Optimal Corn and Soybean Rotations*

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ABSTRACT

We examine crop choice as a dynamic optimization problem over an infinite time horizon, taking into account the effects over time that corn-soybean rotations have on soil quality, which manifest in yield and therefore profit impacts. We show how the efficient decision rule depends on model parameters and how it compares to those characteristic of static models of supply. The model is parameterized for a representative acre of Iowa cropland and used to predict actual crop choices in a panel of over 6500 Iowa plots during 1979–1997 surprisingly well.

* Views expressed are the authors' and not necessarily those of the institutions with which they are affiliated.

A ubiquitous decision in agricultural production is farmers' crop allocation and rotation decisions. These decisions ultimately determine crop supply response. The traditional way of modeling the basic economic problem is to view land as a fixed, unadulterated input at the beginning of each season, a view that works well with static models that are predominantly used in applied work. While crop rotations are incorporated into some programming models, they do not account for the sequential nature of planting decisions. That is, they take multi-year rotation rules as a single decision as if future prices were known in advance, as if the problem were static.

The static view is plainly contrary to a large agronomic literature that shows higher average yields and lower costs resulting from rotating crops, plus the fact that most farmers actually do rotate crops. Indeed, USDA's Agricultural Resource Management Survey data show that farmers rarely depart from planned rotations even amid large changes in relative prices.[1] The most prevalent and salient example is rotation of corn and soybeans: soybeans fix nitrogen that is used by the subsequent year's corn crop, thereby reducing fertilizer costs. Rotating crops also reduces the incidence and severity of pest infestations, because different crops play host to different insects and diseases. Experimental evidence shows higher yields for corn following soybeans than for corn following corn and similarly higher yields for soybeans following corn. Experimental evidence also shows lower marginal productivity of fertilizer inputs to corn yield following soybeans as compared to corn.

When would an optimizing farmer consider altering a planned rotation schedule? Or, more generally, given past plantings and current and expected future prices, what would an optimizing farmer plant in any given year? Given answers to these basic management questions, how would we expect crop supply to change in response to temporary or more persistent changes in commodity demand and/or input supply?

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[1] Information about these data is available at http://www.ers.usda.gov/Briefing/ARMS/.

The agronomy of crop rotations implies that current production opportunities depend on historical plantings and expected future production opportunities depend on current plantings. Crop plantings, and thus supply response, is a fundamentally dynamic problem. The decision problem becomes more interesting and complex in an environment with highly variable and uncertain commodity prices and where price shocks typically persist for many years. Price uncertainties, coupled with irreversible past planting decisions, suggest there are option values associated with choosing crop rotations that maximize soil quality and land disposition, such as susceptibility to pests. Thus, altering a planned rotation in response to a temporary change in relative prices may be profitable in the short run, but may not maximize expected discounted profits over the long run. These short-run vs. long-run tensions suggest supply response in a dynamic model under uncertainty may differ markedly from that implied by static models.

To our knowledge, a systematic analysis of crop rotations under price uncertainty and the implications for supply response have not been examined. There may be considerable interest in the question now due to the recent, sharp increases in commodity prices and the subsidization of corn-based ethanol. Price volatility and uncertainty implied in prices of futures options have also increased. And there is some evidence that some corn-soybean rotations have shifted to corn monoculture.

A general solution to the crop-rotation problem is extremely complex due to its high dimensionality. Rather than solve the general problem, in this article we develop a relatively simple model assuming corn and soybeans are the only two crop choices. We examine optimal policy functions for a representative producer in Iowa, given stochastic price processes estimated for these commodities, and rotation and yield-fertilizer response functions derived from experimental plot data under various rotation schedules (generously provided by Iowa State University). While the dimensionality of this problem is large, it is solvable using modern computers, and incorporates the most characteristic features affecting rotation decisions in the U.S. Corn Belt.

Preliminary findings from our stochastic dynamic programming analysis suggest there is a large region of relative prices where it is optimal to plant soybean after corn and corn after soybean. Policy functions suggest a short-to-medium-term supply response that is more inelastic than static models would indicate. The analysis is useful in two key ways. First, it may provide useful rules of thumb for farmers trying to decide between different rotation systems, as well as planting decisions in any given year. Second, it may serve to influence the way supply response is modeled more generally within agricultural economics. This second application has far-reaching potential implications, influencing the way we assess impacts of many kinds of agricultural policies, including conservation-related policies, biofuel and energy policies, as well as traditional commodity policies.

Model

We consider planting decisions for a standardized unit of land. At a sufficiently small scale, planting decisions on an individual unit of land, such as a field, crop choice is a discrete decision, even though when aggregated across all units for a given farm, or in a county or state, the decision will approximate a continuous decision (e.g. what fraction and which parcels of land to allocate to corn, soybeans, wheat, and so on). To focus squarely on the issue of rotational dynamics, we assume no time or capital allocation constraints, spatial interaction effects, or farm-level liquidity constraints that would force us to approach planting decisions at the farm level. Instead, each unit of land is regarded as an independent “profit center” and, by maximizing profit from harvesting crops on each unit of land, the farm maximizes its value as a whole.

There is some arbitrariness about how big or small an individual unit of land may be. It should usually be treated as a contiguous parcel of land that is typically growing only one crop per season. We might think of this as a "field" where, for agronomic reasons, it would not make sense for a farmer to plant different crops on the same field. If this is not the case, then the "field" should be conceptually subdivided into smaller parcels for which the farmer almost always plants only one crop or the other. For purposes of this first analysis we will abstract away from the size of the unit more carefully. The data to which we compare are model refer to specific points where the discrete crop choice is indicated.

Our agronomic data on crop rotations, which are critical to calibration of the model, are from an experiment station in Northeast Iowa. As such, the model we develop is most salient to that region of the country and nearby regions with similar soils and climate.

The main features we want the model to be capable of predicting are 1) the pattern of planting, 2) total yields, and 3) revenues earned from these yields. We want the model to provide accurate predictions not only for individual farms, but also in aggregate at both the county and state level, and also to match the overall time series properties of planting, yields and revenues earned. Correctly predicting planting is not a sufficient condition for accurate prediction of yields and revenues, because there are macro shocks (weather shocks, or loss of yield due to pests, and so forth) that lead to strong spatial correlation in yields, which also have an effect on the overall market price.

