DOCUMENT TITLE

INTRODUCTORY TEXT

Contents

Clear complete workspace

Its always a good idea to clear the complete workspace and the command window also closing all figures might be helpful. You may also use the header defin some neccessary flags distinguishing bewteen different data sets.

```
close all
clear all
clc

flag='2D';
datadir='data';
```

Read data files

Read in the data files and measure the time for reading. The output of the tic/toc block is in seconds. What you should get from the tic/toc block is that most of the time is spend during data I/O. The actual computation needs only ??? of the time of the I/O operations.

3D

```
if (strcmp('3D',flag))
    tic; % enable timer
    uvel=importdata([datadir,'/',flag,'/uvel']);
    vvel=importdata([datadir,'/',flag,'/vvel']);
```

```
    wvel=importdata([datadir,'/',flag,'/wvel']);
    time_reading = toc; % end timer
end
%%% 2D
if (strcmp('2D',flag))
    tic;
    uvel=importdata([datadir,'/',flag,'/uvel']);
    vvel=importdata([datadir,'/',flag,'/vvel']);
    time_reading = toc;
end
```

Set some neccessary parameters

For further computations it is important to define some parmeters of the DNS simulation such as domain size, grid spacing, and the number of grid points.

3D

```
if (strcmp('3D',flag))
    dim=256; % number of points in one dimension
    Lx=5e-3; % domain size
    Ly=Lx;
    Lz=Lx;
    dx=Lx/dim; % grid spacing
    dy=dx;
    dz=dx;
    nu=1.7e-5; % viscosity
    u=reshape(uvel,dim,dim,dim); % reshape arrays to have them in 3D
    v=reshape(vvel,dim,dim,dim);
    w=reshape(wvel,dim,dim,dim);
end
%%% 2D
if (strcmp('2D',flag))
    dim=1024; % number of points in one dimension
    Lx=1E-2;  % domain size
    Ly=Lx;
    dx=Lx/dim; % grid spacing
    dy=dx;
    u=reshape(uvel,dim,dim); % reshape arrays to have them in 2D
    v=reshape(vvel,dim,dim);
end
```

Compute FFT

This is the most important part of the script. Since the performance of an actual

DFT is rather bad the preferred choice is a FFT. The FFT approach is fastest if the data set to be transformed has a size that is a multiple of two. Thats why the function **nextpow2** is used to get the next powert of two approximating the dimension *dim* of the data set. As a consequence the data set is zero padded or truncated. *Since the output of an FFT operation is symmetric we only need to save half the transform.*

$$\Phi_{ij}(\kappa) = \frac{1}{(2\,\pi)^3} \iiint\limits_{-\infty}^{\infty} R_{ij}(\mathbf{r})\,\mathrm{e}^{-i\kappa r}\,\mathrm{d}\mathbf{r} \tag{1}$$

After the transformation of all velocity components we have to compute the velocity correlation tensor Φ . From theory we know

$$(u_i * u_j) = \int\limits_{-\infty}^{\infty} u_i^*(\mathbf{x})\,u_j(\mathbf{x}+\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{2}$$

Since all our data sets are transformed (and we are in the Fourier space) the last expression can be simply computed by multiplying

$$\mathfrak{F}\{u_i * u_j\} = \alpha \cdot \{\mathfrak{F}\{u_i\}\}^* \cdot \mathfrak{F}\{u_j\}, \tag{3}$$

where α is a normalization factor.

```
if (strcmp('3D',flag))
    tic; % start timer
    NFFT = 2.^nextpow2(size(u)); % next power of 2 fitting the length of u
    u_fft=fftn(u,NFFT)./(2*pi)^3; %2 pi --> definition of FFT
    %
    NFFT = 2.^nextpow2(size(v));
    v_fft=fftn(v,NFFT)./(2*pi)^3;
    %
    NFFT = 2.^nextpow2(size(w));
    w_fft=fftn(w,NFFT)./(2*pi)^3;
    time_fft=toc; % get final time for all transformations

    phi_x=u_fft.*conj(u_fft)/dim^6; % compute velocity correlation tensor
    phi_y=v_fft.*conj(v_fft)/dim^6;
    phi_z=w_fft.*conj(w_fft)/dim^6;
end
if (strcmp('2D',flag))
    tic; %start timer
    NFFT = 2.^nextpow2(size(u));
    u_fft=fft2(u,NFFT(1),NFFT(2))./(2*pi)^2; %2 pi --> definition of FFT
    %
```

```
    NFFT = 2.^nextpow2(size(v));
    v_fft=fft2(v,NFFT(1),NFFT(2))./(2*pi)^2;
    %
    phi_x=u_fft.*conj(u_fft)/size(u,1).^2/size(u,2).^2;
    phi_y=v_fft.*conj(v_fft)/size(v,1).^2/size(v,2).^2;
end
```

