

Cost-Benefit Analysis

Lecture

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Motivation

We often want a quantitative way to compare two or more decisions. Hence, cost-benefit analysis.

It's a simple idea. For a given “decision” (being deliberately vague about what we mean by this), we need some function that tells us how good or bad the decision is. Then, we can compare the goodness of different decisions.

Tip

As a motivating example, consider that we have been asked to help a homeowner decide whether to elevate their home by 5ft to protect against future flooding, or whether to leave it as-is.

Quantifying costs and benefits

We want to pick the decision that maximizes the “goodness” of the decision. We'll call this a “utility” function, but could also use “objective function”, “loss function”, “damage function”, etc. Let's give this some formal notation:

$$u(a, \mathbf{s}) : \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$$

where u is the utility function, $a \in \mathcal{A}$ is the decision, and $\mathbf{s} \in \mathcal{S}$ is the state of the world.

- We'll use “state of the world” as a deliberately vague term that we will use to describe all the uncertainties we want to think about when making a decision.
- You can remember that a is the decision because it's the first letter of “action”.

Often, utility is measured in money. This is not because money is the only thing that matters, but because it is a common unit of account that allows us to compare different things. However, utility can be measured in other units as well – it only requires that we put all the costs and benefits into a common unit.



Figure 1: Floodproofing in Houston, TX ([Houston Public Media](#))

💡 Tip

In our house elevation example, we might consider the following costs and benefits: up-front cost of elevating; cost of flood insurance; change in property value; and cost of damages not covered by flood insurance. These might be well-described in monetary terms. However, putting a value on the peace of mind from not having to worry about future flooding, or the reduced risk of death, is trickier.

Dealing with time

A key feature of nearly all climate adaptation problems is that costs and benefits are spread out over time. How should we weigh costs and benefits that occur at different times?

💡 Tip

For example, the up-front cost of elevating a house is a cost that occurs now, while the benefits of reduced flood risk occur in the future.

The most common way to deal with this is to use net present value (NPV). The idea is to **discount** future costs and benefits to the present day. If our discount rate is $\gamma = 2\% = 0.02$, then we care about a dollar of benefits in one year the same as we care about 98 cents today. More generally, if we have a discount rate of γ then a dollar of benefits in t years is worth $(1 - \gamma)^t$ today.

The net present value of a decision is the sum of the present values of all costs and benefits:

$$NPV = \sum_{t=0}^T (1 - \gamma)^t u(a, \mathbf{s}_t)$$

where T is the time horizon of the decision. Note that we write \mathbf{s}_t to indicate that the state of the world might have time-dependent variables.

Cost-benefit analysis in the real world

Cost-benefit analysis is everywhere! Companies use it to decide whether to invest in new products or technologies, governments use it to decide whether to build new infrastructure or regulate pollution, and much more!

A standard approach is:

1. Come up with a state of the world \mathbf{s} that represents your uncertainties.
2. Write down a utility function $u(a, \mathbf{s})$ that represents your preferences.
3. Choose a discount rate γ
4. Calculate the net present value of each decision
5. Pick the decision with the highest net present value

Handling uncertainty

Bayesian decision theory is a mathematical formalization of a simple concept: decision that gives us the best utility **on average** (i.e., in expectation):

$$a^* = \arg \max_a \mathbb{E}_s [u(a, \mathbf{s})]$$

where $a \in \mathcal{A}$ is the decision, $\mathbf{s} \in \mathcal{S}$ is the **state of the world**, and $u : \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$ is a utility function. “State of the world” is another deliberately vague term that we will use to describe all the uncertainties we want to think about when making a decision.

💡 Tip

For our house elevation problem, a very simple framing would consider the state of the world to be the time series of future flood levels. A more complex framing might also consider the depth-damage function for the house, the cost of elevating the house, the cost of rebuilding a house over time, etc. etc.

Recall the definition of expected value:

$$\mathbb{E}_s [u(a, \mathbf{s})] = \int p(\mathbf{s}) u(a, \mathbf{s}) d\mathbf{s}$$

which requires a probability distribution over states of the world. So applying this theory requires a “subjective” (Savage, 1954) or “personal” probability distribution over states of the world and a utility function.

Reflection

Critiques and limitations

- Simple idea:
 - Add up all the costs
 - Add up all the benefits
- Sometimes it’s hard to combine different things that we care about into a single number!
 - Cost and safety
 - Impacts on different groups of people
 - We will revisit this in the context of multi-criteria decision analysis.
- In practice, this often leads us to care only about costs and benefits that are easy to quantify / monetize
 - Value of ecosystems?
- The limitations of discounting are especially relevant for some types of climate adaptation decisions
 - We will revisit this on Wednesday
- We often deal with “deep” uncertainties for which it’s hard to come up with a probability distribution
 - We will revisit this much later in the semester
- Is Bayesian decision theory a good model of how people actually make decisions?
 - This framework was largely formalized by Savage (1954), who postulated that people often behave as though maximizing expected utility
 - Ellsberg (1961) and others: this is not how people really make decisions!
 - Our goal is to **support** decision-making, not **predict** how people will actually make decisions. So the fact that this is not how people actually make decisions is not a problem for us.

- That said: when our predictions of “what is rational” diverge from what people actually do, we should be curious about why instead of assuming they are stupid!

A defense

Cost-benefit analysis is still useful when applied thoughtfully.

- It forces us to be explicit about our assumptions
 - What we care about and how we are measuring it
 - What we are ignoring
 - How we think about uncertainty
- Allows an apples-to-apples comparison of different decisions

Ultimately, cost-benefit analysis is a great decision-support tool, but it is not a decision-making tool. When applied well, it’s an iterative process through which we repeatedly refine our understanding of the decision problem. When applied poorly, it’s a black-box process that spits out a number that is used to justify a decision that was already made.

Next steps

Wednesday

Digging a bit deeper:

- Discount rates for problems with long time horizons (Arrow et al., 2013)
- Applications of cost-benefit analysis
 - Social cost of carbon
 - Army Corps of Engineers projects
 - EPA regulations

Thursday

Lab: implementing a simple cost-benefit analysis for house elevation.

Arrow, K., Cropper, M., Gollier, C., Groom, B., Heal, G., Newell, R., et al. (2013). Determining benefits and costs for future generations. *Science*, 341(6144), 349–350. <https://doi.org/10.1126/science.1235665>

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics*, 75(4), 643–669. <https://doi.org/10.2307/1884324>

Savage, L. J. (1954). *Foundations of statistics*. New York: Wiley.