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The Determinants of Long-Run Inequality

by

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Abstract

I explore the effect of skill-biased technological change on long-run inequality using a theoretical model where the supply of skilled and unskilled workers, the cost of education, and credit rationing are endogenous. I show that the existence of unequal steady states does not depend on the degree of technological skill bias, but on the credit market, the cost of education, altruism, and the overall growth rate of the economy. However, when unequal steady states exist, economies with a higher technological skill bias have a greater long-run inequality. Therefore, skill-bias technological change is a second-order determinant of long-run inequality: a higher technological skill bias is associated with greater long-run inequality only if long-run inequality exists; the existence of long-run inequality does not depend on skill bias.

Keywords: Endogenous Inequality, Skill Bias, Credit Rationing, Education, Growth.

JEL Numbers: J24, J62, O11, O16, O33.

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1 Introduction

The evolution of the wage structure in the United States between the end of the 1970s and the beginning of the 1990s suggests that technology can increase short-run inequality. During this period, the difference between the average wage of workers with a college degree and of workers with a high school degree increased significantly. The reason was the introduction of the personal computer and the unfolding of the information technology era. This wave of innovations was *skill biased*: it increased the productivity of skilled workers (workers with a college degree), leaving unchanged the productivity of unskilled workers.[1]

However, the long-run impact of technology on inequality is not well understood. In the long-run, the supply of skilled workers may react to variations in the skill premium. The possible reasons are:

- Parents may be willing to spend more on the education of their children when the return on education is higher.
- Student loans have an implicit collateral: the student's future wage. When the wage is higher, more people should be able to access the credit market and finance their education.

In addition, the short-run cost of education is fixed, but the long-run college tuition is likely to be correlated with the skilled wage, since college professors are skilled workers.

In this paper, I explore the effect of skill-biased technological change on long-run inequality by building an overlapping generation model where the supply of different types of skills is endogenous and may respond to variation in the skill bias. The key elements are:

- Altruistic parents who leave bequests to their children.
- An imperfect credit market where young people can raise resources to finance their education.
- An endogenous cost of education.

Note that each of these elements introduces a new potential source of inequality: altruism, credit market imperfections, and the cost of education. These elements can affect the supply of skills both directly and interacted with skill bias. One of the goals of the paper is to understand the interaction between these elements and skill-bias technological change in the evolution of long-run inequality. Finally, the model abstracts away from other determinants of inequality such as ability.

[1] For empirical evidence, see, among many others, Juhn, Murphy, and Pierce (1993), Autor, Katz, and Kearney (2005), and Heckman, Lochner, and Taber (1998).

I show that technological skill bias cannot, by itself, generate long-run inequality. Inequality exists only if some form of non convexities in the investment opportunities exist, so that only agents with wealth above a certain threshold can become skilled workers. The origin of non convexities is in the credit market, in the educational sector, in the degree of altruism and in the overall growth rate of the economy. Skill-biased technological change plays no role in the existence of non convexities and, therefore, in the existence of inequality.

At the same time, I show that, if non convexities exist, skill-bias technology does increase inequality. I do so by focusing on economies in an unequal steady states. I track the convergence of these economies to their new steady state after an arbitrarily small change in skill bias. In order to make a meaningful comparison between economies before and after the technological shock, I build an appropriate measure of long-run inequality that, following Atkinson (1970), depends on the entire distribution of steady-state wealth. I show that after an arbitrarily small increase in skill bias, the long-run skill premium and the long-run inequality both increase. Therefore, skill-bias technology matters, but it is a second order determinant of long-run inequality.

The intuition behind the results is easily captured by looking at two extreme cases. Consider an economy with no credit market. Since agents cannot borrow, the supply of skills will be constrained and the distribution of wealth will be unequal. After an increase in the skill bias there will be no increase in the supply of skilled workers, since the absence of a credit market prevents agents from borrowing. Therefore, any short-run increase in inequality due to skill-biased technological change will be permanent. On the other hand, consider an economy with a perfectly functioning credit market, where the distribution of wealth is equal. Since agents can freely borrow, any increase in the return to schooling will be matched by an increase in the supply of skills. In this case, skill-biased technological change has no long-run impact on inequality.

It follows that the impact of technology on inequality may be permanent: today's inequality may depend on the introduction of the personal computer 30 years ago. In addition, the model suggests an empirical connection between initial inequality and the increase in inequality due to the introduction of skill-biased technology. After a skill-bias technological shock, economies that are already unequal (because of institutional factors like the credit market), should display a low increase in the supply of skill and a permanent increase in inequality. On the other hand, economies that are more equal should experience a greater increase in the supply of skills and no increase in long-run inequality.

Literature

The research on endogenous economic inequality can be roughly divided into two parts. The first one originated with Kuznets (1955), who argued that, when the supply of different types of workers does not adjust immediately, technological shocks can increase short-run inequality by increasing the demand for skills.[2] The second one originated with Banerjee and Newman (1993) and Galor and Zeira (1993). These authors showed that credit market imperfections may constrain the supply of skills and generate long-run inequality. [3]

A recent literature combines these two approaches in order to explain the evolution of income inequality. Piketty (2006) nicely explains why it is important to consider both variations in the supply and in the demand for skills:

> [...] the impact of technology on inequality depends on a large number of institutions, and these institutions vary a great deal over time and across countries. Chief among these are the institutions governing the supply and structure of skills, from formal schooling institutions to on-the-job training schemes. To a large extent, the dynamics of labor market inequality are determined by the race between the demand for skills and the supply of skills. New technologies tend to raise the demand for skills, but the impact on inequality depends as to whether the supply of skills is rising at a faster or lower rate. There is no general presumption that the race should go one way or the other.

Some empirical contributions - such as Goldin and Katz (2008), Heckman, Lochner, and Taber (1998) and Binelli (2009) - decomposed the evolution of the wage structure into variations in the demand for skills and in the supply of skills. With respect to these works, I show that the effect of skill-bias technological change on the supply of skills is, in the long run, too small compared to its impact on the demand for skills. Therefore, the model hints that technology cannot cause an increase in the supply of skilled workers large enough to drive down the skill premium. In fact, Binelli (2009) and Heckman et al. (1998) find that the supply of skills is driven by, respectively, changes in the credit market and on-the-job training.

The only other paper exploring the long-run impact of skill-bias technological change on inequality is Rigolini (2004), who develops an OLG model where the cost of education is endogenous. Rigolini proves that an increase in the *overall* growth rate of the economy may increase or decrease the incentive for unskilled workers to acquire education, measured as the utility difference between an unskilled agent who remains unskilled and an unskilled agent

[2] See also Greenwood and Jovanovic (1990), Townsend and Ueda (2006), and Galor and Moav (2000a).

[3] In this line of research see also Piketty (1997a); Matsuyama (2000); Mookherjee and Ray (2003); Rigolini (2004); Mookherjee and Napel (2007); Mookherjee and Ray (2010); Mookherjee, Prina, and Ray (2010).

who decides to become skilled. He then speculates that the same result should apply with respect to skill-biased technological change. On the contrary, I show that the impact of an increase in skill bias does not depend on the parameters of the utility function. Skilled-bias technological change will always increase inequality if the economy is already in an unequal steady state. A second point of departure from Rigolini is that I define inequality in a rigorous way by building a measure that, following Atkinson (1970), depends on the entire distribution of steady-state wealth. Finally, in my model the existence of credit rationing is determined endogenously, and agents can accumulate assets and leave financial bequests. These elements are absent in Rigolini's model, but are key here because they allow the comparison between different sources of inequality.

