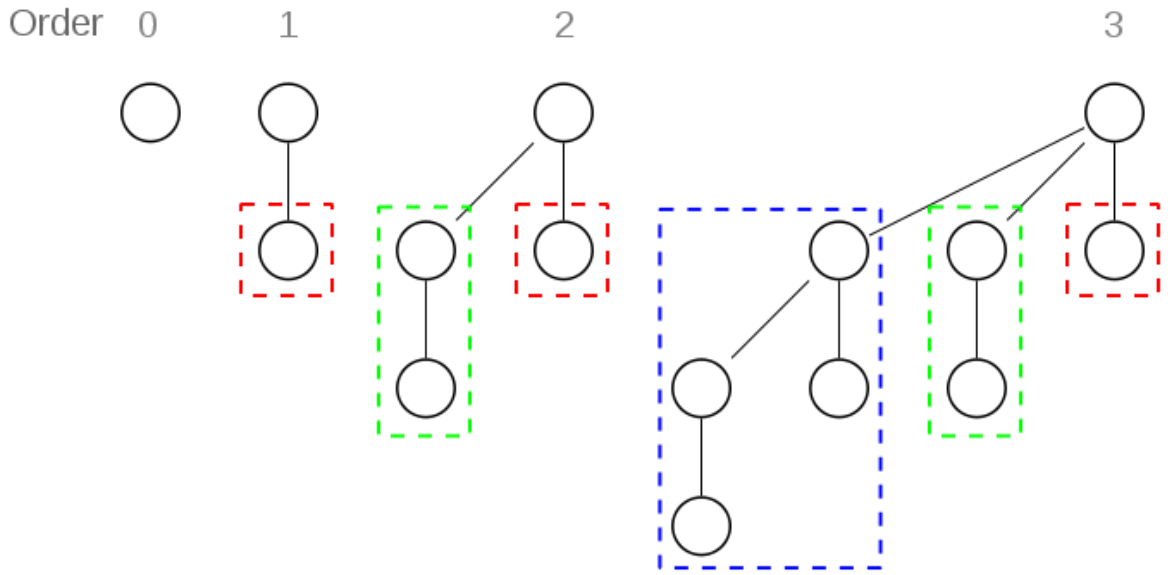
- A **binomial heap** is a specific implementation of the heap data structure.
- Binomial heaps are collections of binomial trees that are linked together where each tree is an ordered heap.
- In a binomial heap, there are either one or zero binomial trees of order k, where k helps describe the number of elements a given tree can have: 2^k

