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Author(s): Kenneth I. Wolpin
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ESTIMATING A STRUCTURAL SEARCH MODEL: THE TRANSITION FROM SCHOOL TO WORK

BY KENNETH I. WOLPIN[1]

This paper presents a finite horizon job search model that is econometrically implemented using all of the restrictions implied by the theory. Following a sample of male high school graduates from the youth cohort of the National Longitudinal Surveys from graduation to employment, search parameters such as the cost of search, the probability of receiving an offer, the discount factor, and those from the wage offer distribution are estimated. Reservation wages and offer probabilities are estimated to be quite low. Simulations are performed of the impact of the parameters on the expected duration of unemployment. For example, it is estimated that an offer probability of unity, as opposed to the estimate of approximately only one per cent per week, would reduce the expected duration of unemployment from 46 weeks to 20 weeks.

KEYWORDS: Duration of unemployment, job search, reservation wages.

THE PURPOSE OF THIS PAPER is to estimate a job search model that is econometrically implemented using all of the restrictions that are implied by job search theory. The usefulness of the structural approach in general is that it provides testable restrictions derived from the theory. With respect to job search, this approach also permits one to distinguish between competing views of the world— on one hand, the view that the labor market environment is one with numerous jobs and few takers, and, on the other, that few jobs are available for numerous takers, and permits the simulation of regime changes, e.g., changes in unemployment subsidies or in the probability of receiving job offers. This structural approach requires that the theory be taken as a very close approximation to the way things actually work rather than as a loose abstraction to be used in organizing thoughts. Because of the simplicity of the theory, however, this paper can be viewed not as a very close approximation of reality, but as a step towards developing the technology to estimate more complex models of job search behavior. My own view is that the empirical results are worthy of attention.

The starting point is a standard discrete-time search model with the following characteristics: the search horizon is finite due to an inability to borrow against future income; the probability of receiving a job offer (in a period of fixed length) is possibly less than unity and may be dependent on the duration of search; there is a cost of obtaining an offer in a given period that is not subject to choice; jobs last a lifetime and once employed capital market constraints are lifted; wage offers are random and prior offers cannot be recalled. The distribution of wage

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offers is known by the individual. The model is, in this form, only incompletely specified; the finite horizon is imposed on the problem rather than being solved for as part of a larger optimization problem. In order to close the model, it is assumed that the individual accepts the first offer received after reaching the end of the search horizon. This assumption may or may not conform to an optimal lifetime search strategy, although it is intuitively appealing. The estimation procedure is analogous to that found in Wolpin (1984); it is based on a recursive numerical solution of reservation wages embedded in a maximum likelihood estimation procedure.[2]

The model is estimated using the 1979 youth cohort of the National Longitudinal Surveys of Labor Market Experience (NLS). The transition from school leaving to the first job is studied using a sample of 144 white males who were graduated from high school in 1979 and who did not return to school thereafter. The longitudinal nature of the data enables individuals to be followed over a three year period from the exact week they were graduated from high school until either the last interview or the date of the first job.

The model is presented in the next section, followed by a discussion of estimation issues. Section 3 presents the data, the next section the results, and the final section a summary.

1. MODEL

Job finding behavior is assumed to correspond to a fairly standard finite horizon search model as presented in Lippman and McCall (1976). It is assumed that individuals who leave school are constrained by an initial asset position to accept an offer within some specified period. The length of the search horizon is taken as given; it is assumed that the individual has somehow decided upon an optimal rate of asset decumulation and thus an optimal consumption profile while searching.

Each period the individual either obtains an offer of employment, summarized by a (lifetime) wage, and accepts or rejects that offer, or does not obtain an offer. If the offer is rejected, the individual continues to search at some fixed cost. An offer arrives randomly each period and the frequency with which offers arrive cannot be influenced by conscious choice. The likelihood of receiving an offer may vary, in a known way, with the duration of unemployment. Wage offers are assumed to be i.i.d. and there is no recall. The job, once accepted, is assumed to last forever so that the wage offer is in terms of the present value of the wage stream; once a job is obtained, financial market constraints are assumed to be no longer binding. If the individual reaches the terminal date without receiving an acceptable offer, the individual is obliged to take the next offer that comes. The length of the horizon in a completely specified model would thus seem to

[2] This paper is part of a growing literature on estimating structural parameters of dynamic programming models of individual behavior. In the search context, recent papers are by Flinn (1982), Lancaster and Chesher (1983), Miller (1984), and Narendranathan and Nickell (1983).

depend among other things upon the level of initial assets, the rate of time preference, and the profile of offer probabilities.[3]

This search problem can be shown to possess the reservation wage property (see Lippmann and McCall, 1976); for each period the individual establishes an acceptance or reservation wage above which the job is taken and below which it is declined. Given the finite search horizon, the reservation wage (if positive) can be shown to be nonincreasing if offer probabilities are nonincreasing with duration and it must fall to the minimum feasible offer at the terminal date.

More formally, the value of search in period t, that is the value of entering period t without a job, is given by

$$V_t = P_t E \max[w_t, -c+\delta V_{t+1}] + (1-P_t)E[-c+\delta V_{t+1}] \qquad (t=1,\ldots,T-1),$$

$$V_T = [P_T Ew-(1-P_T)c] + \delta(1-P_T)[P_{T+1}Ew-(1-P_{T+1})c] + \cdots + \delta^{\tau}(1-P_T)(1-P_{T+1})\cdots(1-P_{T+\tau-1})[P_{T+\tau}Ew-(1-P_{T+\tau})c], \tag{1}$$

where P_t is the probability of receiving an offer at t, w_t is the present value of the wage offer received at t, c is the cost of search for an offer to be received one period ahead, δ is the subjective discount factor, T is the search horizon, and $T+\tau$ is the end of life. The discount factor, δ, reflects the degree of impatience with the unemployment state. All offer probabilities and search costs are known with certainty at the beginning of the search period, as is the distribution of wages. Because the information set does not change over time, the expectations operator in equation (1) is not time-dependent.

