EME 165 Project Spring 2023- Numerical Conduction

Assigned: Saturday May 6th 2023
Due: Thursday, May 25th; by 11:59 pm
Points allocated: 15

1. This is an individual project. Derivations and coding should be the sole work of the individual submitting the work. Discussions with TA, faculty and peers in the class are permitted. Use of textbook is also permitted. However, no other online learning resource/websites/online tutors may be used.
2. You are required to use Matlab or a programming language to solve the problem. Use of *Excel, or a commercially available solver is not permitted*.
3. Begin your project as soon as it is assigned. Do not wait until the last minute. Formulation and debugging of the code can take longer than you may expect.
4. Submit a single pdf of derivation/formulation and the code and solution outputs in gradescope. In a separate gradescope assignment "Project 1 code" submit your code.

Pledge (Project will not be graded without a signed student pledge):

I confirm that my work in this project is **solely my own effort** and that I have not received assistance from **any person OR organization within or outside the class.** I understand that while discussion with friends is permitted, but the formulation and coding are my own work. I furthermore understand that any impropriety will be reported to the UC Davis student misconduct office for disciplinary action.

Student Signature: ______________________

If you are using blank sheets of paper for the solution rather than this printout, acknowledge that you have read the pledge above by signing your name such as:

Pledge: I have read and accept the pledge.
Signature: ______________________

Category	**Points allocated**
a. clear sketch of numerical conduction CV with nodal numbering scheme; identify unique CVs for which equations need to be developed	1+1
b. derivation of CV equations for unique CVs	6
c. Coding and proper solution for T contour- daytime; max T location and discussion of trends; heat loss/gain from/to fruit	3
d. Coding and proper solution for T contour- nighttime; location and discussion of trends; heat loss/gain from/to fruit	2
e. Part 4 answer- clear discussion with sketch; no detailed formulation needed but thought process needs to be clear	1
f. Neatness and organization of the project submission	1
Total	15

Problem Statement

$d_o = 4cm = 0.04m$
$r_o = 0.02m$

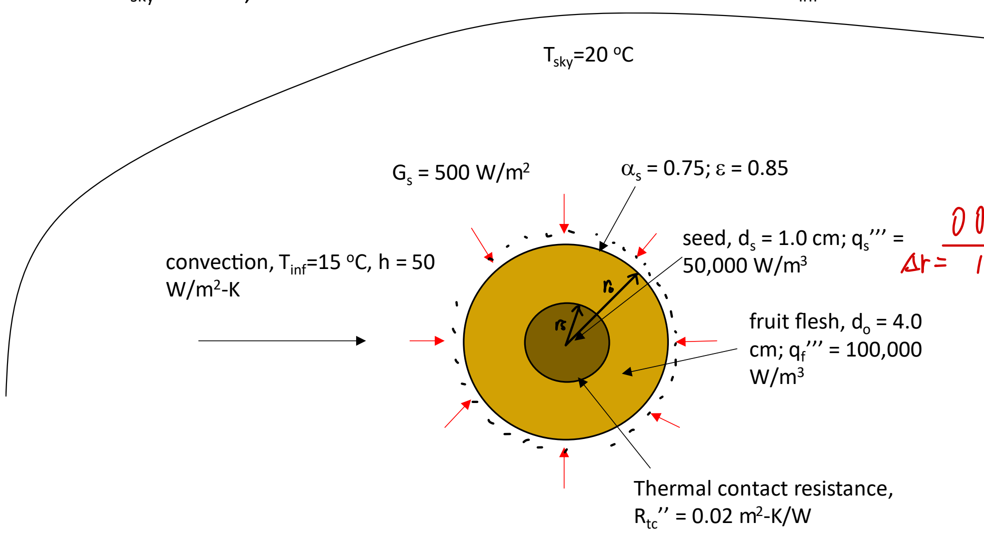
1. An apricot is ripening on a tree. The apricot is $d_o = 4$ cm in diameter and can be approximated to be a sphere. The skin can be assumed to be thin. Its spherical seed is $d_s = 1$ cm in diameter, and its thermal conductivity is $k_s = 0.3$ W/m-K. The seed of the apricot has a volumetrically uniform internal heat generation rate of $q_s''' = 50{,}000$ W/m^3. The flesh of the apricot (i.e., the fruit part) has a volumetrically uniform internal heat generation rate of $q_f''' = 100{,}000$ W/m^3. The thermal conductivity of the flesh, $k_f = 0.6$ W/m-K. There is a contact resistance between the seed and the flesh of the fruit, estimated at $R_{tc}'' = 0.02$ m^2-K/W. A convective breeze blows over the apricot such that the heat transfer coefficient is h = 50 W/m^2-K. The freestream temperature is T_{inf}. The absorptivity to solar radiation of the fruit skin is $\alpha_s = 0.75$, while the emissivity and absorptivity to long-wave radiation is 0.85.

