WAVEGUIDE SIMULATION

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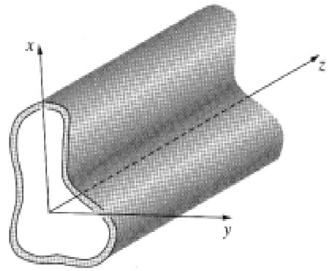
壹、Waveguide 原理

Waveguides generally made of good conductor, so that $\mathbf{E}=0$ and $\mathbf{B}=0$ inside the material.



The boundary conditions at the inner wall are: $\mathbf{E}^{//}=0$ and $B^{\perp}=0$ …

The generic form of the monochromatic waves:

$$\begin{cases} \tilde{\mathbf{E}}(x,y,z,t)=\tilde{\mathbf{E}}_0(x,y)e^{i(\tilde{k}z-\omega t)}=(\tilde{E}_x\hat{\mathbf{x}}+\tilde{E}_y\hat{\mathbf{y}}+\tilde{E}_z\hat{\mathbf{z}})e^{i(\tilde{k}z-\omega t)} \\ \tilde{\mathbf{B}}(x,y,z,t)=\tilde{\mathbf{B}}_0(x,y)e^{i(\tilde{k}z-\omega t)}=(\tilde{B}_x\hat{\mathbf{x}}+\tilde{B}_y\hat{\mathbf{y}}+\tilde{B}_z\hat{\mathbf{z}})e^{i(\tilde{k}z-\omega t)} \end{cases}$$

General Properties of Wave Guides

In the interior of the wave guide, the waves satisfy Maxwell's equations:

$$\nabla\cdot\mathbf{E}=0 \quad \nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$$

$$\nabla\cdot\mathbf{B}=0 \quad \nabla\times\mathbf{B}=\frac{1}{v^2}\frac{\partial\mathbf{E}}{\partial t} \quad \text{where } v=\frac{1}{\sqrt{\varepsilon\mu}}$$

We obtain

$$\text{(i)}\ \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=i\omega B_z \qquad \text{(iv)}\ \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=-\frac{i\omega}{c^2}E_z$$

$$\text{(ii)}\ \frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=i\omega B_x \qquad \text{(v)}\ \frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}=-\frac{i\omega}{c^2}E_x$$

$$\text{(iii)}\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=i\omega B_y \qquad \text{(vi)}\ \frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}=-\frac{i\omega}{c^2}E_y$$

TE, TM, and TEM Waves

Determining the longitudinal components E_z and B_z, we could quickly calculate all the others.

$$E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

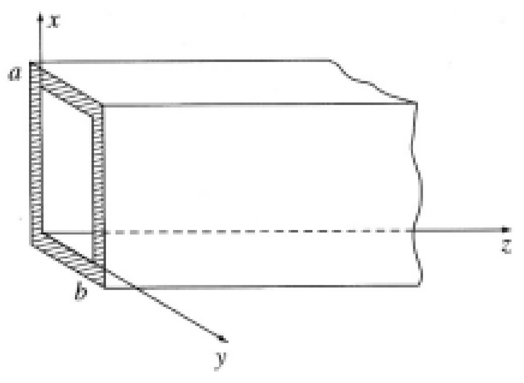
$$B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

We obtain

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{v^2} - k^2\right]E_z = 0$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{v^2} - k^2\right]B_z = 0$$

If $E_z = 0 \Rightarrow$ TE (transverse electric) waves;
If $B_z = 0 \Rightarrow$ TM (transverse magnetic) waves;
If $E_z = 0$ and $B_z = 0 \Rightarrow$ TEM waves.



$E_z = 0$, and $B_z(x,y) = X(x)Y(y) \leftarrow$ separation of variables

$X(x) = A\sin k_x x + B\cos k_x x$

$Y(y) = C\sin k_y y + D\cos k_y y$

TE Waves in a Rectangular Wave Guide (II)

$$E_x \propto \frac{\partial B_z}{\partial y} \propto C\cos k_y y - D\sin k_y y$$

$E_x(@\, y=0) = 0 \Rightarrow C = 0$

$E_x(@\, y=b) = 0 \Rightarrow \sin k_y b = 0, k_y = \frac{n\pi}{b}\ (n = 0,1,2,\ldots)$

$$E_y \propto \frac{\partial B_z}{\partial x} \propto A\cos k_x x - B\sin k_x x$$

$E_y(@\, x=0) = 0 \Rightarrow A = 0$

$E_y(@\, x=a) = 0 \Rightarrow \sin k_x a = 0, k_x = \frac{m\pi}{a}\ (m = 0,1,2,\ldots)$

$B_z(x,y) = B_0\cos(m\pi x/a)\cos(n\pi y/b) \leftarrow$ the TE_{mn} mode

$$k = \sqrt{(\omega/v)^2 - \pi^2[(m/a)^2 + (n/b)^2]}$$

貳、研究方法

目標：

1. 模擬不同模式下波導的運行狀態
2. 做出波導的二維剖面圖與三維向量場分布型態

步驟：

Step1 設定 設定初始形狀為長方體，長寬高分別為L,WD,H，長邊為電磁波傳遞方向。
Step2 計算 寫出電場、磁場，帶入馬克斯威方程式，求出E(電場)，B(磁場)解。
Step3 畫圖 將所得結果分別以3D向量場、2D剖面圖作圖。
step4 延伸 從最初TE10 mode延伸至任一TE、TM mode。

分工：

1. E、B解析解
2. E、B數值解
3. Quiver向量圖像化
4. Yt剖面圖

參、E、B數值解

步驟一：

$$\tilde{B}(x,y,z,t)=\widetilde{B_0}(x,y)e^{i(\bar{k}z-\omega t)}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\omega^2}{v^2}-k^2\right)B_z=0$$

步驟二：差分法解二階微分方程

$$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}+g(x,y)u(x,y)=f(x,y)$$

$$\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta x^2}\cong\left.\frac{\partial^2 u(x,y)}{\partial x^2}\right|_{xj,yi}\qquad\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{\Delta y^2}\cong\left.\frac{\partial^2 u(x,y)}{\partial y^2}\right|_{xj,yi}$$

$$\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta x^2}+\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{\Delta y^2}+g_{i,j}u_{i,j=}f_{i,j}$$