The reservation wage at t, that is, the wage at which the individual is indifferent between acceptance and further search, is

$$\xi_t = -c+\delta V_{t+1} \quad \text{for} \quad t=1,\ldots,T-1, \tag{2}$$
$$\xi_t \leqslant w_{\min} \quad \text{for} \quad t=T, T+1,\ldots,T+\tau.$$

The reservation wage is higher the lower the cost of obtaining the next period offer, the higher the mean of wage offers, the higher the probability of an offer, and the higher the subjective discount factor.[4] The reservation wage can be solved recursively beginning at T using equations (1) and (2). The condition that a job is accepted at t given an offer is

$$w_t \geqslant \xi_t. \tag{3}$$

[3] It may be useful to note at this point that the terminal date is estimated from the data and in principle could be as long as a lifetime.

[4] Some of these results require that the value of search be positive or that the mean wage offer exceed the cost of search. If the cost of search is sufficiently high one might pay not to have to search. The model does not rule out negative reservation wages because no alternative to search other than work is modelled. δ is meant to reflect psychic impatience associated with not having a job. If δ exceeds the market discount rate, then the individual would wait longer for a job than would be warranted by strict wealth maximization and vice versa.

In general, recalling the assumption that wage draws are i.i.d., the value of search at any $t < T$ can be written as[5]

$$V_t = P_t[E(w_t \mid w_t \geq \xi_t) \Pr(w_t \geq \xi_t) + \xi_t \Pr(w_t < \xi_t)] + (1 - P_t)\xi_t. \tag{4}$$

Substituting (4) into (2) yields a general and complex nonlinear difference equation for the reservation wage. Given a suitable distribution for the wage, numerical calculation of reservation wages for all $t = 1, \ldots, T-1$ is possible.

The normal and lognormal distributions are convenient because the conditional expectations in equation (4) necessary to calculate reservation wages have simple forms. Specifically, if u is a normal random variable with zero mean and finite variance, σ_u^2,

$$E(u \mid u > a) \Pr(u > a) = \sigma_u \phi\left(\frac{a}{\sigma_u}\right), \quad \text{and}$$

$$E(e^u \mid e^u > a) \Pr(e^u > a) = e^{(1/2)\sigma^2 u}\left[1 - \Phi\left(\frac{\ln a - \sigma_u^2}{\sigma_u}\right)\right]$$

where ϕ is the standard normal density and Φ the standard normal cumulative. Using these relationships, it is straightforward to show that in the case of a normal wage distribution, the value of search is given by

$$V_t = P_t\left[\bar{w} + \eta_t \Phi\left(\frac{\eta_t}{\sigma_u}\right) + \sigma_u \phi\left(\frac{\eta_t}{\sigma_u}\right)\right] + (1 - P_t)\xi_t, \tag{5}$$

where $w_t = \bar{w} + u_t$ and $\eta_t = \xi_t - \bar{w}$; likewise, in the lognormal case

$$V_t = P_t\left[\tilde{w}\, e^{1/2\sigma_u^2}\left[1 - \Phi\left(\frac{\eta_t - \sigma_u^2}{\sigma_u}\right)\right] + \xi_t \Phi\left(\frac{\eta_t}{\sigma_u}\right)\right] + (1 - P_t)\xi_t, \tag{6}$$

where $w_t = \tilde{w}\, e^{u_t}$ and $\eta_t = \ln \xi_t - \ln \tilde{w}$. Given values for the parameters, numerical solutions for the reservation wages and value functions can be obtained recursively beginning with the calculation of V_T.

2. ESTIMATION ISSUES

Consider a sample of I individuals, each of whom is observed from the date of permanent school leaving either to the date of first job or to an interview date. The only wage offer observed is that for the first job accepted; that is, rejected

[5] The simple reservation wage property of the model is lost if the wage is serially correlated, although the general solution method described in the text would still be valid. The formal complication introduced would be that $E_t V_{t+k}$, the expected value of search at period $t+k$ given the information available at t, would no longer be equal to V_{t+k} since new information about future wages would be obtained given a current wage realization. A similar complication would arise if offer probabilities varied in an unforseen manner, say due to the business cycle. This kind of complication would also be introduced if past offers could be recalled since the acceptable wage offer would depend upon the maximum of previous wage draws. Any of these alternatives would greatly complicate the estimation.

offers are unobserved.[6] For convenience assume that offers arrive weekly, if at all. If an individual's first job is observed to begin t weeks after the week of school separation, then the individual experiences $t-1$ weeks of unemployment prior to accepting the job.[7] If $h_j=1$ indicates employment in period j and $h_j=0$ unemployment, the probability that an individual will be unemployed in period t given unemployment up to t is

$$\Pr(h_t=0 \mid \text{unemployment to } t)=P_t \Pr(w_t<\xi_t)+(1-P_t). \tag{7}$$

The right-hand side is the probability of receiving an offer in period t times the probability of rejecting the offer, plus the probability that no offer is received. Similarly, the joint probability of receiving and accepting an offer of wage w_t given unemployment to t is

$$\Pr(h_t=1, w_t \mid \text{unemployment to } t)=P_t \Pr(w_t \geqslant \xi_t, w_t). \tag{8}$$