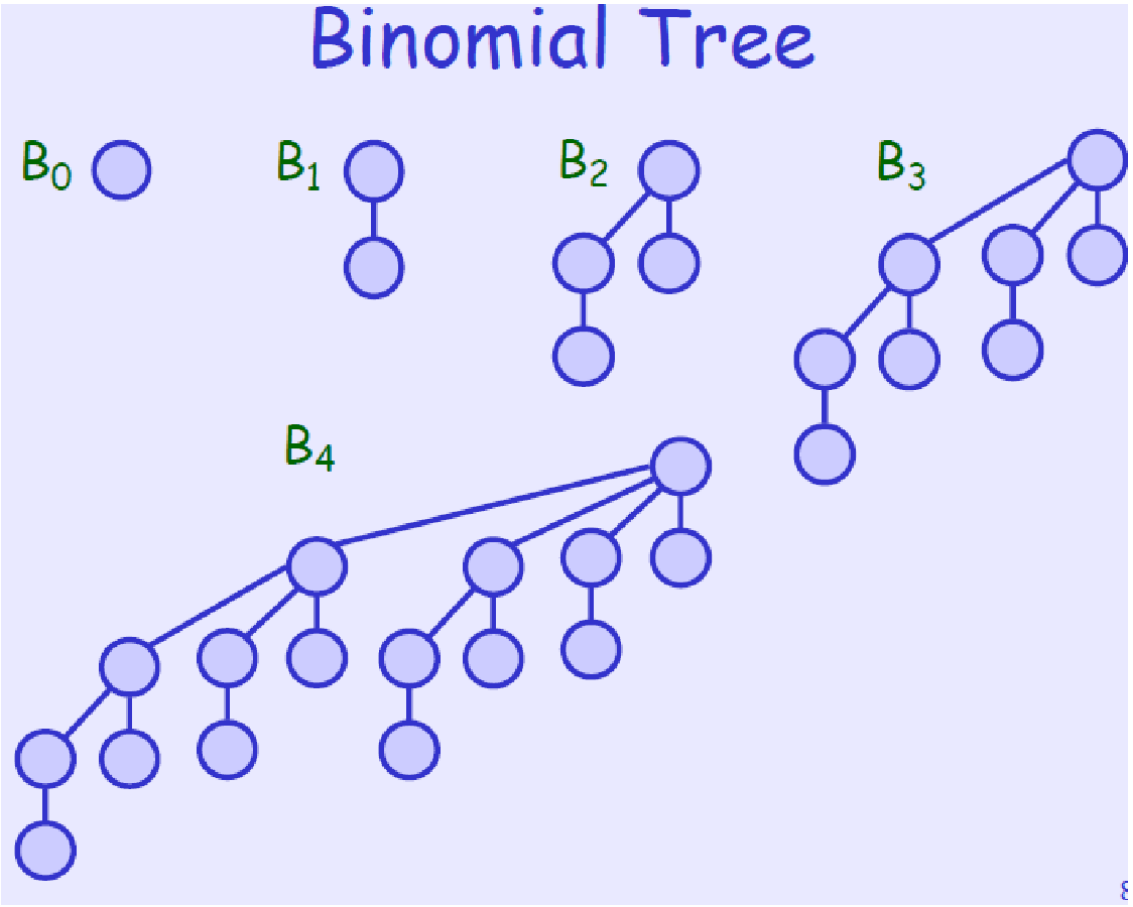
Binomial Tree

Definition:

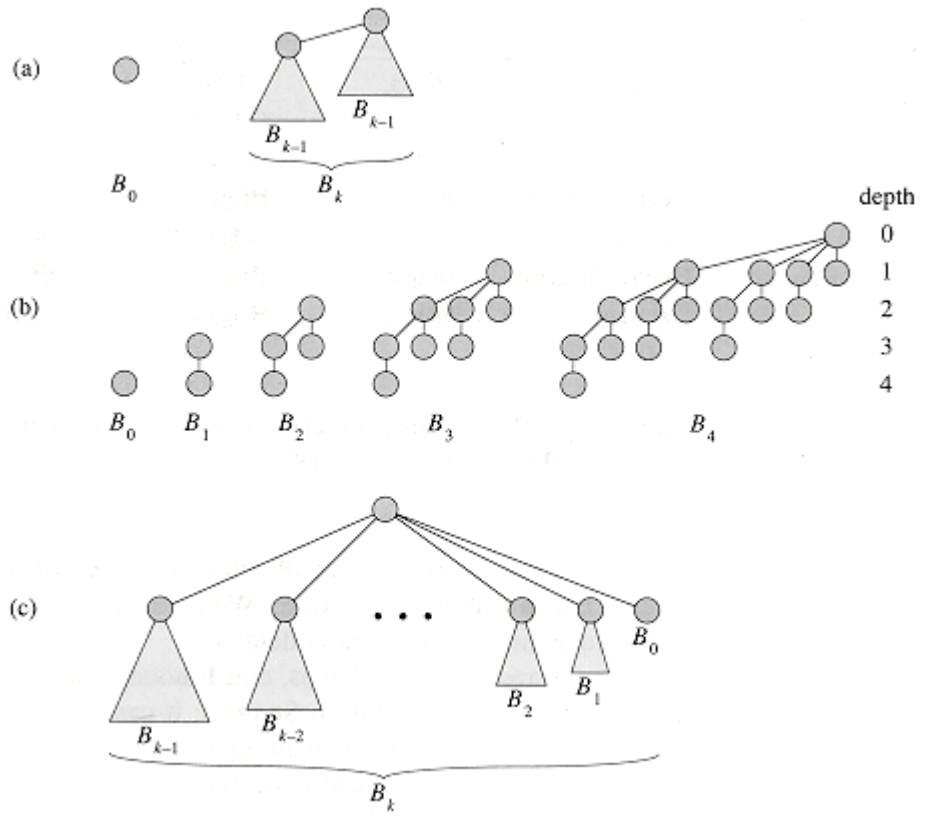
- A binomial tree of order k, denoted by B_k, is defined recursively as follows:
  - ✔ B_0 is a tree with a single node
  - ✔ For k ≥1, B_k is formed by joining two B_{k-1}, such that the root of one tree becomes the leftmost child of the root of the other

