Deriving the Taylor Principle when the Central Bank Supplies Money

by

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Abstract

The paper presents a human-capital-based endogenous growth, cash-in-advance economy with endogenous velocity where exchange credit is produced in a decentralized banking sector, and money is supplied stochastically by the central bank. From this it derives an exact functional form for a general equilibrium `Taylor rule'. The inflation coefficient is always greater than one when the velocity of money exceeds one; velocity growth enters the equilibrium condition as a separate variable. The paper then successfully estimates the magnitude of the coefficient on inflation from 1000 samples of Monte Carlo simulated data. This shows that it would be spurious to conclude that the central bank has a reaction function with a strong response to inflation in a `Taylor principle' sense, since it is only meeting fiscal needs through the inflation tax. The paper also estimates several deliberately misspecified models to show how an inflation coefficient of less than one can result from model misspecification. An inflation coefficient greater than one holds theoretically along the balanced growth path equilibrium, making it a sharply robust principle based on the economy's underlying structural parameters.

1 Introduction

Interest rate rules are widely considered as monetary policy 'reaction functions' that represent how the central bank adjusts a short-term nominal interest rate in response to the state of the economy. The magnitude of the reaction function coefficients are interpreted to reflect a policymaker's attitude towards variation in key macroeconomic variables such as inflation and the output gap. It has been suggested that policymakers ought to adhere to the 'Taylor principle', whereby inflation above target is met by a more-than-proportional increase in the short-term nominal interest rate and hence an increase in the real interest rate. Such an interest rate rule forms one of the three core equations of the prominent New Keynesian modelling framework, such as in Woodford, 2003, Clarida et al., 1999; and Clarida et al. (2000). One well-known finding comes from the latter paper which concludes that the Taylor principle holds for a 'Volcker-Greenspan' sample of U.S. data but that it is violated for a 'pre-Volcker' sample during which the Fed was deemed to be accommodating in its reaction function.

In contrast, a historical strand of literature going back to Poole (1970), and updated for example by Alvarez et al. (2001) and Chowdhury and Schabert (2008), considers interest rate rules and money supply rules as two ways of implementing the same monetary policy. This paper perhaps most closely follows Alvarez et al. (2001) by deriving the equilibrium nominal interest rate in a 'rule' form within a general equilibrium economy in which the central bank conducts policy by stochastically supplying money. Instead of an exogenous fraction of agents being able to use bonds as in Alvarez et al., here the consumer purchases goods with an endogenous fraction of bank-supplied intratemporal credit that avoids the inflation tax on exchange. This cash-in-advance monetary economy is also extended to include endogenous growth, along with endogenous velocity, as in Benk et al. (2010). The resulting equilibrium nominal interest rate condition 'nests' the standard Taylor rule within a more general forward-looking setting that endogenously includes traditional monetary elements, such as the (exogenous) velocity in Alvarez et al., and the money demand in McCallum and Nelson (1999).

The endogenous growth aspect implies that the 'target' terms of the equilibrium 'Taylor condition', such as the inflation rate target or the 'potential' output level, are the balanced growth path (BGP) equilibrium values of the related variables. In addition, the coefficients of the Taylor condition are a function of the model's utility and technology parameters along with the BGP money supply growth rate. This in essence fulfills Lucas's (1976) goal of postulating policy rules with coefficients that depend explicitly upon the economy's underlying utility and technology coefficients plus a key policy choice, in this case the BGP rate of money supply growth. Aesthetic as such a formulation of the Taylor condition may be, Lucas's research agenda provides a solid result here: a theoretical derivation of the 'Taylor principle' where the coefficient on the inflation term always exceeds or equals one. The 'principle' holds for any given non-Friedman (1969) optimum BGP money supply growth rate, it equals one only at the Friedman optimum, and never falls below one. Similarly, the

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inflation coefficient always exceeds one when the endogenous velocity exceeds one since the cash-in-advance velocity rises above one for any non-Friedman optimal rate of money supply growth. In general, the inflation coefficient rises with the BGP velocity level. Another central result is that the expected velocity growth rate itself enters the Taylor condition as an additional term, in contrast to standard Taylor rules. Omitting this term can cause misspecification bias in estimated Taylor rules within the economy.

Having derived the Taylor condition, the paper then estimates it by applying three conventional estimation procedures to one thousand samples of artificial data simulated from the baseline model, where the simulated data is passed through three standard filters prior to estimation. The results verify the theoretical form of the Taylor condition along several key dimensions. In particular, the coefficient on inflation is greater than one and close to its theoretical magnitude for all three estimation techniques and for all three data filters. Satisfying the 'Taylor principle' in this fashion, robustness tests explore the impact of estimating two alternative Taylor conditions. This involves two *ad hoc*, deliberately misspecified equations relative to the 'true' theoretical expression: the first changes just one of the variables in the Taylor condition while the second posits a standard Taylor rule that involves multiple misspecification errors. Using the same artificial data, estimating the two misspecified models results in the coefficient on inflation falling below one, causing the 'Taylor principle' to fail. In the context of actual data, this result would typically be interpreted as the central bank being 'passive' or 'weak' towards inflation. Here, the paper shows that such an interpretation could be spurious in that it could occur simply because of a misspecified estimating equation.[1]

The estimated 'Taylor rule' emerges even though the central bank is merely satisfying fiscal needs through the inflation tax. This implies the central point of the paper: it would be spurious within this economy to associate the Taylor condition with a 'reaction function' for the nominal interest rate since in the model the central bank just stochastically prints money. Second, failure of the so-called Taylor principle in numerous published empirical studies may be a result of model misspecification rather than behavioral changes by the central bank *per se*. Indeed, our current preliminary extension of this work, not presented here, shows that estimation with actual US data of Taylor rules which include the unconventional terms implied by the theory of this paper - particularly velocity growth - can reverse the result that the coefficient on inflation falls below unity during periods of macroeconomic instability.[2]

Related work is vast but includes Taylor (1999), who alludes to the possibility that an interest rate rule can be derived from the quantity theory of money. Sørensen and Whitta-Jacobsen (2005, pp.502-505) present such a derivation under the assumption of constant money growth whereby the coefficients of

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[1] Estimation of simulated data is conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model. We are indebted to Warren Weber for the suggestion to follow such an approach here.

[2] Clarida et al.'s (2000) 'pre-Volcker' sample, for example, corresponds to a period of high and variable inflation.

the 'rule' relate to elasticities of money demand rather than the preferences of policymakers. Fève and Auray (2002) and Schabert (2003) consider the link between money supply rules and interest rate rules in standard cash-in-advance models with velocity fixed at unity. Alternatively, the paper could be viewed in light of Canzoneri et al. (2007) in that it shows how the puzzle of estimating the Euler equation for the nominal interest rate can be solved by combining that equilibrium condition with the stochastic asset pricing kernel to derive an expression for a Taylor condition that can be successfully estimated.

Section 2 describes the economy, as in Benk et al. (2008, 2010). Section 3 derives the model's 'Taylor condition' and Section 4 provides the baseline calibration. Section 5 describes the econometric methodology which is applied to model-simulated data and presents the corresponding estimation results. Section 6 derives special theoretical cases of the more general (Section 2) model to show how alternative Taylor conditions can be derived. Section 7 presents a discussion and Section 8 concludes.

2   Stochastic Endogenous Growth with Banking

The representative agent economy is as in Benk et al (2008, 2010) but with a decentralized banking sector that produces credit as in Gillman and Kejak (2011). By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically in Gillman et al. (2004) and Fountas et al. (2006), for example. Further, money supply shocks can cause inflation at low frequencies, as in Haug and Dewald (2011) and as supported by Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to credit sector productivity exists by virtue of the model's endogenous money velocity via the production function used extensively in the financial intermediation microeconomics literature starting with Clark (1984).

The shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank sector productivity, v_t:

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t\ u_t\ v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\{\varphi_z, \varphi_u, \varphi_v\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\epsilon_{zt}\ \epsilon_{ut}\ \epsilon_{vt}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$. These shocks affect the economy as described below, and as calibrated in Benk et al. (2010).