The likelihood function for the sample of I individuals, the first K of whom have unemployment duration t_i-1 and the next $I-K$ of whom have an incomplete spell length of duration l_i, is

$$\begin{aligned} L=&\prod_{i=1}^{K}\prod_{j=1}^{t_i-1}\Pr(h_j^i=0 \mid \text{unemployment to } j)\\ &\times \Pr(h_{t_i}^i=1, w_{t_i}^i \mid \text{unemployment to } t_i)\\ &\times \prod_{i=K+1}^{I}\prod_{j=1}^{l_i}\Pr(h_j^i=0 \mid \text{unemployment to } j). \end{aligned} \tag{9}$$

Given a sample of homogeneous individuals, it is clear that the reservation wage in any period cannot be greater than the smallest wage observed in the sample in that period; thus, the entire profile of reservation wages will be strongly affected by the minimum accepted wage in the entire sample. For example, if the smallest wage in the sample happens to be a first period wage, then if reservation wages decline with duration, all reservation wages will be lower than the first (and smallest) wage in the sample. In equation (8) $\Pr(h_t=1), w_t \mid \text{unemployment to } t)$ will be zero if the wage that is observed for an individual in the sample does not exceed the calculated reservation wage and would equal the marginal of w_t if it did. To reduce the influence of outliers in the observed wage distribution on the estimated parameters, it is assumed, not unreasonably, that wages are measured with error. In particular, suppose that $w_t=\bar{w}+u_t+\varepsilon_t$ in the normal case or $\ln w_t=\ln\tilde{w}+u_t+\varepsilon_t$ in the lognormal case where ε_t is the random error in measurement assumed to be normal and distributed independently of u_t.

[6] The fact that only accepted wage offers are observed makes it impractical to estimate search models in which wage offers are serially correlated or models with recall.

[7] As noted, the model does not allow for nonparticipation. It is possible to extend the framework to a three state model. There is some evidence that unemployment and nonparticipation are distinct states (Flinn and Heckman, 1982b).

Letting $u_t+\varepsilon_t=\theta_t$, and realizing that the employment decision involves a comparison of the *true* wage with the reservation wage, the likelihood function is given by

$$(10)\qquad L=\prod_{i=1}^{K}\prod_{j=1}^{t_i-1}[P_j Pr(u_j^i<\eta_j)+(1-P_j)]P_t \Pr(u_t^i\geq\eta_t\,|\,\theta_t^i)\Pr(\theta_t^i)$$
$$\times\prod_{i=K+1}^{I}\prod_{j=1}^{l_i}[P_j\Pr(u_j^i<\eta_j)+(1-P_j)]$$
$$=\prod_{i=1}^{K}\prod_{j=1}^{t_i+1}\left[P_j\Phi\left(\frac{\eta_j}{\sigma_u}\right)+(1-P_j)\right]$$
$$\times\left[P_t\left(1-\Phi\left(\frac{\eta_t-\rho(\sigma_u/\sigma_\theta)\theta_t^i}{\sigma_u\sqrt{1-\rho^2}}\right)\right]\frac{1}{\sigma_\theta}\phi\left(\frac{\theta_t^i}{\sigma_\theta}\right)$$
$$\times\prod_{i=K+1}^{I}\prod_{j=1}^{l_i}\left[P_j\Phi\left(\frac{\eta_j}{\sigma_u}\right)+(1-P_j)\right],$$

where $\rho=\sigma_u/\sigma_\theta$ and $\sigma_\theta=\sqrt{\sigma_u^2+\sigma_\varepsilon^2}$.[8] Thus, $1-\rho^2$ is the fraction of the wage variance accounted for by measurement error. A simple and convenient parameterization for P_j which allows for duration dependence is

$$(11)\qquad P_j=\Phi(m_0+m_1 j).$$

A negative (positive) value for m_1 implies that offer probabilities decrease (increase) with duration of unemployment. Introducing individual observed heterogeneity through exogenous determinants of the wage, cost of search, or offer probability causes reservation wages to differ across the population.

In a nonstructural search model where reservation wages are not derived from optimization, identification requires exclusion restrictions—in the case where there are exogenous regressors, some variables would have to appear in the wage equation that do not appear in the cost of search (see Kiefer and Neuman (1979), for example). In the structural model, because the estimation is derived explicitly from the theory (given distributional assumptions), exclusion restrictions are not necessary.[9] Explicitly solving for the reservation wage profile provides sufficient restrictions to permit identification of all of the parameters in the model, because η_t is itself a known function of all of the other parameters of the model.

Although this assertion about identification is demonstrated in the Appendix, the following summary may be given. Consider the duration data alone, i.e., ignore the wage data and for convenience assume a constant offer probability. Now, the data will yield sample fractions of individuals of given unemployment durations. Heuristically, since $\Pr(h_t=1\,|\,\text{unemployment to } t)=P[1-\Phi(\eta_t/\sigma_u)]$,

[8] In deriving (10), the joint normal distribution of u_t and θ_t has been decomposed into the conditional of u_t given θ_t times the marginal of θ_t, both of which are normal. In the case where w_t is lognormal, the likelihood function must additionally be multiplied by $1/w_t$, the Jacobian of the transformation.

