Econometrics 2 (Part 1)

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Regression Discontinuity

Consider a potential outcome model

$$Y_i = (1 - D_i)Y_{0i} + D_i Y_{1i}$$

where in addition to (Y_i, D_i) we observe a variable z_i which defines treatment assignment by a cutoff z_0.

- Cutoff in vote share decides election outcome
- Minimum test score to pass a test
- Minimum number of days employed to be eligible for unemployment benefits

Idea: use the discontinuity created by this rule to identify the treatment effect

2 types of designs

1. Sharp Regression Discontinuity
D is a deterministic function of z

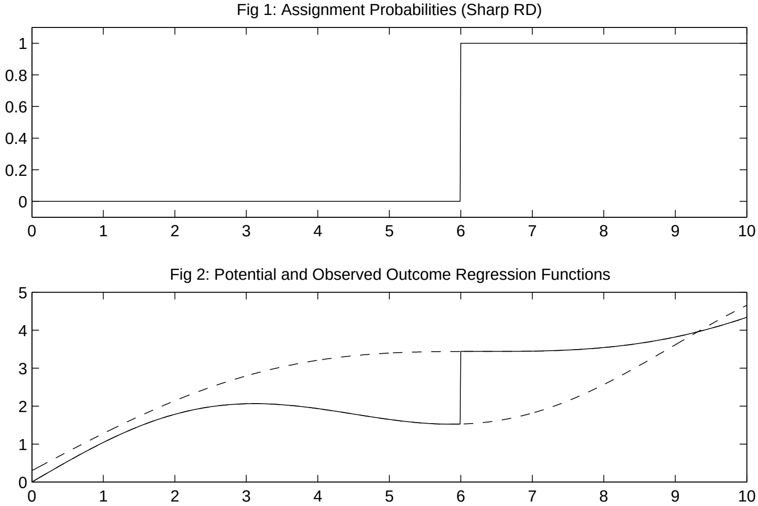
$$D_i = \begin{cases} 1 & z_i \geq z_0 \\ 0 & z_i < z_0 \end{cases}$$

2. Fuzzy Regression Discontinuity
Probability of receiving treatment does not change from 0 to one at the cutoff, but

$$\lim_{z \uparrow z_0} P(D_i = 1 | z_i = z) \neq \lim_{z \downarrow z_0} P(D_i = 1 | z_i = z)$$

other variables also determine treatment assignment, but incentives to participate change discontinuously at the threshold.

Sharp regression discontinuity



Fuzzy design

Fig 3: Assignment Probabilities (Fuzzy RD)

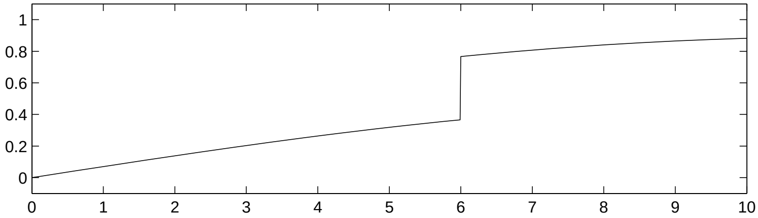
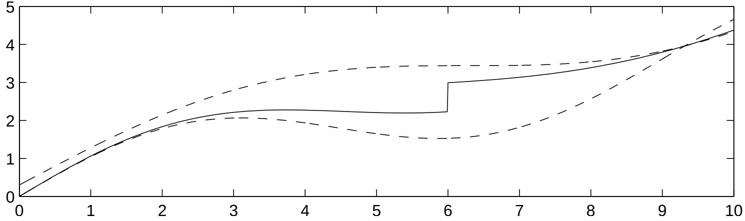


Fig 4: Potential and Observed Outcome Regression (Fuzzy RD)



Notes

Selection on observables:

$$(Y_{1i}, Y_{0i}) \perp D_i | z_i$$

holds trivially. In the sharp design there is no variation in D_i conditional on z_i.

But there is **zero overlap** with respect to z.

Identification Assumptions

RD.1 No difference in the outcome for those close to the cutoff in the absence of treatment. $E(Y_{0i}|z_i = z)$ is continuous at z_0, $E(Y_{1i}|z_i = z)$ is continuous at z_0.