步驟三：將差分法應用於程式，算出Bz

$$u_{i,j}=r_y\left(u_{i,j+1}+u_{i,j-1}\right)+r_x\left(u_{i+1,j}+u_{i-1,j}\right)+r_{xy}\left(g_{i,j}u_{i,j}-f_{i,j}\right)$$

```python
#solve u,Bz
for itr in range(MaxIter):
    for i in range(1,My-1):
        for j in range(1,Mx-1):
            u[i,j] = ry*(u[i,j+1]+u[i,j-1])+rx*(u[i+1,j]+u[i-1,j])+rxy*(G[i,j]*u[i,j]-F[i,j])

    Err = abs(u-u0)


    if (itr>1) & (Err.max()<tol):
        break

    u0=u
```

步驟四：解出Bx、By

$$B_x=\frac{i}{\left(\frac{\omega}{c}\right)^2-k^2}\left(k\frac{\partial B_z}{\partial x}-\frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y=\frac{i}{\left(\frac{\omega}{c}\right)^2-k^2}\left(k\frac{\partial B_z}{\partial y}+\frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

肆、E、B 解析解

1. TE mode

$$B_z = \widetilde{B_0}\, e^{i(\bar{k}z-\omega t)} \cos(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b})$$

$$E_z = 0$$

$$B_x = \widetilde{B_0} e^{i(\bar{k}z-\omega t)} \frac{m\pi}{a} sin\left(\frac{m\pi x}{a}\right) cos(\frac{n\pi y}{b})$$

$$E_x = \widetilde{B_0}(x,y) e^{i(\bar{k}z-\omega t)} \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{n\pi}{b} cos\left(\frac{m\pi x}{a}\right) sin(\frac{n\pi y}{b})$$

$$B_y = \widetilde{B_0} e^{i(\bar{k}z-\omega t)} \frac{n\pi}{b} cos\left(\frac{m\pi x}{a}\right) sin\,(\frac{n\pi y}{b})$$

$$E_y = \widetilde{B_0}(x,y) e^{i(\bar{k}z-\omega t)} \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{m\pi}{a} sin\left(\frac{m\pi x}{a}\right) cos(\frac{n\pi y}{b})$$

$$k_z = \sqrt{(\omega/v)^2 - \pi^2[\left(\frac{m}{a}\right)^2 + (\frac{n}{b})^2]}$$

2. TM mode

$$E_z = \widetilde{E_0}\, e^{i(\bar{k}z-\omega t)} sin(\frac{m\pi x}{a})\, sin(\frac{n\pi y}{b})$$

$$B_z = 0$$

$$E_x = -\widetilde{E_0} e^{i(\bar{k}z-\omega t)} \frac{m\pi}{a} cos\left(\frac{m\pi x}{a}\right) sin(\frac{n\pi y}{b})$$

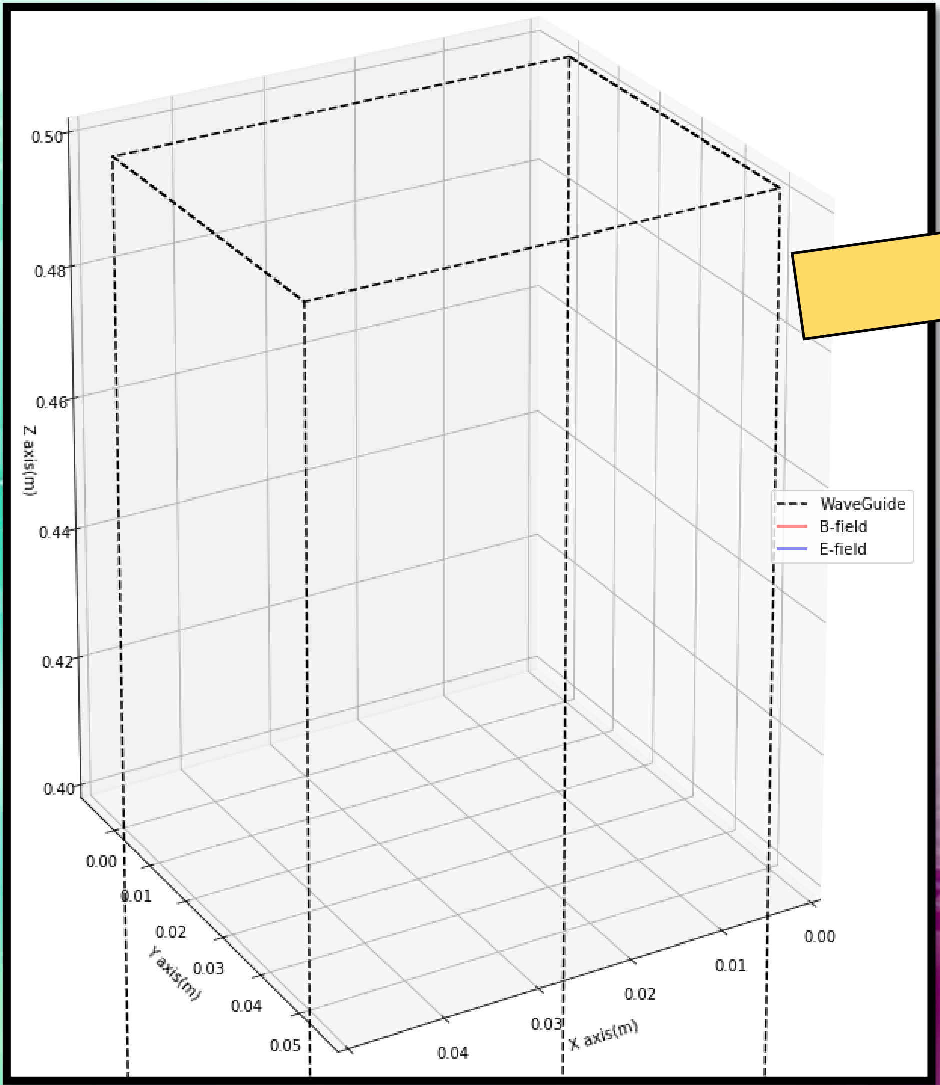
$$B_x = \widetilde{B_0}(x,y) e^{i(\bar{k}z-\omega t)} \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{n\pi}{b} sin\left(\frac{m\pi x}{a}\right) cos(\frac{n\pi y}{b})$$