2.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0, 1)$, $\psi > 0$ and $\theta > 0$, this is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1 - \theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in the goods production (G) and human capital investment (H), whereby:

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_{Gt}, time spent investing in the stock of human capital, l_{Ht}, and time spent working in the bank sector (F subscripts for Finance), denoted by l_{Ft}:

$$l_{Gt} + l_{Ht} + l_{Ft} + x_t = 1. \tag{4}$$

Output of goods can be converted into physical capital, k_t, without cost and so is divided between consumption goods and investment, denoted by i_t, net of capital depreciation. Thus, the capital stock used for production in the next period is given by:

$$k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t. \tag{5}$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $l_{Ht}h_t$, with $A_H > 0$ and $\eta \in [0, 1]$, such that the human capital flow constraint is

$$h_{t+1} = (1 - \delta_h)h_t + A_H(s_{Ht}k_t)^{1-\eta}(l_{Ht}h_t)^{\eta}. \tag{6}$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t (l_{Gt} + l_{Ft}) h_t$ and $P_t r_t (s_{Gt} + s_{Qt}) k_t$, a nominal transfer from the government, T_t, and dividends from the bank. The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Ft}, the consumer receives a nominal dividend income of $P_t R_{Ft} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Ft} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Ft}q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1 - \delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1 + R_t)$, where R_t is the net nominal interest rate, the consumer's budget constraint is:

$$P_t w_t \left(l_{Gt} + l_{Ft}\right) h_t + P_t r_t s_{Gt} k_t + P_t R_{Ft} d_t + T_t \tag{7}$$
$$\geq \quad P_{Ft} q_t + P_t c_t + P_t k_{t+1} - P_t(1 - \delta_k)k_t + M_{t+1} - M_t$$
$$+ B_{t+1} - B_t(1 + R_t).$$

The consumer can purchase the goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is:

$$M_t + T_t + P_t q_t \geq P_t c_t. \tag{8}$$

Since all cash comes out of deposits at the bank, and credit purchases are paid off at the end of the period out of the same deposits, the total deposits are equal to consumption. This gives the constraint that:

$$d_t = c_t. \tag{9}$$

Given k_0, h_0, and the evolution of M_t ($t \geq 0$) as given by the exogenous monetary policy in equation (17) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (7)-(9).

2.2   Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_F > 0$ and $\gamma \in (0, 1)$:

$$q_t = A_F e^{v_t} \left(l_{Ft} h_t\right)^\gamma d_t^{1-\gamma}, \tag{10}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (10), the bank maximizes profit Π_{Ft} with respect to the labor l_{Ft} and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t l_{Ft} h_t - P_t R_{Ft} d_t. \tag{11}$$

Equilibrium implies that:

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma-1} = w_t; \tag{12}$$

$$\left(\frac{P_{Ft}}{P_t}\right)(1-\gamma)A_Fe^{v_t}\left(\frac{l_{Ft}h_t}{d_t}\right)^{\gamma} = R_{Ft}. \tag{13}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t}\left(\frac{l_{Ft}h_t}{d_t}\right)^{\gamma-1}}$, and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by $\left(\frac{P_{Ft}}{P_t}\right)(1-\gamma)\left(\frac{q_t}{d_t}\right)$.

2.3 Goods Producer Problem

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t}(s_{Gt}k_t)^{1-\alpha}(l_{Gt}h_t)^{\alpha}. \tag{14}$$

The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{1-\alpha}, \tag{15}$$

$$r_t = (1-\alpha)A_G e^{z_t}\left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{-\alpha}. \tag{16}$$

2.4 Government Money Supply

It is assumed that the government policy includes sequences of nominal transfers as given by

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1)M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}, \tag{17}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money.

2.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l_G,l_H,l_F,s_G,s_H,q,d,k',h',M'}\{u(c,x) + \beta EV(s')\} \tag{18}$$

subject to the conditions (3) to (9), where the state of the economy is denoted by $s = (k,h,M,B;z,u,v)$ and a prime (') indicates the next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l_G(s)$, $l_H(s)$, $l_F(s)$, $s_G(s)$, $s_H(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

(i) the consumer maximize utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (18);

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(ii) the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (15) and (16);

(iii) the bank firm maximizes profit similarly in equation (11) subject to the technology of equation (10)

(iv) the goods, money and credit markets clear, in equations (7) and (14), and in (8), (17), and (10).

3   General Equilibrium Taylor Condition

The 'Taylor condition' is now derived as an equilibrium condition of the Benk et al. (2010) model described in the previous section. Beginning from the first-order conditions of the model, we obtain:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{19}$$

where R and π are gross rates of nominal interest and inflation, respectively; $\tilde{R}_t$ is 1 plus a 'weighted average costs of exchange, with weights of $\frac{m}{c}$ on the money cost $R_t - 1$ and $1 - \frac{m}{c}$ on the average credit cost of $\gamma(R_t - 1)$:

$$\tilde{R}_t = 1 + \frac{m_t}{c_t}(R_t - 1) + \gamma\left(1 - \frac{m_t}{c_t}\right)(R_t - 1).$$

$\frac{m_t}{c_t}$ is the consumption normalized money demand, i.e. the inverse of the consumption velocity of money. In effect, equation (19) augments a standard consumption Euler equation with the growth rate of this average cost of exchange. If all transactions are conducted using money ($m_t/c_t = 1$) then equation (19) reverts back to the familiar consumption Euler equation which would feature as an equilibrium condition of a standard CIA model without a money alternative.[3]

For any variable z_t, define $\widehat{z}_t \equiv \ln z_t - \ln z$, where the absence of a time subscript denotes a BGP stationary value, and define $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which approximates the growth rate at time $t+1$ for sufficiently small z_t. Consider a log-linear approximation of (19) evaluated around the BGP:

$$0 = E_t \left\{ \theta \widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\pi}_{t+1} \right\}.$$

Rearranging this in terms of $\widehat{R}_t$ gives the Taylor condition expressed in log-deviations from the BGP equilibrium:

$$\begin{aligned} \widehat{R}_t = \quad & E_t \left\{ \Omega \widehat{\pi}_{t+1} + \Omega\theta \widehat{g}_{c,t+1} - \Omega\psi(1-\theta)\widehat{g}_{x,t+1} \right. \\ & \left. + \frac{(1-\gamma)\left(1 - \frac{m}{c}\right)}{R\left[1 - (1-\gamma)\left(1 - \frac{m}{c}\right)\right]} \left[(R-1)\frac{\frac{m}{c}}{1 - \frac{m}{c}}\widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1} \right] \right\}, \end{aligned} \tag{20}$$

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[3] The nominal interest rate and inflation both enter equation (19) with a one period lead. This is consistent with Carlstrom and Fuerst's (2001) "cash in advance timing" which contrasts with their "cash when I'm done timing", where both the nominal interest rate and inflation enter the Euler equation contemporaneously. Carlstrom and Fuerst (2001) reject the latter for CIA models.

where $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1$. The Taylor condition (20) can now be expressed in net rates and absolute deviations from the BGP equilibrium, as demonstrated by the following proposition.

Proposition 1 *An equilibrium condition of the economy is in the form of the Taylor Rule (Orphanides, 2008) that sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of target variables from their targets:*

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\theta\right)E_t\overline{g}_{x,t+1} \tag{21}$$
$$+ \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\left[(R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}}E_t\overline{g}_{\frac{m}{c},t+1} - E_t\left(\overline{R}_{t+1} - \overline{R}\right)\right].$$

where $\Omega \geq 1$, and for a given w, then $\frac{\partial\Omega}{\partial R} > 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$, and the target values are equal to the balanced growth path equilibrium values.[4]

Proof. Since the BGP solution for normalized money demand is:

$$0 \leq \frac{m}{c} = 1 - A_F\left(\frac{(R-1)\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1,$$

then $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1$ and, given w, $\frac{\partial\Omega}{\partial R} \geq 0$ and $\frac{\partial\Omega}{\partial A_F} \geq 0$. ∎

For a linear production function of goods, w is the constant marginal product of labor, but more generally w is endogenous and will change; however this change in w is quantitatively small compared to changes in R and A_F, so that the derivatives above almost always hold true. Note that for a unitary consumption velocity of money, the latter two velocity growth and forward interest terms drop out of the equation (21)

The term in $\bar{\pi}$ in equation (21) can be compared to the inflation target that features in many interest rate rules (e.g. Taylor, 1993; Clarida et al., 2000). This is usually set as an exogenous constant in a conventional rule but represents the BGP rate of inflation in the Taylor condition.[5] The term in consumption growth is similar, but not identical, to the first difference of the output gap that features in the so-called 'speed limit' rule (Walsh, 2003). Alternatively, the term in the growth rate of leisure time can be compared to the unemployment rate which sometimes features in conventional interest rate rules in place of the output gap.[6]

Equation (21) also contains two terms which are not usually found in standard monetary policy reaction functions. First, there is a term in the growth

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[4] This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

[5] Although see Ireland (2007) for an example of a conventional interest rate rule with a time-varying inflation target.

[6] For example, Mankiw (2001) includes the unemployment rate in an interest rate rule and Rudebusch (2009) includes the 'unemployment gap'.

rate of the real (consumption normalized) demand for money. Conventional interest rate rules are usually considered in the context of models which omit monetary relationships and thus money demand does not feature directly in the model, let alone the policy rule.[7] Secondly, the Taylor condition contains a term in the expected future nominal interest rate. This contrasts with the lagged nominal interest term which is often used to capture 'interest rate smoothing' in a conventional rule (e.g. Clarida et al., 2000).

