Bullet Supersonic Flow Analysis: Drag Coefficient and CFD Validation Using Fluent and MATLAB

Burak Yorukcu

1.Introduction

The study of supersonic flow around objects, such as bullets, is critical in aerospace and defense industries due to its impact on aerodynamic performance, stability, and efficiency. Supersonic flows, characterized by Mach numbers greater than 1, introduce complex phenomena such as shock waves, expansion fans, and significant drag forces. The drag coefficient (Cd) is a key parameter in quantifying aerodynamic resistance, directly influencing the design of high-speed projectiles. This project aims to analyze the supersonic flow over a bullet geometry, calculate the drag coefficient and drag forces, and validate the results using computational fluid dynamics (CFD) simulations in ANSYS Fluent and custom MATLAB code.

The motivation for this work stems from the need to bridge analytical and numerical approaches in aerodynamic analysis. By developing a MATLAB code to compute Cd based on theoretical models and comparing it with Fluent's CFD results, this project evaluates the accuracy of simplified analytical methods against industry-standard simulations. The objectives are:

- To model supersonic flow around a bullet using theoretical aerodynamic principles.
- To compute Cd and drag forces using MATLAB.
- To validate the results with Fluent CFD simulations.
- To compare the two approaches and analyze discrepancies.

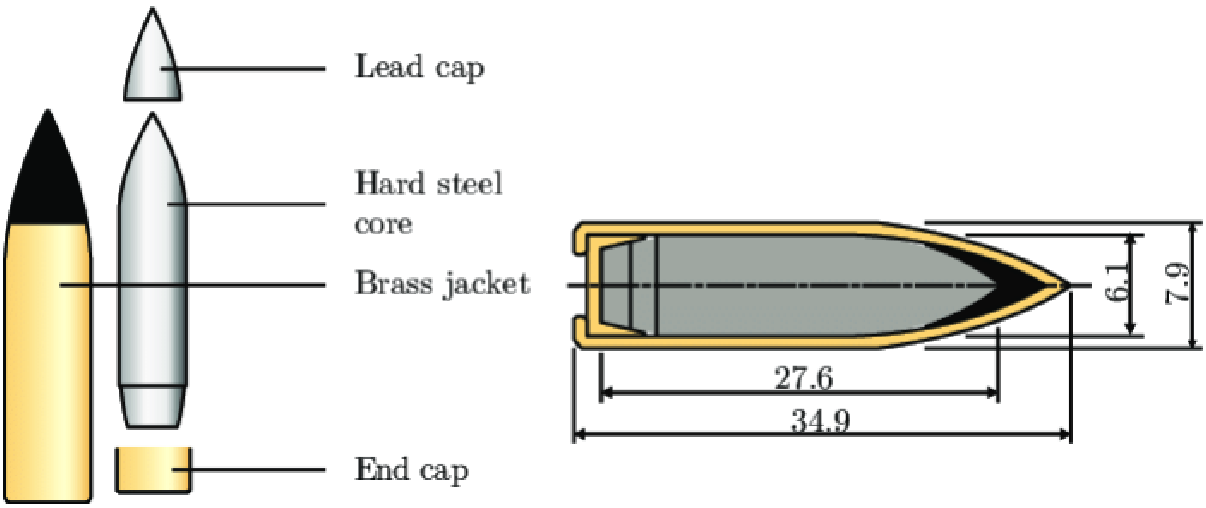

Figure 1.0:Bullet Parts

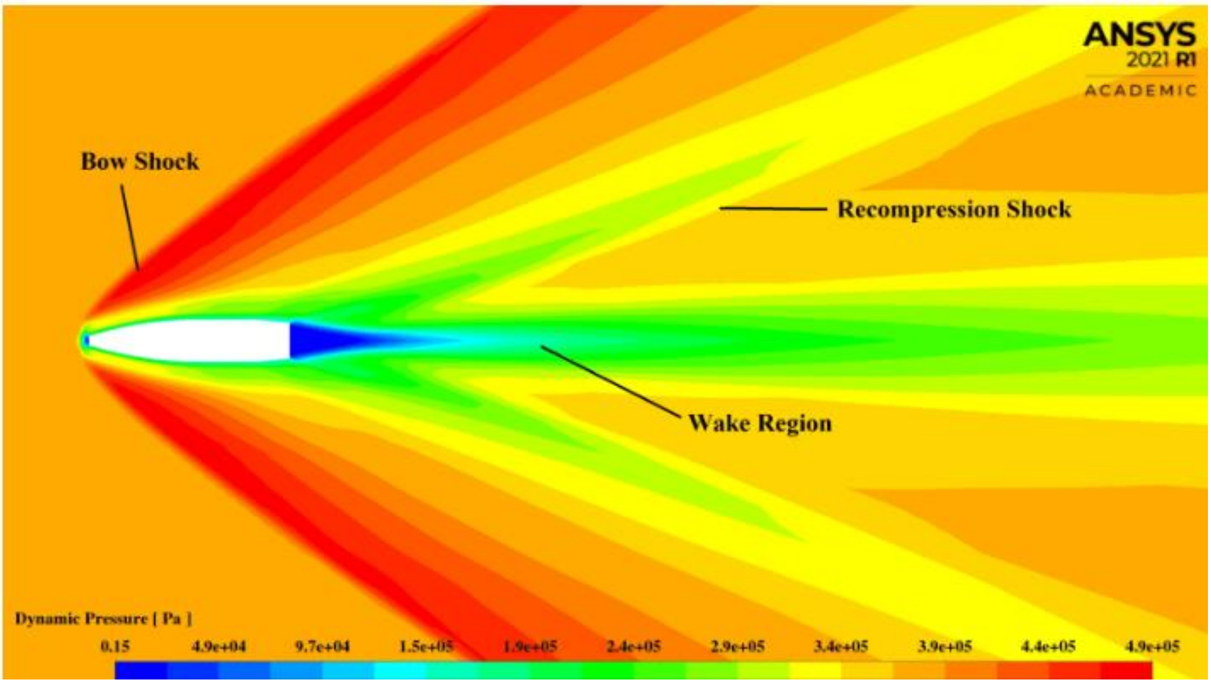

Figure 1.1: Shock waves

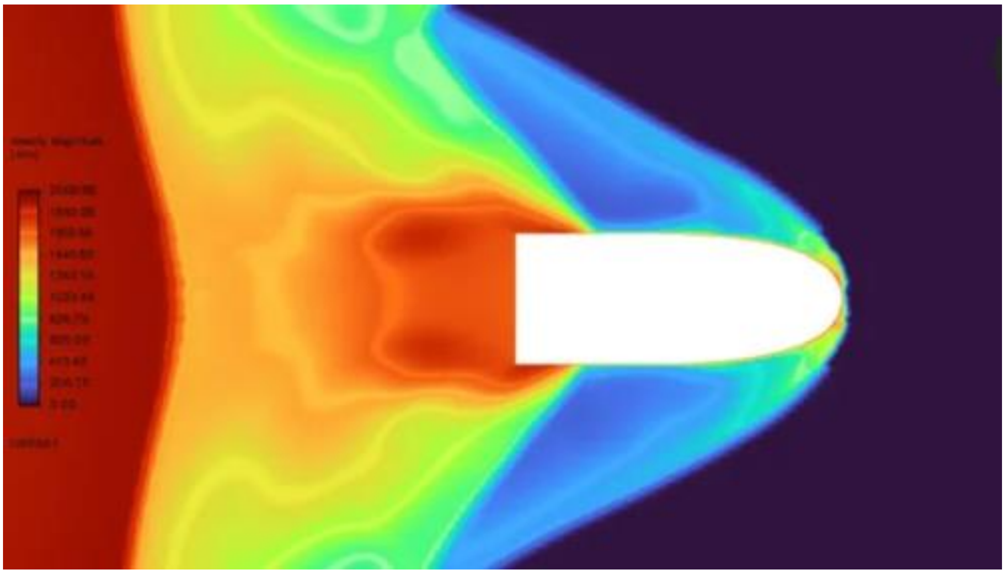

Figure 1.2: Shock waves

2. Theory

Supersonic flow is governed by compressible fluid dynamics, where the Mach number (M) exceeds 1, leading to the formation of shock waves and expansion fans. For a bullet traveling at supersonic speeds, the flow is characterized by an oblique or bow shock at the nose, followed by expansion waves along the body and recompression shocks at the tail. The drag coefficient (Cd) quantifies the total drag force, which includes:

- Wave drag: Caused by shock waves due to compressibility effects.
- Pressure drag: Resulting from pressure differences across the bullet.
- Skin friction drag: Due to viscous effects, though typically minor in supersonic flows.

The drag force (D) is calculated as:

$$D = \frac{1}{2}\rho V^2 A C_d$$

Figure 2.1: Drag Force Formula

The theoretical model in this project relies on simplified assumptions, such as inviscid flow and steady-state conditions, to compute Cd. Key equations include:

- Oblique shock relations: To determine post-shock flow properties (e.g., pressure, density) using the Rankine-Hugoniot relations.
- Prandtl-Meyer expansion: To model flow expansion around the bullet's tapering sections.
- Drag coefficient estimation: Using empirical correlations or analytical models (e.g., modified Newtonian theory) for supersonic projectiles.

These equations are implemented in MATLAB to compute Cd based on the bullet's geometry and freestream conditions (e.g., Mach number, angle of attack). The Fluent simulations, on the other hand, solve the full Navier-Stokes equations numerically, capturing viscous and turbulent effects for a more comprehensive analysis.

3. Methodology

The methodology combines analytical and numerical approaches to analyze supersonic flow over a bullet and compute its drag characteristics. The workflow consists of two main components: MATLAB-based analytical calculations and Fluent-based CFD simulations.

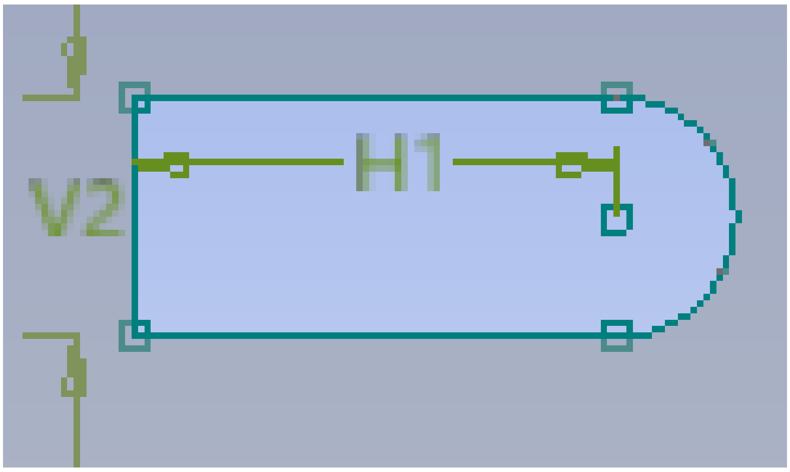

Figure 3.1: My Bullet Geometry

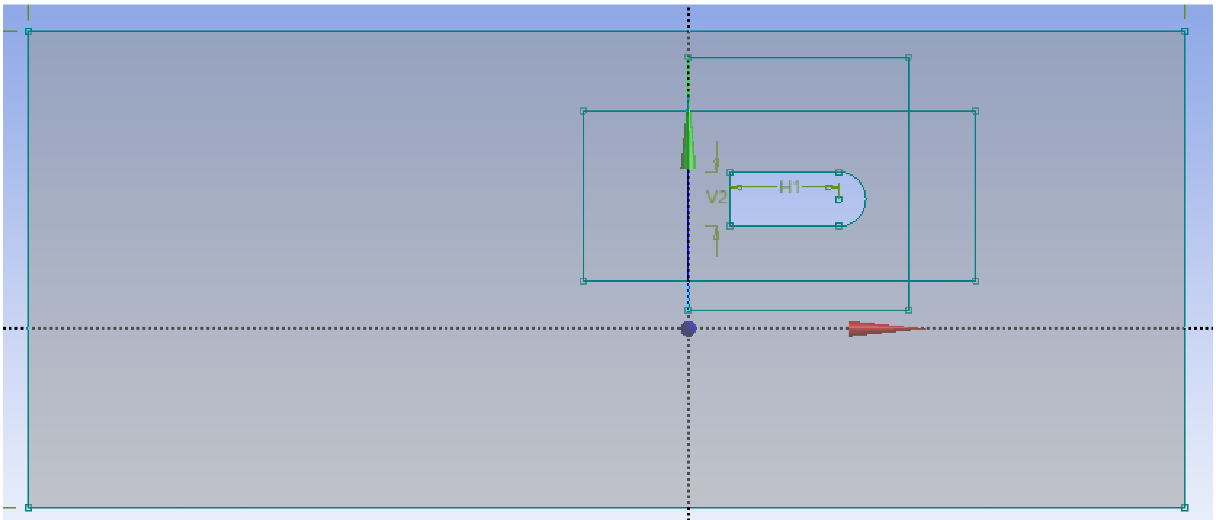

Figure 3.2 : My Control Volume

Due to the limitations of the ANSYS student version, a 2D analysis was conducted for a bullet with dimensions of 8 mm in length and 2 mm in diameter to investigate shock wave formation in supersonic flow.