$d_s = 1\ cm = 0.01m$
$r_s = 0.005m$

$\varepsilon = 0.85$

Consider the following scenarios:

(a) Daytime: During daytime (see sketch below), solar radiation with a value of $G_s = 500$ W/m^2 is incident over the entire fruit. The fruit exchanges radiation with the surrounding sky that is at a temperature of $T_{sky} = 20$ °C, in addition to convection to a freestream at $T_{inf} = 15$ °C.



$\Delta r = \frac{0.005}{10} = 0.0005$

Figure illustrating the daytime scenario

(b) Nighttime: At nighttime, the fruit exchanges radiation with the surrounding sky that is at a temperature of $T_{sky} = 5$ °C, in addition to convection to a freestream at $T_{inf} = 10$ °C.

Determine the following for steady state night time and daytime conditions :

1. *Temperature distribution within the fruit (inclusive of the seed)*
2. *Location and value of the maximum temperature*
3. *Heat gain/loss to/from fruit*
4. *If for the daytime scenario, solar radiation were only incident on one half of the apricot, what changes would you need to make to the formulation to solve the problem? Discuss with a sketch of the control volumes you would need to use.*

Requirements :

1. The code should be in Matlab, python, or other programming language. The code needs to be uploaded onto the gradescope submission portal dedicated for the code
2. A minimum of 10 CVs is needed for the seed, and 20 CVs is needed for the flesh part of the fruit.

1. clearification : / Assumptions:

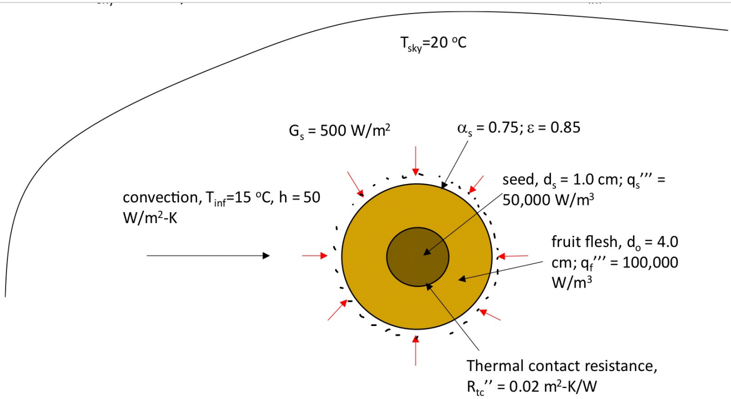


Figure illustrating the daytime scenario

$k_s = 0.3\ W/m^2\text{-}k$

$k_f = 0.6\ W/m\cdot k$

$\sigma = 5.67 \cdot 10^{-8}$

$T_{sky} = 293.15$

$T_{inf} = 288.15$

For skin: No mass, no volume

$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out} + \cancel{\dot{E}_{stored}}^{\,0}$$

$$\Rightarrow \boxed{\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = 0}$$

for spherical: $R_{t,cond} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$r_1 = r_s = 0.005m$

