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AN EMPIRICAL MODEL OF ASSET REPLACEMENT IN DAIRY PRODUCTION

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SUMMARY

Throughout the US dairy farm industry, observed rates of dairy cow replacement consistently exceed the rates prescribed as optimal by dairy economists. We attempt to uncover the causes of this discrepancy by estimating a series of dynamic discrete choice models of dairy cow replacement using historical farm-level data. We also advance the methods used to estimate dynamic discrete choice models by demonstrating how orthogonal polynomial projection methods can be effectively combined with standard maximum likelihood techniques to estimate structural models of dynamic decision making.

1. INTRODUCTION

Livestock production is characterized by complex dynamic biological processes. A decision that must regularly be made by a livestock producer is when to sell an animal that occupies a space in the producer's operation and replace it with a new, younger, and potentially more productive animal. The decision affects both current and expected future returns. Agricultural economists have examined optimal livestock replacement in a variety of contexts using dynamic programming models. Examples of such studies include applications to general livestock production (Burt, 1993), swine production (Chavas *et al.*, 1985), poultry production (McClelland, *et al.*, 1989), and dairy production (Smith, 1972).

The replacement rules prescribed by agricultural economists as optimal often fail to match the replacement rules followed by livestock producers in practice. In the case of dairy production, formal optimization models often prescribe slower replacement rates than are observed on dairy farms. Dairy economists began examining optimal dairy cow replacement in the late 1970s using budgeting and simulation methodologies (e.g. Allaire *et al.*, 1977; Allaire, 1981; Congleton and King, 1985). Latter work used dynamic programming methodologies (e.g. Rogers *et al.* 1988). Regardless of the modeling framework, these studies consistently concluded that dairy producers replace animals too quickly. As a consequence, dairy extension specialists began recommending that producers reduce replacement rates. Dairy producers, however, did not heed these recommendations. If anything, they quickened the pace of replacement between 1980 and 1993 (Ohio Dairy Improvement Association, selected issues).

One possible explanation for the discrepancies between prescribed and actual dairy cow replacement rates is that dairy producers are not rational dynamic profit optimizers. Another explanation is that formal economic models have omitted factors that are important to optimal decision making. In this paper, we hypothesize that omitted factors offer the more likely explanation. To test our hypothesis, we obtain data from five Ohio dairy producers and use dynamic discrete choice estimation methods to test for possible omitted factors. Specifically, we

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test (1) whether maintenance costs increase as the cow ages and (2) whether actual replacement costs are lower than suggested by observable cost data. Detection of either factor would explain the faster replacement rates observed in practice and would aid in developing more realistic models of optimal dairy production. More realistic models would yield improved guidelines for dairy producers as well as more reliable assessments of the potential impacts of new technologies, such as the introduction of bovine Somatotropin (bST).

Econometric estimation of a dynamic discrete choice model of dairy cow replacement presents challenging technical problems due to the relatively large number of state variables and the absence of a closed-form solution for the optimal replacement policy. To estimate the model, Bellman's functional equation must be solved numerically every time that model parameters are perturbed by the hill-climbing routine that seeks the maximum of the likelihood function.[1] We employ Rust's nested fixed-point strategy to estimate our dynamic decision model (Rust 1987, 1988). We advance the technique used by Rust by employing orthogonal polynomial projection methods to solve the embedded stochastic dynamic programming problem (Judd, 1991, 1992). The use of this highly efficient dynamic programming solution method allows us to perform our estimations on a microcomputer in a relatively short amount of time.

In the following sections, we formulate a dynamic optimization model of dairy cow replacement and discuss how to solve and estimate it. In subsequent sections, we present empirical estimates for five Ohio dairy producers and test alternate explanations for the discrepancy between observed and prescribed replacement rates. We also illustrate how the estimated models can be used to assess how replacement decisions would be affected by increases in productivity and changes in milk return and cow replacement costs.

2. A DYNAMIC COW REPLACEMENT MODEL

A dairy manufacturing plant typically has capacity for a fixed number of cows. About 40% of a dairy producer's costs come from maintaining, operating, and financing the plant. These costs are fixed, varying little with respect to the number of cows in the plant. The remaining 60% of costs are variable and include such items as feed, veterinarian services, and milk marketing costs. Revenue from a cow, of which 86% comes from milk sales, virtually always exceeds cow-specific variable costs. Hence, there is typically an economic incentive to operate the plant at maximum capacity.

A typical cow's milk-producing life is divided into lactation cycles, each of which begins when the cow gives birth to a calf. Upon the calf's birth, the dairy cow begins to produce milk, something she continues to do for about eleven months. Approximately four months after the birth of the calf, the producer must decide whether to breed the animal again. If the cow is bred, it enters a nine-month gestation period, making the typical lactation cycle thirteen months long. If the cow is not bred, she is sold for slaughter when she reaches the end of her productive life. The cow is then replaced by a heifer, a female dairy animal that has not yet given birth to a calf. The heifer either comes from a dairy producer's own stock of animals or is purchased from another dairy producer.

A major factor influencing the decision to replace or keep a cow is the pattern of productivity the exists across cows in the herd and, for any given cow, across lactations. Some cows are

[1] Dynamic discrete choice models of labour market participation and asset replacement have been estimated by Wolpin (1984), Eckstein and Wolpin (1989), Berkovec and Stern (1991), Rust (1987), Rosenzweig and Wolpin (1993) and Das (1991).

inherently more productive than others, and will persistently produce above the herd average for cows of comparable age. Conversely, some cows are inherently less productive than others, and this is often detected during the first lactation, typically leading to early replacement. The general pattern of productivity over the life of a dairy cow is for production to increase during the first, second, and third lactations, to reach a peak in the third or fourth lactation, and then to decline over subsequent lactations.