[9] See Flinn and Heckman (1982a) for an extended and illuminating discussion of identification in structural search models with constant reservation wages.

and since all of the η_2's are nonlinear functions of only a few parameters, $\bar{w}$ or $\tilde{w}$, σ_u^2, c, δ, and P, it would seem that there are enough observations to identify all of the parameters. However, in the normal case, $\bar{w}$, c, and σ_u are not separately identified, while in the lognormal case, $\tilde{w}$ and c are not separately identified. With wage data, $\tilde{w}$, σ_u^2, σ_ε^2, and thus ρ, are also identified, and so wage and duration data together enable identification of all parameters. It should be stressed that the fact that the reservation wage changes through time in a systematic and recursive manner is critical for identification. As discussed above, the distributional assumption affects identification, and as shown below, it affects the empirical results as well. Nonparametric estimation of all of the parameters of the model is not, however, feasible.[10]

As just noted, the entire reservation wage profile is determined by a small set of structural parameters. The η_t's are connected through time by a nonlinear difference equation which can be solved numerically. An unrestricted version of the model can be estimated which essentially permits reservation wages to change each period in whatever fashion best fits the observed sample unemployment duration hazards. The number of parameters to be estimated in the unrestricted model is the number of weeks of data (an η is estimated for each week), plus the offer probability parameters, the mean of the wage offer distribution, its variance, and the measurement error variance. The likelihood function for the unrestricted model is given by (1) with η's as parameters. Notice that the unrestricted likelihood function is extremely general and should nest within it a large number of behavioral two-state search models. Although competing restricted models may not be nested within each other, they are all nested within the unrestricted model.

As is well known, positive duration dependence can be obscured by unobserved heterogeneity. Negative duration dependence can arise in the finite horizon search model only if the probability of receiving offers decreases with duration of unemployment (given a constant search cost and mean wage offer). Ignoring unobserved heterogeneity can lead to incorrect inferences in this model. I have not explicitly modelled search in a world with job separations and, therefore, cannot legitimately make use of multiple spells. Thus a fixed effect formulation which requires multiple spells for estimation is ruled out. A random effects model can be estimated either with an additional distributional assumption (e.g., Kiefer and Neuman, 1981; Flinn and Heckman, 1982b) or nonparametrically (as in Heckman and Singer, 1984). The latter is preferable because it is less arbitrary, though in general it is computationally more burdensome and does not yet have an asymptotic distribution theory. Unobserved heterogeneity can arise in several places—in the offer probabilities, in the wage function, or in the cost function. Limiting these possibilities would be necessary for tractability, though arbitrary. If, for example, there were N types of people in terms of some unobservable determinant of the probability of an offer, then the likelihood of observing any

[10] See Flinn and Heckman (1982a) for a discussion of these issues in the context of a constant reservation wage model.

given individual with a particular unemployment duration would be given by the weighted average of terms like those in equation (10), conditional on the person being of a particular type (recall that the reservation wage is affected by the offer probability determinants), where the weights are sample fractions of the different types. The number of types, the sample fractions, and the mass points would be estimated jointly with the other structural parameters of the model.[11] I do not pursue the issue empirically because of its computational burden.

3. DATA

The model is estimated using the 1979 NLS youth cohort. In part, to minimize computational costs, I chose a subsample of white males who were graduated from high school in 1979 and who did not return to school or enter military service by the 1982 interview date. Their duration of search was measured from their week of graduation to the week at which they accepted their first "real" job. A real job is defined to be one for which hours worked per week was at least thirty and which lasted at least three months. This latter restriction was imposed in order to exclude summer jobs held just after graduation and the former because I wanted to consider more or less full-time employment.

Table I gives the distribution of durations for the sample; both complete and incomplete spells are included. Incomplete spells arise for two reasons: (a) the individual had no real job by the 1982 interview, or (b) the individual had missing information which did not permit the dating of the first real job, e.g., the individual took a job but hours worked was missing. Surprisingly, a little more than 30 per

TABLE I

DURATION TO FIRST "REAL" JOB:[a] 1979 WHITE MALE HIGH SCHOOL GRADUATES

Number of Weeks	Frequency All Spells	Frequency Complete Spells	Per Cent All Spells	Cumulative Per Cent
1	45	45	31.1	31.1
2–13	17	13	11.8	43.1
14–26	14	11	9.7	52.8
27–39	9	5	6.2	59.0
40–52	14	9	9.7	68.7
53–65	5	2	3.5	72.2
66–78	1	0	0.7	72.9
79–91	4	2	2.8	75.7
92–104	8	6	5.5	81.2
105–117	5	5	3.5	84.7
118–130	4	3	2.8	87.5
131–143	5	0	3.5	91.0
144–156	12	0	8.4	99.4
157–166	1	0	0.7	100
	144	101		

[a] A "real" job is defined as a job of more than 30 hours per week that lasted at least three months.

[11] Heckman and Singer (1984) implement such a procedure for a continuous time model of unemployment duration. A similar procedure is implemented in Wolpin (1984).

cent of the sample had a real job directly upon leaving high school. However, an equal number did not have a real job within one year of graduation. The large percentage of first week job holders is consistent with the notion that the search process actually begins prior to graduation. It is relatively straightforward to incorporate in-school job search into the likelihood function of the previous section. In particular, if search begins k periods prior to graduation and if P_0 is the per period probability of receiving a job offer while in school, then the probability of observing an individual working the first week after graduation for wage w_{k+1} at that time is given by

$$\begin{aligned}(12)\qquad & P_0 \Pr(w_{k+1} \geq \xi_0, w_{k+1}) \\ & +[(1-P_0)+P_0 \Pr(w_{k+1}<\xi_0)][P_0 \Pr(w_{k+1}>\xi_1, w_{k+1})] \\ & +\cdots+[(1-P_0)+P_0 \Pr(w_{k+1}<\xi_0)]\cdots[(1-P_0)+P_0 \Pr(w_{k+1}<\xi_k)] \\ & \times(P_1 \Pr(w_{k+1} \geq \xi_{k+1}, w_{k+1}).\end{aligned}$$

Conversely, the probability of not working in the first week after graduation is

$$(13)\qquad \prod_{j=0}^{k} [(1-P_0)+P_0 \Pr(w_{k+1}<\xi_j)][(1-P_1)+P_1 \Pr(w_{k+1}<\xi_{k+1})].$$

Note that P_1 is the probability of receiving an offer in the first period after graduation.