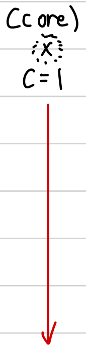
$r_2 = r_o = 0.02m$

$P_{rad} = \alpha_s G_s \cdot A_o$

2. Sketch for CV assumption

- First set "core" temperature or layer to 1
- Second by adding $\Delta r = 0.0005$ for each layer of c, in the name of step $c+1$
- Third by derive correct formulas for each specific node.
    - These nodes are : seed central temperature T_0
      seed → flesh boundary temperature T_1
      flesh → seed boundary temperature T_2
      flesh boundary temperature T_3

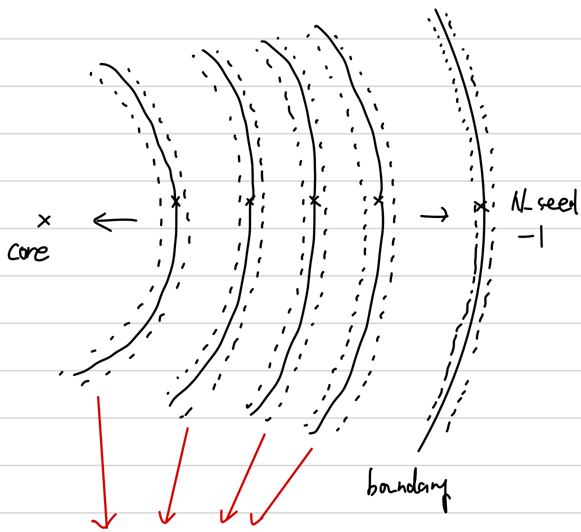
① In seed



unique C.V. ①

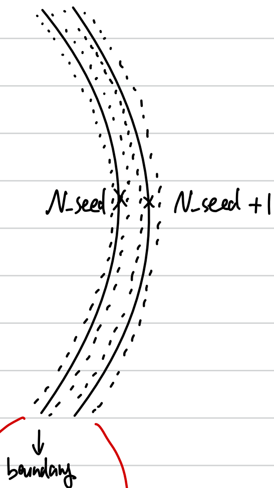
$T(1,c)$

② seed interior



unique C.V. ②
$T(i,c)$

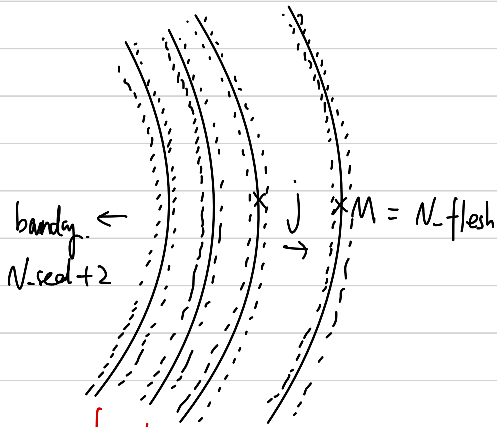
③ boundary condition



unique C.V. ③ unique C.V. ④

$T(N\_seed, c)$ $T(N\_seed+1, c)$

④ interior of flesh

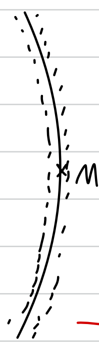


$j = N\_flesh - 1$

unique C.V. ⑤

$T(j, c)$

⑤ most outer layer node / skin



→ unique C.V. ⑥ $T(M, c)$

3. In summary, there are total of 4 specific temperatures that constructed by 5 segments of lines / equations.

That : 1 $T(1, c)$: Temperature equation for most inner node

2 $T(i, c)$: Temperature equation for seed from $1 \rightarrow$ boundary

3 $T(N\_seed, c)$: Temperature equation for boundary in seed side.

4 $T(N\_seed+1, c)$: Temperature equation for boundary in flesh side

5 $T(j, c)$: Temperature equation for flesh / fruit most outer node.

6 $T(M, c)$: Temperature equation for skin node.