In this preliminary model, instead of separating prices and yields we model crop revenues per acre, or price-times-yield. This simplification captures price-yield correlations stemming from spatially-correlated weather and pest outcomes. Historical revenue-per-acre data also appear stationary, despite a significantly increasing trend in yields.

Consider economically efficient crop choice, d_t, and fertilizer use, a_t, for an acre of arable land, on which either soybean, $d_t = 0$, or corn, $d_t = 1$, may be produced. Crop yields and thus revenues expected in the current year are influenced by crop choices made in earlier years. In general, a long history of crop choices and soil management choices may influence current expectations. Incorporating a long crop history would greatly increase the state space and render a solution computationally infeasible.[2] Moreover, conditional on the previous year's crop, crop choices in early years are not statistically significant determinants of yield in simple regression analyses of experimental crop yields. We therefore assume expected revenues depend on an adjustment factor that depends on current and past crop choices, as well as current fertilizer applications, denoted $f(d_t, a_t, d_{t-1})$.

Unit-level expected revenues are also tied to expected prices and the covariances between yield shocks and price shocks. To account for both the autocorrelation of prices and these covariances, we model current expected revenues for the individual unit as being tied to expectations about state-level revenues per acre, which are widely available and observed. State-level revenues per acre equal the average price received in Iowa multiplied by the realized yield, and we denote these by $r_{c,t}$ and $r_{s,t}$ for corn and soybeans, respectively. Crop revenues, like prices, display strong autocorrelation so that past prices strongly influence expectations about current prices. Conditional on the last year's revenue per acre, earlier years are not significant predictors of current price, so we assume current state-level revenues follow a first-order vector autocorrelation process,

(1-a) $$r_{c,t} = \alpha + \beta_1 r_{c,t-1} + \beta_2 r_{s,t-1} + \varepsilon_{c,t}$$

(1-b) $$r_{s,t} = \kappa + \gamma_1 r_{c,t-1} + \gamma_2 r_{s,t-1} + \varepsilon_{s,t}$$

(1-c) $$\mathrm{VAR}(\varepsilon_{c,t}) = \sigma_c^2,\ \mathrm{VAR}(\varepsilon_{s,t}) = \sigma_s^2,\ \ \mathrm{COV}(\varepsilon_{c,t}, \varepsilon_{s,t}) = \sigma_{c,s}$$

Thus, unit-level revenues in period t are given by

[2] Hennessy (2007) considers a framework that can encompass a broader set of possible rotation schedules but does not consider sequential decision making under uncertainty.

$$R\left(a_t, d_t \middle| d_{t-1}, r_{c,t-1}, r_{s,t-1}\right) = E\left[r_{s,t} \middle| r_{c,t-1}, r_{s,t-1}\right] f\left(0, a_t, d_{t-1}\right) \quad \text{if soybean planted in } t \tag{2}$$

$$= E\left[r_{c,t} \middle| r_{c,t-1}, r_{s,t-1}\right] f\left(1, a_t, d_{t-1}\right) \quad \text{if corn planted in } t.$$

The expected revenue functions in (2) are given by the deterministic components of the autoregressive processes in (1-a) and (1-b). We assume the errors $\varepsilon_{c,t}$ and $\varepsilon_{s,t}$ are mean-zero, independent and identically distributed (iid), bivariate-normal random variables.

To obtain the current profit function we need to subtract costs. For this preliminary model we consider only fertilizer expenditures. While other costs and inputs are surely important to crop decisions and unit profits, fertilizer inputs are the largest single expenditure and interact strongly with the agronomic factors associated with crop rotation. A corn monoculture, for example, will require greater levels of fertilizer to obtain the same yield as corn following soybean. Fertilizer is a substitute for rotation and therefore fundamental to our problem. Profit is given by

$$\pi\left(d_t, a_t \middle| d_{t-1}, r_{c,t-1}, r_{s,t-1}\right) = R\left(a_t, d_t \middle| d_{t-1}, r_{c,t-1}, r_{s,t-1}\right) - k a_t, \tag{3}$$

where k is the price of fertilizer, which is assumed fixed and constant over time.[3]

The producer's objective is to maximize the expected present value of profit (3) over an infinite time horizon, subject to the stochastic evolution of revenues. We can write this infinite horizon problem using the recursive Bellman equation that relates the current value function to the future value function:

$$V\left(r_{c,t-1}, r_{s,t-1}, d_{t-1}\right) = \max_{d_t, a_t} E\left[\pi\left(d_t, a_t \middle| p_{c,t-1}, p_{s,t-1}, d_{t-1}\right) + \beta V\left(r_{c,t}, r_{s,t}, d_t\right)\right]. \tag{4}$$

[3] In future research it may be interesting and useful to consider stochastic fertilizer prices that may or may not be associated with past and current commodity prices.

V is the maximum expected present value of profit, a function of three state variables, the previous season's corn and soybean revenues and crop choice, which provide all of the information necessary to form expectations over prices and yields at the end of the current growing season.

Given parameters for the autoregressive process of revenues and a functional form for the agronomy-based adjustment function $f(d_t, a_t, d_{t-1})$, we use value function iteration to find the policy functions $d^*(r_{c,t}, r_{s,t}, d_{t-1})$ and $a^*(r_{c,t}, r_{s,t}, d_{t-1})$, which give optimal crop choice and fertilizer application rates as a function the three state variables. Note that, because of the infinite horizon, the policy functions do not depend on t.

Parameter Estimates

We estimated the key parameters in our model using two key data sources: publicly available data from USDA's National Agricultural Statistics Service (NASS) on Iowa yields from 1950 to 2006 and experimental plot data from Northeastern Iowa generously provided by David Hennessy. These experimental plot data, the same data used in Hennessy (2006), include yield outcomes during 1979-2002 on many plots with different rotation schedules, with each rotation schedule stratified with different fertilizer application rates when corn was planted.