As a maintained hypothesis, we assume that a dairy producer maximizes the discounted sum of current and expected future profits from each cow and its successors. Formally, we write the typical producer's decision problem as:

$$\max E \sum_{t=0}^{\infty} \delta^t [\pi(x_t, i_t) + \varepsilon_t(i_t)] \tag{1}$$

Here, t indexes a thirteen-month period, the typical duration of a lactation; δ is the discount factor implied by a 5% real annual interest rate; $\pi(x_t, i_t)$ is net return at t; $\varepsilon_t(i_t)$ is an unobserved, decision-specific random profit shock at t; x_t is a vector of state variables at t; and i_t is the producer's decision at t.

More explicitly, the state vector $x_t = (a_t, y_t, p_t, c_t)$ consists of four variables:

a_t = age of cow, or its successor
y_t = pounds of milk produced by cow, or its predecessor
p_t = profit contribution per pound of milk
c_t = net replacement cost of cow

and the producer's decision i_t is indicated by:

$$i_t = \begin{cases} 0 & \text{if cow is retained at end of } t \\ 1 & \text{if cow is replaced at end of } t \end{cases}$$

Net return is given by:

$$\pi(x_t, i_t) = p_t y_t - i_t c_t - \gamma_0 - \gamma_1 a_t + \varepsilon_t(i_t)$$

Here, $\gamma_0 + \gamma_1 a_t$ represents unobserved, age-specific maintenance costs. As is standard in binary logit models, we assume that the error terms $\varepsilon_t(0)$ and $\varepsilon_t(1)$ are mutually independent, Gumbel distributed random variables with location parameters μ_0 and μ_1 and common scale parameter λ (see Ben-Akiva and Lerman, 1985).

Age, measured in lactations, evolves as follows:

$$a_{t+1} = \begin{cases} a_t + 1 & i_t = 0 \\ 1 & i_t = 1 \end{cases}$$

Fluid milk production over each lactation is assumed to be composed of a quadratic deterministic part, which represents average production for all cows of age a_t, and a random, cow-specific production shock ξ_t:[2]

$$y_t = \beta_0 + \beta_1 a_t + \beta_2 a_t^2 + \xi_t \tag{2}$$

[2] A quadratic function fits the stylized facts of the milk production cycle by allowing production to rise over the first few lactations and then decline.

Because above or below average milk production tends to be a persistent cow-specific characteristic, we assume that ξ_t follows a first-order autoregressive process with a positive autocorrelation coefficient ρ. We assume that ξ_t is uncorrelated across cows. The innovations in the process are presumed normal with zero mean and variance σ_ξ^2.

The profit contribution of milk p_t represents revenue less variable cost per pound of milk produced. The net replacement cost of a cow c_t represents the opportunity cost of replacing a cow, namely the market price of a new heifer less the slaughter value of the replaced cow. The variables p_t and c_t are assumed to follow a joint stationary first-order vector autoregressive process with normally distributed innovations η_{pt} and η_{ct}:

$$\begin{aligned} p_t &= \alpha_{p0} + \alpha_{pp} p_{t-1} + \alpha_{pc} c_{t-1} + \eta_{pt} \\ c_t &= \alpha_{c0} + \alpha_{cp} p_{t-1} + \alpha_{cc} c_{t-1} + \eta_{ct} \end{aligned} \tag{3}$$

We denote the variances and covariance of the innovations terms by σ_p^2, s_c^2, and σ_{pc}.

In summary, our model of the dairy producer's decision process consists of his or her optimization problem (1), the milk production function (2), and the milk profit-replacement cost process (3). The model is completely characterized by a parameter vector θ, which consists of the parameters of the maintenance cost function γ_0 and γ_1; the parameters of the profit shock distribution μ_0, μ_1, and λ; the parameters of the production function β_0, β_1, β_2, ρ, and σ_ξ^2; and the parameters of the milk profit-replacement cost process α_{p0}, α_{pp}, α_{pc} α_{c0}, α_{cp}, α_{cc} σ_p^2, σ_c^2, and σ_{pc}.

Our model allows us to examine competing hypotheses that would explain why dairy producers replace cows more frequently than prescribed by dairy economists. One hypothesis is that maintenance costs increase with the age of a cow, mainly due to increased incidence of health problems. Significantly positive estimates of γ_1 would support this hypothesis. An alternative hypothesis is that there is an unobserved premium favouring replacing a cow, possibly due to unobserved systematic deviations between quoted market replacement costs and the actual replacement costs faced by the producers. Significantly positive estimates of $\mu_1 - \mu_0$ would support this hypothesis.

3. DERIVING THE LIKELIHOOD FUNCTION

Our empirical problem is to infer the parameter vector θ of the producer's dynamic decision model given a sample (x_t, i_t) of paired observations on the state and decision variables. We accomplish this by finding the vector θ that maximizes the likelihood of the observed sample.

To derive an explicit expression for the likelihood function, we begin by writing the functional equation that characterizes the producer's decision problem. To this end, let $V_\theta(x, \varepsilon)$ denote the maximum discounted sum of current and expected future profit starting from state x and profit shock ε. By Bellman's principle of optimality, the value function V_θ is characterized by Bellman's functional equation:

$$V_\theta(x, \varepsilon) = \max_{i=0,1} \{\pi(x, i) + \varepsilon(i) + \delta E_{x',\varepsilon'|x,i} V_\theta(x', \varepsilon')\} \tag{4}$$

Here, $E_{x',\varepsilon'|x,i}$ denotes the expectation over next period's realizations of the state variable x' and profit shock ε', conditional on the current state x and decision i.

