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The Flat Rental Puzzle

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Why is the price of renting an automobile "flat" as a function of its age or odometer value? Specifically, why is it that car rental companies do not offer customers the option of renting older cars at a discount, instead of offering only relatively new cars at full price? We also tackle a related puzzle: why do car rental companies trade-in their vehicles so early? Most US companies purchase brand new rental cars and replace them after 2 years or when their odometer exceeds 34,000 km. That is a very costly strategy due to the well-known by rapid depreciation in used car prices. We show that in a competitive rental market, prices are a declining function of odometer and cars are rented over their full economic lifespan. Our solution to these puzzles is that actual rental markets are not fully competitive and firms may be behaving *suboptimally*. We provide a case study of a large car rental company that provided us access to its operating data. We develop a model of the company's operations that predicts that the company can significantly increase its profits by keeping its rental cars twice as long as it currently does, discounting the rental prices of older vehicles to induce its customers to rent them. The company undertook an experiment to test our model's predictions. We report initial findings from this experiment which involved over 4500 rentals of over 500 cars in 4 locations over a 5-month period. The results are consistent with the predictions of our model, and suggest that a properly chosen declining rental price function can *increase* overall revenues. Profits also increase significantly, since doubling the holding period of rental cars cuts discounted replacement costs by nearly 40%.

1. INTRODUCTION

Why is the price of renting an automobile "flat" as a function of its age or the odometer when prices of automobiles sold in the used car market decline sharply as a function of age and the odometer? A closely related puzzle is to explain why car rental companies replace their rental cars so early: this greatly increases their operating costs due to rapid depreciation in used car prices. The rapid depreciation in used car prices is well known and the reasons for it are reasonably well understood: it can be due to "lemons problems" (Akerlof, 1970), rapidly increasing maintenance costs, or strong consumer preferences for newer vehicles over older ones (Rust, 1985). However, we are unaware of any previous study that notices the apparent inconsistency between rapid price depreciation in the used car market and the prevalence of flat price schedules in the rental car market, or studies that question the wisdom of replacing rental cars so early.

We show that economic theory predicts that competitive rental prices should decline with age or odometer, and that rental cars should be held and rented by car rental companies for their full economic lifespan. Therefore, it is puzzling why observed behaviour is so much at odds with this theoretical prediction. Our solution to the puzzle is that the competitive model may not be a good approximation to actual rental car markets. Car rental companies may have significant market power (and thus, control over their prices), and may be behaving suboptimally.

We present a detailed case study of the operations of a particular highly profitable car rental company that allowed us to analyse their contract and operating data. We show that its rental prices are indeed flat. We develop a model of the company's operations based on an econometric model of Cho and Rust (2008) (CR (2008) hereafter). The model provides a good approximation to the overall operations and profitability under the company's existing pricing and replacement policy. We use this model and dynamic programming to calculate optimal replacement policies and discounted profits under counterfactual scenarios, including the policy of keeping cars longer than the company currently does. We assume that the company adopts odometer-based discounts of the rental prices of older rental cars to induce customers to rent them, thus avoiding the loss of "customer goodwill" that might occur if the company rented older cars at the same price as new ones.

We show that even under conservative assumptions about maintenance costs and the magnitude of the discounts necessary to induce consumers to rent older cars, the optimal replacement policy involves retaining rental cars for roughly twice as long as the company currently does. Although gains vary by vehicle type, the model predicts that the company's expected discounted profits could be *at least* 6% to over 140% higher (depending on vehicle type) under an alternative operating strategy where vehicles are retained longer and rental prices of older vehicles are discounted to induce customers to rent them. Our alternative strategy is based on conservative assumptions and is not fully optimal itself, so our estimated profit gains constitute *lower bounds* on the amount profits would increase under a fully optimal operating strategy, the calculation of which requires more information about customer preferences than we currently have available.

Our findings convinced the company to undertake an experiment to verify whether this alternative operating strategy is indeed more profitable than what it currently does. The company's main concern is that discounting rental prices of older cars could cause a majority of their customers to substitute older rental cars at discounted rates over rentals of newer car at full price, potentially reducing overall rental revenue. A related concern is that renting cars that are too old could result in a loss of customer goodwill, and/or harm the company's reputation as a high-quality/high-price leader. We report initial findings from this experiment which involved over 4500 rentals of nearly 500 cars in 4 locations over a 5-month period. The results are consistent with the predictions of our model, and demonstrate that a properly chosen declining rental price function can actually *increase* overall revenues.

The experiment revealed that some car rental customers are very responsive to discounts, and that only relatively small discounts are necessary to induce these customers to rent older cars in the company's fleet. Not all rental customers were offered the same discounts: they could range from as much as 40% for customers who had no additional sources of discounts (e.g. using frequent flyer miles, or being a member of the company's "loyal customer club") to as small as a 10% discount for individuals who were eligible for one or more of these other discounts. The average decrease in the rental prices of cars over 2 years old was 13%. This discount caused total rental revenue for the discounted older cars to *quadruple,* more than offsetting a 16% decline in revenues from rentals of new cars (i.e. cars less than 2 years old).

There might not be a puzzle if the rental market as a whole offers consumers a declining rental price function, even if individual firms offer only a limited range of vintages and adopt flat rental price schedules. For example, although most car rental companies focus on renting cars that are very close to being brand new, there is a US-based car rental company, *Rent-A-Wreck,* that specializes in renting older, higher-mileage used cars at discounted rates. Thus, if a customer wants a high-priced new rental car they can go to companies such as Hertz, but if they are willing to rent a less expensive older car, they can go to *Rent-A-Wreck*.

However *Rent-A-Wreck* is only the ninth-largest US car rental company, and thus, has a relatively small market share relative to the top four car rental companies (Hertz, National, Budget, and Avis), which specialize in purchasing and renting new cars, and control nearly two-thirds of the US rental car market. This is particularly true of the lucrative "airport markets" (constituting about one-third of the overall US car rental revenues), where *Rent-A-Wreck's* presence is almost non-existent.

Thus, we do not see the existence of *Rent-A-Wreck* as providing a solution to the flat rental puzzle. Even though it is commendable that this company provides a unique service that the top four car rental companies do not provide consumers, *Rent-A-Wreck* is still regarded as serving a relatively small niche market. The same puzzle remains: is it really more profitable for the top four firms to restrict their portfolios to brand new cars, and why are they unwilling to offer their customers the option to rent older cars at a discount?

The fact that most cars owned by individuals are significantly older than the cars owned by car rental companies is another aspect of the puzzle we are attempting to address: one might expect that the age or odometer distribution for rental cars would be roughly the same as the corresponding distributions for cars owned by individuals. However, the vast majority of cars rented under short-term rental contracts by the major car rental companies are under 2 years old. In effect, car rental companies follow a vehicle replacement strategy that most consumers find far too costly to do themselves, i.e. to buy brand new cars and hold them only for one or two years before trading them in.

The company we are studying is already highly profitable: the average pre-tax internal rate of return on the cars in its rental fleet that we analysed was 50%. We are not suggesting that the primary explanation for these high rates of return is that this company keeps its cars longer than other leading US car rental companies: other factors, including market power, may be more important reasons why it is so extraordinarily profitable. However, the improvements in profitability that we predict from relatively small deviations in this company's operating policy may seem surprising: how could such a successful company overlook such seemingly obvious opportunities to make even higher profits?

Of course there is an alternative solution to the "puzzle" that must be seriously considered: perhaps our model and our thinking about this problem is wrong, and that the gains our model predicts would not be realized in practice. We certainly acknowledge that our model is indeed "wrong" in the sense that it is simplified in several key respects. In particular, while we have a huge amount of data on individual contracts and track the rentals of individual cars in the company's fleet, we do not have adequate data on its *customers*. Thus, we know relatively little about their other options and their preferences and demands for rental cars.

We deal with our lack of information about customers in the following ways. First, we do not claim that the counterfactual operating strategy our model predicts is fully optimal. We only claim that this strategy can increase profits relative to the company's existing operating strategy. Second, given our lack of knowledge of "the demand function" that this firm faces, we restrict attention to relatively minor counterfactual changes to the firm's price schedule by assuming that discount schemes can be designed that keep the company's customers approximately indifferent (at least on average) between renting new cars at full price versus older cars at a discount. If this is true, then it is reasonable to assume that there will not be significant changes in the firm's overall scale of operations, or in the stochastic pattern of operations (including durations in the lot and in rental spells) for the vehicles in the company's fleet over their lifetimes. The econometric model we use does not account for the effects of large changes in rental prices on the overall demand and pattern of utilization of vehicles.

We acknowledge that the assumed *invariance* in stochastic utilization patterns of vehicles with respect to modest changes in the rental price structure may not hold in practice, and it is a key limitation of our approach. Since we have no data on past rental price changes, particularly in the dimension of discounts for older cars, we are limited in our ability to model how customers would change their rental behaviour under the counterfactuals we consider. In addition, given our incomplete understanding of the company's customers and its overall operations, we may have overlooked some crucial aspects of the situation that could invalidate our predictions. However, our assumptions and predictions are something that can be tested, so long as the company is sufficiently convinced by our reasoning to undertake a field experiment.

In fact, the company did previously experiment with discounting the rental prices of its older vehicles, but it discontinued the experiment because *too many* customers were choosing older vehicles. The experiment involved a 20% discount off the daily rental price for any vehicle over 2 years old. While we do not have any data from this previous experiment (and thus, cannot determine if it raised or lowered overall rental revenue), if total revenue had declined, we would simply view this as evidence that the 20% discount was more than necessary to induce customers to rent older vehicles. With lower discounts, the company may still be able to increase revenues from rentals of older cars, without losing significant revenues from rentals of its newer vehicles.

Thus, we argue that a moderate age or odometer-based discounting strategy could lead to a win–win situation: it could enable the company to increase its profits while at the same time providing a wider range of choice and benefits to its customers. Customers would benefit since they can always choose to rent a new car at the full rental rate. However many customers may prefer to rent an older vehicle at a reduced rental rate, and these customers will be strictly better off under this alternative rental rate structure. The company would benefit from being able to keep vehicles in its fleet longer, and thus, earn more rental revenue over a longer holding period that would help to amortize the high trading costs it incurs due to the rapid depreciation in new car prices. Thus, by appropriately discounting its rental prices, the company should be able to significantly increase its profits without risking its reputation and customer goodwill.

2. THE FLAT RENTAL PUZZLE: THEORY

This section uses a simple model of competitive car rental markets of Rust (1985) to argue that competitive rental prices should be a declining function of a vehicle's age or odometer value. In other words, this theory suggests that rental car prices should *not* be flat. The intuition for this result is quite simple: it is because of a combination of maintenance costs that increase with the age or odometer of the vehicle and consumer preferences for newer cars. In equilibrium, both rental prices and secondary market prices for autos must decline in order to induce consumers to buy or rent older cars instead of newer ones.

There is another possible explanation for declining secondary market and rental prices—the well known "lemons problem" (Akerlof, 1970). However, while informational asymmetries can lead to complete market failure, they are obviously not severe enough to kill off the secondary market for used cars. With improved technology including electronic sensors and warning systems and computerized engine monitoring and diagnostics, modern vehicles are generally better maintained and are easier to monitor than vehicles at the time Akerlof published his classic article. In fact, the empirical evidence on the severity of the lemons problem is quite mixed, and even studies that do find evidence supportive of lemons problems (e.g. Genesove, 1993; Emons and Sheldon, 2009) find only weak effects: "Although present, the lemons problem does not appear to be widespread, however" (p. 2884). Other recent work by Engers, Hartmann and Stern (2009) and Adams, Hoskens and Newberry (2006) do not find evidence supportive of a lemons problem, “Overall, for used Chevrolet Corvettes sold on eBay, there is little empirical support for the hypothesis presented in Akerlof (1970)”. Finally, theoretical work dating back to Kim (1985) shows that average quality of non-traded used cars could be either higher or lower than traded cars. Furthermore, common sense suggests that reputational effects would provide strong motivation for car rental companies to adequately maintain their vehicles and avoid renting lemons to their customers, similar to the way reputation works to discourage auto dealers from selling lemons to their customers (Genesove, 1993).

For all these reasons, we do not view the lemons problem as the most promising avenue to explain the flat rental puzzle. However, if rental companies do succeed in maintaining their rental cars in top condition, and if consumers are nearly indifferent in choosing between a new car and a clean, well maintained used car, and if older cars are no more expensive to maintain than newer ones, then Rust’s (1985) theory does imply flat rental prices. However, even in this scenario, there is still a puzzle as to why car rental companies replace their vehicles as early as they do. In Section 3, we provide evidence that *maintenance costs are flat*—at least over the range of odometer values observed in the cars in our dataset. If maintenance costs are literally flat, and if consumers are literally indifferent between old and new cars (assuming old cars that are well maintained can be as safe and reliable, and perform as well as new cars), then the theory predicts that car rental companies should *never* replace their rental vehicles. Instead, similar to the London taxi cabs, the optimal replacement policy is to maintain and keep each rental vehicle *forever.*

This *reductio ad absurdum* should convince the reader that there really is a puzzle here. It seems unlikely that consumers are indifferent between new cars and very old cars, or that maintenance costs for new cars are the same as for cars that are very old. Instead, what we find is that it is hard to detect vehicle aging effects among cars that are sufficiently new and well maintained. Owing to the frequent cleaning and regular maintenance the car rental company performs on its vehicles, a somewhat older car is likely to be viewed as a close substitute to a brand new car of the same make and model provided it is not “too old”. However this leads to yet another puzzle, since, if consumers really are approximately indifferent between new cars and relatively new and well maintained used cars, then why is this firm penalized so heavily (the low sale values it receives from sales of its used vehicles, as documented in Section 3) when it tries to sell its cars in the used car market? One possible explanation is that buyers in the used car market anticipate that this company will be trying to sell them its lemons, and they are willing to pay far less as a result. However, if the cars this company is trying to sell are not really lemons, but are only *perceived* to be lemons by buyers in the used car market, we still have the puzzle of why this company insists on selling its cars so soon to suspicious buyers instead of continuing to rent them to their trusting rental customers.

