Sequential Decision Problems

Lecture

Dr. James Doss-Gollin

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This lecture borrows heavily from Herman et al. (2020).

Introduction

> **!** Important
>
> I am actively adding more material to this lecture!

House elevation problem

Thus far, we have looked at a single *static* decision: how high to elevate the house. However, in many cases, decisions are not static, but rather *sequential*. For example, we don't necessarily need to make this decision today! Instead, we could wait and see how fast local sea-levels are rising, and then make a decision later **with more information**.

General dynamic decision problems

Dynamic planning problems identify policies to select actions in response *to new information over time*. Policy design involves choosing the sequence, timing, and/or threshold of actions to achieve a desired outcome. This typically involves a combination of optimal control and adaptive design.

Optimal control

Optimal control is a family of mathematical techniques used to optimize sequential decision problems. The most important concept in optimal control is the **state** of the system. We can think of the state as the information available to the decision maker at any given time, although we can also consider formulations where some state variables are not directly observable. The state evolves over time according to a dynamics model, which describes how the state changes in response to the decision maker's actions and external factors:

$$\mathbf{x}_{t+1} = f_t(\mathbf{x}_t, a_t, e_{t+1}),$$

where - $\mathbf{x}_t$ is the state at time t, e.g. the sea level at time t - a_t is the decision at time t, e.g. whether we elevate and if so how high - e_t is some forcing, e.g., - f_t is assumed deterministic, but it can evolve over time.

The optimization problem is to choose a **policy** that selects actions to minimize some cost function (equivalently, maximize some reward function, etc.) over time.

Expected future rewards

Tip

For more a detailed introduction to optimal control or reinforcement learning, see Sutton & Barto (2018).

Herman, J. D., Quinn, J. D., Steinschneider, S., Giuliani, M., & Fletcher, S. (2020). Climate adaptation as a control problem: Review and perspectives on dynamic water resources planning under uncertainty. *Water Resources Research*, e24389. https://doi.org/10.1029/2019wr025502

Sutton, R. S., & Barto, A. G. (2018). *Reinforcement learning: An Introduction* (Second Edition). Cambridge, Massachusetts and London, England: MIT Press.