The model that is closer to mine is Mookherjee and Ray (2010). Like in their model, I assume that agents care about their offspring's wealth (paternalistic altruism), that the return on education is endogenous, and that parents leave both financial and educational bequests. However, Mookherjee and Ray (2010) focus on the existence of long-run inequality and multiple steady states when the number of occupation in the economy increases. They show that, as the number of occupation approaches a continuum, the steady state becomes unique, but inequality may not disappear. Because of its different goal, my paper departs from Mookherjee and Ray (2010) in a number of ways. Most important, in my model the cost of education and the existence of credit rationing evolve endogenously, while these two elements are exogenous in their model.

Finally, following Galor and Zeira (1993), several authors developed models where economic inequality is due to heterogeneity in ex-ante ability, in order to explain the observed dynamics of wage inequality. Among these works, Guvenen and Kuruscu (2007) establish that skill-biased technological change leads to higher long-run inequality. However, these authors define 'long run' as the lifetime earnings of an agent living for a finite number of periods. Here instead I focus on the intergenerational transmission of inequality to distant generations.

In the next section I illustrate the model. In the third section I derive the steady state of the economy. I build a measure of inequality in the fourth section. I derive the dynamics and compare steady states before and after a skill-biased technological shock in the fifth section. Finally, in the last section I conclude with a brief summary of my main results.

2 The Model

A small open economy is composed of a measure one of agents, all identical but starting their lives with different levels of wealth.

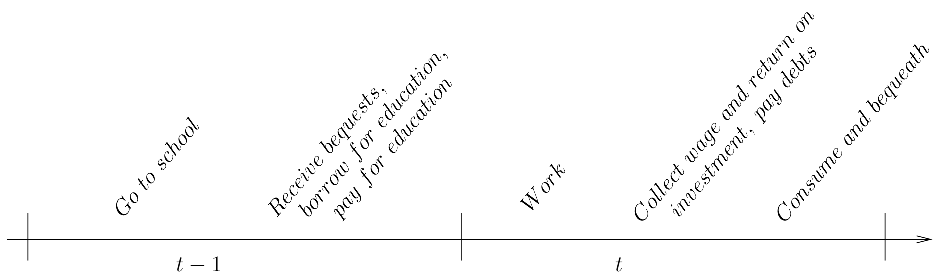

Fig. 1: Timeline

2.1 The Households.

Each individual is alive for two periods. During the first one, she receives a bequest from her parent and decides whether to get an education or not. If she chooses to go to school, she'll be a skilled type; otherwise she'll be an unskilled type. In the second period she works, earns a wage, consumes and bequeaths to her only child (see figure 1). The end-of-life utility of an agent active at time t depends on her own consumption as well as on the wealth of her child.[4] Define w_t^s and w_t^u as the wage of a skilled and of an unskilled worker active in period t. Define e_t^i as the bequest made by the member of household i active in period t (agent $\{i, t-1\}$) to the one born in period t (agent $\{i, t\}$, active in period $t+1$)). Define the total wealth of agent $\{i, t\}$ as the sum of the resources available to her in period $t+1$, before consuming and bequeathing:

$$m_{t+1}^i = \begin{cases} (e_t^i - \xi_t)(1+r) + w_{t+1}^s & \text{if } \{i,t\} \text{ is high skilled} \\ e_t^i(1+r) + w_{t+1}^u & \text{if } \{i,t\} \text{ is low skilled} \end{cases}$$

where ξ_t is the cost of education in period t. It follows that a parent's utility is given by:

$$U_t^i = u(c_t^i) + \beta v(m_{t+1}^i) \tag{1}$$

I assume that both functions $u(.)$ and $v(.)$ are identical CES:

$$U_t^i = \frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(m_{t+1}^i\right)^{1-\sigma}}{1-\sigma}$$

Children can become unskilled workers for free, but becoming a skilled worker is costly. In each period some skilled workers will become teachers, and each teacher can teach to $\frac{1}{\lambda}$ number of students, where λ measures the efficiency of the educational technology. Since

[4] This form of altruism is called *paternalistic altruism.* It implies that parents care about their direct offspring but not about distant generations. For details, see Mookherjee and Ray (2010). In section 6 I discuss how the results of the model would change under different forms of altruism.

teachers are skilled workers, they must receive the skilled wage so that the cost of education is given by:

$$\xi_t = \lambda w^s_{t-1}$$

where w^s_{t-1} is the wage of a skilled worker born in period $t-1$ earn when adult in period t.

In order to finance their education, people can borrow on the capital market using the bequest received and their future wage as collateral. If a young individual goes to school without borrowing, or if she borrows and repays her loan, her budget constraint is:

$$c^i_t + e^i_{t+1} = w^s_t + \left(e^i_t - \lambda w^s_{t-1}\right)(1+r) \tag{2}$$

However, agents may choose not repay their loans. In this case, they will lose only a fraction $1-\theta$ of their wealth when old: there is limited liability. Thus, if an agent borrows for school and does not repay, her budget constraint is:

$$c^i_t + e^i_{t+1} = \theta w^s_t \tag{3}$$

If she does not go to school, her budget constraint is:

$$c^i_t + e^i_{t+1} = w^u_t + e^i_t(1+r) \tag{4}$$

Given this, banks will lend to agents only if the RHS of 2 is greater than the RHS of 3: access to the credit market and to school is determined by the bequest received at the beginning of life. More precisely, people can become skilled if:

$$e_t \geq \lambda w^s_{t-1} - w^s_t \left(\frac{1-\theta}{1+r}\right) \tag{5}$$

Note that, although the parameter θ is taken as given, the existence of credit rationing in the economy is determined endogenously. An economy with very severe credit constraints in some periods could evolve toward a perfect credit market. Similarly, an economy with a perfect credit market could later on develop some imperfections. Since inequality and credit market imperfections are strictly interconnected, most of the analysis that follows will focus on the long-run evolution of equation 5.

Given the utility function and the budget constraint, it is possible to derive the first order conditions. In principle, there are several different cases, depending on the profession of the parent and on the profession of the child. However, the first order conditions that will be relevant in steady state are: skilled parents with skilled son, interior solution:

$$e_{t+1}^{i,s} = \frac{1}{\rho(1+r)+1}\left(\left(e_t^i - \lambda w_{t-1}^s\right)(1+r) + w_t^s\left(1+\rho(1+r)\lambda\right) - \rho w_{t+1}^s\right) \tag{6}$$

skilled parents with skilled son, corner solution:

$$e_{t+1}^{i,s} = \max\left\{\lambda w_{t-1}^s - w_{t+1}^s\left(\frac{1-\theta}{1+r}\right), 0\right\} \tag{7}$$

unskilled parent with unskilled son, interior solution:

$$e_{t+1}^{i,u} = \frac{1}{\rho(1+r)+1}\left(e_t^i(1+r) + w_t^u - \rho w_{t+1}^u\right) \tag{8}$$

and unskilled parent with unskilled son, corner solution:

$$e_{t+1}^{i,u} = 0 \tag{9}$$

where $\rho \equiv \left[\beta\left(1+r\right)\right]^{-\frac{1}{\sigma}}$.