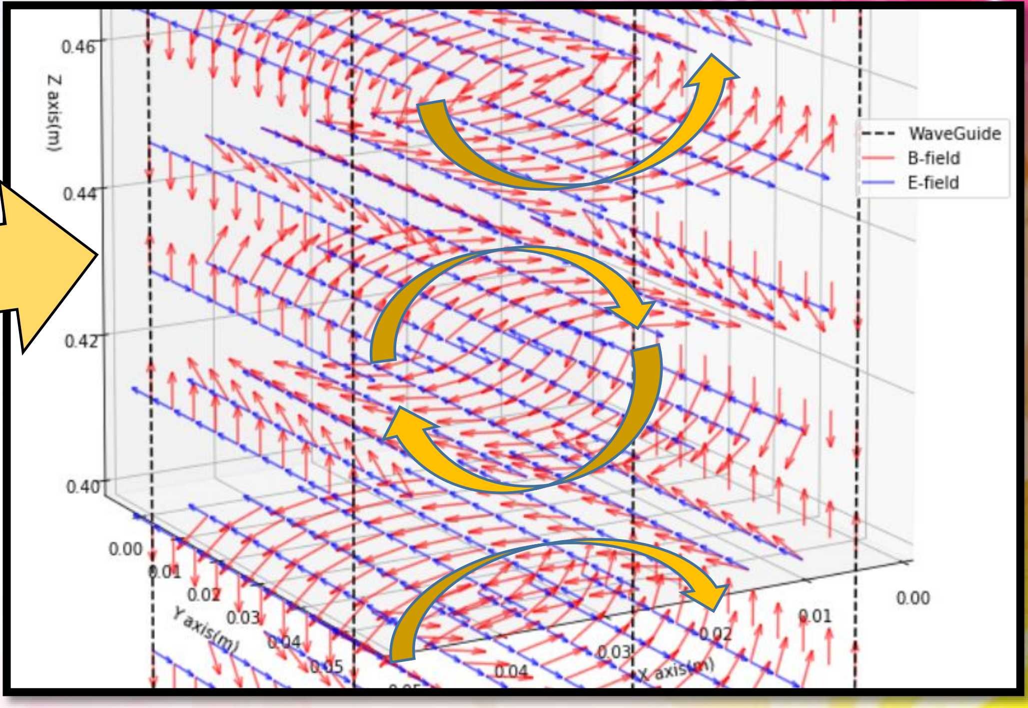
$$E_y = -\widetilde{E_0} e^{i(\bar{k}z-\omega t)} \frac{n\pi}{b} sin\left(\frac{m\pi x}{a}\right) cos\,(\frac{n\pi y}{b})$$

$$B_y = \widetilde{B_0}(x,y) e^{i(\bar{k}z-\omega t)} \frac{-i}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{m\pi}{a} cos\left(\frac{m\pi x}{a}\right) sin(\frac{n\pi y}{b})$$

$$k_z = \sqrt{(\omega/v)^2 - \pi^2[\left(\frac{m}{a}\right)^2 + (\frac{n}{b})^2]}$$