In general, the coefficient on inflation in (21) exceeds unity ($\Omega > 1$). This replicates the 'Taylor principle' whereby the nominal interest rate responds more than one-for-one to (expected future) inflation deviations from 'target'. However, the inflation coefficient in the Taylor condition is a function of the BGP nominal interest rate (R), the consumption normalized demand for real money balances (m/c) and the efficiency with which the banking sector transforms units of deposits into units of the credit service, as reflected by the magnitude of $(1-\gamma)$. Furthermore, higher productivity in the banking sector (A_F) causes a higher velocity and implies a larger inflation coefficient in the Taylor condition. The magnitude of Ω clearly does not reflect a policymaker reaction to inflation in the conventional, 'reaction function' sense.[8]

Equation (21) can alternatively be rewritten in terms of the consumption velocity of money, $V_t \equiv \frac{c_t}{m_t}$, and the productive time, or 'employment', growth rate ($l \equiv l_G + l_H + l_F = 1 - x$). Using the fact that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$:

$$
\begin{aligned}
\overline{R}_t - \overline{R} =\ & \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi} \right) + \Omega \theta E_t \left(\overline{g}_{c,t+1} - \overline{g} \right) + \Omega \psi \left(1 - \theta \right) \frac{l}{1-l} E_t \overline{g}_{l,t+1} \\
& - \Omega_V E_t \overline{g}_{V,t+1} - (\Omega - 1) E_t \left(\overline{R}_{t+1} - \overline{R} \right).
\end{aligned}
\tag{22}
$$

Where overbarred terms again denote net rates and

$$
\Omega_V \equiv \frac{(R-1)}{R} \left(\frac{(1-\gamma)\frac{m}{c}}{\gamma + (1-\gamma)\frac{m}{c}} \right).
$$

Proposition 2 *For the Taylor condition of equation (22), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

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[7] Shifts in the demand for money are perfectly accommodated by adjustments to the money supply in order to maintain the rule-implied nominal interest rate. This, it is claimed, renders the evolution of the money supply an operational detail which need not be modelled directly (e.g. Woodford, 2008).

[8] Unlike Sørensen and Whitta-Jacobsen's (2005, pp.502-505) quantity theory based equilibrium condition, the inflation coefficient in (21) exceeds unity for any (admissible) interest elasticity of money demand. In their expression, the inflation coefficient falls below unity if the interest (semi) elasticity of money demand exceeds one in absolute value. In the Benk et al. (2010) model, the coefficient on inflation would exceed unity even in this case but the central bank would not wish to increase the money supply growth rate to this extent because seigniorage revenues would begin to recede as the elasticity increases beyond this point.

Proof.

$$\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R[1-(1-\gamma)\left(1-\frac{m}{c}\right)]} \geq 1; \quad \frac{m}{c} = 1 - A_F^{\frac{1}{1-\gamma}}\left[\frac{(R-1)\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1;$$

$$1 \geq (1-\gamma)\left(1-\frac{m}{c}\right) \geq 0; \Rightarrow 0 \leq \Omega_V \equiv \frac{(R-1)(1-\gamma)}{R}\left(\frac{\frac{m}{c}}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right) \leq 1;$$

$$\Rightarrow 0 \leq \Omega_V \leq 1 \leq \Omega.$$

∎

Note that at the Friedman (1969) optimum of (gross) $R = 1$, then $\frac{m}{c} = 1$, $\omega = 0$, and the velocity coefficient is $\Omega_V = 0$. The velocity growth term only matters when the nominal interest rate and inflation differ from the Friedman (1969) optimum and fluctuate. In turn, this has implications for $\Omega = 1 + \left(\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R[1-(1-\gamma)\left(1-\frac{m}{c}\right)]}\right)$, since when $R = 1$, then $(1-\gamma)\left(1-\frac{m}{c}\right) = 0$, and $\Omega = 1$. For $\frac{m}{c}$ below one (i.e. velocity above one), which is true for most practical experience, the model's equivalent of the 'Taylor principle', $\Omega > 1$, holds.

Corollary 3 *Given w, then $\frac{\partial \Omega}{\partial R} \geq 0, \frac{\partial \Omega_V}{\partial R} \geq 0, \frac{\partial \Omega}{\partial A_F} \geq 0, \frac{\partial \Omega_V}{\partial A_F} \leq 0.$*

Proof. This comes directly from the definitions of parameters above. ∎

A higher target R can be accomplished only by a higher BGP money supply growth rate. This would in turn make the inflation coefficient Ω larger, and so also the consumption growth coefficient ($\Omega\theta$), and the forward interest rate and velocity coefficients would become more negative. A higher credit productivity factor A_F, and so a higher velocity, causes a higher inflation coefficient and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

Note that with exogenous growth, the above Taylor condition would appear to look identical. However, under exogenous growth the targeted inflation rate and growth rate of the economy are unrelated and exogenously specified. Under endogenous growth, the targets are instead the endogenously determined BGP values: for inflation, the growth rate, and the nominal interest rate. And all of these are determined in part by the long run stationary money supply growth rate Θ^*, which is exogenously given. In turn, this Θ^* translates directly into a long run inflation target accepted by the central bank, such as two percent. So the model assumes only this target of a long run money supply growth, or alternatively, the long run inflation rate target.

3.1 Misspecified Taylor Condition with Output Growth

It is not surprising to find that the growth rate of consumption appears in equation (22) rather than the output growth rate given that the derivation of the Taylor condition begins from the consumption Euler equation (19). However, the Taylor condition can be rewritten to include an output growth term and thus correspond more closely to standard Taylor rule specifications, in particular the

'speed limit' rule considered by Walsh (2003). To derive this alternative rule, consider that the equation, $y_t = c_t + i_t$, implies that $\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t$, with $\widehat{i}_t = \frac{k}{i}\left[\widehat{k}_t - (1-\delta)\widehat{k}_{t-1}\right]$. The growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. The Taylor condition rewrites as

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] \quad (23)$$

$$+ \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right).$$

A term in investment growth does not appear in standard, exogenously specified Taylor rules but plays a role as part of what is interpreted as growth in the output gap in this Taylor condition with output growth. Equation (23) forms the basis for the two misspecified estimating equations considered in Section 5. The first misspecified estimating equation simply replaces the consumption growth term in equation (22) with an output growth term as follows:

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \quad (24)$$

$$+ \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right).$$

As the comparison between equation (23) and equation (24) shows, such an estimating equation erroneously overlooks the weighting on the output growth rate ($\frac{y}{c}$) and omits the term in the investment growth rate. Replacing consumption growth with output growth without an additional term in investment therefore misrepresents the structure of the underlying, Benk et al. (2010) model and as such equation (24) is misspecified. Note that with no physical capital in the economy, such a Taylor condition as above would be the correct equilibrium condition of the economy.

3.2 Misspecified Standard Taylor Rule

The second misspecified model erroneously imposes yet more restrictions on equation (23). Imposing the same restrictions used to arrive at equation (24) but also dropping the terms in productive time and velocity gives:

$$\overline{R}_t - \overline{R} = \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \quad (25)$$

$$- \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right).$$

This can be interpreted as a conventional interest rate rule with a forward-looking 'interest rate smoothing' term; the additional restriction that $\Omega = 1$ would replicate a standard interest rate rule without interest rate smoothing. Once again, equation (25) does not accurately represent an equilibrium condition of the Benk et al. (2010) economy and is therefore misspecified. The

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_k	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_h	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

first two terms would indeed be the correct equilibrium Taylor condition if the economy had neither physical capital or the ability to use exchange credit to avoid the inflation tax. Then $\Omega = 1$, there is no velocity or forward interest rate term, and the output growth term would enter as above.

4   Calibration

We follow Benk et al. (2010) in using postwar U.S. data to calibrate the model (Table 1) and calculate a series of 'target values' consistent with this calibration (Table 2); please see Benk et al. for the shock process calibration.

Subject to this calibration, we derive a set of theoretical 'predictions' for the coefficients of the Taylor condition (22). These values will subsequently be compared to the coefficients estimated from artificial data simulated from the model. Consider first the inflation coefficient (Ω). According to the calibration and target values presented in tables 1 and 2, its theoretical value is

$$\Omega = 1 + \frac{(1-\gamma)\left(1 - \frac{m}{c}\right)}{R\left[1 - (1-\gamma)\left(1 - \frac{m}{c}\right)\right]} = 1 + \frac{(1-0.11)(1-0.38)}{1.0944\left(1 - (1-0.11)(1-0.38)\right)} = 2.125$$

And for $R = 1$, only cash is used so that $\frac{m}{c} = 1$ and Ω reverts to its lower bound of one. This also happens with zero credit productivity ($A_F = 0$), in which case only cash is used in exchange.

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1 - x$	0.45	Productive time

<div align="center">Table 2: Target Values</div>

The remaining coefficients, except for velocity, are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$, which with $\theta = 1$ for log-utility should simply take the same magnitude as the coefficient on inflation ($\theta\Omega = 2.125$). The coefficient on the productive time growth rate is $0 = \Omega\left(1 - \theta\right)\psi\frac{l}{1-l}$ because of log utility. However with leisure preference calibrated at 1.84, and productive time ($1 - x \equiv l$) along the BGP equal to 0.45, the estimated value of the productive time coefficient can be interpreted as implying a certain θ as factored by $\Omega\psi\frac{l}{1-l} = (2.125)(1.84)\frac{0.45}{0.55} = 3.199$. Given the magnitude of the inflation coefficient, the coefficient on the forward interest term is simply $-(\Omega - 1) = -1.125$; the velocity coefficient $-\Omega_V$ is -0.065 :

$$-\frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = -\frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)(1-0.38))}\right).$$

At the Friedman (1969) optimum ($R = 1$), $\Omega_V = 0$. In this case the omission of the term in velocity growth in the estimation exercises that follow would be innocuous but this is not true in general.

