

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1 \bar{x}_2 \bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1 \bar{x}_2 x_3$	$M_1 = x_1 + x_2 + x_3$
2	0	1	0	$m_2 = \bar{x}_1 x_2 \bar{x}_3$	$M_2 = x_1 + x_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1 x_2 x_3$	$M_3 = x_1 + x_2 + x_3$
4	1	0	0	$m_4 = x_1 \bar{x}_2 \bar{x}_3$	$M_4 = x_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1 \bar{x}_2 x_3$	$M_5 = \bar{x}_1 + x_2 + x_3$
6	1	1	0	$m_6 = x_1 x_2 \bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$$\begin{aligned} m_0 &= M_0 \\ (\bar{x}_1 \bar{x}_2 \bar{x}_3) &= (\bar{x}_1) + (\bar{x}_2) + (\bar{x}_3) \\ &= x_1 + x_2 + x_3 = M_0 \end{aligned}$$

## Example 1:

Express the Boolean function  $F = A + BC$  as a sum of minterms. The function has three variables: A, B, and C.

The first term A is missing two variables; therefore,

$$A + AB + B' + AB' \rightarrow B + B' = 1, A \cdot 1 = A$$

This function is still missing one variable,

$$so A + AB(C + C') + AB(C + C') = ABC + ABC + ABC' + ABC'$$

The second term BC is missing one variable;

Table 2.2 Truth Table for $F = A + BC$				
	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

hence,

$$BC + B'C(A + A') = ABC + ABC'$$

Combining all terms, we have

$$F = A + BC + ABC + ABC' + ABC' + ABC = ABC + ABC'$$

$$F(m) = m_0 + m_1 + m_3 + m_4 + m_5$$

$\{A, B, C\}$  mean of minterms {1, 4, 5, 6, 7}

$$= \sum m(1, 4, 5, 6, 7)$$

## Example 2:

Express the Boolean function  $F = xy + x'y$  as a product of maxterms. First, convert the function into OR terms by using the distributive law:

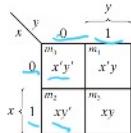
$$\begin{aligned} F = xy + x'y &= (xy + x')(xy + z) \\ &= (x + x')(y + x')(y + z)(y + z) \\ &= (x' + y)(y + z)(y + z) \end{aligned}$$

The function has three variables: x, y, and z. Each OR term is missing one variable; therefore,

$$\begin{aligned} x' + y &= (x + y) + zz = (x' + y + z)(x' + y + z') \\ x + z &= x + z + y' = (x + y + z)(x + y' + z) \\ y + z &= y + z + x' = (x + y + z)(x' + y + z) \end{aligned}$$

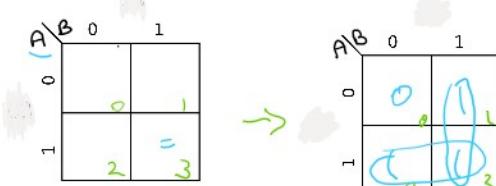
Combining all the terms and removing those which appear more than once, we finally obtain

$$\begin{aligned} F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z') \\ &= M_0 M_2 M_4 M_5 \end{aligned}$$

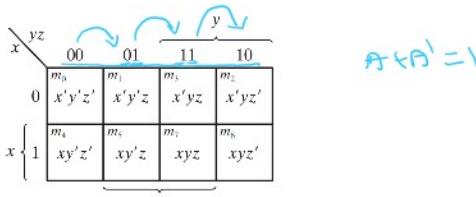


$$\sum m(1, 2, 3)$$

$$\begin{aligned} a+b+c &= (a+b)(a+c) \\ RHS: \\ (a+b) \cdot (a+c) &= a \cdot a + a \cdot c + b \cdot a + b \cdot c \\ &= a + a \cdot c + b \cdot a + b \cdot c \\ &\rightarrow a \cdot (1+c) + b \cdot a + b \cdot c \\ &= a \cdot 1 + b \cdot a + b \cdot c \\ &= a(1+b) + bc = a + b \cdot c // \end{aligned}$$



$$\begin{aligned} \text{Problem: } F &= m_1 + m_3 = A'B + AB \\ &= B(A + A') = B \\ \text{①} \rightarrow B & \\ \text{②} \rightarrow A & \\ \text{Final exprn} &= A + B \end{aligned}$$