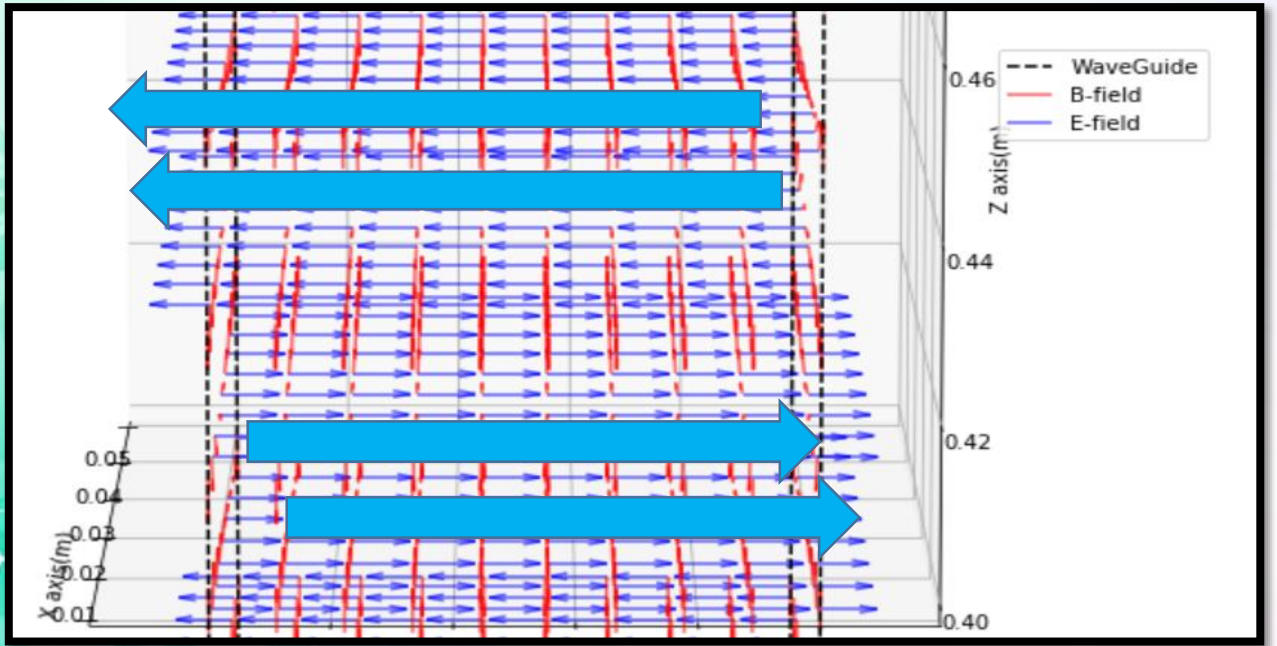
伍、QUIVER 向量圖形化

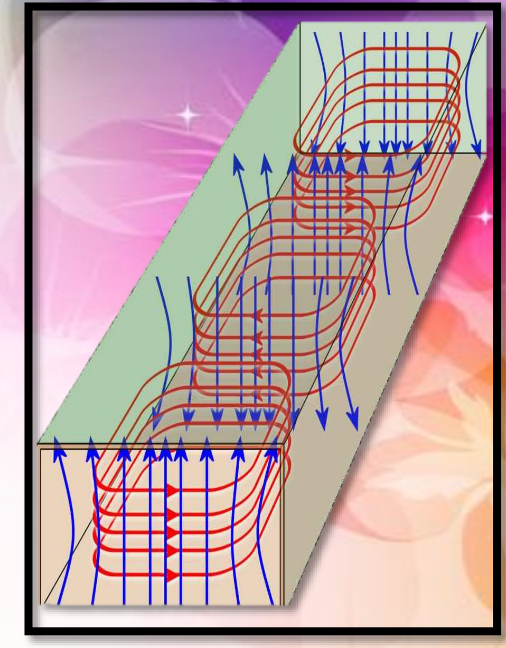


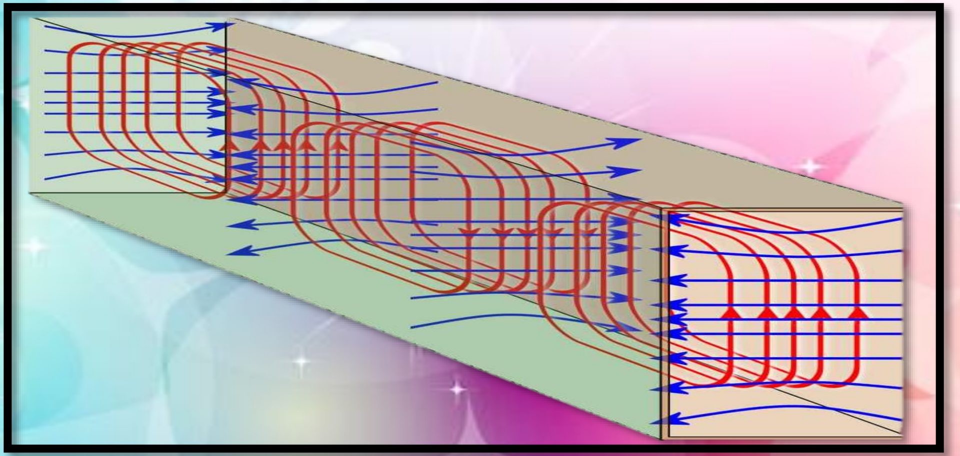


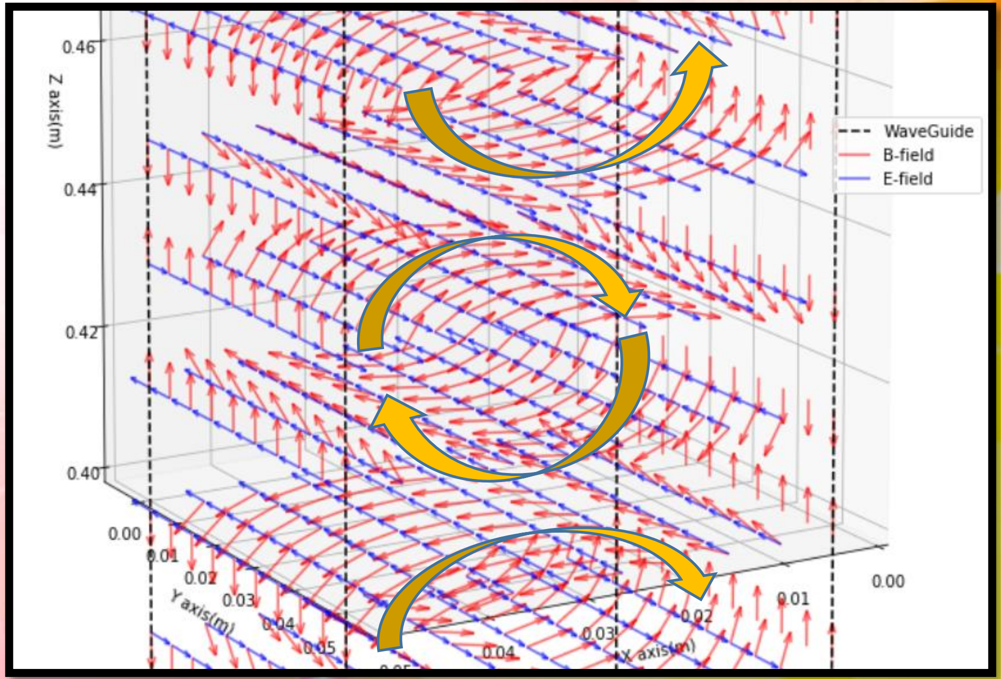

```
#Constructing Space
x=np.linspace(0+delta, H-delta , numberx)
y=np.linspace(0+delta, WD-delta, numbery)
z=np.linspace(0+delta, L-delta , numberz)

X, Y, Z = np.meshgrid(x, y, z)
```