RD.2 No sorting based on anticipated gains from treatment near the cutoff $E(x_i|z_i = z)$ is continuous at z_0.

Identification of the treatment effect

sharp design

$$\hat{\rho}_s = \lim_{z \downarrow z_0} E(Y_i | z_i = z) - \lim_{z \uparrow z_0} E(Y_i | z_i = z)$$

fuzzy design

$$\hat{\rho}_f = \frac{\lim_{z \downarrow z_0} E(Y_i | z_i = z) - \lim_{z \uparrow z_0} E(Y_i | z_i = z)}{\lim_{z \downarrow z_0} E(D_i | z_i = z) - \lim_{z \uparrow z_0} E(D_i | z_i = z)}$$

Notes:

- We identify a *local* treatment effect around the discontinuity
- Limited external validity, but strong internal validity of the result.
- The fuzzy RD estimator can be interpreted as an IV estimator (compare to Wald estimator)

Estimation of the treatment effect

1. **Graphical analysis**: Important part of any RD analysis!
   - Group the data in bins with similar values of z_i to the left and right of $z = z_0$.
   - Plot mean *outcomes* Y_i in each bin against z.
   - Is there a jump in Y at $z = z_0$?
   - Are there jumps in mean outcomes at other values of $z \neq z_0$?
   - First stage: Plot mean values of D_i in each bin against z, sharp or fuzzy design?
   - Testable assumptions: Plot mean values of *covariates* X_i in each bin.
   - Distribution of z: Plot the number of observations in each bin.

Estimation of the treatment effect

2. Regression analysis Local linear regression at the boundary, only use observations with values of z close to the discontinuity

$$Y_i = \alpha + \rho D_i + \boldsymbol{\beta}\mathbf{z}_i + \varepsilon_i$$

Include more observations and control for flexible functions in z to the left and right of $z = Z_0$

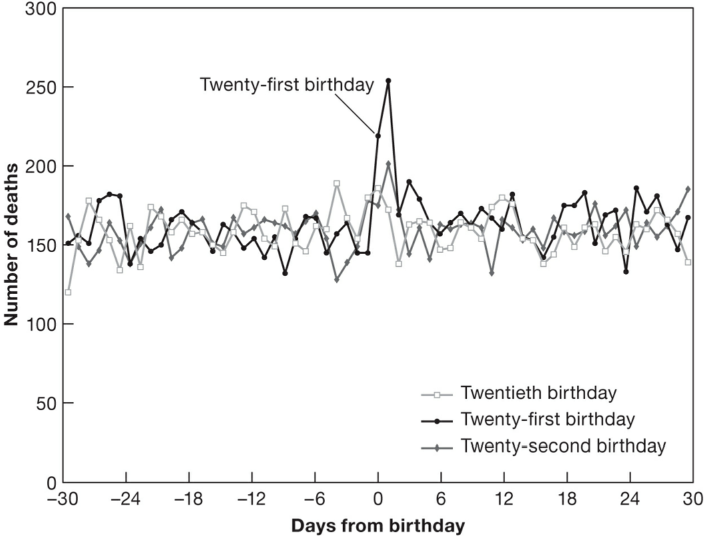
$$Y_i = \alpha + \rho D_i + f_1(z_i - z_0) + D_i * f_2(z_i - z_0) + \boldsymbol{\beta}\mathbf{x}_i + \varepsilon_i$$

- f_1 and f_2 are polynomials in $(z - z_0)$
- δ measures the jump in y at $z = z_0$

