Review of Economic Studies (1992) 59, 277-297 0034-6527/92/00150277$02.00

A Micro-Econometric Model of Capital Utilization and Retirement: The Case of the U.S. Cement Industry

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First version received March 1990; final version accepted April 1991 (Eds.)

The paper presents a micro-econometric model of capital utilization and retirement. Estimates of a firm's discrete decision problem with regard to an existing piece of capital—whether to operate, hold idle or retire it—are obtained, in the context of the U.S. cement industry, by solving a discrete-choice stochastic dynamic programming model. The estimates are then used to simulate the effects of product and input price changes, and changes in the size and age of a cement kiln on a firm's propensity to operate, hold idle and retire a kiln.

1. INTRODUCTION

The study of capital utilization and retirement is important both at the macro- and the micro-level of an economy. At the macro-level it can be used to explain and predict business cycle patterns and the productivity of factors of production. At the micro-level such studies may be used to analyze the impact of demand and cost shocks, induced by policy changes and external factors, on the general performance of particular industries in terms of factor employment and productivity, and temporary or permanent plant closures.

While most studies of capital utilization and retirement are macro-oriented this paper attempts to develop and estimate a model of capital utilization and retirement at the (micro-) level of individual production units. Specifically, the U.S. cement industry is the object of our analysis. The most important piece of capital in cement production that defines a production unit is called a "kiln"—a huge rotary steel tube lined with fire brick which is very fuel-intensive in operation. The data set, which covers 1972–1980, provides an interesting account of the nature of kiln utilization and retirement and how the utilization and retirement of kilns responded to the oil price shock in the seventies.

Table I displays the distribution of annual kiln utilization rates over all kilns and the entire sample period. Clearly, kiln utilization may be at the two extremes. Note that 21% of kilns in the data set have zero annual utilization rates while 46% have a 100% utilization rate (see Section 2 for details). Furthermore, due to the existence of such corner solutions at the kiln level, the appropriate level of disaggregation for studying utilization in the cement industry is the kiln level rather than a more aggregated level. The occurrence of the corner solutions at the level of individual production units implies

TABLE I

Frequency distribution of utilization rates

Annual utilization rate (*aur*)	Proportion of kilns (Out of 987)
$aur = 0$	0·21
$0 < aur \leqq 0{\cdot}2$	0·01
$0{\cdot}2 < aur \leqq 0{\cdot}4$	0·02
$0{\cdot}4 < aur \leqq 0{\cdot}6$	0·02
$0{\cdot}6 < aur \leqq 0{\cdot}8$	0·09
$0{\cdot}8 < aur \leqq 0{\cdot}99$	0·19
$aur = 1{\cdot}0$	0·46

non-smooth changes in the rate of utilization of capital at the plant level (averaged over its production units). (In the context of labour employment decisions by firms, Hamermesh (1989) makes a similar point.)

Table II depicts the distribution of kilns according to use within various age groups. It is interesting to observe that the proportion of operated (idled or retired) kilns does not decrease (increase) monotonically with age. Thus there is little evidence of "phasing out" of kilns as they get older, which indicates that future price expectations may be more important than the age of kilns in determining utilization and retirement choices.

Table III shows the year-by-year average cement and input prices in real terms as well as the distribution of kiln use. It is evident that the oil price shock of 1973–1974 resulted in a significant increase in the real fuel price (in terms of cement) in 1974–1975. It stayed high until 1979 and fell off in 1980. Over the sample period the real wage did not change much, while the real electricity price had an increasing trend and was particularly high in 1978 and 1980. Following the first oil price shock, kiln idleness and retirement increased whereas kiln utilization decreased from 1975. Kiln retirement stayed high relative to the pre-oil-price shock era, and in two particular years, 1978 and 1980, there were "blips" in retirement.

Of course, the obvious task is to empirically explain some of these behavioural changes. Although there are numerous macro-oriented models of capital utilization,[1]

TABLE II

Distribution of kilns according to use within various age groups

Age (in years)	Proportion of kilns operated	Proportion of kilns idled	Proportion of kilns retired
$0 \leqq \text{age} < 10$	1·00	0·00	0·00
$10 \leqq \text{age} < 20$	0·92	0·07	0·01
$20 \leqq \text{age} < 30$	0·83	0·12	0·05
$30 \leqq \text{age} < 40$	0·75	0·19	0·06
$40 \leqq \text{age} < 50$	0·72	0·20	0·08
$50 \leqq \text{age} < 60$	0·56	0·25	0·19
$60 \leqq \text{age} < 70$	0·64	0·34	0·01
$70 \leqq \text{age} < 80$	0·00	0·53	0·47

1. Existing papers, both theoretical and empirical, that endogenize capital utilization typically assume that the rate of utilization is continuous and smooth and is necessarily positive at the optimum, e.g. Nadiri and Rosen (1969), Epstein and Denny (1980), Abel (1981), Berndt and Morrison (1981), Bernstein (1983), Merrick (1984) and Morrison (1985). Bentancourt and Clague (1981) regard utilization as a positive but discrete-valued variable. The "standard" neoclassical model of Jorgenson (1963, 1974) and its numerous extensions that do not endogenize capital utilization presume full capital utilization, and moreover, capital retirement choice is ignored in them.

TABLE III

Summary statistics of the data

Year	Proportion of kilns operated	Proportion of kilns idled	Proportion of kilns retired	Mean age of kilns (in years)	Mean cement price* (in $/ ton)	Mean fuel price* (in $/ (mbtu)	Mean electr. price* (in $/ mbtu)	Mean wage rate* (in $/ hour)
1972	0·89	0·11	0·00	31	15·64	0·33	3·07	3·93
1973	0·88	0·12	0·00	32	15·75	0·33	3·08	3·95
1974	0·86	0·14	0·00	32	17·31	0·50	3·81	3·82
1975	0·64	0·30	0·06	33	18·70	0·64	4·33	3·80
1976	0·72	0·21	0·07	33	19·29	0·63	4·36	4·13
1977	0·75	0·19	0·06	34	19·33	0·63	4·65	4·21
1978	0·69	0·13	0·18	35	20·16	0·61	4·79	4·23
1979	0·86	0·10	0·04	31	21·12	0·63	4·53	4·21
1980	0·71	0·11	0·18	31	20·31	0·56	5·02	4·10

Note: *These are real prices, deflated by the CPI with 1967 as the base year.

they do not seem suitable for such a task for the following reasons. One is the lumpiness of the utilization rate at 0 and 100%—which is utterly unrealistic at a macro level—but not so at a micro-level (as seen in Table I). Another reason is that the capital retirement choice in the macro-analysis is typically tied up with either the replacement decision (e.g. Feldstein and Foot (1971) and Feldstein and Rothschild (1974)) or embodied technical progress (e.g. Salter (1960) and Solow (1969)),[2] whereas it may be quite feasible—and indeed optimal—for an individual firm to retire a piece of capital without replacing it or without technical progress being relevant. For instance, in the data set, 58 kilns were retired while 3 new ones were bought, and also, plants that retired kilns were not necessarily the ones that bought new kilns. Furthermore, there has not been much technical progress in cement production over the last two decades or so, and yet, retirement of kilns is observed in the mid to late 1970s (see Table III), presumably due to the oil price shocks.

Hence with the aim of explaining kiln utilization and retirement, a discrete-choice micro model is developed in which a cement firm faces three choices with regard to its existing kilns: operate, hold idle or retire. The simultaneity of these decisions and the methodology used in estimating the parameters of the decision rules are considered novel. The estimates are then used to compare the predicted and the observed patterns in kiln utilization and retirement. Then the model is used to simulate the impact of variations in output and input prices on the probabilities of operating, holding idle or retiring a kiln.

Next, we present a brief description of the cement technology and the nature of the available data, which in turn guide some of the assumptions of our model.

2. CEMENT TECHNOLOGY AND DATA

2.1. *Technology*[3]

The chemical components of cement are calcium oxide, aluminum oxide, iron oxide and

2. Malcomson (1975) and Nickell (1975) are exceptions. Rust (1987), which is a micro study, also treats retirement and replacement as simultaneous decisions.