伍、QUIVER 向量圖形化



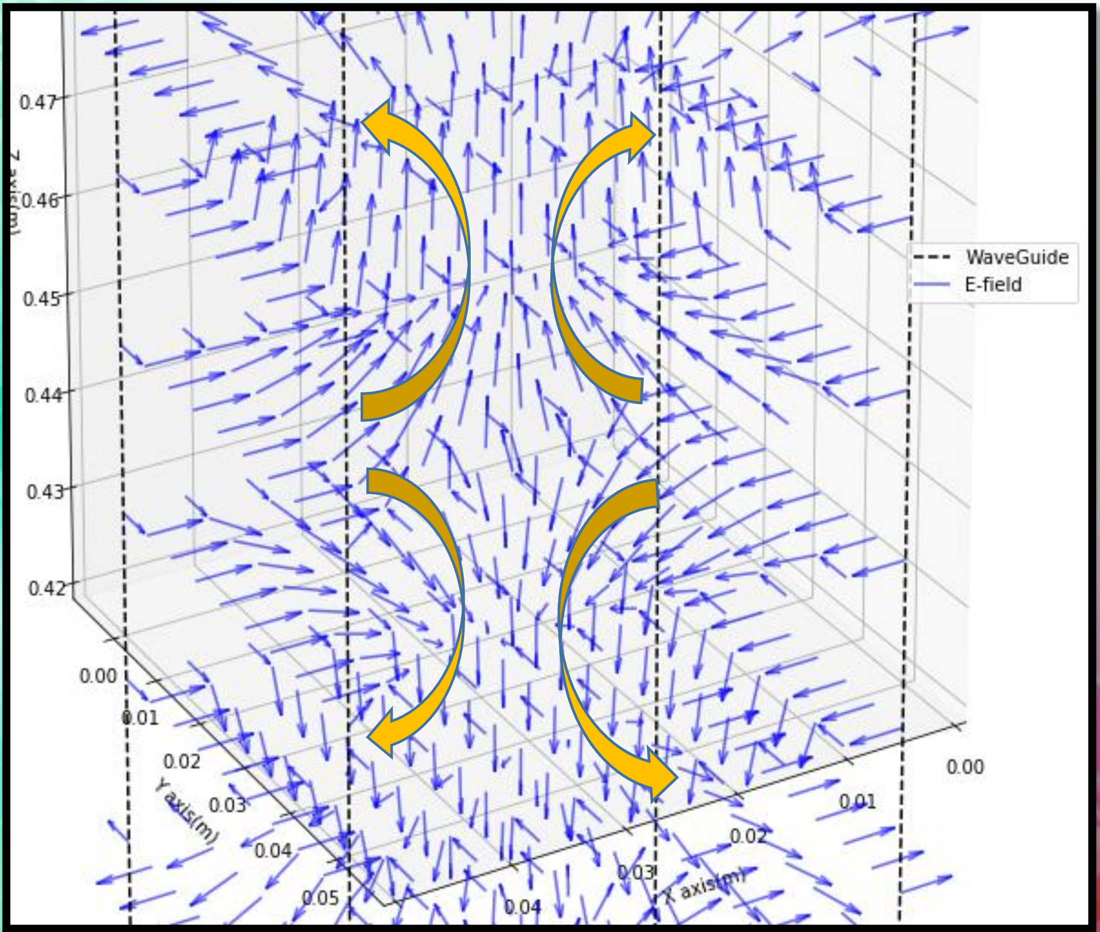


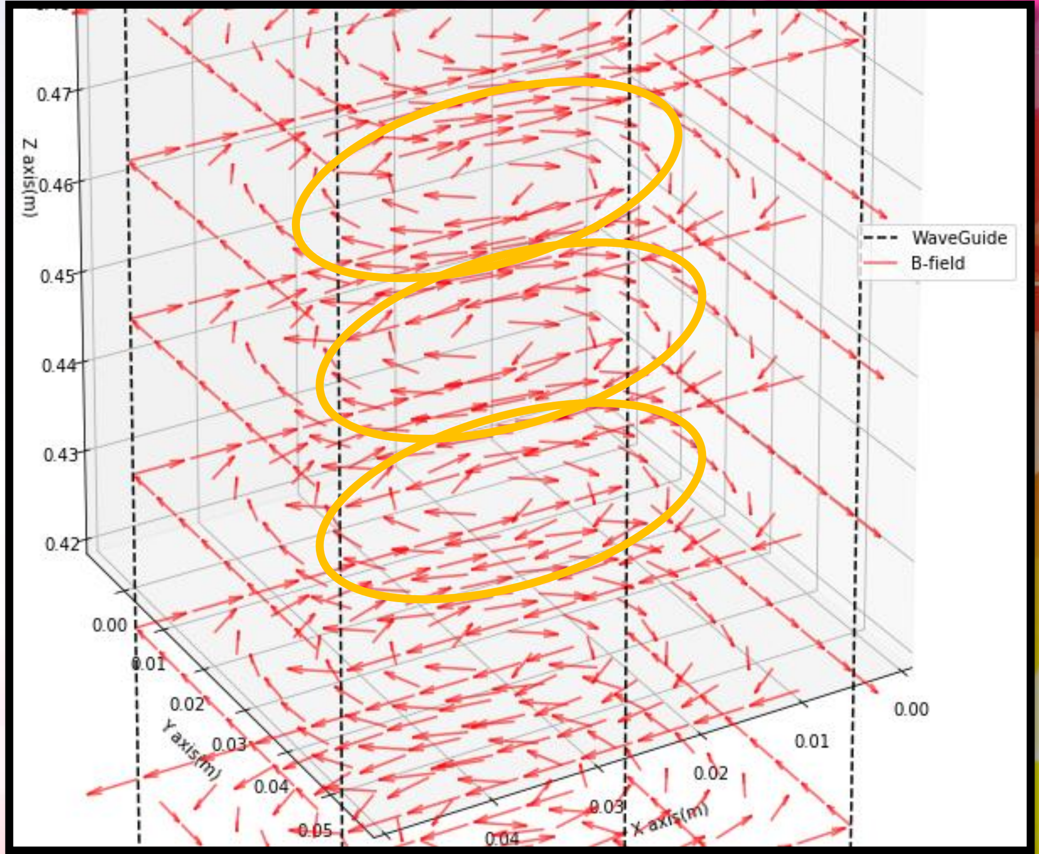




TE10 mode

TM11 mode


0.47
0.46
0.45
0.44
0.43
0.42
Z axis(m)
WaveGuide
E-field
0.00
0.01
0.02
0.03
0.04
0.05
Y axis(m)
0.04
0.03
0.02
0.01
0.00
X axis(m)