We used the NASS data for Iowa yields and prices to construct time-series of revenues for corn and soybeans ($r_{c,t}$ and $r_{s,t}$) and used these to estimate the first-order vector autoregressive processes in (1-a and 1-b). We estimated higher order autoregressive processes as well as autoregressive-moving average (ARMA) models, but found higher-order coefficients were not statistically significant and had AIC and BIC selection criteria that were inferior to the simple first-order process. We chose the first-order model for these reasons plus the fact that adding additional state variables to the model greatly increases computational expense. Coefficient estimates and standard errors are reported in table 1.

To solve the model current revenues assumed 20 values, where as last season's crop choice assumed only two values, either 0 for soybean or 1 for corn. As a result, there were 800 possible combinations of the state variables. Endpoints for the revenue states were determined by simulating the estimated vector autoregressive process over 1000 years and choosing values above and below the highest and lowest values realized in the simulation. We allowed each of the revenue disturbance terms assume 15 values and, as a result, the 15-by-15 Markovian transition process assumed 225 possible values.

We used the experimental plot data to estimate the revenue adjustment function $f()$. These data include yields and fertilizer inputs from a series of test plots. These show clear evidence of yield benefits from planting corn following soybeans and soybeans following corn. We estimated these effects by pooling all data from all rotation schedules and then limiting it the data set to observations with either corn or soybeans in the current and previous year and regressing yield on a dummy variable indicating the prior crop. In the corn regression, we also included a quadratic function of the fertilizer application rate and rate-squared, and an interaction with the previous year crop dummy. The interaction captures differences in marginal fertilizer productivity depending on whether corn follows corn or corn follows soybeans. We also include year fixed effects to capture weather variation and reduced standard errors of the parameter estimates. While fertilizer application rates take on just four discrete values, we estimated a continuous function to facilitate modeling of continuous application rates within our dynamic model. OLS regression results are summarized in table 2.

The last two parameters are the price of fertilizer, k, and discount factor, β. Average real (2006) prices ($/lb) for anhydrous ammonia ($0.2261-$0.2512), nitrogen solutions (30%) ($0.1174-$0.1294), urea (45-46% nitrogen) ($0.1799-$0.1961), ammonium nitrate ($0.1697-$0.1827), and sulfate of ammonium ($0.1372-$0.1589) during 1994-2006 were used to specify the base model's fixed nitrogen price per pound at k = $0.1749. The BLS PPI was used to convert nominal prices to real prices, and the fertilizer data are from NASS as reported by Huang (www.ers.usda.gov/Data/FertilizerUse/). The base model's discount factor is from Lence, β = 0.94931. We vary these parameters to test the sensitivity of the results to these assumptions.

We solved the model using value-function iteration. The computer code was written in Matlab and used several functions written by Paul Fackler, which he provides free of charge on his personal web page (http://www4.ncsu.edu/~pfackler/) site. (For more information about these functions, as well as a general treatment of the theory and practice of constructing and solving stochastic dynamic programming models and arbitrage models see Miranda and Fackler). Convergence of the value function was achieved after an average of roughly 200 iterations.

Optimal Policy Functions

The baseline policy functions for crop choice are displayed in all of the figures presented below, together with policy functions for different model parameters. Figure 1-A shows the optimal crop choices across the revenue state space when the previous year's crop is corn, and Figure 1-B shows the optimal choices when the previous year's crop is soybean. In 1-B, it is almost always optimal to plant corn after soybean. Increasing the fertilizer price, because fertilizer is only used in corn, increases only slightly the incidence of double-cropping soybean. In 1-A, however, it is usually optimal to plant soybeans, but there more revenue combinations where double-cropping is optimal. For larger previous season corn revenues and smaller previous season soybean revenues, it is optimal to double-crop corn. The range of revenue-state combinations contracts for higher fertilizer prices, because it costs more to plant corn.

Increasing the discount rate by a factor of two and then by a factor of three had no impact on the optimal decision rule and, surprisingly, reducing and increasing the elements of the covariance of revenue disturbances had very little impact on the decision rule. However, modifying the parameters of the revenues equations has a profound impact on the optimal decision rule. The is because the mean and the variance of conditional expectations are affected simultaneously. Figures 2A and 2B show the effects of modifying the soybean revenue equation, and figures 3A and 3B show the impacts of changing the corn revenue equation. All of the parameters were modified in accordance with 95% confidence intervals and correlation coefficients of the parameter estimates.

Notice how the first modification (indicated with a dot) of the soybean revenue equation almost completely eliminates the incidence of double cropping corn. The change reduces expected end-of-season soybean revenues by 1.4%, on average, but reduces the variance of expected end-of-season soybean revenues by 8.8%. This very slight reduction in volatility significantly reduces the value of waiting to plant soybean, even though the foregone gains of planting corn are increased. The second modification (indicated with a circle) has the opposite effects on the mean and variance of conditional soybean revenue expectations. This change increases expected end-of-season soybean revenues by 2.5%, on average, and increases the variance of expected end-of-season soybean revenues by 11.5%. This a larger jump in volatility, which increases the value of waiting to plant soybean, an option value that is further enhanced by the increase in the foregone gains associated with planting corn now. Similar effects occur with respect to modifications of the corn revenue equation.

Figures 4A and 4B show the decision rules for various forms of myopic decision rules. The first modification (indicated with a dot) examines a static, single-period, expected profit maximization model of crop rotation that accounts for last season's crop choice. The second modification (indicated with a circle) examines a static, single-period, expected profit maximization model that does not account for last season's crop choice. Notice how corn is double cropped much more as the degree of myopia is increased, demonstrating quite remarkably the benefit of viewing the rotation problem in its correct context, as a dynamic optimization problem.

Finally, figure 5 demonstrates how our parameterized model agreed with actual crop choices. Crop choices were simulated using the base model's optimal policy function and compared to the crop choices made by individuals on over 6,500 plots reported in National Resources Inventory data for Iowa during 1979-1997. Not all years are reported so, for those years the policy function was used to estimate the crop choice. Overall, our simple model did an astounding job of correctly predicting actual crop rotations.