3. A detailed description of the cement production process is given in Witt (1966). The basic production technology has not changed since 1952.

silicon oxide. These components occur naturally as deposits in limestone, shale, iron ore, clay or sand quarries. Hence cement production begins with the quarrying and crushing of the raw material deposits. The raw materials are then mixed into a "kiln-feed" either by adding water (the wet process) or without adding water (the dry process). The newer dry process preheats the kiln-feed before it enters the kiln. So there are effectively three processes of cement production: wet, dry and dry-with-preheater. The production process of a kiln is irreversible. The kiln-feed enters the kiln at one end and as it moves down the rotating kiln, heated by fuel (coal, oil or gas) at about 2700°F, its chemical composition changes. The product that exits the kiln is called "clinker" which is then ground to get cement.

The kiln is the largest and the most important piece of equipment in cement production. All other auxiliary equipment is built to be consistent with the operation of the kiln. The capacity of each kiln and the number of kilns determine the capacity of a cement plant. Thus, kiln is the "unit of production".

The following features of kiln technology and costs will be relevant for the econometric model to be specified.

First, kilns operate continuously twenty four hours a day if they are operated at all. This is because frequent heating and cooling of a kiln can damage its firebrick lining, and the start-up heating cost is high. In fact, once heated up, firms prefer to operate a kiln almost all year round, except for a few days for maintenance, because they face capacity constraints during peak demand periods of summer and fall. Thus, typically, the annual output of a kiln is approximately either zero or its capacity level—which is consistent with the utilization rates reported in Table I. Furthermore, even though inventory accumulation is undertaken within a year to smooth out production, inventory holding is not significant between years due to the high storage cost of cement.

Second, there is no jointness of cement production across kilns, which implies that a cement plant's total cost function is additively separable in kilns.

Third, there are essentially five variable inputs in cement production—labour, fuel (coal, oil or gas), electricity, raw material and maintenance. Labour is mainly used in the quarry and for packing. Fuel is largely consumed by the kilns. Electricity is consumed mainly by the auxiliary equipment. Part of the maintenance cost is variable and part of it is fixed (e.g. "winterizing" the kilns each winter). The variable cost function then includes the cost of these inputs.

Fourth, as variable inputs are not substitutable, there is a fixed coefficient technology. Accordingly, the total variable cost function is linear in the output and input prices or in other words, the average variable cost is independent of the output. However, the marginal cost may increase with the age of the kiln.[4]

We now discuss the available data.

2.2. *Data*

The data consist of 987 annual observations over the period 1972 to 1980 on dry process kilns spread over 32 cement plants. A particular plant typically tends to have kilns of one process only. (There was only one exception in which a plant had dry process kilns as well as a newer dry-with-preheater process kiln.) The number of kilns in a plant ranged from 1 to 19. Multi-kiln plants tend to have kilns of similar sizes. Data include each

4. This has been recognized by McBride (1983, p. 1013).

plant's location, clinker production, fuel and electricity consumption and the total number of kilns. For each kiln it includes the vintage, the kind of fuel it uses and its capacity.[5]

Kiln-level cement production data are not available although plant-level cement production data are. But since our study is at the level of kilns, the kiln-level production data are generated from the plant-level production data. This is done in the following way.[6]

Because the marginal cost of a kiln increases with its age and multi-kiln plants have kilns of almost identical sizes, a profit-maximizing firm will use kilns of a process in the reverse order of their ages. Thus, given the plant-level cement production data, the output of the youngest kiln equals the minimum of its capacity and the plant output. If the capacity of the youngest kiln is less than the plant output, then the youngest kiln is assigned all the output and the rest of kilns in the plant, if any, are assigned zero output. Otherwise, while the youngest kiln is assigned its capacity output, the remaining plant output is assigned toward the capacity of the next to the youngest kiln. Any left-over plant output is then assigned to the youngest of the remaining kilns, and so on. In the case of two kilns of the same age, if the output to be assigned is less than the capacity of one kiln then only one kiln is assigned all the output. This is because the high temperature required for kiln operation means a high start-up cost and the frequent heating and cooling of kilns damages their firebrick lining. Hence it is efficient to produce the total output by heating one kiln only. But if the output to be assigned is greater than the capacity of one kiln so that both kilns are used, then they are assumed to be utilized to the same extent.

The annual utilization rate of a kiln is calculated as the ratio of its output over its annual capacity. The frequency distribution of the kiln utilization rate is given in Table I. As expected, the utilization rate is concentrated at the two extremes: 86% of the observations have either a zero or greater than 80% utilization rate.

Our econometric analysis requires information on whether during a given year a kiln in the data is operated, held idle or retired. The information on kiln retirement is directly given in the data set (58 out of 987 kilns were retired over the sample period). But kilns that are either operated or held idle have to be classified according to the utilization rates. Although, ideally, kilns that have a zero utilization rate but are not retired should be classified as held idle and the rest operated, the cut-off point for "held idle" and "operated" is taken at 20%, i.e. kilns whose utilization rate ranged from 0 to 0·2 and which are not retired are identified as held idle and the rest operated. This is because in the data set there are two out of the fifty eight retired kilns which have utilization rates of 0·17 and 0·2. (This happened because an old kiln may be used part of a year and then retired in the same year.) The cut-off point at 0·2 rather than at the zero utilization rate is hardly significant because, as Table I shows, only 1% of kilns have utilization rates which are positive but less than 0·2. In other words, the cement technology implies that capacity utilization is either nearly zero or nearly full; however, the choice between the two is an economic one.[7]

Given the above classification, the proportions of kilns operated, held idle and retired each year in the sample period are reported in Table III. Table III also contains summary statistics of the age of kilns, cement price and input prices such as electricity price, fuel

5. Thus capacity here is an engineering notion—a name-plate rating.

6. Experts in cement plants with whom I have discussed the cement technology agree with the following procedure.

7. This notion of optimal capacity or capital utilization is different from that in the standard literature in which the optimal utilization choice arises when there are other quasi-fixed inputs like labour, besides capital, and it is costly to change output and utilization by changing such inputs (see, for example, Abel (1981) and Shapiro (1986)).

price and wage rate (all relative to CPI).[8] However, data on the prices of the remaining variable inputs—raw material and maintenance—are not available and hence the sum of these two costs will be regarded as unobserved costs.

We now present the theoretical model.

3. A MODEL OF KILN UTILIZATION AND RETIREMENT

3.1. *Assumptions*

*A*1. A firm has one or more kilns and the cost function associated with a kiln is independent of any other.

*A*2. There are fixed costs of maintaining or "holding" a kiln (e.g. electrical and mechanical maintenance costs).

In general, such costs are necessary for retirement to be a non-trivial choice, because otherwise there would be no incentive for a firm to retire a production unit that is currently making a loss as long as there is a positive probability of making positive profits in the future.

*A*3. Firms are risk-neutral and hence maximize expected discounted sum of profits.

*A*4. Firms are perfectly competitive in the output and input markets.

*A*5. There is uncertainty about future output and input prices and they follow a joint first-order Markov process.[9]

*A*6. Kiln depreciation is in the form of "input decay" (in the terminology of Feldstein and Rothschild), i.e. it requires an increasing amount of one or more variable inputs to produce a unit of output as it gets older.[10]

Assumptions A1 and A4 imply that a cement plant's profit function is separable in kilns. As risk neutrality (A3) further implies that the profitability of any kiln is independent of that of any other, existing or new. In other words, unlike the existing models in which the interdependence of marginal revenue products of existing and new capital implies jointness of utilization, retirement and new investment decisions, these decisions are

8. The data on cement price and input prices were obtained from the following sources. The cement prices in each state were obtained from the *Minerals Yearbook.* The hourly wage rates were derived from the earnings and hours data given for the cement industry in *Employment and Earnings* published by the Bureau of Labor Statistics. State-level fuel and electricity prices were obtained from the *Energy Price and Expenditure Data Report (1970–1980)* which is published every ten years by the U.S. Department of Energy. In the case where a kiln could use more than one source of fuel, the fuel price relevant for the kiln in any year is taken to be the one that is the cheapest in that year in the state in which the kiln was located. In order to abstract from the differences in price due to differences in the general price level, the cement price and all the input prices are deflated by the CPI in each state.

