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Structural Estimation of the Stochastic Dynamic Decision Problems of Resource Users: An Application to the Timber Harvest Decision

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This paper examines a method for structural estimation of the stochastic dynamic decision problems of resource users. The method involves nesting a stochastic dynamic programming algorithm within a maximum likelihood routine. An application to the timber harvest decision indicates that owners of slash pine plantations in southeast Georgia use a low discount rate and treat timber price as a random walk. Moreover, the harvest decision is apparently dominated by profit shocks overlooked in the usual models of the harvest decision.

I. INTRODUCTION

Resource economists employ fairly sophisticated models to explain how a resource should be allocated over time. So, for instance, economists turn to recent versions of Hotelling's theory of the mine when discussing how to allocate a finite stock over time, and to various derivatives of the Faustmann model to examine the issue of when to cut a timber stand. On the other hand, in statistical inference concerning the actual decisions of resource managers these models are treated rather allegorically, with the analyst turning to them for inspiration—to determine, for instance, what variables to use in linear regression—but never estimating the models directly. There are a number of explanations for this discrepancy between theory and applied research, but certainly among the most important is the conceptual and computational difficulty of estimating dynamic structural models under uncertainty. Over the past 10 years a literature concerning the estimation of such models has emerged, and no doubt will continue to grow in the years ahead. Reviews of this literature are contained in Rust [22], Eckstein and Wolpin [8], and Rust and Pakes [23]. Miranda and Schnitkey [20] and Fafchamps [10] provide recent examples in agricultural economics.

Structural estimation of dynamic models of resource use under uncertainty would seem a natural extension of this literature. This paper estimates a stochastic dynamic structural model of the timber harvest decision. The analysis provides estimates of the real discount rate used by forest owners in their harvest decisions, and of the price process used to forecast future timber prices. In its investigation of the discount rate, the paper returns to an issue first examined by Berck [2], the seemingly perennial charge that timber firms cut their timber "too quickly," as reflected in an implicit discount rate that is too high. Berck examined this issue via estimation of a set of aggregate demand and supply equations. Here the issue is revisited with firm-level data and a firm-level model.

For various stationary processes the optimal harvest policy is a reservation price policy characterized by the result that harvest proceeds if the observed price of

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timber exceeds a state-dependent reservation price (where the state variable is the volume or age of the timber stand), and is postponed otherwise. The logic underlying a reservation price policy is that when prices are relatively low it is better to wait for higher prices. Brazee and Mendelsohn [5], Lohmander [17], and Haight and Smith [14] find that with stationary timber prices a reservation price policy may yield significant gains to forest owners compared to the usual "fixed rotation" harvest policy derived from a deterministic (Faustmann) representation of the harvest problem. Washburn and Binkley [27] respond that if the market for timber stumpage is efficient, the expected speculative gain from postponing harvest in anticipation of higher future prices is zero. Their empirical results suggest markets are indeed efficient. Haight and Holmes [13] find that the empirical results of Washburn and Binkley may be due to averaging of quarterly prices, and they reiterate the conclusion of previous authors that a reservation price policy may provide substantial gains to forest owners. Finally, various papers have examined the optimal harvest policy when prices follow a random walk (the aforementioned Haight and Holmes, as well as Thompson [24], Clark and Reed [6], and Reed and Clark [21]). This nonstationary price process is of particular interest because it is consistent with informational efficiency in the stumpage market. Results indicate that when stand management costs are zero and timber growth is monotonic, the optimal harvest policy is to cut at the deterministic Faustmann rotation age—the problem engenders a certainty equivalence analog. Otherwise the optimal policy is state-dependent, although it takes a form quite different from the reservation price policies derived when the price process is stationary.

Lost in the discussion about the best price process to use in the harvest decision is the question addressed below: what price process do forest owners actually use? The next section introduces an estimable model of the timber harvest decision. Section III discusses the application of the model in estimation of the pulpwood harvest decision in southeast Georgia, with special emphasis on some of the methodological issues peculiar to the study. Section IV presents estimation results. Results generally indicate that forest owners use a low discount rate, and treat the current price of timber as the best estimate of future prices. The hypothesis that forest owner price expectations are rational, in the sense that in their harvest decisions they use the "true" price process derived from independent time-series estimation, is rejected. Results also indicate that the harvest decision is dominated by a profit shock observed each period by the timber owner but not by the analyst. In other words, despite the emphasis in the recent literature on the optimal timing of harvest with respect to price, other factors embodied in the profit shock—perhaps inventory scheduling at the mill site, cash flow management, and so on—seem to be more important in the harvest decision.

The concluding section examines the policy implications of estimation results, and offers several remarks concerning the opportunities and limitations of structural estimation of stochastic dynamic models of resource use.

II. AN ESTIMABLE MODEL OF THE TIMBER HARVEST DECISION

Rotation forestry is a complicated enterprise involving decisions about site preparation, tree planting, precommercial and commercial thinning, and final harvest for pulpwood or sawtimber. By focusing on mature timber stands, and using the market price of bare forestland as a proxy for the expected net present value of future earnings from forestry after the initial timber stand is cut, a simple albeit empirically useful model of the timber harvest decision can be developed. This is the approach taken here. Consider a planted timber stand that is already merchantable and sufficiently thinned, as indicated by characteristics of the stand and accepted forestry management practices, so that the only remaining decision for the forest owner at time t is whether to clearcut his timber or to postpone harvest for another period. The model below supposes that all trees in the stand are either identical or sufficiently similar so that the harvest decision can be cast as a matter of whether or not to cut the "representative tree"; put another way, the state of the forest is sufficiently represented by mean volume per stem. Let i denote the harvest decision, with $i = 0$ if harvest is postponed, and $i = 1$ if the stand is cut. The information the forest owner uses to make his harvest decision is contained in three state variables. The first is the net price of timber stumpage, p_t. Here we assume that forest owner price expectations about future timber prices are characterized by a first-order Markov process,

$$p_t = \alpha_0 + \alpha_1 p_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is a normal random variable, independently and identically distributed over time, with mean zero and variance σ_ε^2.