0.47
0.46
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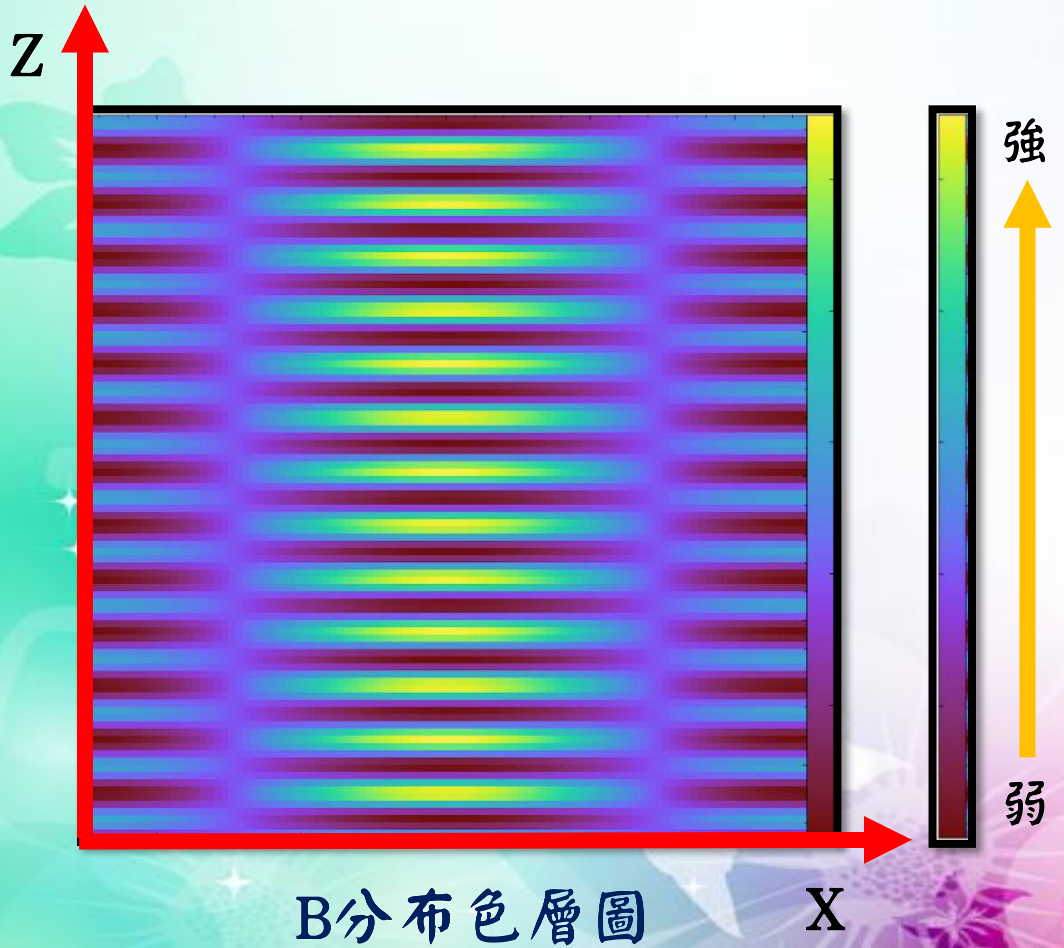
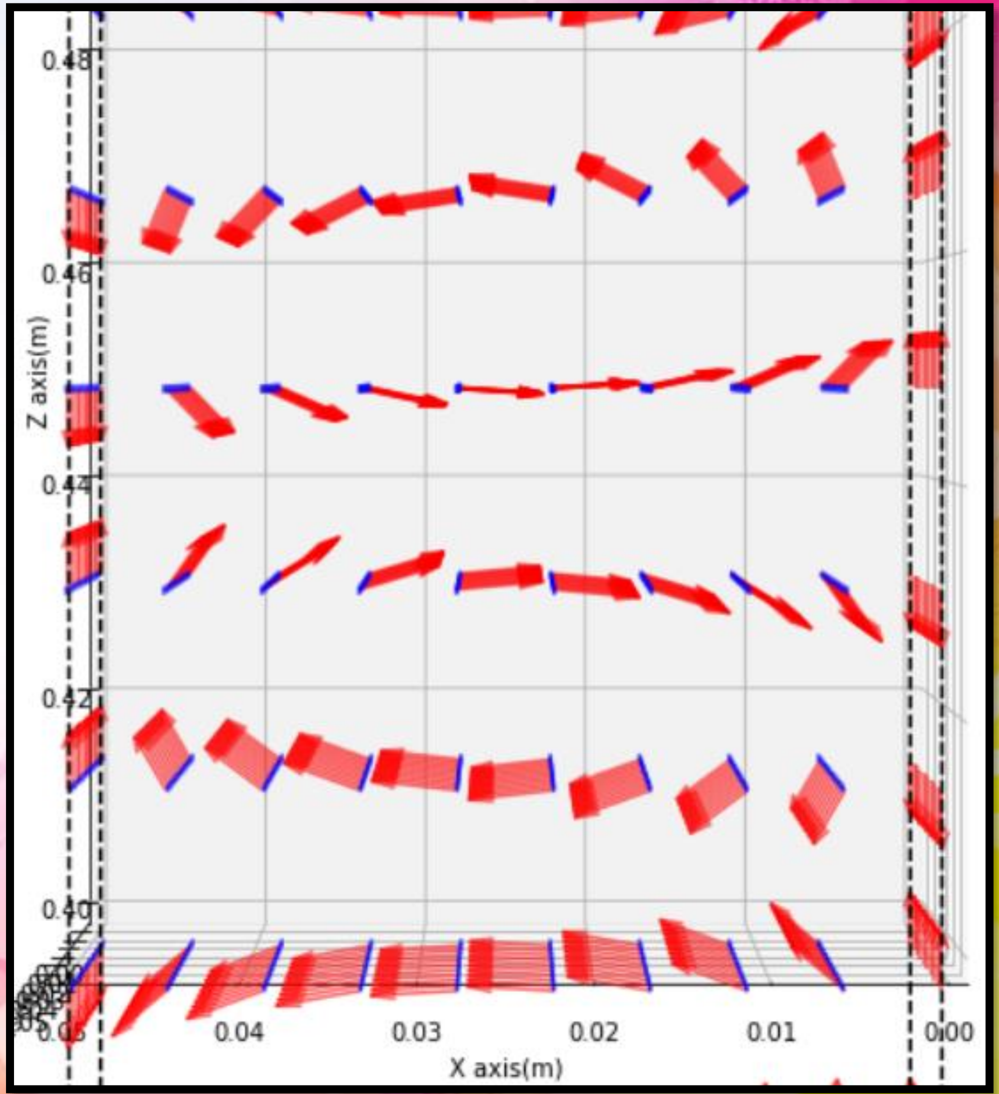
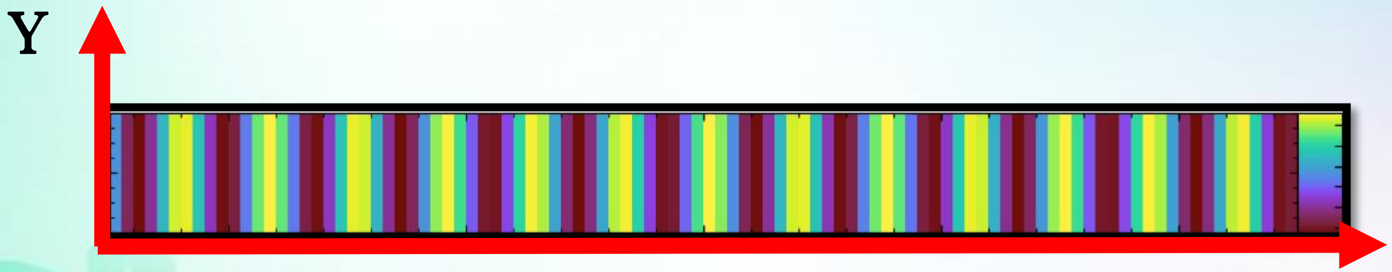
0.44
0.43
0.42
Z axis(m)
WaveGuide
B-field
0.00
0.01
0.02
0.03
0.04
0.05
Y axis(m)
0.04
0.03
0.02
0.01
0.00
X axis(m)