Conclusion

We examine crop choice as a dynamic optimization problem over an infinite time horizon, taking into account the effects over time that corn-soybean rotations have on soil quality, which manifest in yield and therefore profit impacts. We show how the efficient decision rule depends on model parameters and how it compares to those characteristic of static models of supply. The model is parameterized for a representative acre of Iowa cropland and used to predict actual crop choices in a panel of over 6500 Iowa plots during 1979–1997 surprisingly well.

References

Hennessy, David A., "On Monoculture and the Structure of Crop Rotations." *American Journal of Agricultural Economics*. 88(2006):900–14.

Lence, S.H. "Using Consumption and Asset Return Data to Estimate Farmers' Time Preferences and Risk Attitudes." *American Journal of Agricultural Economics*, Vol. 82(2000):934–47.

Miranda,M.J., and P.L. Fackler. *Applied Computational Economics and Finance*. Cambridge MA: MIT Press, 2002.

Table 1.

Summary of first-order vector autoregressive model of Iowa state-level revenues-per-acre

	Corn ($r_{c,t}$)			Soybeans ($r_{s,t}$)	
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
α	70.43306	43.411	κ	60.60741	35.987
β_1	0.359455	0.1609	γ_1	0.03023262	0.1334
β_2	0.6093376	0.1916	γ_2	0.815847	0.1588
VAR($\varepsilon_{c,t}$)	9341.519		VAR($\varepsilon_{s,t}$)	6419.619	

COV($\varepsilon_{c,t}$, $\varepsilon_{s,t}$) = 4952.483

Note: The errors are assumed iid so estimates were obtained using OLS. Standard errors for the variance and covariance estimates were obtained using a non-parametric bootstrap of the residuals.

Table 2.

OLS regression results: Dependent variable the natural log of yield

	Corn ($r_{c,t}$)			Soybeans ($r_{s,t}$)	
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
Intercept	4.4632	0.02514	*Intercept*	3.5091	0.018267
ln(*Fertilizer*)	0.082018	0.0030006			
d_{t-1} ln(Fertilizer)^2	0.019051	0.00075647			
d_{t-1}	-0.6678	0.016757	d_{t-1}	0.21327	0.0082209
Year F.E.	Yes		*Year F.E.*	Yes	
R^2 = 0.8127, Observations =2100, Error degrees of freedom = 2072, Std. Error = 0.20089			R^2 = 0.83668, Observations =1020, Error degrees of freedom = 994, Std. Error =0.11913		

Notes: Experimental plot data for northeast Iowa during 1979-2002.

Figure 1-A.

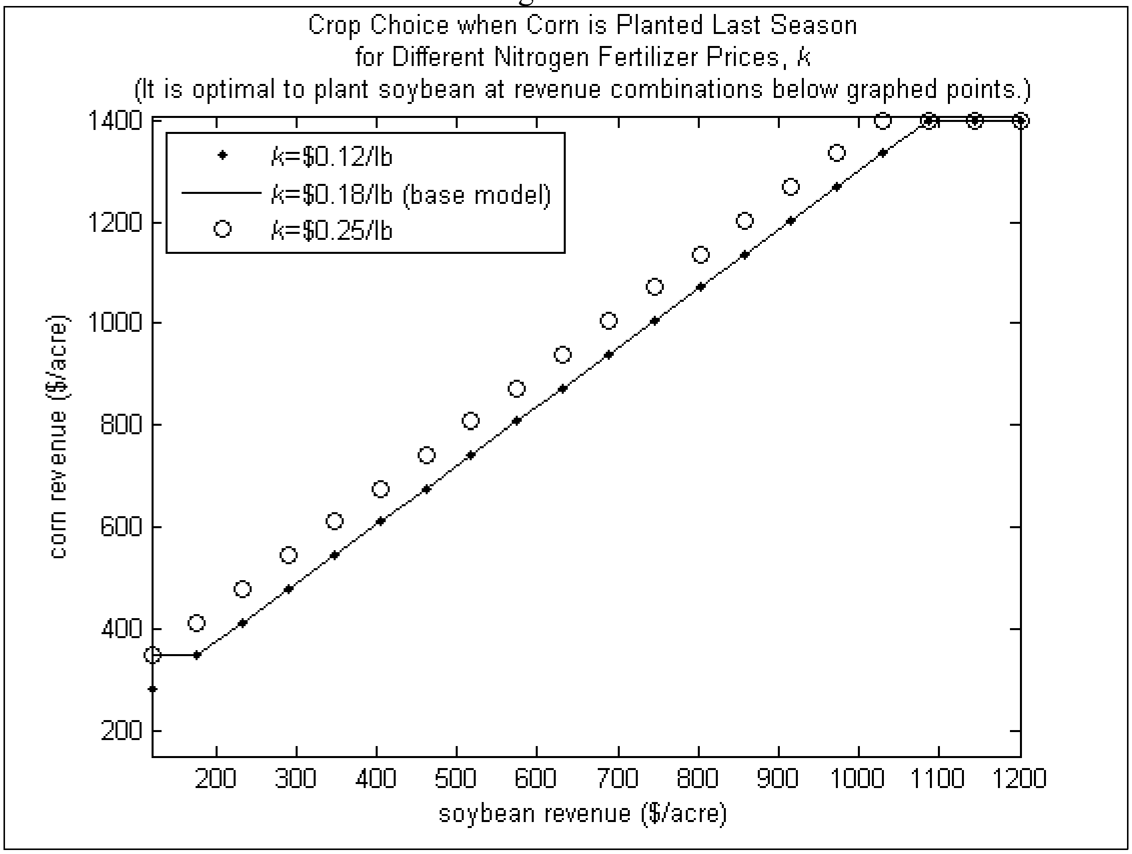


Figure 1-B.