Finally, I will show in the next section that, under fairly weak assumptions, both professions are always employed in production. This implies that becoming a skilled worker must always be at least as profitable as becoming an unskilled worker. In other words, the RHS of 2 is always greater or equal than the RHS of 4:

$$w_t^s - \lambda w_{t-1}^s(1+r) \geq w_t^u \tag{10}$$

2.2 The Production Function.

The final good is produced according to the following function:

$$Y_t = K_t^{\alpha}\left(a_t S_t^{\epsilon} + b_t U_t^{\epsilon}\right)^{\frac{1-\alpha}{\epsilon}} \tag{11}$$

where K is capital, S and U are skilled and unskilled workers employed in production, and a_t and b_t represent the productivity of the two types of workers. If both a_t and b_t change by the same amount, this translates into a change in the overall productivity on the economy. Instead, variations to $\frac{a_t}{b_t}$ reflect variations in the skill bias of the economy.

Markets are competitive and all inputs receive their marginal product:

$$r = \frac{\partial Y_t}{\partial k_t} = \alpha\left(\frac{\left(a_t S_t^{\epsilon} + b_t U_t^{\epsilon}\right)^{\frac{1}{\epsilon}}}{K_t}\right)^{1-\alpha} \tag{12}$$

$$w_t^s = \frac{\partial Y_t}{\partial S_t} = K_t^\alpha (1-\alpha)(a_t S_t^\epsilon + b_t U_t^\epsilon)^{\frac{1-\alpha}{\epsilon}-1} a_t S_t^{\epsilon-1} \tag{13}$$

$$w_t^u = \frac{\partial Y_t}{\partial U_t} = K_t^\alpha (1-\alpha)(a_t S_t^\epsilon + b_t U_t^\epsilon)^{\frac{1-\alpha}{\epsilon}-1} b_t U_t^{\epsilon-1} \tag{14}$$

I assume that $\epsilon < 1$. Under this condition the marginal product of labor at zero is infinity: no matter how high wages are, firms will always demand a strictly positive amount of each type of labor. Note that if $\epsilon \leq 0$ no production can occur unless all inputs are used. If instead $0 < \epsilon < 1$ production can, in principle, occur using only one type of workers. Finally, whenever $1-\alpha > \epsilon$ the two types of workers are complements, while if $1-\alpha < \epsilon$ they are substitutes.

Consistent with the literature, let's call the ratio of the two wages *skill premium*:

$$\frac{w_t^s}{w_t^u} = \frac{a_t}{b_t}\left(\frac{U_t}{S_t}\right)^{1-\epsilon} \tag{15}$$

2.3 Market Clearing Conditions.

I assume that the economy is small and that it can freely borrow and lend on the international capital market. Under these assumptions, the domestic market clearing interest rate is equal to the international one, and the capital market is always in equilibrium. In addition, by Walras' law I can ignore the consumption good's market. It follows that the economy is in equilibrium if the two labor markets and the education market clear.

The demand for skilled and unskilled workers is given by equations 13 and 14. The supply depends on whether the returns on the two professions are equal or not. If agents prefer to be skilled, the supply of skilled workers is given by the number of workers with wealth satisfying condition 5. If instead they are indifferent, any agent with wealth satisfying condition 5 can be either skilled or unskilled.

Proposition 1. *Define $F_t(e)$ as the c.d.f of the wealth distribution across the generation active at period t, and call T_t the number of teachers available in period t. Whenever*

$$w_t^s - \lambda w_{t-1}^s(1+r) > w_t^u \tag{16}$$

labor market clears if

$$U_t = F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right) \tag{17}$$

$$U_t = 1 - S_t - T_t \tag{18}$$

and equations 13 and 14 hold. If instead

$$w_t^s - \lambda w_{t-1}^s(1+r) = w_t^u \tag{19}$$

The labor market clears if there exists a measure μ such that

$$U_t = F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right) + \mu \tag{20}$$

$$U_t = 1 - S_t - T_t$$

where $0 \leq \mu \leq 1 - F_t\left(\lambda w_{t-1}^s - \frac{w_t^s(1-\theta)}{(1+r)}\right)$ *and, again, 13 and 14 are satisfied.*

The existence of a competitive equilibrium is shown in the appendix. The proof is quite standard except for one point. In every period the economy starts with a given number of skilled and unskilled adults. However, the number of skilled agents employed as workers or as teachers is determined endogenously. The cost of education will increase when more teachers are required (since less skilled workers are employed in production) and decrease otherwise. Hence, when the demand for education increases, the skilled wage decreases and the cost of education increases as well.

3 Steady State

Let's assume that the productivity parameters a_t and b_t grow at the common constant rate g.

Definition 2. The economy is in a *steady state* if the number of each type of workers is constant, and aggregate output, aggregate capital, wages, and consumption grow at a constant rate.

Definition 3. A steady state is *equal* if the returns on the two profession are equal. A steady state is *unequal* if the two professions yield different returns.

Since a steady state must also be a competitive equilibrium, it is quite straightforward to see that in an unequal steady state the following properties must hold:

- The return on the skilled profession is higher than the return on the unskilled profession.
- Unskilled workers do not have access to credit (no-credit constraint).
- Either unskilled workers do not want to bequeath enough so that their children can become skilled (no-deviation constraint), or it is impossible for them to do so (no-negative-consumption constraint).

Lemma 4. *In an unequal steady state the descendant of a skilled worker will be skilled, and the descendant of an unskilled worker will be unskilled.*

In other words, in a steady state each child inherits the profession of his parent,[5] and the economy is populated by skilled households and unskilled households.

Lemma 5. *Assume that a_t and b_t grow at a constant rate g. In a steady state, consumption, bequests, wages, and capital grow at a gross rate:*

$$\gamma \equiv 1 + g$$

Using lemma 4 and lemma 5, I can rewrite the first order conditions 6, 7, 8, and 9 as:

$$e^s_{t+1,ss} = w^s_{t,ss} \max\left\{ \frac{(1-\rho\gamma)(\gamma - \lambda(1+r))}{\gamma - (1+r)(1-\rho\gamma)}, \lambda - \frac{\gamma(1-\theta)}{1+r}, 0 \right\} \tag{21}$$

and

$$e^u_{t+1,ss} = w^u_{t,ss} \max\left\{ \frac{\gamma(1-\rho\gamma)}{\gamma - (1+r)(1-\rho\gamma)}, 0 \right\} \tag{22}$$

In order to guarantee the existence of a steady state, I need to impose some restrictions on the parameters.

Assumption 6. *The net return on the skilled profession is positive:*

$$\gamma > \lambda(1+r) \tag{23}$$

Assumption 6 is equivalent to

$$\frac{w^s_{t+1,ss}}{1+r} > \lambda w^s_{t,ss}$$

where the RHS is the cost of education in a steady state, and the LHS is the skilled wage discounted by one period. This assumption implies that, in order for a steady state to exist, the cost of education shouldn't be too high.