4 derive each equation :

Properties: $r_i = 0.005\ m$ $\quad k_s = 0.3\ W/m\cdot k$ $\quad q'''_s = 50000\ W/m^2$

$r_o = 0.02\ m$ $\quad k_f = 0.6\ W/m\cdot k$ $\quad q'''_f = 100000\ W/m^2$ $\quad R_{tc} = 0.02\ m^2\cdot k/w$

$T_{\infty,d} = 288.15\ °k$ $\quad T_{sky,d} = 293.15\ °k$

$T_{\infty,n} = 283.15\ °k$ $\quad T_{sky,n} = 278.15\ °k$

$\Delta r = 0.0005$ $\quad$ number of nodes : $N_1 = \#seed = \frac{r_i}{\Delta r} + 1$

$N_2 = \#flesh = \frac{r_o}{\Delta r} + 2$

For $T(1,c)$ which the center temperature ①

$$T(1,C) = [q_{gen} + q_{cond} \cdot T(2,c-1)] / [q_{cond}]$$

$$T(1,C) = \frac{\left[q_s''' \cdot \frac{4}{3}\pi\left[\frac{\Delta r}{2}\right]^3 + \frac{k_s \cdot 4\pi\left[\frac{\Delta r}{2}\right]^2}{\Delta r} \cdot T(2,C-1) \right]}{\frac{k_s \cdot 4\pi\left[\frac{\Delta r}{2}\right]^2}{\Delta r}}$$

For $T(2,C)$ has same setup that produce conduction to $T(1,C)$

For $T(i,c)$ which are the interior nodes in seed ②

$$T(i,c) = \frac{\left[T(i+1,c-1)\cdot 4\pi r_o^2 \cdot \frac{k_s}{\Delta r}\right] + \left[T(i-1,c-1)\cdot 4\pi r_i^2 \cdot \frac{k_s}{\Delta r}\right] + \left[q_s''' \cdot \frac{4}{3}\pi (r_o^3 - r_i^3)\right]}{\left[4\pi r_o^2 \cdot \frac{k_s}{\Delta r} + 4\pi r_i^2 \cdot \frac{k_s}{\Delta r}\right]}$$

For $T(N\_seed, c)$ which is the inner boundary. ③

$$T(N\_seed, c) = \frac{q_{gen} + q_{cond_1} \cdot T(N\_seed-1, c-1) + q_{contact} \cdot T(N\_seed+1, c-1)}{q_{cond_1} + q_{contact}}$$

$$T(N\_seed, c) = \frac{\left[q_s''' \cdot \frac{4}{3}\pi \left(r_i^3 - \left(\frac{dr}{2}\right)^3\right)\right] + \left[\left(4\pi r_i^2 \cdot \frac{k_s}{\Delta r}\right)\cdot T(N\_seed-1, c-1)\right] + \left[\frac{4\pi r_i^2}{R_{tc}} \cdot T(N\_seed+1, c-1)\right]}{\left[4\pi r_i^2 \cdot \frac{k_s}{\Delta r} + \frac{4\pi r_i^2}{R_{tc}}\right]}$$

For $T(N\_seed+1, c)$ which is the outer boundary. ④

$$T(N\_seed+1, c) = \frac{q_{gen} + q_{cond_2} \cdot T(N\_seed+2, c-1) + q_{contact} \cdot T(N\_seed, c-1)}{q_{cond_2} + q_{contact}}$$

$$T(N\_seed+1, c) = \frac{\left[q_s''' \cdot \frac{4}{3}\pi \cdot \left(\left(r_i + \frac{\Delta r}{2}\right)^3 - r_i^3\right)\right] + \left[4\pi \left(r_i + \frac{\Delta r}{2}\right)^2 \cdot \frac{k_f}{\Delta r} \cdot T(N\_seed+2, c-1)\right] + \left[\frac{4\pi r_i^2}{R_{tc}} \cdot T(N\_seed, c-1)\right]}{\left[4\pi \left(r_i + \frac{\Delta r}{2}\right)^2 \cdot \frac{k_f}{\Delta r} + \frac{4\pi r_i^2}{R_{tc}}\right]}$$