Now consider a simple model of used car markets and car rental intermediaries—Rust’s (1985) durable asset pricing model. This is an idealized market with complete information and zero transactions costs. The theory also covers the case where there are heterogeneous consumers (and thus, strict gains from trade from the operation of a secondary market), but as Rust (1985) shows, such a market is observationally equivalent to a homogeneous consumer market when an appropriate “representative consumer” is chosen. Konishi and Sandfort (2002) have extended this theory to account for transactions costs. Rental prices are non-flat even in heterogeneous agent markets with positive transactions costs. However, the main conclusions are easiest to illustrate in the zero-cost, homogeneous consumer case.

There is only one make/model of car, and the only feature that distinguishes cars is their odometer value x, which we assume is readily observable, so there is no “lemons problem” in this model. Consumers prefer newer cars to older ones, and there is an infinitely elastic supply of new cars at price $\overline{P}$ and an infinitely elastic demand for used cars at a scrap price $\underline{P}$. The per-period mileage travelled by each car is represented by a Markov process with transition density $f(x'|x)$, where x' is the odometer reading next period given that the odometer is x this period. There are a continuum of cars and a continuum of homogeneous infinitely lived consumers, each of whom has an inelastic demand for exactly one automobile, with a quasi-linear utility function over income I and the odometer value (newness) of the car given by $U(I, x) = I - q(x)$, where $q(x)$ is an increasing function of x satisfying the normalization $q(0) = 0$. Consumers have a common discount factor $\beta \in (0, 1)$. The per-period expected cost of maintaining a car with odometer value x is $m(x)$.

It is not difficult to prove that with zero transactions costs, a vehicle owner would want to trade their vehicle every period for a preferred vehicle z. The per-period expected cost to the consumer for holding vehicle z is the expected maintenance cost $m(z)$, plus the dollar equivalent utility cost of holding a used car z instead of a new car, $q(z)$, plus the expected depreciation in the vehicle value, $P(z) - \beta EP(z)$, where $P(z)$ is the price of a car with odometer value z in the used car market, and $EP(z)$ is the expected sale price next period,

$$EP(z) = \int P(z')f(z'|z)dz'. \tag{1}$$

If all consumers are homogeneous, then in order to induce them to hold cars of every possible odometer value z, they must be indifferent between all available odometer values z, so the following equation must be satisfied:

$$M(z) + P(z) - \beta EP(z) = K, \tag{2}$$

for some constant K, where $M(x) = m(x) + q(x)$ is the sum of the per-period maintenance cost and the disutility opportunity cost of owning a car with odometer value x. Since the supply of new cars is infinitely elastic at price $\overline{P}$ we must have $P(0) = \overline{P}$, and since there is an infinitely elastic demand for scrapped cars at price $\underline{P}$, there is a scrapping threshold γ such that $P(x) = \underline{P}$ for $x \geq \gamma$. These conditions can be condensed into a single functional equation that the equilibrium price function must satisfy:

$$P(x) = \max\left[\underline{P}, \overline{P} + M(0) - \beta EP(0) - M(x) + \beta EP(x)\right]. \tag{3}$$

It is easy to show that this equilibrium defines P as the fixed point of a contraction mapping, and hence, there exists a unique equilibrium price function. Under fairly general conditions, this function will be a downward sloping function of x, provided that M is an increasing function of x. Figure 1 provides examples of the equilibrium price functions when $\beta = 0.99$, $\overline{P} = 50{,}000$, $\underline{P} = 5000$, and $f(x'|x) = \lambda \exp\{-\lambda(x' - x)\}$ for $x' \geq x$, and $f(x'|x) = 0$ otherwise, where $\lambda = 0.1$. The left hand panel of Figure 1 plots equilibrium price functions for two different $M(x)$ functions, $M(x) = 40x$ and $M(x) = (3000\sqrt{x})(x < 15) + (200\sqrt{x})(x \geq 15)$ (the latter $M(x)$ has a kink at $x = 15$, and the corresponding $P(x)$ also has a kink at $x = 15$, as can be seen from the left hand panel of Figure 1). Depending on assumptions we make about maintenance costs and consumer utility for new versus used cars, the model is capable of generating the rapid early depreciation in car prices that we observe almost universally in used car markets around the world (specific evidence for the company we study in this article will be presented in the next section).

Now we add rental car intermediaries to this market. Let $R(x)$ be the per-period rental price of a car with odometer value x. We assume that the rental market is competitive, so that $R(x)$ is determined by equilibrium considerations and is beyond the control of the rental company. The rental company's main decision is when to replace the current rental car with a new one.

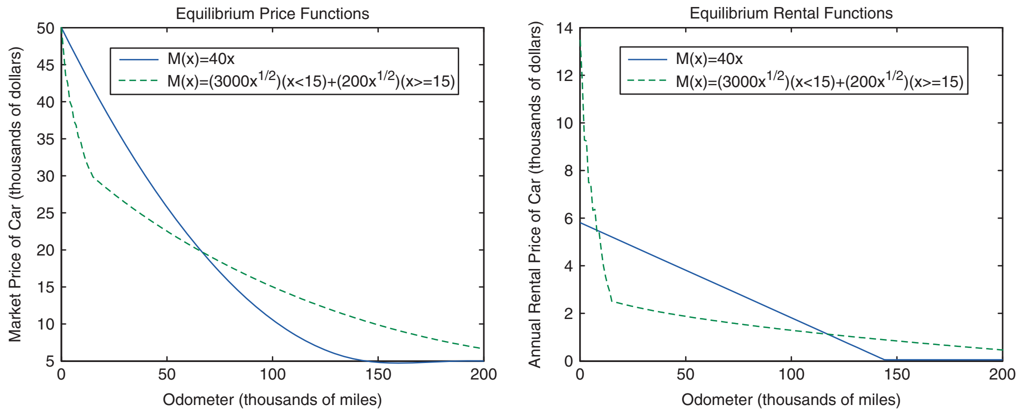

FIGURE 1
Equilibrium price and rental functions

Let $V(x)$ denote the expected discounted profits that a rental company expects to earn, given that it owns a rental car with an odometer value of x. The Bellman equation of V is

$$V(x) = \max\left[R(x) - m(x) + \beta EV(x), \underline{P} - \overline{P} + R(0) - m(0) + \beta EV(0)\right]. \tag{4}$$

In our formulation of the Bellman equation, we have assumed that if the rental company sells their current rental car, they only receive scrap value for it, and that they always replace this car with a brand new one. It is not hard to show that our conclusions are unchanged if we allow the rental company to sell a rental car with odometer value x on the secondary market for $P(x)$ and then purchase another car with odometer value z, where z is not necessarily a brand new car (i.e. $z = 0$). The Bellman equation in this latter case is

$$V(x) = \max\left[R(x) - m(x) + \beta EV(x), P(x) + \max_z\left[-P(z) + R(z) - m(z) + \beta EV(z)\right]\right]. \tag{5}$$

It turns out that in equilibrium giving the firm these extra options does not enable it to increase its profits any more than a "buy brand new and hold until scrap" strategy. In a competitive market, free entry of rental companies will ensure that each company makes zero net profits. The cost of entering the rental market is simply the cost $P(x)$ of purchasing a vehicle in the secondary market. Therefore, the zero profit condition is

$$V(x) = P(x), \quad x \in [0, \gamma], \tag{6}$$

where γ is the equilibrium scrapping threshold, i.e. the smallest x satisfying $P(x) = \underline{P}$. This implies

$$R(x) = m(x) + P(x) - \beta EP(x). \tag{7}$$

That is, in competitive equilibrium, rental prices equal the sum of per-period expected maintenance costs, $m(x)$, plus the expected depreciation on the vehicle, $P(x) - \beta EP(x)$. It should be reasonably clear from the convex shapes of the equilibrium price functions that expected depreciation is a declining function of x. Of course, maintenance costs could be an increasing function of x, so it is not immediately apparent whether rental prices are increasing, decreasing, or flat. However, note that there is another restriction that must be satisfied in a competitive equilibrium. In order for homogeneous consumers to be willing to rent vehicles of all possible odometer values x, *the "gross" rental price must be flat* i.e.

$$R(x) + q(x) = K, \tag{8}$$

for some constant K. That is, when a consumer considers whether to rent a vehicle with odometer value x, they consider the sum of the rental price $R(x)$ and the dollar equivalent disutility cost $q(x)$ of having a vehicle with odometer x (versus the alternative of renting a newer vehicle). As long as $q(x)$ is an increasing function of x, this simple chain of arguments leads us to the following conclusion:

Theorem 1. *In a competitive rental market with homogeneous consumers who strictly prefer new cars to older ones, rental prices cannot be flat.*

The right hand panel of Figure 1 illustrates this theorem by computing the equilibrium rental functions corresponding to the same two $M(x)$ functions used to compute the equilibrium price function shown in the left hand panel of Figure 1 (i.e. $M(x) = 40x$ and $M(x) = (3000\sqrt{x})(x < 15) + (200\sqrt{x})(x \geq 15)$, where the rental price function for the latter $M(x)$ is the kinked rental price curve in the right hand panel of Figure 1). It is clearly evident that rental prices are a declining function of x.

We conclude that if the car rental market is competitive, then under fairly general and reasonable assumptions about consumer preferences, rental prices cannot be flat. We do not believe this conclusion will be overturned by extending the model in various realistic directions, such as allowing heterogeneous consumers (Rust, 1985) or allowing for transactions costs (Konishi and Sandfort 2002).

One possible objection to the theoretical argument outlined in the previous section is that the car rental market may not be competitive. In particular, car rental companies do appear to have at least some limited market power and the ability to control their own rental prices. Furthermore, the fact that the company we are studying earns such high internal rates of returns on its rental vehicles is further direct evidence against the competitive rental market assumption: far from earning zero profits, our calculations show that the profits this company earns are many times the initial cost of a new car.

However even when the firm has market power and can choose a rental price function $R(x)$, it is hard to see conditions where it is optimal for it to choose a flat rental price structure. Indeed, it is not difficult to redo the calculations in the previous section under the hypothesis that there is no secondary market and only a monopolist renter. Rental prices will still be a declining function of x under this situation. Further, a monopolist renter will keep durables until they are scrapped, and the monopolist will scrap its rental vehicles at the socially optimal scrapping threshold γ^* (Rust, 1986).[1] The argument also holds in an oligopolistic or monopolistically competitive setting as well. Let K be the lowest expected gross rental price that a consumer can obtain from renting a car at the best alternative rental company. With quasi-linear utility,

1. The socially optimal scrapping threshold is derived from the optimal replacement policy to maximize consumer utility, i.e. it is derived from the solution to the Bellman equation $J(x) = \min[M(x) + \beta EJ(x), \overline{P} - \underline{P} + M(0) + \beta EJ(0)]$ and the optimal scrapping threshold is the solution to $J(\gamma) = \overline{P} + \underline{P} + J(0)$. In a competitive secondary market, we have $P(x) = \overline{P} - J(x) + J(0)$ and $P(\gamma^*) = \underline{P}$. That is, the competitive equilibrium implies a socially optimal replacement policy for cars.

the highest rental price that a rental company could charge this consumer to rent a car with odometer x is

$$R(x) = K - q(x), \tag{9}$$

which will be a declining function of x if q is increasing in x (i.e. whenever consumers prefer newer cars to older ones).

In comments on this paper, one of the leading experts on nonlinear pricing, Robert Wilson, observed: "I have given some thought to the absence of price differentiation by age or odometer but cannot come up with anything fundamental that might explain it. The usual explanation is that offering lower rental prices for older cars will increase market penetration or market share, but with a loss of revenue from those customers who switch to the older cars. But still this implies moderated price differentiation, not complete exclusion of any differentiation on this dimension."

Wilson suggested the following reputational argument, "that a premium brand like Hertz cannot sustain its share of business travellers if it is offering a 'discounted' line of older cars that somehow reek of inferiority" that might provide a link and possible explanation of the two puzzles raised in this paper. The leading car rental companies might have established a reputation as providers of high-quality goods that enables them to support a high rental price, high profit outcome by buying new cars and replacing them early. Given the short holding duration for their cars, the degree in differentiation between a brand new car and a one-year-old car may not be large enough to justify price discrimination along this dimension.

This reputational argument does not answer the question of why the major car rental companies do not form *Rent-A-Wreck* subsidiaries to capture the extra profits that can be earned from renting used cars. This would insulate these companies from reputational damage, and at the same time enable them reduce the losses they incur from the rapid depreciation in car cost. Perhaps the major car companies are concerned that such rental subsidiaries could create a form of internal competition by providing a close substitute that could limit the amount they can charge for brand new rental cars.

There may also exist imperfectly competitive market equilibrium explanations for the flat rental puzzle as well. In previous work, Wilson developed a theoretical model with two equilibria for oligopolistic airline pricing, one with no time-of-purchase differentiation, and another with complete differentiation. Wilson used numerical methods to show that for normally distributed valuations the first brings higher profits. A similar result appears in a labour market context, as shown by Wilson (1988).