9. Even though the theory below holds for any joint first-order Markov process, the jointness assumed in the empirical analysis is limited due to insufficient variation in our data on prices. Details are given in Section 4.

10. It may be remarked that this way of modelling depreciation is no less reasonable than the usual Jorgenson-type assumption of 'output decay'. Output decay implies that same amounts of capital of different ages differ in the quantity of services they provide, whereas input decay implies that they differ in quality (even in the absence of technical progress). As Nickell recognizes, the difference in quality implicit in the notion of input decay is critical for capital retirement.

separated in our model.[11] While such separation in the decision making is plausible as they directly follow from A1, A2 and A3 it may not be realistic in many situations. For instance, if a firm possessed some market power its marginal revenue would be dependent on the total output from all the kilns and hence the separation property would not hold. But for our purpose it is attractive as it simplifies an otherwise high-dimensional (and computationally highly intractable!) joint decision problem of utilization and retirement of all existing kilns and new investment to a trinomial choice—operate, hold idle or retire—for each existing kiln.[12]

3.2. *The discrete-choice dynamic programming model*

The decision problem of a firm is then to choose a sequence of decision rules $I = \{i_t = f_t(x_t, \varepsilon_t, \theta)\}_{t=0}^{T}$ to maximize the expected discounted sum of profits from each kiln given by

$$\operatorname*{Max}_{I} E_0\{\textstyle\sum_{t=0}^{T} \beta^t u(x_t, \varepsilon_t, i_t, \theta)\}, \tag{1}$$

where E_0 = expectation based on information available at the current period 0, T = end of the physical lifetime of a kiln (about 80 years), β = the discount rate, $0 \leqslant \beta \leqslant 1$, $u(\cdot)$ = (real) instantaneous profit function of a kiln, x = vector of observed exogenous variables—the output price, input prices and the observed characteristics of a kiln, ε = the vector $(\varepsilon_0, \varepsilon_1, \varepsilon_2)$ where ε_i is the unobserved random component of costs associated with choice i, $i \in C \equiv$ {operate, hold idle or retire), θ = parameters of the profit function to be estimated.

The instantaneous profit function $u(\cdot)$ equals

$$u(\cdot) = \begin{cases} P_tQ_t - AVC_tQ_t - F_t + \varepsilon_{0t} & \text{if } i = \text{operate} \\ -F_t + \varepsilon_{1t} & \text{if } i = \text{hold idle} \\ SV_t + \varepsilon_{2t} & \text{if } i = \text{retire} \end{cases} \tag{2}$$

where P = (real) output price, Q = profit maximizing output when i = operate, AVC = average variable cost function (to be specified later), F = fixed costs, SV = scrap value of a kiln net of the cost of removing the scrapped unit.

Expression (1) is the value function $V_t(x_t, \varepsilon_t, \theta)$, which is the recursive solution to the Bellman equation:

$$V_t(x_t, \varepsilon_t, \theta) = \text{Max}_{i \in C}\, [u(x_t, i_t, \varepsilon_t, \theta) + \beta EV_t(x_t, i_t, \varepsilon_t, \theta)] \tag{3}$$

where

$$EV_t(x_t, i_t, \varepsilon_t, \theta) = \int_{x_{t+1}} \int_{\varepsilon_{t+1}} V_{t+1}(x_{t+1}, \varepsilon_{t+1}, \theta)\, dp(x_{t+1}, \varepsilon_{t+1} \mid x_t, i_t, \varepsilon_t). \tag{4}$$

Under certain regularity conditions described in Rust (1988) the optimal choice is given by

$$f(x_t, \varepsilon_t, \theta) = \operatorname*{argmax}_{i \in C} \{u(x_t, i_t, \varepsilon_t, \theta) + \beta EV_t(x_t, i_t, \varepsilon_t, \theta)\}. \tag{5}$$

Our objective is to estimate the parameter vector θ and then assess the impact of changes in output and input prices on the kiln utilization and retirement choices implied by (5). The choices being discrete, there are no closed-form solutions for the decision rules that solve (1). However, the nature of the solution can be illustrated in terms of a two-period special case.

11. More specifically, the jointness in the existing literature is implied from the strictly concave revenue function or the total equilibrium output being strictly concave in capital or both.

12. It is however recognized that with further development in computing technology and more experience with estimating stochastic dynamic programming models, the formulation and estimation of the simultaneous choices of capital utilization and retirement and new investment in capital could be feasible in the near future.

3.3 *The two-period special case: an illustration*

Let the two periods be denoted by $t=0$ (present) and 1 (future). Thus T (the terminal period) $=1$. Consider Figure 1. The current product price P_0 and the expected future product price are measured respectively along the horizontal and vertical axes. If P_0 exceeds the current minimum average total cost, ATC_0, the optimal choice is to operate (at the profit-maximizing level of output); this is indicated by the region to the right of *YNZ*.

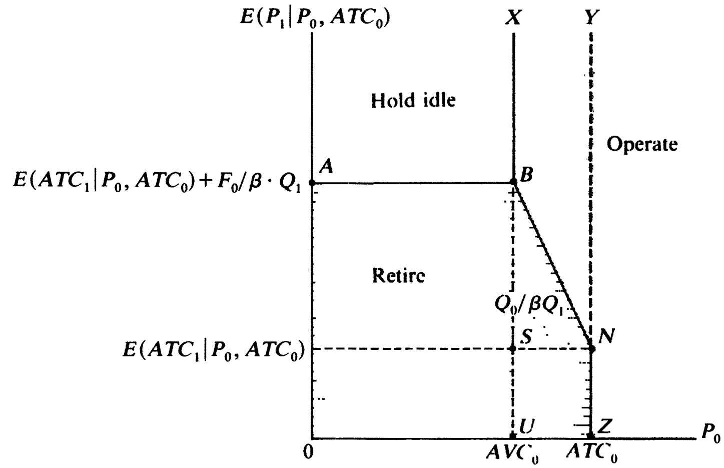

FIGURE 1

If $P_0 < ATC_0$, i.e. losses are incurred in the current period, then future profit expectations play a crucial role. Two sub-cases have to be considered: (a) $AVC_0 \leqq P_0 \leqq ATC_0$ and (b) $P_0 \leqq AVC_0 \leqq ATC_0$, where AVC_0 is the current minimum average variable cost. In sub-case (a), the optimal current choice is either operate or retire. The reasoning is as follows. If the firm expects to also make a loss in the future, i.e. $E(P_1|P_0, ATC_0) < E(ATC_1|P_0, ATC_0)$, then it is optimal to retire the kiln *now*. This is indicated by the area *USNZ*. If the firm expects to make a profit in the future period it will operate or retire depending on whether the discounted future profits exceeds the current loss or not. The dividing line BN whose slope equals $Q_0/(\beta Q_1)$ depicts this choice. In sub-case (b), the optimal current choice is either hold idle or retire. If the unit is held idle the current loss equals the fixed costs. Thus it would be worthwhile to incur this loss now by holding the unit idle *rather than retire it* if the discounted future period profit exceeds the current fixed costs, i.e.

$$\beta Q_1 E(P_1 - ATC_1 | P_0, ATC_0) > F_0,$$

or

$$E(P_1|P_0, ATC_0) > E(ATC_1|P_0, ATC_0) + F_0/(\beta Q_1).$$

This is indicated by the region above the line *AB*. Otherwise, if the reverse of the inequality holds, it is optimal to retire the kiln now, indicated by the area below the line *AB*.

Figure 1 illustrates the two main features of our analysis: the discreteness of the choices as shown by the distinct regions and the role of future expectations.