This specification is sufficiently general to include an appropriate version of the rational expectations hypothesis for a storable commodity (see, for instance, Washburn and Binkley [27]). For such commodities, rational expectations ultimately assure that prices are "smoothed" over time, in the sense that anticipated future price shocks are captured in current prices. Typically the market price evolves according to

$$p_t = E_t\{p_{t+1} - c_t\}e^{g_t - r}, \tag{1a}$$

where r is the discount rate, c_t is the cost of storing the marginal unit of timber at time t, and g_t is the marginal rate of timber growth. In general, estimation of a model with this price process requires an additional state variable, the stock of timber in storage, because both c_t and g_t depend on the size of the commodity stock. However, as argued by Washburn and Binkley, timber stocks change only slightly over time, and certainly this change across quarters or even years is virtually imperceptible. For instance, in the interval 1982–1987 the timber stock in Georgia fell at an annual rate of less than one-tenth of one percent. In this case, c_t and g_t may be treated as constants, so that (1a) is representable by (1).

The issue of rational expectations aside, one might take exception to so simple a specification as (1). While it may be true that forest owner price expectations are considerably more elaborate than this, it also seems reasonable that in a decision as complicated as the harvest decision, forest owners distill their price expectations to something representable by (1). For much more elaborate specifications the problem is unmanageable.

The second state variable is the merchantable volume of timber per stem (per tree) at time t, v_t. When the stand is not harvested at time t, volume per stem at time $t + 1$ is given by,

$$v_{t+1} = f(v_t), \tag{2}$$

where the tree is always growing over time, at least in the economically relevant range; formally, $f(\cdot)$ is monotonically increasing over v_t, with $f(\cdot) > v_t$.

The last state variable is a random revenue shock associated with the decision to harvest timber. The value of this shock per stem is denoted by θ_t. This disturbance term is a normal random variable, independently and identically distributed over time, with mean zero and variance σ_θ^2. At time t it is observed by the forest owner but *not* by the econometrician.

Formally, let $\pi_t^i(p_t, v_t, \theta_t)$ denote the expected net present value per stem of timber management given decision i at time t. The forest owner's maximization problem at time t can be stated,

$$J(p_t, v_t, \theta_t) = \max\{\pi^0(p_t, v_t, \theta_t), \pi^1(p_t, v_t, \theta_t)\}, \tag{3}$$

subject to (1) and (2), and with

$$\pi^0(\cdot) = \beta E\{J(p_{t+1}, v_{t+1}, \theta_{t+1})\}, \tag{4}$$

$$\pi^1(\cdot) = p_t v_t + \gamma + \theta_t, \tag{5}$$

where β is a discount factor and γ is the expected value of the bare land required to grow a tree; it is equivalent to the soil expectation value (SEV) that foresters use to denote the imputed value of bare forestland. Initially it might seem surprising that the price of bare land is not expressed as a function of the current stumpage price p_t. Insofar as the value of bare forestland reflects the capitalized value of future timber harvests, and a single timber stand requires at least fifteen years to reach merchantable (saleable) size, this is a reasonable assumption.

The expression in (4) reflects Bellman's principal of optimality [1]; in particular, the expected value of not harvesting in period t is derived with the understanding that optimal decisions about harvesting are made in the future. The expression in (5) indicates that the profit shock θ_t arises from a variety of sources. It captures deviations from the mean price of bare land (γ), as well as revenue shocks directly associated with the harvest operation. It can also be understood to represent a (random) shadow price of harvesting arising in vertically integrated timber firms.

The harvest problem (1)–(5) can be solved via the recursive methods of dynamic programming (DP). With an upper limit on stem volume, the problem is especially tractable because the value of bare land is not endogenous to the problem and timber volume increases monotonically over time. In this case the optimal harvest policy may be found by backward recursion over a finite horizon problem in which the timber volume in the first stage of the recursion (last period of the problem) is the upper limit on timber volume per stem, $v_{\max}$, and timber volumes in earlier periods (later stages) are obtained by inverting (2). At each stage the state variables explicitly considered in the usual way are p_t and θ_t; the dimensionality of the problem is reduced from three state variables to two. Supposing that it requires N stages to work back to the lower limit of merchantable stem volumes, $v_{\min}$, this algorithm provides expected values of forestland at N stem volumes, which then serve as the basis for approximating $J(\cdot)$.

The reduction in the dimensionality of the DP problem comes at the cost of an additional parameter that ultimately requires estimation. The abbreviated finite horizon problem begins with a comparison of the value of harvesting timber at stem volume $v_{\max}$ and a fixed terminal value in the event of no harvest, ψ.