It is more convenient to express the likelihood function, not in terms of the value function V_θ, but in terms of the function

$$U_\theta(x, i) = E_{x',\varepsilon'|x,i} V_\theta(x', \varepsilon') \tag{5}$$

which gives the maximum expected *future* profit if decision i is undertaken in state x. Substituting this expression into equation (4) allows us to rewrite Bellman's functional equation as

$$V_\theta(x, \varepsilon) = \max_{i=0,1} \{\pi(x, i) + \varepsilon(i) + \delta U_\theta(x, i)\} \tag{6}$$

This expressions implies that a producer observing state x and profit shock ε will make the decision i that maximizes the sum of current profits $\pi(x, i) + \varepsilon(i)$ and discounted expected future profits $\delta U_\theta(x, i)$.

Using standard properties of the Gumbel distribution (Ben-Akiva and Lerman, 1985), the probability of observing decision i conditional on state x and parameter vector θ is

$$\Pr(i \mid x, \theta) = \frac{\exp \lambda[\pi(x, i) + \mu_i + \delta U_\theta(x, i)]}{\sum_{j=0,1} \exp \lambda[\pi(x, j) + \mu_j + \delta U_\theta(x, j)]} \tag{7}$$

The likelihood of the entire sample of observed replacement decisions and state variable realizations as a function of the parameter vector θ is thus:

$$\begin{aligned} \mathcal{L}(\theta) &= \prod_t \Pr(i_t \mid x_t, \theta)\Pr(x_t \mid \theta) \\ &= \prod_t \frac{\exp \lambda[\pi(x_t, i_t) + \mu_{i_t} + \delta U_\theta(x_t, i_t)]}{\sum_{j=0,1} \exp \lambda[\pi(x_t, j) + \mu_j + \delta U_\theta(x_t, j)]} \cdot \Pr(x_t \mid \theta) \end{aligned} \tag{8}$$

Here $\Pr(x_t \mid \theta)$ is the likelihood of the observed realizations of the production process (2) and milk profit-replacement cost process (3).[3]

Given specific values for the model parameters θ and an expression for the expected future profit function U_θ, one could in principle evaluate the sample likelihood function (8). The function U_θ, however, lacks a known closed-form expression and is only implicitly characterized by a functional equation. Using standard properties of the Gumbel distribution, this characterization is obtained by integrating both sides of equation (6) with respect to ε'

$$E_{\varepsilon'|x'} V_\theta(x', \varepsilon') = \frac{1}{\lambda} \log\left\{ \sum_{j=0,1} \exp \lambda[\pi(x', j) + \mu_j + \delta U_\theta(x', j)]\right\} \tag{9}$$

and substituting into equation (5):

$$U_\theta(x, i) = E_{x'|x,i} \frac{1}{\lambda} \log\left\{ \sum_{j=0,1} \exp \lambda[\pi(x', j) + \mu_j + \delta U_\theta(x', j)]\right\} \tag{10}$$

To evaluate the likelihood function, we must solve this functional fixed-point equation for the expected future profit function U_θ. The method we use to solve for U_θ must be particularly efficient because U_θ must be rederived every time that the model parameter vector θ is perturbed by the hill-climbing routine seeking the maximum of the likelihood function.

[3]The production and milk profit-replacement costs processes are standard linear Gaussian models with conventional likelihood functions. For the sake of brevity, we do not explicitly present the likelihood functions for these processes here.

4. FIXED-POINT SOLUTION PROCEDURE

Because equation (10) generally does not possess a closed-form solution, $\dot{U}_\theta$ must be approximated numerically. Previous empirical studies of capital asset replacement have discretized the state space of the underlying decision problem and derived approximations for U_θ using standard discrete-space dynamic programming methods (Rust, 1987; Das, 1991). This approach works well for problems of low dimensionality, but becomes increasingly inefficient as the dimensionality of the state space rises.

To compute an approximation for U_θ, we turn to the polynomial projection method (Judd, 1991, 1992). The first step in implementing this method is to approximate U_θ using a finite linear combination of basis polynomials $\phi_1, \phi_2, \phi_3, \ldots, \phi_m$:

$$U_\theta(x, i) \approx \sum_{j=1}^{m} c_{ij}\phi_j(x) \tag{11}$$

Here, the basis polynomials ϕ_j are chosen by the analyst so that their linear combinations span the set of all polynomials of given degree n over the state space.

The second step in implementing the polynomial projection method is to relax the requirement that the functional fixed-point equation (10) be satisfied exactly at every possible state. In particular, the approximation for U_θ is asked to satisfy equation (10) at only m points selected by the analyst. The points, denoted $x_1, x_2, x_3, \ldots, x_m$, are called the collocation nodes. If the collocation nodes are wisely selected, equation (10) will be very nearly satisfied at the other states.

The polynomial projection strategy causes the original infinite-dimensional functional fixed-point problem (10) to be replaced with a finite-dimensional nonlinear root-finding problem. Specifically, to compute the coefficients c_{ij} of the polynomial approximant of U_θ, we must solve the $2m$ equation system

$$\sum_{j=1}^{m} c_{ij}\phi_j(x_k) = E_{x'|x_k,i}\,\frac{1}{\lambda}\log\left\{\sum_{i'=0,1}\exp\lambda\left[\pi(x', i') + \mu_{i'} + \delta\sum_{j=1}^{m} c_{i'j}\phi_j(x')\right]\right\} \tag{12}$$

for $i = 1, 2$ and $k = 1, 2, \ldots, m$. We solve the $2m$ equation system using Newton's method because it converges at a quadratic rate in the vicinity of the solution and because it generates, as a by-product, the analytic derivatives of the likelihood function. The availability of the analytic derivatives accelerates the maximum likelihood estimation process and makes it numerically more stable. Implementation of the Newton method and computation of the analytic derivatives are discussed further in the Appendix.