Note that assumption 6 would not be necessary if the cost of education were fixed. In that case, when the skilled wage increases, so does the return on education and the supply of skilled workers. In this model however, the cost of education is endogenous and increases with the skilled wage. Assumption 23 guarantees that the overall effect of an increase in the skilled wage on the return on education is positive.[6]

[5] I assume, without loss of generality, that this is the case also when the two professions yield the same return.

[6] This assumption wasn't necessary when deriving the existence of a competitive equilibrium. The reason is that, in the short run, the cost of education is given (i.e. it depends on the previous period), so that when the skilled wage increases the supply of skilled workers increases.

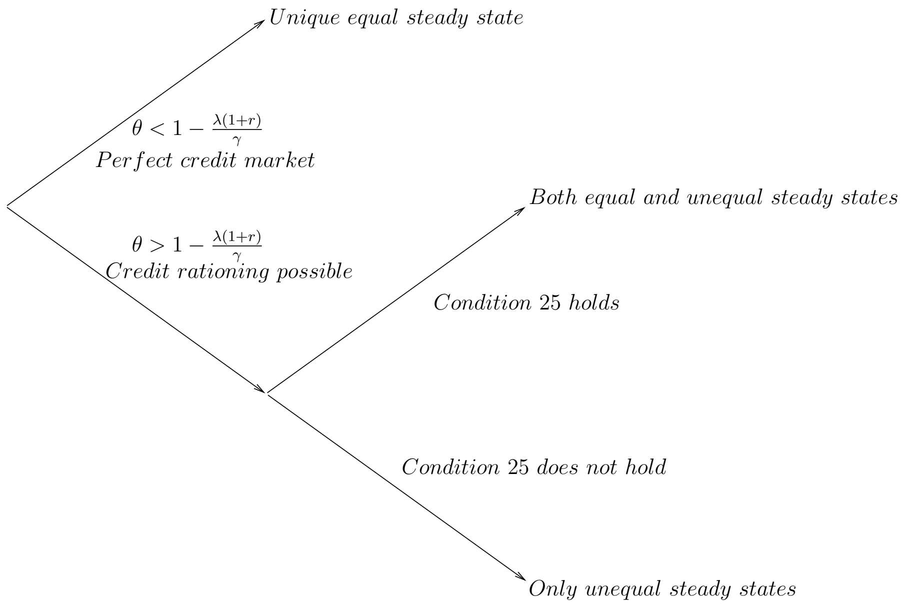

Fig. 2: Steady states.

Proposition 7. *Under assumption 6, if the credit market works well enough*

$$\theta < 1 - \frac{\lambda(1+r)}{\gamma}$$

the economy has a unique steady state that is equal.

When proposition 7 holds, in a steady state everybody is able to borrow and pay for education, even if they have zero wealth. It follows that the only possible steady state is the equal one.

Proposition 8. *Assume the that credit market is imperfect:*

$$\theta > 1 - \frac{\lambda(1+r)}{\gamma}$$

Under assumption 6, if

$$\frac{1+r-\gamma}{1+r} < \gamma\rho < 1 \tag{24}$$

then the economy has a continuum of unequal steady states and one equal steady state. Otherwise, the economy has a continuum of unequal steady states, but no equal steady state.

The proof of proposition 8 shows that, when credit market is imperfect, there are always

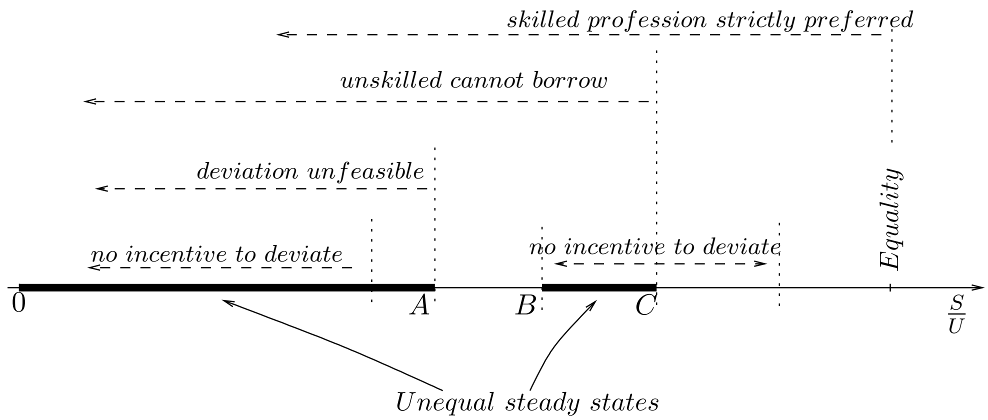

Fig. 3: Example of Steady-State Set.

steady states where unskilled agents are too poor to borrow and too poor to leave bequests high enough to allow their children to access education.

Propositions 8 and 7 deliver the first result of the paper. They show that skill-biased technology has nothing to do with the existence of long-run inequality. Quite intuitively, unequal steady states exist if:

- The credit market functions poorly (high θ).
- The growth rate of the economy γ is low.
- The interest rate r is high.
- The cost of education λ is high.

Therefore, any impact that skill-biased technology may have on inequality is of secondary order: it is relevant only if the other parameters are such that there is inequality in the first place.

Condition 24 determines whether in the presence of credit market imperfection there can be an equal steady state. When it holds, the agents' optimization problem has an interior solution, in the sense that the optimal bequest level is computed using the first order conditions. In other words, unskilled parents leave positive bequests and skilled parents leave bequests strictly greater than the one necessary for their children to access education. Using Mookherjee and Ray (2010) framework, one could say that parents leave both an *educational bequest* and a *financial bequest*: they leave the money required to access a profession plus some assets. This implies that, if the returns on the two professions are equal, both types of workers bequeath the same amount, and equality is possible in steady state. If instead condition 24 does not hold, the agents' optimization problem has a corner solution where skilled workers bequeath just enough so that their children can go to school, and unskilled workers do not leave any bequest (again, one could say that parents leave only educational bequests: only the money required to access a given profession). In this case there cannot be an equal steady state, since a skilled parent would be better off by switching to zero bequest, increasing their own consumption, and leaving his child income unchanged (but forcing him to be unskilled).[7]

Lemma 9. *The set of steady state skill premia does not depend on* $\frac{a}{b}$.

The proof of lemma 9 shows that each of the constraints characterizing an unequal steady state defines a set of skill premia. In other words, whether education is desirable, whether it is possible to send children to school, or whether it is possible to access credit depends on the skill premium. Since technology affects the constraints only through the skill premium, the steady-state skill premia are independent on the skill bias. Figure 3 depicts one possible set of unequal steady states. When skill bias increases, the points A, B, and C shift to the right and the distance between A and B, and B and C expand, so as to keep the skill premium at the three points constant. Similarly, when skill bias decreases, the set of unequal steady states shrinks. If inequality exists, changing $\frac{a}{b}$ will change the shape of the unequal steady-states set, but the unequal steady-states set will continue to exist.