4. THE ECONOMETRIC ANALYSIS

In this section we specify the current profit function $u(\cdot)$, identify the parameter vector θ that enters the decision rule (5) with regard to each kiln, and discuss its estimation. Using the estimates the model is then simulated to provide predictions; these are presented in Section 5.

4.1. *Specification of the profit function*

If the kiln is held idle, the current revenues from it are obviously zero. If it is operated, the current revenues equal $P \cdot K$ (where $P \equiv$ (real) cement price and $K \equiv$ capacity output) since, as explained in Section 2, it is optimal to produce the capacity output if the kiln is operated at all. If the kiln is retired the current revenues are equal to SV, the net scrap value of the kiln.

Turn next to the cost side. Among the five variable inputs, electricity, fuel, labour, raw materials and maintenance, data are available only on the electricity and fuel prices and the wage rate. Accordingly, the (real) average variable cost function, AVC—which is independent of the output—is specified as:

$$AVC = ACU + AVLI \cdot W + AVFI \cdot PF + AVEI \cdot PE \tag{6}$$

where ACU = the (real) average cost of unobserved variable inputs (raw material and maintenance), $AVLI$, $AVFI$, $AVEI$ = average variable labour, fuel and electricity input, W, PF, PE = (real) hourly wage rate, (real) fuel price, (real) electricity price.

With respect to ACU, we assume that it is proportional to the price of cement, the rationale being that the price of cement reflects the strength of demand for cement, which will be positively related to the demand for raw material and maintenance inputs. Hence,

$$ACU = \alpha P,^{13} \tag{7}$$

where α is a positive constant. The constancy of α is justified by the fixed coefficient nature of the cement technology.

Furthermore, as kiln operation is very fuel-intensive, it is likely that $AVFI$ increases with the age of a kiln—a form of "input decay". In keeping with the existing literature (e.g. Feldstein and Rothschild), we assume a constant rate, δ, of input decay, i.e. $AVFI_{t+1} = (1+\delta)AVFI_t$. This difference equation implies

$$AVFI_t = (1+\delta)^{A_t} AVFI_0, \tag{8}$$

where A_t is the age of the kiln at time t and $AVFI_0$ is the average variable fuel input of a new kiln.[14]

Substituting (7) and (8) in (6), the average variable cost function is written as

$$AVC_t = \alpha P_t + AVLI \cdot W_t + (1+\delta)^{A_t} AVFI_0 \cdot PF_t + AVEI \cdot PE_t. \tag{9}$$

Now consider the fixed costs, which consist largely of mechanical and electrical maintenance of kilns. These costs are assumed to increase proportionately with the

13. The inclusion of a constant term in (7) led to identification problems in estimating the AVC function. The estimation procedure is discussed in the next sub-section.

14. Since labour is used mainly for quarrying and packing, the labour requirement is unlikely to change as a kiln ages. Also, electricity is used mainly by the auxiliary equipment and for lighting buildings, and its usage is unlikely to vary with the age of a kiln.

capacity of the kiln and the age of the kiln. Hence

$$F = h \cdot K + d \cdot A, \qquad h, d > 0 \tag{10}$$

where F is the fixed costs, h can be interpreted as the holding cost per unit of kiln capacity and A is the age of a kiln.

Collecting the expressions for total revenue, variable costs given in (9) and fixed costs given in (10), the instantaneous profit function of a kiln can now be written as

$$u(\cdot) = \begin{cases} P \cdot K - AVC \cdot K - F + \varepsilon_0 & \text{if } i = \text{operate} \\ -F + \varepsilon_1 & \text{if } i = \text{hold idle} \\ SV + \varepsilon_2 & \text{if } i = \text{retire.} \end{cases} \tag{11}$$

The exogenous variables are: K, A, P, W, PE and PF, and the estimable parameters of the profit function are: $\theta = (\alpha, \delta, AVLI, AVFI_0, AVEI, h, d, SV)$.

4.2. *Outline of the estimation procedure*

The parameter vector θ is estimated in the context of the dynamic programming model given in (1), where $u(\cdot)$ is specified in (8), (9), (10) and (11). The choices being discrete there are no closed-form solutions or explicit estimating equations. Instead, the estimation involves solving the dynamic programming model numerically.

Compared to the existing literature on the estimation of discrete-choice dynamic programming models, e.g. Miller (1984), Wolpin (1984, 1987), Pakes (1986) and Rust (1987), there are three distinctive features of our estimation exercise. First, our model involves more choice variables and more exogenous variables, which are implied by the nature of our decision problem. However, the "computational burden" and the "curse of dimensionality" of dynamic programming models are well-known. Hence, all the parameters are not estimated simultaneously. A two-step procedure is used instead (discussed below). Second, in discrete-choice models the parameters are typically identifiable only up to a positive scale constant. But the two-step procedure enables us to obtain consistent estimates of *individual* parameters. Third, while discrete-choice models are typically estimated by assuming specific distributions for the disturbances, we obtain estimates of as many parameters as feasible, given the data and the existing econometric methodologies, without assuming specific distributions.

Step 1 involves estimating the parameters of the AVC function—α, δ, $AVLI$, $AVFI_0$ and $AVEI$—which does not require solving the dynamic programming model for the following reasons. Since plant level data are available for fuel and electricity consumption (but not for other inputs), δ, $AVFI_0$ and $AVEI$—the parameters that are associated with fuel and electricity inputs—are estimated by regression. Conditional on the regression estimates the remaining parameters, α and $AVLI$, of the AVC function are then estimated by identifying a static decision rule nested in the firm's dynamic programming model: "do not operate a kiln if its current AVC exceeds the current price P"—which is illustrated by the region to the left of UX in Figure 1. This decision rule is discrete, but because it is static, semi-parametric estimates could be obtained. Manski's (1985) maximum score procedure was used for this purpose (for details, see Das (1991)). The estimates of the parameters of the AVC function are reported in Table IV.

Given these estimates, in this paper the remaining parameters—h, d and SV—are estimated by using Rust's (1988) method that solves the dynamic programming model at each iteration of a maximum likelihood routine. This is step 2. A drawback of this two-step procedure is that the standard errors of h, d and SV are biased downward and

TABLE IV

Estimates of the unobserved cost parameter, and electricity, fuel and labour coefficients

	α	$AVEI$	$AVFI_0$	δ	$AVLI$
Estimate	0·4965	0·3667	5·0832	0·0087	0·5744
Standard error	(0·0847)	(0·0146)	(0·4657)	(0·0005)	(0·3385)

cannot be corrected because a feasible closed form of the standard errors of the maximum score estimates are not yet known. Hence the significance of the estimates should be cautiously evaluated.[15]

We now present the procedure for estimating h, d and SV—a subset of the vector θ in (1). These estimates are obtained, conditional on the estimated AVC function, by maximizing the likelihood function for the controlled stochastic process $\{i_t, x_t\}$ for a sample of N kilns with the nth kiln having T_n observations:

$$L=\prod_{n=1}^{N}\prod_{t=1}^{T_n} p(i_{nt}\,|\,x_{nt},\,\theta), \tag{12}$$

where $p(i_{nt}\,|\,x_{nt},\theta)$ are the optimal choice probabilities, given x_{nt} and θ. These choice probabilities are obtained from the optimal decision rule $i_{nt}=f(x_{nt},\varepsilon_{nt},\theta)$, as given in (5), and by specifying a distribution for ε.

However, as (5) shows, the choice probabilities depend on the value at $t+1$ as expected at t—$EV_t(x_t, i_t, \varepsilon_t)$—which does not have a closed-form solution. Hence at each iteration of a maximum likelihood routine, $EV_t(\cdot)$ and its derivatives are to be numerically computed by backward induction and then used to evaluate the likelihood function L and its derivatives.

In our model the $EV_t(\cdot)$ function involves a nine-dimensional integral as $\varepsilon=(\varepsilon_0, \varepsilon_1, \varepsilon_2)$ and there are six exogenous variables, $x=(K, A, P, PE, PF, W)$. For simplifying computation, we make the following assumption on the joint distribution of x_{t+1} and ε_{t+1}, as in Rust.