3.2 Main Meshes

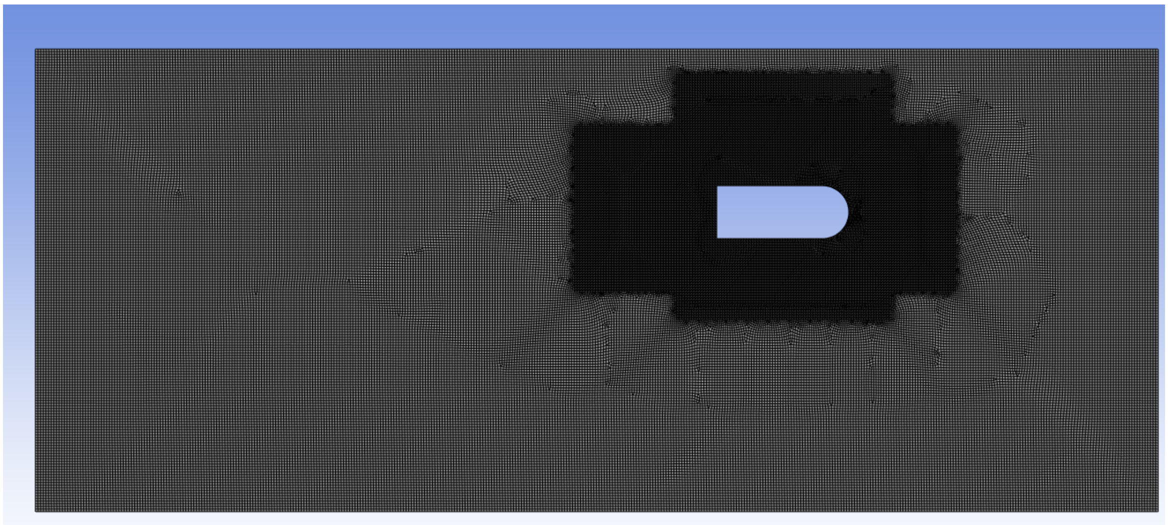

Figure 3.3: 100k Mesh with Quad Dominant Method

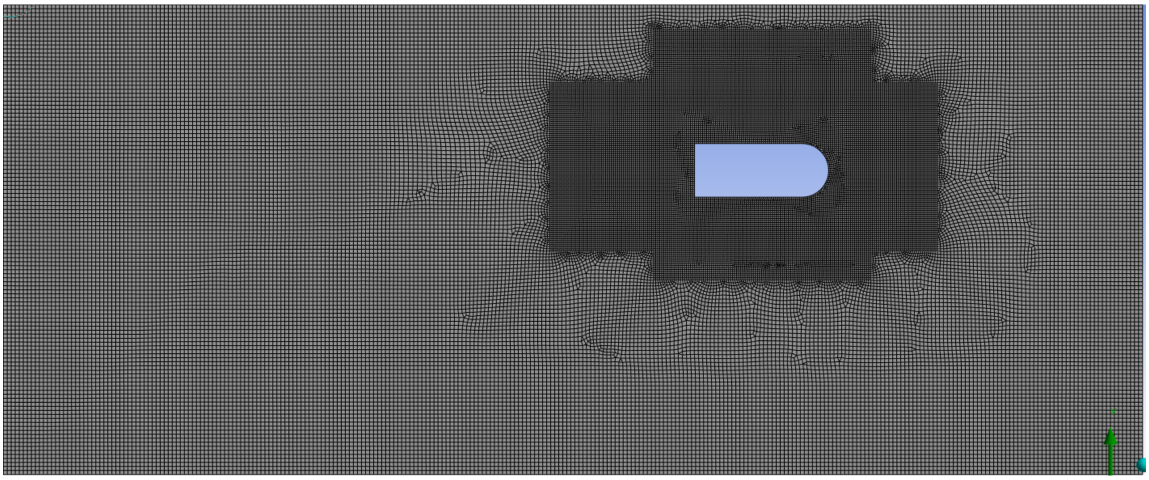

Figure 3.4: 50k Mesh with Quad Dominant Method

Figure 3.5: 300k Mesh With Quad Dominant Method

3.3 Solver Settings in ANSYS

The primary objective of this study is to analyze shock wave formation, drag force, and drag coefficient for a bullet in supersonic flow while minimizing computational cost. To achieve this, a 2D analysis was performed for a bullet with dimensions of 8 mm in length and 2 mm in diameter, leveraging the restrictions of the ANSYS Fluent student version. The k-epsilon standard turbulence model was selected over the k-omega model due to its computational efficiency and suitability for the small bullet geometry, where near-wall effects are less critical. Air was modeled as an ideal gas to enable simulations at two different Mach numbers, 1.5 and 2, ensuring flexibility in analyzing compressibility effects. A pressure-based, steady-state solver was employed to obtain results without the computational overhead of transient animations, focusing on steady shock structures, drag force, and drag coefficient. To optimize computational efficiency, various mesh densities configurations were tested to determine the optimal mesh size that balances accuracy and computational cost. This approach demonstrates that reasonable results can be achieved without complex solvers or settings, making the methodology practical for resource-constrained environments.