Another possible explanation might be that there is some sort of collusive equilibrium strategy followed by the major rental companies, and in this equilibrium "simple" flat rental price schedules might be more conducive to supporting the collusive equilibrium (e.g. via "trigger strategies" that are invoked if any member of the cartel cuts costs) than more complex nonlinear rental pricing schedules that are functions of the age or odometer value of the vehicle (see, e.g. Green and Porter, 1984). Thus, the large car rental companies may be deterred from adopting more complex rental price schedules because adopting them could unleash a higher-dimensional competition over rental price schedules, opening up the possibility of difficult-to-observe cost-cutting by rivals, and this could undermine a high rental price, high profit collusive equilibrium under the *status quo.*

While we cannot dismiss any of these possible explanations for the flat rental puzzle on *a priori* grounds, we do not think any of them are especially plausible explanations of the behaviour of the car rental company we have studied. For example, we dismiss the collusion argument, at least for the particular firm we are studying, by the mere fact that this firm is willing to undertake our suggested experiment of keeping its rental cars longer and providing discounts on its older vehicles. If this firm were engaging in an implicit or explicit collusive strategy, it seems unlikely that it would be interested in risking retaliation of its competitors by undertaking this experiment.

The remainder of this paper will present our preferred explanation for this puzzle, at least for the particular firm we have studied: flat rental prices and rapid replacements represent a suboptimal strategy that has arisen from a combination of *satisficing behaviour* and *imitation* of the strategies followed by other leading firms in the car rental industry.

3. CASE STUDY OF A LARGE CAR RENTAL COMPANY

We now try to shed further light on the flat rental puzzle by undertaking a detailed case study of a particular car rental company that was generous enough to share their data with us and give us access to their key operating officers to answer our questions. The company provided us with data on over 3900 individual vehicles at various rental locations. These do not represent the entire fleet at any point in time, but they do represent a significant share of the company's holdings. All these vehicles were first acquired (i.e. registered) after 1999, and almost all these vehicles were purchased brand new from auto manufacturers. Purchase times of individual vehicles are fairly evenly spread out in time. While there are occasional "group purchases" of particular brands and models of vehicles on the same date, when these group purchases did occur, they typically amounted to only four or five vehicles of the same brand/model at the same time. Thus, this company, by and large, follows an *individual* vehicle replacement and acquisition strategy, as opposed to "block acquisitions and replacements", i.e. simultaneously acquiring and disposing of large numbers of vehicles of the same make and model at the same time.

The data consist of information on date and purchase price for each vehicle it acquired, the date and odometer value when the vehicle was sold, and the complete history of accidents, repairs, maintenance, and rentals between the purchase and sale dates. The rental contract data record the dates when each contract started and ended, and (sometimes) the odometer value of the vehicle at the start and end of the rental contract.

Figure 2 illustrates the well known rapid early depreciation in resale values of used cars by scatterplots of the actual resale values received for a particular popular make and model of luxury car in the company's fleet.[2] This figure also plots the best fitting regression line using a regression equation

$$\log(P_t/\overline{P}(\tau)) = \alpha_1(\tau) + \alpha_2(\tau)o_t + \epsilon_t, \tag{10}$$

where τ denotes the type of car (compact, luxury, or RV), $\overline{P}(\tau)$ is the new cost of car type τ, P_t is the actual resale price received by the company, and o_t is the odometer value on the car when it was sold.

Cho and Rust (2008) present regressions with other explanatory variables to predict resale prices, including the age of the car, number of accidents and accident repair costs, vehicle-specific maintenance costs (on a per-day basis), and a measure of the vehicle's "capacity utilization", i.e. the fraction of days the car was rented. The regression results show that both age and odometer value are significant predictors of the resale price of used cars, however the incremental predictive power of adding age in addition to odometer value is not huge. Nevertheless, we cannot reject the hypothesis that age is a significant predictor of used car

2. We have also analysed other makes and models including compact cars and SUVs and have come to similar conclusions. However, due to space limitations, we only show the results for the luxury car below.

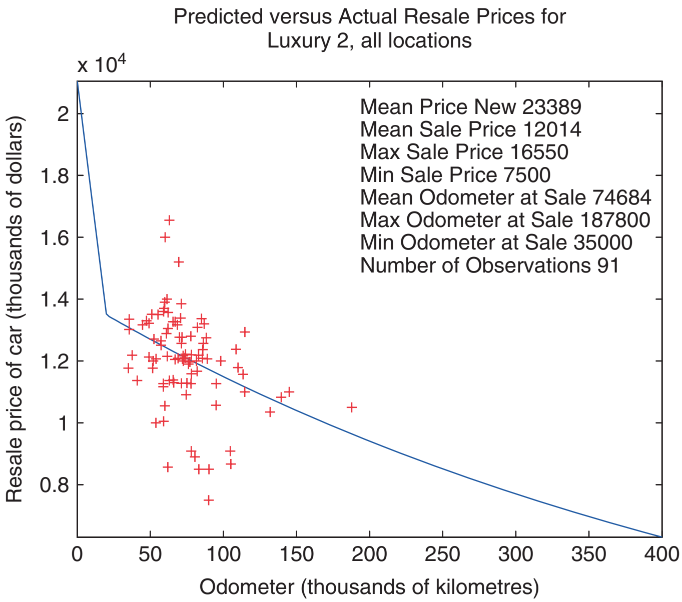

FIGURE 2

Predicted versus actual resale prices: luxury—all locations

prices. However beyond age, no other variables, including (perhaps surprisingly) number of accidents and total maintenance costs, are significant predictors of the resale price of a vehicle.[3]

The constant term in the regressions is a measure of how much depreciation a vehicle experiences the "minute it goes off the new car lot". We see that this predicted "instantaneous depreciation" is *huge* for all vehicle types. The luxury car we are focusing on loses 40% of its value after driving off the lot! Thus, the rapid early depreciation in car costs is evident in these data. While a number of cars are sold quite "early" after their initial purchase (measured in terms of either their age or odometer value), we do not have any observations of sale costs the company might have received if it were to have sold vehicles in only a matter of a few weeks or months after the initial purchase. For the purposes of our modelling of counterfactual replacement strategies in Section 4, we did not feel we could trust the regression extrapolations for used vehicle prices for age or odometer values very close to zero. Therefore, we made a simple, but *ad hoc* extrapolation of what we think a very new used car (i.e. one with less than 20,000 km) would sell for, instead of using the estimated regression intercepts. We assumed that the "instantaneous depreciation" for a brand new car would be only 10%, and then used a straightline interpolation from this value to the resale values implied by our regressions at an odometer value of 20,000 km. Above this value, we relied on the regression prediction, since it does accurately predict the mean resale price in the range of odometer values where most of our observations lie.

3. The insignificant effect of accidents could reflect the impact of insurance, which is supposed to result in repairs following an accident that restores the vehicle to its pre-accident condition, and the insignificant effect of maintenance costs may be due to the fact that the company is not required to disclose total maintenance costs to a buyer (although it must disclose the vehicle's accident history). Thus, a potential purchaser may not have the information to ascertain whether a certain vehicle was a "lemon" (as might be reflected by very high maintenance costs). The only option available to a buyer is to take the car to a mechanic to have it inspected prior to purchase.

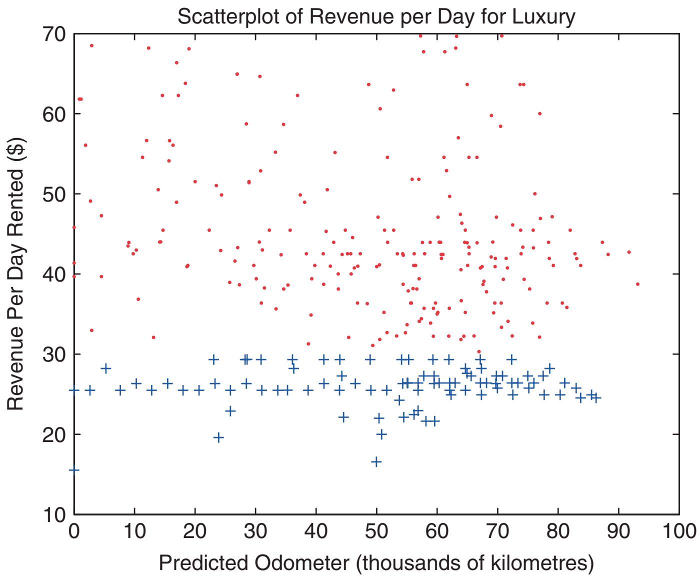

FIGURE 3

Scatterplots of daily rental rates by odometer for luxury model type prices

Now we provide evidence that unlike used car costs, the rental prices this company charges are flat. This seems obvious from merely inspecting the tariff schedules on the company's website, or checking rental price on the website of any major car rental company: while there are variations in rentals based on type of vehicle and contract (e.g. long or short term), *we do not observe any discounts based on the age or odometer of the vehicle one actually rents.* However, it might be possible that even if the published price schedules do not reveal any age or odometer-based discounts, there may be informal, unpublicized discounts or rebates, especially for customers who complain after being assigned an older rental vehicle.

Figure 3 provides direct evidence that rentals are flat, *using the actual recorded rental revenues from this company's records.* The figure contains scatterplots of the daily rental prices received on each rental episode for the luxury vehicle, where the data are from a specific location. The "+" symbols correspond to daily rental prices for long-term contracts, and the dots "." correspond to short-term rental prices. We see that the daily rental prices for long-term contracts are lower than for short-term rental contracts, but in both cases *rental prices are flat as a function of odometer.* Figure 3 plots rental prices as a function of *predicted* odometer values, which was necessitated by the fact that the company does not always record the odometer values at the beginning and end of each rental spell.[4] However, we see the same pattern when we plot the data as a function of age at time of rental instead of predicted odometer value, and the conclusion is robust if we do regressions and include other

4. The firm's contract data do accurately measure the beginning and starting *dates* and thus elapsed time of each rental contract. We also accurately observe the odometer reading when the car is purchased new (it is essentially equal to 0 then) and when the car is sold. Using the total time spent in long- and short-term rental contracts as covariates, CR (2008) were able to compute the average kilometres travelled per day during a short- or long-term rental contract. Using these estimated values, they were able to accurately predict a car's odometer over time, based on the elapsed total time spent in short- and long-term rental contracts. We have followed this procedure to calculate predicted odometer values in Figures 3 and 4.

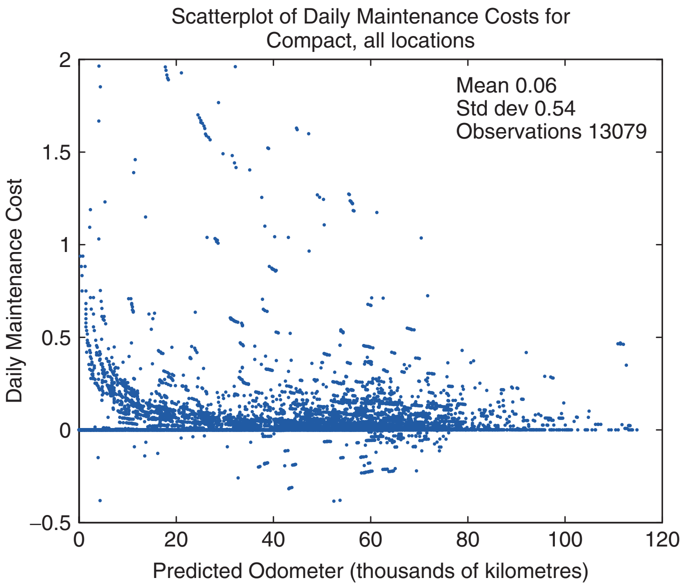

FIGURE 4

Scatterplot of average daily maintenance costs in previous 30 days: compact—all locations

covariates to try to explain some of the variability in rental prices. Thus, Figure 3 confirms that realized daily rental prices are flat, and the flatness is not just something advertised on the company's website, but it is actually something we can verify from *ex post* rental revenues as well.

Figure 4 provides a scatterplot of average daily maintenance costs (over the preceding 30 days) as a function of predicted odometer value. The figure shows maintenance costs as function of predicted odometer (we see essentially the same pattern when we plot maintenance costs as a function of vehicle age). It is clear that at least over the range of odometer values that we observe in our dataset, there is no evidence that maintenance costs increase with age or odometer. Thus, we also conclude that *maintenance costs are flat.*

4. A MODEL OF THE CAR RENTAL COMPANY'S OPERATIONS

We now describe an econometric model of the car rental company's operations developed and estimated by CR (2008). This is a highly detailed model that enables us to describe the operations of the company at the level of individual rental cars and rental contracts. We show via stochastic simulations that the aggregate operating behaviour implied by this model provides a good approximation to the company's actual operating behaviour (including number of rentals, revenues, costs and profits). We summarize the model in this section, and then use this model in the next section to make predictions of the effect of counterfactual changes on operating strategy on the overall profitability of this company.

CR (2008) developed a semi-Markov model that recognizes that any rental car that the company owns will be in one of three possible states at any given time: (1) in a long-term rental contract (i.e. a "long-term rental spell"); (2) in a short-term rental contract (i.e. a "short-term rental spell"); (3) in the lot waiting to be rented when the previous rental state was a long-term rental spell; and (4) in the lot waiting to be rented when previous rental state was a short-term rental spell. We refer to the latter two states, 3 and 4, as *lot spells.* We differentiate between these states since it turns out empirically that the duration distribution of a car in a lot spell is quite different depending upon whether it had previously been in a long- or short-term rental contract. In particular, hazard rates for type 3 lot spells are lower and mean durations are higher. In plain language, if a car had previously been in a long-term rental that did not immediately roll over, one would expect the car to be on the lot for a longer period of time compared to the case where the car has returned to the lot from a previous short-term rental.