To implement the polynomial projection method, the analyst must specify the polynomial basis functions and the collocation nodes. Numerical analysis theory suggests that in most applications, the Chebychev nodes yield superior approximation results. Chebychev nodes are known to minimize the maximum approximation error under general conditions. Given this choice of nodes, the Chebychev polynomials make the best choices for the polynomial basis because they are orthogonal at the Chebychev nodes. Computing the coefficients c_{ij} of the interpolating polynomial will thus be significantly faster and numerically more stable than for other polynomial bases. Chebychev polynomial approximation is discussed in most numerical analysis textbooks (e.g., Atkinson, 1984; Judd, 1991). The Chebychev polynomial projection method for solving dynamic programs is illustrated in Judd (1992).

For the current econometric exercise, we implemented the Chebychev collocation method using a 1250-point grid constructed by using 10 different lactations and five Chebychev nodes each for milk production, profit contribution, and replacement cost. We assessed the accuracy afforded by this approximation strategy by numerically estimating, *ex post*, the root-mean-squared percentage error in equation (10) over the entire state space. The percentage error ranged from 0·0025% to 0·0091% for the models encountered in our exercise. The root-mean-squared errors computed over the states in our data set were even smaller. The results gave us high confidence that our polynomial projection algorithm solves the nested fixed-point problem with acceptable accuracy.

5. DATA COLLECTION AND CONSTRUCTION

Empirical estimation of the model required observations on cow-specific data (i.e. replacement decision, age, and milk production) matched with contemporaneous observations on the net profit contribution of milk and net cow replacement costs. The cow-specific data consisted of 2340 observations drawn from five Ohio dairy farms over the period 1989–92. The data were provided by the five dairy producers through the Dairy Herd Improvement Association.

The data exhibit a natural partial ordering. Each cow is observed over a series of lactations, then is replaced, and its successor is observed over a series of lactations, and so on. Cows that are linked by the predecessor–successor relation belong to the same natural sequence and may be thought of as occupying the same stall in the producer's dairy operation, though at different points in time. There are many sequences associated with each producer. At any point in time, different dairy cows will be observed at different stages of the lactation within a given dairy farm.

Net profit contributions, measured in dollars per pound of fluid milk, reflected the average profit contribution of a pound of milk for a lactation beginning on that particular month. Since milk production typically occurs over a 13-month lactation cycle a deflation procedure was used to state net profit contribution at the beginning of a lactation. To accomplish this, milk production, variable feed requirements, and other variable costs for a lactation were divided into monthly quantities using monthly lactation factors (Arbaugh, 1989) and other production coefficients (Ohio Cooperative Extension Service, 1991). Monthly milk production, feed quantities and quantities for other variable items were multiplied by their respective prices reflecting Ohio conditions (US Department of Agriculture, selected issues). Variable feed costs and other variable costs were subtracted from milk revenue to arrive at monthly revenue less variable costs. Monthly revenue less variable costs then were discounted back to the first month of the lactation and divided by total milk production during a lactation to determine net profit contribution. Data were collected for 1984 though the ninth month of 1992 and were inflated to reflect 1992 conditions using the gross national product implicit price deflator.

The monthly net replacement cost, measured in dollars per cow, equalled the price of a new heifer minus the value of the cow sold for slaughter. Prices of new heifers were obtained from the Ohio Holstein association. The value of slaughter cows was based on 14 cwt the average weight of Holstein cows leaving the herd, times the per cwt cow price reported in *Agricultural Prices*. Net replacement costs for 1984 to the ninth month of 1992 were inflated using the gross national product implicit price deflator.

6. ESTIMATION RESULTS

Our maximum likelihood estimation algorithm was developed using the Lahey F77L-EM/32 FORTRAN compiler version 5.1 and was implemented on a Gateway 2000, 90 MHz Pentium microcomputer running MS-DOS version 6.22. The sequential quadratic hill-climbing algorithm included in the NPSOL version 4.0 numerical optimization package was used to maximize the likelihood function (Gill *et al.*). The algorithm had little trouble converging to maximum of the likelihood function when initiated at the ordinary least squares estimates of the milk production function parameters and at zero levels for other parameters. In no instance did convergence require more than 12 minutes of computer time. The convergence criterion demanded that partial derivatives not exceed 10^{-9} in any direction.

Estimates for the milk profit contribution and cow replacement cost equations are presented in Table I.[4] As can be seen in Table I, estimates of the parameters relating profit contribution and replacement cost to their own lagged values are 0·93 and 0·97, respectively, both highly statistically significant. Profit contribution and replacement cost, however, are not significantly related to each other's lagged values. The equation disturbances possess variances of 0·55 and 31·77, respectively, and covariance 1·75.

The parameters of the production processes for each farm are also given in Table II. As can be seen in Table II, all production function parameter estimates β_i are significantly different from zero. The estimated production functions achieve a maximum in either the third or fourth lactation, a result that conforms to historical experience concerning milk production. The production disturbance variance σ_ξ^2 varies between 7·510 and 15·590 lb squared, suggesting considerable cross-cow variations in production. Cow-specific production shocks exhibit significant positive autocorrelation with ρ ranging from 0·52 to 0·57, confirming that above- and below-average productivity tend to be persistent cow-specific characteristics.

Table II also reports estimates of parameters that more directly impact the rate of dairy cow replacement. The first of these is the difference between the means of the producer-specific profit shock distributions: $\mu_1 - \mu_0$. As is typical of discrete choice models, only the difference of the means is identified. The estimated differences $\mu_1 - \mu_0$ are all positive, varying from a \$10 replacement premium per cow to a \$411 replacement premium per cow. Of these replacement premiums, two are statistically significant at the 0·05 level (farms 3 and 5).