3.4 Results in ANSYS

Results of 50k Mesh with 1.5 Mach Number

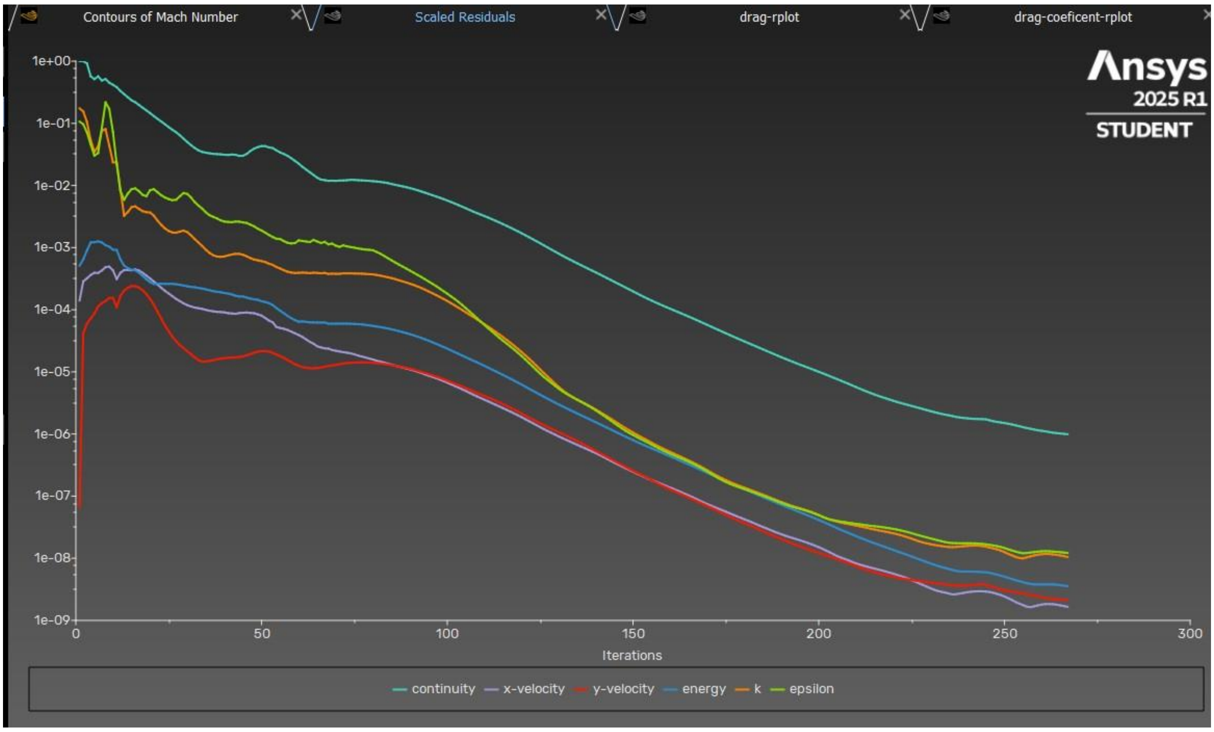

Figure 3.6: Scaled Residuals

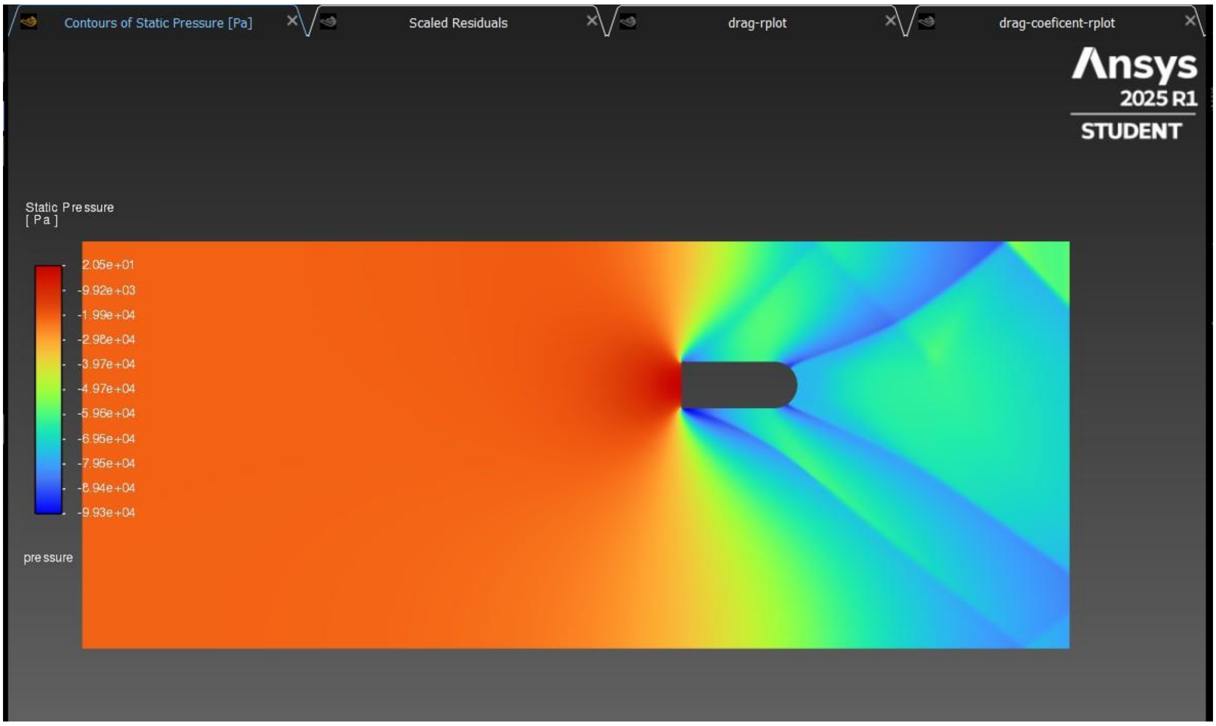

Figure 3.7: Pressure Contour

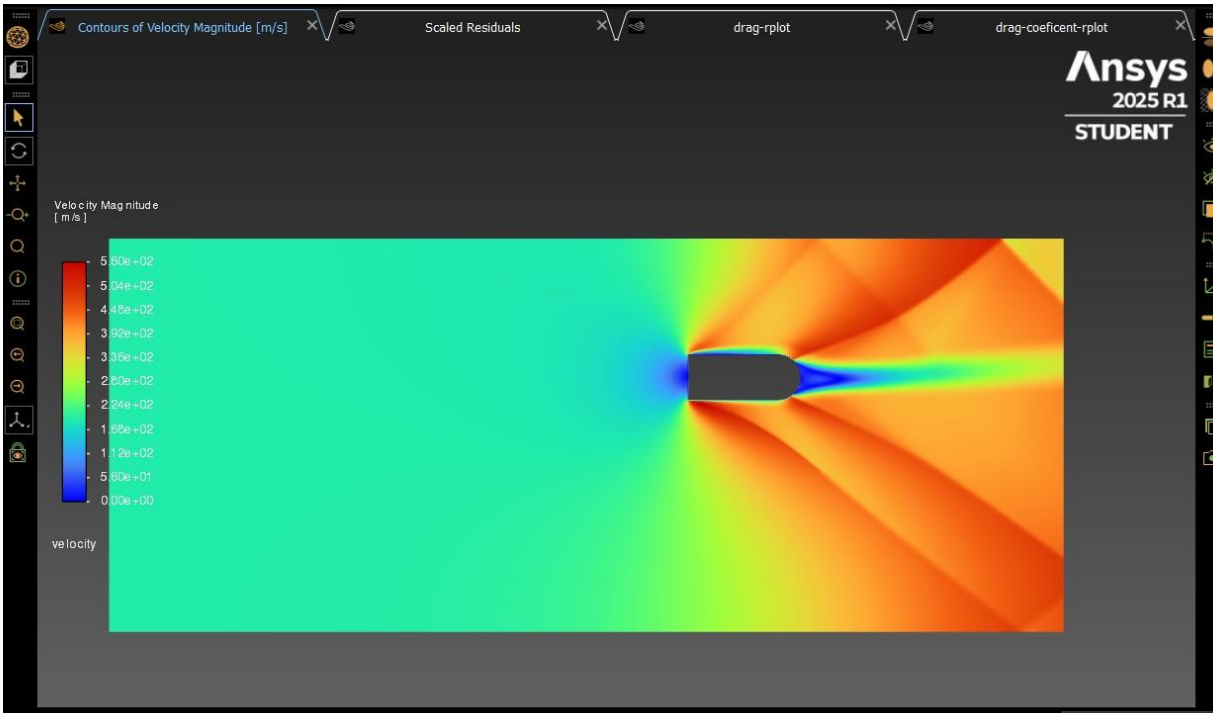

Figure3.8: Velocity Contour

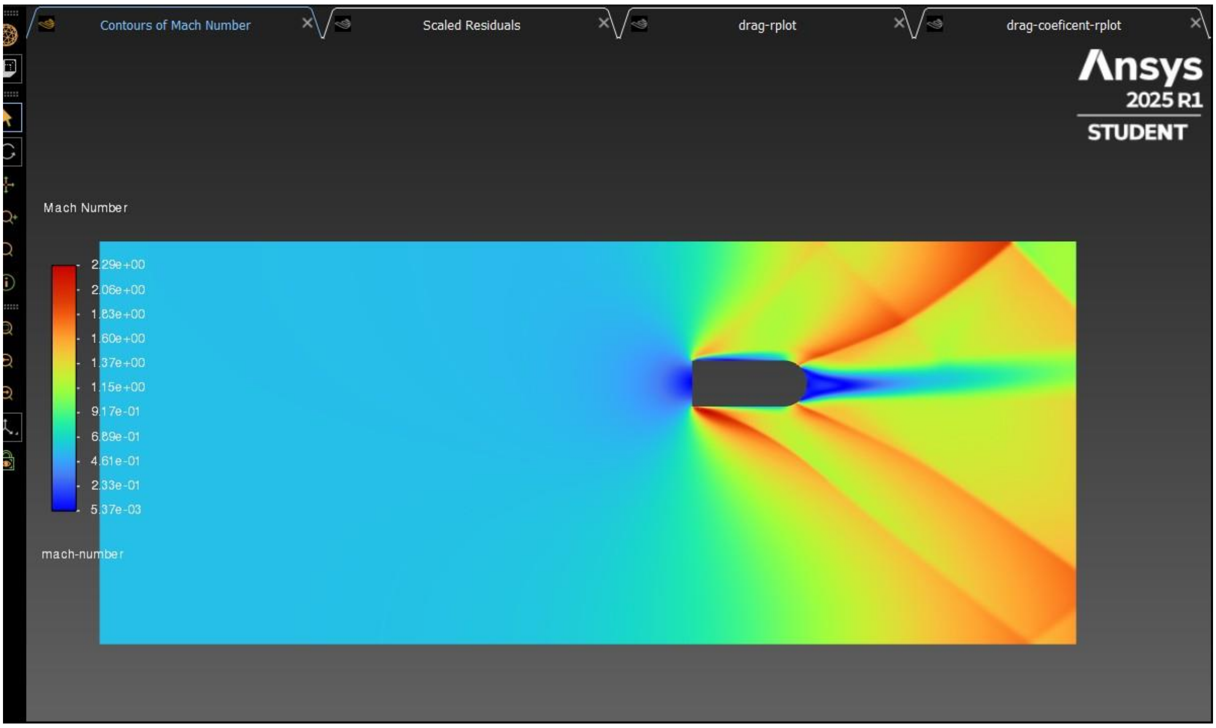

Figure 3.9: Mach Number Contour

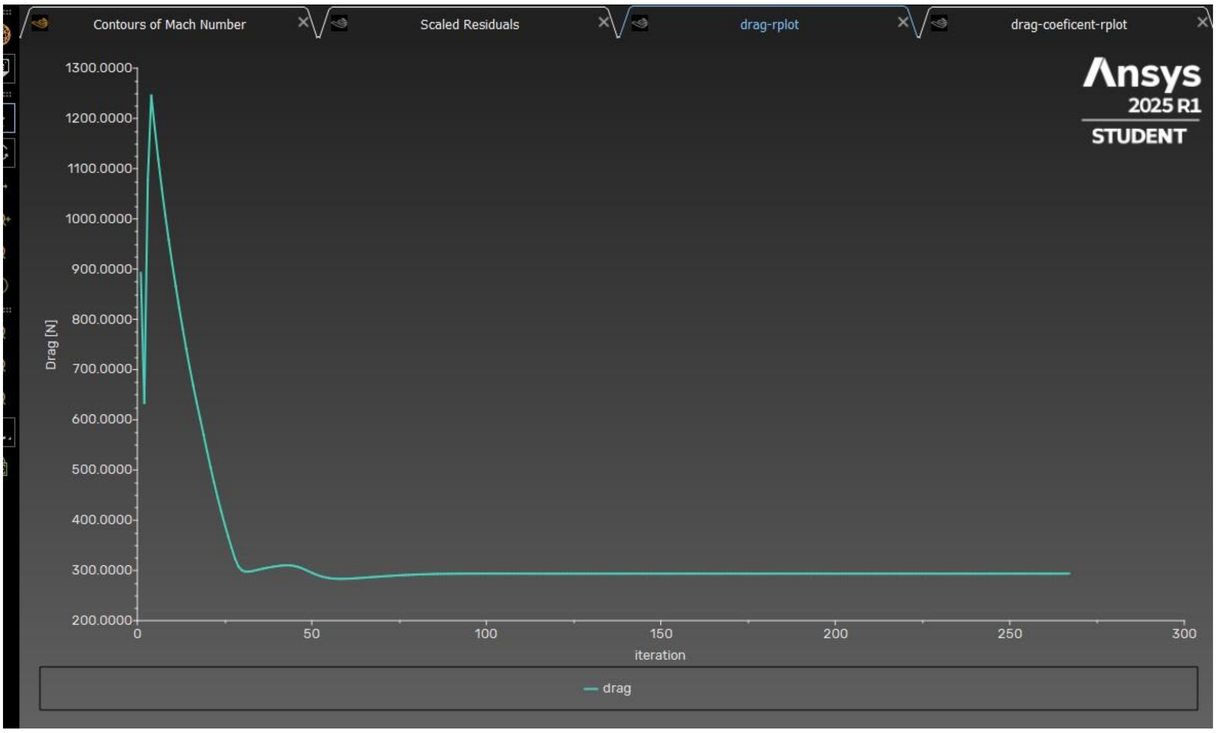

Figure 3.10: Drag Force

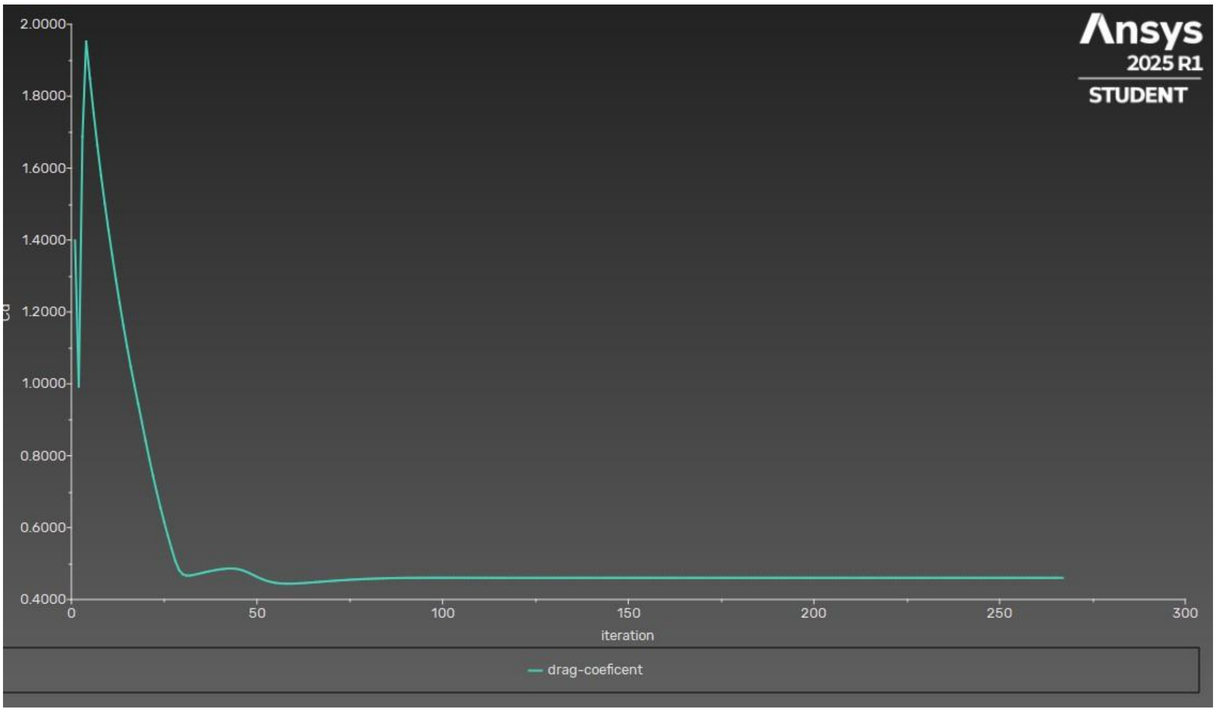

Figure 3.11: Drag Coefficent

Results of 50k Mesh with 2.0 Mach Number

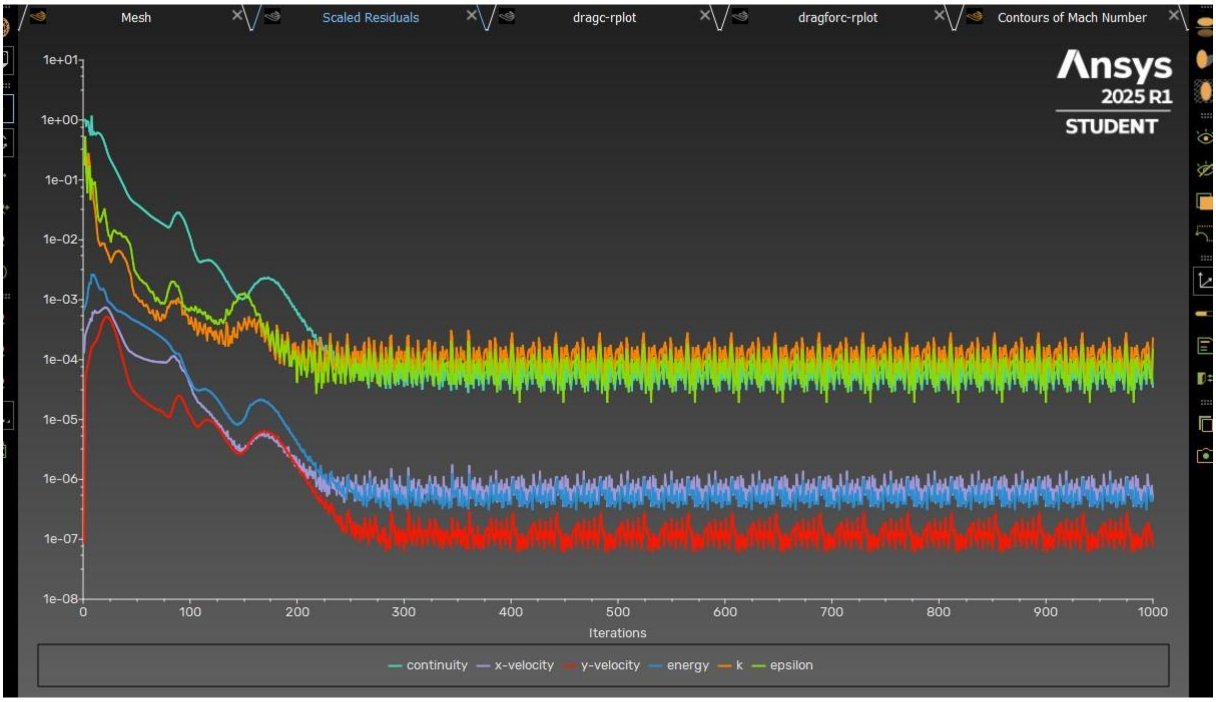

Figure 3.12: Scaled Residuals

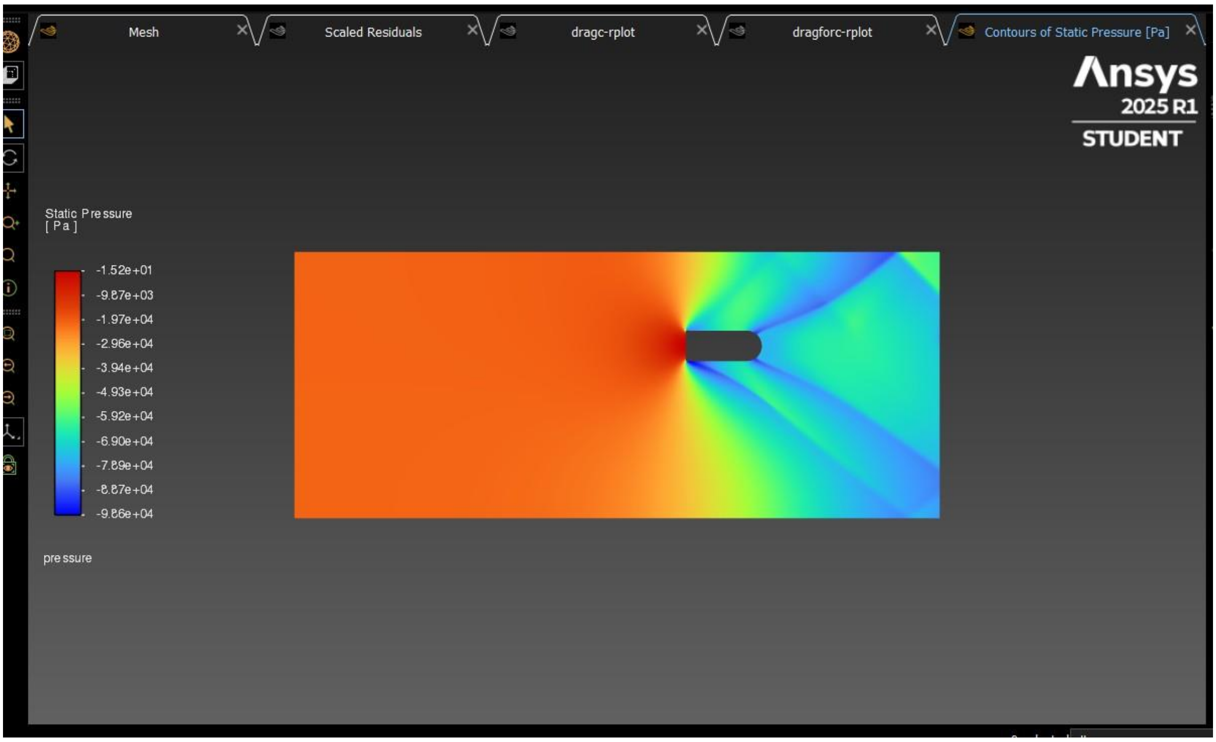

Figure 3.13: Pressure Contour

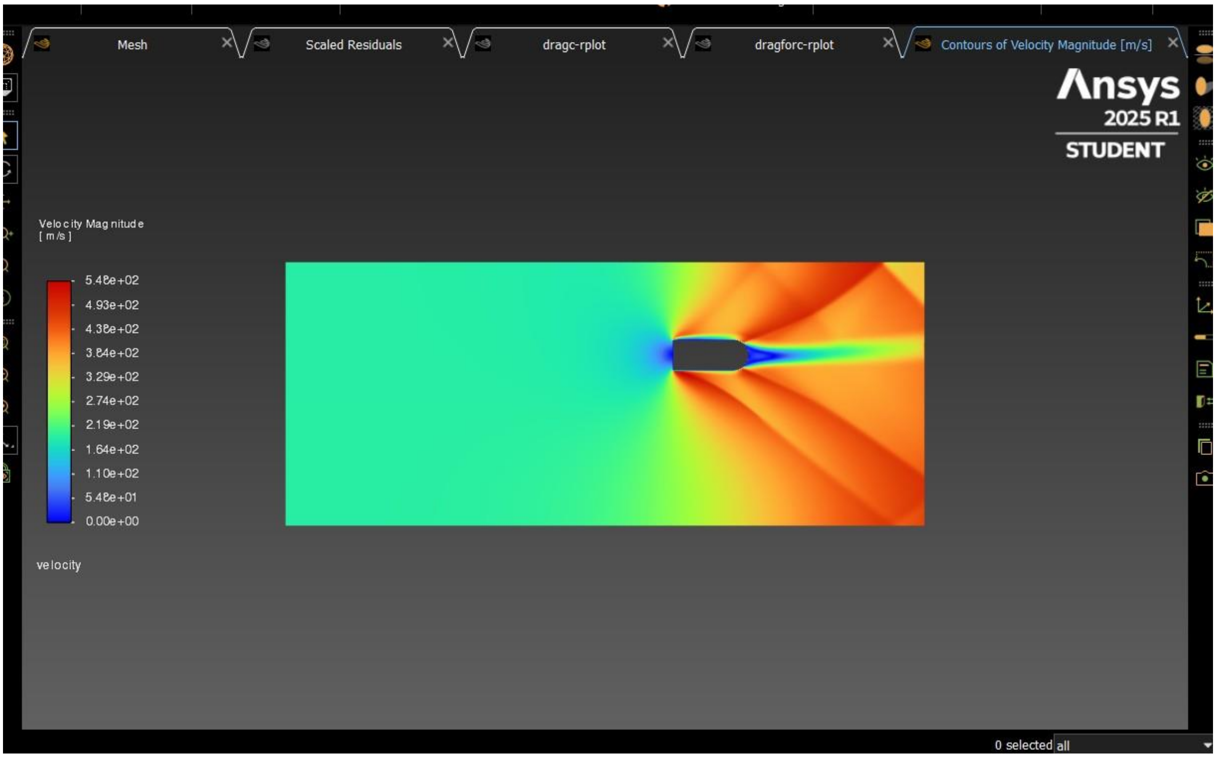

Figure 3.14: Velocity Contour

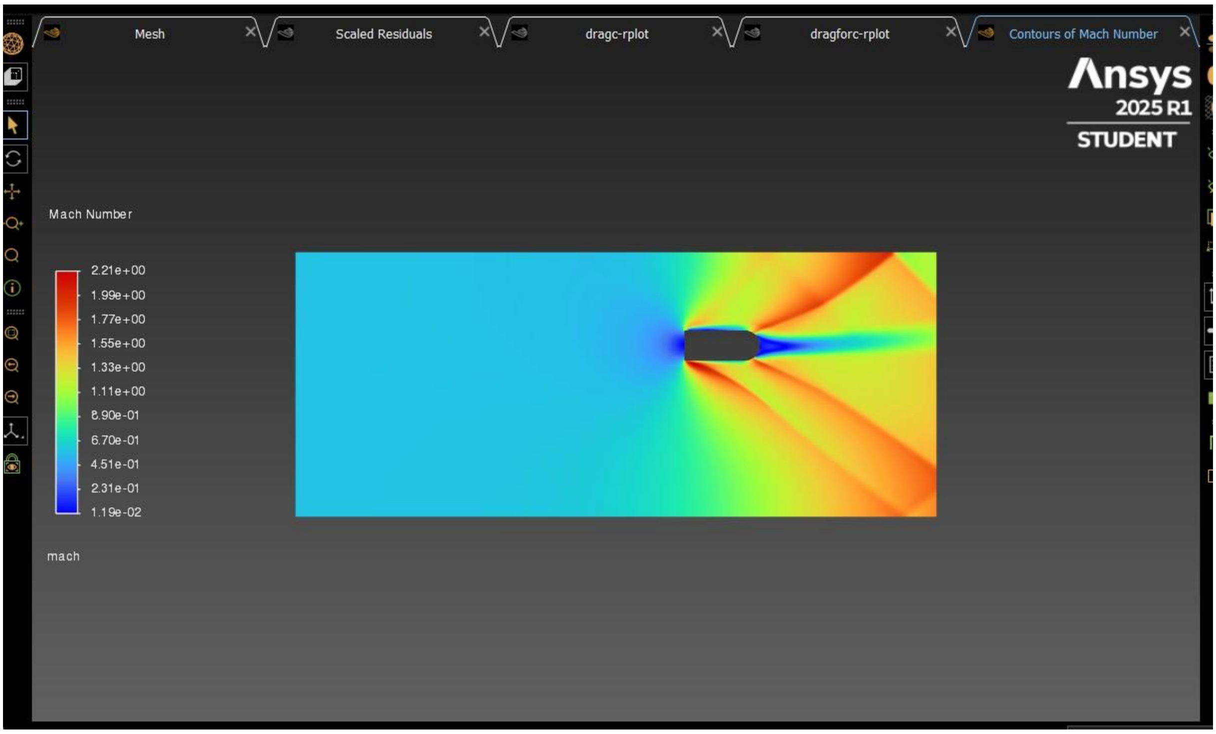

Figure 3.15: Mach Number Countour

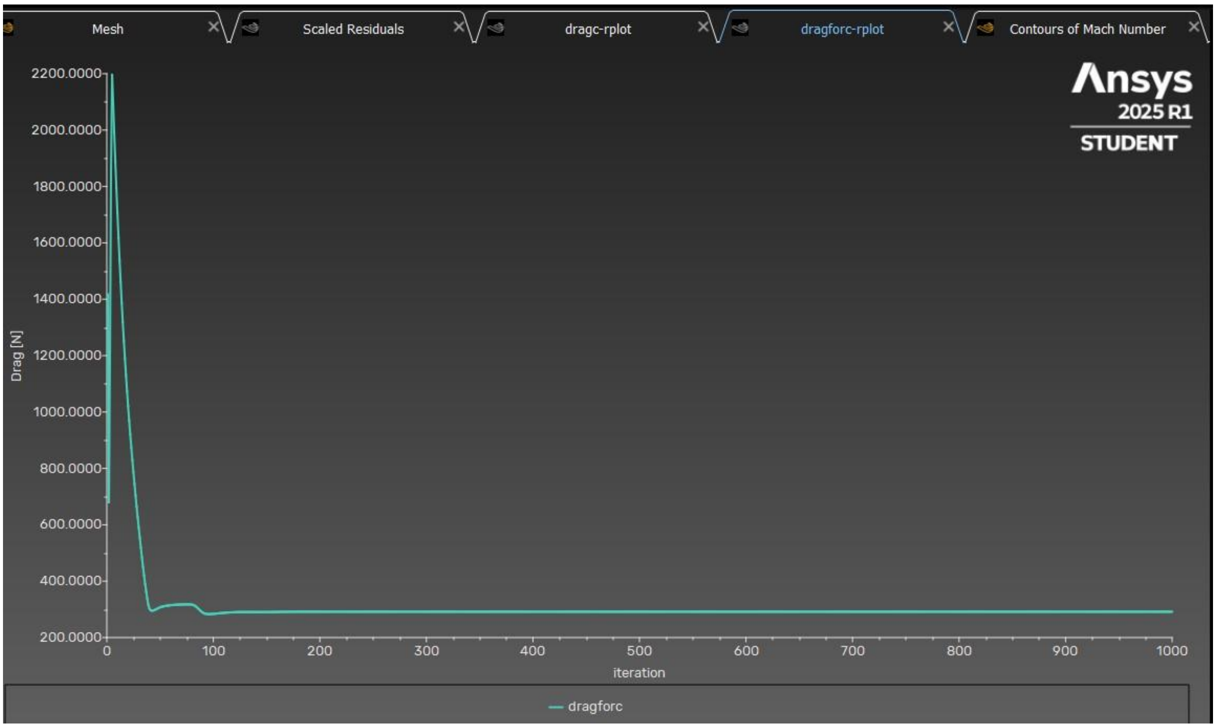

Figure 3.16: Drag Force

Figure 3.17: Drag Coefficent

Results of 100k Mesh with 1.5 Mach Number

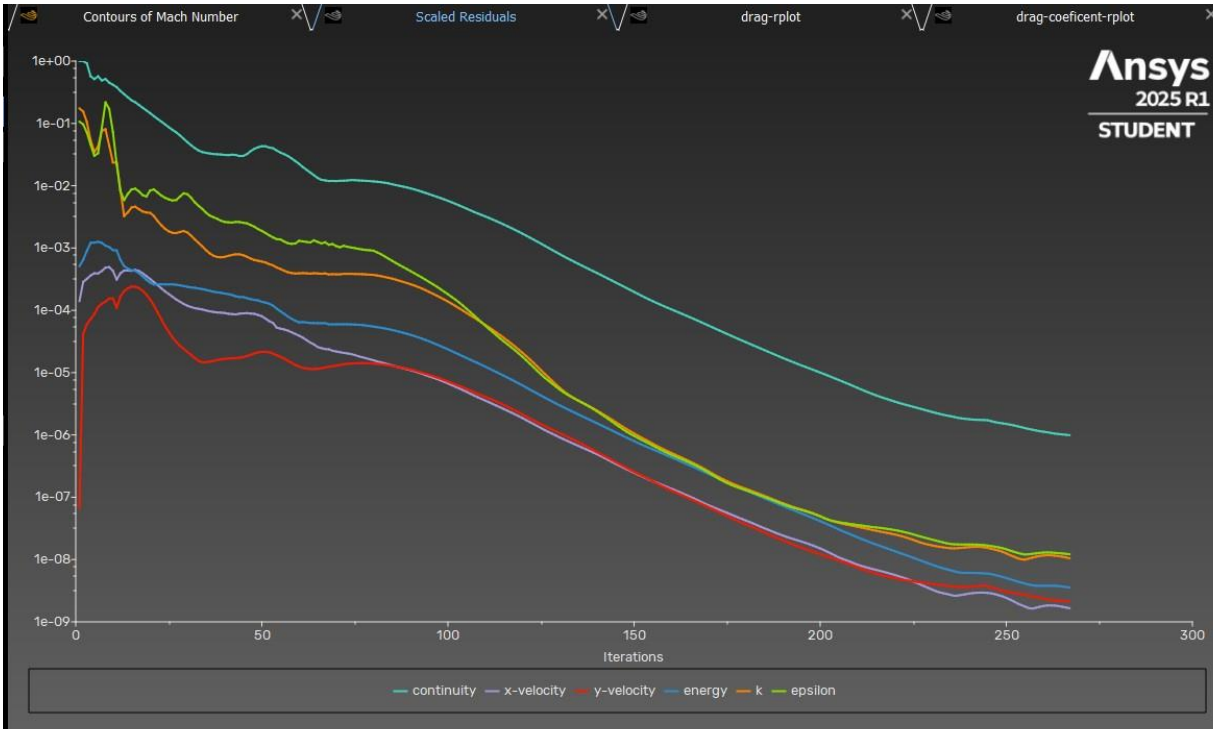

Figure 3.18: Scaled Residuals

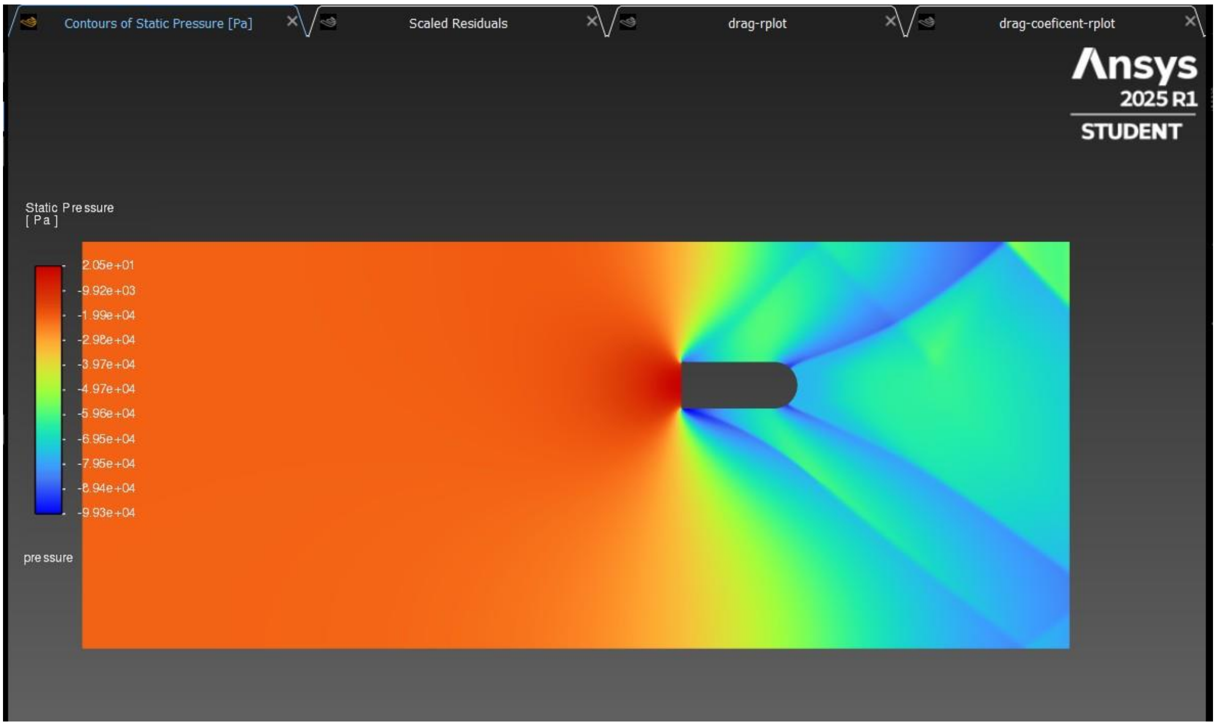

Figure 3.19: Pressure Contour

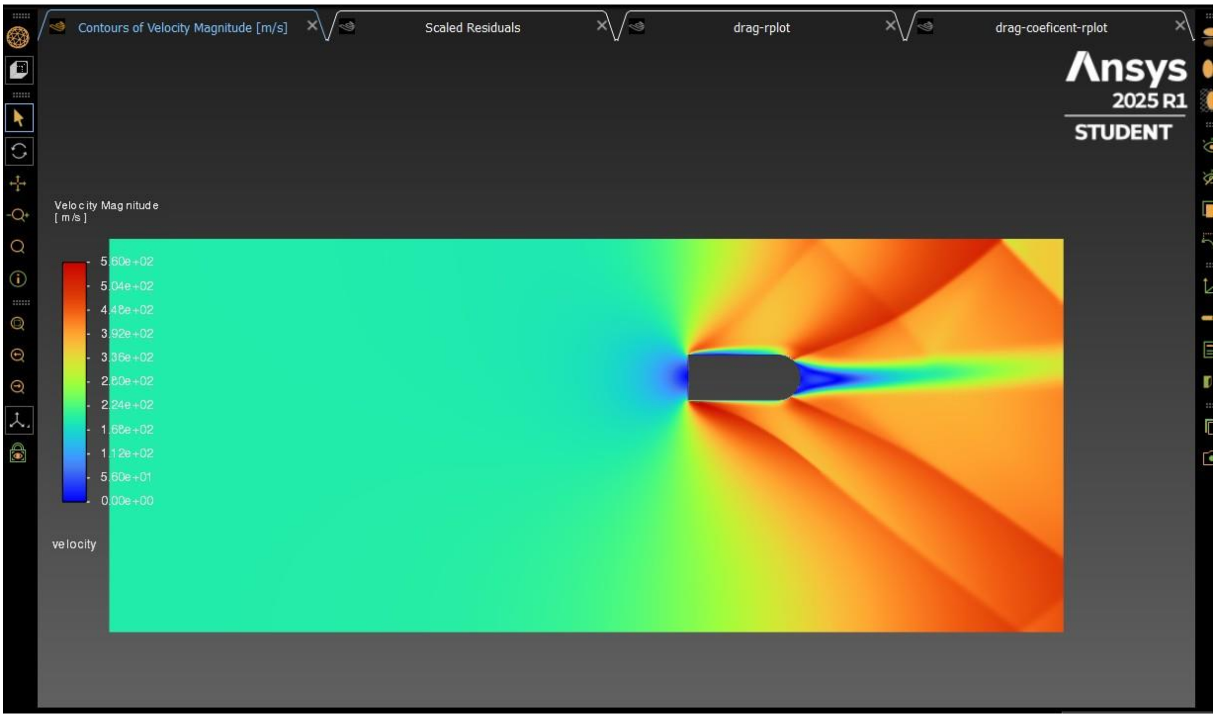

Figure 3.20: Velocity Contour

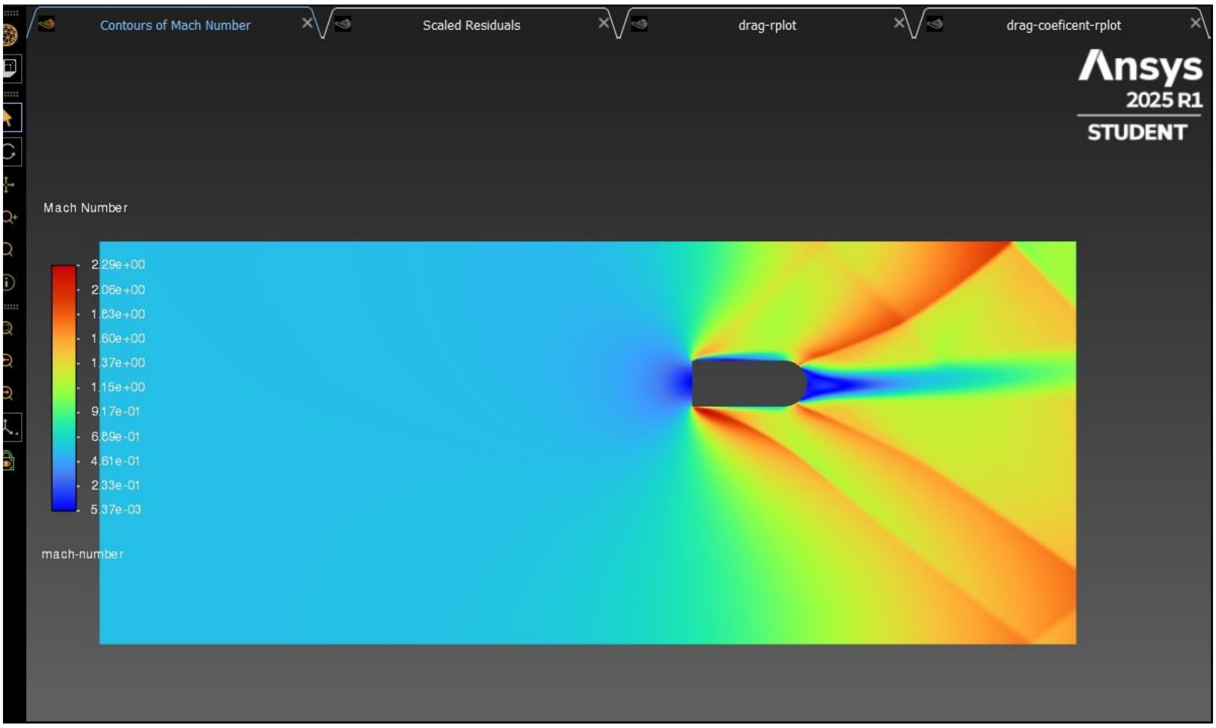

Figure 3.21 : Mach Number Contour

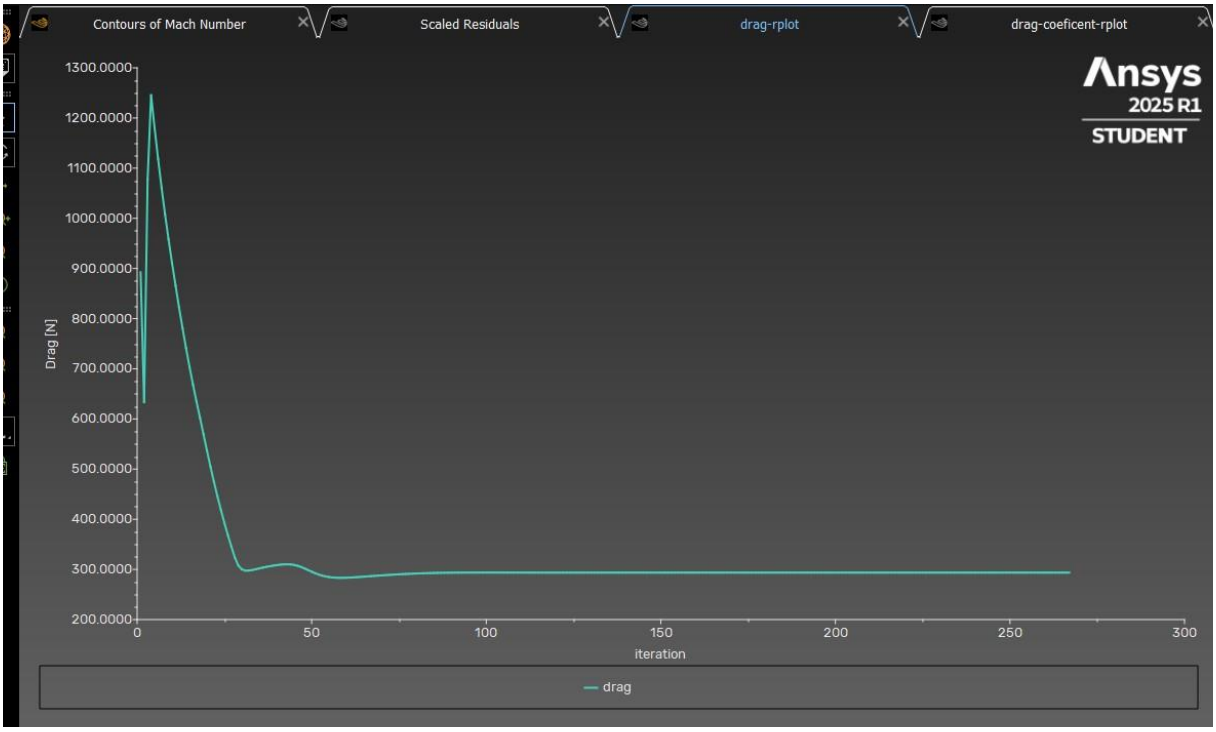

Figure 3.22 : Drag Force

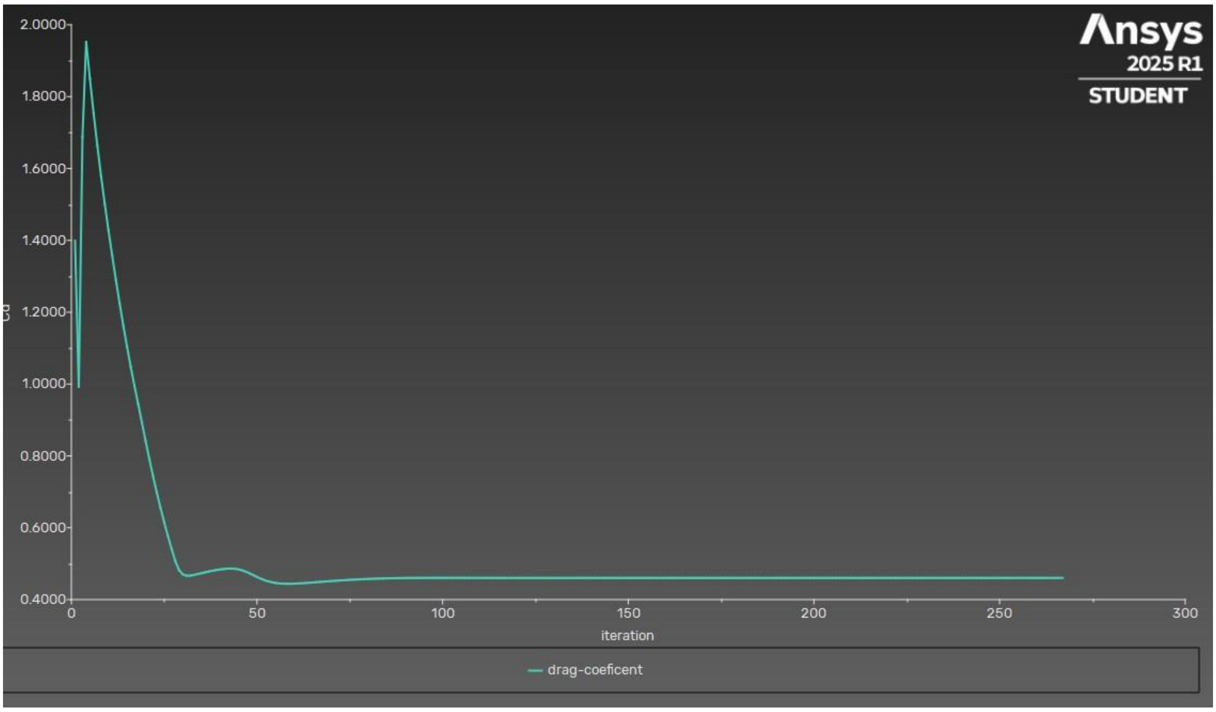

Figure 3.23: Drag Coefficent