Let r_t denote the rental state of a given car on day t. From the discussion above, r_t can assume one of the four possible values $\{1, 2, 3, 4\}$. In addition to the rental state, other relevant state variables for modelling the decisions of the rental company are the vehicle's *odometer value,* which we denote by o_t, and the *duration in the current rental state,* which we denote by d_t. Thus, an individual rental car can be modelled as a realization of the stochastic process $\{r_t, o_t, d_t\}$. The remaining state variable of interest, the vehicle's *age* a_t, can be derived as a simple by-product from realizations of this process, via the simple accounting identity $a_{t+1} = a_t + 1$ if the vehicle is not replaced, or $a_t = 0$ otherwise.[5]

Since a vehicle's age a_t is strongly correlated to its odometer value o_t there is a "collinearity problem" that makes it difficult to identify the independent effects of these two variables on decisions to sell a car, or on maintenance costs, state transition probabilities, durations in states, and even on the resale value of used vehicles. Since there are numerical and computational advantages to minimizing the number of different variables we include in our dynamic programming model, we have opted to follow CR and exclude vehicle age from the list of variables that we use to predict the company's selling decision, vehicle resale values, transitions and durations in spells, and so forth. This facilitates our use stationary, infinite horizon dynamic programming methods to analyse the profitability of alternative operating strategies.

The semi-Markov model of the firm's rental operations consists of the following objects: (1) a model of the resale value the company receives if it were to sell one of its cars; (2) a model of the random durations of a car in each of the rental and lot states; (3) a model of a car's transition to the next rental spell at the end of the current rental or lot spell; (4) a model of the utilization (kilometres driven) on a particular car during a long- or short-term rental contract; (5) a model of rental revenues received and maintenance costs incurred by the company over the life of the car; and (6) a model of the company's *selling decision,* i.e. the factors that motivate it to sell a given car at a particular point in time. We refer the reader to CR for details on the functional forms and estimation results for each of these objects.

One of the conclusions from CR's empirical analysis is that the only major "aging effect" observed in the data is the sharp decline in resale prices of the cars the company sells. Maintenance costs, rental prices, and durations of rental spells and lot spells are all essentially flat as a function of age or odometer value over the range of age and odometer values that we observe in the data.

5. The constraint that $a_t = 0$ implies that the company always replaces an old vehicle with a brand new one. This is in fact what the company does and we take it as a given in our counterfactual calculations. However, profits can be increased further if the company were to purchase slightly used vehicles instead of brand new ones, since as we have seen in Section 3, the rapid early depreciation of vehicles implies that savings of at least 40% off the initial purchase price can be obtained by purchasing almost new but slightly used vehicles from other car rental companies that follow rapid replacement strategies. However we felt that if we were to recommend this strategy in addition to keeping vehicles longer, the company executives would be more likely to dismiss our strategy as one that involves purchasing competitors' "hand-me-downs" that could jeopardize the company's reputation as a high-quality leader.

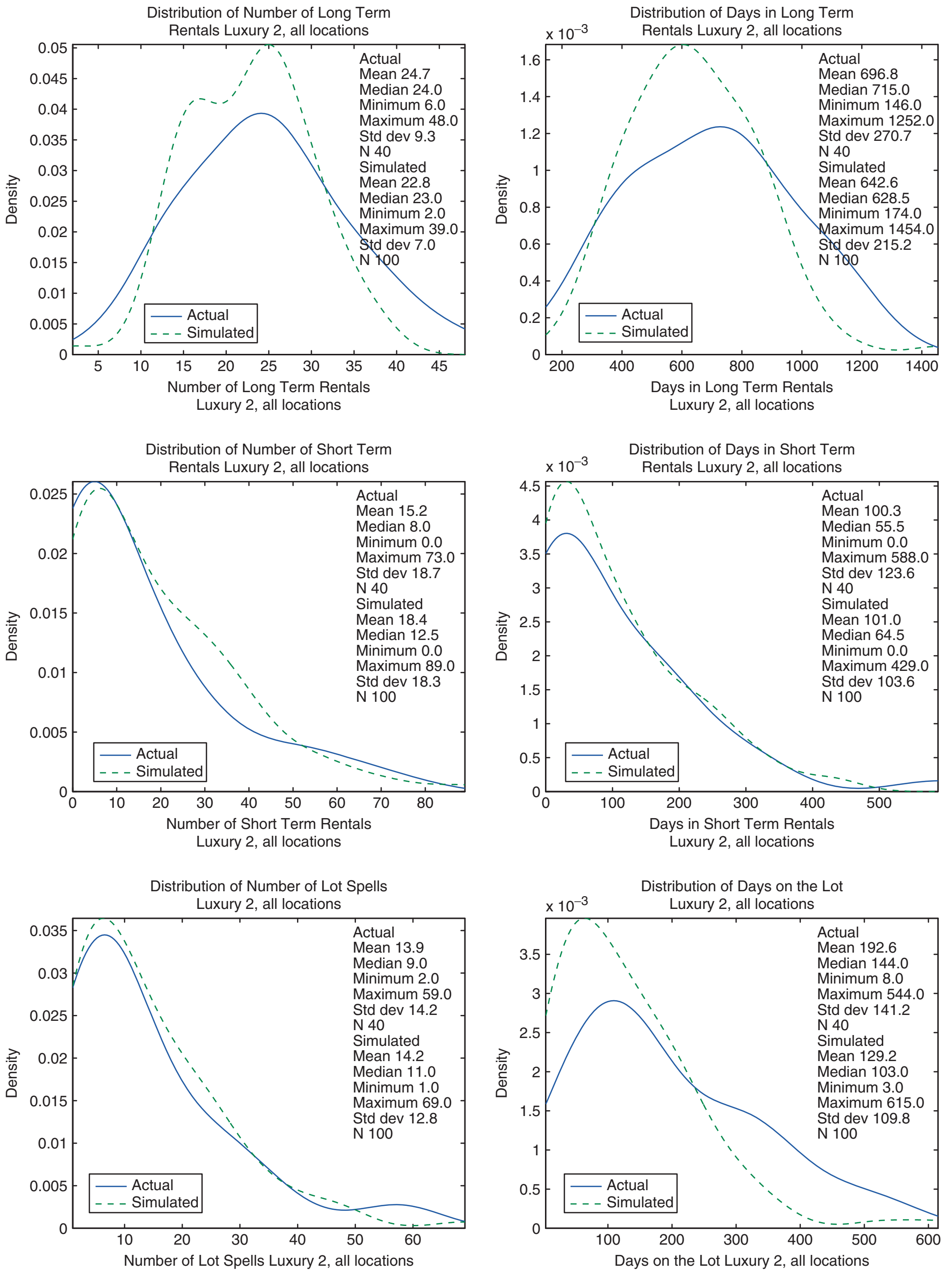

FIGURE 5

Simulated versus actual number of, and days in spells: luxury—all locations

In order to convince the reader that the econometric model developed by CR is a good one, we conclude this section with Figures 5 and 6, which compare a variety of simulated outcomes from their model to the actual distribution of outcomes in the dataset. Figure 5 shows that the econometric model provides a good approximation to the distribution of different rental

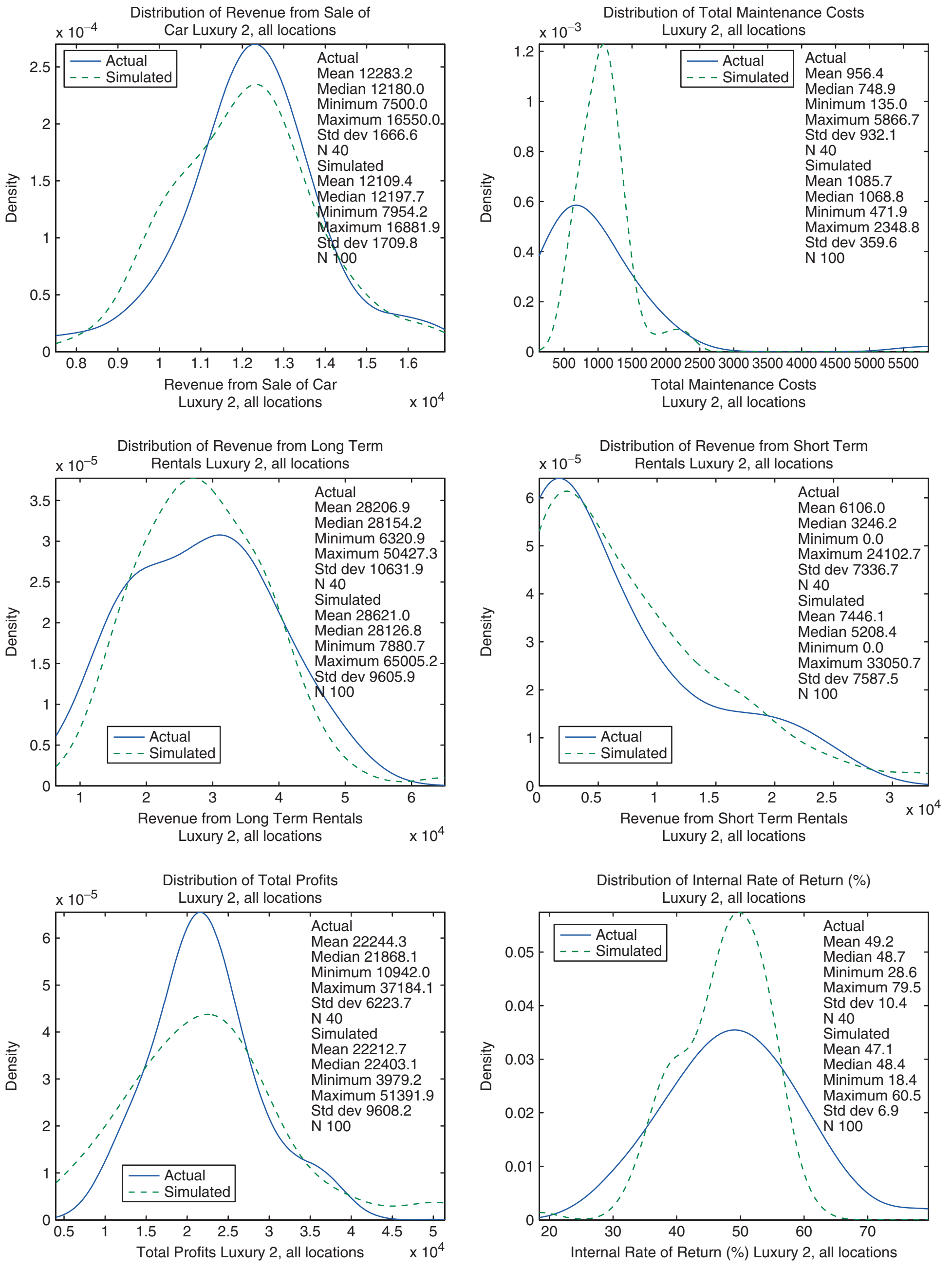

FIGURE 6

Simulated versus actual costs, revenues and profits: luxury—all locations

outcomes over a vehicle's service life. The left hand panels compare the simulated versus actual distributions of the number of long-term and short-term rental spells, and the number of lot spells. The right hand panels compare simulated versus actual distributions of total days spent in each of these spells. We see that the model not only does a good job of matching the mean values of the number of spells and durations of each spell type, but it is also able to approximate the overall distribution of outcomes as well. The ability to match both the number of spells and the duration of the various types of spells turns out to be the key to accurate predictions of revenues, profits, and returns.

Figure 6 plots comparisons of simulated versus actual distributions of the relevant financial variables. The top left panel of Figure 6 shows that the econometric model results in a distribution of proceeds from sales of cars that is quite close to the actual distribution. This is evidence that the lognormal regression model of vehicle cost depreciation (equation (10) above) is a good one. The bottom two panels of Figure 6 compare the actual and simulated distributions of total profits and internal rates of return (IRRs). The econometric model provides a good approximation to mean total profits and mean IRR, although the simulated distribution of total profits has a larger variance than the actual distribution, and the simulated distribution of IRRs has a lower variance than the actual distribution. Since the dynamic programming model we develop in the next section assumes that the company is an expected profit maximizer (i.e. the company is not risk averse), only the mean values of profits matters: an expected profit-maximizing firm would be indifferent between two different operating strategies that result in the same mean profits, even though one of the strategies results in a larger variance of profits. Thus, we do not think it is as important for the econometric model to accurately approximate the entire distribution of actual outcomes as long as it provides a good approximation to the *expected values of revenues, costs, and profits.*

5. USING DYNAMIC PROGRAMMING TO OPTIMIZE REPLACEMENT POLICY

While it is possible to evaluate *specific hypothetical alternatives* to the company's *status quo* operating policy using simulation methods, there are more efficient methods available for characterizing the *optimal replacement policy* that involve searching over what is effectively an infinite dimensional space of *all possible replacement policies.* Mathematically, the optimal replacement problem is equivalent to a specific type of *optimal stopping problem* known as a *regenerative optimal stopping problem* (see Rust, 1987). The term "regenerative" is used, since the decision to replace a vehicle does not stop or end the decision process, but rather, results in a "regeneration" or "rebirth", i.e. a replacement of an old vehicle by a brand new one.

However the counterfactual strategies we consider are not completely unrestricted. First, we are not simultaneously optimizing over rental prices and replacement policies for reasons discussed below. Also, as noted above, we assume that the company always replaces an existing car with a brand new version of the same make and model as opposed to purchasing a slightly used version of the same car at a substantially discounted purchase cost. Thus, when we refer to "optimal replacement policy" the reader should understand that the policy is only optimal relative within a restricted class of operating strategies.

The optimal replacement strategies that we calculate are *optimal stopping rules* that take the form of *threshold rules.* That is, the optimal time to replace a car occurs when its odometer value o exceeds a threshold value $\overline{o}(d, r, \tau)$ that depends on the current rental state r, the duration in that state d, and the car type τ. Using numerical methods, we solve the dynamic programming problem and calculate the optimal stopping thresholds $\overline{o}(d, r, \tau)$ for the compact, luxury and RV car types and the associated optimal *value functions* $V(r, d, o, \tau)$. This function provides the expected discounted profits (over an infinite horizon) under the optimal replacement policy for a vehicle that is in state (r, d, o).