Proposition 9 questions one of Rigolini's claims. In a model with a continuum of occupations, he shows that the only unequal steady state is the one where parents are indifferent between educating their children or not (as in point B of figure 3). He shows that, after an increase in the overall growth rate of the economy, the steady state skill premium may increase or decrease, depending on the parameters of the utility function. The reason is that the return on education is higher: whether the demand for education increases or not depends on the strength of the income and substitution effects. He then claims that the same holds true for a skill-biased technological change.

Here, Rigolini's claim does not hold because skill-biased technological change has an additional effect on the wealth of unskilled workers. Unskilled parents do not want to leave a bequest that is high enough to allow their children to go to school if

$$\begin{aligned} U(e^u_{t,ss}(1+r)+w^u_{t,ss}-e^u_{t+1,ss})+\beta U(e^u_{t+1,ss}(1+r)+w^u_{t+1,ss}) \quad &\geq \\ U(e^u_{t,ss}(1+r)+w^u_{t,ss}-e^\star_{t+1,ss})+\beta U(w^s_{t+1,ss})& \end{aligned}$$

[7] One of the nice technical features of the model is that, in steady state, whether there is a corner solution or not is independent on the wage level. This also implies that either both types of workers face a corner solution or neither of them.

where

$$e_{t+1}^{\star} = w_{t+1,ss}^{s} \left(\lambda - \frac{(1-\theta)\gamma}{1+r} \right)$$

is the amount of wealth a young agent needs to access education. The above expression can be rewritten as

$$T \geq \frac{1}{1-\sigma} \left[\left(R(1+r) + 1 - \frac{w^s \gamma}{w^u} O \right)^{1-\sigma} + \beta \left(\frac{\gamma w^s}{w^u} \right)^{1-\sigma} \right] \tag{25}$$

where

$$R = \max \left\{ \frac{\gamma(1-\rho\gamma)}{\gamma - (1+r)(1-\rho\gamma)}, 0 \right\}$$

$$O = \lambda - \frac{(1-\theta)\gamma}{1+r}$$

$$T = \frac{1}{1-\sigma} \left[((1+r-\gamma)R+1)^{1-\sigma} + \beta \left((1+r)\gamma R + 1 \right)^{1-\sigma} \right]$$

Therefore, the constraint depends on skill bias only through the skill premium. Also, the RHS of inequality 25 is strictly concave in $\frac{w^s}{w^u}$: the constraint is satisfied for higher skill premium and lower skill premium. It follows that an increase in the skill premium decreases the incentive to acquire education for low $\frac{S}{U}$ and increases it for high $\frac{S}{U}$.

4 Inequality

Most of the papers dealing with skill-biased technological change use skill premium as a measure for inequality. However, the features of the model presented here make skill premium a bad proxy for long-run inequality. For example, the cost of education is endogenous: if skill premium goes up, but so does the cost of education, can we say that the economy is less equal? Skilled workers will earn more but will pay more for becoming skilled workers. They may not gain anything.

I build a measure of inequality based on Atkinson (1970). For a given social welfare function W, define x_W^{eq} as the wealth level that, if equally distributed across all agents, would achieve the current social welfare. Inequality can be measured as

$$\mu = 1 - \frac{x_W^{eq}}{E(x)} \tag{26}$$

where $E(x)$ is the average wealth. This measure is always between zero and one, and it's exactly zero when wealth is constant across agents. Furthermore, it is invariant with respect to shifts in the average wealth. The interpretation of this measure is as follows: if $\mu = 0.7$, by distributing wealth equally we could achieve the very same level of social welfare, saving

70% of the current average wealth.

Finally, I specify a particular social welfare function. I use a Rawlsian criterion:

$$W = \min_i U(x^i) \tag{27}$$

This way, inequality measures the amount of wealth relative to the average wealth a social planner can collect if he expropriates the surplus of everybody who owns more than the poorest agent in the economy.

Lemma 10. *In steady state, inequality is given by:*

$$\mu\left(\frac{U}{S}\right) = \begin{cases} \frac{(1+\lambda)\left[\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1-\frac{(1+r)\lambda}{\gamma}\right)-1\right]}{(1+\lambda)\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1-\frac{(1+r)\lambda}{\gamma}\right)+\frac{U}{S}} & \textit{if } \frac{1+r-\gamma}{1+r} < \gamma\rho < 1 \\ \frac{(1+\lambda)\left[\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1+\frac{(1+r)\lambda(\gamma-1)}{\gamma}-\gamma(1-\theta)\right)-1\right]}{(1+\lambda)\frac{a}{b}\left(\frac{U}{S}\right)^{1-\epsilon}\left(1+\frac{(1+r)\lambda(\gamma-1)}{\gamma}-\gamma(1-\theta)\right)+\frac{U}{S}} & \textit{otherwise} \end{cases} \tag{28}$$

Following Atkinson's intuition, the distribution of wealth can be seen as a lottery, and the measure of inequality can be seen as a normalized risk premium. Using the inequality measure to compare steady states having the same number of skilled and unskilled workers but with different wealth distributions is equivalent to comparing lotteries with two possible outcomes, where the probability of each outcome stays constant but the particular realization varies. Since the normalized risk premium and the inequality measure are invariant to shifts in the mean of the distribution, this is simply a mean-preserving spread. Whereas any risk averse agent has the same preference ranking over all the mean preserving spreads of a given distribution, any concave social welfare function delivers the same ranking of steady states in terms of inequality. Hence, the following lemma.

Lemma 11. *In comparing inequality across economies with the same $\frac{S}{U}$, the use of a Rawlsian social welfare function is without loss of generality.*

Note also that, for given $\frac{S}{U}$, inequality is a strictly increasing function of the difference between the wealth of a skilled workers and the wealth of an unskilled worker, expressed as fraction of total wealth. This implies that, fixing $\frac{S}{U}$, if inequality increases, the resulting Lorentz curve will be nowhere above the original one. Hence the following lemma.

Lemma 12. *Consider two economies in steady state having the same $\frac{S}{U}$ but different wealth distributions. The following statements are equivalent:*

- *Inequality is greater in economy 1 than in economy 2.*
- *The Lorenz curve of economy 1 is nowhere above the Lorentz curve of economy 2.*

- *The Gini coefficient of economy 1 is greater than the Gini coefficient of economy 2.*[8]

However, when using the inequality measure to compare steady states with different skilled-to-unskilled ratios, the results will be specific to the Rawlsian criterion and may not generalize to other social welfare functions.

5 Stability

In models with a continuum of steady states, it is usually very difficult to determine what initial conditions lead to a particular steady state. Luckily, here it is possible to derive the dynamics of the economy for some specific initial conditions.

Proposition 13. *Consider an economy in a steady state having a skilled-to-unskilled ratio* p*. If* p *is in the interior of the set of unequal steady states, after a permanent shock the economy will converge again to* p*.*

A permanent shock is, by definition, an arbitrarily small variation to one of the parameters of the economy. If the variation is very small, and p is in the interior, p remains a steady state. The intuition behind proposition 13 is that, after a shock, unskilled workers remain credit constrained. It follows that the number of skilled and unskilled workers remains constant along the transition to the new steady state. Therefore, the economy can only converge back to p. Let's now look at one specific type of shock: a variation in the skill bias.