Supposedly the terminal value is actually a function of the state variables, but to the extent that $v_{\max}$ is chosen from the upper tail of the observed frequency distribution of harvest volumes, state-dependent fluctuations in the terminal value would be expected to have little effect on the optimal timing of harvest, and by extension the estimation of parameters. In other words, while it may be true that the current state of nature affects current expectations about the terminal value, in the case where $v_{\max}$ is large relative to the usual harvest volume, the feedback effect of the terminal value on the optimal timing of harvest is reasonably captured with a fixed parameter because other forces—the discount rate, the probability of observing relatively high values of θ_t before $v_{\max}$ is reached, and so on—serve to dilute the impact of state-related variability in the terminal value.[1]

Let $J(\cdot|\Gamma)$ denote the value function for the set of parameters $\Gamma = \{\gamma, \alpha_0, \alpha_1, \sigma_\varepsilon, \sigma_\theta, \beta, \psi\}$. With reference to (3), the likelihood that the analyst observes a harvest given state values v_t and p_t is simply

$$\begin{aligned} \Pr(i_t = 1|p_t, v_t, \Gamma) &= \Pr\big(p_t v_t + \gamma + \theta_t > \beta E\{J(p_{t+1} v_{t+1}, \theta_{t+1}|\Gamma)\}\big) \\ &= \Pr(\theta_t > \beta E\{J(\cdot|\Gamma)\} - p_t v_t - \gamma) \\ &= \int_A^\infty g(\theta)\, d\theta, \end{aligned} \tag{6}$$

where $A = \beta E\{J(\cdot|\Gamma)\} - p_t v_t - \gamma$, and $g(\theta)$ is the probability density function of θ. Given a time series of observations of the harvest decision on forest plot k, denoted by i_{kt}, $t = 1, \ldots, T$, the likelihood of the entire sample of K plots is

$$\begin{aligned} L(\Gamma) &= \prod_{k=1}^{K} \prod_{t=1}^{T} \Pr(i_{kt}|p_t, v_{kt}, \Gamma) \\ &= \prod_{k=1}^{K} L_k(\Gamma). \end{aligned} \tag{7}$$

Maximum likelihood estimates are obtained via a search over the likelihood function $L(\Gamma)$. Note, however, that at every iteration of the search the calculation of the likelihood value is possible only after the stochastic DP problem (1)–(5) is solved to obtain the expected value function $E\{J(\cdot|\Gamma)\}$ associated with the new set of parameters. In other words, the likelihood of observing a particular harvest given a set of parameters Γ cannot be determined without first solving the dynamic harvest problem implied by the parameters. This nesting of an "inner" stochastic dynamic optimization routine within an "outer" search for maximum likelihood estimates is the defining feature of the new generation of estimable stochastic dynamic models of behavior. It also explains the emphasis on parsimony in these models; computational feasibility places a premium on low dimensionality, both in terms of the number of estimable parameters and the number of state variables.

[1] The structure of the DP model is identical to that of Haight and Holmes [13] in two important respects. First, the value of bare land after harvest is represented by the fixed value γ. And second, there exists an upper bound on volume per stem. In Haight and Holmes, the fixed terminal value ψ is set equal to zero.

III. AN APPLICATION OF THE MODEL TO THE PULPWOOD HARVEST DECISION IN SOUTHEAST GEORGIA

A quarterly model of the harvest decision was estimated using data drawn from the U.S. Forest Service's Eastwide Forest Inventory Data Base [25]. This data base is constructed from periodic inventories of forest resources conducted in eastern states every five to fifteen years. In each inventory, fixed plots are revisited by Forest Service crews who record data concerning the state of the plot—timber volume, timber growth since the last inventory, timber harvests, year of harvest, tree mortality, forest biomass, species composition, and so on. The standard data base generated by the sixth inventory of the state of Georgia, completed in 1989, provided observations of the harvest decision for the six-year period beginning in the first quarter of 1982 and ending in the last quarter of 1987. The nature of these observations raises several issues in estimation; these are discussed below. Quarterly price data were obtained from Timber Mart South. The net price used in estimation includes a 31% tax rate on harvested timber, the average of the capital gains tax rate before and after passage of the 1986 Tax Reform Act. Numerical experimentation with the DP algorithm supports previous results in the literature that modest changes in tax rates have virtually no effect on the timber harvest decision [11], and so using a fixed tax rate simplifies the analysis without apparent bias.

Selection of Observations

A maintained hypothesis of the estimation procedure is that the basic structure of the harvest problem is the same across forest owners and over time. In an effort to make credible this hypothesis, estimation involved a relatively narrow set of observations. In particular, observations were limited to timber plots owned by forest industry firms in southeast Georgia. The focus on southeast Georgia was intended to control for differential growth due to climate, and to control for variability in economic conditions. The focus on forest industry firms (firms involved in the manufacture of forest products like paper and lumber) to the exclusion of the non-industry landowners, who own the majority of forestland in Georgia, reflects the predominant view that non-industry forest owners manage their forests for both timber production and recreational activities like birdwatching and hunting [3, 4, 7, 15, 18].

The set of observations was further restricted by considering only slash pine plantations on site indexes of 60 to 70 on a 50-year base.[2] Restricting the set of observations to a particular species and a narrow range of land quality makes more acceptable the assumption that the state equation concerning volume growth (Eq. (2)) is the same across observations. The decision to examine planted stands rather than natural stands also makes this assumption more acceptable, and makes good sense for two additional reasons. First, the act of planting the seedlings of a single species signals the forest owner's intention to manage a stand strictly for timber. Second, the variability in stem growth within a planted stand is generally lower than in a natural stand, which frequently includes a variety of species and a variety

[2] A site index provides a general sense of land quality. A site index of 60 on a 50-year base indicates that at full stocking a southern pine will reach a height of 60 ft. at age 50.

of age classes. Reducing this variability is important because within the model volume growth is deterministic; the lower the variability in growth, the more reasonable the argument that forest owners behave as if growth is deterministic.