It is also possible to compute the value of any alternative operating strategy μ, which can include *mixed* or probabilistic operating strategies where the decision to replace a car is given by a conditional probability distribution $\mu(r, d, o, \tau)$. We let $V_\mu(r, d, o, \tau)$ denote the expected discounted profits (again over an infinite horizon) under the alternative replacement policy μ. We will calculate both V and V_μ where μ is an approximation to the company's current or *status quo* operating policy. Thus, the difference $V(r, d, o, \tau) - V_\mu(r, d, o, \tau)$ will represent our estimate of the gain in profits from adopting an optimal replacement policy. As we noted in the introduction, the optimal policy entails keeping cars significantly longer than the company currently keeps them, but by doing this, we show that the company can increase its expected discounted profits by over 10%.

Accounting for the additional complexity of the two types of rental contracts (i.e. compare this to the Bellman equation (4) when only a simple rental one-period rental contract is available), the Bellman equation for the more realistic version of the rental problem is given by

$$V(r, d, o) = \max\left[EP(o) - \overline{P} + \beta EV(r_0, 0, 0), ER(r, d, o) - EM + \beta EV(r, d, o)\right] \quad (11)$$

where we suppress the τ notation under the understanding that separate Bellman equations are solved for each of the three car types $\tau \in \{\text{compact, luxury, RV}\}$. In the Bellman equation (11), $\overline{P}$ denotes the cost of a new car, $EP(o)$ is the expected resale value of a car with odometer value o, $ER(r, d, o)$ is the expected rental revenue from renting the car to customers, EM is the expected daily cost of maintaining the car including the cost of cleaning cars at the end of rental contracts, and EV is the expected discounted value of future profits from operating a sequence of rental cars (possibly until the infinite future).

There are two EV functions in the Bellman equation, $EV(r, d, o)$ and $EV(r_0, 0, 0)$. The term $EV(r, d, o)$ denotes the expected value of an *existing* car which has an odometer value of o and has been in rental state r for a duration of d days. The term $EV(r_0, 0, 0)$ denotes the expected value of a *new car* just after it has been purchased when it is on the lot waiting for its first rental. The notation r_0 denotes the first lot spell. To economize on states, we actually assume that this function can be represented in terms of lot states $r = 3$ and $r = 4$ (where, recall, these are lot states where the previous rental spell was either a long-term contract or a short-term contract, respectively), as

$$EV(r_0, 0, 0) = [\eta EV(3, 0, 0) + (1 - \eta)EV(4, 0, 0)] \quad (12)$$

where the parameter $\eta \in (0, 1)$ is chosen so that the weighted average duration distributions and the transition probability for the initial lot spell matches the mean duration for initial lot spells that we observe in the data.

The left hand term on the right hand side of (11) is the expected value of replacing a current vehicle with a new one. Thus, $EP(o) - \overline{P}$ is the expected cost of replacement, i.e. the expected resale value of the existing car (which has odometer value o) less the cost of a new replacement car $\overline{P}$, plus the expected discounted value from tomorrow onward, $\beta EV(r_0, 0, 0)$. We assume that a brand new car has an odometer value of $o = 0$ and starts its life in a lot spell with a duration of $d = 0$.

The Bellman equation (11) actually applies only when the car is in a lot spell ($r \in \{3, 4\}$) or before the first day of a rental spell ($d = 0$ if $r \in \{1, 2\}$), since we assume that the company will not interrupt an ongoing rental contract to sell a vehicle. Thus, for cars in the midst of a rental spell, ($r = 1$ or $r = 2$ and $d > 0$), we have

$$V(r, d, o) = ER(r, d, o) - EM + \beta EV(r, d, o). \quad (13)$$

The $EV(r, d, o)$ function is a conditional expectation of the value function $V(r, d, o)$. For lot spells, $r > 2$, we have

$$\begin{aligned} EV(r, d, o) \quad = \quad & h(d, r)\,[V(1, 1, o)\pi(1|r, d, o) + V(2, 1, o)[1 - \pi(1|r, d, o)]] \\ & +[1 - h(d, r)]V(r, \min(d + 1, 31), o). \end{aligned} \quad (14)$$

What this equation says is that with probability $h(d, r)$ the lot spell ends and the car will transit either to a rental spell under a long-term contract $r = 1$ with probability $\pi(1|r, d, o)$ or a rental spell under a short-term contract with probability $1 - \pi(1|r, d, o) = \pi(2|r, d, o)$ since we have ruled out self-transitions back to the lot, $\pi(r|r, d, o) = 0$ for $r > 2$. With probability $1 - h(d, r)$, the lot spell continues and the value function in this case will be $V(r, d + 1, o)$, reflecting an increment of 1 more day to the duration counter, unless $d \geq 31$ in which case d remains at the absorbing state value of $d = 31$ as reflected in the term $d_{t+1} = \min(d_t + 1, 31)$ in the equation for the value function above.

For rental spells, $r \in \{1, 2\}$, we have

$$\begin{aligned} EV(r, d, o) \quad = \quad & h(d, r) \int_{o'} \left[\sum_{r'} V(r', 0, o') \pi(r'|r, d, o) \right] f(o'|r, d, o) do' \\ & + [1 - h(d, r)] V(l(r), d + 1, o), \end{aligned} \tag{15}$$

where $l(r)$ is the lot state following a termination of rental state r, i.e. $l(1) = 3$ and $l(2) = 4$, and $f(o'|r, d, o)$ is the conditional density of the number of kilometres on the odometer of a car returning from a rental spell of type r that has lasted d days and started with an odometer value of o. Thus, if a car is in a rental spell, it will either remain in the rental spell for another day with probability $1 - h(d, r)$ (unless $d \geq 31$, in which case $h(d, r) = 1$), or with probability $h(d, r)$ the rental spell ends and the car transits to a new rental state r', which will either be a lot spell, $r' > 2$, or a rental spell under a short contract $r' = 2$, or a long-term contract $r' = 1$. If a car remains in a rental spell, the company will not know the odometer reading until the car returns from the spell. Thus, we keep the odometer state variable o fixed at its original value as long as a car continues its current rental spell. However if a car returns from a rental spell, the company learns the mileage travelled by the customer, $o' - o$, and thus, the odometer state variable increases from o to o'. Following CR, we assume that the total mileage driven under a rental contract that has lasted d days and is of type $r \in \{1, 2\}$ is an Erlang distribution with parameters d and λ_r, where λ_r is the mean mileage driven per day by customers under contract type r.

Finally, we specify the expected rental revenue function, $ER(r, d, o)$. Initially, we assume that long-term and short-term contracts allow unlimited kilometres and are charged a daily cost, except with an early return penalty for long-term contracts. Thus, for short-term contracts, $r = 2$, we have

$$ER(2, d, o) = h(d, 2) EDR(2) d, \tag{16}$$

where $EDR(2)$ is the expected daily rental price for a short-term contract. We multiply by the hazard since we assume that the rental is paid only at the end of the rental spell, but no revenue is received otherwise (i.e. if the rental contract continues another day). This expected value accounts for cases where cars are rented multiple times in the same day as "chauffeured vehicles" and reflects the expected sum of all rental revenue received on such days less the amount paid to the chauffeur.

For long-term rentals, there is a lower per-day cost, $EDR(1)$ provided the vehicle is rented for a sufficient number of days, say $\overline{d}$. Otherwise, the car is treated as an early return and there is a per-day penalty, ρ, added on for such early returns. Thus, the expected revenue function is

$$ER(1, d, o) = \begin{cases} h(d, 1) EDR(1) d & \text{if } d \geq \overline{d} \\ h(d, 1)[EDR(1) + \rho] d & \text{if } d < \overline{d}. \end{cases} \tag{17}$$

The optimal stopping threshold is the value of o where the firm is indifferent between keeping the car and replacing it. That is, it is the solution to the equation

$$EP(\overline{o}(d,r)) - \overline{P} + \beta EV(r_0, 0, 0) = ER(r, d, \overline{o}(d,r)) - EM + \beta EV(r, d, \overline{o}(d,r)). \quad (18)$$

As we noted above, if we were to solve the regenerative optimal stopping problem under the assumption that the only aging effects are (1) the depreciation in vehicle resale values, and (2) the "rental contract composition effect" (see Section 4), then the optimal stopping thresholds is $\overline{o}(d,r) = \infty$, i.e. it is *never optimal to sell an existing vehicle.* These results are due to our assumption that average daily maintenance costs EM do not increase as a function of odometer value, and that rental prices do not decrease as a function of odometer values. While there is substantial empirical justification for these assumptions *over the range of our observations* (see the discussion in Section 3), it is questionable that these assumptions will continue to be valid as a vehicle's odometer and age increases indefinitely, far beyond the range for which we have any observations.

In order to make headway, we proceed to calculate the optimal replacement policy under *extremely conservative assumptions about increases in maintenance costs and decreases in rental prices beyond the range of our data.* That is, we will assume that beyond the range of our observations, maintenance costs increase at a very rapid rate as odometer increases, and that to induce customers to rent older vehicles, daily rental prices must be steeply discounted. Since maintenance costs are flat for $o \in [0, 130{,}000)$ km, we assume daily maintenance costs do not increase until $o > 130{,}000$ km, after which they start increasing at a very rapid rate, reaching a level that is *11 times* the daily maintenance costs of vehicle with 130,000 km by the time the vehicle reaches 400,000 km.

For rental prices, we assume that in order to induce consumers to rent older vehicles, the company must reduce the daily rental prices on the older vehicles in its fleet at a rate that is linear in the vehicle's odometer value. We assume a very steep decline in rental prices, so that at the point a vehicle reaches 400,000 km the daily rental price would be *zero.* For a vehicle with 265,000 km, the rental price it can charge is only half the price it charges for vehicles that have 130,000 or fewer kilometres on their odometers. We assume that when such discounts are in effect, *customers are indifferent between renting newer versus older rental cars, and as a result, there is no net increase in the frequency of rentals of older cars at a discount at the expense of rentals of newer cars at the full rental value.* In addition, we assume that the *stochastic pattern of utilization of vehicles, including the durations of short- and long-term rental contracts is unaffected by the discounts provided to customers.* These assumptions imply that the duration and transition models estimated by CR (2008) can be used to predict utilization patterns under the counterfactual replacement and rental pricing scenarios that we calculate below.

As we noted, the firm does in fact have a small number of vehicles in its fleet with odometer values in the range (130,000, 265,000), yet it does not offer discounts on rentals of these vehicles, and nevertheless, still succeeds in renting them to customers. We view this as evidence that the rental discount function that we have assumed is actually much steeper than necessary to induce some of the firm's customers to rent older vehicles. Indeed, as we noted in the introduction, the company conducted an experiment to discount rental prices for vehicles over 2 years old, but stopped it when it found that virtually all customers preferred to rent an older car at a 20% discount rather than a newer car at the full daily price. This suggests that discounts could be popular with many customers and that required discounts could be much less drastic than what we have assumed. For this reason, we view the calculations below as providing a *lower bound* on the increase in profits that the firm could obtain from adopting a fully optimal pricing and replacement strategy.

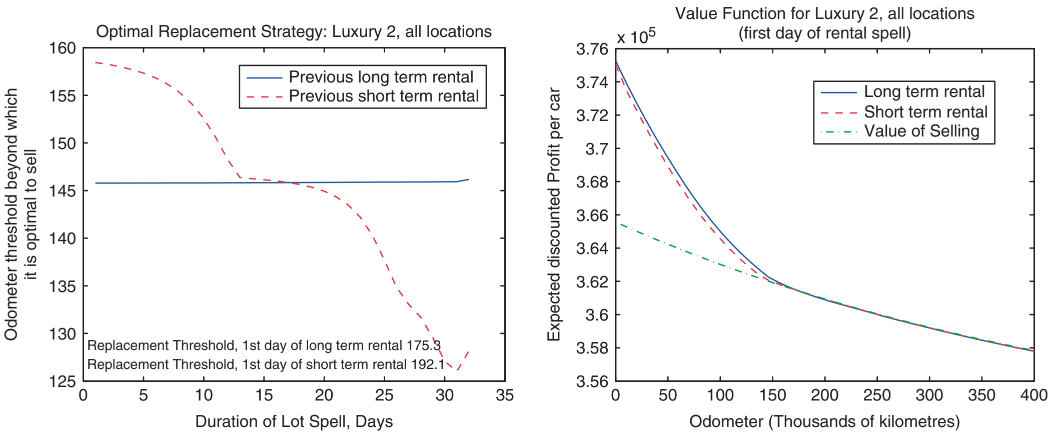

FIGURE 7

Optimal decision rules and value functions: luxury—all locations

Figure 7 shows our calculated optimal replacement thresholds $\overline{o}(d, r, \tau)$ and the value functions for the luxury car (results are similar for other car types). The left hand panel displays the replacement thresholds $\overline{o}(d, r, \tau)$. The solid line is the replacement threshold for the case where a car is in a lot spell of type $r = 3$, i.e. the previous rental spell was a long-term contract, and the dashed line is for a car that is in a lot spell of type $r = 4$, i.e. where the previous rental was a short-term contract. In addition, the firm can decide whether or not to replace a car at the start of a rental spell, and the figures also present these thresholds, $\overline{o}(0, 1, \tau)$ (the threshold applicable at the start of a long-term rental spell) and $\overline{o}(0, 2, \tau)$ (the threshold applicable at the start of a short-term rental spell).

We see that for type $r = 3$ lot spells, the replacement threshold is approximately equal to 145,000 km for all three car types. This threshold is basically flat as a function of the duration in the spell, except that in the case of the RV, the threshold starts out at about 157,000 km and then decreases with the duration on the lot to about 146,000 km for cars that have been on the lot 30 days or more. The fact that the replacement threshold is essentially flat as a function of duration in the lot for a type $r = 3$ lot spell is due to two factors: (1) the hazard rate for exiting the lot is essentially flat as a function of duration on the lot d; and (2) there is a very high probability that at the end of the lot spell the car will transit to another long-term rental spell. These two factors imply an absence of "duration dependence" (i.e. length of time on the lot does affect the chances that the vehicle will leave the lot the next day) which in term implies an absence of duration dependence in the replacement threshold $\overline{o}(d, 3, \tau)$.