- A binomial tree of order k has a root node whose children are roots of binomial trees of orders k−1, k−2... 2, 1, 0 (in this order).
- The order represents how many children the root node is able to have.
- For example, there are three children coming out of the order 3 node and no children coming out of the order 0 node.


Order
0
1
2
3


Binomial Tree
B0
B1
B2
B3
B4

Binomial trees B_0 through B_4.



A Binomial Tree of order k has following properties.

a) It has exactly 2^k nodes.

b) It has depth as k.

c) There are exactly ${}^kC_i$ nodes at depth i for i = 0, 1, . . . , k.

d) The root has degree k and children of root are themselves Binomial Trees with order k-1, k-2,.. 0 from left to right.

Combination Formula

$$ {}^nC_r = \frac{n!}{(n-r)!r!} $$

$$_5C_3 = \frac{5!}{3!(5-3)!} \quad \frac{5*4*3*2*1}{3*2*1(2)!} \quad \frac{5*4*\cancel{3}*\cancel{2}*1}{\cancel{3}*2*1*\cancel{2}*1}$$

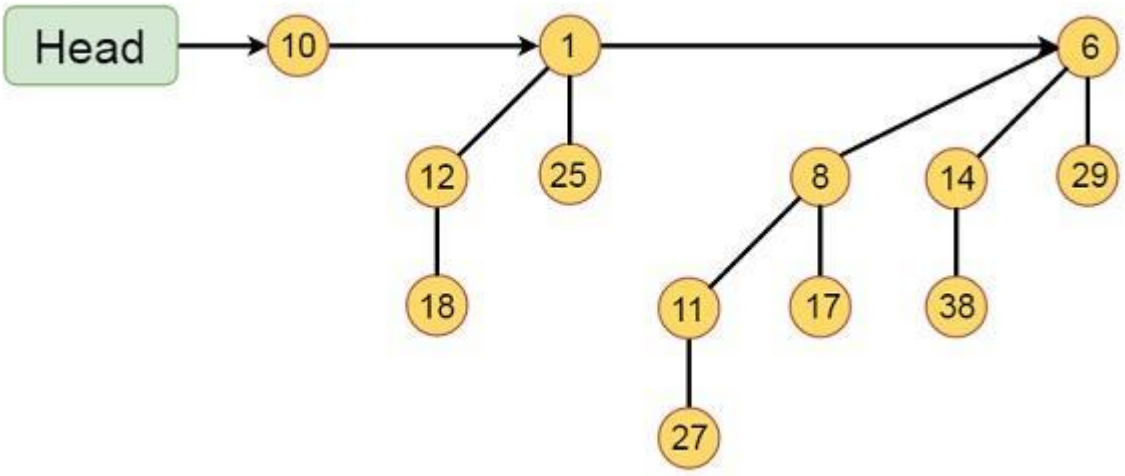
$$\frac{5*4}{2*1} = 10$$

A binomial heap H is a set of binomial trees that satisfies the following binomial heap properties.

1. Each binomial tree in H obeys the min-heap property: the key of a node is greater than or equal to the key of its parent. We say that each such tree is min-heap-ordered.

2. For any nonnegative integer k, there is at most one binomial tree in H whose root has degree k.

The first property tells us that the root of a min-heap-ordered tree contains the smallest key in the tree



This binomial Heap H consists of binomial trees B0, B2 and B3,which have 1, 4 and 8 nodes respectively. And in total n = 13 nodes.