The decision to limit observations to timber stands dominated by slash pine reflects an additional modeling consideration. Standard silvicultural treatment of slash pine plantations does not include commercial thinnings [26]. While slash pine responds well to precommercial thinnings in the sense that the growth rate of the remaining stems increases significantly, it does not respond well to thinning once it reaches a merchantable size. Thus the decision about when to harvest a merchantable stand of slash pine is uncomplicated by the possibility of thinning the stand.[3]

Estimated Parameters

A stem volume of one cubic foot corresponds to a stem measuring five inches in diameter at breast height, which is generally regarded as the minimum tree size for which harvest is economically feasible.[4] Trees with volumes between 1 ft.^3 (5-in. dbh) and 10 ft.^3 (9-inch dbh) can be harvested for pulpwood only; once a tree attains 10 ft.^3 it can be harvested for sawtimber. In other words, 10 ft.^3 is the tree size at which the timber stand evolves from one where the only timber output is pulpwood to one where the most valuable output is sawtimber. Explicitly modeling this transition is not conceptually difficult, but it does involve additional state variables and thus a significant increase in the dimensionality of the problem. A perusal of the relevant data suggests that modeling this transition is not a worthwhile endeavor; in southeast Georgia timber firms clearly manage their slash pine plantations for pulpwood. Of 4540 plot observations meeting the selection criteria described above, only 125 exceed the mean stem volume of 10 ft.^3. Of 62 plots harvested in the observation period 1982–1987, only six were harvested after mean stem volume reached 10 ft.^3; in all these cases, mean volume per stem was less than 12.5 ft.^3 (mean stem dbh was less than 10 in.).

The estimation below takes the perspective that slash pine plantations in southeast Georgia are managed for pulpwood. It avoids the complexity of modeling a timber stand's transition to sawtimber while recognizing the possibility that circumstances—for instance, consistently negative values of the profit shock θ_t—may cause the forest owner to grow its timber to sawtimber size. In the last period of the pulpwood harvest problem (first stage of the DP recursion) mean volume per stem is 10 ft.^3, and the fixed terminal value ψ denotes the value of the representative tree in the event that the stand grows to sawtimber size.

Eckstein and Wolpin [8] discuss problems with the identification of dynamic stochastic choice models, noting in particular that when an optimal policy is a function of a ratio or sum of parameters, only the ratio or sum can be identified. As with most nonlinear models, underidentification or quasi-underidentification is discernible only through numerical experimentation. Such experimentation with the parameter set Γ revealed quasi-underidentification of the parameters γ and ψ.

[3] This is supported by the data used in estimation. No commercial thinnings were observed on the slash pine plantations meeting the criteria described above.

[4] Diameter at breast height (dbh) is the traditional measure of tree size; it is the diameter of a tree 4.5 ft above the ground.

Although the *difference* between these parameters is identifiable, the actual values of these parameters is not, as indicated by a "ridge" in ψ–γ space, along which likelihood values were virtually identical. The source of the problem probably lies in the last period (first stage) of the DP formulation. Denoting the last period by T (not to be confused with the last quarter in the time series used in estimation), the probability of harvest is

$$\Pr(i_T = 1|\Gamma) = \Pr(\theta_T > \psi - \gamma - p_T v_{\max}), \tag{8}$$

and so in the terminal period at least, only the difference between ψ and γ can be identified. Apparently the additional information provided by the data concerning the harvest decision in earlier periods (later stages) improves matters only slightly, transforming the problem from one of strict underidentification to one of quasi-underidentification. In estimation the problem was avoided by setting γ equal to the market price of bare forestland, leaving six parameters for estimation.[5]

Estimation of the Volume State Equation

Estimation of the stochastic DP problem (1)–(5) begins with separate estimation of state equation (1). Unfortunately, the Forest Service's Eastwide Data Base does not contain quarterly data on timber volumes. However, the sixth inventory of Georgia provides a plot's mean stem volume, as well as growth during a remeasurement period between the fifth and sixth inventories; with these data mean stem volume at the start of the remeasurement period is obtained. In southeast Georgia the sixth inventory was conducted from January of 1988 to July of 1988. Plots were previously surveyed in the fifth inventory from November of 1980 to October of 1981. The average length of the remeasurement period is thus 28 quarters (seven years).[6] Estimation begins with a linear specification of (1), which implies a nonlinear rate of growth:

$$v_{t+1} = c_0 + c_1 v_t. \tag{9}$$

Forward substitution in (9) yields

$$\begin{aligned} v_{t+28} &= c_0 \sum_{i=0}^{28} c_1^i + c_1^{28} v_t \\ &= \hat{c}_0 + \hat{c}_1 v_t. \end{aligned} \tag{10}$$

The growth parameters $\hat{c}_0$ and $\hat{c}_1$ were estimated using 54 unharvested plots from the sixth inventory satisfying the selection criteria described above, and for which mean stem dbh was at least 5 in. (minimum size for a commercial harvest) at the start of the remeasurement period. Small variations in the remeasurement period

[5] According to the Georgia State Forestry Commission, the market price for bare forestland in southeast Georgia, site index 70, is \$300/acre. In the sample the average stand density is 228 trees/acre, and so the market price of the land required to produce a tree is \$1.31.