陸、YT剖面圖(TE10 MODE)

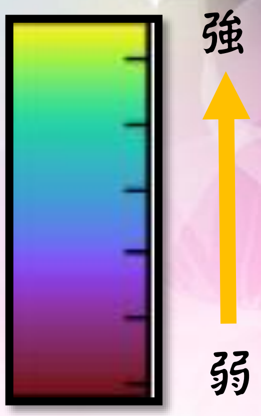


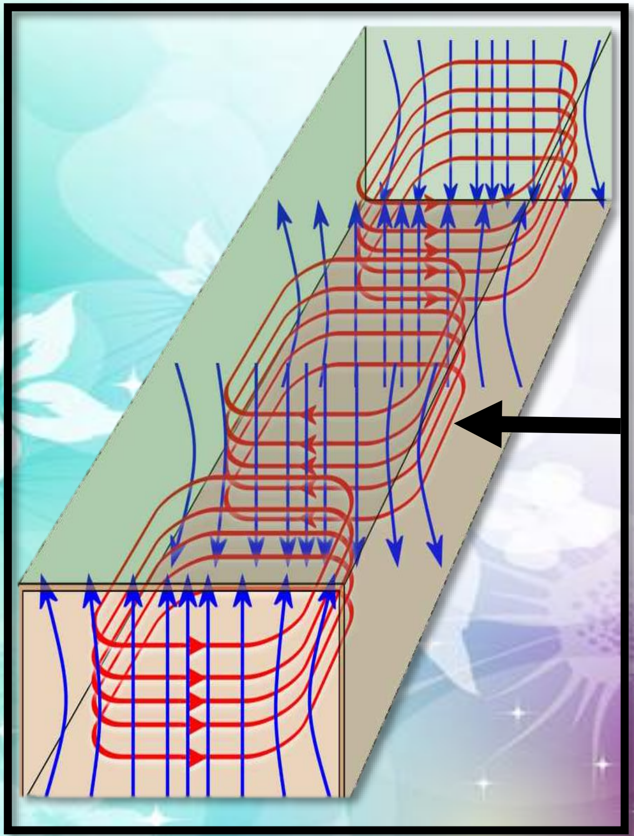
B分布色層圖

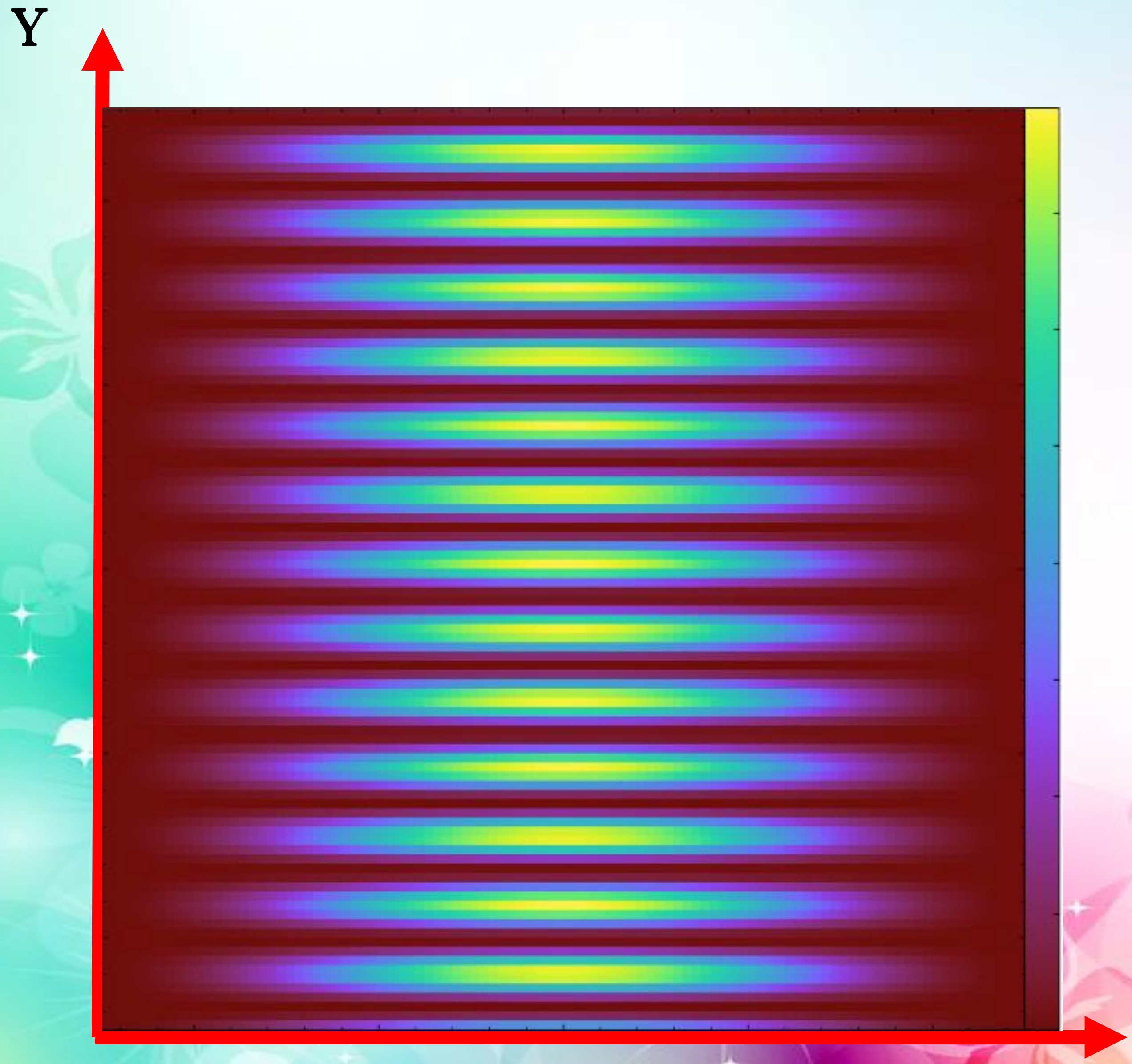




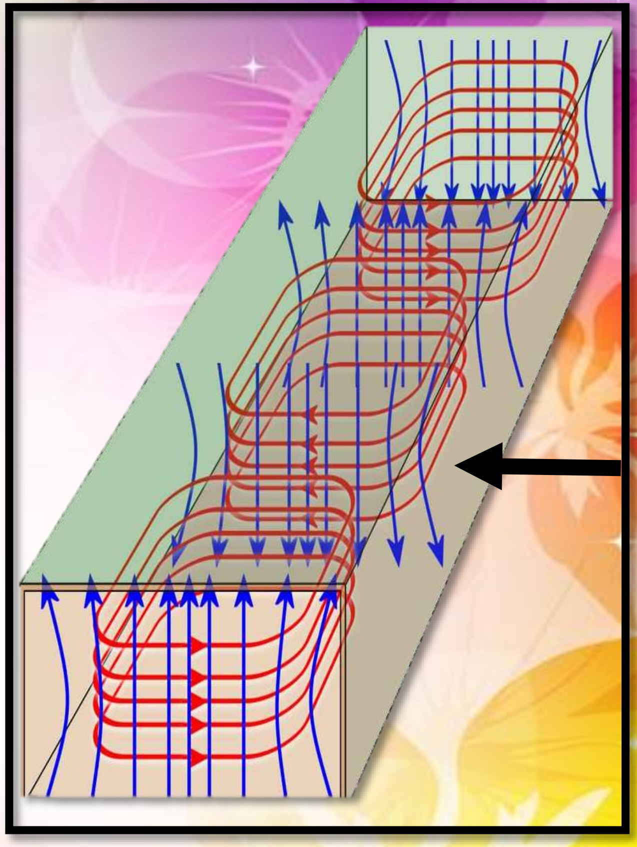
B分布色層圖