It is not possible to separately identify k, P_0, and the reservation wage path, $\xi_0, \ldots, \xi_k$ only from data on the fraction of the population working in the first period after graduation. One cannot estimate more than one parameter from a single sample statistic. The reservation wage path from $t=k+1$ to $T+k$, where $T+k$ is the total search horizon including in-school search, is unaffected by k or p_0 for fixed T (the horizon after graduation) and given k, P_0 will adjust so that the probability of working the first period after graduation given by (12) will exactly equal the sample proportion no matter what is the reservation wage path from $t=0$ to k. Using the exact week the job was taken prior to graduation, as is available in the data, would require the presumption that all such individuals taking these jobs knew that they would be the first real job after graduation, but many individuals work during school in jobs that do not carry over to the post-graduation period. I chose k to be 61 weeks because that was the longest period prior to graduation observed for an individual working the first week after graduation. The offer probability while in school is thus predetermined by this choice. An arbitrary reduction in k would increase P_0; if k were set to zero, $P_0 \Pr(w_{k+1}>\xi_{k+1})$ would be the sample fraction observed working in the first week after graduation.

It is assumed that there are only two states after graduation, working or searching. As already noted, to accommodate time out of the labor force would require further modelling or the optimization problem. In addition, about 40 per cent of the sample had at least one non-real job (less than 30 hours per week and/or less than three months duration) after graduation, but before the real job (if they even had one). Any rationale for taking such jobs, such as to finance search for a real job, would also have to be explicitly incorporated into an optimal search model. To do so would seem to require an explicit characterization of the consumption process or a different search model, e.g., job matching.

4. RESULTS

The model was estimated separately under normal and lognormal assumptions for the true wage and measurement error distributions. The results for the normal model were not reasonable: the mean of the wage offer distribution was negative, the cost of search was negative and unrealistically large, and the discount factor was almost zero. For this reason attention is restricted to the lognormal model. One should recognize that estimation of search models requires some distributional assumption because the observed wage distribution does not represent that of the population (Flinn and Heckman, 1982a).

Table II presents estimates of two versions of the model, one without regressors and the second with regressors in the wage equation. In the estimation, an annual discount rate of five per cent was applied to weekly earnings to form a lifetime wage and τ was set to 500 weeks in calculating the final period value function. The length of the horizon, along with the other parameters, was estimated in specification one and then assumed to be the same in specification two.[12] The

TABLE II

MAXIMUM LIKELIHOOD ESTIMATES OF SEARCH MODEL PARAMETERS

	(1)		(2)	
	Coefficient	Asymptotic Standard Error	Coefficient	Asymptotic Standard Error
Wage				
Constant ($\tilde{w}$)	166	4.90	157	8.3
AFQT (β_1)	—	—	.038	.0001
Father's schooling (β_2)	—	—	.327	.001
Offer Probability				
Constant (m_0)	−2.08	0.038	−2.10	.06
Duration (m_1)	−.0025	.00009	−.0023	.0002
Search Cost (c)	104	2.29	223	19.6
σ_u	.499	.014	.439	.03
ρ	.994	.0002	.953	.066
δ	.999	.00001	.999	.00014
T[a]	54	—	54	—
k[b]	61	—	61	—
Log Likelihood[c]	−434.22		−430.97	

[a] T is the search period after graduation.
[b] k is the search period prior to graduation.
[c] p_0 is also estimated, but as noted, its value is determined exactly by the choice of k.

[12] With different values of T the entire reservation wage path shifts. With offer probabilities that are independent of duration the reservation wage is the same for any given number of periods prior to T for all values of T. See the Appendix for further discussion.

GQOPT program was used for the numerical optimization—first NMSIMP, a simplex search routine, was used to reach the neighborhood of the optimum; then GRADX, a gradient method, was used to generate the final values and the standard errors. First and second derivatives of the likelihood function were calculated numerically in the GRADX routine.[13] To economize on computation, only two explanatory variables were used and both were transformed to be dichotomous. Father's schooling was set to unity if the youth's father was at least a high school graduate and was zero otherwise, while the AFQT score, an ability measure, was set to unity if it was above the average of youths in the sample and was zero otherwise. Restricting the study to only these four types of individuals allows us to calculate reservation wages only four times for each evaluation of the likelihood function, whereas if the variables had been permitted to vary continously, each individual would have had a unique reservation wage path. Descriptive statistics are presented in Table III.

An initial model was estimated in which both regressors appeared not only in the wage equation ($\ln w = \ln \tilde{w} + \beta_1 AFQT + \beta_2 FS$) but also as determinants of the post-high school offer probability and the cost of search. However, the restrictions implied by specification two in Table II were not rejected by an appropriate likelihood ratio test.[14] In addition, the log likelihood value for a model which rules out any search behavior, that is, one in which any job is taken once offered, was -437.51. The search model, although it nests this Bernoulli-type model, requires restrictions on the boundary of the parameter space, $\delta = 0$, so that the likelihood ratio test is not valid. Loosely speaking, nonzero reservation wages derived from an explicit search model seem to significantly improve the fit.