5 Artificial Data Estimation

The structural model which underpins the general equilibrium Taylor condition forms the basis for the data generating process of the simulated data. In particular, the model introduced in Section 2 is simulated using the calibration provided in Section 4 in order to generate 1000 alternative 'joint histories' for each of the variables in equation (22), where each history is 100 periods in length. To do so, 100 random sequences for the shock vector innovations are generated. Control functions of the log-linearized model are then used to compute sequences for each variable. Each observation within a given history may be thought of as an annual period given the frequency considered by the Benk et al. (2010) model. The data set can therefore be viewed as comprising of 1000, '100-year', samples of artificial data.

5.1 Estimation Methodology

This section presents the results of estimating a 'correctly specified' estimating equation based upon the 'true' theoretical relationship (22) against artificial data generated from the Benk et al. (2010) model.[9] In a similar manner, two alternative estimating equations are evaluated using this same data set. Since these alternative estimating equations differ from the expression based upon the 'true' theoretical relationship, they necessarily constitute misspecified empirical models.[10]

Prior estimation, the simulated data is filtered alternatively by 1) a Hodrick-Prescott (HP) filter with a smoothing parameter selected in accordance with Ravn and Uhlig (2002); 2) a Christiano and Fitzgerald (2003) band pass filter which uses a 3-8 window that is standard for the 'business cycle' frequency; and 3) a Christiano and Fitzgerald (2003) band pass filter which uses a 2-15 window in order to retain lower frequency trends in the data as well, in particular as in Comin and Gertler's (2006) 'medium-term cycle'.[11] *A priori*, the 2-15 band pass filter may be regarded as the 'most relevant' to the underlying theoretical model because shocks in the model can cause low frequency events during the business cycle such as a change in the permanent income level without it returning to its previous level.[12]

The first estimation technique considered is OLS, as used by Taylor (1999) in the context of a contemporaneous interest rate rule. However, because expected future variables may be correlated with the error term we seek a suitable set of instruments to proxy for these terms.[13] Two instrumental variables (IV) techniques are considered and each differs by the instrument set employed. The first is a two stage least squares (2SLS) estimator under which the first lags of inflation, consumption growth, productive time growth and velocity growth and the second lag of the nominal interest rate (since the first lag is the dependent variable) are used as instruments. Adding a constant term to the instrument set yields a 2SLS estimator with no over-identifying restrictions. In using lagged

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[9] The exercise conducted here is similar to those conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model.

[10] We acknowledge that in a full information maximum likelihood estimation that uses all of the equilibrium conditions of the economy we may be able to recover almost exactly the theoretical coefficients of the Taylor condition; we leave that exercise as an important part of future research that encompasses the entire alternative model; and then we could also compare it to the standard three equation central bank policy model.

[11] Their 'medium-term cycle' is defined using a 2-200 band pass filter for quarterly data; the general principle is to retain elements of the data that the HP and 3-8 filters would ordinarily consign to the 'trend'.

[12] The filtering procedure takes account of the Siklos and Wohar (2005) critique of empirical Taylor rule studies which do not address the non-stationarity of the data. Standard ADF and KPSS tests (results not reported) suggest that the filtered data considered here is stationary. Accordingly, the filters do not implement a de-trending procedure.

[13] Empirical studies usually deal with expected future terms either by replacing them with realised future values and appealing to rational expectations for the resulting conditional forecast errors (e.g. Clarida et al., 1998, 2000) or by using private sector or central bank forecasts as empirical proxies (e.g. Orphanides, 2001; Siklos and Wohar, 2005).

terms as instruments we exploit the fact that such terms are 'pre-determined' and thus not susceptible to the simultaneity problem which motivates the use of IV techniques. The 2SLS procedure applies a Newey-West adjustment for heteroskedasticity and autocorrelation (HAC) to the coefficient covariance matrix.

The second IV procedure is a generalized method of moments (GMM) estimator under which three additional lags of inflation, consumption growth, productive time growth and velocity growth and two further lags of the nominal interest rate are added to the instrument set.[14] Expanding the instrument set in this manner reduces the sample size available for each of the 1000 estimation runs but enables the validity of the instrument set to be assessed using the Hansen J-test. The GMM estimator employed iterates on the weighting matrix in two steps and applies a HAC adjustment to the weighting matrix using a Bartlett kernel with a Newey-West fixed bandwidth.[15] A similar HAC adjustment is also applied to the covariance weighting matrix.

Results are presented in three sets of tables, one set for each estimating equation, and are further subdivided according to the statistical filter applied to the data. Alongside the estimates obtained from an 'unrestricted' estimating equation, each table also reports estimates obtained from a 'restricted' estimating equation which arbitrarily omits the forward interest rate term ($\beta_5 = 0$). This arbitrary restriction demonstrates the importance of the dynamic term in equation (22). Each table of results presents mean coefficient estimates along with the standard error of these estimates (as opposed to the mean standard error). The figures in square brackets report the number of coefficients estimated to be statistically different from zero at the 5% level of significance and this term is used as an indication of the 'precision' of the estimates rather than the mean standard error. An 'adjusted mean' figure is also reported for each coefficient; this is obtained by setting non statistically significant coefficient estimates to zero when calculating the averages. The tables also report mean R-square and mean adjusted R-square statistics along with mean P-values for the F-statistic for overall significance (these cannot be computed for the GMM estimator) and mean P-values for the Hansen J-statistic which tests the va-

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[14]Carare and Tchaidze (2005, p.15) note that the four-lags-as-instruments specification is the standard approach in the interest rate rule literature (e.g. Orphanides, 2001). However, we preserve the sample size of each estimation run rather than add an additional (fifth) lag of the expected future nominal interest rate to the GMM instrument set. The 2SLS and GMM estimators yield identical estimates if the instrument set for the latter is restricted to match that of the former. Note that although the GMM procedure in general corrects for autocorrelation and heteroskedasticity for actual data estimation, in estimating with simulated data we use lags as instruments for pre-determined variables that are free from simultaneity bias. The instruments may be 'good' because the data is serially correlated but no further lags are needed for the estimating equation itself. For actual data, Clarida et al. (QJE, 2000, p.153) use a GMM estimator "with an optimal weighting matrix that accounts for possible serial correlation in [the error term]" but they also add two lags of the dependent variable to their estimating equation on the basis that this "seemed to be sufficient to eliminate any serial correlation in the error term." (p.157), implying that the GMM correction was insufficient.

[15]Jondeau et al. (2004, p.227) state that: "To our knowledge, all estimations of the forward-looking reaction function based on GMM have so far relied on the two-step estimator." They proceed to consider more sophisticated GMM estimators but nevertheless identify advantages to the "simple approach" (p.238) adopted in the literature.

lidity of the instrument set (these can only be calculated in the presence of over-identifying restrictions), and mean Durbin-Watson (D-W) statistics which test for autocorrelation. The number of estimation runs for which the null hypothesis of the F-statistic is rejected and the number for which the J-statistic is *not* rejected are reported alongside the relevant P-values and the number of estimation runs for which the D-W test statistic exceeds its upper critical value is reported alongside this test statistic.[16]

5.2   General Taylor Condition

Tables 3-5 present estimates obtained from the following 'correctly specified' estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (26)$$

Expected future variables on the right hand side are obtained directly from the model simulation procedure and are instrumented for as described above.

The key result is that Tables 3-5 consistently report an inflation coefficient which exceeds unity for the empirical model which most accurately reflects the underlying theoretical model. This result is found to be robust to the statistical filter applied to the data and to the estimator employed, subject to the estimator providing a 'precise' set of estimates. The forward interest rate term is also found to be important in terms of generating a coefficient on inflation consistent with the underlying, Benk et al. (2010), model. Arbitrarily omitting this dynamic term yields much smaller estimates of the inflation coefficient to the extent that most of the estimates now fall below the economically significant threshold of unity.

In terms of the general features of the results obtained from the unrestricted specification, the OLS and GMM procedures tend to generate a greater number of statistically significant estimates than the 2SLS estimator. Focusing on Table 5, the 2SLS estimator provides a statistically significant estimate for the inflation coefficient for only 580 of the 1000 simulated histories in Table 5 while the OLS and GMM estimators both return 1000 significant estimates. The constant term, for which very few statistically significant estimates are generated, stands as an exception but this finding is consistent with equation (22). The OLS and GMM procedures generate reasonably large R-square and adjusted R-square statistics, whereas negative R-square statistics are obtained from the simple 2SLS estimator, possibly symptomatic of an inadequate instrument set. Expanding the instrument set in order to implement the GMM procedure leads to 1000 rejections of the J-test for instrument validity across all three filters. One might also be wary of the low number of decisive D-W statistic rejections produced by the OLS procedure, although the mean D-W statistic remains 'reasonably large' in each case; 1.56 for the 2-15 filter, for example. The results for

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[16] The D-W count includes only decisive rejections, i.e. it excludes test statistics which lie in the inconclusive region of the test's critical values.

the 3-8 band pass filter in Table 4 are unusual in the sense that all three estimation procedures produce a very low number of D-W statistic rejections. For the other two filters, this undesirable result is specific to the OLS estimator.