Results of 100k Mesh with 2 Mach Number

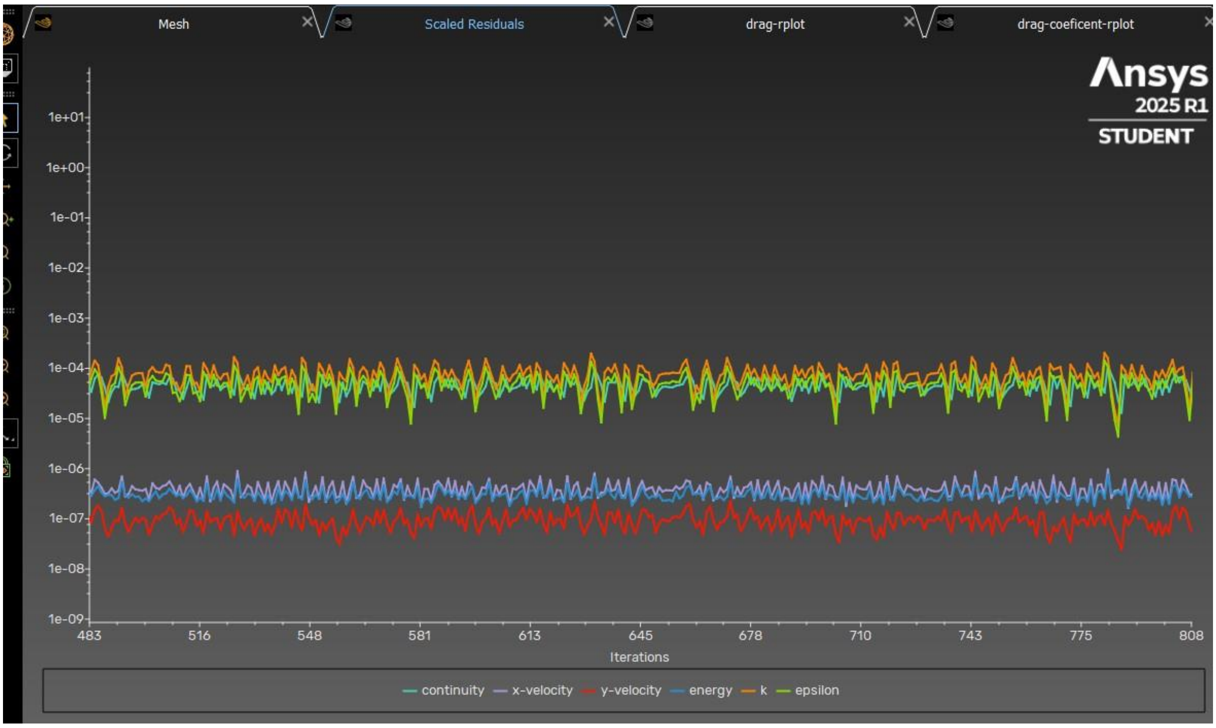

Figure 3.24: Scaled Residuals

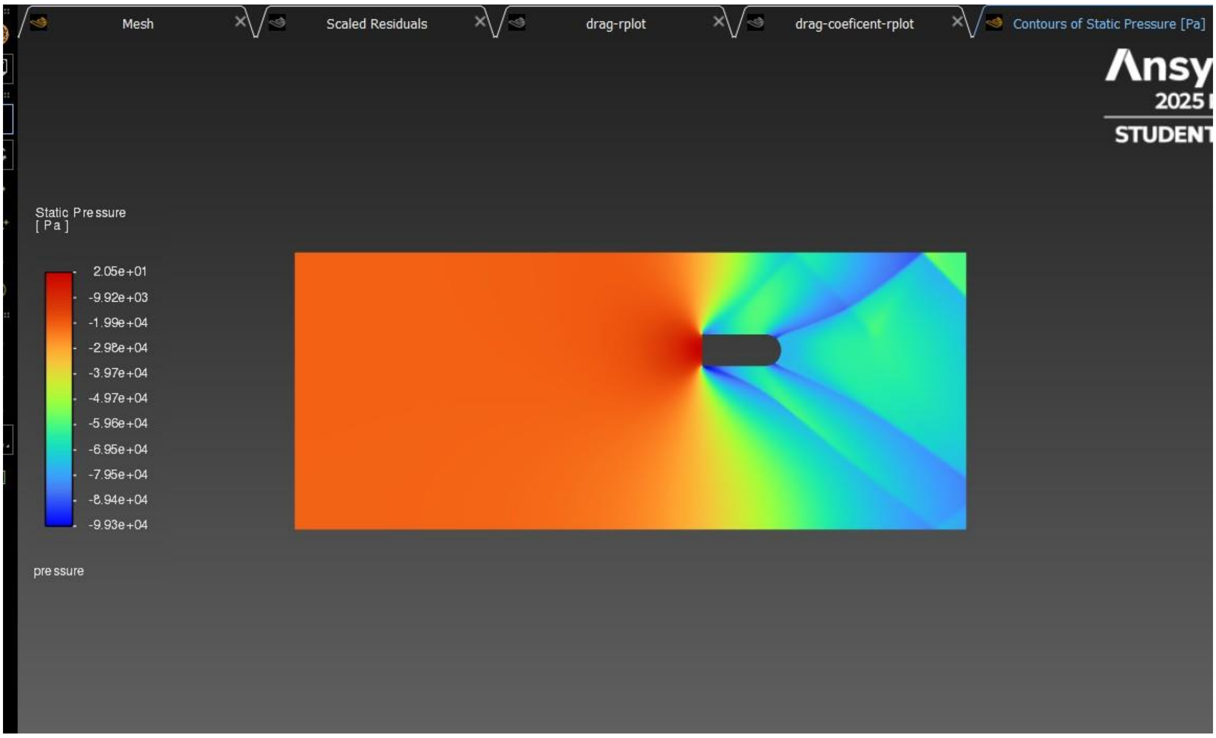

Figure 3.25: Pressure Contour

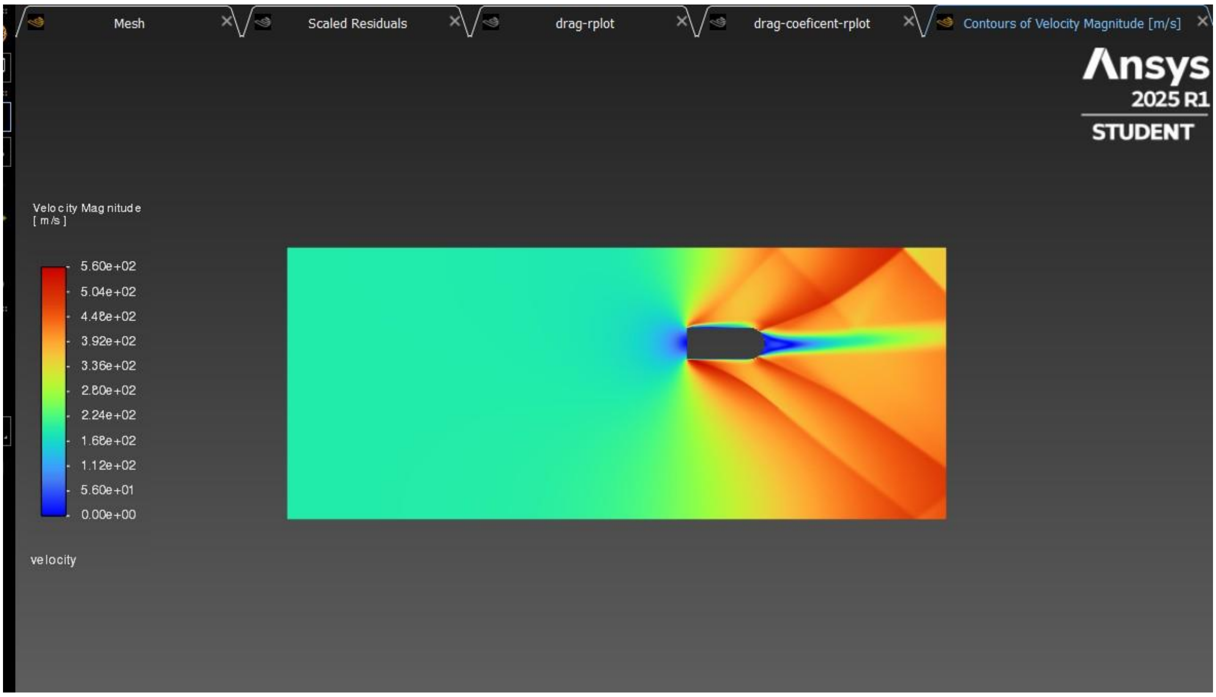

Figure 3.26: Velocity Contour

Figure 3.27: Drag Force

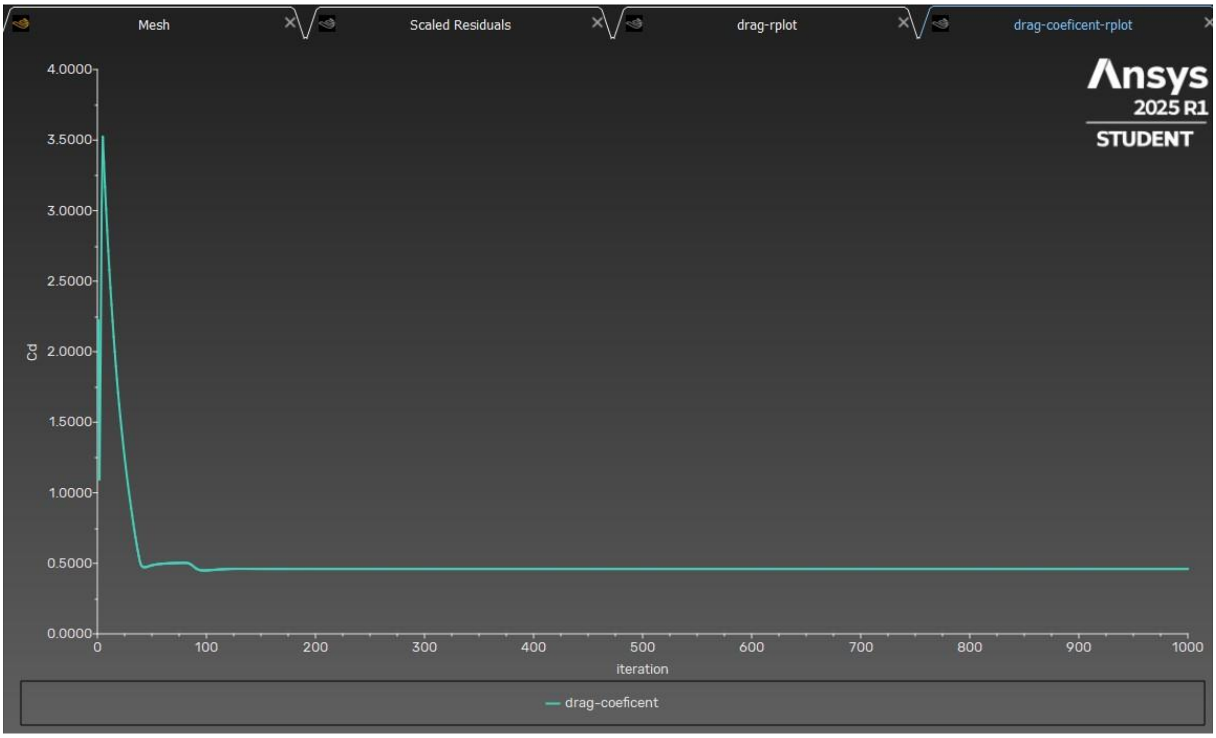

Figure 3.28: Drag Coefficent

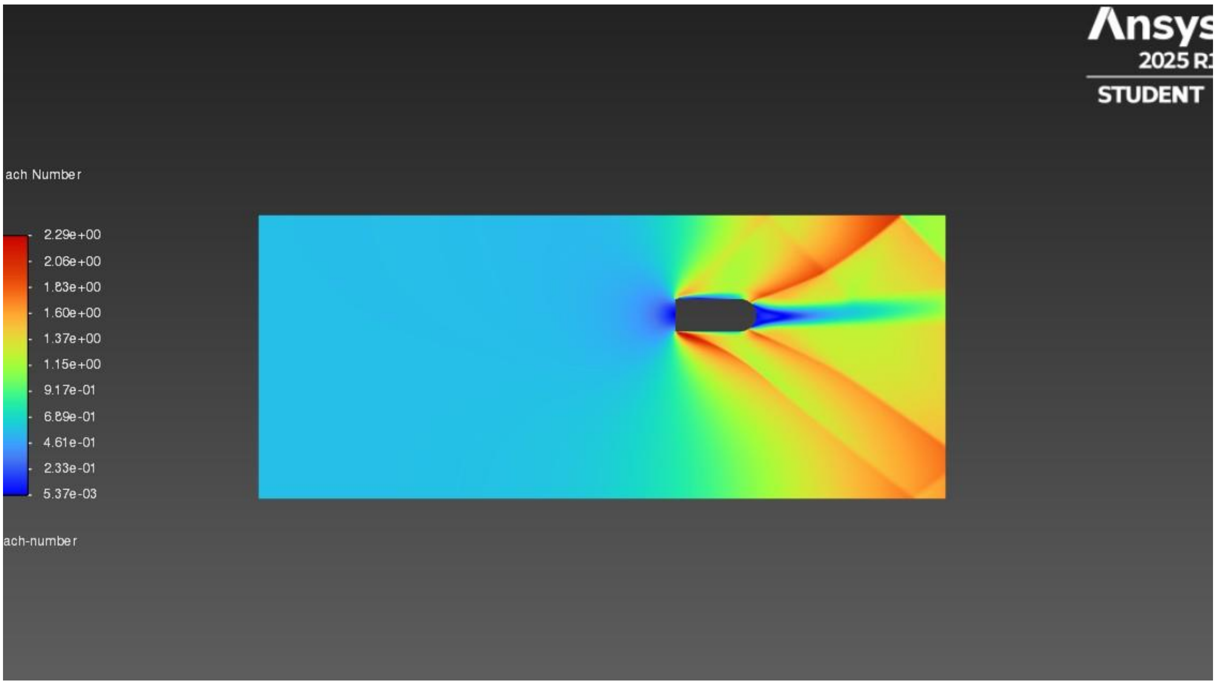

Figure 3.29: Mach Number Counter

Results of 300k Mesh with 1.5 Mach Number

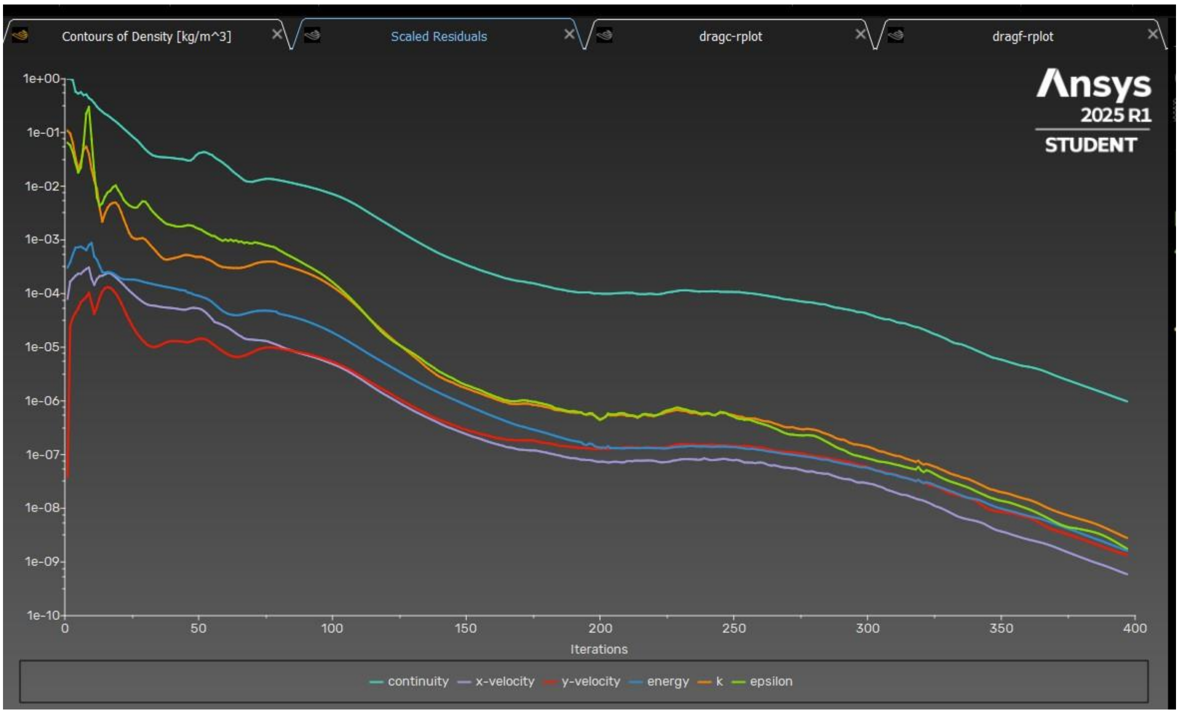

Figure 3.30: Scaled Residuals

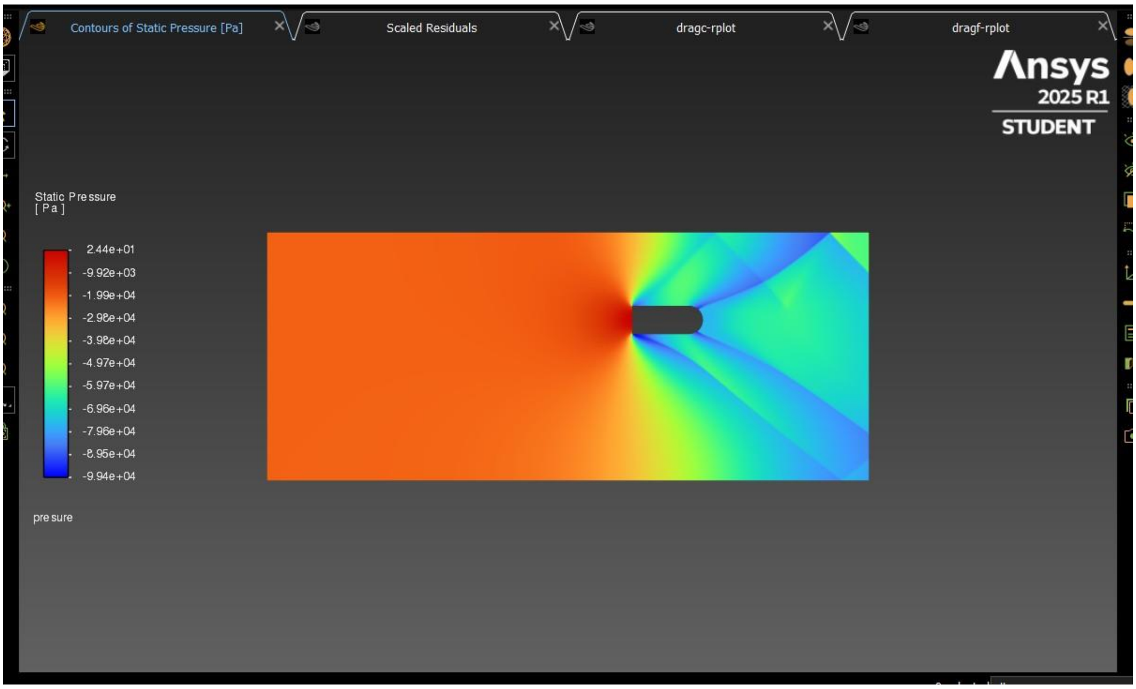

Figure 3.31: Pressure Counter

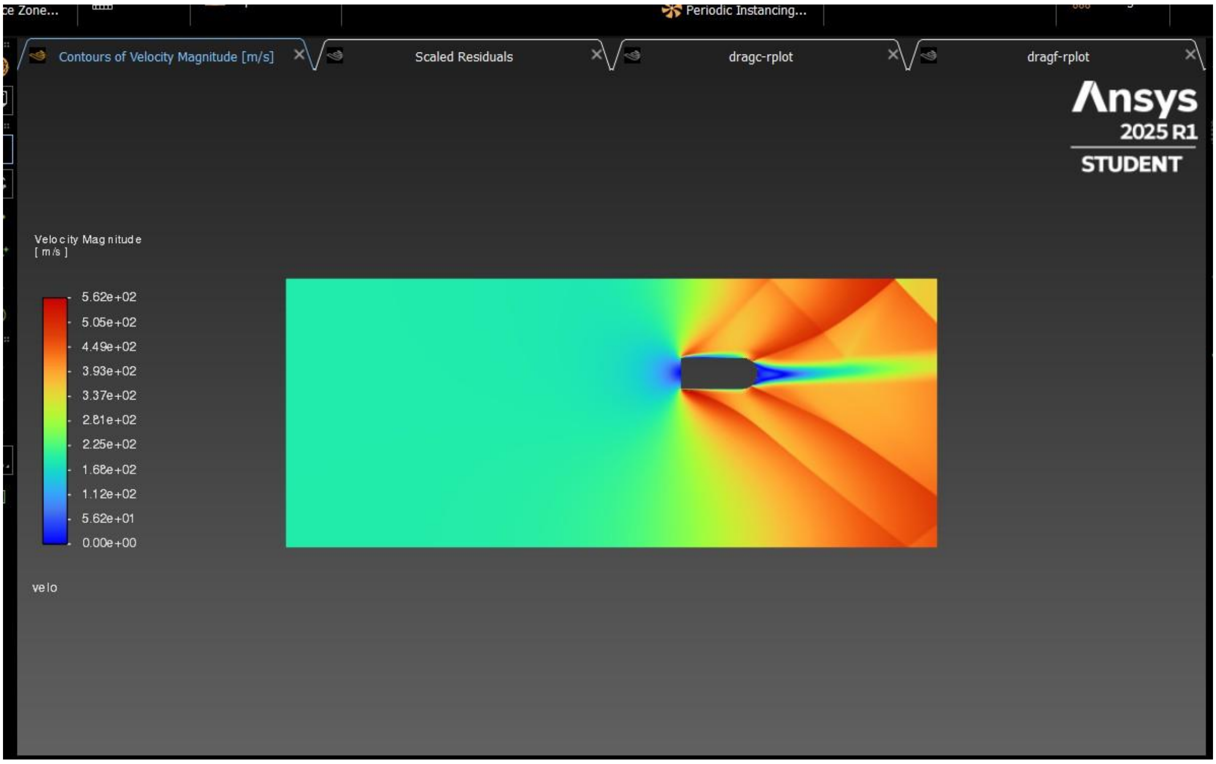

Figure 3.32: Velocity Counter

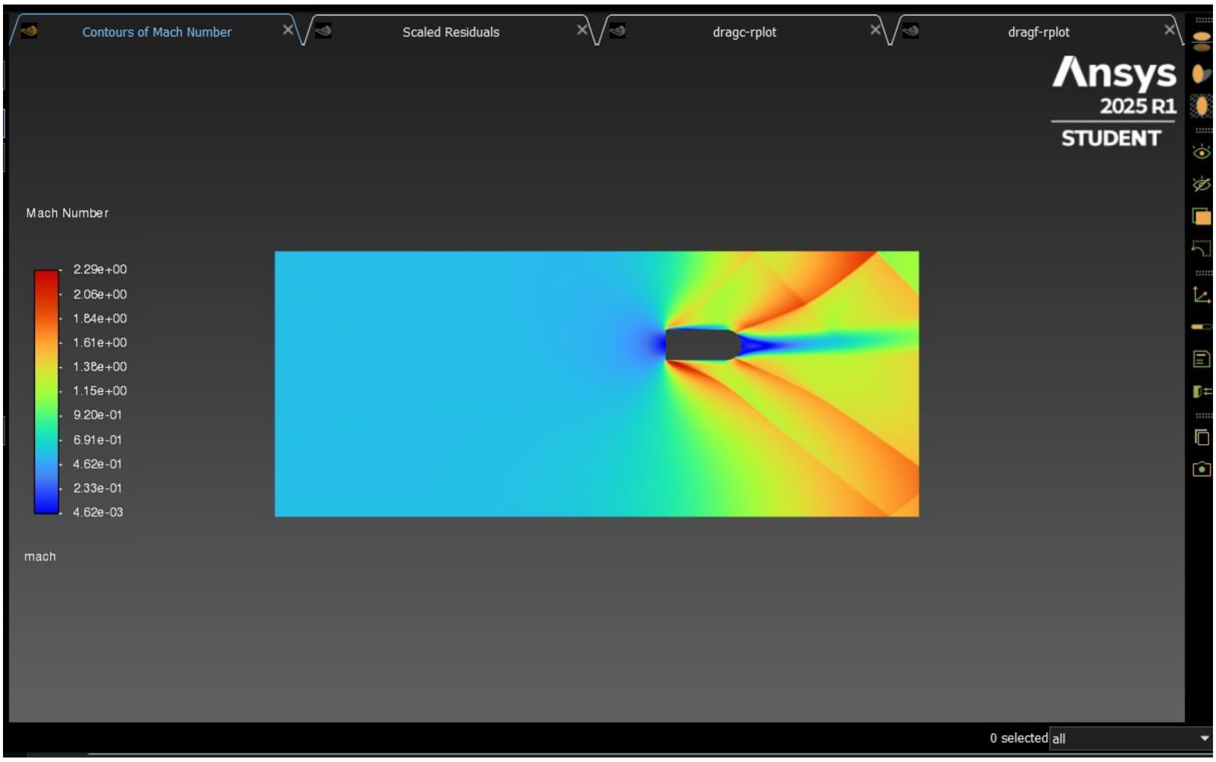

Figure 3.33: Mach Number Counter

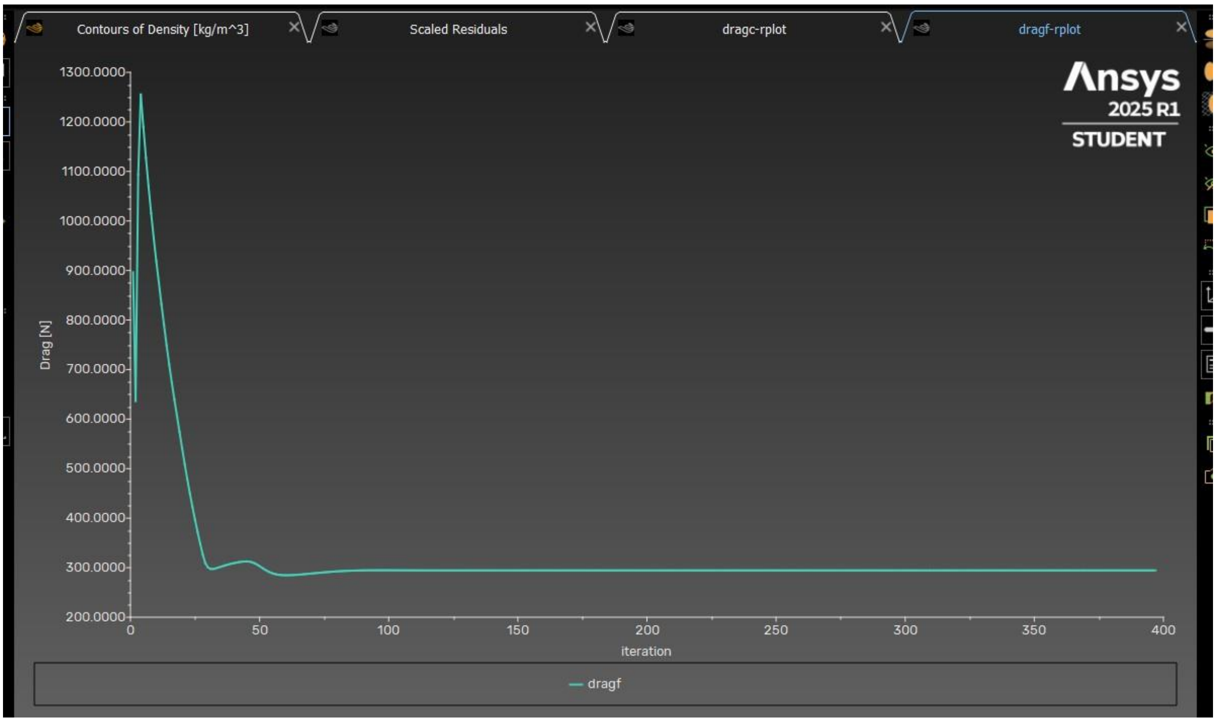

Figure 3.34: Drag Force

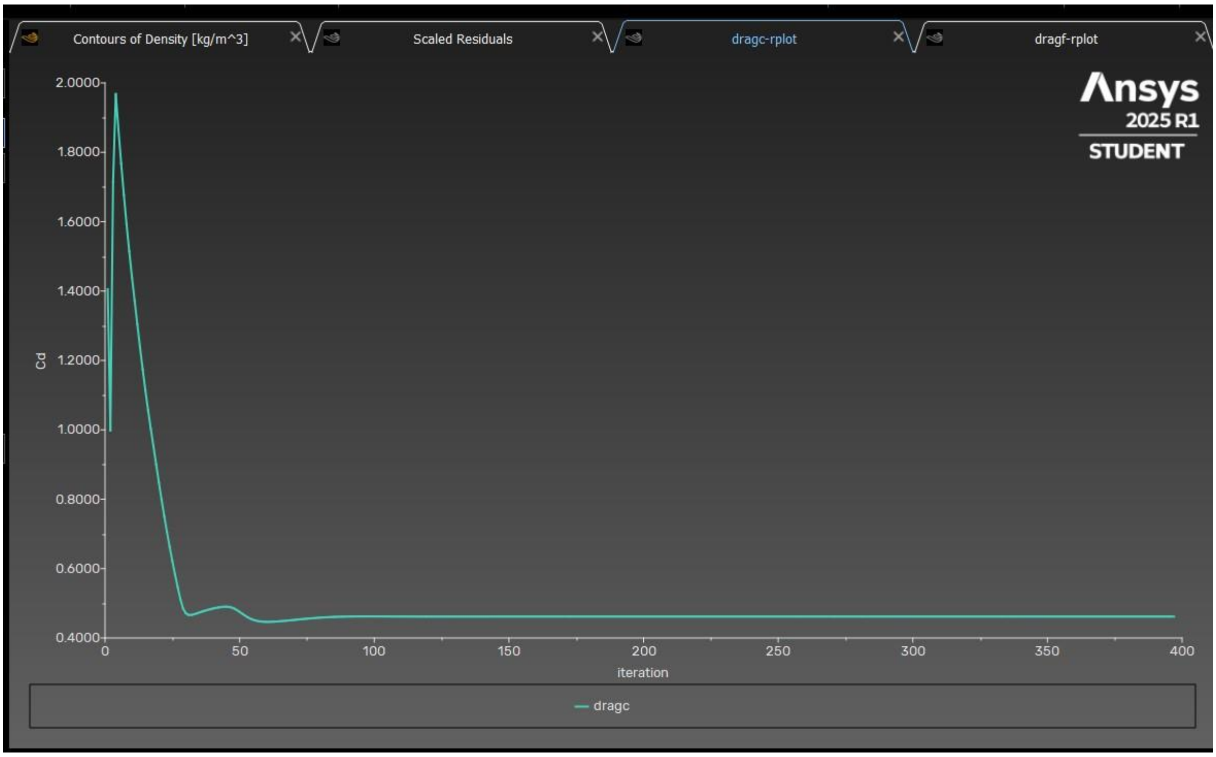

Figure 3.35: Drag Coefficent

Results of 300k Mesh with 2 Mach Number

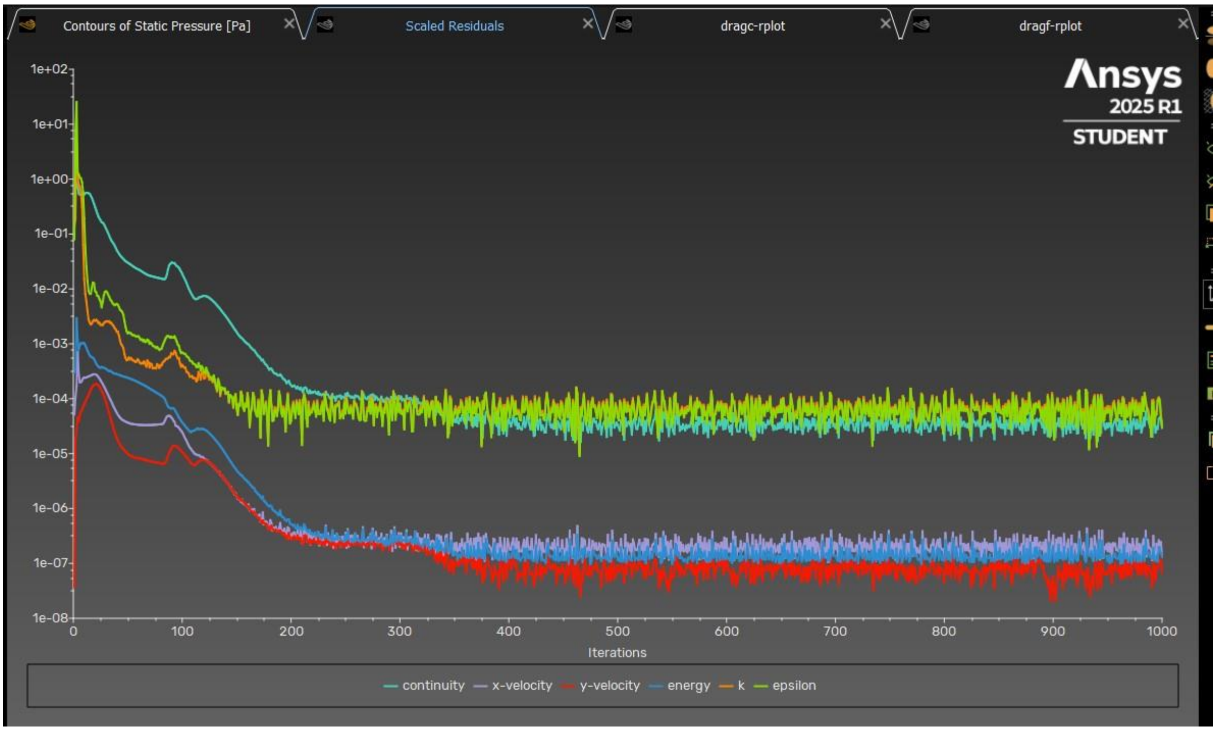

Figure 3.36: Scaled Residuals

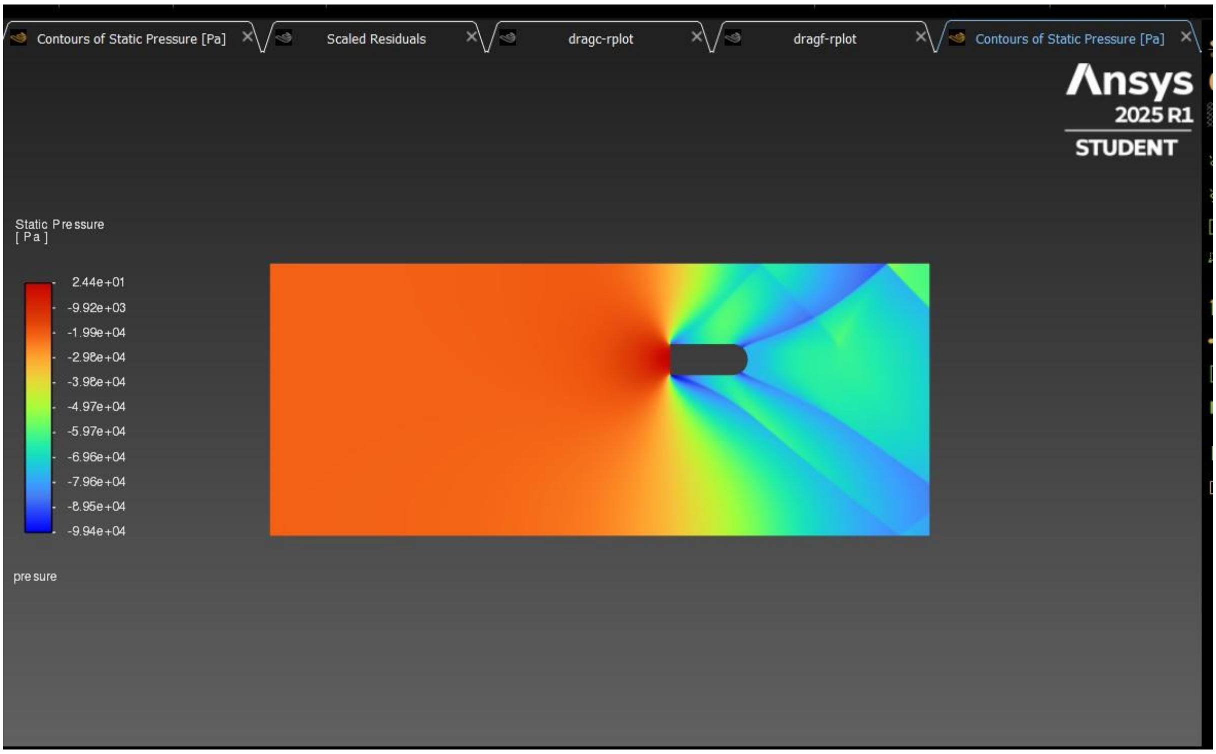

Figure 3.37: Pressure Contour

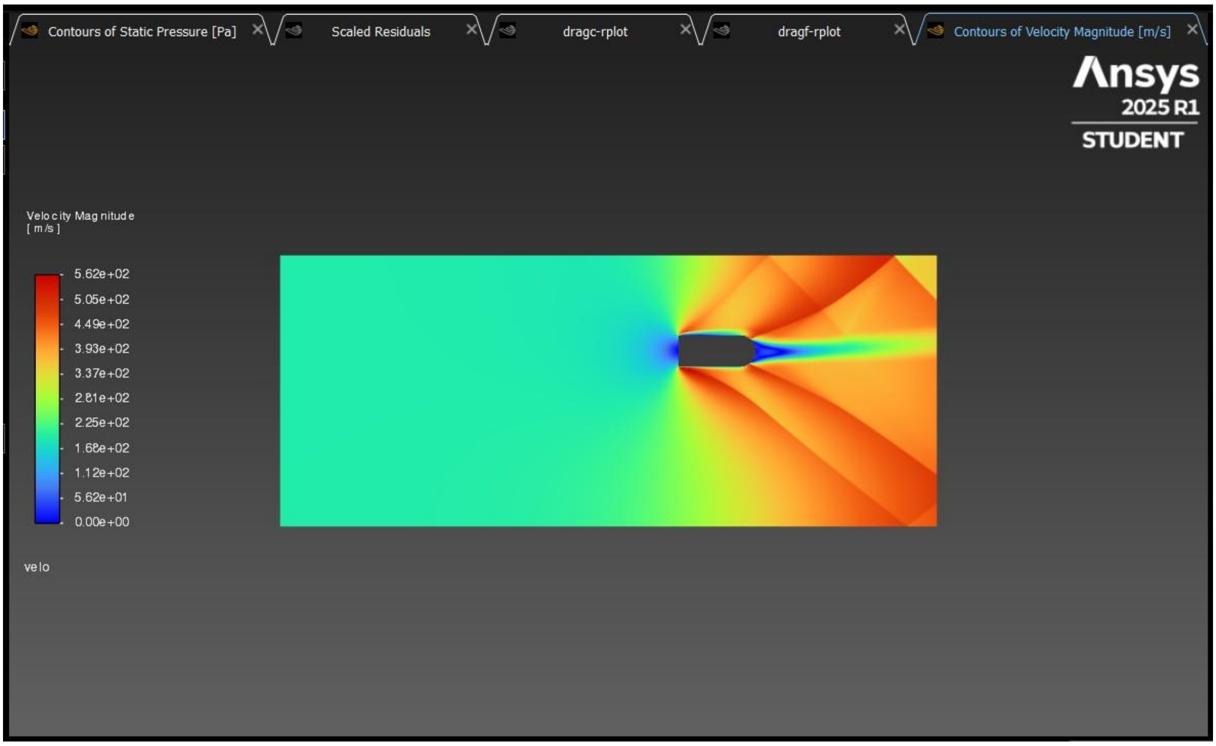

Figure 3.38: Velocity Contour

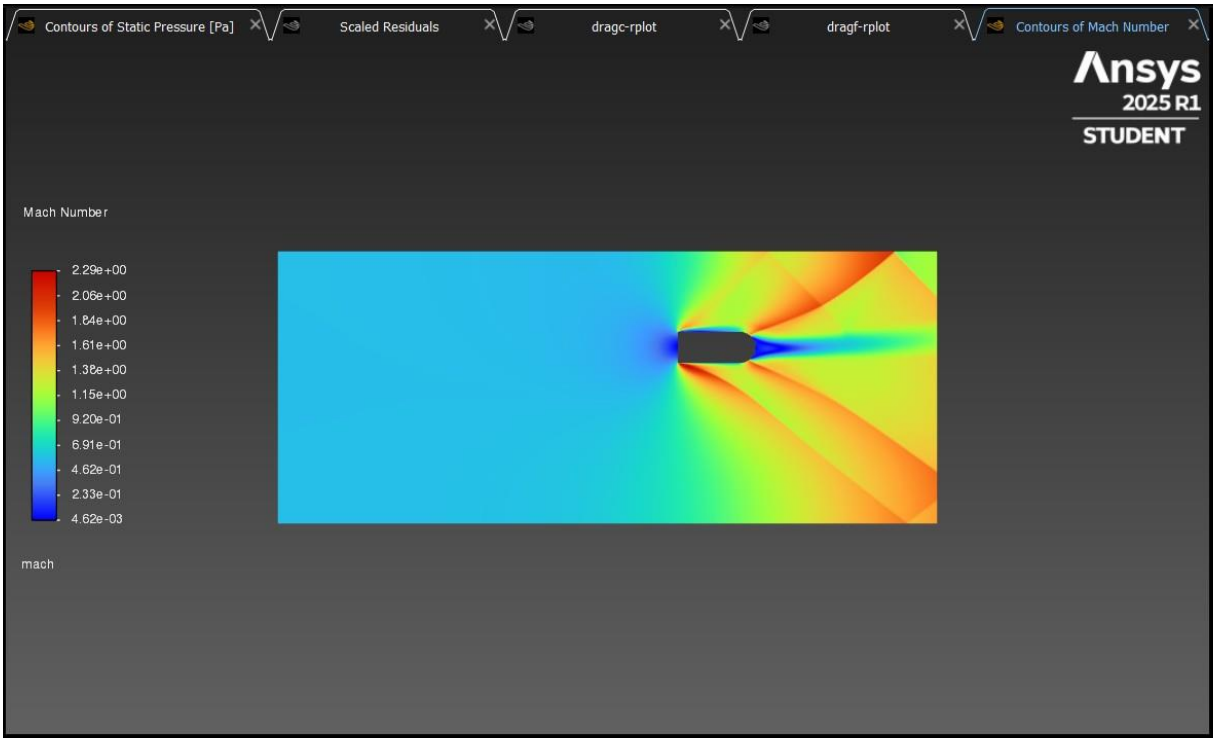

Figure 3.39: Mach Number Contour

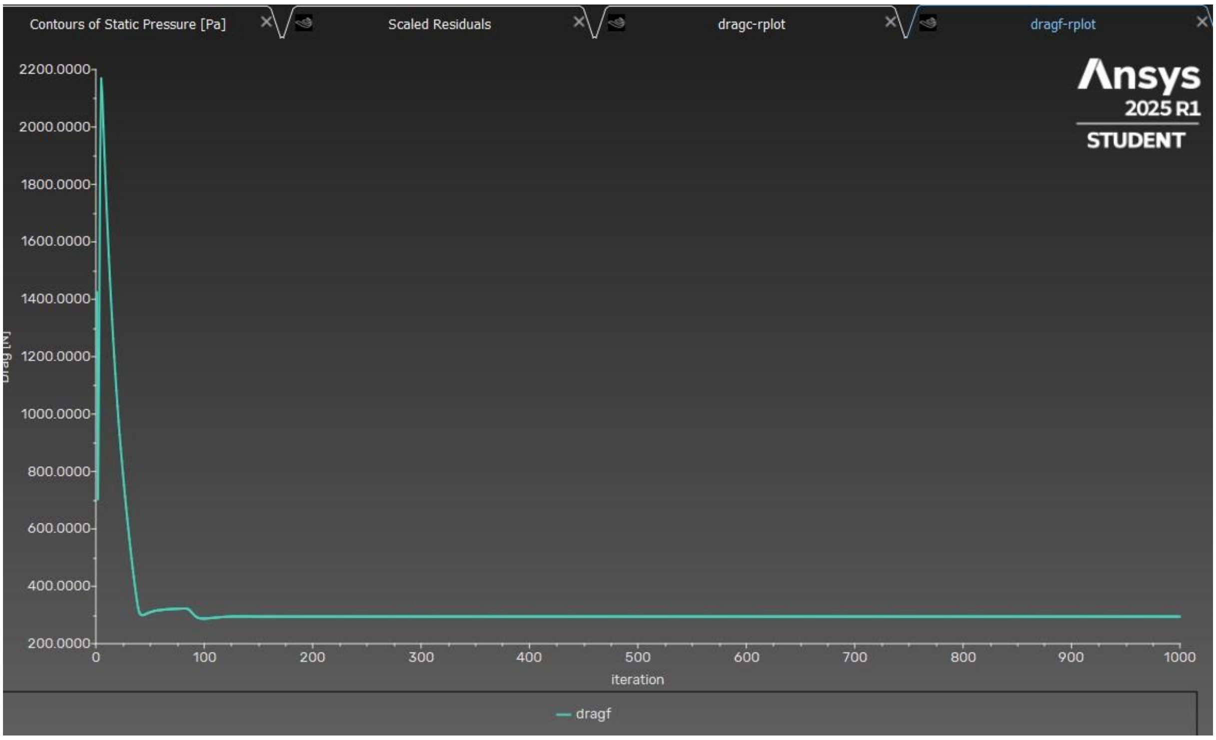

Figure 3.40: Drag Force

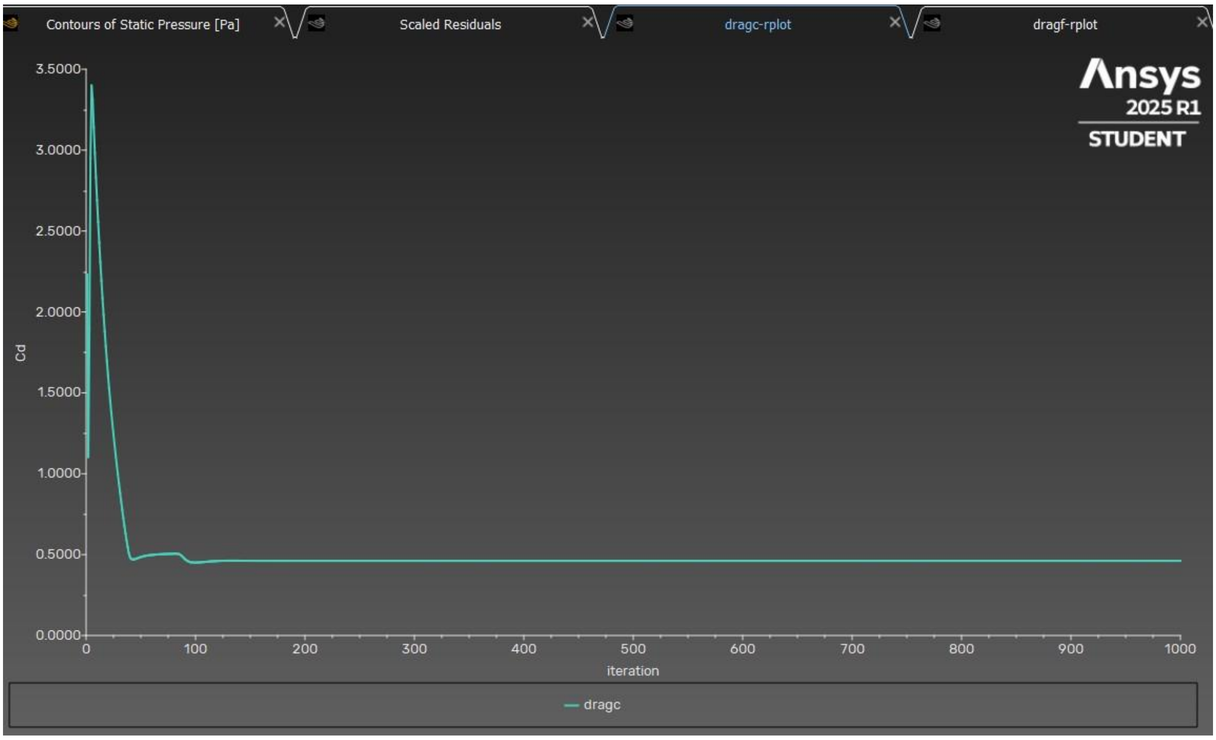

Figure 3.41: Drag Coefficent