TABLE III
DESCRIPTIVE STATISTICS

	Mean	Standard Deviation
Weekly earnings: completed spells	205	121
Duration of unemployment: completed spells	27.3	37.9
Duration of unemployment: incomplete spells	88.6	54.8
Father's schooling more than 11 Years (FS = 1)	.56	
AFQT score above sample mean (AFQT = 1)	.58	
Weekly earnings (FS = 1, AFQT = 1)	221	130
Weekly earnings (FS = 1, AFQT = 0)	204	85
Weekly earnings (FS = 0, AFQT = 1)	206	168
Weekly earnings (FS = 0, AFQT = 0)	177	77

[13] A single iteration of GRADX took approximately four minutes of CPU on an AMDAHL 470 for the model with no regressors and several times longer with regressors. Standard errors seem too small and should be viewed with caution because the numerically calculated second derivatives from GRADX may be inaccurate. The convergence level in GRADX was set at 10^{-6} in terms of the relative change in the likelihood value.

[14] The log likelihood for the unrestricted model was -429.38 so that the appropriate chi-square statistic is 3.2 which, given 4 degrees of freedom, is less than the critical chi-square at conventional levels.

The parameters themselves are not unreasonable. From specification one the estimated mean wage offer ($\tilde{w}\, e^{(1/2)\sigma_u^2}$) is 188 dollars per week, which is about 17 dollars per week less than the mean of observed wages. The estimated mean wage for the group with (a) low AFQT and low father's schooling is 173; (b) low AFQT and high father's schooling is 240; (c) high AFQT and low father's schooling is 180; and (d) high AFQT and high father's schooling is 250.[15] The cost of search was estimated to be 104 dollars per week when there were no regressors and 223 with regressors; the latter estimate especially would seem high if it reflects only direct search costs such as transportation, meals, etc. Little of the total wage variance (1.2 per cent in specification one and 9.2 per cent in specification two) is attributable to measurement error. The probability of receiving an offer is 1.28 per cent per week upon graduation from high school; it declines thereafter quite gradually to .91 per cent at T. Recall that the declining offer probabilities may arise because of unobserved heterogeneity. The estimated weekly discount factor is very close to unity in either specification. The search horizon is estimated to extend for approximately one year (54 weeks) after graduation, at which time it becomes optimal to accept any positive offer. From the limited experimentation that I have done, it appears that for $T > 54$, reservation wages become negative around week 55 after graduation. Thus, for individuals with an unemployment duration of 55 weeks or longer, the hazard rate is unchanged for any $T \geqslant 54$. Because reservation wages are so low and the reservation wage path changes little for longer horizons, the results are not very sensitive to T.

To assess the fit of the model, the observed hazard rates are compared to the hazard rate pattern predicted by the model. Table IV shows this comparison only for specification one because cell sizes are two small when the four groups in specification two are distinguished. The first week predicted hazard is by construction exactly equal to the observed hazard. The general decline in the observed hazard rate is picked up by the model, although the peculiarly large hazard rates around one year and two years of leaving high school are not predicted by the model. It is possible that some of this clustering in reported job taking at these

[15] A simple log-linear regression of the wage on AFQT and FS yields the following parameter estimates and standard errors:

constant:	162
	(1.1)
AFQT:	.047
	(.09)
FS:	.389
	(.12)

The constant term is very close to the maximum likelihood estimate, the AFQT coefficient is overestimated by 24 per cent in the OLS regression, and the FS coefficient is overestimated by 19 per cent. Although for the constant term the standard error in the regression is almost an order of magnitude smaller than in the maximum likelihood estimation, for the AFQT and FS coefficients, the regression standard errors are several order of magnitude larger. As already noted, it is prudent to view the GRADX generated standard errors with caution.

TABLE IV
PREDICTED AND OBSERVED HAZARD RATES

Week	Observed Proportion Employed by Week i Given Unemployed Up to Week i	Predicted Proportion Employed by Week i Given Unemployed Up to Week i
1	.313	.313
2–13	.131	.141
14–26	.134	.135
27–39	.074	.127
40–52	.153	.117
53–65	.044	.105
66–78	.000	.097
79–91	.051	.090
92–104	.171	.083
105–117	.185	.076
118–130	.136	.070
131–143	.000	.064
144–156	.000	.059
157–166	.000	.054

durations is not real, but either a result of the small sample or of recall error. Comparing the predicted accepted mean wage given the sample freqency of duration, i.e., $\sum f_t E(w_t \mid w_t \geqslant \xi_t)$ where f_t is the sample proportion of individuals who searched exactly t weeks (completed spells in Table I), with the observed mean wage provides another indication of how the model conforms to the data. In particular, for the entire sample the predicted accepted mean wage is 193 dollars, which is quite close to the observed figure of 205 dollars in Table III. The calculated figures for the four groups used in specification 2 are not quite as close to the respective groups; reading down the column in Table III, they are 261, 250, 182, and 175, as compared to 221, 204, 206, and 177.

The maximum number of periods that any individual searched is 166. Thus, as noted, an unrestricted model in which reservation wages are not necessarily related period to period would contain those 166 parameters plus $\tilde{w}$, Π_0, Π_1, σ_u, and ρ. The restricted model given by specification one in Table II, however, contains only eight parameters, not including T. The log likelihood of the unrestricted model was estimated to be -373.06. The χ^2 statistic for the likelihood ratio test with respect to specification one is thus 122.32 with 161 degrees of freedom, so the restrictions of the search model are not rejected at conventional significance levels.[16]

Reservation wages are quite low and decline continuously. For the specification without wage regressors, the reservation wages in the first period of search is 113

[16] In 127 of the weeks no individual took a job. In forming the unrestricted likelihood the probability of working was assumed to be zero in those periods, and implicitly an infinite reservation wage was estimated. Because an infinite reservation wage fits the data perfectly in the periods in which no individual was observed to take a job while the restricted model does not fit those data perfectly, one degree of freedom is used up in the restricted model for each of those periods.