A7. $$p(x_{t+1}, \varepsilon_{t+1}\,|\,x_t, i_t, \varepsilon_t)=q(\varepsilon_{t+1}\,|\,x_{t+1})\,p(x_{t+1}\,|\,i_t, x_t),$$

where $q(\cdot)$ is the density of ε and $p(x_{t+1}\,|\,i_t, x_t)$ is the Markovian law of motion for the observed state variables.

Rust calls this the "conditional independence" assumption. Assumption A7 involves two restrictions. First, x_{t+1} is a sufficient statistic for ε_{t+1}, which implies that any serial correlation in ε_{t+1} is transmitted entirely through the vector x_{t+1}. Second, the distribution of x_{t+1} depends only on x_t, *not on* ε_t.[16]

The assumption A7 implies that $EV_t(x_t, i_t, \varepsilon_t, \theta)$ is no longer a function of ε_t, i.e. $EV_t(x_t, i_t, \varepsilon_t, \theta)=EV_t(x_t, i_t, \theta)$.[17] This greatly simplifies the computation; $EV_t(\cdot)$ can be

15. Although our estimates of h, d and SV are consistent there may be some doubt about whether they converge at rate $\sqrt{N}$ because the rate of convergence of our semi-parametric estimates is less than $\sqrt{N}$. We still expect our estimates to converge at rate $\sqrt{N}$, based on the work of Ahn and Manski (1990). They have proved that, in binary choice dynamic optimization models, when non-parametrically obtained estimates of some parameters or functions (which have a rate of convergence less than $\sqrt{N}$) are used to estimate other parameters, the second-stage estimates still converge at the rate $\sqrt{N}$ and are asymptotically normal. We expect that this holds in our case although its formal proof is a topic of future research.

16. In our model this is reasonable because among the components of x for a given kiln, K (capacity) does not change over time, A (age) changes in a deterministic manner and the (real) observed prices are determined either unilaterally or jointly in their respective markets.

17. See Rust (1988) for a proof.

computed only on the state space for x by using finite-grid approximation. Hence we have

$$EV_t(x_t, i_t, \varepsilon_t, \theta) = \begin{cases} EV_t(x_t, i_t, \theta) \equiv EV_t(x_t, \theta) & \text{if } i_t = \text{operate or hold idle} \\ 0 & \text{if } i_t = \text{retire.} \end{cases} \tag{13}$$

Using (13) and the profit function $u(\cdot)$ in (11), the value function (3) can now be written as

$$V_t(x_t, \varepsilon_t, \theta) = \text{Max}\,\{V_{0t}(x_t) + \varepsilon_{0t},\ V_{1t}(x_t) + \varepsilon_{1t},\ SV + \varepsilon_{2t}\}, \tag{3$'$}$$

where

$$V_{0t}(x_t) \equiv P_t \cdot K_t - AVC_t \cdot K_t - F_t + \beta EV_t(x_t, \theta)$$

$$V_{1t}(x_t) \equiv -F_t + \beta EV_t(x_t, \theta).$$

We have the following closed-form choice probabilities by assuming that $q(\varepsilon \mid x)$ is given by a multivariate extreme value distribution:

$$\begin{aligned} p(\text{operate} \mid x_t, \theta) &= \exp(V_{0t}(x_t))/H_t \\ p(\text{hold idle} \mid x_t, \theta) &= \exp(V_{1t}(x_t))/H_t \\ p(\text{retire} \mid x_t, \theta) &= \exp(SV)/H_t, \end{aligned} \tag{14}$$

where $H_t \equiv \exp(V_{0t}) + \exp(V_{1t}) + \exp(SV)$ and $EV_t(x_t, \theta)$ is given by the unique solution to the functional equation:

$$EV_t(x_t, \theta) = \int \log\left(e^{V_{0t+1}(x_{t+1})} + e^{V_{1t+1}(x_{t+1})} + e^{SV}\right) p(x_{t+1} \mid x_t, i_t)\, dx_{t+1}. \tag{15}$$

In (15), $p(x_{t+1} \mid x_t, i_t)$ is the law of motion of the observed state vector x, where $x_t = (K_t, A_t, P_t, PE_t, PF_t, W_t)$. Its individual components are discussed below.

For a given kiln, K_t—the capacity—does not change over time.

The age of a kiln, A_t, evolves in a deterministic manner:

$$p(A_{t+1} \mid A_t, i_t) = \begin{cases} 1 & \text{for } A_{t+1} = A_t + 1 \text{ if } i_t \neq \text{retire} \\ 1 & \text{for } A_{t+1} = A_t = \text{the absorbing state if } i_t = \text{retire or } A_t = 80 \text{ years} \\ 0 & \text{Otherwise.} \end{cases}$$

It would be ideal to estimate the price process $\{P_t, PE_t, PF_t, W_t\}$ jointly and without making any distributional assumptions. However, as explained below, we allow for very limited jointness due to insufficient variation in our price data.[18]

Since electricity and fuel markets are not specific to the cement industry their (real) prices are assumed to be determined in their respective markets. Hence the transitions of their prices from t to $t+1$ are assumed to follow a univariate first-order Markov process. These are denoted by $p(PE_{t+1} \mid PE_t)$ and $p(PF_{t+1} \mid PF_t)$ respectively.

Labour is highly unionized in the cement industry and hence the wage rate (W) and the cement price (P) are likely to be jointly determined. We assume that they follow a joint first-order Markov process, denoted by $p(P_{t+1}, W_{t+1} \mid P_t, W_t)$.

18. As noted in Section 1 the wage rate varies over time but not across states, while the other prices vary over time and across states but not much across the locations of kilns.

Hence in (15), *for each given value of capacity,* we have

$$p(x_{t+1}|x_t, i_t) = p(A_{t+1}|A_t, i_t) \cdot p(PE_{t+1}|PE_t) \cdot p(PF_{t+1}|PF_t) \cdot p(P_{t+1}, W_{t+1}|P_t, W_t). \tag{16}$$

The actual estimation task begins with (16). Notice that $p(A_{t+1}|A_t, i_t)$ does not contain any unknowns. The rest of the transition probabilities are non-parametrically estimated using the corresponding sample relative frequencies for the grid values of x_t at which $EV(x_t, \theta)$ was evaluated.[19]

Given the estimated $p(x_{t+1}|x_t, i_t)$, the estimates of h, d and SV are obtained by a maximum likelihood algorithm that consists of two loops, an outer loop and an inner loop.[20,21,22]

It should be emphasized here that while in typical discrete-choice models the parameters are identifiable only up to a positive scale, in this model we are able to obtain individual estimates by using the estimated AVC function. This can be seen by writing the instantaneous profit function up to a positive scale, say c, as

$$u(\cdot) = \begin{cases} c\cdot(P - A\hat{V}C)K - c\cdot h\cdot K - c\cdot d\cdot A + \varepsilon_0 & \text{if } i = \text{operate} \\ -c\cdot h\cdot K - c\cdot d\cdot A + \varepsilon_1 & \text{if } i = \text{hold idle} \\ c\cdot SV + \varepsilon_2 & \text{if } i = \text{retire} \end{cases} \tag{11$'$}$$

where $A\hat{V}C$ is already known and used as data.

4.3. *Estimates of the fixed cost parameters* (h *and* d) *and the scrap value* (SV), *and the overall fit of the model*

The procedure outlined above generated the estimates of c, $c\cdot h$, $c\cdot d$ and $c\cdot SV$. Then c was eliminated to obtain (individual) estimates of h, d and SV. These estimates are reported in Tables V and VI respectively.

In interpreting the significance of the estimates in Table V, it should be recalled that the standard errors are underestimated, because these standard errors are obtained by assuming that the estimated AVC is the true function. Hence the significance of the estimates is evaluated accordingly.

The parameter c which indicates the significance of the estimated AVC function is positive and highly significant. Its t-ratio indicates that it would continue to be significant (at 5%) even if the true variance is 150 times higher than the one reported.