[6] The exact length of the remeasurement period for a plot is not known from the standard data base, because the dates on which a plot is surveyed in the fifth inventory, and resurveyed in the sixth inventory, is not known. This raises several estimation issues discussed below.

(on the order of several months) do not present a problem in estimation. This is apparent if one views recorded mean stem volumes in the sixth inventory as imperfect measures of volumes at the end of a 28-quarter remeasurement period. In other words, v_{t+28} is observed with (small) random error in the sixth inventory, in which case the estimation of (10) proceeds as usual. Results are reported in Table I. Estimates of the primitive coefficients c_0 and c_1 were recovered from the estimates of $\hat{c}_0$ and $\hat{c}_1$ via (10). Other attempts to use the Forest Service's Eastwide Data Base to estimate yield relationships are reported elsewhere [9, 12, 19].

State equation (9) enters the estimation of the pulpwood harvest decision in two ways. First, it is used in a nested DP algorithm to find the optimal harvest policy for a given choice of the parameter set Γ. To the extent that forest owners use this state equation in their own harvest decisions, its use in the DP algorithm is appropriate, even if "wrong" in the sense that growth is actually stochastic, or that for a particular site, state equation parameters deviate from those estimated. The possibility that forest owners use a representative, deterministic expression of volume growth in the harvest decision seems reasonable, especially in light of the cost of gathering detailed, site-specific growth data.

State equation (9) is also used to derive estimates of quarterly mean stem volumes for sample plots in the years between the fifth and sixth inventories (the observation period used in estimation), 1982–1987. Estimates are obtained via backward recursion in (9) from the observed "terminal" value recorded in the sixth inventory. Mean stem volumes generated in this manner can be considered unbiased estimates of "true" volumes, obtained analytically using the observed biological process generating stem volumes (the estimated state equation). In light of the good fit obtained for Eq. (10), approximation errors are presumed to be sufficiently small that their influence on likelihood calculations—arising via limits of integration, as in (6)—is essentially zero.

Estimation with Imperfect Observation of the Harvest Decision

The data base generated by the sixth inventory of Georgia provides an unbiased sample of observations of the harvest decision in southeast Georgia from the first quarter of 1982 (the first full quarter following completion of the fifth inventory in southeast Georgia) to the last quarter of 1987 (the last full quarter before initiation of the sixth inventory in southeast Georgia). Unfortunately, estimation using these observations involves several complications. The most important of these concerns the observation of harvest dates. For harvested plots, the data base provides mean volume per stem at harvest, $\bar{v}_k$, and the year of harvest (1982–1988), but not the

TABLE I

Estimation Results for the Volume State Equation

Parameter	Estimate	t-Statistic
$\hat{c}_0$	2.6825	15.79
$\hat{c}_1$	1.0430	19.93
c_0	1.0015	—
c_1	0.1020	—

quarter of harvest.[7] The appropriate response to this uncertainty is to treat the quarter of harvest as a random variable that may assume any of the four values indexing the quarters of the harvest year, in which case the likelihood value for harvested plot k, $L_k(\Gamma)$, is calculated slightly differently than indicated by (6) and (7). Let $t = 1, \ldots, 24$ index the sequence of 24 quarters comprising the observation period, and let T_k denote the last quarter of the year plot k is harvested. Given that harvest occurs in quarter $T_k - s$, $s = 0, 1, 2, 3$, volumes in preceeding quarters are known via backward recursion in (9) using the terminal volume $\bar{v}_k$, and estimation of conditional likelihood values (conditional on harvest in period T_{k-s})) proceeds as outlined in (6) and (7). The unconditional likelihood value is then the appropriately weighted sum of the conditional values.

Formally, denote the likelihood of observations on plot k given harvest at time $T_k - s$ by

$$L_k\left(\Gamma | v_{k, T_k - s} = \bar{v}_k\right), \qquad s = 0, 1, 2, 3.$$

The unconditional likelihood is a weighted sum of these values, where weights are the respective conditional probabilities

$$\Pr\left(v_{k, T_k - s} = \bar{v}_k \,\middle|\, \sum_{j=0}^{3} i_{k, T_k - j} = 1 \right). \tag{11}$$

Applying Bayes Theorem to (11) yields

$$\Pr\left(v_{k, T_k - s} = \bar{v}_k \,\middle|\, \sum_{j=0}^{3} i_{k, T_k - j} = 1 \right) = \frac{\Pr\left(v_{k, T_k - s} = \bar{v}_k\right)}{\sum_{j=0}^{3} \Pr\left(v_{k, T_k - j} = \bar{v}_k\right)}. \tag{12}$$

Presumably the probability of observing a plot with a particular mean stem volume does not change over the course of a year, and so from (12) the probability weights used to calculate the unconditional likelihood value for plot k are each .25.

The Estimation Algorithm

The algorithm begins with an initial guess of the six parameters to be estimated. These estimates are used in a dynamic programming (DP) subroutine to determine for all admissible states the optimal harvest decision, and more importantly for estimation, the expected value (per stem) of the timber resource given the decision *not* to harvest timber. A hill-climbing algorithm then searches for the parameter set that maximizes the likelihood function by iterating over the DP and integration algorithms.

The hill-climbing algorithm used in estimation is the Davidon–Fletcher–Powell algorithm available in GQOPT.[8] The integration algorithm is also in GQOPT. The

[7] Because of the way the data is recorded, the assertion that the year of harvest is known is perhaps problematic. Nonetheless, there is an interval in which harvest surely occurred. The approach taken here readily applies to whatever interval is deemed appropriate. Estimation with a six-quarter interval had no substantive effect on results.