For T(j, c). which are the interior nodes in flesh ⑤

$$T(j,c) = \frac{[q_{cond,o} \cdot T(j+1, c-1) + q_{cond,i} \cdot T(j-1, c-1) + q_f''' \cdot \forall]}{[q_{cond,o} + q_{cond,i}]}$$

$$T(j,c) = \frac{\left[4\pi\left((j-\frac{3}{2})\Delta r\right)^2 \cdot \frac{k_f}{\Delta r} \cdot T(j+1, c-1)\right] + \left[4\pi\left((j-\frac{5}{2})\Delta r\right)^2 \cdot \frac{k_f}{\Delta r} \cdot T(j-1, c-1)\right] + \left[\frac{4}{3}\pi\left((j-\frac{3}{2})^3 - (j-\frac{5}{2})^3\right) \cdot q_f'''\right]}{\left[4\pi\left((j-\frac{3}{2})\Delta r\right)^2 \cdot \frac{k_f}{\Delta r} + 4\pi\left((j-\frac{5}{2})\Delta r\right)^2 \cdot \frac{k_f}{\Delta r}\right]}$$

[Day condition] ⑥

For T(M, c), for easier reading in code changed to T(N_flesh, c) as skin node Ts

$$T(M,c) = q_{cond,f} + q_f''' \cdot \forall - q_{conv,d} + q_{emi,d} + q_{radi}$$

in form of $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = 0$
and for Ts temperature assumption.

$$T(M,c) = q_{cond,f} \cdot [T(M-1, c-1) - T_s] + \left[q_f''' \cdot \frac{4}{3}\pi(r_o^3 - (r_o - \frac{\Delta r}{2})^3)\right] - q_{conv} + q_{emi}$$

$$\begin{aligned} T(M,c) = & \left[4\pi r_o^2 \cdot \frac{k_f}{\Delta r} \cdot (T(M-1, c-1) - T_s)\right] + \left[q_f''' \cdot \frac{4}{3}\pi(r_o^3 - (r_o - \frac{\Delta r}{2})^3)\right] \\ & - \left[h \cdot 4\pi r_o^2 \cdot (T_s - T_{\infty,d})\right] + \left[\varepsilon \cdot \sigma \cdot 4\pi r_o^2 (T_{sky,d}^4 - T_s^4)\right] \\ & + \left[\alpha \cdot G \cdot 4\pi r_o^2\right] \end{aligned}$$

with Ts setted unknown for MATLAB solver.

[Night condition] ⑥'

For $T(M,c)$, for easier reading in code changed to $T(N\_flesh, c)$ as skin node T_s

$$T(M,c) = q_{cond,f} + q_f''' \cdot \forall + q_{conv,n} + q_{emi,n}$$ | in form of $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = 0$ and for T_s temperature assumption.