19. Recently, the common lack of a closed-form value function for dynamic optimization problems has spawned many such grid approximation numerical solution routines. For instance, see the series of eleven articles in the *Journal of Business & Economic Statistics*, January 1990. In our case, for each value of K in the data, the grid dimension of x_t used was 11520, which included 80 possible values for A_t and 144 values for (P_t, PE_t, PF_t, W_t).

20. For given values of the parameters generated by the outer loop, at each iteration the inner loop computes the value function and its derivatives by backward induction. Using the value function and its derivatives the outer loop computes the likelihood function and its derivatives and generates new estimates of parameters to be given to the inner loop for the next iteration.

21. A general problem with estimating dynamic models such as ours is that since the likelihood function in the sample is not globally concave for a positive value of the discount rate β, convergence may be quite difficult to obtain. The trick is to start at $\beta = 0$, maximize the likelihood (which is globally concave at $\beta = 0$) and use the converged values as initial values for maximizing the likelihood for slightly increased value of β from zero and so on. This is closely related to the homotopy method described in Zangwill and Garcia (1981). This method provides the maximum likelihood estimate of β by grid search but does not provide its standard errors, which is not a significant loss since estimating β is not our focus here.

The maximized likelihood value was obtained at $\beta = 0{\cdot}9$ and the likelihood ratio statistic for the null $\beta = 0$ was equal to 19·8 and hence the null was rejected at 0·5% level of significance. This indicates that the dynamic model, $\beta > 0$, explains the data better than a static model, $\beta = 0$.

22. Again, based on the work of Ahn and Manski, we expect that our estimates to converge at rate $\sqrt{N}$.

TABLE V

Estimation of the fixed cost parameters and the scrap value

	c	$c \cdot h$	$c \cdot d$	$c \cdot SV$
Estimate	33·5966	0·4292	0·0009	3·2018
Standard error	(1·6640)	(0·0689)	(0·0026)	(1·6221)
T-ratio	20·1903	6·2293	0·3462	1·9739
		$\chi^2 = 1{\cdot}1214$		

TABLE VI

Rescaled Estimates

	h	d	SV
Estimate	0·0128	0·00003	0·0953

With regard to the holding cost parameter, $c \cdot h$, the estimate is positive and would continue to be significant (at 5%) even if the true variance is 14 times higher than the one reported. The value of c and $c \cdot h$ imply that h—the holding cost—equals 0·0128 real dollars per unit of capacity (see Table VI). Since the minimum capacity of a kiln in the data is 42,000 tons/year and the maximum is 567,000 tons/year, it implies the fixed cost $(K \cdot h)$ to be between 538 and 7258 real dollars per year (with 1967 as the base year). In current dollars, the fixed costs would of course be much higher.

The increase in fixed cost, d, as a kiln get older is surprisingly not significant (at 10%). It may indicate that with regular maintenance of the kilns, as indicated by the significance of the parameter h, age does not have a significant effect on the fixed cost.

Lastly, the net scrap value, SV, though significant (at 5%) in Table V, would not continue to be so if the true variance is 1·4 times the one reported. Besides, as seen in Table V, the value of SV (equal to 0·0953 real dollars per year) is not economically significant. Note that SV defines the scrap value, net of the cost of removal of the scrapped kiln which may be substantial. Since secondary markets for kilns do not exist it is not surprising that the cost of removal matches up to the scrap value, as implied by the insignificance of SV.

Now using all the available estimates from steps 1 and 2 we can compare the model prediction to the data. One way is to compare the observed proportion of kilns operated, held idle and retired in each year to the respective mean probability (over kilns) predicted by the model for that year. These comparisons are given Figures 2–4. It is seen in Figures 2 and 3 that the model tends to underpredict utilization ("operated") and overpredict idleness, but, except during 1975–1978, the predicted trends are close to those observed in the data. In Figure 4 we see that the model tracks retirement quite well until 1977 but fails to explain the "blips" in retirement in 1978 and 1980. Although the real electricity price rose steadily during 1978–80, it is unlikely to explain the blips since cement production is not electricity-intensive. The model accordingly does not predict the blips. Given that cement production is fuel-intensive, a more plausible explanation may lie in the lags in learning about fuel prices by cement firms. The initial jump in the fuel price in 1974 may not have been regarded as a permanent phenomenon. But as the real fuel price stayed high until 1978, the industry may then have updated its long-run expectation and thus reacted to such expectation. Since our empirical model does not capture well