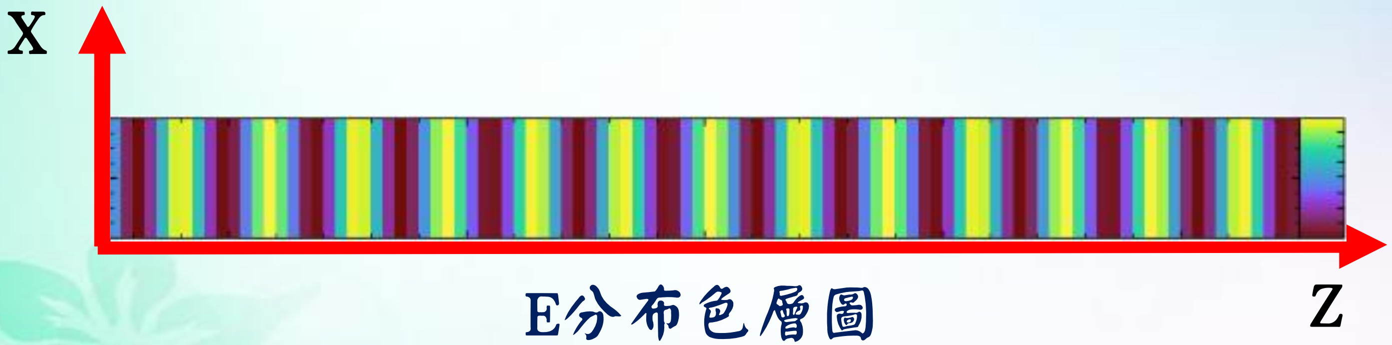




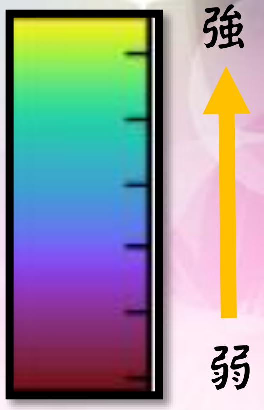


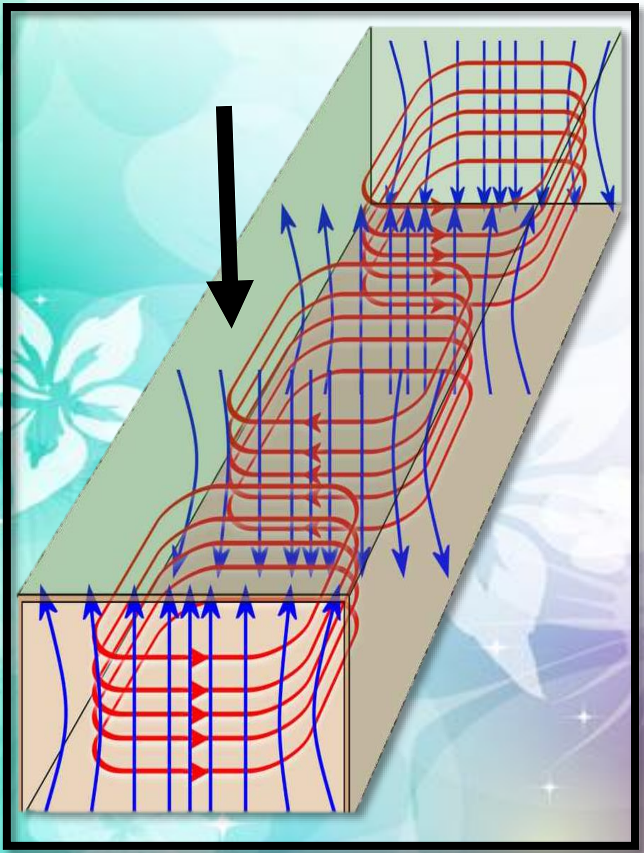
E分布色層圖





E分布色層圖





柒、參考資料

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謝謝聆聽

祝大家暑假快樂！