Results of Analysis

A mesh convergence study was conducted to assess the adequacy of different mesh densities for capturing supersonic flow characteristics at Mach 1.5 and Mach 2, as illustrated in Figures 3.6 to 3.41. For Mach 1.5, scaled residuals indicate that meshes with 50,000, 100,000, and 300,000 elements are sufficient to achieve converged solutions. However, at Mach 2, these mesh sizes prove inadequate, suggesting that higher Mach numbers require increased mesh refinement to accurately resolve shock wave structures and flow gradients. To further evaluate mesh performance, drag force and drag coefficient will be calculated for both Mach numbers using the k-epsilon turbulence model in ANSYS Fluent. Error percentages between the computed values and reference solutions (or finer mesh results) will be analyzed to quantify accuracy. Based on this analysis, the optimal mesh size will be selected, balancing computational cost and solution accuracy for practical supersonic flow simulations.

4.Matlab Calculations

For Mach Number 1.5

```
=== Mach 1.5 ===
 - Velocity          : 510.39 m/s
 - Density           : 1.225 kg/m^3
 - Cd                : 0.4167
 - Drag Force        : 265.9781 N
>>
```

Figure 4.1: Mach Number 1.5 Results

For Mach Number 2

```
=== Mach 2.0 ===
 - Velocity          : 680.53 m/s
 - Density           : 1.225 kg/m^3
 - Cd                : 0.3000