Another parameter that directly impacts the rate of dairy cow replacement is the slope of the maintenance cost function γ_1. The constant term γ_0 is not identified because it enters as a fixed

Table I. Parameter estimates for milk profit contribution and cow replacement cost processes (standard errors in parentheses)

Milk profit contribution		Cow replacement cost	
α_{p0}	0·3912	α_{c0}	−0·3485
	(0·2431)		(14·2540)
α_{pp}	0·9345	α_{cp}	0·9704
	(0·0301)		(2·0330)
α_{pc}	0·0004	α_{cc}	0·9793
	(0·0005)		(0·0184)
α_p^2	0·5477	α_c^2	31·7710
α_{pc}	1·7476		

[4] The parameters of the production function and the milk profit-replacement cost equations were estimated separately from other model parameters to reduce the computational complexity of the estimation. The attendant loss of statistical efficiency should be small, however, because all producers live in the same geographical region and thus face essentially the same market prices.

Table II. Structural estimates and log likelihood for dynamic cow replacement model (standard errors in parentheses)

Parameter	Farm 1	Farm 2	Farm 3	Farm 4	Farm 5
$\mu_1 - \mu_0$	208·1254	177·8837	163·9458	9·8078	411·1992
	(144·9901)	(134·7364)	(78·0016)	(117·7283)	(83·1939)
γ_1	−44·1998	48·7311	15·9518	37·4285	4·9562
	(32·8634)	(55·9757)	(23·2785)	(28·7245)	(26·1608)
λ	0·0084	0·0086	0·0070	0·0064	0·0096
	(0·0019)	(0·0018)	(0·0009)	(0·0010)	(0·0017)
β_0	12·3087	16·0886	16·6089	17·1548	11·1466
	(0·6740)	(0·8668)	(0·4327)	(0·5464)	(0·4835)
β_1	4·5712	3·7157	3·1931	3·9387	4·2629
	(0·6272)	(0·9888)	(0·4319)	(0·5196)	(0·4594)
β_2	−0·7152	−0·5122	−0·4801	−0·4912	−0·4803
	(0·1251)	(0·2506)	(0·0935)	(0·1074)	(0·0920)
σ_ξ^2	9·6964	15·1132	11·0667	15·5892	7·5110
	(1·0319)	(1·2991)	(0·6341)	(0·9951)	(0·6492)
ρ	0·5683	0·5309	0·5158	0·5403	0·5562
	(0·0567)	(0·0556)	(0·0318)	(0·0346)	(0·0385)
Observations	252	335	763	643	347
Log likelihood	−444·9024	−708·9583	-1526·3895	-1320·0694	−586·8380

cost that is invariant across decisions and thus is irrelevant to the replacement decision. As seen in Table II, estimated maintenance cost increases associated with each lactation range from −\$44 to \$49. However, all the estimates are statistically insignificant, suggesting that maintenance costs do not increase with age and thus cannot explain the high replacement rates of our five dairy producers.

To further investigate potential causes of higher replacement rates, we performed a likelihood ratio test of the joint hypotheses that maintenance costs are constant and the replacement premium equals zero (i.e. $\gamma_1 = \mu_1 - \mu_0 = 0$). The test statistics, distributed Chi-square with 2 degrees of freedom, for farms 1 to 5 are 2·07, 9·18, 9·67, 2·99, and 21·30. Of these, three are statistically significant at the 0·05 significance level: farms 2, 3, and 5. The result for farm 2 is interesting, given that the hypotheses of constant maintenance cost and zero replacement premium could not be rejected independently. Convolution of the parameter estimates make it difficult to distinguish which factor is the more dominant. However, additional evidence suggests that unobserved replacement premiums are the more likely candidate. We re-estimated the model for farm 2 imposing the constraint $\gamma_1 = 0$ and found $\mu_1 - \mu_0$ to be significantly positive.

7. ADDITIONAL ANALYSIS

To test the validity of our model, we performed Chi-squared goodness-of-fit tests for each farm (Rao, 1973, Chapter 6). The test was performed by partitioning observations into separate categories according to lactation of the dairy cow and whether the milk profit contribution and the cost of replacement were above or below steady-state levels. The actual proportion of cows replaced is then compared to the predicted proportion of cows replaced within each category to derive a Chi-squared statistic with degrees of freedom equal to the number of categories containing observations, less the number of estimated parameters (8), less one. If actual and predicted replacement rates differ substantially, the Chi-squared statistic will be large and the model is deemed to provide a poor fit. As can be seen in Table III, the null hypothesis that our model is true could not be rejected for any of the five farms. We can thus conclude statistically that our model describes our data well. This is more informally confirmed through a comparison of the actual replacement rates to the replacement rates predicted by the model, which across farms do not differ by more than 0·2%.

To further test the validity of our model, we conducted a series of likelihood ratio tests that shed light on the replacement behavior of our producers. The first was a 'myopia' test, in which our dynamic optimization specification was tested against an alternative static specification in which the producer ignores future profits in making replacement decisions (i.e. δ in Bellman's equation (4) is zero). The likelihood ratio test statistics for farms 1 to 5 are 63·7, 97·7, 144·9, 134·7, and 59·8. Each of these test statistics, which are distributed Chi-square with 1 degree of freedom, are overwhelmingly significant, soundly rejecting the hypothesis that producers behave myopically and providing evidence that producers are dynamic optimizers. We also tested whether the eight parameters reported in Table II are homogeneous across producers. This test, which is based on a Chi-square statistic with 32 degrees of freedom, yielded a highly significant value of 653, confirming that there are significant differences in production and replacement patterns across producers.