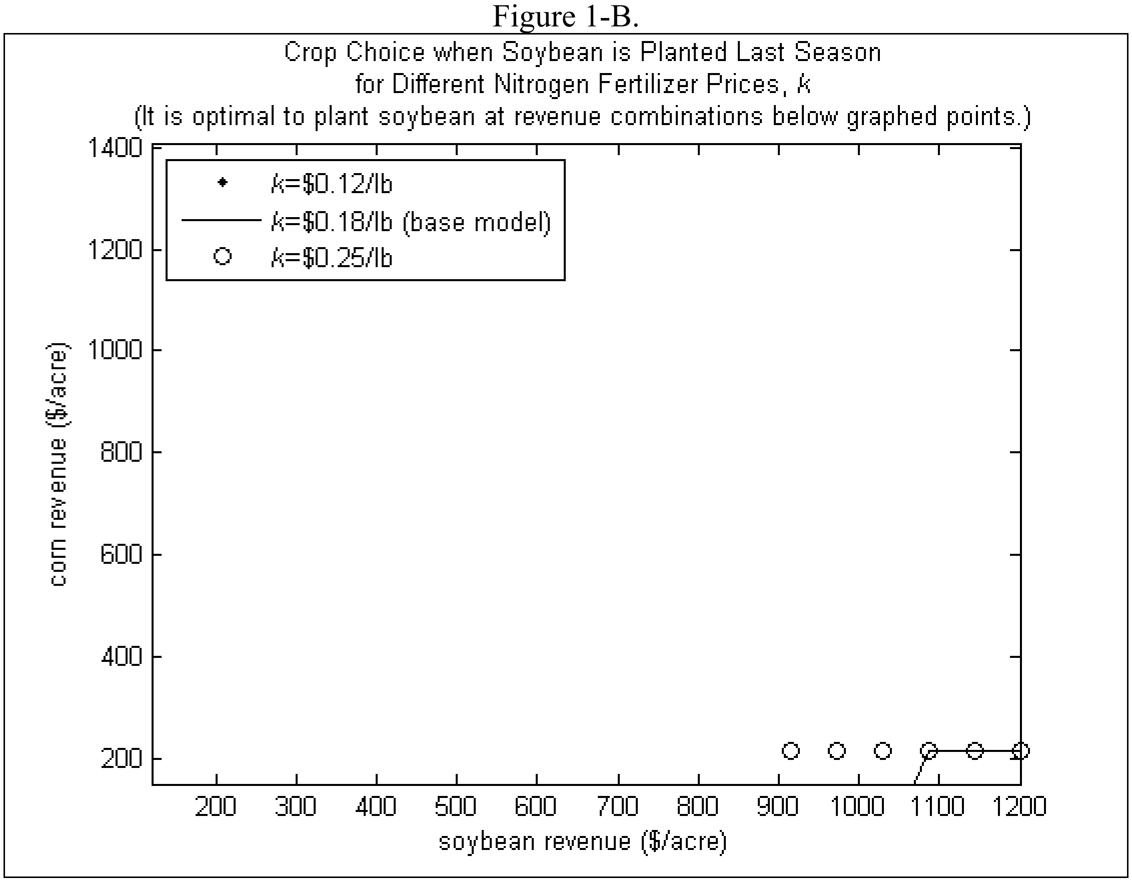


Figure 2-A

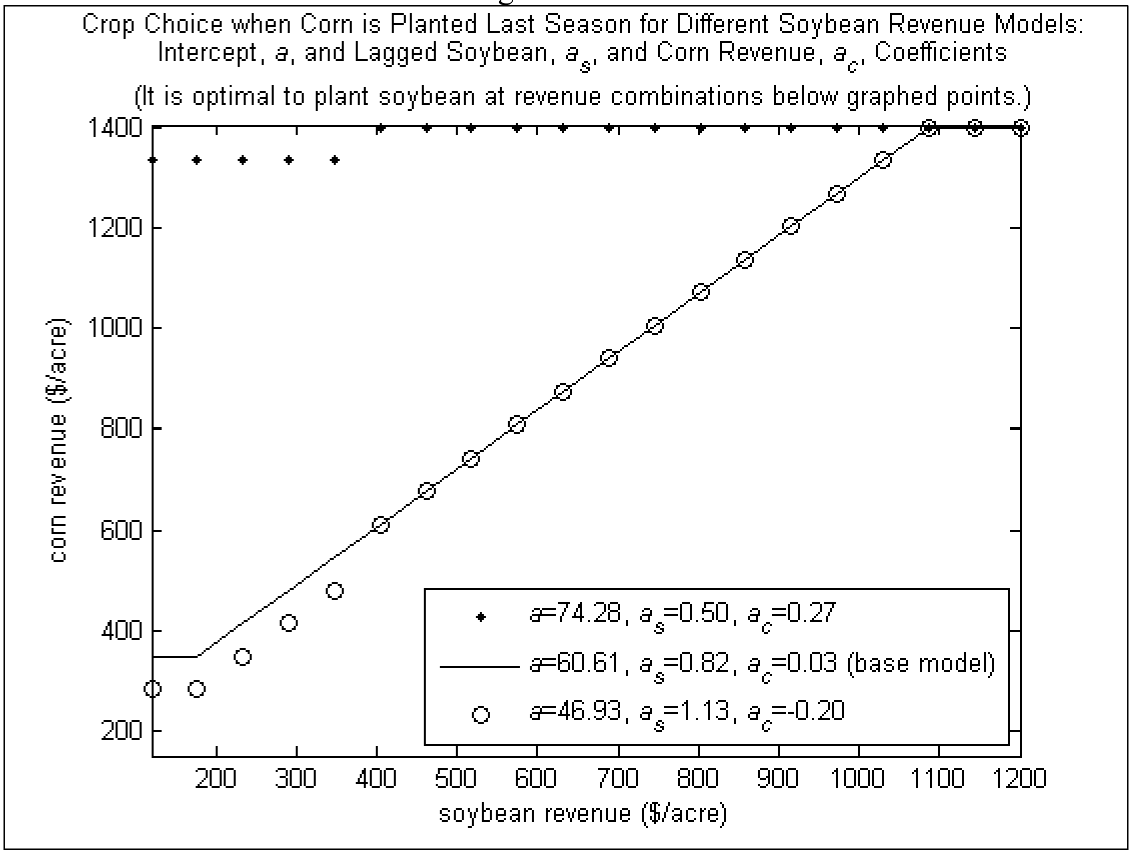


Figure 2-B

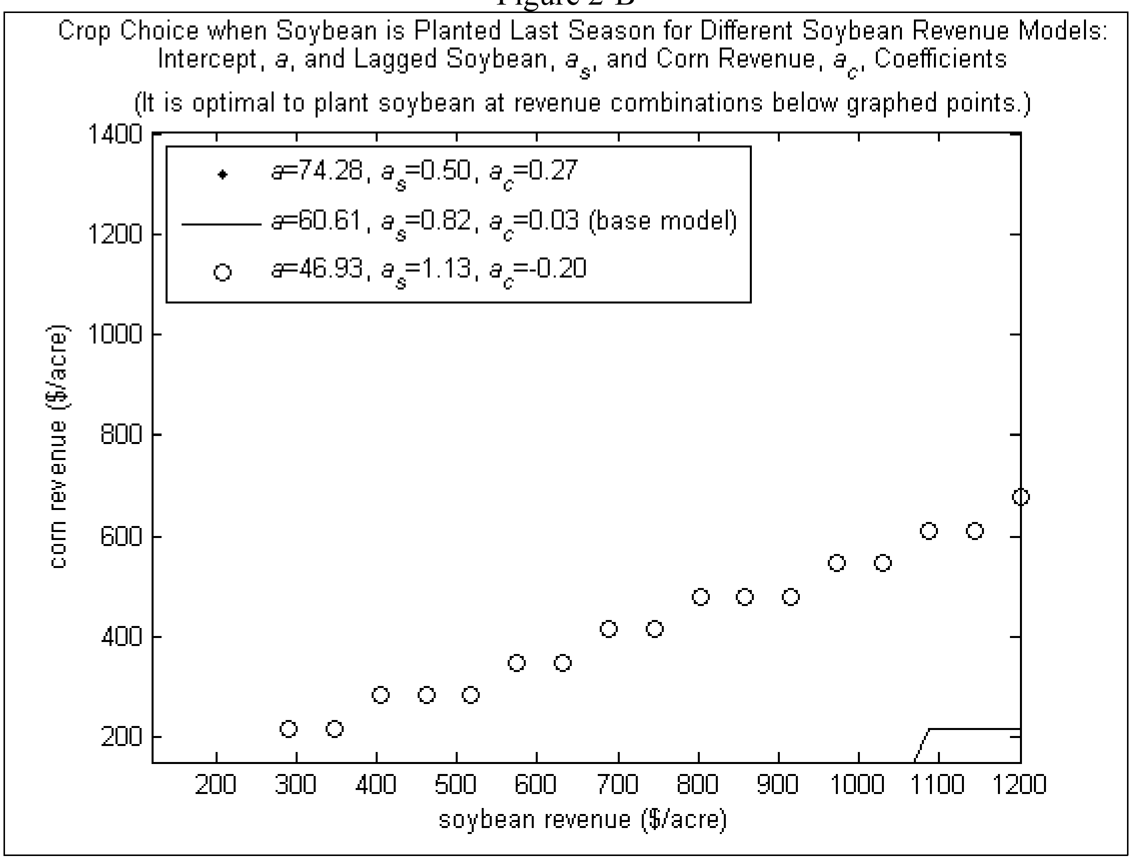


Figure 3-A