[8] GQOPT is a package of FORTRAN code written by Richard E. Quandt and Stephen M. Goldfeld, Princeton University.

DP algorithm is original code. It differs significantly from the traditional approach in which continuous random variables are represented by a set of discrete values and an associated probability transition matrix. The structure of the problem enables its solution with only stumpage price and the profit shock θ entering the algorithm as state variables in the usual way. Because both the value of bare land and the value of the forest when it reaches sawtimber size are treated parametrically, and stem growth is deterministic, the harvest decision can be treated as a finite-horizon problem solved recursively from the largest admissible stem size (10 ft.^{3}) to the smallest admissible size (1 ft.^{3}); the 85 values for timber volume used in the algorithm correspond to the 85 stages of the recursion.

Of the two remaining state variables, the profit shock is independently distributed over time, and so it is readily integrated out of the problem at each stage via numerical quadrature, leaving stumpage price as the only explicit state variable in $E\{J(\cdot)\}$. One problem with using the traditional approach to deal with this remaining state variable is that the variance associated with the disturbance term changes with each iteration of the "outer" hill-climbing algorithm, implying that the appropriate grid used to discretize the state space must also change. The obvious solution is to use a very fine grid that yields reliable results for all reasonable values of the parameter set, but in general this involves a large set of states and excessive computer run times. An alternative is to approximate value functions via Chebyshev polynomials, as advocated by Judd [16]. This is the approach taken here. Value functions were approximated by degree 50 Chebyshev polynomials. Experimentation indicated these approximations give virtually the same results as a traditional approach involving a very fine grid over stumpage price, while reducing computer run times by a factor of 25.[9]

IV. ESTIMATION RESULTS

Estimation results are presented in Tables II and III, and Fig. 1. The unrestricted model (model 1) is characterized by a negative discount rate (the discount factor β is greater than one) and a price process fairly close to a random walk. In a search for a more believable model, estimation was restricted to the parameter space where $\beta < 1$. The best model in this subspace (model 2) is characterized by a discount rate of 2.6% and a price process that is indistinguishable from a random walk. A likelihood ratio test indicates that at any reasonable level of significance model 2 cannot be rejected as the "true" model. Moreover, table 3 indicates that the in-sample forecasts of models 1 and 2 are about the same. Both models seem to fit the data reasonably well, except in 1986, when both severely underpredict the number of harvests. Nonetheless, the models do distinguish between high harvest years (1986–1987) and low harvest years (1982–1984). One explanation for the discrepancy in 1986 is the passage of the 1986 Tax Reform Act. Although modest, unanticipated changes in the capital gains tax rate on harvested timber have virtually no effect on the optimal harvest policy, the *anticipation* of change can have a significant effect. Arguably this explains the high rate of harvest for 1986. Anticipating the increase in the capital gains tax rate in 1987, more firms than usual found it optimal to harvest in 1986.

Given the similarity of the in-sample forecasts of models 1 and 2, and the relatively small difference in their statistical performance, it seems reasonable to

[9] All code is available from the author.

TABLE II

Estimation Results[a]

Model no.	Model	Parameter β	ψ	α_0	α_1	σ_ε	σ_θ	Log like value	Like-ratio test statistic[b]
1.	Unrestricted	1.0201 (.009)	1.669 (.636)	−2.040 (.509)	1.062 (.021)	.004 (.495)	.4340 (.132)	−249.85	—
2.	Alternative "best"	.9742 (.0003)	2.057 (.234)	0.009 (.045)	1.000 (.002)	1.689 (.266)	.0616 (.012)	−253.76	7.82
3.	$\beta = .96$	.9600[c]	2.401	−9.702	1.209	13.732	.2425	−270.97	42.24
4.	Faustmann	1.0136	1.851	0.0[c]	1.000[c]	0.0[c]	9.0077	−262.66	25.62
5.	"True" price process	1.0140	1.866	9.080[c]	.654[c]	2.099[c]	8.8284	−262.50	25.30

[a]To simplify the presentation, the discount factor is presented in annual terms. So, for instance, a reported discount factor of .99 corresponds to an annual discount rate of 1%. Standard errors for the unrestricted model and the alternative "best" model are in parentheses.

[b]The test statistic is distributed as a chi-square variable with degrees of freedom equal to the number of restrictions. At the .01 significance level the null hypothesis is rejected if the value of the test statistic exceeds the following (degrees of freedom in parentheses): $\chi^2(1) = 6.635$; $\chi^2(2) = 9.210$; $\chi^2(3) = 11.345$; $\chi^2(6) = 16.8$. At the .1 significance level the null hypothesis is rejected if the value of the test statistic exceeds the following: $\chi^2(1) = 2.706$; $\chi^2(2) = 4.605$; $\chi^2(3) = 6.251$; $\chi^2(6) = 10.6$.

[c]Parameter value is not estimated in the model; it represents a restriction.

accept model 2 as the better model of the harvest decision; it has the advantage of a positive discount rate.