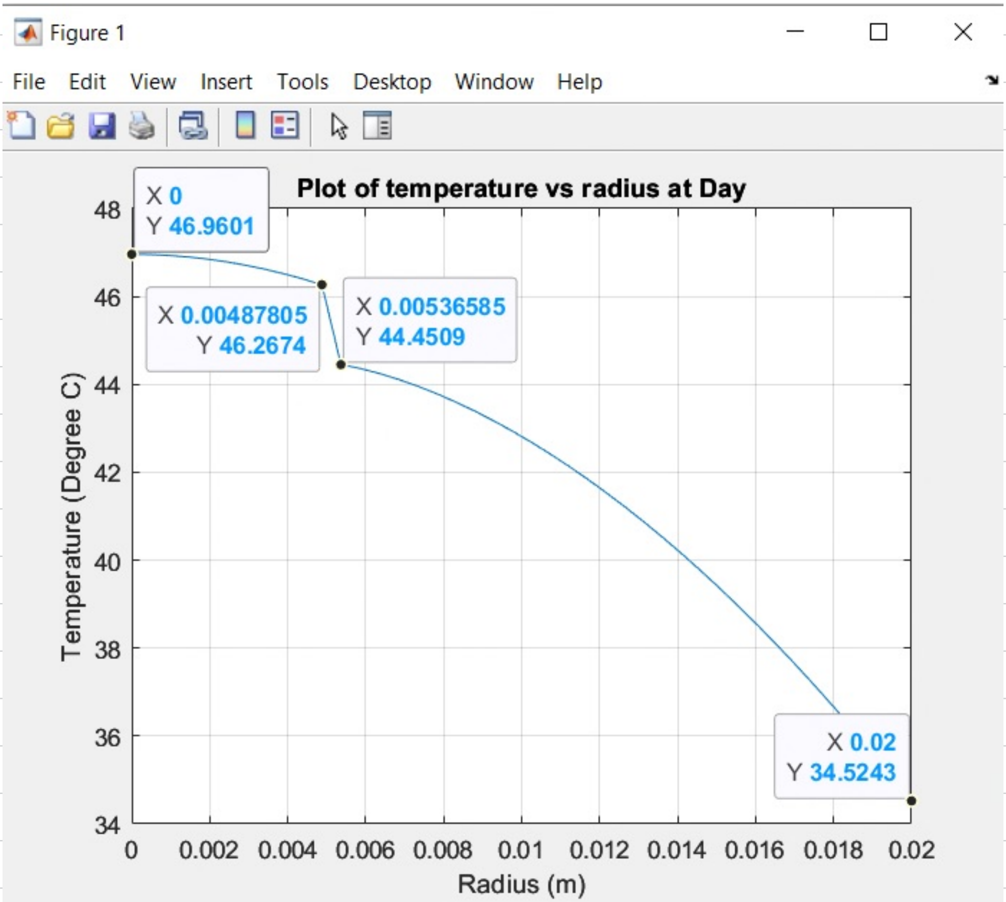
$$T(M,c) = q_{cond,f} \cdot [T(M-1, c-1) - T_s] + \left[q_f''' \cdot \frac{4}{3}\pi\left(r_o^3 - \left(r_o - \frac{\Delta r}{2}\right)^3\right)\right] - q_{conv} + q_{emi}$$

$$T(M,c) = \left[4\pi r_o^2 \cdot \frac{k_f}{\Delta r} \cdot (T(M-1, c-1) - T_s)\right] + \left[q_f''' \cdot \frac{4}{3}\pi\left(r_o^3 - \left(r_o - \frac{\Delta r}{2}\right)^3\right)\right]$$
$$- \left[h \cdot 4\pi r_o^2 \cdot (T_s - T_{\infty,n})\right] + \left[\varepsilon \cdot \sigma \cdot 4\pi r_o^2 (T_{sky,n}^4 - T_s^4)\right]$$

with T_s setted unknown for MATLAB solver.

Answer to Questions :