 - Drag Force        : 340.4520 N
```

Figure 4.2: Mach Number 2 Results

5. ANSYS Fluent Results vs Matlab Results

Effect of Mesh Resolution:

- For both Mach 1.5 and Mach 2, the drag coefficient (CdC_dCd) and drag force (FdF_dFd) show a decreasing trend with an increase in mesh resolution. This is expected as higher mesh densities lead to more accurate simulation results, and the initial results are likely influenced by mesh-related inaccuracies.
- Mach 1.5: The drag coefficient decreases from 0.45 at 50k mesh to 0.42 at 300k mesh, while the drag force decreases from 290 N to 270 N.
- Mach 2: The drag coefficient decreases from 0.49 at 50k mesh to 0.46 at 300k mesh, and the drag force increases from 295 N to 310 N.

These results indicate that higher mesh resolutions provide a better approximation of the drag behavior of the bullet.

For Mach 1.5, the drag coefficient and drag force from the CFD simulations (0.41, 265 N) are in good agreement with the MATLAB results (0.42, 270 N). The small differences can be attributed to the approximations in the MATLAB model and the mesh resolution in the CFD simulations.

For Mach 2, the CFD results show a significant deviation in the drag coefficient (0.46 vs. 0.30) and drag force (310 N vs. 340 N) compared to the MATLAB results. This discrepancy could be due to the simplifications in the MATLAB model, such as the assumption of a constant drag coefficient (CdC_dCd) and lack of detailed flow features in the MATLAB calculations.