Compute correlations

Computing a correlation can be a tedious work (requireing tremendeous effort) especially if you have large data sets. From theory it is well known that the multiplication of the transform of a data set and its complex conjugate are an accurate representation of the correlation function. Using the FFT approach this gives an enormeous speed advantage. Since we already computed the veloity correlation tensor we may use this result in order to compute the correlation tensor.

$$R_{ij} = \frac{cov(U_i, U_j)}{\sqrt{\sigma_i^2\,\sigma_j^2}} = \frac{(u_i' - \mu_i)\,(u_j - \mu_j)}{\sqrt{\sigma_i^2\,\sigma_j^2}} \tag{4}$$

```
if (strcmp('3D',flag))
    R11=ifftn(u_fft.*conj(u_fft))/dim^3/std2(u)^2;
    R22=ifftn(u_fft.*conj(u_fft))/dim^3/std2(v)^2;
    R33=ifftn(u_fft.*conj(u_fft))/dim^3/std2(w)^2;
    %
    R11=R11(1:round(size(R11,1)/2),1,1);
    R22=R22(1:round(size(R22,1)/2),1,1);
    R33=R33(1:round(size(R33,1)/2),1,1);
    %
    r = linspace(0,Lx/2,dim/2)/(Lx/2);
end
if (strcmp('2D',flag))

    NFFT = 2.^nextpow2(size(u_fft));
    R1 = ifft2(u_fft.*conj(u_fft),NFFT(1),NFFT(2))...
                ./NFFT(1)./NFFT(2)./std2(u)^2 ...
                .*(2*pi)^4; % scaling due to division by 2*pi
    %
    NFFT = 2.^nextpow2(size(v_fft));
    R2 = ifft2(v_fft.*conj(v_fft),NFFT(1),NFFT(2))...
                ./NFFT(1)./NFFT(2)./std2(v)^2 ...
                .*(2*pi)^4; % scaling due to division by 2*pi
    %
    R11 = (R1(1:round(size(R1,1)/2),1) + ...
```

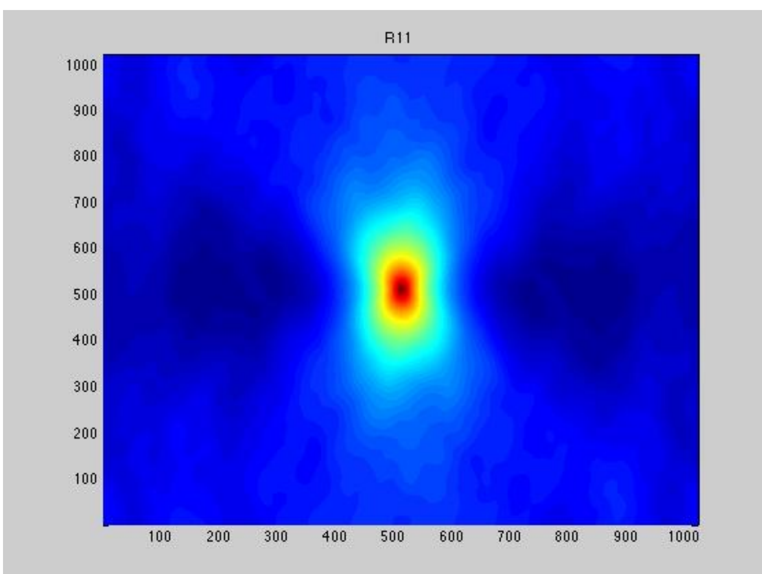
```
        R2(1,1:round(size(R2,1)/2))')/2; % build the mean
    R22 = (R2(1:round(size(R2,1)/2),1) + ...
        R1(1,1:round(size(R1,1)/2))')/2;
    %
    r = linspace(0,Lx/2,dim/2)/(Lx/2); % get the radius
```

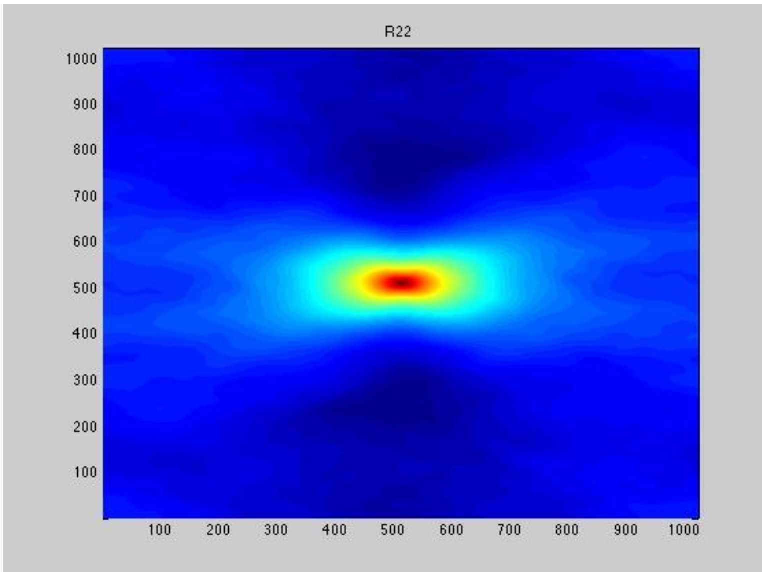
From theory we know that the transversal correlation could also be computed from the longitudinal correlation by