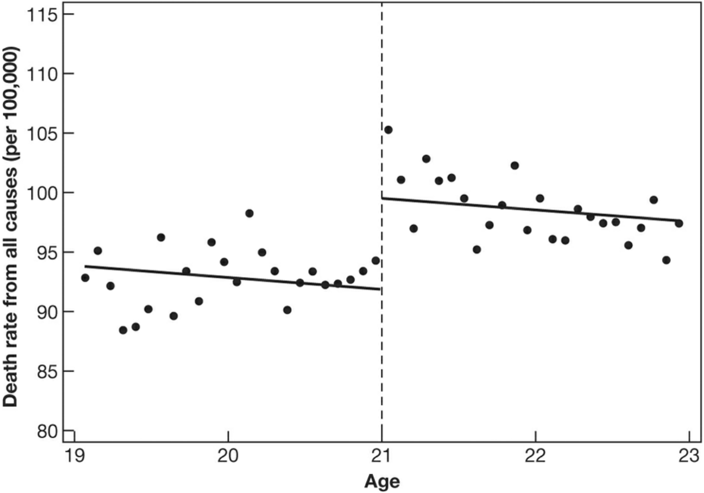
Applications: Sharp Regression Discontinuity

- Americans over 21 can drink legally
- American college presidents have lobbied to return the Minimum Legal Drinking Age (MLDA) at 18
- Theory (Amethyst Initiative): legal drinking at 18 avoids binge drinking contrasting the view that the age-21 MLDA reduces youth access to alcohol, preventing some harm

Birthdays and funerals



Minimum Drinking Age and Mortality Effects



Minimum Drinking Age and Mortality Effects

D_a is a deterministic function of *age*. Age is the running variable. MLDA is a sharp function of age

$$D_a = \begin{cases} 1 & a \geq 21 \\ 0 & a < 21 \end{cases}$$

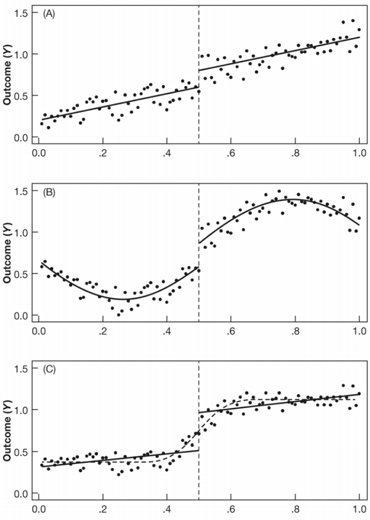
- Treatment status is a deterministic function of a. Once we know a, we know D_a
- Treatment status is a discontinuous function of a, because no matter how close a gets to the cutoff, D_a remains unchanged until the cutoff is reached.

Estimation of the treatment effect

$$M_a = \alpha + \rho D_a + \gamma a + \varepsilon_a$$

- where M_a is mortality in month a defined as 30 day interval
- RD tools aren't guaranteed to produce reliable causal estimates.
  - Get as close to the cutoff as possible
  - Model nonlinearities directly as polynomial functions of the running variable on both sides of the discontinuity (no need to be the same degree of polynomial on both sides)

Three cases



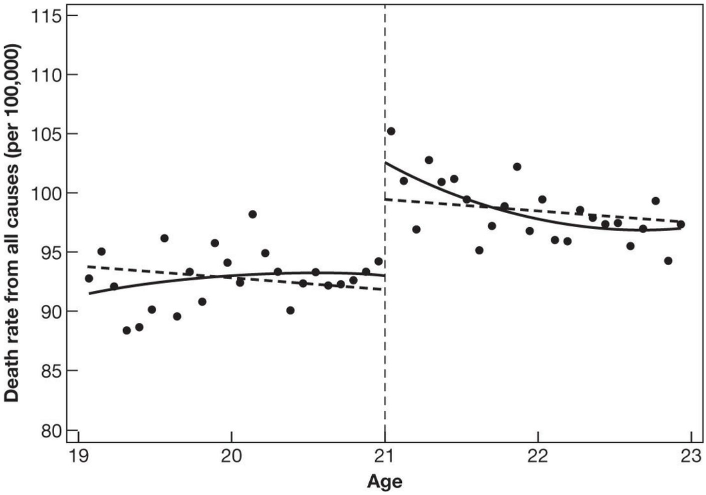
z_i

What polynomial degree?