Consider in particular the results presented in Table 5 for the 2-15 band pass filter, i.e. the filter which retains more of the low frequency components of the simulated data. The mean coefficient on inflation is estimated to be 2.179 using the OLS estimator and 2.306 using the GMM estimator.[17] These estimates compare favorably to the theoretical value of $\Omega = 2.125$. The right hand side of Table 5 shows that the mean of the estimated inflation coefficients falls below unity when the the forward interest rate term is arbitrarily omitted from the estimating equation; a precise mean estimate of 0.614 is obtained from the OLS procedure and a similarly precise mean estimate of 0.964 is obtained from the GMM procedure. Similar OLS and GMM estimates are obtained for the coefficient on inflation under the two alternative band pass filters in Tables 4 and 5, both in terms of the mean coefficient estimates for the unrestricted specification and in terms of the decline in magnitude induced when $\beta_5 = 0$ is arbitrarily imposed upon the estimating equation.

Compared to the estimated inflation coefficients, the estimated coefficients on consumption growth and productive time growth diverge more from their theoretical predictions for the 'unrestricted' estimating equation. Under log utility ($\theta = 1$), the former should take the same magnitude as the coefficient on inflation and the latter should take a value of zero. The mean estimates of both of these theoretical parameters can be used to 'back-out' an estimate of the coefficient of relative risk aversion (θ). Firstly, using the mean GMM estimate for the coefficient on consumption growth of 0.302 (Table 5) and the corresponding estimate of Ω, an implied estimate of θ can be calculated as $\frac{\beta_2}{\beta_1} = \frac{0.302}{2.306} = 0.131$, which is smaller than the baseline calibration of $\theta = 1$. Alternatively, the relationship $\beta_3 = \beta_1 \psi (1 - \theta) l / (1 - l)$, which is obtained from equation (22) with Ω replaced by its estimate β_1, can also be used to obtain an implied value of θ. Using the estimates presented in Table 5, the implied estimate of θ is 1.103, which is much closer to the calibrated value of $\theta = 1$.

Table 5 also reports that both the OLS and GMM procedures generate 1000 statistically significant estimates for the coefficient on velocity growth under the unrestricted estimating equation and that the mean of the estimated coefficients is correctly signed for both estimators. The mean of the point estimates are reported as -0.196 and -0.269 for OLS and GMM respectively; these are smaller than the theoretical prediction of -0.065. Similar estimates are obtained under the HP and 3-8 filters. Finally, Table 5 reports mean estimates of -1.761 (OLS) and -1.729 (GMM) for the forward interest rate coefficient compared to a theoretical value of -1.125. The mean estimates are therefore correctly signed but, again, smaller than the theoretical prediction.

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[17] The discussion focuses on the OLS and GMM estimators because they produce more 'precise' estimates and also because the OLS estimator tends to reject the null hypothesis of the F-statistic more frequently than the 2SLS estimator (1000 vs. 907 rejections in Table 5, for example). The OLS regressions are possibly afflicted by autocorrelation, however, as discussed above.

The standard, 'reaction function' interpretation of a coefficient on inflation in excess of unity is that policymakers adhere to a rule which reflects their intolerance towards inflation deviations from target. However, this interpretation is not applicable to the Taylor condition. The general result that the coefficient on inflation exceeds unity is a consequence of a simple money growth rule not a measure of policymakers' attitude towards inflation. Analogously, the break-down of the Taylor principle under the 'restricted' estimating equation in the following subsection cannot be interpreted as a softening of policymakers' attitude towards inflation; it simply emanates from model misspecification.

5.3 Taylor Condition with Output Growth

The same estimation procedure is now applied to an estimating equation which replaces the term in the consumption growth rate in equation (26) with a term in output growth as follows:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \quad (27)$$

Crucially, the simulated data remains unchanged therefore equation (27) represents a misspecified version of the 'correct' estimating equation, which continues to be equation (26). In particular, equation (27) can be seen to correspond to the misspecified Taylor condition (24).

The results are similar across the HP, the 3-8 band-pass and the 2-15 band-pass filters. As the latter gives the most statistically significant results for variable estimation, here only that table is presented, as Table 6, for equation (27).[18] Comparing the general features of the results to those presented in Tables 3-5, there is a decline in the precision with which the coefficients are estimated, a decline in the magnitude of the R-square and adjusted R-square statistics and a decline in the number of rejections of the null hypothesis of the F-statistic for joint significance. This is unsurprising given that an element of misspecification has been introduced into the estimating equation. The number of decisive rejections of the null hypothesis of the D-W test statistic also tends to decline although the GMM procedure applied to 2-15 filtered data still rejects for 94.5% of the estimation runs.

The estimated coefficients on inflation are now found to be substantially greater than the coefficients obtained from the 'correctly specified' estimating equation (26). For instance, the GMM estimate for the unrestricted estimating equation rises from 2.306 in Table 5 to 5.274 in Table 6 (or 5.235 according to the adjusted mean). Similarly, the OLS estimate increases from 2.179 to 4.219 (or 4.185 adjusted). Corresponding upward shifts in the estimated inflation coefficient are found when comparing Table 4 to the 3-8 band pass filter results (not shown) and even larger increases are found for the HP filtered data (not shown). Therefore, the estimates diverge further from the theoretical value of $\Omega = 2.125$ under this particular form of misspecification.

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[18]The instrument sets used for the 2SLS and GMM estimators are modified by replacing consumption growth with output growth but remain unchanged in terms of the number of lags included.

HP filtered data,	Unrestricted			Assumed $\beta_5 = 0$		
where HP $\lambda = 6.25$	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-9.68E-07 [0]	-2.04E-07 [0]	8.09E-07 [17]	-7.60E-07 [0]	-4.78E-07 [0]	-7.57E-08 [8]
Standard error	2.87E-05	2.15E-05	3.15E-05	2.39E-05	1.67E-05	4.29E-05
Adjusted mean	-	-	-4.27E-08	-	-	2.59E-07
$E_t\pi_{t+1}$	2.019 [1000]	2.309 [691]	2.299 [1000]	0.315 [830]	0.757 [397]	0.621 [925]
Standard error	0.248	1.488	0.268	0.126	0.856	0.265
Adjusted mean	2.019	1.800	2.299	0.293	0.475	0.614
$E_t g_{c,t+1}$	0.251 [1000]	0.336 [959]	0.293 [1000]	0.172 [1000]	0.313 [989]	0.231 [1000]
Standard error	0.024	0.096	0.020	0.020	0.048	0.025
Adjusted mean	0.251	0.324	0.293	0.172	0.311	0.231
$E_t g_{l,t+1}$	-0.243 [890]	-0.536 [774]	-0.374 [997]	-0.281 [864]	-0.530 [774]	-0.427 [996]
Standard error	0.094	0.321	0.079	0.100	0.231	0.111
Adjusted mean	-0.236	-0.448	-0.374	-0.265	-0.453	-0.427
$E_t g_{V,t+1}$	-0.137 [990]	-0.267 [800]	-0.212 [1000]	-0.098 [889]	-0.317 [888]	-0.190 [992]
Standard error	0.031	0.228	0.033	0.036	0.109	0.056
Adjusted mean	-0.137	-0.229	-0.212	-0.093	-0.293	-0.190
$E_t R_{t+1}$	-1.819 [1000]	-2.338 [646]	-2.005 [1000]	N/A	N/A	N/A
Standard error	0.221	2.282	0.277			
Adjusted mean	-1.819	-1.692	-2.005			
$Mean;$						
R-square	0.789	<0	0.796	0.544	<0	0.482
Adjusted R-square	0.778	<0	0.785	0.525	<0	0.459
Pr(F-statistic)	2.35E-15 (1000)	0.015 (974)	N/A	3.93E-09 (1000)	0.003 (992)	N/A
Pr(J-statistic)	N/A	N/A	0.258 {1000}	N/A	0.159 {482}	0.269 {1000}
Durbin-Watson	1.474 <151>	2.243 <1000>	2.194 <970>	1.732 <419>	2.145 <999>	2.047 <882>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· [ ] records the number of statistically significant coefficient estimates, ( ) the number of F-statistic rejections,

and {} the number of J-statistic non-rejections (all at the 5% level of significance).

· <> records the number of times the D-W statistic exceeds its upper critical value (i.e. rejects null of positive A.C.)