Figure 1 presents the harvest policy implied by model 2 as a contour mapping of profit shocks in the volume–price dimension. Results are presented for the full range of pulpwood volumes, and for prices ranging from \$10 per cord to the highest price observed in the sample, \$30.55 per cord. Contours denote standard deviations from the mean profit shock. For a given profit shock harvest occurs at all volume–price pairs that lie "below" the associated contour level. Alternatively, for a given volume–price pair, harvest occurs if the observed profit shock exceeds the value implied by the contour passing through the corresponding point in Fig. 1. So, for instance, when mean volume per stem is 5.0 ft.^{3} and the observed price per cord is \$15, harvest occurs if the profit shock is +1.5 standard deviations from its mean value (0) or greater, but not if the profit shock is +1.4 standard deviations or smaller.

The contour mapping displays a distinctive saddle shape. Unlike the typical stationary autoregressive model, the harvest policy is not characterized by a simple reservation price policy, where a stand with a given volume per stem is harvested if the observed price exceeds a reservation price. Here the harvest policy is more complicated, and often counterintuitive, reflecting the influence of profit shocks as well as random walk price expectations. For instance, at the high price of \$30/cord the contour for 1.5 standard deviations is intercepted at a stem volume of 7.0 ft.^{3}, indicating that at this price approximately 6.7% of slash pine plantations with a

TABLE III

Sample Harvests and the Mean Number of Harvests Predicted by Each Model

		Harvests						
Model no.	Model	1982 (sample = 4)	1983 (sample = 8)	1984 (sample = 5)	1985 (sample = 7)	1986 (sample = 19)	1987 (sample = 13)	Total (sample = 56)
1.	Unrestricted	3.57	5.24	6.16	10.12	12.52	12.38	49.99
2.	Alternative "best"	4.34	5.79	5.55	10.11	11.47	12.97	50.23
3.	$\beta = .96$	5.31	6.92	7.10	9.55	10.35	9.66	48.89
4.	Faustmann	5.54	6.88	6.43	8.06	8.16	7.31	42.38
5.	"True" price process	5.59	6.92	6.38	7.84	7.80	6.85	41.38

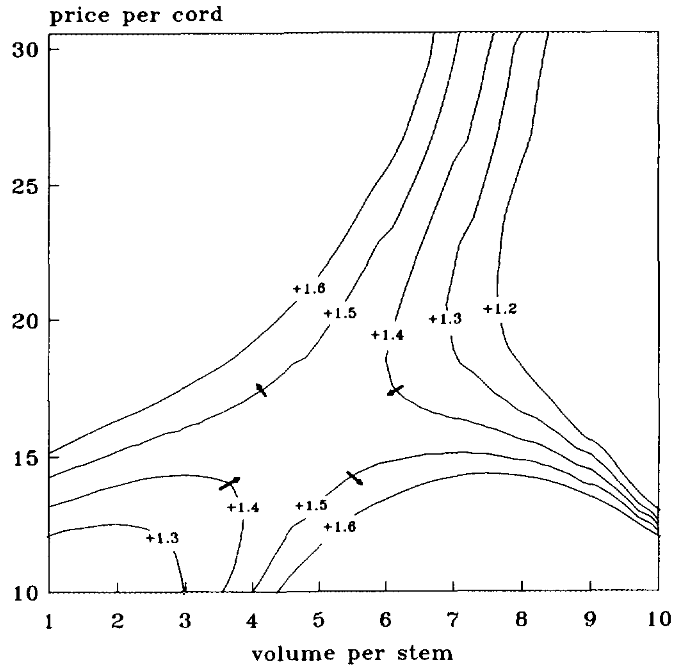

FIG. 1. Profit shock contour mapping of the optimal policy.

mean stem volume of 7.0 ft.^{3} are harvested.[10] By comparison, at the relatively low price of \$20/cord (which is lower than the lowest price in the sample, \$21.71/cord) the contour passing through 7.0 ft.^{3} is +1.3, indicating that at this price approximately 9.7% of slash pine plantations with a mean stem volume of 7.0 ft.^{3} are harvested. The small change in harvest despite the substantial change in price reveals the minor effect of price on the harvest decision; clearly the profit shock is the dominating factor. Surprisingly, what small effect price does have on the harvest decision is negative; the proportion of stands harvested is *higher* at the lower price. This result coincides with that found by Haight and Holmes [13] for a random walk.

Tables II and III also provide estimates for three restricted models. In model 3 the discount rate is restricted to 4%. This restriction is rejected at any reasonable level of significance. By comparison, Berck [2] estimated a discount rate of 5%. While the present analysis suggests a discount rate that is lower than commonly assumed, it nevertheless lends support to the general conclusion of Berck that the implicit discount rate used by timber firms in their harvest decisions is lower than the return available for other private investments. To the extent that rapid harvesting of timber is associated with high discount rates, the frequent criticism of timber firms for cutting timber too quickly is not supported.

Model 4 presents the case where timber owners solve a sequence of deterministic "Faustmann" decision problems. In each period they assume that with certainty

[10] The proportion of plantations for which the observed profit shock exceeds +1.5 standard deviations from the mean is 6.7%.

the observed price of timber will remain the price of timber in the future. This model yields high estimates for the standard deviation of the profit shock (recall that the value of bare land in the model is \$1.31) and is rejected at reasonable levels of significance.