$$g(r) = f + \frac{r}{2}\frac{\partial f}{\partial r} \tag{5}$$

```
    g_r = R11 + r'/2.*gradient(R11,max(diff(r)));

end
plot(r,R11,r,R22,r,g_r)
legend('R11','R22','g_r');
h=line([0 1],[0 0],'Color',[0 0 0],'LineWidth',1.0);
% 2D graphs of correlation function
pcolor(fftshift(R1));shading interp;title('R11');
figure
pcolor(fftshift(R2));shading interp;title('R22');
```





Compute length scales

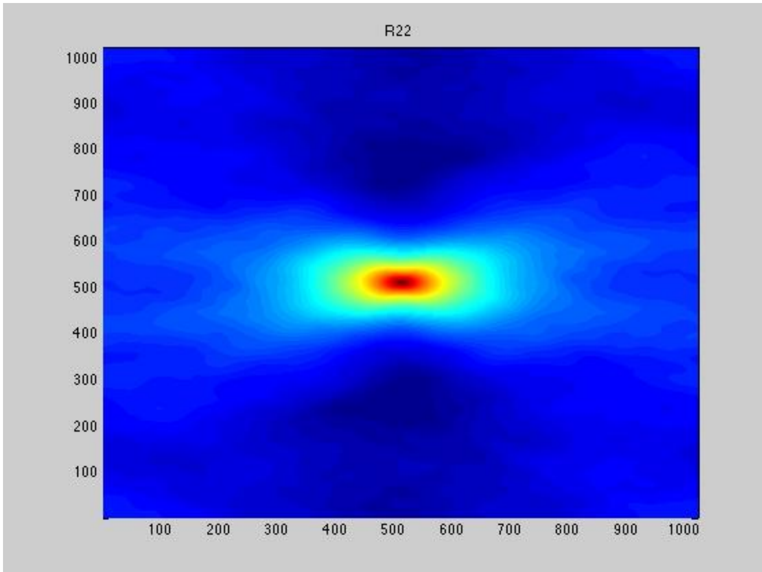
Computing the length scales is rather easy. The longitudinal and transversal length scale are

$$L_{11} = \int_0^\infty R_{11}\,\mathrm{d}r \tag{6}$$

$$L_{22} = \int_0^\infty R_{22}\,\mathrm{d}r \tag{7}$$

$$\tag{8}$$

```
L11=trapz(r,R11);
L22=trapz(r,R22);
hold on
rectangle('Position',[0,0,L11,1],'LineWidth',2,'LineStyle','--')
```



compute 1D spectrum

```
L=length(R1);
NFFT=2^nextpow2(L);
spec_1D=fft(R1(:,1),NFFT)/L.*2/pi;

f = linspace(0,1,NFFT)*2*pi/dx;

slope=1.5*664092^(2/3)*(f.^(-5/3));
% loglog(f,2*abs(spec_(1:NFFT/2+1)));
% hold on
% loglog(f,slope);
```

compute spectrum

spec = zeros(round(dim*dim*dim/8),1);

```
if (strcmp('3D',flag))
%     phi = u_fft;
%     phi(:,:,:)=0.0;
%     for k=1:dim
%         for j=1:dim
%             for i=1:dim
    %             kappa = sqrt(i*i+j*j+k*k);
    %             kappa_pos=int16(kappa);
    %             if (kappa_pos <= size(spec,1))
```

```
    %                 spec(kappa_pos) = spec(kappa_pos)+kappa*kappa*(...
    %                 + real(u_fft(i,j,k))*real(u_fft(i,j,k))+imag(u_fft(i,j,k))*imag(u_fft(
    %                 + real(v_fft(i,j,k))*real(v_fft(i,j,k))+imag(v_fft(i,j,k))*imag(v_fft(
    %                 + real(w_fft(i,j,k))*real(w_fft(i,j,k))+imag(w_fft(i,j,k))*imag(w_fft(

    %             end
    %             spec(kappa_pos) = spec(kappa_pos) + kappa*kappa*0.5*(phi_x(i,j,k).^+phi_y(
                  phi = 0.5*(phi_x+phi_y+phi_z);
                  phi = phi(1:round(size(phi_x,1)/2),...
                            1:round(size(phi_y,1)/2),...
                            1:round(size(phi_z,1)/2));
%             end
%         end
%     end
else
%     phi = u_fft;
%     phi(:,:)=0.0;
%     for j=1:dim
%         for i=1:dim
%             phi(i,j) = phi(i,j) +(phi_x(i,j)+phi_y(i,j));
%         end
%     end
    phi = 0.5*phi_x+phi_y;
    phi = phi(1:round(size(phi_x,1)/2),...
              1:round(size(phi_y,1)/2));
%     phi = phi(1:round(size(phi,1)));
end
```