Table 3: Taylor Condition Estimation, HP Filtered Data, Ravn and Uhlig (2002)
Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-7.57E-07 [0]	6.54E-06 [0]	-7.10E-07 [3]	7.73E-07 [0]	-2.86E-06 [0]	-1.51E-06 [1]
Standard error	1.65E-05	3.24E-04	2.09E-05	1.45E-05	4.37E-05	2.68E-05
Adjusted mean	-	-	-1.83E-07	-	-	-9.81E-08
$E_t\pi_{t+1}$	2.166 [998]	2.484 [724]	2.423 [1000]	0.633 [969]	2.417 [965]	0.682 [974]
Standard error	0.391	32.906	0.298	0.195	1.125	0.222
Adjusted mean	2.166	2.141	2.423	0.628	2.291	0.679
$E_t g_{c,t+1}$	0.283 [1000]	0.304 [623]	0.314 [1000]	0.155 [1000]	0.175 [834]	0.168 [1000]
Standard error	0.043	7.324	0.027	0.029	0.074	0.030
Adjusted mean	0.283	0.231	0.314	0.155	0.160	0.168
$E_t g_{l,t+1}$	-0.237 [827]	-0.573 [430]	-0.312 [982]	-0.222 [685]	-0.595 [666]	-0.268 [870]
Standard error	0.131	10.199	0.099	0.133	0.361	0.135
Adjusted mean	-0.229	-0.193	-0.312	-0.195	-0.455	-0.259
$E_t g_{V,t+1}$	-0.152 [982]	-0.453 [351]	-0.174 [998]	-0.174 [976]	-0.604 [973]	-0.194 [984]
Standard error	0.043	15.068	0.039	0.052	0.252	0.057
Adjusted mean	-0.151	-0.128	-0.174	-0.173	-0.578	-0.193
$E_t R_{t+1}$	-2.026 [994]	-1.532 [424]	-2.289 [1000]			
Standard error	0.435	116.012	0.339			
Adjusted mean	-2.024	-1.211	-2.289			
$Mean;$						
R-square	0.789	<0	0.842	0.576	<0	0.590
Adjusted R-square	0.778	<0	0.833	0.558	<0	0.572
Pr(F-statistic)	4.75E-10 (1000)	0.077 (874)	N/A	2.08E-07 (1000)	0.005 (981)	N/A
Pr(J-statistic)	N/A	N/A	0.213 {1000}	N/A	0.344 {848}	0.249 {1000}
Durbin-Watson	1.568 <54>	1.653 <330>	1.517 <49>	1.728 <333>	1.728 <378>	1.715 <306>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· [ ] reports the number of statistically significant coefficient estimates, ( ) reports the number of F-statistic rejections, and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

· <> records the number of times the D-W statistic exceeds its upper critical value (i.e. rejects null of positive A.C.)

Table 4: Taylor Condition Estimation, Band Pass Filtered Data (3-8 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-2.26E-06 [0]	4.22E-05 [0]	2.54E-07 [22]	-1.82E-06 [0]	-2.46E-06 [0]	3.44E-06 [21]
Standard error	4.01E-05	0.001	4.82E-05	3.27E-05	4.83E-05	6.26E-05
Adjusted mean	-	-	4.25E-07	-	-	9.03E-07
$E_t\pi_{t+1}$	2.179 [1000]	3.816 [580]	2.306 [1000]	0.614 [999]	1.127 [763]	0.964 [999]
Standard error	0.195	51.040	0.272	0.108	0.640	0.169
Adjusted mean	2.179	1.402	2.306	0.614	0.936	0.963
$E_t g_{c,t+1}$	0.277 [1000]	0.570 [730]	0.302 [1000]	0.170 [1000]	0.262 [851]	0.207 [1000]
Standard error	0.016	5.546	0.025	0.017	0.103	0.026
Adjusted mean	0.277	0.265	0.302	0.170	0.230	0.207
$E_t g_{l,t+1}$	-0.295 [997]	-0.737 [526]	-0.359 [999]	-0.210 [732]	-0.263 [405]	-0.277 [935]
Standard error	0.067	8.208	0.085	0.088	0.203	0.111
Adjusted mean	-0.294	-0.242	-0.359	-0.182	-0.152	-0.272
$E_t g_{V,t+1}$	-0.196 [1000]	-0.347 [807]	-0.269 [1000]	-0.158 [998]	-0.307 [944]	-0.236 [1000]
Standard error	0.024	0.271	0.031	0.032	0.077	0.042
Adjusted mean	-0.196	-0.273	-0.269	-0.158	-0.292	-0.236
$E_t R_{t+1}$	-1.761 [1000]	-5.586 [335]	-1.729 [1000]			
Standard error	0.201	114.905	0.322			
Adjusted mean	-1.761	-0.712	-1.729			
$Mean;$						
R-square	0.830	<0	0.782	0.625	<0	0.522
Adjusted R-square	0.821	<0	0.770	0.609	<0	0.501
Pr(F-statistic)	2.50E-24 (1000)	0.051 (907)	N/A	5.04E-10 (1000)	0.003 (985)	N/A
Pr(J-statistic)	N/A	N/A	0.315 {1000}	N/A	0.298 {757}	0.298 {1000}
Durbin-Watson	1.558 <141>	2.059 <972>	2.040 <881>	1.954 <864>	2.052 <998>	2.205 <977>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· [ ] reports the number of statistically significant coefficient estimates, ( ) reports the number of F-statistic rejections, and {} reports the number of J-statistic non-rejections (all at the 5% level of significance).

· <> records the number of times the D-W statistic exceeds its upper critical value (i.e. rejects null of positive A.C.)

Table 5: Taylor Condition Estimation, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

The incorrectly specified estimating equation also induces a substantial decrease in the estimated coefficients for the productive time growth rate and the forward nominal interest rate. These estimates therefore tend to diverge further from the theoretical predictions of the model. The estimated coefficient on the productive time growth rate decreases from -0.294 to -2.073 (both adjusted means) between Table 5 and Table 6 according to the OLS estimator and from -0.359 to -2.790 for the GMM estimator, compared to the theoretical prediction of $\beta_3 = 0$. The GMM estimates of the forward interest rate term also decrease from -2.005, -2.289 and -1.729 under the HP filter, 3-8 and 2-15 filters respectively to -12.868, -6.203 and -4.372 (adjusted means where appropriate; HP and 3-8 results not shown). Again, the estimates diverge further from theoretical value of -1.125.

The estimated coefficients for output growth in Table 6 are comparable to those for consumption growth presented in Tables 3-5, despite the impact that the misspecification has on the other estimates. For example, the OLS estimate for β_3 in Table 6 is 0.300 (adjusted) compared to the corresponding estimate of 0.277 in Table 5. For the GMM estimator the coefficient on output growth is 0.402 (adjusted) in Table 6 compared to the corresponding estimate of 0.302 reported in Table 5.

The velocity growth term is estimated precisely by the GMM estimator even after the modification to the estimating equation. Estimates of β_4 retain the correct sign and are of a similar magnitude as under the correctly specified estimating equation; for example, a GMM estimate of -0.190 in Table 6 compared to a corresponding estimate of -0.269 in Table 5.

Considering the restricted specification ($\beta_5 = 0$), the estimates undergo similar changes as those obtained from the restricted version of the 'correct' estimating equation (26). The OLS and GMM estimators generate inflation coefficients which often fall below unity in a manner incompatible with the theoretical model from which the Taylor condition is derived, although there is one notable exception to this for the GMM estimator in Table 6.

In short, the estimates obtained from applying equation (27) to the simulated data show that adapting the estimating equation in a seemingly minor way can have a substantial impact upon the reported estimates. The erratic results obtained from this misspecified estimating equation provide an illustration of the fundamental difference between the Taylor condition and a conventional interest rate rule. Unlike a Taylor rule, the Taylor condition cannot be modified in an *ad hoc* fashion.[19] In order to make the progression from (26) to (27) in a legitimate manner, one would need to alter the underlying model in some way (excluding physical capital in particular). A new set of artificial data would then need to be simulated from this alternative model prior to re-estimation.