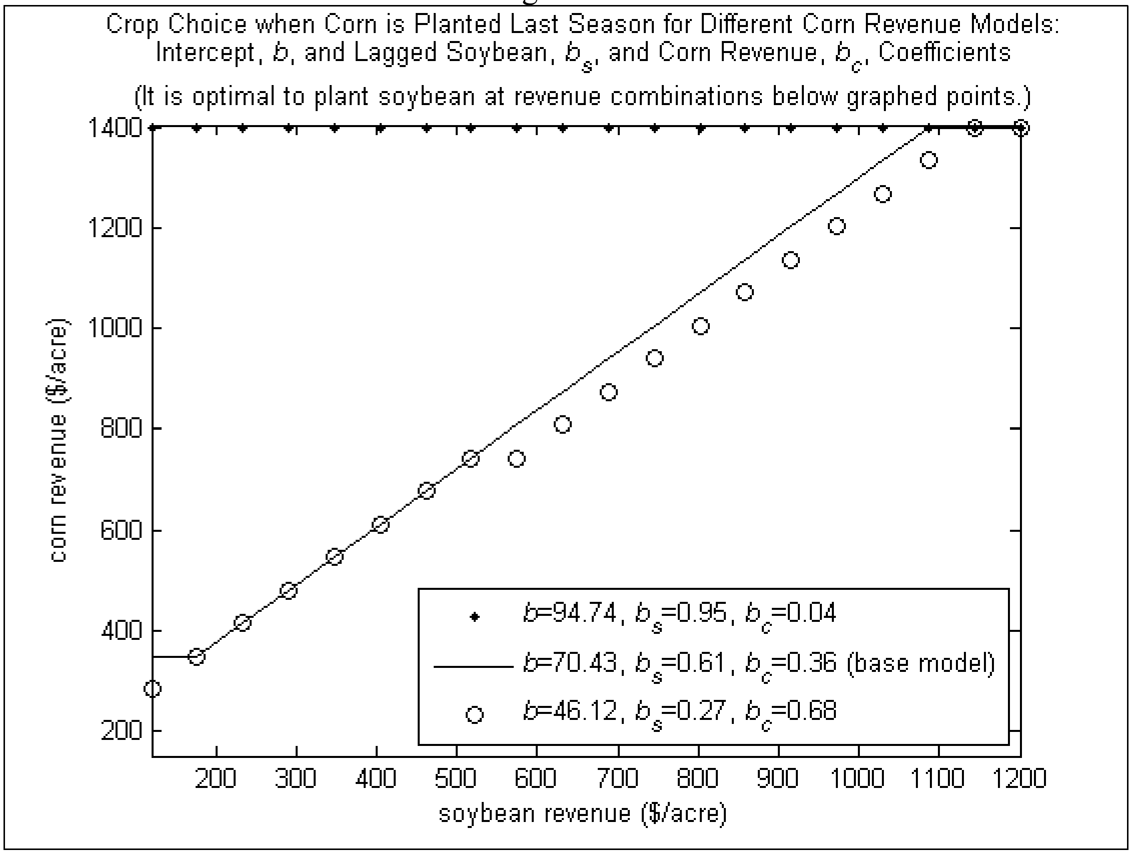


Figure 3-B

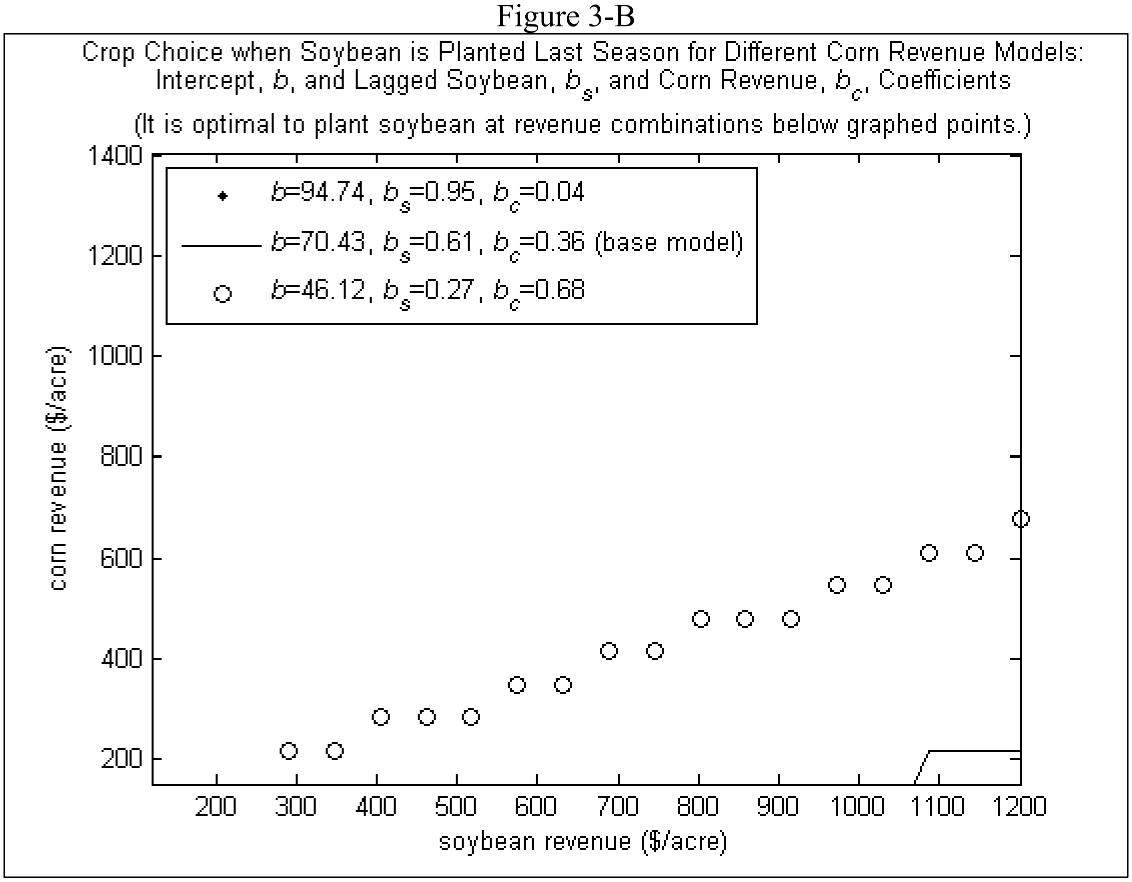


Figure 4-A

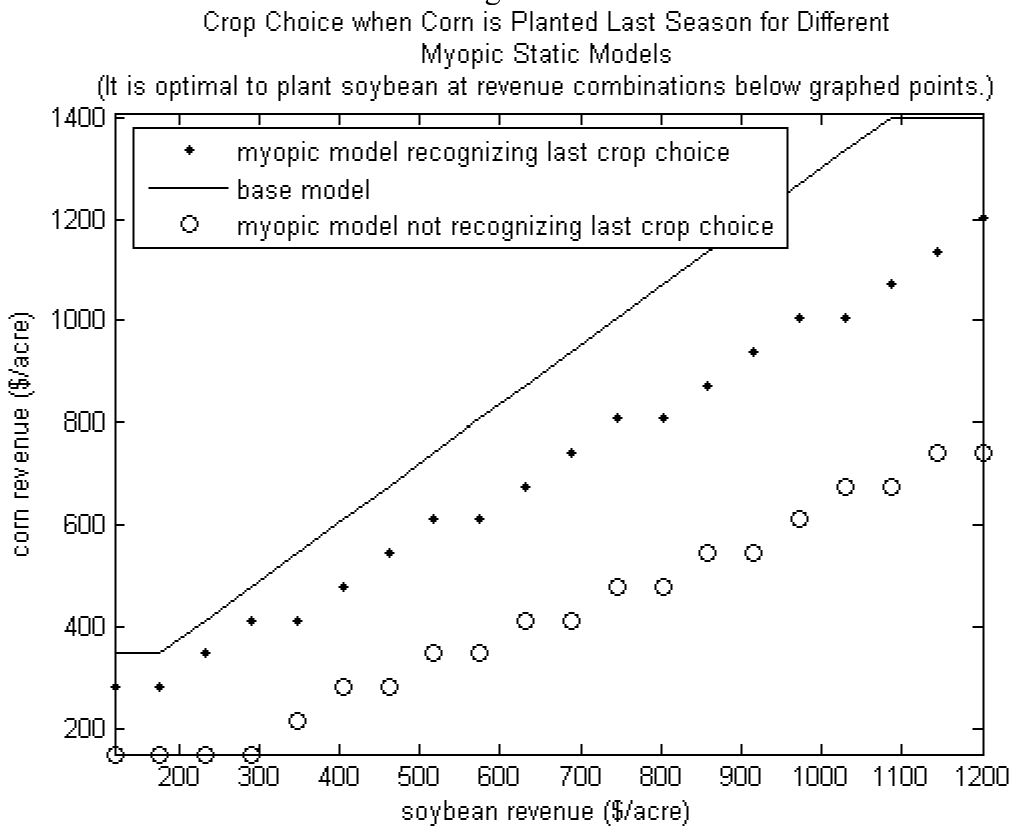