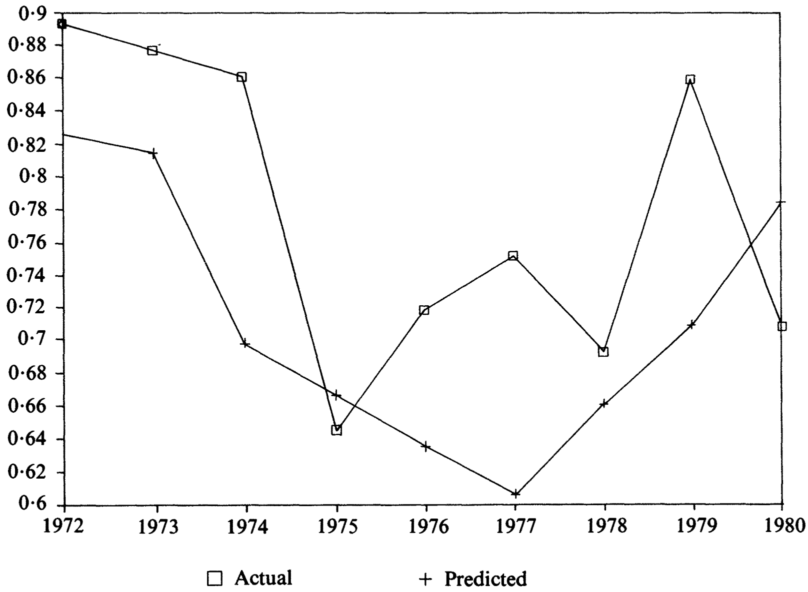

FIGURE 2

Probability of operating: actual and predicted

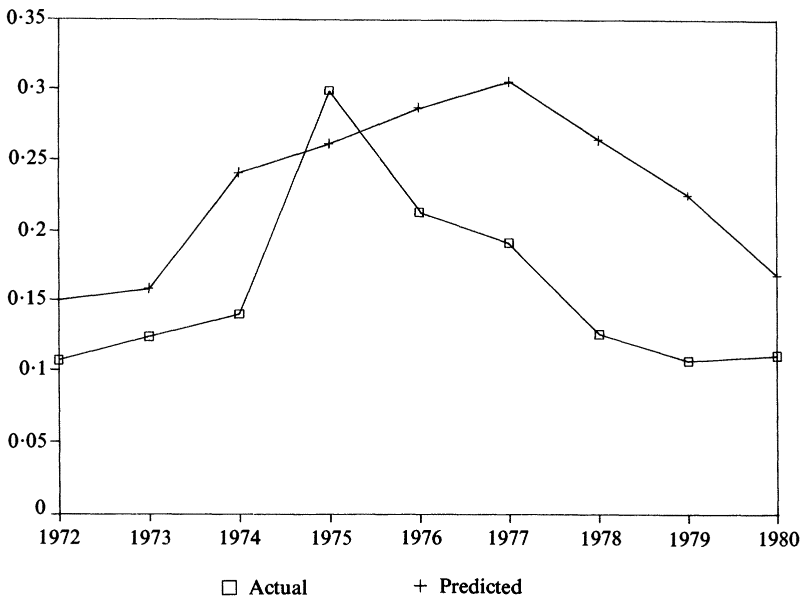

FIGURE 3

Probability of holding idle: actual and predicted

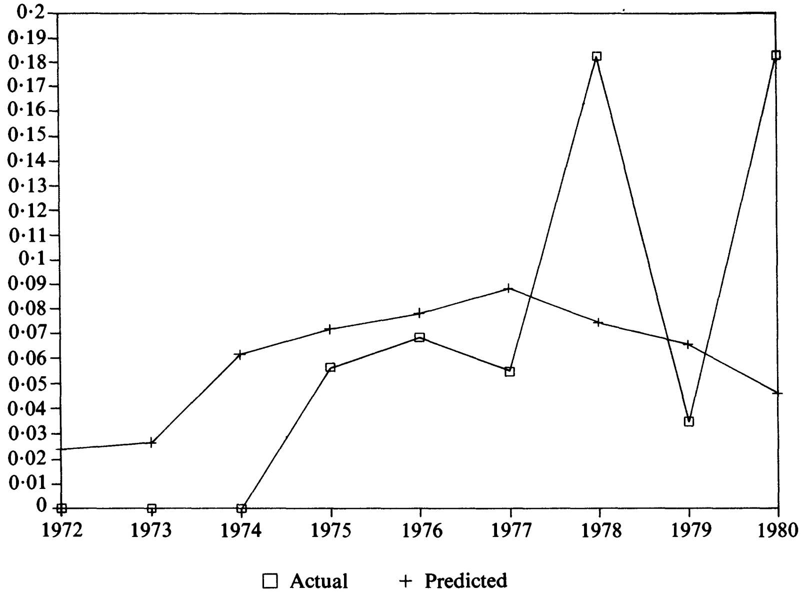

FIGURE 4

Probability of retiring: actual and predicted

the effects of a lag in prices of longer than one period it is not able to explain the blips in retirement.

These are all qualitative statements on the fitness of the model. The overall statistical fit can be evaluated by the χ^2 statistic, obtained by the formula (see Rao (1973, Chapter 6)):

$$\sum_i \sum_t (p_{dit} - p_{mit})^2 / p_{mit},$$ [23]

where i = operate, hold idle or retire, $t = 1972, 1973, \ldots, 1980$, p_{dit} = observed proportion of kilns with the i-th choice in the data (given in Table III), p_{mit} = mean (over kilns) of the i-th choice probability in year t predicted by our model.

Its value equals 1·1214. In our model, the χ^2 statistic has 17 degrees of freedom (the number of cells (27) minus the number of estimated parameters including the parameters of the AVC function (9) minus 1). Hence the null hypothesis that our model is true cannot be rejected at 0·5% level of significance. Thus it may be concluded that statistically our model describes the data well.

We now use the structural estimates obtained to simulate the response of the probabilities of operating, idling and retiring a kiln to changes in output and input prices.

5. SENSITIVITY ANALYSIS

The model permits a number of interesting simulations with respect to changes in prices, age and size (capacity) of the kiln at various possible combinations. For the sake of brevity, only a few simulation results are presented. It may be remarked here that the

23. Note that even though p_{mit} depends partly on semi-parametric estimates, this expression is still expected to have a chi-square distribution, because, as shown in Rao, it has a chi-square distribution when p_{mit} is nonparametrically estimated and also when p_{mit} is parametrically estimated. Hence this should hold in the intermediate case of semi-parametric estimation.

qualitative impacts of a change in size on the decisions to operate, hold idle or retire are not obvious from the theoretical model. Nor are the qualitative impacts of changes in age or prices on the decisions to hold idle and retire as they are similar alternatives to operating the capital in the current period. In general however, the qualitative as well as the quantitative impacts are of interest here.

Over the sample period the variations in cement, fuel and electricity prices much exceed the variation in the wage rate. Hence the model is simulated with respect to these