Lemma 14. *Consider an economy in an unequal steady state. Assume that its skilled-to-unskilled ratio is in the interior of the set of unequal steady states. After an arbitrarily small increase in skill bias, this economy converges to a steady state where skill premium and inequality are higher than before. Similarly, after an arbitrary decrease in skill bias, this economy converges to a steady state where skill premium and inequality are lower.*

Intuitively, keeping $\frac{S}{U}$ fixed, inequality increases when skill bias (and, therefore, skill premium) increases. The steady state wealth is a linear function of the wages of skilled and unskilled workers, and skill bias matters only through the wages. Relative wealth depends on skill bias only through the skill premium, exactly like in the short run case.

Therefore, for the steady states in the interior of the steady state set, skill bias and inequality move together. Note also that this result does not depend on the particular social welfare function chosen, since, as discussed on page 16, we are comparing inequality levels across economies having the same $\frac{S}{U}$.

[8] This, again, proves that using a Rawlsian criterion is WLOG. See Atkinson (1970), p. 247.

What happen after after a technological shock in economies at the boundary of the steady state set, such as point A, B and C in figure 3? Clearly, if either A, B or C remain a steady state, the lemma just discussed applies and the economy will converge back to the same steady state, maintaining the skilled-to-unskilled ratio constant during the transition. If instead the original steady state is not a steady state anymore, the economy will transition to a different steady state skilled-to-unskilled ratio. The following lemma provides the answer for the case of an increase in skill bias:

Proposition 15. *Consider economies at the boundary of the steady state set. If, after an increase of skill bias, the original steady state is not a steady state anymore, these economies will converge to a new steady state having the same skill premium.*

Given the previous discussion, the only case still to be addressed in the proof corresponds to point B in figure 3 (if it exists). The proof of the lemma is based on the fact that, before the shock, at point B unskilled parents are indifferent between sending their children to school or not. After the shock, all unskilled parents strictly prefer to purchase education (both cost and benefit of education increase, but the overall effect is positive). The influx of students will decrease the benefit of education, and increase the cost of education (since some skilled adult will have to leave production and become teachers) until the point when parents are again indifferent. Hence, the new steady state will have the same starting skill premium (so to keep unskilled parents indifferent) but more skilled workers. This implies that, also in this case, long-run inequality increases following an increase in the skill bias. However, it is important to remember that the result depends on the specific social welfare function chosen. Since a Rawlsian criterion is a special case, even if the conjecture is correct this result is somehow fragile.

Given the fact that the economy has a continuum of unequal steady states, what happens at point B may seem almost irrelevant. However, in models with an exogenous cost of education by introducing a continuum of occupations (as in Mookherjee and Ray (2010)), or random ability shocks (as in Mookherjee and Napel (2007)), or different fertility rates (as in Mookherjee, Prina, and Ray (2010)), the surviving unequal steady states are the ones where unskilled agents are indifferent between purchasing education or not, exactly like at B. In light of these other works, it is important to fully understand the evolution of long-run inequality at steady states such as point B, using a general class of social welfare functions. Answering this question is left for future work.

6 Discussion

How robust are the results derived so far to other specifications of the model, such as other forms of altruism? The two most obvious variations that should be considered are:

- *dynastic altruism*, where parents care about the utility of their offspring. In this case, individual well being is affected by the consumption of all future generations.
- *warm-glow altruism*, where parents derive utility from the amount of bequests left. In this case the occupational choice of the direct offspring does not affect the parents' utility. The amount of bequests does not respond to variations in the return on education.

Proposition 7 - showing that when the credit market works sufficiently well there is no inequality, so that skill-biased technological change does not matter in the long run - is clearly robust to the two other types of altruism. The reason is that when credit markets are perfect, even agents with no wealth can access education, so it doesn't matter how bequests are determined (for a more formal treatment, see Loury (1981) and Galor and Zeira (1993)).

Proposition 14 - showing the convergence of unequal steady states to even more unequal steady states following an episode of skill-biased technological change - is also robust to this other form of inequality. The intuition for the proof is that, when unskilled workers are credit constrained and the change in the skill bias is arbitrary small, unskilled workers will remain credit constrained. The key point here is that the amount of bequests left by parents is a continuous function of technological skill bias, so that, at a small increase in skill bias corresponds a small increase in bequests, not enough to access the credit market. Any form of altruism that preserves this feature will deliver the same result.

7 Conclusion

There is convincing evidence showing that, in the short run, skill-biased technological change can increase inequality. However, very little is known about the long-run effect of the introduction of a skill-biased innovation. My paper aims at filling this gap.

I build an OLG model with several potential sources of inequality: skill-biased technology, imperfect credit market, exogenous productivity growth, altruism, and education technology. I show that long-run inequality exists if the credit market functions poorly, productivity growth is low, altruism is low, or the education technology is inefficient. Surprisingly, the existence of long-run inequality does not depend on the degree of skill bias of the economy.

I build a measure of inequality based on Atkinson (1970) and I show that, if inequality exists, after an increase in skill bias almost all the economies will converge to steady states with a higher skill premium and higher inequality. Therefore, if long-run inequality exists, short-run and long-run inequality move in the same direction. This result shows that the introduction of a skill biased technological innovation decades ago such as the computer may partly be responsible for today's inequality.

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A Proofs of section 2.

A.1 Existence of the competitive equilibrium.

Consider a given sequence of bequests distributions $\{F_t(e)\}_{t=1}^{\infty}$. At the beginning of period t, the state of the economy is given by the number of skilled and unskilled adults (note that these agents were born in period $t-1$). Note that:

$$w_{t-1}^{s}\left(S_{t-1},U_{t-1}\right)=a\chi\left(a+b\left(\frac{U_{t-1}}{S_{t-1}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}-1}$$

$$w_{t}^{u}\left(S_{t-1},U_{t-1}\right)=b\chi\left(a\left(\frac{S_{t-1}}{U_{t-1}}\right)^{\epsilon}+b\right)^{\frac{1}{\epsilon}-1}$$

where

$$\chi=\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

This implies that the cost of education in period t is determined in period t by allocating skilled agents between production and teaching.

In order to show that a competitive equilibrium exists, I need to show that there exists a$U_t^\star$ and $S_t^\star$ with

$$U_t^\star+S_t^\star=1-T_t^\star<1$$

$$S_t^\star=\frac{1}{\lambda}T_{t-1}^\star$$

$$T_{t-1}^\star\in[0,S_{t-1}]$$

such that either:

$$w_t^s\left(S_t^\star,U_t^\star\right)-\lambda(1+r)w_{t-1}^s\left(S_t^\star\right)\geq w_t^u\left(S_t^\star,U_t^\star\right)\tag{29}$$

$$F_t\left(\lambda w_{t-1}^s\left(S_t^\star\right)-\frac{1-\theta}{1+r}w_t^s\left(S_t^\star,U_t^\star\right)\right)=U_t^\star\tag{30}$$

or

$$w_t^s\left(S_t^\star,U_t^\star\right)-\lambda(1+r)w_{t-1}^s\left(S_t^\star\right)=w_t^u\left(S_t^\star,U_t^\star\right)\tag{31}$$

$$F_t\left(\lambda w_{t-1}^s\left(S_t^\star\right)-\frac{1-\theta}{1+r}w_t^s\left(S_t^\star,U_t^\star\right)\right)\leq U_t^\star\tag{32}$$

Consider a S_t', U_t', and assume that condition 30 is satisfied, but condition 29 is not satisfied, so that S_t', U_t' is not an equilibrium. Consider now a S_t'', U_t'' such that condition 31 holds. Note the following

- either $S_t''<S_t'$ or $U_t''>U_t'$ or both (i.e. when fewer agents choose the skilled profession, or more agents choose the unskilled profession, the skilled profession becomes more attractive relatively to the unskilled one).