Figure 4-B

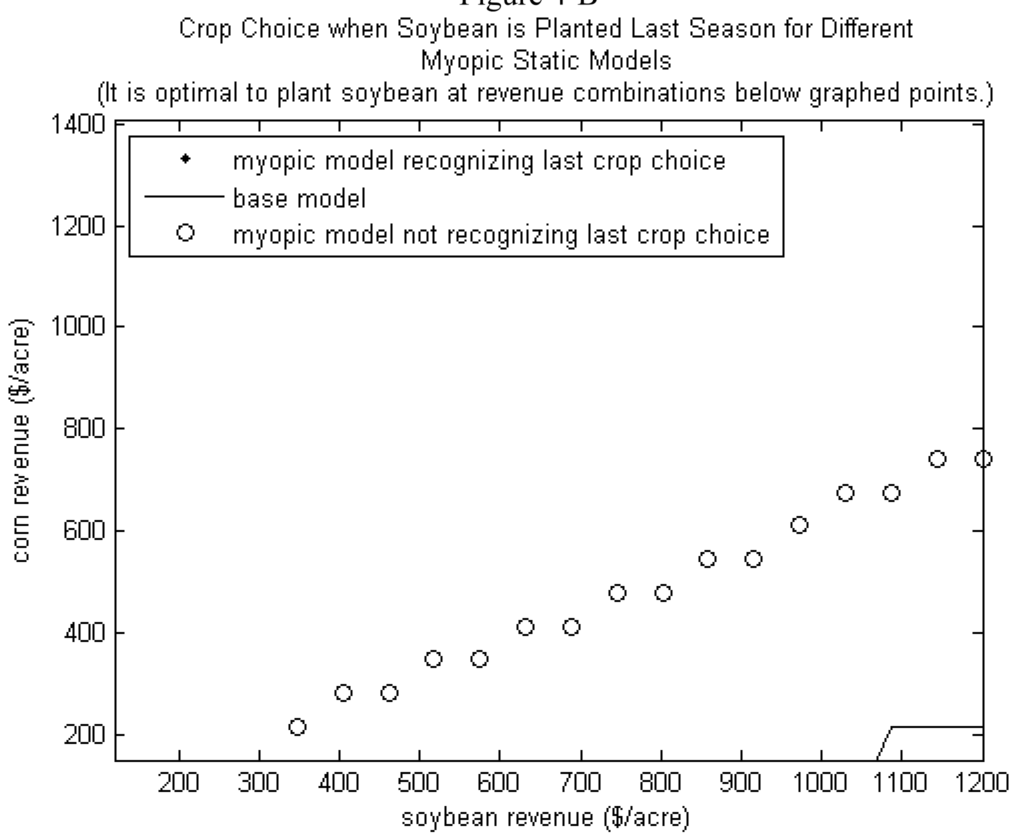
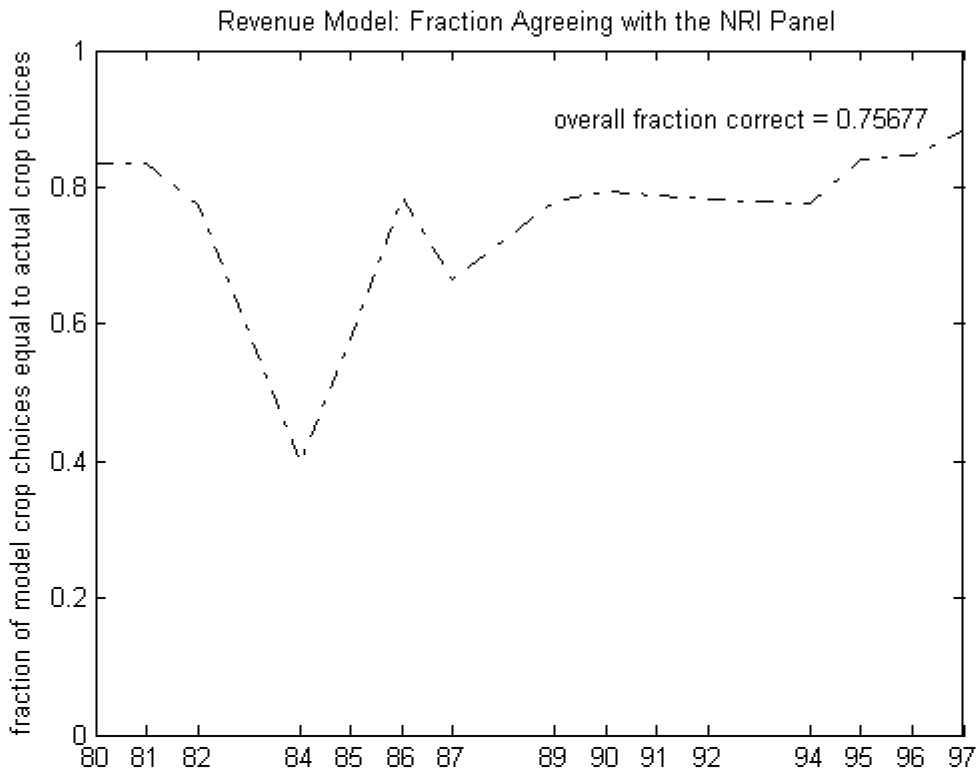


Figure 5



Revenue Model: Fraction Agreeing with the NRI Panel
overall fraction correct = 0.75677
fraction of model crop choices equal to actual crop choices
1
0.8
0.6
0.4
0.2
0
80 81 82 84 85 86 87 89 90 91 92 94 95 96 97