- Gelman and Imbens (2018) reccomend polynomials up to a quadratic to avoid the problem of overfitting. Usually, researchers report estimates with various degrees.
- To make the model with interactions easier to interpret, we center the running variable by subtracting the cutoff

$$\begin{aligned} M_a = \quad & \alpha + \rho D_a + \gamma_1 (a - 21) + \gamma_2 (a - 21)^2 + \\ & \delta_1 [D_a (a - 21)] + \delta_2 [D_a (a - 21)^2] + \varepsilon_a \end{aligned}$$

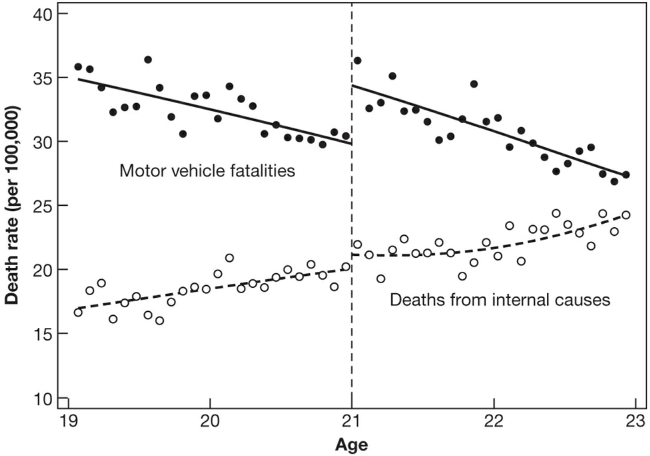
Quadratic control in an RD design



Covariate Balance

Dependent variable	Ages 19–22		Ages 20–21	
	(1)	(2)	(3)	(4)
All deaths	7.66 (1.51)	9.55 (1.83)	9.75 (2.06)	9.61 (2.29)
Motor vehicle accidents	4.53 (.72)	4.66 (1.09)	4.76 (1.08)	5.89 (1.33)
Suicide	1.79 (.50)	1.81 (.78)	1.72 (.73)	1.30 (1.14)
Homicide	.10 (.45)	.20 (.50)	.16 (.59)	−.45 (.93)
Other external causes	.84 (.42)	1.80 (.56)	1.41 (.59)	1.63 (.75)
All internal causes	.39 (.54)	1.07 (.80)	1.69 (.74)	1.25 (1.01)
Alcohol-related causes	.44 (.21)	.80 (.32)	.74 (.33)	1.03 (.41)
Controls	age	age, age^2, interacted with over-21	age	age, age^2, interacted with over-21
Sample size	48	48	24	24

RD estimates of MLDA effects on mortality by cause of death



How close to the boundary?

- As close as possible
- Drawback: few observations left. This causes the resulting estimates are likely to be too imprecise to be useful.
- Solution: Non-parametric RD
- To make the model with interactions easier to interpret, we center the running variable by subtracting the cutoff

$$M_a = \quad \alpha + \rho D_a + \gamma a + \varepsilon_a$$

- in a sample such that: $a_0 - b \leq a \leq a_0 + b$
- b is the bandwidth which is a function of sampling size
- Theoretical econometricians have proposed strategies for making such bias-variance trade-offs efficiently. The bandwidth selection algorithm is not completely data-dependent and requires researchers to choose certain parameters.

Local linear nonparametric regression

- Kernel regression as a weighted regression restricted to a window (bandwidth)
- The kernel provides the weights to that regression. A rectangular kernel would give the same result as taking $E[Y]$ at a given bin on z. The triangular kernel gives more importance to the observations closest to the center.

$$(\hat{\alpha}, \hat{\rho}) = \underset{\alpha,\rho}{argmin} \sum_{i}^{n} (Y_i - \alpha - \rho(z_i - z_0))^2 K(\frac{z_i - z_0}{h}) \mathbb{1}(z_i > z_0)$$

RD recommendation

- Hahn, Jinyong, Petra Todd, and Wilbert Van der Klaauw. "Identification and estimation of treatment effects with a regression-discontinuity design." Econometrica 69.1 (2001): 201-209.
- Imbens, Guido W. and Thomas Lemieux (2008) "Regression Discontinuity Designs: A Guide to Practice" Journal of Econometrics, 142, 615-635.
- Gelman, Andrew, and Guido Imbens. "Why high-order polynomials should not be used in regression discontinuity designs." Journal of Business & Economic Statistics (2018): 1-10.