compute k vector

```
if (strcmp('3D',flag))
    maxdim = sqrt(dim^2*(2*pi/Lx)^2+dim^2*(2*pi/Ly)^2+dim^2*(2*pi/Lz)^2);
    E=zeros(uint64(sqrt(3*dim^2)),1);
    kk=zeros(uint64(sqrt(3*dim^2)),1);
    dim = size(phi,1);
    for k=1:dim
        for j=1:dim
            for i=1:dim
                kappa=sqrt(i*i*(2*pi/Lx)^2+j*j*(2*pi/Ly)^2+k*k*(2*pi/Lz)^2);
                kappa_pos=uint64(sqrt(i*i+j*j+k*k));
                E(kappa_pos) = E(kappa_pos) + phi(i,j,k);
                kk(kappa_pos) = kappa;
            end
        end
    end
    E=E*4*pi;
```

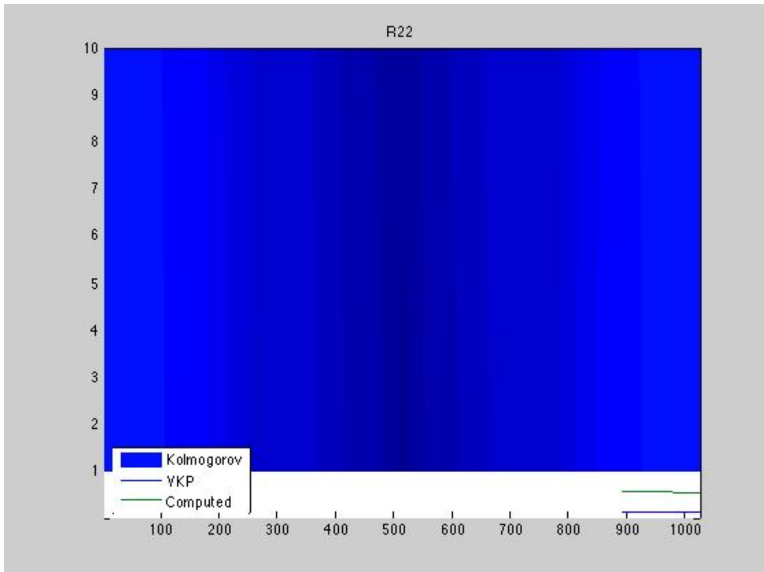
```
% E=E.*kk.^2;
else
    dim = size(phi,1);
    maxdim = sqrt(dim^2*(2*pi/Lx)^2+dim^2*(2*pi/Ly)^2);
    E=zeros(uint64(sqrt(dim^2+dim^2)),1);
    kk=zeros(uint64(sqrt(dim^2+dim^2)),1);
    bin_counter=zeros(uint64(sqrt(dim^2+dim^2)),1);
    for j=1:dim
        for i=1:dim
            kappa=sqrt(i*i*(2*pi/Lx)^2+j*j*(2*pi/Ly)^2);
            kappa_pos=uint64(sqrt(i*i+j*j));
            E(kappa_pos) = E(kappa_pos) + phi(i,j);
bin_counter(kappa_pos) = bin_counter(kappa_pos) + 1;
            kk(kappa_pos) = kappa;
        end
    end
    EE=E*2*pi.*kk./bin_counter;
%     EEE = E*2*pi.*kk;
end
```

compute 1D spectrum

close all

```
slope=1.5*664092^(2/3)*(kk.^(-5/3));
% test=importdata('INPUT/2D/CTRL_TURB_ENERGY');
%
% dissip=664092;
dissip=664092;
up=17;
L=Lx;
kkke=kk./(2*pi)*L;
kkkd=kk./(2*pi*100)*L;
VKP = 1.5*17^5/dissip.*(kkke).^4./(1+kkke.^2).^(17/6).*exp(-3/2*1.5.*(kkkd).^(4/3));
%
loglog(kk,slope,kk,VKP,kk(2:end),E(2:end))
ylim([1e-14 10]);
h=legend('Kolmogorov','VKP','Computed');
set(h,'Location','SouthWest')
```

```
Warning: Legend not supported for patches with
FaceColor 'interp'
```


R22
10
9
8
7
6
5
4
3
2
1
100
200
300
400
500
600
700
800
900
1000
Kolmogorov
VKP
Computed