dollars per week, at the time of graduation 79 dollars per week, and one year after graduation, only 4 dollars per week. For those with father's education under 12 years, the low AFQT individuals have an initial reservation wage of only 96 dollars per week while for the high AFQT individuals the reservation wage begins at 98 dollars per week. For those with father's education at least 12 years, the low AFQT individuals have an initial reservation wage of only 155 dollars and the comparable figure for high AFQT individuals is 163 dollars per week. At the time of graduation, approximately one year after beginning search, reservation wages are 72, 77, 131, and 140 dollars per week, respectively. Only for the last two groups does the reservation wage ever exceed the minimum wage for a 30 hour week. Acceptance probabilities given an offer are conversely very high and continuously increasing. At the time of initial search the acceptance probability ranges from a high of .97 for group one to a low of .88 for group four. The probability of being employed given prior unemployment (the hazard rate), that is, the probability of receiving an offer times the probability of acceptance given an offer, is only .012 (specification one) the first week after graduation and falls continuously thereafter to .009 one year after graduation—the declining reservation wage is more than offset by the declining offer probabilities. Given that all groups have the same offer probability schedule, the groups with the highest reservation wages have the lowest working probabilities.

5. DISCUSSION AND CONCLUDING REMARKS

Given the parameter estimates, it is straightforward to simulate the effect of parameter changes on unemployment. The unemployment duration hazards can be summarized by calculating the expected duration of unemployment after graduation, namely,

$$E(d)=\sum_{d=1}^{55+\tau} df(d)=.31+(1-.31)\sum_{d=2}^{55+\tau} d \times \prod_{j=2}^{d-1}(1-P_j \Pr(u_j \geqslant \eta_j))P_d \Pr(u_d \geqslant \eta_d),$$

where the .31 figure corresponds to the probability of a duration on one week, $d=55$ is the last week of search with a positive reservation wage, and τ is set at 500 as in the estimation.[17] The expected duration of post-graduation unemployment is 46.2 weeks. Since this figure is very close to the average duration of all spells (complete and incomplete), 45.6 weeks, either the incomplete spells of short duration are in reality complete spells (or would have shortly been completed), or the incomplete spells of long duration would have been closed very quickly, or both. Recall that the majority of incomplete spells arise due to missing information.

[17] Recall that the proportion of individuals employed in the first period after graduation (.31) is perfectly predicted in the estimation.

TABLE V

SIMULATED CHANGES IN THE EXPECTED DURATION OF UNEMPLOYMENT AND SELECTED RESERVATION WAGES

	Maximum Likelihood Estimate	Mean Wage Offer		Cost of Search		Offer Probability		
		206	150	50	0	.01	.05	.10
Expected Duration of Unemployment (weeks)	46.2	46.4	45.8	50.5	66.4	86.1	33.9	26.8
Reservation Wage (dollars)								
At Graduation	79	97	43	134	195	191	248	287
One Year After Graduation	4	21	0	92	174	176	185	187

It is interesting to see how sensitive the expected duration of unemployment is to changes in the wage offer distribution, the offer probability distribution, and the cost of search. Table V provides such simulations.[18] Increasing the mean wage offer theoretically may increase or decrease the expected duration of unemployment because the increased probability of acceptance for a given reservation wage is offset by the increased reservation wage induced by the increase in the mean wage offer. Evidently, the increase in the reservation wage is sufficiently great that the expected duration of unemployment actually increases, although the elasticity of response is very small.[19] Reducing the cost of search increases the reservation wage and thus the expected duration of unemployment. A 50 per cent reduction increases the unemployment duration by nine per cent while eliminating the cost of search, say through an unemployment compensation subsidy, would increase the duration by 44 per cent. To evaluate the sensitivity of search to changes in the offer probability, constant offer probability profiles at different levels were compared. Increasing the offer probability may either increase or reduce the duration of unemployment depending upon the responsiveness of reservation wages to offer probabilities. As Table V shows, increasing the offer probability from one per cent to five per cent reduces the expected duration of unemployment by approximately 60 per cent. A further doubling of the offer probability however, only reduces the duration by another 21 per cent. If the probability of receiving an offer each period was unity, the simulated expected duration of unemployment would be 20 weeks, so that in a sense approximately one-half of the mean duration of 46 weeks is due to the lack of job offers. All of these simulations should be viewed cautiously since in a general equilibrium model it is not clear that these ceteris paribus experiments are feasible.

As demonstrated above, estimating a structural model of search provides means for evaluating the impact of potential policy instruments. Although one may wish

[18] The probability of a duration of one week is fixed at .31 in all simulations.

[19] The increase in the mean wage offer in Table V was obtained by increasing $\tilde{w}$.

to perform further tests of the robustness of the implications of the model to changes in the basic assumptions of the theory, this procedure yields policy relevant results in a behaviorally consistent framework.

Center for Human Resource Research, The Ohio State University, 650 Ackerman Road, Columbus, Ohio 43202, U.S.A.

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APPENDIX

This appendix demonstrates conditions for identification and discusses some general properties of the solution.