Figures 1–3 illustrate the post-estimation analysis that can be conducted with an empirically estimated structural model of dairy cow replacement. The figures exhibit weighted averages across all five producers in our study. Figure 1 shows the probability that a typical dairy cow from our collective sample will be replaced given the cow's milk production relative to the average of other cows her age. The figure indicates that the probability of replacement declines with productivity, but increases with age. A cow producing 25% below average in the first lactation has a high probability of being kept; a cow producing slightly above average in the fifth lactation, on the other hand, has a high probability of being replaced. The actual numbers, of course, will vary across producers, although the same general patterns persists.

Figure 2 shows the expected value of keeping versus replacing a cow as a function of the cow's milk production, across lactation stages, for the average producer. For the purposes of illustration, we assume that the unit profit contribution of milk and the cost of replacement are at their expected long-run values and that the profit shocks are at their respective means. The figure indicates that the value of keeping a cow rises with productivity, but falls with age. The points at which the curves intersect the horizontal axis indicate the break-even production level, below which a cow is likely to be replaced and above which a cow is likely to be kept. The

Table III. Chi-squared goodness of fit statistic, actual and predicted replacement rates, and simulated replacement rate under a bST-induced 10% increase in production

			Percentage replacement		
	df	χ^2	Actual	Predicted	bST
Farm 1	13	1·31	12·3	12·4	12·9
Farm 2	9	0·17	18·5	18·4	18·3
Farm 3	15	2·25	18·2	18·1	18·0
Farm 4	17	2·95	12·8	12·7	12·7
Farm 5	17	1·39	12·4	12·2	11·4

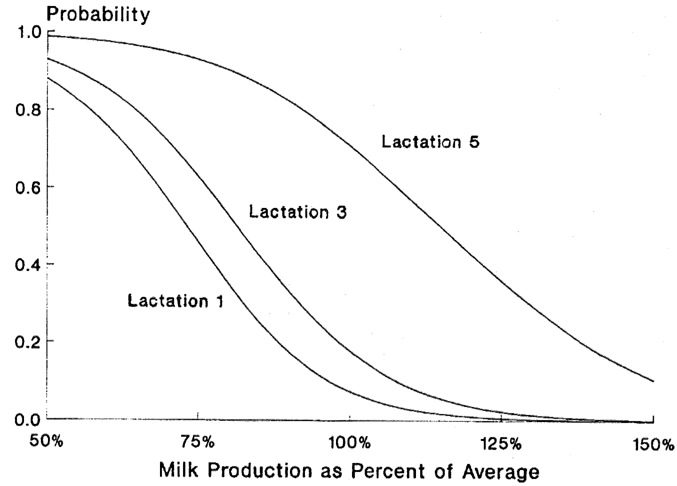

Figure 1. Probability of replacement

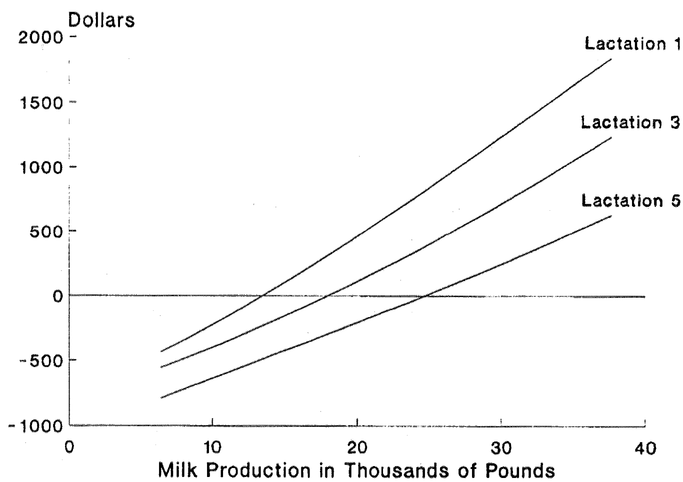

Figure 2. Expected value of keeping a cow

break-even level rises with age, from approximately 14,000 lb in the first lactation to 24,000 lb in the fifth lactation. This pattern is sensible because the productivity of a young cow is expected to rise, whereas the productivity of an old cow is expected to fall.

Figure 3 shows the rate of voluntary cow replacements as a function of the cow replacement cost and the unit profit contribution of milk. This figure indicates that cows are replaced more frequently, the lower the replacement cost and the lower the profit contribution. At a replacement cost 50% below the long-run mean of $374 per cow, our five dairy producers, on average, will replace approximately 28% of their herd annually; at a cost 50% above average, the turnover falls to approximately 20%. These estimates imply an elasticity of demand for replacement heifers of approximately −0·17. Replacement rates are less sensitive to variations in the unit profit contribution of milk. The turnover falls from 25% to 22% as the contribution

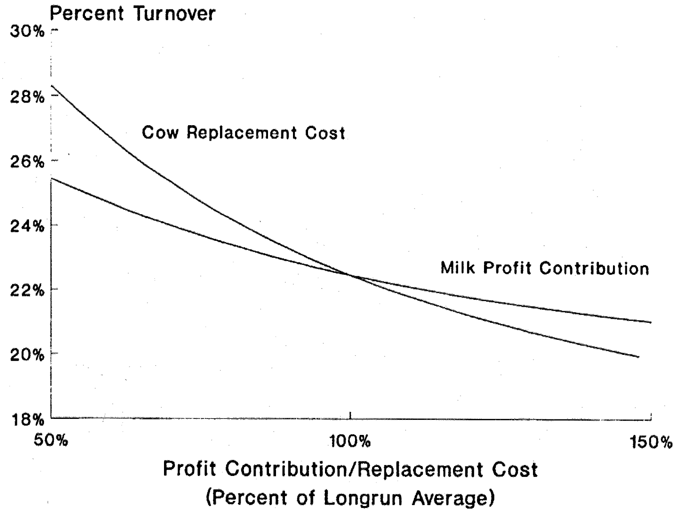

Figure 3. Annual turnover of dairy herd

rises from 50% below to 50% above the long-run mean of 8·33 cents per pound, indicating an elasticity of $-0{\cdot}07$.