1. MATLAB code graph output , Temperature distribution (Day time)



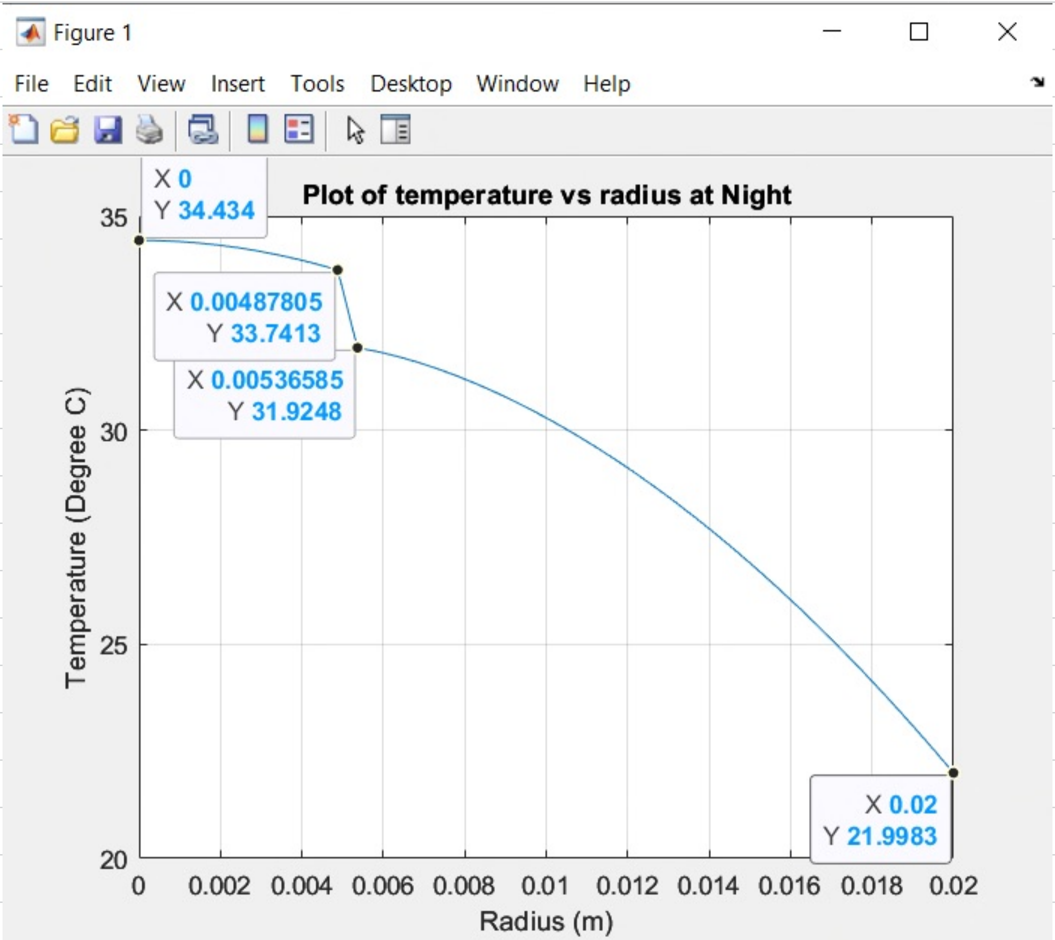
$T_0 = T_{seed,\,core} = 46.91\ °C$

$T_1 = T_{bound,\,seed} = 46.27\ °C$

$T_2 = T_{bound,\,flesh} = 44.45\ °C$

$T_3 = T_s = 34.52\ °C$

Night time



$T_0 = T_{seed, core} = 34.43\ °C$

$T_1 = T_{bound, seed} = 33.74\ °C$

$T_2 = T_{bound, flesh} = 31.92\ °C$

$T_3 = T_s = 21.99\ °C$

2. Location and value for maximum temperature:

At day time : $T_{seed,core} = 46.91\,°C$

At night time : $T_{seed,core} = 34.43\,°C$

3. In order to determine either heat lose or in, we need to find that $\dot{E}_{in} = \dot{E}_{out}$ for $q_{in} = \dot{E}_{in} + \dot{E}_{gen}$,

$$\dot{E}_{in} = \alpha G \cdot 4\pi r_o^2 + \frac{k_f}{\Delta r} \cdot 4\pi r_o^2 \left(T_{(m-1,c-1)} - T_s\right) + q_f''' \cdot \frac{4}{3}\pi \left(r_o^3 - \left(r_o - \frac{\Delta r}{2}\right)^3\right)$$

$$\dot{E}_{out} = 4\pi r_o^2 h \cdot \left(T_s - T_{\infty,d}\right) - \varepsilon \cdot \sigma \cdot 4\pi \cdot r_o^2 \left(T_{sky,d}^4 - T_s^4\right)$$