At Mach 1.5, the drag coefficient is relatively higher (0.45) than at **Mach 2** (0.49), suggesting that the bullet experiences more resistance at lower Mach numbers.

The drag force is slightly higher for Mach 1.5 (290 N) compared to **Mach 2** (295 N) at the lowest mesh density (50k), which is a counterintuitive result. This could be due to the specifics of the flow characteristics at these different Mach numbers and how the mesh resolution affects the accuracy of the simulation.

6. Conclusion

The CFD simulations show consistent trends across mesh densities, but small variations in drag coefficient and drag force can be observed as the mesh density increases.

MATLAB results for Mach 1.5 align quite well with the CFD results, while the Mach 2 MATLAB results show greater discrepancies. This suggests that the MATLAB approach is more accurate for lower Mach numbers or needs further refinement for high-speed regimes like Mach 2.

Further improvements in the MATLAB model and better mesh resolution could potentially reduce the discrepancies observed for higher Mach numbers.

7.REFERENCES

Anderson, J. D. (2010). Fundamentals of Aerodynamics (5th ed.).

McGraw-Hill. Houghton, E. L., & Carruthers, N. (2009). Aerodynamics for Engineers. Pearson.

Liepmann, H. W., & Roshko, A. (1957). Elements of Gas Dynamics. Wiley.