However, for type $r = 4$ lot spells, i.e. lot spells that were preceded by a short-term rental spell, there is duration dependence: i.e. the probability of leaving the lot spell is a decreasing function of the length of time spent on the lot d. In addition, there is a significantly lower chance that the next rental spell will be a long-term rental, and thus, a high chance that the vehicle will have a relatively short rental spell in a short-term contract and will return to the lot again. Thus, a vehicle that is currently in a type $r = 4$ lot spell is more likely to be idle in the future, and this fact, combined with the decreasing chance of exiting the lot as lot duration d increases, causes the replacement threshold $\overline{o}(d, 4, \tau)$ to decline with d. For all three car types, we see fairly steep declines in $\overline{o}(d, 4, \tau)$. For example, for the compact, the replacement threshold starts out at approximately 164,000 km on the first day of the spell, and declines to about 144,000 km by the 31st day on the lot.[6]

It is also not surprising to find that the replacement thresholds are significantly higher at the start of a short- or long-term rental spell. The company has the option to sell the car at this point, or to let the renter take it. It seems intuitively obvious that the replacement threshold should be higher when the company has "a bird in the hand" than for a car that is on the lot waiting to be rented. For the compact, the replacement threshold at the start of a long-term rental contract is $\overline{o}(0, 1, \tau) = 156{,}200$ km, which exceeds the 145,000 km threshold for a car in the first day of a lot spell that has just emerged from a long-term rental spell. We see that the replacement threshold at the start of a short-term rental contract is even higher. In the case of the compact the threshold is $\overline{o}(0, 2, \tau) = 201{,}600$ km which is significantly higher than the 164,000 km threshold for a car that is fresh on the lot, having just emerged from a short-term rental spell.

The disparity in replacement thresholds at the start of long- and short-term rental spells is not as great for the other two car types. The explanation is that short-term contracts are much more lucrative than long-term contracts for the compact car type compared to the other two car types, since the ratio of daily rental rates for short-term contracts to long-term contracts is significantly higher for the compact car type than for the other two car types. Thus, the company is relatively more eager to take advantage of a short-term rental opportunity for its compact cars, and this makes it optimal for it to have a significantly higher replacement threshold at the start of a short-term rental contract.

The right hand panel of Figure 7 plots the calculated value function for the luxury car as a function of odometer o. We plot three functions: the value of keeping a car on the first day of a rental spell, $V(r, 1, o)$ for $r = \{3, 4\}$, and the expected value of selling the vehicle, $EP(o) - \overline{P} + \beta EV(r_0, 0, 0)$ as a function of its current odometer value o. The first point where the value of keeping the car equals the value of selling it constitutes the optimal replacement threshold $\overline{o}(d, r)$. Thus, if we were to enlarge the top right hand panel of Figure 7 we would see the dashed line (the value of keeping a car when it is in lot spell type $r = 4$) first intersects the dash-dotted line at 201,600 km, so this constitutes the optimal replacement threshold that is appropriate at the start of a short-term rental spell that we noted previously. The solid line is the value of keeping a car when it is in lot spell type $r = 3$, and it first intersects the dash-dotted line at 156,200 km, which is the optimal replacement threshold at the start of a long-term rental spell.

For all three car types, the "rental value", i.e. the difference in the value of keeping and selling it for a new one is a relatively steeply decreasing function of odometer value. While we noted that the resale value of a car is a sharply decreasing function of the vehicle's odometer (except in the case of the RV where the rate of depreciation is much flatter after the initial rapid depreciation that occurs in the first 20,000 km), *we see that the depreciation in a vehicle's rental value is an even steeper function of its odometer value than its resale value.* This result is an interesting contrast to the relatively mild "aging effects" that CR found from their econometric analysis. Note that our assumed sharp drop-off in rental prices and the sharp rise in maintenance costs do not start until after 130,000 km, yet the decline in value of a car occurs immediately. But the "rental contract composition aging effect" that CR found is not steep enough to explain the sharp declines in values of rental vehicles that we see in Figure 7.

6. Due to our constant hazard assumption for lot spell durations longer than $d = 31$, this 144,000 km threshold applies to all durations over $d = 31$.

The key explanation for the rapid drop in the value of a rental car as a function of odometer is the *horizon effect*. Essentially the instant a company purchases a new car, it represents a large investment that will be earning the company a stream of profits for a finite period of time until the car reaches its replacement threshold at which time the company will have to incur another large expenditure to buy another new vehicle. Thus, the values of keeping an existing car in Figure 7 represent *expected future profits* over the life of the car, but the new purchase price of the *current* vehicle is treated as a sunk cost (or bygone expense). Thus, as the vehicle's odometer increases from zero towards the optimal replacement threshold, the expected discounted value of remaining profits *on the current car* necessarily decreases since there is a shorter remaining life over which this profit stream will be collected. When the company replaces the vehicle (the value of which is represented by the dash-dotted lines in Figure 7), the firm must incur the cost of buying a new replacement vehicle and the process starts over again.

The difference between the value of keeping a (just purchased) new vehicle and immediately trading it for another new vehicle, $V(r_0, 0, 0) - [EP(0) - \overline{P} + \beta EV(r_0, 0, 0)]$, represents the expected discounted profit that the firm expects to earn on the current vehicle over its lifetime. For the luxury car type, we have $V(r_0, 0, 0) = \$375{,}000$ whereas the value of immediately selling a newly purchased car is approximately $EP(0) - \overline{P} + EV(r_0, 0, 0) = \$366{,}000$. Thus, *the company expects to make a net discounted profit of approximately \$9000 over the service lifetime of a single luxury vehicle*. The total discounted profits are higher, \$375,000, since this is the expectation of discounted profits earned from an infinite sequence of rental vehicles.

In order to learn more about the implications of the optimal replacement policy, particularly about the distribution of ages at which replacements occur, we simulated the optimal replacement policy using the same basic approach that we used to simulate the econometric model under the *status quo* replacement policy in Section 4. Figure 8 presents a comparison of the simulation results (for 100 simulated luxury cars) and the firm's actual outcomes under its *status quo* replacement policy. The left hand panel compares the simulated and actual distribution of odometer values of cars at replacement, and the right hand panel compares the distributions of ages at replacement.

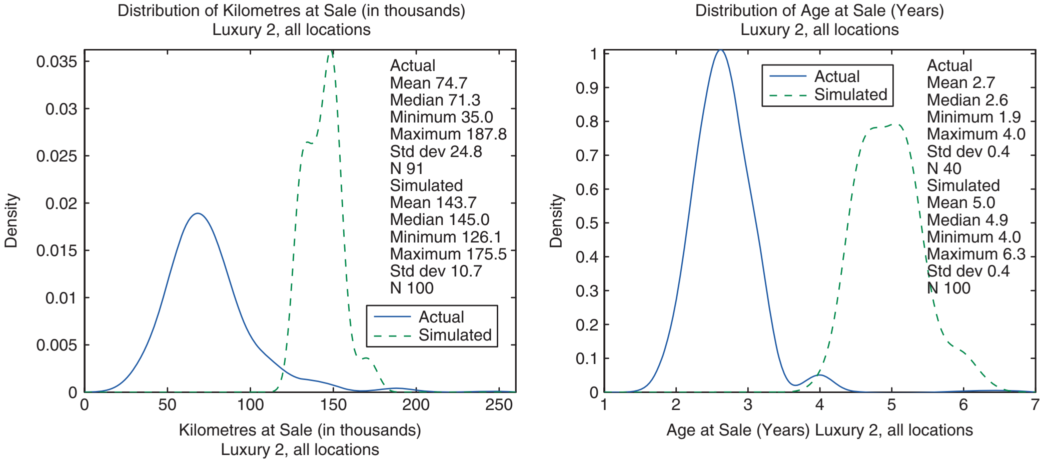

FIGURE 8

Simulated versus actual ages and odometers at replacement: luxury—all locations

We see that under the optimal replacement policy, the mean odometer value at replacement is more than twice as large as the mean value under the *status quo*. The variance in odometer values about the mean value is also less under the optimal replacement policy than under the *status quo*. The right hand panel of Figure 8 compares the actual distribution of ages at replacement to the distribution predicted to occur under the optimal replacement policy. The optimal replacement policy, the mean age at replacement, is almost twice as high for all three car types we analysed, ranging from 4.6 to 5.0 years under the optimal replacement policy versus being between 2.6 and 2.7 years under the *status quo*.

We emphasize that it is optimal to keep these vehicles longer despite the rather substantial increases in maintenance costs and reductions in rental prices that we have assumed occurs after 130,000 km. We can see from the right hand panel of Figure 8 that almost all replacements that occur under the optimal replacement policy occur well after 130,000 km, when these "adverse" aging effects have kicked in. Note, however, that all the cars are replaced before they reach 265,000 km, which is the point where rental prices are discounted to 50% of the price charged for a vehicle with 130,000 km. Also, according to our assumptions daily maintenance costs are about 5 times higher for vehicles at 265,000 km than the values we observe for vehicles that have fewer than 130,000 km. So the combination of the rental discounts and rapid increase in maintenance cost does take its toll, and greatly alters the optimal replacement policy. Instead of it being optimal to *never* replace its existing vehicles, once we make the assumptions about rapidly rising maintenance costs and rapidly declining rental prices after 130,000 km, it is no longer optimal for the company to keep and maintain its existing stock of cars indefinitely. However what is surprising is that despite our conservative assumptions, the optimal replacement policy still entails keeping cars about twice as long as the company currently keeps them.

Figure 9 compares the actual distributions of internal rates of returns that the company realized for the particular make/model of luxury vehicle we analysed to the distributions of

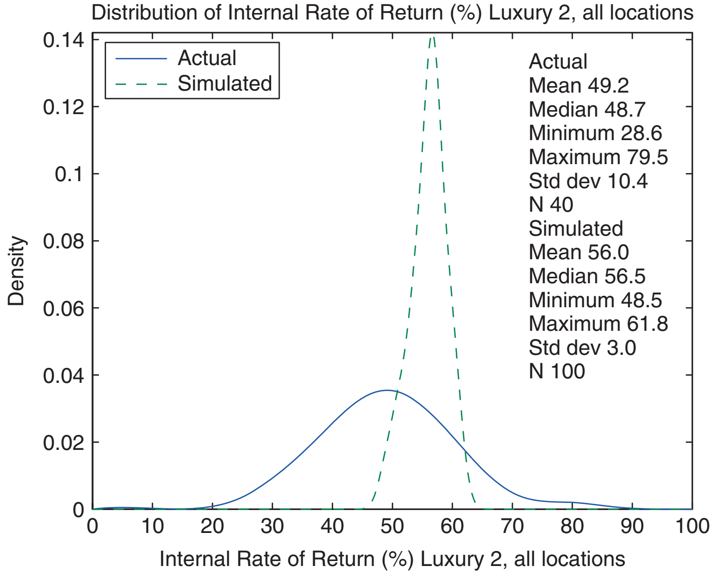

FIGURE 9

Simulated versus actual internal rates of return: luxury—all locations returns implied by the optimal replacement policy. We see that mean returns are uniformly higher, and this is true for all three car types we analysed, not just the luxury segment. For example, the mean IRR for the compact increased from 77% to 94% and the mean IRR for the RV increased from 53 to 64% under the optimal replacement policy. We see that the distribution of returns has a greater variance under the *status quo* replacement policy, and due to this higher variance, a small fraction of cars, roughly 5%, achieve rates of return under the *status quo* that are higher than the highest possible return earned under the optimal replacement policy. On the other hand, the upper 95% of the distribution of returns under the optimal replacement policy is higher than the median return earned under the *status quo.* Overall, we conclude that adoption of the optimal replacement policy results in a fairly significant upward shift in the distribution of realized returns.

However a more straightforward way to compare the company's *status quo* operating policy with the optimal replacement policy is to compute and compare the *expected discounted profits* under the two operating strategies. We feel that a "fair" comparison requires calculating the expected discounted profits *over an infinite horizon* rather than comparing profits for only a single generation of vehicles. The reason is that a comparison that looks only at a single generation will always be biased towards strategies that keep vehicles longer since, as we discussed previously, the company incurs a large "up front" capital cost when it buys a new car. Thus, by keeping vehicles longer the company earns more rental revenues that help to increase profits by "amortizing" the initial capital expense over a longer service life. However a "single generation" analysis fails to account for the fact that postponing a vehicle replacement also postpones *the profits from subsequent generations of vehicles, and the postponement of these future profits can represent a large "opportunity cost" that can outweigh the increased short-term profits from keeping vehicles longer.*

However, to compare the discounted profits under an infinite horizon, we need to make extrapolations of the firm's *status quo* replacement policy into the indefinite future, since our data obviously only covers a relatively short time span of the firm's operations. To make this calculation, we solved the following analogue of the Bellman equation for the value functions $V_\mu(r, d, o, \tau)$, $\tau \in \{\text{compact}, \text{luxury}, \text{RV}\}$. Dropping the dependence on τ in the equations below to economize on space, we have

$$\begin{aligned} V_\mu(r, d, o) \quad = \quad & \mu(r, d, o)[EP(o) - \overline{P} + \beta EV_\mu(r_0, 0, 0)] + [1 - \mu(r, d, o)][ER(r, d, o) \\ & -EM + \beta EV_\mu(r, d, o)], \end{aligned} \tag{19}$$

where $\mu(r, d, o)$ is the conditional probability that a car in state (r, d, o) is replaced under the *status quo* replacement policy. This is a linear functional equation that we solved using the same numerical techniques that we used to solve for the value function under the optimal replacement policy. By definition, it must be the case that the optimal replacement policy results in higher expected discounted profits than under any alternative strategy μ, so we have

$$V(r, d, o) \geq V_\mu(r, d, o), \quad \forall\ (r, d, o). \tag{20}$$

However the question is "how far is the company's existing replacement policy from optimality?" We can answer this by computing the ratio $V(r, d, o)/V_\mu(r, d, o)$, which represents the factor by which the firm can increase its discounted profits by adopting an optimal replacement policy. If this ratio is not too much bigger than 1, then the company can rest comfortably in the knowledge that its *status quo* replacement policy is "almost" optimal.