Finally, we used the estimated models to examine the potential impacts of the introduction of bovine Somatotropin (bST). The addition of the growth hormone bST is expected to increase the milk production of cows between 5% and 10%. We rescaled all of the milk production parameters by the larger of these two factors, 10%, while holding all other model parameters constant, and recomputed each farmers optimal decision rule. The optimal decision rule was then used to compute the expected number of replacements across the states observed in our data sample. The expected number of replacements with bST, which are reported in Table III under the column heading 'bST', are not appreciably different from the rates that were observed without bST. The small impact is not surprising: although bST will increase the productivity of a cow, it will also increase the productivity of her replacement. The introduction of bST will not affect production differentials across cows, and thus will not have a significant impact on marginal incentives to exchange one cow for another.

8. CONCLUSION

We estimated dynamic discrete choice models of dairy cow replacement using historical farm-level data to identify reasons why observed replacement rates on dairy farms are higher than the rates typically prescribed by agricultural economists. From our estimation results, we find support that unobserved replacement premiums explain high replacement rates, at least on some farms. We find no support, on the other hand, that higher replacement rates are due to maintenance costs that rise with age.

Two explanations for a replacement premium come to mind. First, all the farms in our study grew most of their own replacement heifers, which we valued at the posted market rate. The market rate, however, may overstate the true opportunity cost of a replacement heifer, because it ignores transaction costs. Transaction costs in the market for heifers is significant because well-organized markets for replacement heifers do not exist and the cost of finding a willing buyer is high. Our observed replacement premium, therefore, may be directly due to an overvaluation of the true cost of replacement. Second, genetic progress occurs over time and replacement heifers are expected to produce more milk than their predecessors. Hence, a heifer's expected net return may be higher than predicted by our model, again leading to a replacement premium.

We draw three implications based on our empirical results. First, future research should more thoroughly examine the sources of replacement premiums. The importance of transaction costs could be assessed by collecting data on search costs associated with selling heifers. A modification of our model and a longer time series of replacement decisions could be used to assess the importance of genetic progress. Second, we caution dairy economists when offering replacement advice to dairy producers. Formal dynamic optimization models may be ignoring important factors that would imply faster optimal rates of replacement. Third, we suggest that future dynamic programming studies have stronger empirical underpinnings derived from structurally estimated models of decisionmaking of profitable producers. Carefully estimated models are more likely to reveal factors that are important to optimal decisionmaking, but that have been overlooked by the economic analyst.

Finally, in our estimation exercise, we have demonstrated how orthogonal polynomial projection methods can be effectively combined with standard maximum likelihood techniques to estimate structural models of stochastic dynamic decision making. In so doing, we effectively addressed two problems often associated with the estimation of dynamic decision models. First, the fixed-point functional equation approximation error afforded by the Chebychev polynomial projection method appears to be substantially lower than that afforded by more common space-discretization methods. Second, the computational effort required to estimate the model using these methods appears to be substantially lower than those previously reported in studies using space-discretization methods.

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APPENDIX: NEWTON METHOD AND ANALYTIC DERIVATIVES

Let

$$c = (c_{01}, c_{02}, \ldots, c_{0m}, c_{11}, \ldots, c_{1m})$$

denote the $2m$-vector of basis coefficients in the polynomial approximation for U_θ given in equation (11). Also let

$$f(c, \theta) = (f_{01}(c,, \theta), f_{02}(c, \theta), \ldots, f_{0m}(c, \theta), f_{11}(c, \theta), \ldots, f_{1m}(c, \theta))$$

denote the $2m$-vector-valued function, where

$$f_{ik}(c, \theta) = \sum_{j=1}^{m} c_{ij}\phi_j(x_k) - E_{x'|x_k, i}\frac{1}{\lambda}\log\left\{\sum_{i'=0,1}\exp\lambda\left[\pi(x', i') + \mu_{i'} + \delta\sum_{j=1}^{m} c_{i'j}\phi_j(x')\right]\right\}$$

for $i = 0, 1$ and $k = 1, 2, \ldots, m$.

For any given vector of model parameters θ, the vector of basis coefficients c in the polynomial approximation of U_θ are obtained by solving the $2m$ by $2m$ nonlinear equation problem

$$f(c, \theta) = 0 \tag{A1}$$

We solve the nonlinear equation system using Newton's method. This requires the formation of a sequence of iterates $c^{(n)}$ according to the updating rule $c^{(n+1)} = c^{(n)} + \Delta c^{(n)}$ where $\Delta c^{(n)}$ solves the linear equation system

$$\frac{\partial f}{\partial c}(c^{(n)}, \theta)\Delta c^{(n)} = -f(c^{(n)}, \theta) \tag{A2}$$

Suppose that the iterates $c^{(n)}$ converge to the value c and we wish to compute the derivatives of the likelihood function (8) at that point. Note that most steps required to differentiate equation (8) are straightforward and analytic, with the exception of differentiating U_θ with respect to 0. From equation (11), it is clear that the critical derivation is to compute the derivative of the coefficient vector c with respect to θ. Totally differentiating equation (A1), it follows that

$$\frac{\partial f}{\partial c}(c, \theta)\frac{\partial c}{\partial \theta} = -\frac{\partial f}{\partial \theta}(c, \theta) \tag{A3}$$