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[19] In contrast, conventional interest rate rules are exogenously specified and thus amenable to arbitrary modifications. Clarida et al. (1998), for example, add the exchange rate to the standard Taylor rule and Cecchetti et al. (2000) and Bernanke and Gertler (2001) consider whether policymakers should react to asset prices.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-3.66E-06 [0]	0.001 [0]	3.07E-06 [16]	-9.34E-07 [0]	-4.46E-06 [0]	2.80E-06 [5]
Standard error	6.19E-05	0.031	8.58E-05	2.43E-05	9.65E-05	6.42E-05
Adjusted mean	-	-	-9.38E-07	-	-	4.23E-07
$E_t\pi_{t+1}$	4.219 [971]	-25.003 [189]	5.274 [961]	0.541 [941]	2.338 [882]	1.101 [990]
Standard error	1.715	1290.303	2.582	0.170	1.151	0.264
Adjusted mean	4.185	2.522	5.235	0.532	2.010	1.100
$E_t g_{y,t+1}$	0.303 [967]	-2.019 [206]	0.406 [940]	0.038 [563]	0.211 [262]	0.082 [956]
Standard error	0.125	122.643	0.204	0.020	0.304	0.029
Adjusted mean	0.300	0.244	0.402	0.029	0.077	0.081
$E_t g_{l,t+1}$	-2.098 [959]	14.698 [211]	-2.815 [954]	-0.284 [424]	-1.462 [353]	-0.610 [941]
Standard error	0.892	825.197	1.417	0.189	1.740	0.232
Adjusted mean	-2.073	-1.672	-2.790	-0.189	-0.655	-0.598
$E_t g_{V,t+1}$	-0.118 [884]	0.069 [310]	-0.191 [970]	-0.095 [733]	-0.246 [587]	-0.158 [923]
Standard error	0.042	16.979	0.064	0.043	0.164	0.061
Adjusted mean	-0.113	-0.117	-0.190	-0.084	-0.161	-0.156
$E_t R_{t+1}$	-3.878 [907]	29.503 [128]	-4.498 [849]			
Standard error	1.812	1437.910	2.802			
Adjusted mean	-3.767	-1.757	-4.372			
$Mean;$						
R-square	0.361	<0	0.127	0.246	<0	<0
Adjusted R-square	0.327	<0	0.079	0.214	<0	<0
Pr(F-statistic)	0.001 (995)	0.379 (411)	N/A	0.020 (924)	0.055 (829)	N/A
Pr(J-statistic)	N/A	N/A	0.226 {1000}	N/A	0.260 {682}	0.264 {1000}
Durbin-Watson	1.882 <699>	1.982 <868>	2.342 <945>	2.295 <1000>	2.145 <997>	2.728 <999>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· [ ] reports the number of statistically significant coefficient estimates, ( ) reports the number of F-statistic rejections and { } reports the number of J-statistic non-rejections (all at the 5% level of significance).

· <> records the number of times the D-W statistic exceeds its upper critical value (i.e. rejects null of positive A.C.)

Table 6: Output Growth instead of Consumption Growth, Band Pass Filtered
data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

5.4 A Conventional Interest Rate Rule

The estimation procedure is now re-applied to the following estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{28}$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth; equation (25). This can be interpreted as a 'dynamic forward-looking Taylor rule' for $\beta_5 \neq 0$ or a 'static forward-looking Taylor rule' under the restriction $\beta_5 = 0$. Notably, the term in velocity growth is absent from this expression. This omission would be expected to have a bearing on the estimates because equations (26) and (27) produced many statistically significant estimates for this term.

Results are again similar across the HP and two band-pass filters so only the 2x15 band-pass results are presented, in Table 7, for equation (28).[20] The estimates are generally found to be poor in terms of the number of statistically significant cases produced and in terms of mean R-square and adjusted R-square statistics. This is unsurprising given that we have added yet another source of misspecification to the estimating equation. However, the number of D-W statistic rejections remains high for the HP (not shown) and the 2-15 filters, while we obtain the same result of fewer decisive rejections for the 3-8 filter as was obtained from the 'correctly specified' estimating equation (not shown).

The estimated coefficient on inflation does not exceed unity for the three filters and estimators considered according to the adjusted mean, these being the appropriate figures to consult given the low number of statistically significant estimates. The results are also comparatively weak in terms of the frequency with which the null hypothesis of the F-statistic is rejected, the HP filtered data gives particularly poor results in this regard, (not shown) and in terms of the number of non-rejections of the null hypothesis of the Hansen J-test (not shown). The latter finding calls into question the validity of the standard instrument set used for the GMM estimator under equation (28).

The estimation results are similarly imprecise under the restricted specification ($\beta_5 = 0$) for the HP filter (not shown), although we now obtain a reasonable number of significant estimates for the coefficient on inflation for the OLS and GMM estimators in the 3-8 band-pass filter (not shown) and in Table 7. These estimates are similar for the 3-8 band pass filter, 0.227 (adjusted mean, OLS) and 0.282 (adjusted mean, GMM), but differ quite substantially for the 2-15 filter (0.317, OLS, compared to 0.892, GMM).

In short, replicating a conventional Taylor rule restricts the 'true' estimating equation to such an extent that the theoretical prediction that the coefficient on expected inflation exceeds unity is not recovered even under the unrestricted estimating equation. Results such as these when derived from an estimating

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[20] The instrument set now comprises of four lags of expected future inflation, four lags of expected future output growth, the second, third and fourth lags of the nominal interest rate and a constant term for the GMM estimator or just the shortest of these lags and a constant term for the exactly identified 2SLS estimator.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-6.03E-07 [0]	-8.11E-05 [0]	-2.84E-06 [7]	-5.32E-07 [0]	1.85E-07 [0]	-2.73E-07 [11]
Standard error	2.60E-05	0.008	1.16E-04	2.59E-05	2.98E-04	1.13E-04
Adjusted mean	-	-	-1.12E-06	-	-	4.92E-07
$E_t\pi_{t+1}$	0.311 [238]	12.975 [21]	0.142 [349]	0.327 [929]	1.493 [412]	0.895 [980]
Standard error	0.446	480.114	0.950	0.099	4.468	0.315
Adjusted mean	0.185	0.020	0.110	0.317	0.814	0.894
$E_t g_{y,t+1}$	0.020 [281]	-0.666 [40]	0.019 [236]	0.021 [404]	0.027 [162]	0.031 [456]
Standard error	0.017	33.878	0.027	0.012	0.825	0.027
Adjusted mean	0.011	0.012	0.012	0.013	0.041	0.024
$E_t R_{t+1}$	0.007 [140]	-5.714 [27]	0.909 [524]	N/A	N/A	N/A
Standard error	0.473	387.875	0.999			
Adjusted mean	0.010	0.101	0.781			
$Mean$;						
R-square	0.168	<0	<0	0.153	<0	<0
Adjust R-square	0.142	<0	<0	0.136	<0	<0
Pr(F-statistic)	0.030 (882)	0.527 (162)	N/A	0.025 (889)	0.146 (602)	N/A
Pr(J-statistic)	N/A	N/A	0.050 {340}	N/A	0.351 (677)	0.058 (448)
Durbin-Watson	1.827 <848>	2.013 <974>	2.237 <991>	1.817 <780>	2.047 <996>	2.186 <990>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if over-identifying restrictions).

· [ ] records the number of statistically significant coefficient estimates, ( ) the number of F-statistic rejections and {} the number of J-statistic non-rejections, at 5% level of significance.

· <> records the number of times the D-W statistic exceeds its upper critical value (i.e. rejects null of positive A.C.)

Table 7: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

equation such as (28) might erroneously be interpreted to signify that the Taylor principle is violated but this result is simply a product of a misspecified estimating equation in the present context. Only if the model excluded physical capital and set velocity to one, with $\beta_5 = 0$, would such a model be valid and so be expected to be well-estimated from simulated data.

6 Alternative Interpretations of the Taylor Condition

Consider two alternative expressions mathematically of the Taylor condition of equation (22). First a backward-looking model is specified and discussed. Then a credit version of the model is stated and briefly discussed.

6.1 Backward Looking Taylor Condition

Mathematically, the Taylor condition can also be formulated to have a lagged dependent on the right hand side instead of the lead independent variable which appears in equation (22). This yields a similar expression written in terms of R_{t+1} instead of R_t:

$$
\begin{aligned}
\overline{R}_{t+1} - \overline{R} \;=\; & \frac{\Omega}{(\Omega-1)} E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \frac{\Omega\theta}{(\Omega-1)} E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) \\
& - \frac{\Omega\psi(\theta-1)\frac{l}{1-l}}{(\Omega-1)} E_t\overline{g}_{l,t+1} - \frac{\Omega_V}{(\Omega-1)} E_t\overline{g}_{V,t} - \frac{1}{(\Omega-1)}\left(\overline{R}_t - \overline{R}\right).
\end{aligned}
\tag{29}
$$

While equation (29) compares better to interest rate rules which feature the lagged dependent variable on the right hand side as an 'interest rate smoothing' term, the lead nominal interest rate now is the dependent variable. Such a postulated model is more akin to a forecasting equation for the nominal interest rate than to an interest rate condition that compares to the Taylor rule. More fundamentally such a transformation raises the issue identified by McCallum (2010). He argues that the equilibrium conditions of a structural model stipulate whether any given difference equation is forward-looking ("expectational") or backward-looking ("inertial") and that the researcher is not free to alter the direction of causality implied by the model as is convenient. The forward looking representation of the Taylor condition (22) is the long accepted rational expectations version; for example, Lucas (1980) suggests that the forward looking "filters" suit models which feature an optimizing consumer. In fact, we would argue that the timing of the cash-in-advance economy is such that our forward-looking rule in equation (22) is the exact correct model, while the above equation is consistent with the alternative "cash-when-your done" timing which we do not employ (see Carlstrom and Fuerst, 2001).