To simplify our analysis, we focus on comparing the value of a newly purchased brand new car that has just entered the lot. Thus, in Table 1 we report $V(r_0, 0, 0)$, the value of a

TABLE 1
Comparison of profits/returns: optimal policy versus status quo

	Compact all locations	Luxury all locations	RV all locations
Quantity	Value	Value	Value
$\overline{P}$	9668	23,389	18,774
Expected discounted values under optimal replacement policy			
$V(r_0, 0, 0)$	268,963	374,913	327,057
$(1-\beta)V(r_0, 0, 0)$	22.11	30.81	26.88
$V(r_0, 0, 0)/\overline{P}$	27.8	16.0	17.4
Expected discounted values under *status quo* replacement policy			
$V_\mu(r_0, 0, 0)$	196,589	318,247	136,792
$(1-\beta)V_\mu(r_0, 0, 0)$	16.16	26.16	11.24
$V(r_0, 0, 0)_\mu/\overline{P}$	20.3	13.6	7.3
Ratio of expected values: optimal policy versus *status quo*			
$V(r_0, 0, 0)/V_\mu(r_0, 0, 0)$	1.37	1.18	2.39

new car that has just arrived in the lot under the optimal replacement policy, and $V_\mu(r_0, 0, 0)$, the value of a new car that has just arrived in the lot under the the firm's *status quo* operating strategy μ. For μ we used the cumulative distribution function for replacements as a function of odometer o only: our approximation to the firm's replacement policy does not depend on spell type r or duration in the spell d. This is justified by CR's findings that once we account for odometer o, neither d nor r provide huge enhancements in our ability to predict when the firm replaces one of its vehicles. Table 1 also presents an "equivalent daily profit rate" which is approximated as $(1-\beta)V(r_0, 0, 0)$ and $(1-\beta)V_\mu(r_0, 0, 0)$ where $\beta = \exp\{-r/365\}$ is the daily discount factor. For our calculations we have assumed that $r = 0.03$, and this implies a daily discount factor that is quite close to 1, $\beta = 0.99988679886$.[7]

The first section of Table 1 presents the expected discounted values and the daily expected profit equivalent values for the optimal replacement policy for each of the three car types that we analysed. Also, to provide context for these numbers, the top line also presents the average cost of a new vehicle for each car type. We see that for the compact car, for example, the expected present discounted value of profits is $268,963, which is 27.8 times the cost of a new compact car. Applying the final value theorem, we find that this discounted profit is equivalent to about $22.11 in profits on a daily basis.

The second section of Table 1 presents the expected discounted value of profits under the *status quo*. The expected discounted value of profits over an infinite horizon for the luxury car type is $318,247, which is equivalent to $26.16 on a daily basis. Thus, we see that according to our predictions, the firm could increase its discounted profits by 18% (i.e.

7. According to the *final value theorem* (see, e.g. Howard, 1971, p. 46) for any convergent sequence $\{a_t\}$ we have

$$\lim_{\beta \to 1}(1-\beta)\sum_{t=0}^{\infty}\beta^t a_t = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}a_t. \tag{21}$$

There are stochastic extensions of this result that imply that for β close to 1, $(1-\beta)V(r, d, o)$ is close to the "long run average profits", which in our case corresponds to an equivalent daily profit.

$V(r_0, 0, 0)/V_\mu(r_0, 0, 0) = 1.18$), if it adopted the optimal replacement policy, in combination with discounts in the rental prices of older vehicles.[8]

We find that for the luxury car type, the firm's replacement strategy is closer to optimality: its profits would increase by 18% under the optimal replacement strategy. However for the RV, the firm's existing policy appears to be far from optimal: the present discounted profits are predicted to be *2.4 times higher under the optimal replacement strategy*.

We have calculated discounted profits under even more pessimistic assumptions about the rate of increase in maintenance costs and the required discounts in rental prices that would be necessary if the company kept its cars longer than it currently does. We do not have the space to discuss this even more pessimistic scenario in any detail, but as expected, it is optimal to replace cars even sooner. Nevertheless, the optimal replacement policy still entails keeping cars significantly longer than the company currently keeps them, and the optimal replacement policy still results in significantly higher profits than the status quo. In particular, the model predicts that profits will still increase by 25% for the compact, 6% for the luxury, and 101% for the RV. We conclude that our predictions are quite robust to variations in our assumptions. Our estimates of the gains that can result from delaying the replacement of rental vehicles are likely to be extremely conservative: as we show in the next section, the company does not need to discount rental prices of older cars anywhere near as much as we have assumed. Thus, the gains it will realize from keeping its cars longer should be even larger than the values predicted above.

The main limitation of the model and the calculations performed in this section is that we have assumed that the semi-Markov process governing vehicle utilization is *invariant* to changes in the rental price schedule. We do not have observations on past changes in the company's rental price schedule that would enable us to model how change in the price schedule will alter vehicle rental and utilization patterns. Thus, we caution readers that the counterfactual predictions presented in this section are likely to be valid only for relatively small deviations from the company's current flat rental schedule, where relatively small discounts are given to customers to induce them to rent older vehicles. If discounts are small, it seems reasonable to assume that utilization patterns of older vehicles rented at discounted prices will be approximately the same as for newer vehicles rented at the full price. However if the large discounts that we have assumed are not actually necessary, the firm could lose revenue if most of its customers switch from renting new vehicles at full price to renting older vehicles at a discount. We have assumed that large discounts are necessary to keep customers approximately indifferent, but the results of a field experiment reported in the next section suggest much smaller discounts would be necessary. Thus, potential improvements in profitability that we have estimated in this section are likely to be very conservative and represent a lower bound on the amount profits could increase under an intelligently designed alternative operating strategy.

8. Note that the \$26.16 predicted average daily profits is higher than the average daily profits calculated from the actual data of \$22.57 ($= 22244/(365 * 2.7)$), using the mean total profits from luxury cars in the lower left hand panel of Figure 6 and the 2.7 mean holding duration reported in the right hand panel of Figure 8. This discrepancy can be accounted for by the fact that the value \$26.16=$(1-\beta)V_\mu(r_0, 0, 0)$ is only an *approximation* to the average daily profit that holds in the limit as $\beta \uparrow 1$. We calculated the value \$26.16 using $\beta = \exp\{-0.03/365\} = 0.99988679886$. As $\beta \uparrow 1$ the value of $(1-\beta)V_\mu(r_0, 0, 0)$ decreases towards the \$22.57 value. For example when $r = 3.5 \times 10^{-6}$ and $\beta = \exp\{-r/365\}$, we calculate $(1-\beta)V_\mu(r_0, 0, 0) = \$22.64$ which is very close to the observed value of \$22.57.

6. RESULTS FROM A FIELD EXPERIMENT

Prior to our interactions with this company, management had conducted an experiment that involved a 20% discount for cars over 2 years old. The company found that there was a very strong substitution from new cars to old cars in response to the discount. Specifically the number of rentals of cars over 2 years old increased significantly, but at the expense of rentals of vehicles under 2 years old. The company did not have records of this previous experiment, and so we were unable to analyse it in any detail. In particular, it is not clear whether the discount caused total rental revenues to rise or fall. However the memory of this experiment was not a positive one in the mind of the executive that we interacted with, and he was sceptical that any new insights could be learned from running another experiment.

Overall, this previous experiment reinforced the company's main concern that by discounting rental prices for older vehicles, the firm is effectively engaging in price competition with itself, risking lowering overall revenues. However as we noted above, if the company maintains the same overall number of cars, then the policy of keeping cars longer before replacing them necessarily results in some overall reduction in total revenues whenever the company reduces rental prices of older vehicles to induce its customers to rent them. For the luxury car analysed in the previous section, Table 1 predicts a \$4.65 increase in daily profits from keeping these cars an average of 5 years rather than the current 2.7-year-mean duration to replacement under the company's current replacement policy. Stretching the holding times in this fashion accounts for nearly all of the increase in the daily profits: average daily replacement costs fall from \$11.54 to \$6.84, or a \$4.70 increase in profits per car on a daily basis.[9] Mean revenues per day under the *status quo* amounted to \$34.81. Thus, mean rental revenues would have to fall by more than 13% in order to exceed the reduction in replacement costs that result from a longer holding period for vehicles. If revenues fall by less than this amount or even increase, then adopting a declining rental function and keeping rental cars longer should cause total profits to increase.

The executive understood these arguments and he felt that the risks that revenues would reduce by such a large amount were sufficiently small to be willing to undertake another limited scale experiment. The executive conducted the experiment at four smaller rural rental locations during the low season from January to May, 2007. We refer to these four locations below as the "treatment locations". We were not involved in the design of the experiment, although the company did allow us to analyse individual contract data from these locations and the company also provided us total rental revenues (broken down by location and month) from 6 "control" locations that were chosen to match the characteristics of the treatment locations as closely as possible. Total rental revenue at the 6 control locations increased by 7.5% during the first 5 months of 2007 over to the first 5 months of 2006, reflecting general price inflation (approximately equal to 2.5% per year) as well as growth in population and income per capita, and other factors that have caused steady expansion in the company's customer base and overall revenues over time.

Our analysis of the contract data from the 4 treatment locations shows that total revenue actually *increased* by 12.4% during the first 5 months of 2007 over the same period in 2006.

9. The average daily replacement costs on the current replacement policy can be calculated as $11.54 = (23{,}389-12{,}014)/(365 * 2.7)$, where \$23,389 is the cost of a new luxury car, \$12,014 is the mean resale value of these cars after the mean holding period of 2.7 years. If the mean holding period is increased to 5 years, the resale value is predicted by the regression equation (10) in Section 3 to fall to \$10,500. So on a daily basis, replacement costs fall to $6.84 = (23{,}389-10{,}900)/(365 * 5)$, a saving of \$4.70 per day.

This increase is nearly 5 percentage points *higher* than the increase in the revenues experienced at the 6 control locations.

Before we provide more detail on the results of the experiment, we note that the "treatment" (i.e. discount) was not the same for all customers at these locations. The 40% nominal discount that the company offered to all consumer for renting vehicles over 2 years old must be evaluated relative to discounts most customers already receive from the company. A majority of the firm's customers are members of its "frequent renters" programme that provides its members a 30% discount on the rental price of *all* cars. These customers thus received only a 10% additional discount if they rented a car that was over 2 years old, so the effective discount for renting an older car for many of the firm's customers would be 10%, not the full 40%. Customers who are not members of the company's frequent renters club, but who are members of a rewards programme offered by a consortium of businesses with which this company is allied (including customers who have certain types of credit cards) were offered a 20% discount on certain cars. Thus, for these customers, the effective discount for cars over 2 years old was 20% (= 40 − 20%). Only a minority of customers who were not members of the frequent renters club or the rewards programme (and thus would not receive any discount on new rental cars) would receive the full 40% discount from renting a car over 2 years old. For this minority of the firm's customers, the discount from renting a car over 2 years old is the full 40% discount relative to the price of new cars, and thus is a particularly strong incentive for these customers.

Unfortunately, the firm did not provide us with records of individual customers, only the contract data and the actual price paid. As a result, we cannot tell which customers were eligible for a 40% discount, which were eligible for a 20% and which were eligible for a 10% discount for renting a car over 2 years old. It is not easy to determine from the actual contract revenues how much the discount was, since the contract cost also includes certain extra charges such as rentals of additional items (e.g. baby seats, GPS devices, cell phones), and the costs also reflect discounts based on the rental duration (e.g. lower daily costs for renting a car on a weekly rather than daily basis). The complicated nature of discounts (not including the discount offered for cars over 2 years old) is evident from the variability in daily rental prices that we observe in Figure 3 for a particular make/model. Although our analysis here is complicated by the heterogeneous nature of the treatments, we believe there is sufficient data to reveal with a high degree of confidence that the "average treatment effect" (i.e. the effect of the discounts on rentals of older vehicles) was substantial.

Table 2 compares the average daily rental prices received for 2029 rental contracts in the 4 treatment locations in the first 5 months of 2006 with 2481 contracts made for the equivalent period in 2007. The daily rental prices for individual contracts were computed by dividing the total revenue reported for a contract by the number of days the vehicle was rented, adjusting for the company's practice to charge either a half day or full day of extra charges depending

TABLE 2
Daily rental rates in treatment locations; 2006 versus 2007

Year	Item	New cars (under 2 years old)	Old cars (over 2 years old)
2006 (Jan–May)	Daily rental rate	54.10	57.01
	(standard error)	(22.75)	(32.53)
	Number of contracts	1858	171
2007 (Jan–May)	Daily rental rate	56.53	49.40
	(standard error)	(25.19)	(23.88)
	Number of contracts	1658	823

on the number of hours in excess of 24 when a car is actually returned. We see that in 2006, the average daily rental price for cars over two years old is actually slightly *higher* that the average daily rental price for new cars. However, we note that there are only 171 contracts for old cars in 2006, and thus, the slightly higher price is likely just an artifact of sampling variability in the estimated mean due to the relatively small numbers of observations in this cell.