Memory representation of Binomial Heap

- Binomial heaps are collection of binomial trees stored in ascending order of size.
- The root list in the heap is a linked list of roots of the Binomial heap.
- The degree of the nodes of the roots increase as on traversing the root list.

Binomial Heap Node:

Each node in a binomial heap has 5 fields :

1. Pointer to parent
2. Key
3. Degree
4. Pointer to child (leftmost child)
5. Pointer to sibling which is immediately to its right

A Binomial Heap Node

Pointers in each node:

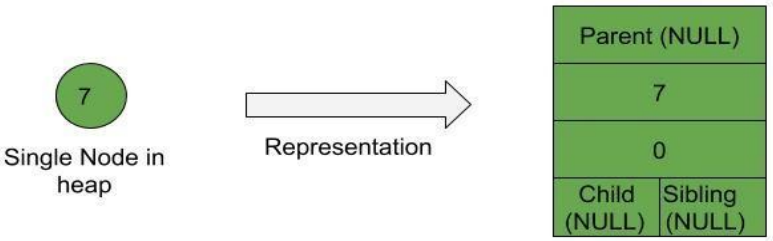
Each node has the following pointers:

1. A parent pointer pointing to the immediate parent of the node

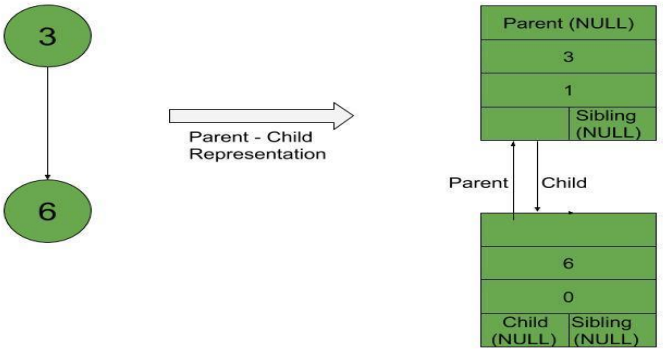
2. A left pointer pointing to the first child of the node
3. A right pointer pointing to the next sibling of the node.

Types of nodes and their representations

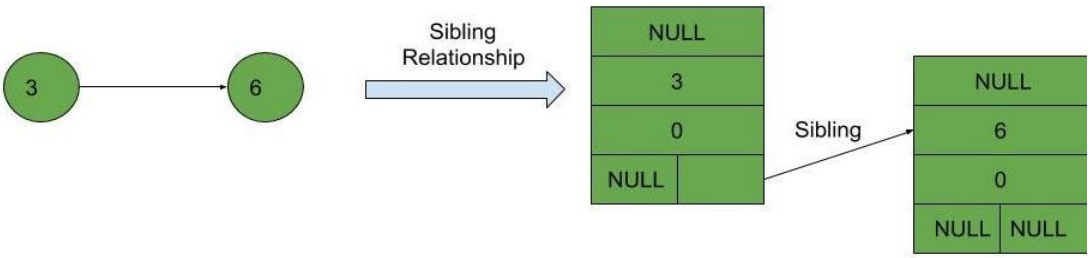
- **Single node in the Heap**



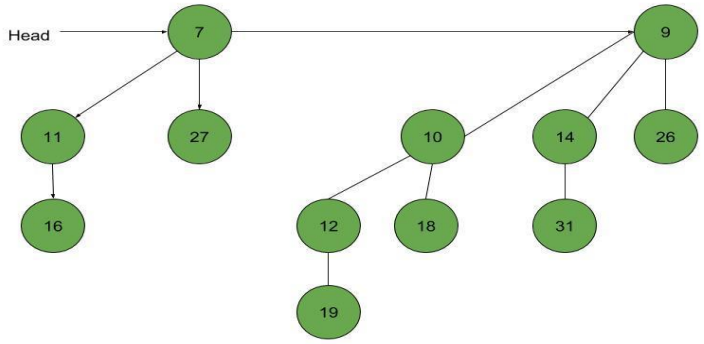
- **Parent – Child relationship between nodes**



- **Sibling relationship between nodes**



- **Representation of Full binomial heap:**



The memory representation of each node of the Binomial heap given above can be illustrated using the following diagram: