ECBS 6060: International Trade
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Lecture 12: The Melitz model

The Melitz model

- We first describe the equilibrium of a closed economy.
- We then turn to the open economy.
- We conduct two comparative statics:
  - autarky vs open economy
  - change in trade costs

The basics of the model

- Firms differ in their productivity.
- There are fixed costs of exporting.

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- Firms differ in their productivity.
- There are fixed costs of exporting.
- Such a model is *qualitatively* consistent with the two mentioned facts:
  - Fixed costs prevent many firms from exporting.
  - More productive firms are bigger, and bigger firms can recover the fixed cost more easily $\rightarrow$ they are more likely to export.

Strict sorting property

- There is a *strict sorting* property:
  - all firms below a cutoff are non-exporters
  - all firms above that cutoff are exporters

Key implications

In the event of a trade liberalization (falling trade costs),

1. more firms will export ("extensive margin"),
2. labor is reallocated from less productive non-exporters to more productive exporters,
3. and least productive non-exporters will exit.

Consumers

- Consumers have a CES demand structure:

$$U = \left[\int_{\Omega} (y(\omega))^{\rho}\, d\omega\right]^{1/\rho}$$

  with $\rho \in (0,1)$.
    - elasticity of substitution: $\theta = 1/(1-\rho)$
    - set of varieties: Ω (continuum)
- Demand for good ω:

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\theta},$$

  where

$$P = \left[\int_{\Omega} p(\omega)^{1-\theta}\, d\omega\right]^{\frac{1}{1-\theta}}.$$

Firms

- There is a continuum of firms, indexed by $\omega \in \Omega$.
- They produce differentiated products and compete monopolistically.
- Firms differ in productivity $\varphi(\omega)$.

Technology

- Each unit of good ω requires $1/\varphi(\omega)$ units of labor.
- There is a fixed cost of production, f.

$$l(\omega) = \frac{y(\omega)}{\varphi(\omega)} + f$$

  - overhead costs of marketing, management, distribution etc.

Pricing and demand

- Because demand is isoelastic, firms charge a constant markup $1/\rho$,

$$p(\omega) = \frac{w}{\rho\varphi(\omega)}.$$

  - More productive firms have lower prices.
- We normalize the wage rate to 1.
- Revenue of firm ω is

$$r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\theta} = R\frac{\varphi(\omega)^{\theta-1}}{\int_{\Omega}\varphi(\omega)^{\theta-1}\,d\omega}$$

  - More productive firms have higher revenue.

Profits

- A constant $1-\rho$ fraction of revenue is retained as *operating* profits:

$$\pi(\omega) = (1-\rho)r(\omega) - f = (1-\rho)R\frac{\varphi(\omega)^{\theta-1}}{\int_{\Omega}\varphi\left(\omega\right)^{\theta-1}d\omega} - f.$$

Aggregation

- The aggregate price index:

$$P = \left[\int_{\Omega} p\left(\omega\right)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}.$$

- Substituting in $p(\omega) = 1/(\rho\varphi)$:

$$P = \frac{1}{\rho} \left[\int_{\Omega} \varphi\left(\omega\right)^{\theta-1} d\omega \right]^{\frac{1}{1-\theta}}.$$

- Since each firm is fully characterized by φ, let us do a change of variables:

$$P = \frac{1}{\rho} \left[\int_{\Omega} \varphi^{\theta-1} M \mu(\varphi) d\varphi \right]^{\frac{1}{1-\theta}}.$$

  - M: number ("mass") of firms
  - $\mu(\varphi)$: fraction of firms with productivity φ

Average productivity

- Introduce a *geometric average* of productivities,

$$\tilde{\varphi} = \left[\int_{\Omega} \varphi^{\theta-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\theta-1}} .$$

- Clearly,

$$P = \frac{1}{\rho \tilde{\varphi}} M^{1/(1-\theta)}.$$

- In fact, $\tilde{\varphi}$ completely summarizes the productivity distribution of firms.
- All aggregate variables are as if the economy were populated by
  - M *identical* firms
  - with productivity $\tilde{\varphi}$

Average profits

- Recall
$$\pi(\tilde{\varphi}) = (1-\rho)r(\tilde{\varphi}) - f.$$
- Aggregate revenue is R:
$$R = Mr(\tilde{\varphi}).$$
- So the profit of the average firm is
$$\pi(\tilde{\varphi}) = \frac{(1-\rho)R}{M} - f.$$

Firm-level variables

- Firm-level variables depend on $\varphi/\tilde{\varphi}$.
- Revenue of firm φ is

$$r(\varphi) = r(\tilde{\varphi}) \left(\frac{\varphi}{\tilde{\varphi}}\right)^{\theta-1}$$

- Profits:

$$\pi(\varphi) = (1-\rho) r(\tilde{\varphi}) \left(\frac{\varphi}{\tilde{\varphi}}\right)^{\theta-1} - f$$

- All else equal, if the average firm (the competition) is better, firm i makes less revenue and less profits.

Firm entry

- Before entering, the firm doesn't know its true productivity. (Otherwise only the best firms would enter.)
- (Everything we say about productivity can be understood as "product appeal". Why?)
- They pay a fixed cost, f_e.
  - setting up a plant
  - hiring workers
  - producing the first prototype
- After entry, they draw a productivity from a continuous distribution $G(\varphi)$.
- Clearly, if $\pi(\varphi) < 0$ (remember the overhead costs), the firm exits right away.

Zero-profit cutoff

- There exists a cutoff probability below which firms make negative profit:

$$\varphi^* : \frac{(1-\rho)R}{M}\left(\frac{\varphi^*}{\tilde{\varphi}}\right)^{\theta-1} - f = 0$$

- This clearly depends on average productivity, $\tilde{\varphi}$.
  - Higher $\tilde{\varphi}$ reduces the profit of each firm.
  - Even more productive firms cannot recover their fixed cost.
  - φ^* rises.

Average productivity

- Average productivity depends on the distribution of firms.
- Firms below φ^* exit right away, $\mu(\varphi) = 0$ for all $\varphi < \varphi^*$.
- Firms above φ^* are drawn from a *truncated distribution*:

$$\mu(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*)}$$

- Substitute into average productivity:

$$\tilde{\varphi} = \left[\int_\Omega \varphi^{\theta-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}.$$

Expected profit

- The expected profit of a new entrant is

$$G(\varphi^*) \cdot 0 + (1 - G(\varphi^*)) \cdot \pi(\tilde{\varphi}).$$

- If there is free entry, this should equal the entry cost:

$$(1 - G(\varphi^*))\pi(\tilde{\varphi}) = f_e.$$

$$(1 - G(\varphi^*)) \left[\frac{(1-\rho)R}{M} - f \right] = f_e.$$

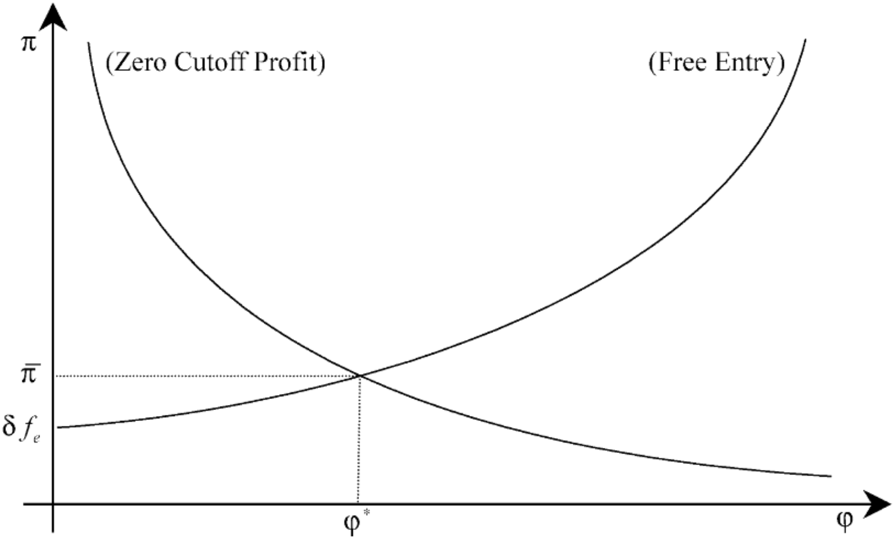
Equilibrium

- In equlibrium, both the zero-profit-cutoff and the free-entry conditions hold.
- We also have labor market clearing.
- Because *in expectation* firms make zero profit,

$$L = R.$$

- This pins down the *number of firms*, M.

Equilibrium production cutoff



Pareto example

- Suppose the distribution of firms is Pareto:

$$G(\varphi) = 1 - \left(\frac{\varphi}{\varphi_{\min}}\right)^{-k}.$$

- Then $\varphi^{\theta-1}$ is also pareto with exponent $K = k/(\theta - 1)$.
- The Pareto distribution has a simple truncated mean

$$\tilde{\varphi}^{\theta-1} = \frac{K}{K-1}\varphi_*^{\theta-1}$$

- This simplifies many of the formulas.

Pareto example

- Zero-profit cutoff:

$$\frac{(1-\rho)R}{M}\frac{K-1}{K} - f = 0,$$

so that

$$\frac{(1-\rho)R}{M} = \frac{K}{K-1}f.$$

- Free entry:

$$\left(\frac{\varphi^*}{\varphi_{\min}}\right)^{-k}\left[\frac{(1-\rho)R}{M} - f\right]$$

$$= \left(\frac{\varphi^*}{\varphi_{\min}}\right)^{-k}\frac{1}{K-1}f = f_e.$$

The open economy

The open economy

- We take a small open economy with trade costs τ.
- (We would have similar conclusions with GE of large economies.)
- Opening up to trade basically amounts to increasing the size of the market.

Foreign demand

- Foreign demand is a similarly isoelastic,

$$q_x(\varphi) = Ap_x(\varphi)^{-\theta}.$$

- Because we are in a small open economy, we treat A as a *constant* demand shifter.
- (It would move around in GE.)

Exports vs domestic sales

- Exporting has a marginal, "iceberg" cost $\tau > 1$.
  - This includes tariffs as well as transportation costs.
- We can conduct comparative statics wrt τ.

Export pricing

- Foreigners have to pay the shipping cost,

$$p_x(\varphi) = \frac{\tau}{\rho\varphi},$$

  (again, wage is normalized to 1).
- Foreign price depends on τ.

Revenues

- Foreign revenues are

$$r_x(\varphi) = A\rho^{\theta-1}\tau^{-\theta}\varphi^{\theta-1}$$

  - More productive firms are bigger both at home and abroad.
  - Foreign revenue decreases in τ.
- Total revenue of an exporter:

$$r(\varphi) = r_x(\varphi) + r_d(\varphi) = \varphi^{\theta-1}\left[A\rho^{\theta-1}\tau^{-\theta} + \frac{R_d}{M}\tilde{\varphi}^{1-\theta}\right]$$

  - Total revenue is proportional to $\varphi^{\theta-1}$, just like before.
  - In fact, export / domestic sales is constant across all exporters.

The decision to export

- To send any positive exports, a firm also has to pay a fixed cost f_x.
  - setting up distribution networks
  - conforming to regulation
  - catering to foreign tastes
- This can deter some firms from exporting.
- To proceed step by step, we first assume $f_x = 0$ so that everybody exports.

Everybody exports

Profits

- Recall that operating profits are $(1-\rho)$ fraction of revenue:

$$\pi(\varphi) = (1-\rho)r(\varphi) = (1-\rho)\varphi^{\theta-1}\left[A\rho^{\theta-1}\tau^{-\theta} + \frac{R_d}{M}\tilde{\varphi}^{1-\theta}\right] - f$$

- Importantly,

$$\frac{\pi(\varphi)+f}{\pi(\varphi')+f} = \left(\frac{\varphi}{\varphi'}\right)^{\theta-1}$$

for any φ and φ' so that

$$\pi(\varphi) = [\pi(\varphi')+f]\left(\frac{\varphi}{\varphi'}\right)^{\theta-1} - f.$$

- Using this relationship for the *representative firm* $(\tilde{\varphi})$, we can write down
  1. the zero-profit cutoff
  2. and the free entry condition.

Zero-profit cutoff

- The firm at the margin makes zero profit:

$$0 = \pi(\varphi^*) = [\pi(\tilde{\varphi}) + f] \left(\frac{\varphi^*}{\tilde{\varphi}}\right)^{\theta-1} - f.$$

- Average profit:

$$\pi(\tilde{\varphi}) = f \left[\left(\frac{\varphi^*}{\tilde{\varphi}}\right)^{1-\theta} - 1\right]$$

Expressing average productivity

- Again, average productivity depends on the cutoff:

$$\tilde{\varphi} = \tilde{\varphi}(\varphi^*).$$

  - This function only depends on the distribution of φ, $G()$.
  - For the *Pareto distribution*, it is

$$\tilde{\varphi}(\varphi^*) = k\varphi^*$$

    where $k > 1$ is related to the shape parameter of the distribution.

Zero-profit cutoff

- Under the Pareto distribution, average profit is independent of the cutoff:

$$\pi(\tilde{\varphi}) = [k^{\theta-1} - 1]f.$$

- This is in fact the same equation as under closed economy.
- (This latter would also hold for other distributions.)

Free entry

- Free entry equates *expected profits* with entry costs:

$$[1 - G(\varphi^*)]\pi(\tilde{\varphi}) = f_e.$$

- Or, writing in average profits from the ZCF:

$$[1 - G(\varphi^*)][k^{\theta-1} - 1]f = f_e.$$

- This pins down φ^*.
- This is the same φ^* as before.

Labor demand

- Given zero profits (in expectation), wages still equal revenue

$$L = R_d + R_x.$$

- Total revenue is also the same as before.
- Because average profits are the same, so are average revenues.
- But then the number of firms, M, is also the same.
- Nothing changed. Why?

Fixed cost of exporting

Fixed cost of exporting

- To send any positive exports, a firm also has to pay a fixed cost f_x.
  - setting up distribution networks
  - conforming to regulation
  - catering to foreign tastes

Fixed cost of exporting

- Now only some firms export.
- These are not a random selection: they are the better ones.
- There is a *strict sorting* property:
  - all firms below a cutoff are non-exporters
  - all firms above that cutoff are exporters
- Importantly,

$$\frac{\pi(\varphi) + f}{\pi(\varphi') + f} = \left(\frac{\varphi}{\varphi'}\right)^{\theta - 1}$$

  no longer holds.

Exporters vs nonexporters

- Revenue:

$$r(\varphi) = \begin{cases} r_d(\varphi) & \text{if not an exporter} \\ r_d(\varphi) + r_x(\varphi) & \text{if an exporter} \end{cases}$$

- Employment:

$$l(\varphi) = \begin{cases} \rho r_d(\varphi) & \text{if not an exporter} \\ \rho[r_d(\varphi) + r_x(\varphi)] & \text{if an exporter} \end{cases}$$

- Exporters are bigger in both respect (reallocation!).

The decision to export

- A firm with productivity φ exports if export profits exceed fixed costs,

$$(1-\rho)r_x(\varphi) = A\rho^{\theta-1}\tau^{-\theta}\varphi^{\theta-1} \geq f_x.$$

- Firms above a threshold φ_x^* will export, others will not.

Aggregation

- $\tilde{\varphi}_x$ is the average productivity of *exporters*:

$$\tilde{\varphi}_x = \left[\frac{1}{M_x}\int_{\Omega_x}\left(\varphi\left(\omega\right)\right)^{\theta-1} d\omega\right]^{\frac{1}{\theta-1}}$$

- Relative export sales:

$$\frac{r_x(\varphi)}{r_x(\tilde{\varphi}_x)} = \left(\frac{\varphi}{\tilde{\varphi}_x}\right)^{\theta-1}$$

- This implies for *export profits* (but not for total profits)

$$\frac{\pi_x(\varphi) + f_x}{\pi_x(\tilde{\varphi}_x) + f_x} = \left(\frac{\varphi}{\tilde{\varphi}_x}\right)^{\theta-1}$$

Zero-profit cutoffs

- Now we have two zero-profit cutoffs: one for operation, one for exports.

$$\begin{aligned} \pi(\varphi^*) = \pi_d(\varphi^*) &= 0 \\ \pi_x(\varphi_x^*) &= 0 \end{aligned}$$

- (The first equation follows from the fact the the smallest firm does not export.)
- Using the previous methods, these pin down average export and domestic profits:

$$\begin{aligned} \pi_d(\tilde{\varphi}) &= [k^{\theta-1} - 1]f, \\ \pi_x(\tilde{\varphi}) &= [k^{\theta-1} - 1]f_x. \end{aligned}$$

- Up till now, profits from the two markets could be treated more or less independent from one another.
- However, not all firms receive π_x.

Free entry

- Ex ante expected profit is now the sum of two terms:

$$E(\pi) = E(\pi_d) + E(\pi_x) = [1 - G(\varphi^*)]\pi_d(\tilde{\varphi}) + [1 - G(\varphi_x^*)]\pi_x(\tilde{\varphi}_x)$$

- This has to equal entry costs, f_e.
- Or, writing in average profits from the ZCF:

$$[k^{\theta-1} - 1]\left\{[1 - G(\varphi^*)]f + [1 - G(\varphi_x^*)]f_x\right\} = f_e.$$

- This now pins down a *combination* of φ^* and φ_x^*.

Export and exit

- Lower φ_x^* implies higher φ^*.
- If more firms export, this is because export profits are greater.
- This encourages more new firms to enter the industry.
- The least productive firms cannot compete with these new entrants and hence they exit.

Closing the model

- We still have 2 unknowns for 1 equation.
- We introduce 1 more unknown: M, the number of firms.
- We then use 2 market clearings: domestic and foreign.

Export demand

- For the marginal exporter,
$$\pi_x(\varphi_x^*) = 0.$$
- But we can actually use *export demand* to express its profits:
$$(1-\rho)A\rho^{\theta-1}\tau^{-\theta}\varphi_x^{*\,\theta-1} = f_x.$$
- This pins down the cutoff as
$$\varphi_x^* = \left(\frac{f_x}{(1-\rho)A}\right)^{1/(\theta-1)}\frac{\tau}{\rho}.$$
  - This is increasing τ,
  - decreasing in A.

Domestic cutoff

- The domestic cutoff is then
  - decreasing τ,
  - increasing in A.

Labor demand

- Because *in expectation* firms make zero profit,

$$L = R_d + R_x.$$

- Total revenues are average times number of firms:

$$L = R_d + R_x = M r_d(\tilde{\varphi}) + M_x r_x(\tilde{\varphi}_x)$$

- Conditional on the total number of firms, the number of exporters:

$$\frac{M_x}{M} = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}.$$

- The labor market clearing then pins down M.

The three effects

In the event of a trade liberalization (falling trade costs),

1. more firms will export (”extensive margin”),
2. labor is reallocated from less productive non-exporters to more productive exporters,
3. and least productive non-exporters will exit.

The three effects

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1. more firms will export ("extensive margin"),
2. labor is reallocated from less productive non-exporters to more productive exporters,
3. and least productive non-exporters will exit.
   We conduct comparative statics wrt τ.

Extensive margin

- The fraction of exporters M_x/M is decreasing in the export cutoff.
- Recall that export cutoff,

$$\varphi_x^* = \left(\frac{f_x}{(1-\rho)A}\right)^{1/(\theta-1)} \frac{\tau}{\rho},$$

  is increasing in τ.
- Hence M_x/M increases with trade liberalization.

Labor reallocation

- A constant ρ fraction of revenue goes to labor:

$$l(\varphi) = \rho r(\varphi).$$

- Relative employment for an exporter and a non-exporter:

$$\frac{l(\varphi|\varphi > \varphi_x^*)}{l(\varphi'|\varphi' < \varphi_x^*)} = \frac{r_d(\varphi) + r_x(\varphi)}{r_d(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\theta-1} \frac{A\rho^{\theta-1}\tau^{-\theta} + \frac{R_d}{M}\tilde{\varphi}^{1-\theta}}{\frac{R_d}{M}\tilde{\varphi}^{1-\theta}}$$

- This is clearly *decreasing* in τ.
- After trade liberalization, exporters expand relative to non-exporters.

Exit

- As discussed before, export profit opportunities generate more entry.
- New entrants squeeze out less productive firms.