White, F. M. (2011). Fluid Mechanics (7th ed.). McGraw-Hill.

Versteeg, H. K., & Malalasekera, W. (2007). ***An Introduction to Computational Fluid Dynamics*** **(for k-epsilon model and CFD basics).**

ANSYS Fluent User's Guide (2023) (for solver settings and ideal gas model).

Anderson, J. D. (2016). ***Fundamentals of Aerodynamics*** **(for Mach number effects in supersonic flow).**

ANSYS Inc. (2023). ***ANSYS Fluent User's Guide*****. [For CFD simulation setup and solver settings]**

Versteeg, H. K., & Malalasekera, W. (2007). ***An Introduction to Computational Fluid Dynamics: The Finite Volume Method*** **(2nd ed.). Pearson. [For CFD methodology]**

MATLAB Documentation. (2023). ***MATLAB Programming Fundamentals*****. MathWorks. [For MATLAB coding reference]**

Anderson, J. D. (2016). ***Fundamentals of Aerodynamics*** **(6th ed.). McGraw-Hill Education. [For shock wave and expansion fan theory]**

Zucrow, M. J., & Hoffman, J. D. (1976). ***Gas Dynamics*** **(Vol. 1). Wiley. [For compressible flow equations and drag calculations]**

Carlucci, D. E., & Jacobson, S. S. (2013). ***Ballistics: Theory and Design of Guns and Ammunition*** **(2nd ed.). CRC Press. [For bullet-specific aerodynamics]**

Bertin, J. J., & Cummings, R. M. (2013). ***Aerodynamics for Engineers*** **(6th ed.). Pearson. [For drag coefficient and projectile aerodynamics]**