6.2 Credit Interpretation of the Taylor Condition

Christiano et al. (2010) have considered how the growth rate of credit might be included as part of a Taylor rule so that "allowing an independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices." The term on the growth of velocity can be inversely interpreted as a growth rate of credit in the following way. Since $V_t =$

$\frac{c_t}{m_t} = \frac{1}{1 - \left(1 - \frac{m_t}{c_t}\right)}$, then $V\widehat{V}_t = \left(1 - \frac{m}{c}\right)\left(1 - \widehat{\frac{m_t}{c_t}}\right)$ and $\overline{g}_{V,t} = \frac{m}{c}\left(1 - \frac{m}{c}\right)\overline{g}_{\left(1 - \frac{m}{c}\right),t}$ where $\overline{g}_{\left(1 - \frac{m}{c}\right),t}$ is the growth rate of normalized credit. The equivalent Taylor condition to equation (22) is

$$
\begin{aligned}
\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1 - \theta\right)\frac{l}{1 - l}E_t\overline{g}_{l,t+1} \\
&\quad - \Omega_{\left(1 - \frac{m}{c}\right)}E_t\overline{g}_{\left(1 - \frac{m}{c}\right),t+1} - \Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right). \quad\quad (30)
\end{aligned}
$$

The credit coefficient can be derived as $\Omega_{\left(1 - \frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma)\left(\frac{m_t}{c_t}\right)^2}{(1-\gamma)^2}\frac{(1-\gamma)\left(1 - \frac{m_t}{c_t}\right)}{R\left[1 - (1-\gamma)\left(1 - \frac{m_t}{c_t}\right)\right]} \geq$ 0. A positive expected credit growth rate decreases the current net nominal interest rate $\overline{R}_t$. With velocity set at one as in a standard cash-in-advance economy, then neither the credit or velocity would enter the equation since either it does not exist (credit) or it cannot change (velocity) over time.

7   Discussion

Expressing the monetary policy process in terms of the nominal interest rate has the advantage of reconciling the language of economists who have traditionally depicted the money supply as the instrument of monetary policy with the language of central bankers, who are more accustomed to conducting policy deliberations in terms of a short-term interest rate (Mehrling, 2006). Alvarez et al. (2001) caution that modelling monetary policy solely in terms of a nominal interest rate rejects the quantity theory in spite of the strong empirical link between money growth, inflation and interest rates. Schabert (2003), for example, uses the equilibrium conditions of a standard cash-in-advance (CIA) model in order to derive the conditions under which a money supply rule and an interest rate rule are 'equivalent', while Fève and Auray (2002) generate simulated data from a similar model and demonstrate that an interest rate rule can be spuriously recovered from this data even though monetary policy is modelled in terms of a money growth rule.

This paper has derived an expression similar to a conventional interest rate rule as an equilibrium condition of an endogenous growth model with endogenous velocity in which monetary policy is characterized as a stochastic money supply rule. The theoretical model underpinning this expression implies that the coefficient on inflation exceeds unity in general, takes a value of unity as a special case at the Friedman (1969) optimum but that it may not fall below unity. Simulation exercises support the theoretical restriction placed on this coefficient, so long as the estimating equation accurately reflects the equilibrium condition.

Our results can be interpreted in several ways. First, the derivation could be said to represent an 'equivalence proposition' between the money supply process modelled and an 'interest rate rule', which actually represents an equilibrium condition of the model. This would be similar to the interpretation adopted in Alvarez et al. (2001), Végh (2002) and Schabert (2003), and

Second, the Taylor condition can be interpreted as the interest rate rule which results from the money supply process in the context of the Benk et al. (2010) model. Woodford similarly derives the interest rate rule which "implements" strict inflation targeting in the New Keynesian model (Woodford, 2003, pp.290-295). However, the money supply does not enter that model. Changes in the velocity of money therefore play no role and thus cannot be used to help explain why traditional Taylor rule estimations might use misspecified estimating equations in finding an inflation coefficient of less than one. The fact that our framework assigns a central role to money potentially implies that the money growth rule can offer guidance to policymakers at times when the conventional monetary policy instrument encounters the zero lower bound, as is the case at the present time.

Third the Taylor condition contrasts with the equilibrium condition for the nominal interest rate derived from a standard Euler equation: Canzoneri et al. (2007, p.1866), for example, derive an expression for the nominal interest rate from a conventional Euler equation in which the coefficient on the term in inflation is one.[21] For post-1966 U.S. data, they show that the Euler-equation-implied nominal interest rate fits poorly to the observed nominal interest rate. On the other hand, a conventional Taylor rule with a coefficient on inflation in excess of unity has often been found to fit the observed nominal interest rate well (e.g. Taylor, 1993). For example, Clarida et al. (2000) find such an estimate for 'post-Volcker' subsamples (but not pre-Volcker). The Taylor condition (22) therefore represents an equilibrium condition which contains a coefficient on inflation consistent with empirical results which find evidence of a 'Taylor principle', while suggesting that results which fail to find support for the Taylor principle may omit potentially important variables such as the velocity of money.

The Taylor condition derivation here also resonates with Hetzel (2000) who warns that empirical correlations between a short-term interest rate and macroeconomic variables such as output and inflation cannot be interpreted to reveal the behavior of policymakers (i.e. their policy rule) unless the relationship obtained can be declared as structural. It is also consistent with Cochrane (2011), who argues that the Taylor rule suffers from an identification problem in the New Keynesian model. Our contribution has been to offer one very particular explanation based on a neoclassical monetary model extended to include endogenous growth and endogenous velocity in order to shed light on the structural relationships which might underpin the reduced form expressions to which Hetzel (2000) refers.

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[21] Their expression is a log-normal approximation to a standard Euler equation and is written in terms of the inverse of the gross nominal interest rate. Therefore, it also contains second moments and the coefficient on inflation has a theoretical coefficient of *minus* one.

8 Conclusion

The paper has derived a general equilibrium dynamic Taylor condition for a constant relative risk aversion economy with leisure, Lucas (1988) endogenous growth, and with endogenous velocity through production of exchange credit in a financial intermediary. The importance of a fluctuating velocity in reproducing the 'Taylor principle' is consistent with the role for velocity reported by Reynard (2004, 2006) and with Benk et al. (2010).

While providing a theoretical means to overview the empirical literature relating to the Taylor rule, as reviewed by Siklos and Wohar (2005), here the focus is first to show that estimation of a Taylor rule may result in a spurious inference that the central bank is engaged in Taylor principle behavior, rather than simply supplying money. This is established here by generating artificial data as simulated from the model and then successfully estimating a theoretical Taylor condition. This condition is simply an equilibrium condition in the economy in which the central bank stochastically makes changes in the money supply growth rate to finance government spending. For example, such money supply changes tend to occur whenever the government needs to depart from its stationary money supply growth rate and resort to the 'fiscal inflation tax'. This typically can occur during banking crisis, recession, or war.

Money velocity growth itself enters as a variable and ends up playing a potentially significant role; in particular this occurs when velocity is changing, such as during the recent banking crisis and during the 1930s when US velocity cycled downwards, as identified in Benk et al. (2010), and in the "pre-Volcker" US high inflation of the 1970s. Velocity is endogenized in the model following the banking financial intermediation microeconomic literature, where financial services are produced according to a Cobb-Douglas production function that includes deposited funds as an input. This approach implies a bank service sector value-added that is consistent with the US national income accounting treatment of the bank service sector.

The paper exhibits how the banking production of exchange credit is surprisingly crucial to the derivation of a Taylor principle whereby the coefficient on the inflation term is in fact greater than one. This results only through an endogenous velocity of money; a simple CIA (cash-only) constraint with a constant velocity of one is shown to provide an inflation coefficient of unity. Through endogenous growth, we can derive an output gap measure not inconsistent with Taylor and Wieland's (2010) emphasis on changes in output as a measure for the output gap. In our model, the output growth term does not enter directly unless we also include an investment growth term; otherwise the consumption growth is the 'output gap' term of the model's Taylor condition.

Estimation results are also given for two misspecified models using the simulated data from the correct model. One includes output growth without including investment growth. The second is a standard Taylor rule that exists in the model economy only if there is no physical capital and if there cannot exist exchange credit as a substitute to money ($A_F = 0$). Omitted variables cause significant misspecification bias in the reported results. The implication

is that the results hold promise for explaining disparate estimated rules across different periods and countries, as well as during bank crises, sudden financial deregulation, or times of other significant shifts in money velocity. This could help organize and show greater robustness for this literature.

By simulating data of the model and estimating successfully a 'Taylor rule' from the data, the paper implies that identification of such a rule econometrically can be achieved as part of the economy's asset pricing behavior when the central bank simply prints money stochastically. In that case it would be spurious to claim that such Taylor estimations show how the central bank actually conducts policy through interest rate targeting rather than through simply satisfying its fiscal needs via direct and indirect taxes, including the inflation tax. Put differently, if this economy were representative of the actual economy, then estimating a standard Taylor (1993) model using actual data would be expected to produce an above-one Taylor-principle inflation coefficient only if velocity (or exchange credit) and investment did not change over the sample period. Reynard (2004, 2006) and Benk et al. (2010) for example put doubt on a constancy of velocity while large business cycle fluctuations in investment are a well-documented feature of business cycle research.

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