- This follows that, if condition 30 holds at S_t', U_t', then condition 32 must hold at S_t'', U_t''.

Therefore S''_t, U''_t is an equilibrium. Finally, note that the return on education can be made arbitrarily large or arbitrary small. It will be arbitrary large when $S_t \to 0$, it is arbitrary small when $S_t \to \frac{1}{\lambda}(1 - U_{t-1})$ (since all the skilled agents in generation $t-1$ will be teachers, and the skilled wage w^s_{t-1} will diverge to infinity) or when $U_t \to 0$. Hence the equilibrium always exists.

To conclude, note that the above argument may not work if the distribution $F_t(e)$ is discontinuous. In general, there may be a discontinuity in the supply of skilled workers whenever there is a positive mass of agents with wealth equal to:

$$\overline{e} = \lambda w^s_{t-1}(S^\star_t) - \frac{1-\theta}{1+r} w^s_t(S_t, U_t)$$

in this case banks are indifferent about lending to agents with wealth $\overline{e}$. We can assume that the banking sector lends to any arbitrary fraction of agents with wealth equal to $\overline{e}$. This way, a competitive equilibrium exists no matter the distribution of wealth.

B Proofs of section 3.

B.1 Proof of lemma 4.

Since the total number of skilled and unskilled workers is a constant in steady state, there can be social mobility only if two households swap occupations. Suppose that there is one household with a skilled father and an unskilled son, and another household with a unskilled father and a skilled son. Since agents prefer to be skilled, it must be the case that the son of the unskilled worker has access to the credit market and the son of the skilled worker does not. This implies that the unskilled father must be wealthier than the skilled father, leading to a contradiction: if the unskilled father had access to the credit market he would have chosen the skilled profession.

B.2 Proof of lemma 5.

Consider equation 12 in steady state at time t. Take the log of both sides and subtract the log of the same equation in $t+1$. The gross growth rate of capital is given by:

$$\gamma = 1 + g$$

Similarly, using equations 13 and 14, it is possible to show that both wages grow at the same rate γ. Finally, consider the budget constraint of an unskilled agent in a steady state, in period t:

$$e_{t+1,ss} + c_{t,ss} = w_{t,ss} + e_{t,ss}(1+r)$$

that is equivalent to:

$$e_{t,ss}\gamma_e + c_{t-1,ss}\gamma_c = w_{t-1,ss}\gamma + e_{t-1,ss}(1+r)\gamma_e$$

Since the budget constraint must also hold in period $t-1$:

$$e_{t,ss} + c_{t-1,ss} = w_{t-1,ss} + e_{t-1,ss}(1+r)$$

in general we have that:

$$N = \left(\frac{\gamma}{\gamma_e}\right)^s - M\left(\frac{\gamma_c}{\gamma_e}\right)^s \quad \forall s \geq 0 \tag{33}$$

where $N = \frac{e_{t,ss}-e_{t-1,ss}(1+r)}{w_{t-1,ss}}$ and $M = \frac{c_{t-1,ss}}{w_{t-1,ss}}$. Consider condition 33 for $s=0$ and $s=1$, and plug back the results. We get:

$$\left[1-\left(\frac{\gamma}{\gamma_e}\right)^s\right]\left[1-\left(\frac{\gamma_c}{\gamma_e}\right)^s\right]^{-1} = \left(1-\frac{\gamma}{\gamma_e}\right)\left(1-\frac{\gamma_c}{\gamma_e}\right)^{-1} \quad \forall s \geq 0$$

which implies either $\gamma = \gamma_e$ or $\frac{\gamma}{\gamma_e} = \frac{\gamma_c}{\gamma_e}$. In either case, looking back at condition 33 we conclude that $\gamma = \gamma_c = \gamma_e$.

B.3 Proof of propositions 7, 8 and 9.

Lemma 16. *Under assumption 6, if condition 24 holds, agents are unconstrained:*

$$e^s_{t+1,ss} = w^s_{t,ss}\frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}$$

$$e^u_{t+1,ss} = w^u_{t,ss}\frac{\gamma(1-\rho\gamma)}{\gamma-(1+r)(1-\rho\gamma)}$$

Otherwise, the constraints are binding:

$$e^s_{t+1,ss} = w^s_{t,ss} max\left\{\lambda - \frac{\gamma(1-\theta)}{1+r}, 0\right\}$$

$$e^u_{t+1,ss} = 0$$

Proof. Skilled workers are unconstrained when

$$\lambda - \frac{\gamma(1-\theta)}{1+r} < \frac{(1-\rho\gamma)(\gamma-\lambda(1+r))}{\gamma-(1+r)(1-\rho\gamma)}$$

or

$$\theta < \left(\frac{(1-\rho\gamma)(\gamma - \lambda(1+r))}{\gamma - (1+r)(1-\rho\gamma)} \right) \frac{(1+r)}{\gamma} + 1$$

by assumptions 6, when condition 24 holds the LHS is greater than one and the inequality holds. Finally, for unskilled workers the conclusion follows simply by condition 24. □

B.3.1 Proposition 7.

It follows simply because if

$$\theta < 1 - \frac{\lambda(1+r)}{\gamma}$$

everybody can borrow.

B.3.2 Proposition 8.

Assume that credit market is imperfect:

$$\theta > 1 - \frac{\lambda(1+r)}{\gamma}$$

- Case 1: agents are unconstrained (condition 24 holds).

There is an equal steady state if the returns on the two professions is equal:

$$1 - \frac{\lambda(1+r)}{\gamma} = \frac{w^u}{w^s} \tag{34}$$

There is an unequal steady state if the two professions yield different returns:

$$\frac{w^u}{w^s} < 1 - \frac{\lambda(1+r)}{\gamma}$$

unskilled workers cannot access the credit market:

$$\frac{w^u}{w^s} < \left(\lambda - \frac{(1-\theta)\gamma}{1+r} \right) \left(\frac{\gamma - (1+r)(1-\rho\gamma)}{\gamma(1-\rho\gamma)} \right)$$

and the no-negative-consumption constraint holds:

$$\frac{w^u}{w^s} \frac{1}{\gamma} < \left(\lambda - \frac{(1-\theta)\gamma}{1+r} \right) \left(\frac{\gamma - (1+r)(1-\rho\gamma)}{(\gamma-1)(1-\rho\gamma)(1+r)+\gamma} \right)$$

Because of assumption 6, if the credit market is imperfect, there is always some $\frac{S}{U}$ that satisfies the three inequalities above. However, note that the above constraints do not define the set of unequal steady states. The extra constraint missing is the no-deviation constraint:

parents should not want to bequeath enough so that their children cannot go to school, whenever this deviation is possible. However, if a $\frac{w^u}{w^s}$ satisfies all the above constraints, this $\frac{w^u}{w^s}$ is an unequal steady state, since the no-negative-consumption constraints requires such deviation not to be feasible.