Consider the case where T, the length of the search horizon, is less than the longest observed spell of unemployment, complete or incomplete. Given data on accepted wages, along with data on unemployment duration, the parameters of the wage offer distribution $\bar{w}$ or $\tilde{w}$, σ_u^2 and σ_ε^2 (or ρ) are clearly identified. Further, because any offer after T is accepted, by definition, the offer probability P, which for convenience is assumed here to be independent of duration, is identified from spell lengths of individuals still unemployed at or beyond T. Now, equating the sample hazard rates, Π_t, i.e., the proportion of individuals employed at $t-1$ and completing their spell at t, to the theoretical hazards yields

$$
\text{(A.1)}\qquad
\begin{aligned}
&\Pi_{T+\tau} = P,\\
&\vdots\\
&\Pi_T = P,\\
&\Pi_{T-1} = P\left(1-\Phi\left(\frac{\eta_{t-1}}{\sigma_u}\right)\right),\\
&\Pi_{T-2} = P\left(1-\Phi\left(\frac{\eta_{T-2}}{\sigma_u}\right)\right),\\
&\vdots\\
&\Pi_{T-k} = P\left(1-\Phi\left(\frac{\eta_{T-k}}{\sigma_u}\right)\right),
\end{aligned}
$$

where $1-\Phi(\cdot)$ is the probability of accepting an offer given an offer is received and Φ is the standard normal cumulative. As seen from (A.1), P is identified from $\Pi_T, \ldots, \Pi_{T+\tau}$. Note that although in (A.1) all spells are treated as complete, incomplete spells would serve equally well to identify P because the probability of remaining unemployed an extra period is $1-P$. With P identified, the η_t's are identified from the Π_t's for $t<T$. It is necessary to demonstrate that the cost of search, c, and the discount factor δ are separately identified from the η_t's and that the η_t's are uniquely determined by c and δ.

CASE I: w normal. It can be shown from (2) and (5) that the η_t's have the following representation ($\tau=\infty$):

$$
\text{(A.2)}\qquad
\begin{aligned}
&\frac{\eta_{T-1}}{\sigma_u} = \frac{\bar{w}(\delta-1)-c}{\sigma_u[1-\delta(1-P)]},\\
&\frac{\eta_{T-k}}{\sigma_u} = \frac{\eta_{T-1}}{\sigma_u} - \delta(1-P)\left(\frac{\eta_{T-1}}{\sigma_u} - \frac{\eta_{T-k+1}}{\sigma_u}\right) + \delta P\frac{\eta_{T-k+1}}{\sigma_u}\,\Phi\left(\frac{\eta_{T-k+1}}{\sigma_u}\right)_{k>1} + \delta P\phi\left(\frac{\eta_{T-k+1}}{\sigma_u}\right).
\end{aligned}
$$

From (A.2) it is clear that for any $k>1$ the second equation together with the estimate of P uniquely identifies δ. The first equation in (A.2) then uniquely identifies c. Further, c and δ uniquely determine the η_t's. The parameters of the model are indeed overidentified if at least two periods prior to $T-1$ are available. $\delta=0$ will result from (A.2) only if the η_t's and thus the Π_t's, are independent of duration. Thus, $\delta=0$ mimics an infinite horizon model in the sense of constant hazard rates with the reservation wage, ξ_t, equal to $-c$. Without wage data $\bar{w}$, c and σ_u evidently cannot be separately identified given η_{T-1}/σ_u.

CASE II: w lognormal. It can be shown from (2) and (6) that the η_t's have the following representation ($\tau = \infty$):

$$\text{(A.3)} \qquad e^{\eta_{T-1}} = \frac{\delta P \tilde{w}\, e^{1/2\sigma_u^2} - c}{\tilde{w}[1-\delta(1-P)]},$$

$$e^{\eta_{T-k}} = \frac{-c}{\tilde{w}} + \delta Z_{T-k}, \qquad k > 1,$$

where

$$Z_{T-k} = P e^{1/2\sigma_u^2}\left[1 - \Phi\left(\frac{\eta_{T-k+1}}{\sigma_u} - \sigma_u\right)\right] + P e^{\eta_{T-k+1}} \Phi\left(\frac{\eta_{T-k+1}}{\sigma_u}\right) + (1-P)\, e^{\eta_{T-k+1}}.$$

As in the normal case, (A.3) is a system of linear simultaneous equations in c and δ. A unique solution for c and δ exists if and only if $(1-P)\,e^{\eta_{T-1}} + P e^{1/2\sigma_u^2} \neq Z_{T-k}$. Further, because of the recursive nature of (A.3), c and δ uniquely determine the η_t's. If Π_t's were all equal, and equal to P, then the solution would be $\delta = c = 0$. As in the normal case, the parameters are overidentified. Without wage data $\tilde{w}$ and c cannot be separated.

In both cases as T changes, so does the estimate of P, and thus, of course, c and δ. Because P is uniquely defined for each T, T is also identified as the time at which the hazards no longer change, at least in the case where T is less than the longest observed spell.

If T is longer than the longest observed spell, identification is more difficult to demonstrate because P and the η_t's are not separately identified from the Π_t's. If there is no measurement error in the wage, then the η_t's are identified from the smallest accepted wage at each t (Flinn and Heckman (1982a)) and so P is identified. Although η_{T-1} cannot be estimated from Π_{T-1} if those data are unavailable, in the lognormal case it is unnecessary to identify η_{T-1} to obtain all of the structural parameters. In the normal case, differencing any two η_{T-k}'s for $k > 2$ (from the second equation in (A.2)) eliminates η_{T-1} and allows identification of δ which can then be used to identify η_{T-1} from either of the differenced equations; c is then identified from η_{T-1}. The horizon T is identified because it uniquely determines the set of η_t's through the Π_t's.

It should be apparent that a critical feature of the model in terms of identification is the fact that reservation wages change systematically with duration due to the finite horizon. In the infinite horizon model the η_t's are independent of t and the same exact function of c and δ. It is, therefore, not possible in that case to separately identify those two parameters.

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