In fact, a two-sample t-test of equality of the mean daily rental price in the old car and new car subsamples is unable to reject the hypothesis that the two rental prices are the same ($t = -1.14$, p-value of 0.87). This is consistent with the flat rental price schedule in place in 2006 before the experiment took place. However for 2007, we see that the average daily rental price for old cars drops to 49.40, which is 13% lower than the average daily rental price for new cars, 56.53. When we perform a two sample t-test of the equality of mean rental prices for 2007, we strongly reject the hypothesis that the rental price for new cars is the same as for old cars ($t = 6.88$, p-value of 2.910^{-12}).

Figure 10 compares the distributions of daily rental prices in 2006 and 2007 for the "treatment group" and the "control group" (i.e. cars over and under 2 years old, respectively, in 2006 and 2007). The left hand panel of Figure 10 shows that there is a clear leftward shift in the distribution of rental prices for cars over 2 years old resulting from the discounts to older cars at the four treatment locations. However the right hand panel of Figure 10 shows that distributions of daily rental prices for cars under 2 years old are nearly identical in the treatment and control locations, which of course is a result of the fact that no discounts were provided for rentals of newer cars.

Table 3 shows the effect of the discount on mean duration of rentals and total rental revenues. As we noted above, total revenues increased by 12.4% in the first 5 months of 2007 over the corresponding period in 2006. We can see from Table 3 that this was due to the fact that revenue from rentals of the older, discounted cars more than quadrupled between 2007 and 2006, which far outweighed a 16% decrease in revenues from new car rentals. Thus, the experiment provides evidence of high elasticity of substitution for *certain* customers in response to the discount that the company offered. We see that the number of contracts for older rental cars increased by more than 10-fold, and the average duration of a rental contract increased nearly 5-fold, to 4.9 days. This increase in rental duration actually raised the average revenue earned per rental contract slightly, to $130.11 in 2007 from $128.79 in 2006. This is

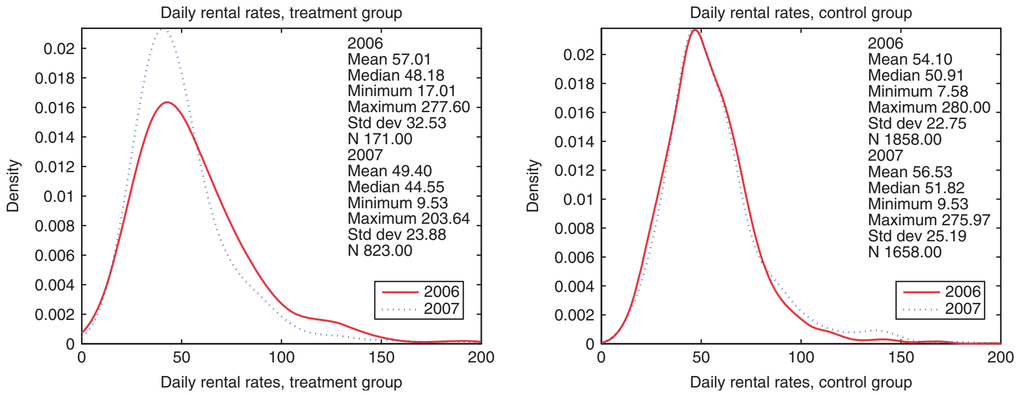

FIGURE 10

Comparison of average daily rental rates in treatment locations: 2006 versus 2007

TABLE 3

Days rented and rental revenues in treatment locations: 2006 versus 2007

Year	Item	New cars (under 2 years old)	Old cars (over 2 years old)
2006 (Jan–May)	Average days rented	11.8	1.0
	Total days rented	5755	492
	Total revenues	239,304	22,248
	Revenue per contract	128.79	130.11
	Number of contracts	1858	171
2007 (Jan–May)	Average days rented	9.4	4.9
	Total days rented	4560	2367
	Total revenues	202,060	91,933
	Revenue per contract	121.87	111.70
	Number of contracts	1658	823

the reason why total rental revenues from rental of old cars nearly quadrupled even though the average daily rental prices for older cars decreased by 13%.

Total revenue from new car rentals did decrease by 16%. This was evidently due to a substitution from new car rentals into rentals of the discounted older cars. The average duration of a rental contract decreased by 20% to 9.4 days, and this caused the average revenue per contract to decrease to \$121.87 from \$128.79 in 2006. This 5% decline in revenue per contract, coupled with the 11% decline in the total number of contracts explains the 16% decline in revenues from new car rentals during 2007.

It is clear that there has been some loss in revenues from new car rentals due to the discounts provided to customers who rent older cars. Lacking data on choices made by individual customers and the differential discounts they received, it is hard for us to determine whether most of the response was due to customers who faced the largest (40%) discounts, and whether most of the discounts were provided to these "pre-existing customers" or whether the discounts (which were not advertised but knowledge of which could have spread by word of mouth) might have attracted new customers to the firm.

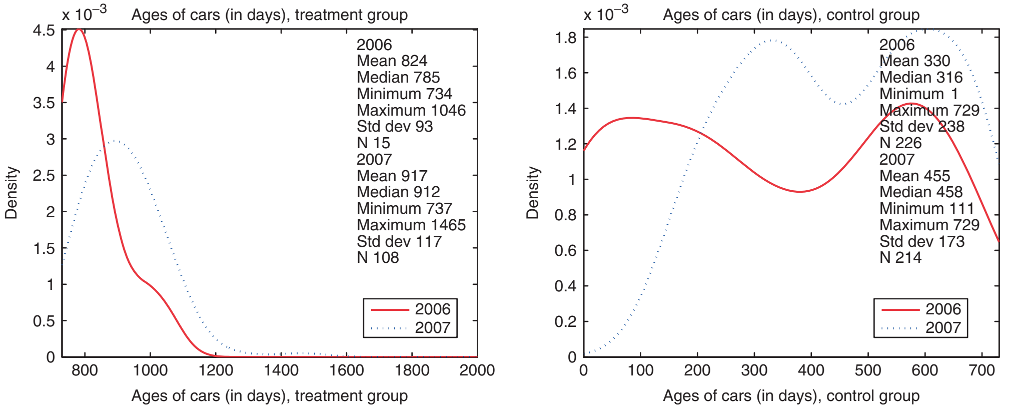

FIGURE 11

Comparison of average age of cars in treatment locations: 2006 versus 2007

We conclude this section with Figure 11 which shows significant aging in the set of cars that were rented in 2007 compared to 2006. The mean age of cars rented from the treatment group increased in age by over 3 months, to an average of 2.5 years old in 2007. We would expect to see more rentals of older cars if the discounts lead customers to demand a greater fraction of older cars that were previously on the lot not being rented. However the mean age of cars rented from the control group (i.e. the cars under 2 years old) also increased by over 4 months to 1.2 years old. We do not know if the fact that the "new cars" were slightly older in 2007 compared to 2006 was something customers noticed, or if this had any effect on their decision about whether to rent a car under 2 years old at full cost versus the same make and model of car over 2 years old at a discount. However the increased age of the vehicles that were rented in 2007 did not result in any significant increase maintenance costs, consistent with our finding that maintenance costs are flat.

What have we learned from this experiment? First, it is clear that our assumption that rental car usage is invariant to changes in the rental price schedule is incorrect. We observed a significant increase in the average rental duration of older cars and a more moderate reduction in the usage of new cars in this experiment. However when we resolved the firm's optimal replacement problem taking into account the induced shifts in vehicle utilization in our semi-Markov model, we found the optimal replacement thresholds and the predicted increases in profitability were essentially unchanged. This is largely due to the offsetting nature of the utilization patterns: newer vehicles are used somewhat less, whereas used vehicles are used significantly more intensively and the net effect on discounted revenues and maintenance costs is close to zero.

The most important conclusion is that the experiment supports our contention that the firm can provide discounts to customers who rent older vehicles without significantly reducing overall revenues. Although the discounts do cause a significant substitution away from new car rentals to old car rentals, the increase in revenues from rentals of older cars more than compensated for the decline in new car rentals. We cannot tell from the contract data alone whether most of this substitution effect was driven by the subset of customers who were eligible for the largest discounts (the full 40% discount), but the fact that the average discount was 13% suggests that most of the substitution came from the company's "frequent renters" who were eligible for the smaller 10% discount (since they already received a 30% discount from the frequent renter programme).

7. CONCLUSION

We have introduced the "flat rental puzzle" and shown that economic theory implies that rental prices for cars should decrease with age or odometer *in a competitive rental market.* Our proposed solution to this puzzle is that the competitive model may not be a good approximation to actual rental car markets. Car rental companies may have significant market power (and thus control over their prices), and may be behaving *suboptimally.* Actually, we have identified two closely related puzzles in this paper: (1) the "short holding period puzzle", and (2) the "flat rental puzzle". Car rental companies appear to justify using flat rental price schedules because they replace their vehicles so quickly, presumably based on the belief that cars are virtual perfect substitutes for each other when they are sufficiently new.

We have been fortunate to obtain data and access to top managers at a particular large and highly successful car rental company. We verified that both puzzles hold for this firm. Based on our discussions with executives at this company and our analysis of its operating data, we have concluded that the most plausible explanation of these puzzles, at least for this firm, is that it has adopted a suboptimal strategy.

This conclusion is supported by the results of a controlled experiment that the company undertook to test the qualitative predictions of our model. While it is relatively straightforward to see that keeping rental cars longer will significantly reduce replacement costs, the company's main concern is that discounts in rental prices necessary to induce its customers to rent older vehicles in its fleet could significantly reduce overall revenues if enough of its customers choose to rent older cars at discounted costs instead of new cars at the full rental price. However, the experiment revealed that rental revenues actually *increased* as a result of the discounts it provided for older vehicles. There was a 16% reduction in rental revenue from new cars, but the rental revenues from the older, discounted cars *quadrupled* causing overall revenues to increase significantly more at the four "treatment locations" where the discounting experiment was conducted in comparison to six control locations where the flat rental price schedules were maintained.

Should we be concerned if a particular highly profitable car rental company is not maximizing profits? We believe that this study does have broader significance for economics. First, we think that Herbert Simon's notion of *satisficing* may be relevant for understanding behaviour of companies that face very difficult, high-dimensional dynamic optimization problems. It might be the case that it is very hard for firms to find *optimal* strategies, but relatively easy for them to find *good* strategies. If this is the case, the potential range of observed outcomes in markets may be much larger than we would expect if we assume that firms are perfect dynamic profit maximizers. In particular, we would expect *imitation* to play an important role since it makes sense for new firms to copy strategies that seem to have proven successful by other firms. We hypothesize that imitation is a major reason for the prevalence of flat rental prices and rapid replacement of vehicles by so many car rental companies around the world.

The second general message is that economic and econometric modelling, backed up by small-scale experimentation to test whether the counterfactual predictions of the models are accurate or not, may be useful tools to help firms find more profitable strategies. Even though this company is highly profitable, and already keeps its vehicles approximately twice as long as most US car rental companies, our model correctly predicted that the company's profits can be increased significantly by a further doubling of the holding period for its vehicles combined with discounts to the rental prices of older vehicles to induce its customers to rent them.

We emphasize that we do not claim that the counterfactual replacement and rental discount strategy we suggested is fully optimal either. As we discuss below we believe we need to develop a much more sophisticated and comprehensive model of the firm and its customers to derive optimal pricing and operating strategies. However, for purposes of demonstrating that the firm is behaving suboptimally, it is only necessary for us to show that there exists an alternative counterfactual operating strategy that yields higher profits than the firm earns under its current pricing and operating strategy.

The other general observation is that *experimentation may be costly,* and this may explain why some firms do not seem to undertake as much data gathering and experimentation as we in academia might expect them to do. When experimentation is sufficiently costly, it becomes more plausible that firms may fail to discover better strategies via an evolutionary process of trial and error learning, and instead, rely more on imitating the behaviour of successful rivals.

Economists like to develop extremely sophisticated economic theories, and assert that their models are relevant for understanding firm behaviour based on the assumption that firms are rational profit maximizers. However puzzles such as this one force us to consider whether some of our theories, particularly those that presume that firms have no bounds on their ability to make difficult calculations, are out of touch with reality.

At the same time, we also need to be aware of our own computational limitations as economic modellers. It is typically the case that real situations involve complications that are difficult for us to model, or result in models that are difficult or impossible for us to solve. So we typically rely on overly simplistic models that might ignore critical aspects of reality. In these situations, we err when we confuse the limitations of our models as a "problem" in the firms or consumers we are studying.

We certainly keep this latter admonition in mind in this case. We acknowledge that our model of the company's operations has several important limitations, most important of which is our limited knowledge of how customers will react to changes in the firm's rental price schedule. We have employed a model that assumed utilization patterns of rental vehicles is invariant to changes in the rental price schedule, but the experiment the company conducted revealed that this is a poor assumption. Although in this particular experiment, we found that accounting for the induced changes in customer utilization patterns in new and used vehicles had small effects on the model's predictions (largely due to offsetting revenue effects, with the loss in revenue from reduced rentals of new cars counterbalancing increased revenues from more intensive use of older vehicles), it is hazardous to rely on overly simplistic models that ignore endogenous behavioural responses and hope that the models will result in accurate predictions of counterfactual policy changes.

It is clear that we need a more comprehensive model that can address the question of the overall *scale and allocations of cars in the company's "portfolio"* including the optimal number and geographical location of rental locations. Such a model needs to account for the rate of arrival of rental customers, how customer arrivals and choices are affected by the overall price structure that the company chooses for its entire portfolio of cars, as well as the price structures that competing firms offer for their rental car portfolios. Such a model could tell us the optimal number of vehicles of various ages, makes and models to have on hand at each of the firm's rental locations, and how changes in rental price schedules interact with these vehicle allocation decisions.

However, our initial attempts to address these deeper, harder questions only seem to raise new puzzles. In particular, we find large discrepancies in the profitability of different makes and models of cars assigned to a given rental location. If one make and model is significantly more profitable than another, why does the firm not increase its holdings of the most profitable makes/models and decrease or eliminate its holdings of the least profitable makes and models of vehicles?

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