

# CFD 100 Series

## Lecture 03:

**Navier-Stokes Equations:  
Unsteady diffusion**



**An introduction  
to Computational  
Fluid Dynamics**

## 1.1

# 有限差分法 Finite Difference Method 控制方程

$$\frac{d^2 T}{dx^2} = H(T - T_{\infty})$$

- 空间项：中心插分

$$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + S \rightarrow \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$$

- 时间项：一阶向前插分
- 空间项：中心插分

## 1.1

# 有限差分法 Finite Difference Method

## 控制方程的离散

$$\frac{\partial u}{\partial t} \quad ?$$

$$u_i^{n+1} = u_i^n + \Delta t \frac{\partial u}{\partial t} \Big|_i^n + \frac{\Delta t}{2} \frac{\partial^2 u}{\partial t^2} \Big|_i^n + \dots$$

$$\frac{\partial u}{\partial t} \Big|_i^n = \frac{u_i^{n+1} - u_i^n}{\Delta t} + \mathcal{O}(\Delta t)$$

$$\frac{\partial^2 u}{\partial x^2} \quad ?$$

$$\frac{\partial^2 u}{\partial x^2} \Big|_i^n = \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} + \mathcal{O}(\Delta x^2)$$

$$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \mu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

## 1.1

# 有限差分法 Finite Difference Method

## 控制方程的离散

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \mu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$u_i^{n+1} = u_i^n + \mu \frac{\Delta t}{\Delta x^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