From equations (A2) and (A3) it is clear that the most time-consuming and demanding step in computing the coefficients c of the U_θ approximation (11) and in computing the derivatives of the likelihood function (8) involve the derivation and factorization of the $2m$ by $2m$ matrix of partial derivatives $(\partial f/\partial c)(c, \theta)$. To compute the matrix $(\partial f/\partial c)(c, \theta)$, note that

$$\frac{\partial f_{ik}}{\partial c_{i'j}}(c, \theta) = \begin{cases} -\delta E_{x'|x_k,i}[P_\theta(x', i'; c)\phi_j(x')] + \phi_j(x_k) & \text{for } i = i' \\ -\delta E_{x'|x_k,i}[P_\theta(x', i'; c)\phi_j(x')] & \text{for } i \neq i' \end{cases}$$

for $k = 1, 2, \ldots, m$, where

$$P_\theta(x, i; c) = \frac{\exp \lambda\left[\pi(x, i) + \mu_i + \delta \sum_{j=1}^{m} c_{ij}\phi_j(x)\right]}{\sum_{i'=0,1} \exp \lambda\left[\pi(x, i') + \mu_i' + \delta \sum_{j=1}^{m} c_{i'j}\phi_j(x)\right]}$$

Finally, to compute the vector $(\partial f/\partial \theta)(c, \theta)$, which is required to compute the derivative of the likelihood function, note that

$$\frac{\partial f_{ik}}{\partial \theta}(c, \theta) = -E_{x'|x_k,i} \sum_{i'=0,1}\left[P_\theta(x', i'; c)\left(\frac{\partial \pi}{\partial \theta}(x', i') + \frac{\partial \mu_{i'}}{\partial \theta}\right)\right]$$

REFERENCES

Atkinson, K. E. (1989), *An Introduction to Numerical Analysis*, 2nd edition, John Wiley, New York.

Allaire, F. R. (1981), 'Consequences of replacing cows with genetically improved heifers', *Journal of Dairy Science*, **64**, 1985–1995.

Allaire, F. R., H. F. Sterwerf and T. M. Ludwick (1977), 'Variations in removal reasons and culling rates with age for dairy females', *Journal of Dairy Science*, **60**, 254.

Arbaugh, D. (1989), *Economic Losses Associated with Clinical Mastitis on Nine Well-Managed Ohio Dairy Herds*, Unpublished masters thesis, Department of Agricultural Economics and Rural Sociology, The Ohio State University.

Ben-Akiva, M., and S. R. Lerman (1985), *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press, Cambridge, Mass.

Berkovec, J., and S. Stern (1991), 'Job exit behavior of older men', *Econometrica*, **59**, 189–210.

Burt, O. R. (1993), 'Decision rules for the dynamic animal feeding problem', *American Journal of Agricultural Economics*, **75**, 190–202.

Chavas J.-P., J. Kliebenstein and T. D. Crenshaw (1985), 'Modeling dynamic agricultural production response: the case of swine production', *American Journal of Agricultural Economics*, **67**, 636–646.

Congleton, Jr, W. R., and L. W. King (1985), 'Culling cows on predicted future profitability with a variable planning horizon', *Journal of Dairy Science*, **68**, 2970–2983.

Das, S. (1991), 'A microeconometric model of capital utilization and retirement: the case of the U.S. cement industry', *Review of Economic Studies*, **59**, 277–97.

Eckstein, Z., and K. I. Wolpin (1989), 'The specification and estimation of dynamic discrete choice models: a survey', *Journal of Human Resources*, **29**, 562–598.

Gill, P. E., W. Murray, M. A. Saunders and M. H. Wright (1986), 'User's guide for NPSOL (Version 4.0): a Fortran package for nonlinear programming', Department of Operations Research, Technical Report SOL 86-2.

Judd, K. L. (1991), *Numerical Methods in Economics*, Manuscript, Hoover Institution, Stanford University.

Judd, K. L. (1992), 'Projection methods for solving aggregate growth models', *Journal of Economic Theory*, **58**, 410–452.

McClelland, J. W., M. E. Wetzstein and R. K. Noles (1989), 'Optimal replacement policies for rejuvenated assets', *American Journal of Agricultural Economics*, **71**, 147–157.

Ohio Dairy Improvement Association, *Annual Summary of Milk Production of Ohio Herds on Record*, selected issues.

Ohio Cooperative Extension Service (1991), *1991 Ohio Dairy Enterprise Budgets*, Department of Agricultural Economics, The Ohio State University.

Rao, C. R. (1973), *Linear Statistical Inference and Its Applications*, 2nd edition, John Wiley, New York.

Rogers, G. W., J. A. M. Van Arendonk and B. T. McDaniel (1988), 'Influence of production and prices on optimum culling rates and annualized net revenue', *Journal of Dairy Science*, **71**, 3453–3462.

Rosenzweig, M. R., and Wolpin, K. I. (1993), 'Credit market constraints, consumption smoothing, and the accumulation of durable production assets in low-income countries: investments in bullocks in India', *Journal of Political Economy*, **101**, 223–244.

Rust, J. (1987), 'Optimal replacement of GMC bus engines: an empirical model of Harold Zurcher', *Econometrica*, **55**, 999–1033.

Rust, J. (1988), 'Maximum likelihood estimation of discrete control processes', *SIAM Journal of Control and Optimization*, **26**, 1006–1024.

Smith, B. J. (1972), 'Dynamic programming of the dairy cow replacement problem', *American Journal of Agricultural Economics*, **55**, 100–104.

US Department of Agriculture, National Agricultural Statistical Service, *Agricultural Prices*, PR-1, selected issues.

Wolpin, K. I. (1984), 'An estimable dynamic stochastic model of fertility and child mortality', *Journal of Political Economy*, **92**, 852–874.