setting up the equation in MATLAB $\dot{E}_{in} = \dot{E}_{out}$

```
q_in =

    5.2888

q_out =

    5.2888
```

```
%% Engergy Balance Calculation Base on Read Numbers for T_s
T_s=307.674284659641; %Surface temperature base on graph (DAY)

q_in= (k_f/dr)*4*pi*r_o^2*(T(N_flesh -1, c-1) - T_s)   + q_f_gen*(4/3)*pi*(r_o^3 - (r_o - dr/2)^3) + alpha_s*G_s*4*pi*r_o^2
q_out= h*4*pi*r_o^2*(T_s-T_infd) - (alpha_emi*(5.67*10^-8)*4*pi*r_o^2*(T_skyd^4 - T_s^4))
```

Zoom: 80% | UTF-8 | CRLF | script | Ln 131 Col

For Day time condition

Therefore when replace T_M in T_s for surface temperature, $\dot{E}_{in} = \dot{E}_{out}$ so it prove the calculation is correct and the code is running properly.

therefor we can assume 1-D, steady state, control volume and read the graph:

Base on graph output:
Heat is moving out of the fruit, therefor $\boxed{\text{Heat loss from fruit}}$

In order to determine either heat lose or in, we need to find that $\dot{E}_{in} = \dot{E}_{out}$ for $q_{in} = \dot{E}_{in} + \dot{E}_{gen}$,

$$\dot{E}_{in} = \frac{k_f}{\Delta r} \cdot 4\pi r_o^2 \left(T(n-1,c+1) - T_s\right) + q_f''' \cdot \frac{4}{3}\pi \left(r_o^3 - \left(r_o - \frac{\Delta r}{2}\right)^3\right)$$

$$\dot{E}_{out} = 4\pi r_o^2 h \cdot \left(T_s - T_{\infty,d}\right) - \varepsilon \cdot \sigma \cdot 4\pi \cdot r_o^2 \left(T_{sky,d}^4 - T_s^4\right)$$

setting up the equation in MATLAB $\dot{E}_{in} = \dot{E}_{out}$

```
q_in =

     3.4038

q_out =

     3.4038
```

```
%% Engergy Balance Calculation Base on Read Numbers for T_s
T_s=295.148300077858; %Surface temperature base on graph (NIGHT)

q_in= (k_f/dr)*4*pi*r_o^2*(T(N_flesh -1, c-1) - T_s)   + q_f_gen*(4/3)*pi*(r_o^3 - (r_o - dr/2)^3)
q_out= h*4*pi*r_o^2*(T_s-T_infn) - (alpha_emi*(5.67*10^-8)*4*pi*r_o^2*(T_skyn^4 - T_s^4))
```

Zoom: 80% UTF-8 CRLF script

For Night time condition

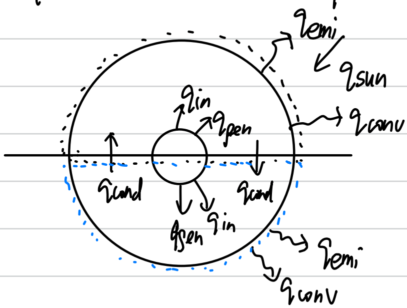
Therefore when replace T_M in T_s for surface temperature, $\dot{E}_{in} = \dot{E}_{out}$ so it prove the calculation is correct and the code is running properly.

therefor we can assume 1-D, steady state, control volume and read the graph:

Bace on graph output:
Heat is moving out of the fruit, therefor $\boxed{\text{Heat loss from fruit}}$

Total iteration = 7780 times

4\. 1. We can first divide the apricot into two hemispheres with no insulation so we need two control volume to fix the value.



2\. We should still consider the thermal resistance R_{tc}'' in to account

3\. Conduction : as the heated side that face the sun, conduction beome more factored and also consider the conduction between two hemisphere parts

4\. solar radiation is different as the top hemisphere will have solar radiation heat flux while the lower hemisphere doesn't

5\. convection part will remain the same as it all expose to the sky.