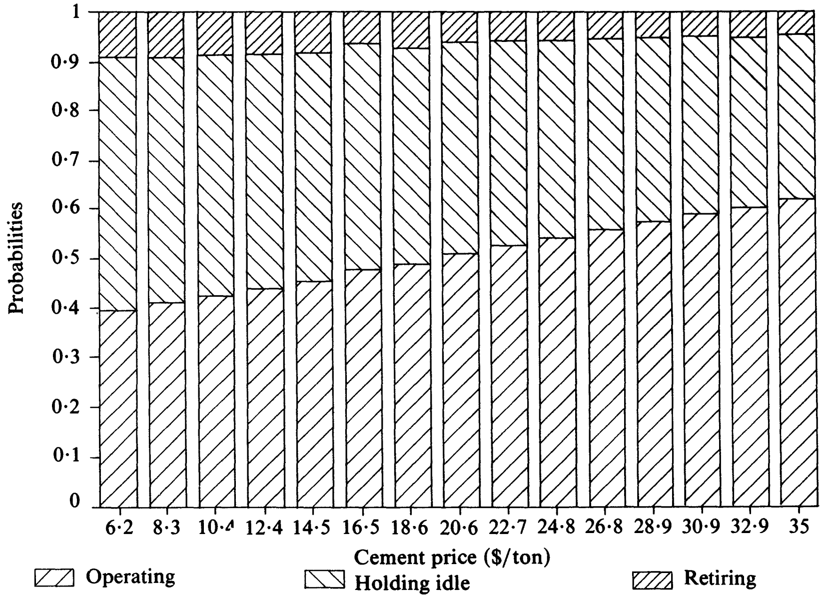

FIGURE 5

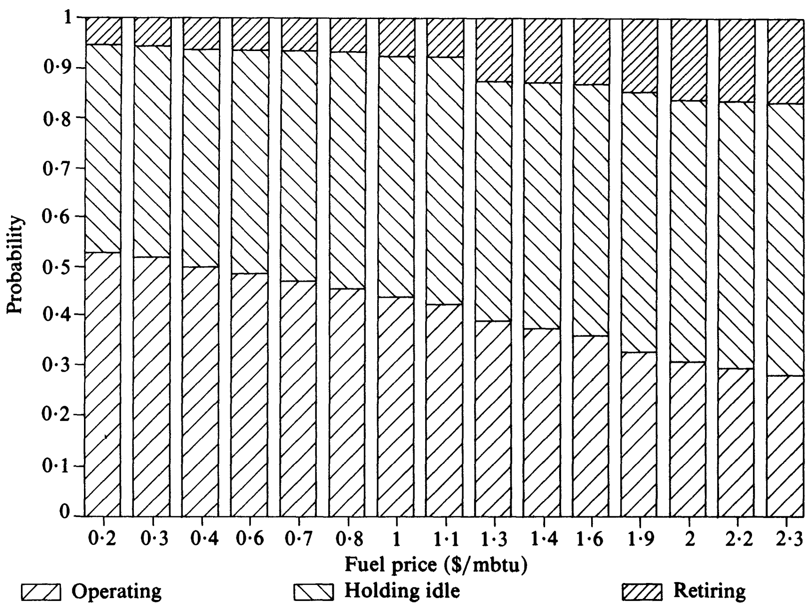

FIGURE 6

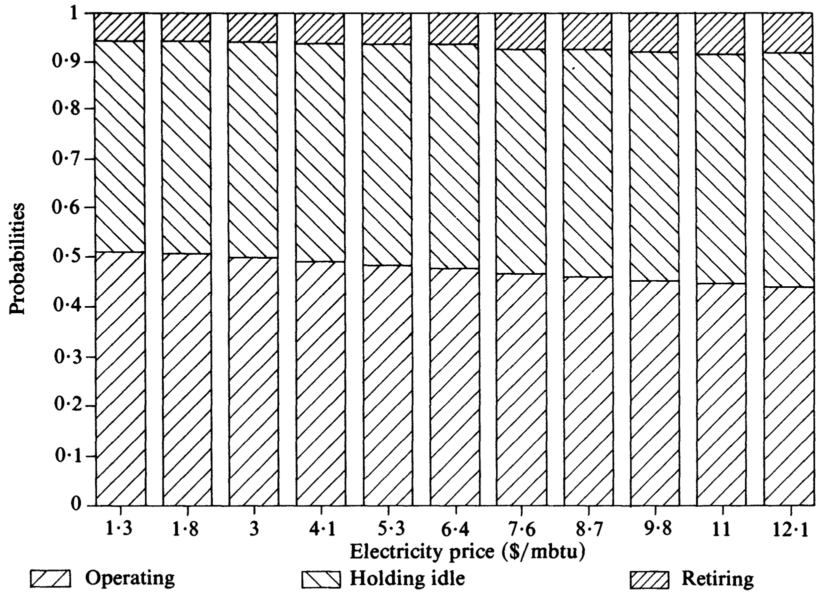

FIGURE 7

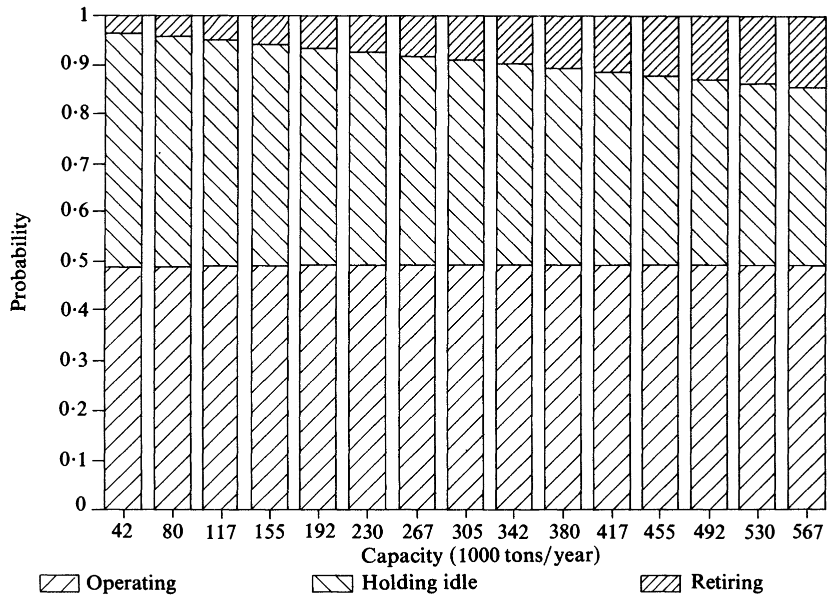

FIGURE 8

prices individually while keeping other prices, the wage rate and age and size of the kiln fixed at their respective sample means.

In Figures 5-9 we graph how the probabilities of operating, holding idle and retiring a cement kiln change as the cement price, fuel price, electricity price, kiln size and kiln age vary. In Figure 5, as the (real) cement price varies through the range of \$6·25/ton to \$35/ton, the probability of operating increases from 39·4% to 61·8%, that of holding

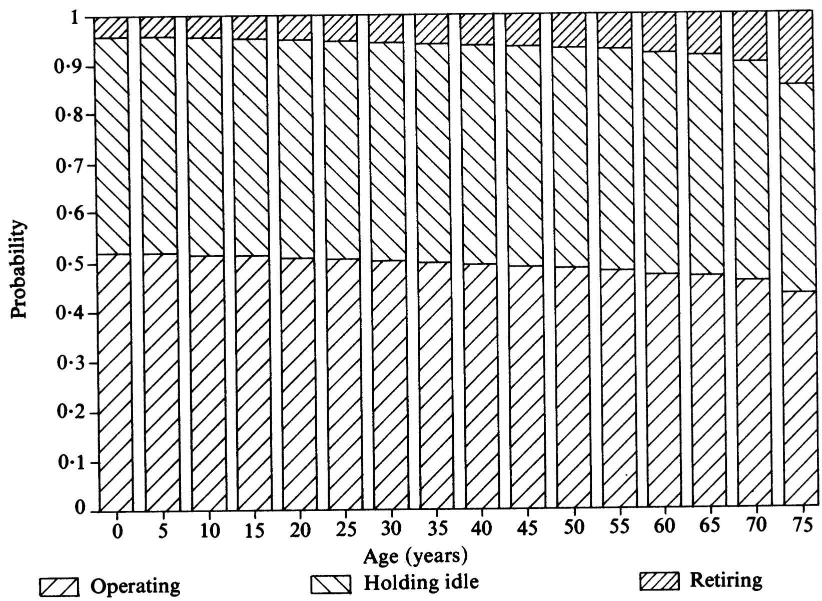

FIGURE 9

idle decreases from 51·7% to 33·8% and that of retiring decreases from 9% to 4%; thus changes in the cement price mostly affect the choice between operating and holding idle.

In Figure 6, as the (real) fuel price varies through the range \$0·25/mbtu to \$2·3/mbtu, the probability of operating decreases from 52·8% to 28·2%, that of holding idle increases from 42% to 55% and that of retiring increases from 5% to 16·8%. Thus each decision is quite sensitive to fuel price changes, which is consistent with the observed behaviour reported in Table III.

In Figure 7 the (real) electricity price ranges from \$1·27/mbtu to \$12·14/mbtu. Correspondingly, the probability of operating decreases from 51% to 44%, that of holding idle increases from 43·4% to 48% and that of retiring increases from 5·6% to 7·8%. Hence the effects are the largest on the decision to operate and smallest on the decision to retire.

Comparing the sensitivity with respect to the three prices, it is seen that the decisions are most sensitive to fuel price changes, next to changes in the cement price and least to changes in the electricity price. It is also found (but not shown) that the impacts of these price changes are more pronounced for bigger and/or older kilns.

Figure 8 shows the marginal impact of kiln size (while the kiln age and all the prices are kept at their sample means). The probability of operating hardly changes with size. The probability of retiring increases and almost "crowds out" the probability of holding idle as the size increases.

Finally, in Figure 9 we see that the age of a kiln has relatively little impact on the decision to hold idle. But it has significant impact on the decisions to operate and retire.

6. CONCLUDING REMARKS

This paper has analyzed a micro-econometric model of capital utilization and retirement choice. At any given point in time, a firm has a discrete decision problem with respect to an existing piece of capital stock: operate, hold idle or retire. The paper has obtained structural estimates of the parameters of such discrete decision rules in the context of cement kilns.

A two-step estimation procedure was used. Estimation of some of the relevant parameters obtained previously constitutes step 1. Conditional on these estimates, the estimation of the remaining parameters (step 2) is done in this paper by solving a stochastic dynamic programming model. A method similar to Rust (1988) was used, which solved the dynamic programming model numerically at each iteration of the maximization of a likelihood function. In contrast to the discrete-choice dynamic programming models in the existing literature that obtain estimates unique only up to a positive scale, our two-step procedure enabled us to obtain individual parameters of the decision rules, though the standard errors of the step 2 estimates are biased.

Given the estimates, the probabilities of operating, holding idle and retiring a cement kiln that are predicted by our model were computed. These predicted probabilities compare well with the observed proportions in the data in accordance with the chi-square test. Finally, various simulations were conducted to determine the nature of variations in the predicted probabilities due to changes in cement price, fuel price, kiln size and kiln age.

Although the estimates per se are specific to the cement industry, the general methodology of the paper that deals with generic features such as discreteness or lumpiness of choices toward existing capital at the micro level, size and age structure of capital, stochastic processes of exogenous variables in the decision making, numerical solution of the underlying dynamic programming model etc. transcends the cement industry and can be used, with suitable modifications, in other micro-econometric studies of capital management.

Another feature of the model that goes beyond the particular example of the cement industry is the conceptual distinction between capital idling and capital retiring that can imply persistence of random shocks or "hysteresis". External shocks at a point in time (policy-induced or not) may induce a firm to retire *older* capital and shocks of the opposite kind in a later period may induce a firm to buy *newer* capital. Hence there may be a permanent change in the capital structure of a firm although the external environment may be the same as in an earlier period. Such effects could have potential implications for policy changes or business-cycle analysis.

Acknowledgement. The first version of this paper was presented at the Econometric Society Winter Meetings in December 1988 at New York under a slightly different title. I am thankful to the Amdahl Corporation and Mr. Sharad Gavali for running my programs on the AMDAHL 1200 VECTOR PROCESSOR. Discussions with and comments from Satya Das, Jeffrey Dubin, Charles Manski, Ariel Pakes and John Rust are appreciated. I am also thankful to two referees of this journal for many constructive comments. Remaining errors are mine.

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