Finally, model 5 restricts the price process used by forest owners to the "true" process obtained by estimating (1) independently of the decision model. The data used in estimation consists of a 15-year quarterly price series (1977–1991) for southeast Georgia reported in Timber Mart South. The sample autocorrelation function indicated stationarity of the price series. Diagnostic tests further indicated that an AR(1) process provides a good description of the underlying price process. The only reasonable alternative is an ARMA(1,1) process; residual autocorrelations from AR(1) estimation indicate that serial correlation in the disturbance term is possible.[11] Nonetheless, the evidence for the ARMA(1,1) is weak, and so parsimony dictates choosing the AR(1) process as the "true" process.[12]

The hypothesis that timber firms use this price process in the harvest decision is rejected at the .01 significance level. Moreover, Table II indicates that the model underpredicts harvests and fails to distinguish high harvest years (1986 and 1987) from low harvest years. For whatever reason, firms with slash pine plantations in southeast Georgia apparently do not use this price process in the harvest decision, instead preferring to treat prices as a random walk.

Finally, in all models the estimate of the terminal land value ψ appears to indicate that sawtimber harvesting is not an important consideration on slash pine plantations. By definition this value is the sum of the value of the bare land required to grow a tree and the imputed expected value of a tree when it reaches sawtimber size (10 ft.^{3}). Multiplying the sample average net stumpage price for pulpwood by ten cubic feet yields a value of \$1.35 for a sawtimber-sized tree. Adding this to the value of bare land used in estimation ($\gamma = 1.31$) yields a computed value for ψ of 2.66. The low value of ψ obtained in estimation provides weak evidence that in southeast Georgia slash pine plantations on land with a site index of 60 or 70 are managed for pulpwood even after reaching sawtimber size. At the very least the estimated values suggest that timber owners foresee no great advantage to holding timber to sawtimber size.

V. CONCLUSIONS

This paper presents an initial attempt to directly estimate a stochastic dynamic structural model of a resource extraction decision. Unlike previous attempts at statistical inference using micro-level observations of the timber harvest decision, the analysis provides estimates of the "primitive" parameters of the harvest decision, including the implicit real discount rate used by firms, and the parameters defining the price process used by firms to forecast future timber prices. Underlying the model is the maintained hypothesis that all timber owners follow the same decision process all the time. This position is generally untenable, and a more liberal and useful interpretation of results is that *on average* timber owners act as

[11] The p-value for the $Q(1)$ statistic is .091. The p-value for the $Q(5)$ statistic is .462. The p-value for the $Q(10)$ statistic is .813.

[12] For the AR(1) process, $R^2 = .4228$. T-statistics for the intercept and slope coefficients are 3.509 and 6.657, respectively.

indicated by the estimated model, though this possibility is not explicit in estimation and remains to be explored formally. Another significant modeling issue is the specification of the profit shock. It is possible that the true profit shock is not characterized by the simple distribution used in the analysis, in which case parameter estimates are distorted. With these caveats in mind, results seem to indicate that the decision to harvest slash pine plantations for pulpwood is characterized by a low discount rate and a price process close to a random walk. Moreover, results reveal the presence of a random profit shock that significantly impacts the harvest decision, suggesting that the optimal harvest policies usually suggested in the literature are incomplete; the decision about whether to cut a timber stand of a particular volume depends not only on the current price of stumpage, but on other variables that forest economists will find difficult to identify.

A number of issues are raised by estimation results; all are left for later investigation. Most prominent is the failure of forest owners to use the "true" price process in the harvest decision. Haight and Holmes [13] established that using a random walk to forecast prices when a stationary autoregressive price process is appropriate can lead to a considerable loss in expected net revenues. The authors note, however, that when price information is costly a simple fixed rotation policy may be optimal, and so one explanation of the random walk used by forest owners is that it simplifies the harvest decision and thereby reduces information costs. A related issue is whether government should begin forecasting timber prices. Ostensibly this would increase social welfare by improving the harvest decision. But the problem is more difficult than it first appears; if large numbers of forest owners used government price forecasts based on time series analyses, the underlying process generating prices would itself change, rendering the government forecasts obsolete.

Two obstacles stand in the way of widespread use, among natural resource economists, of the basic methodology presented above. The first is the difficulty of developing a model that captures the essential features of the resource decision problem while remaining computationally feasible. The ever-increasing speed of modern computers notwithstanding, parsimony will continue to be the defining feature of these models for the foreseeable future. Such parsimony implies compromises in the development of models.[13] The second obstacle is the substantial programming required for estimation. Although the GQOPT programming package was used in estimation, writing the DP algorithm and nesting this algorithm within the nonlinear optimization routine provided by GQOPT required considerable time and effort.

Still, the potential rewards from direct estimation of micro-level models of behavior in a dynamic, stochastic setting are obvious. By providing estimates of the primitive parameters of resource decision problems, direct estimation provides insights to firm behavior that are simply unobtainable via traditional "reduced form" techniques. Moreover, structural estimation avoids many of the pitfalls of

[13] Of course, compromises are often struck in "traditional" estimation, too, in the form of omitted variables, ad hoc specifications of functional forms, and so on. Perhaps the only real difference between the methodology presented above and more traditional forms of estimation is that the estimation presented above is so closely linked to a firm-level behavioral model that such compromise is less abstract than in traditional estimation.

reduced form estimation that arise due to the disengagement of estimation from an underlying theoretical model. A simple example makes this point. Recall from Fig. 1 that when forest owners forecast prices as a random walk, they are about as likely to harvest timber at low prices as at high prices. In traditional linear discrete choice regression analysis, this result is revealed as a nonsignificant coefficient on price. Such a result can mislead the analyst to either argue that timber firms ignore stumpage price in their harvest decisions, or more likely, rationalize the result as the outcome of various, often obscure countervailing effects. The real explanation is that the analyst fails to correctly link the statistical investigation to the appropriate model of behavior. Establishing this link more firmly should be a priority in empirical investigations of resource use.

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