- Case 2: agents are constrained (condition 24 does not hold).

Unskilled agents bequeath zero wealth and are not able to borrow: the no-credit constraint is always satisfied. Note that there cannot be an equal steady state. This would correspond to a situation where skilled workers bequeath just enough so that their children can to go to school and unskilled workers leave no bequests. In this situation, if the returns on the two professions are equal, skilled parents prefer to leave zero wealth: they can increase their own consumption leaving their children income unchanged.

There is an unequal steady state if the return on one profession is strictly bigger than the return on the other one:

$$\frac{w^u}{w^s} < 1 - \frac{\lambda(1+r)}{\gamma}$$

and unskilled workers cannot bequeath enough so that their children can go to school:

$$\frac{w^u}{w^s}\frac{1}{\gamma} < 1 - \frac{\lambda(1+r)}{\gamma}$$

Note that by assumption 6, if credit market is imperfect and agents are constrained, there is always some $\frac{S}{U}$ that satisfies both conditions. This $\frac{S}{U}$ is an unequal steady state.

B.3.3 Proof of proposition 9.

All the constraints relevant in an unequal steady state have already been derived in the proof of proposition 8, except for the no-deviation constraint that is discussed in the main text on page 14. All of them are a function of skill bias only through the skill premium.

It is interesting to note here that the set of skill premia satisfying the no-deviation constraint has the shape $(0, x'] \cup [x'', x''']$, while the set of skill premia satisfying the other constraints has shape $(0, x]$. This implies that the set of unequal steady state that is between B and C in figure 3 may not exist.

C Proofs of section 4.

C.1 Proof of lemma 10.

The result follows from simple, but tedious, manipulations. By definition:

$$\mu = \frac{S(1+\lambda)\left[(e^S_{t,ss} - \lambda w^S_{t-1,ss})(1+r) + w^S_{t,ss} - e^u_{t,ss}(1+r) - w^u_{t,ss}\right]}{S(1+\lambda)((e^s_{t,ss} - \lambda w^s_{t-1,ss})(1+r) + w^s_{t,ss}) + U(e^u_{t,ss}(1+r) + w^u)}$$

write $w^s_{t-1,ss} = \frac{w^s_{t,ss}}{\gamma}$ and $e^i_{t,ss} = \frac{e^i_{t+1,ss}}{\gamma}$ for $i = s, u$. Assume that condition 24 holds and plug in the first order conditions 21 and 22 (using the fact that agents are unconstrained):

$$\mu = \frac{S(1+\lambda)\left[w^s_{t,ss}\left(A(\gamma - \lambda(1+r))\frac{(1+r)}{\gamma} - \frac{\lambda(1+r)}{\gamma} + 1\right) - w^u_{t,ss}\left(A(1+r)+1\right)\right]}{S(1+\lambda)w^s_{t,ss}\left(A(\gamma - \lambda(1+r))\frac{(1+r)}{\gamma} - \frac{\lambda(1+r)}{\gamma} + 1\right) + Uw^u_{t,ss}\left(A(1+r)+1\right)}$$

where

$$A = \frac{(1-\rho\gamma)}{\gamma - (1+r)(1-\rho\gamma)}$$

Simplify the expression dividing both numerator and denominator by $w^u\left(A(1+r)+1\right)$:

$$\mu = \frac{(1+\lambda)\left[\frac{w^s}{w^u}\left(1 - \frac{\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma}}{A(1+r)+1}\right) - 1\right]}{(1+\lambda)\frac{w^s}{w^u}\left(1 - \frac{\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma}}{A(1+r)+1}\right) + \frac{U}{S}}$$

and note that $\frac{A\lambda(1+r)^2}{\gamma} + \frac{\lambda(1+r)}{\gamma} = (A(1+r)+1)\left(\frac{(1+r)\lambda}{\gamma}\right)$.

In case condition 24 does not hold following the same steps, the solution is:

$$\mu = \frac{(1+\lambda)\left[\frac{w^s}{w^u}\left(1 + \lambda(1+r)\left(1 - \frac{1}{\gamma}\right) - \gamma(1-\theta)\right) - 1\right]}{(1+\lambda)\frac{w^s}{w^u}\left(1 + \lambda(1+r)\left(1 - \frac{1}{\gamma}\right) - \gamma(1-\theta)\right) + \frac{U}{S}}$$

D Proofs of section 5.

D.1 Proof of proposition 13.

By assumption, before the shock the credit-constraint condition holds with a strict inequality:

$$e^u_t > \lambda w^s_{t-1} - w^s_t\left(\frac{1-\theta}{1+r}\right)$$

in the moment when the shock hit, there is an increase in w_t^s. Clearly, because of the strict inequality and because the shock is arbitrarily small, the above condition hold also after the shock. It follows that, immediately after the shock, unskilled workers are unable to borrow.

For as long as unskilled workers cannot borrow, wages remain constant at their post-shock level and bequests converge monotonically to the steady state bequests corresponding to the same m and the new parameters. Since in the new steady state they cannot borrow, and because the convergence is monotonic, there is no moment during the transition when unskilled workers have access to the credit market. The economy must converge to the steady state corresponding to the new parameters and the old m.

D.2 Proof of lemma 14.

Rewrite expression 28 as

$$\mu = \frac{(1+\lambda)\left[\frac{w^s}{w^u}\Psi - 1\right]}{(1+\lambda)\frac{w^s}{w^u}\Psi + \frac{U}{S}}$$

where

$$\Psi = \left(1 + \lambda(1+r)\left(1 - \frac{1}{\gamma}\right) - \gamma(1-\theta)\right)$$

taking derivative, and using the fact that $\frac{w^s}{w^u} = \frac{a}{b}\frac{U}{S}$ it is possible to show that

$$\frac{\partial \mu}{\partial\left(\frac{a}{b}\right)} = \frac{(1+\lambda)\Psi}{\Psi(1+\lambda)\frac{w^s}{w^u} + \frac{U}{S}} - \frac{\Psi(1+\lambda)^2\left(\Psi\frac{w^s}{w^u} - 1\right)}{\left(\Psi(1+\lambda)\frac{w^s}{w^u} + \frac{U}{S}\right)^2} > 0$$

This implies that, for given $\frac{S}{U}$, increasing $\frac{a}{b}$ increases inequality.

D.3 Proof of Proposition 15

The only case still to be addressed corresponds to point B in figure 3 (if it exists). Suppose anytime during the transition to the new steady state, parents prefer their children to be educated: the skilled to unskilled ratio increases so to keep unskilled parents indifferent. In periods where parents strictly prefer their children not to be educated, nothing happen. Hence, during the transition, the skilled-to-unskilled ratio (weakly) increases over time, with an upper bound at the steady state skilled-to-unskilled ratio where parents are indifferent (call it B', and note that skill premium at B before the shock, and at B' after the shock are the same). Since no skilled to unskilled ratio between B and B' is a steady state, the economy must converge to B'.