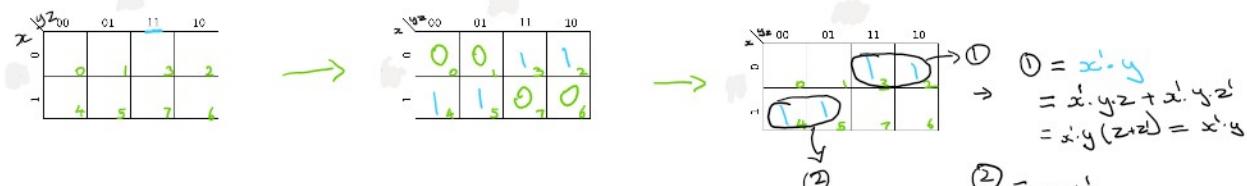
$$A + A' = 1$$

Problem:  
 $F(a, b, c) = \text{sum of minterms}\{2, 3, 4, 5\}$



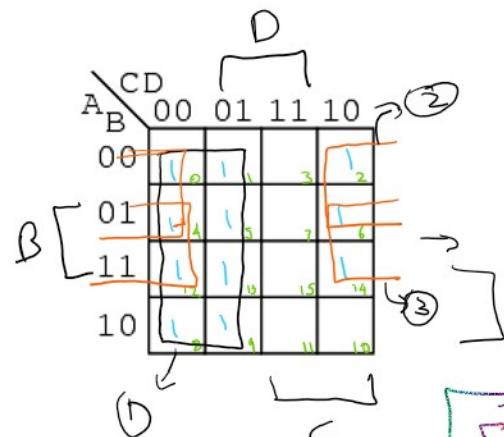
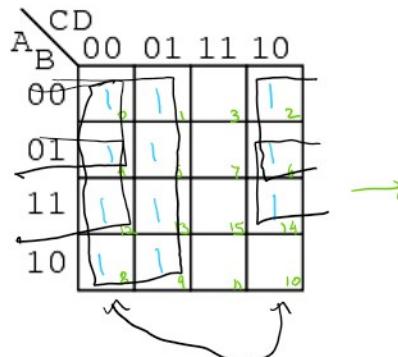
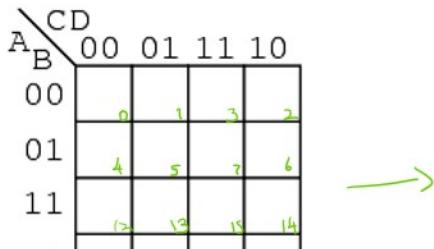


$F(a, b, c) = \text{sum of minterms}(2, 3, 6, 5)$



		yz	00	01	11	10
		x	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'
		y	m <sub>0</sub>	m <sub>1</sub>	m <sub>3</sub>	m <sub>2</sub>
(0)	00	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'	①
01	01	w'xy'z'	w'xy'z	w'xyz	w'xyz'	②
	11	wxy'z'	wxy'z	wxyz	wxyz'	④
	10	wx'y'z'	wx'y'z	wx'yz	wx'yz'	③

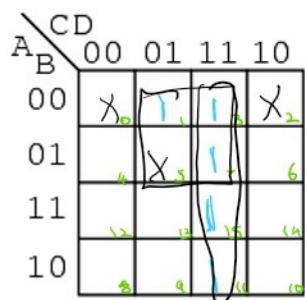
Problem:  
 $F(a, b, c, d) = \text{sum of minterms } \{0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14\}$



Don't care:

Problem:

$F(a, b, c, d) = \{1, 3, 7, 11, 15\}$   
 $d(a, b, c, d) = \{0, 2, 5\}$



$$\textcircled{1} \Rightarrow C'$$

$$\textcircled{2} \Rightarrow D' \cdot A' = A \cdot D'$$

$$\textcircled{3} \Rightarrow B \cdot D$$

$$\textcircled{1} + \textcircled{2} + \textcircled{3} = C' + A' \cdot D' + B \cdot D'$$