Estimating Dynamic Models of Quit Behavior: The Case of Military Reenlistment

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We estimate the effect of financial incentives for reenlistment on military retention rates using a stochastic dynamic programming model. We show that the computational burden of the model is relatively low even when estimated on panel data with unobserved heterogeneity. The estimates of the model show strong effects of military compensation, especially of retirement pay, on retention rates. We also compare our model with simpler-to-compute models and find that all give approximately the same fit but that our dynamic programming model gives more plausible predictions of policy measures that affect military and civilian compensation at future dates.

I. Introduction

Military retention is a major topic in the economics of military manpower. The military controls the size of its force and the relative mix of senior and junior personnel not only by controlling the rate of new enlistments but also by altering compensation incentives for reenlistment after certain fixed terms. Reenlistment bonuses are often offered explicitly for this purpose since they are a more flexible method of altering reenlistment incentives than changes in basic military pay or retirement benefits.

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A key parameter to the military is thus the elasticity of the retention response to the compensation package offered for reenlistment. In the current environment of military downsizing, for example, where reductions in force are being achieved by reductions in retention rates as well as through lowered initial enlistment goals, knowing the value of the elasticity is especially important.

In this article, we provide new estimates of this elasticity for the case of army reenlistment at the end of first and second terms of service. Compared to the past literature on this topic, the major contribution of our study is to provide estimates of a dynamic reenlistment model adapted from the literature on stochastic dynamic programming models (for surveys, see Eckstein and Wolpin 1989; Rust 1991). The retention decision is inherently dynamic because the enlistee reaching the end of a term must consider the alternative future streams of income that would obtain if he were to stay in the military and if he were to leave.

Most past studies of military retention have instead obtained estimates from what is called the Annualized Cost of Leaving, or "ACOL," model (e.g., Warner and Goldberg 1984). This model, which we discuss in more detail in our article, is a simplified version of the traditional dynamic programming model which imposes certain restrictions on the form of uncertainty, restrictions that are difficult to reconcile with standard assumptions of time consistency. The model has also been applied to the study of the effects of pensions on retirement by Stock and Wise (1990), who use a related version of the ACOL model which we call the "TCOL" model. The major advantage claimed for the ACOL model is its computational simplicity, an argument buttressed by the study of Gotz and McCall (1984). Gotz and McCall estimated a dynamic programming model of air force officer retention but found the estimation to be sufficiently difficult that only three parameters could be estimated, no exogenous covariates were allowed, the discount rate was fixed a priori, and no standard errors were calculated.

We show that dynamic retention models are considerably less difficult to estimate than this literature implies. At least for the case of a simple leave-stay decision—the case with the smallest possible state space—we show that the retention decision is a linear function of a simple weighted sum of current and future wage differences. While the solution requires backwards recursion, the recursion formula is of a very simple form. We report estimates of a model with 11 parameters, seven exogenous covariates, and an estimated discount rate, with standard errors for all parameters. In addition, we permit unobserved heterogeneity in the form of a random individual effect.

The results show strong effects on retention of the military-civilian income differential over the lifetime and of the timing of that differential with the date of departure from the military. Military retirement benefits are found to be particularly important. In addition, our model is found to be superior to the ACOL and TCOL models in some respects. While our model does not provide a better in-sample fit than those models (all have approximately the same fit), our model yields very accurate out-of-sample predictions. In addition, our estimated dynamic programming model provides more plausible predictions of the effects of some changes in military pay policy than do the ACOL model and its variants. In particular, we use our model to simulate the effect of recently announced army policies aimed at reducing reenlistment rates.

In the next section, we lay out our dynamic retention model. Following that, we compare our model to those in the past literature, particularly the ACOL model. The subsequent section reports our data and results, and a final section provides a summary.

II. A Dynamic Retention Model

Consider an army enlistee at time t, at the end of a term of service, considering whether to leave the military for the civilian sector or to reenlist.[1] Assume that he cannot return to the military if he leaves and that future income streams and the time horizon are known with certainty. Let W^m_τ be the military compensation at time τ—including basic military pay and bonuses—and let W^c_τ be civilian compensation (including military retirement pay). Letting L denote the choice to leave, S denote the choice to stay, and V denote the present value of the alternatives, we have:

$$V^L_t = W^c_t + \sum_{\tau=t+1}^{T} \beta^{\tau-t} W^c_\tau + \varepsilon^c_t, \tag{1}$$

$$V^S_t = W^m_t + \beta E_t(V_{t+1}) + \varepsilon^m_t, \tag{2}$$

$$V_{t+1} = \mathrm{Max}(V^L_{t+1}, V^S_{t+1}), \tag{3}$$

where T is the time horizon, β is the discount rate, and ε^c_t and ε^m_t are sources of uncertainty. We assume that the individual knows the distribution function for the error terms as well as their current values but not their actual future values, apart from an individual effect we specify below. The optimal nature and internal consistency of the decision process assumed are reflected in equations (2) and (3), showing that current valuations are based on the assumption of optimal future decisions and that both are governed by the same process and parameters. The individual leaves if V^L_t is greater than V^S_t.

[1] We have exposited and estimated the 1-period version of this model in Daula and Moffitt (1991). The major difference in the model we provide here is the inclusion of a second period and the consequent provision of unobserved heterogeneity to avoid dynamic selection bias.

This simple dynamic programming model has a clear and intuitive solution. Solving (2) forward to T, the model can be reformulated as follows:[2]

$$\left.\begin{aligned} S_t &= 1 \quad \text{if } S_t^* \geq 0 \quad \text{(stay)}, \\ S_t &= 0 \quad \text{if } S_t^* < 0 \quad \text{(leave)}, \end{aligned}\right\} \tag{4}$$

$$\begin{aligned} S_t^* &= V_t^S - V_t^L \\ &= a_t + \varepsilon_t, \end{aligned} \tag{5}$$

where, assuming $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and letting f and F be the standard normal probability density function and cumulative distribution function, respectively,

$$\varepsilon_t = \varepsilon_t^m - \varepsilon_t^c, \tag{6}$$

$$a_t = W_t^m - W_t^c + \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_\tau (W_\tau^m - W_\tau^c) + \sigma_\varepsilon \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_{\tau-1} f_\tau, \tag{7}$$

$$f_\tau = f(a_\tau/\sigma_\varepsilon), \tag{8}$$

$$\begin{aligned} r_\tau &= \prod_{k=t+1}^{\tau} \text{prob}(S_k^* \geq 0) = \prod_{k=t+1}^{\tau} F(a_k/\sigma_\varepsilon) \quad \text{for } \tau \geq t+1 \\ &= 1 \quad \text{for } \tau = t. \end{aligned} \tag{9}$$

[2] To solve the model forward requires that we obtain an explicit solution for $E_t(V_{t+1})$ in (2). We can obtain it by recognizing that (3) implies a simple recursion relation in $E_t(V_s)$ for all s:

$$\begin{aligned} E_t(V_s) = {} & \text{prob}(V_s^S < V_s^L)[W_s^c + \sum_{\tau=s+1}^{T} \beta^{\tau-s} W_\tau^c + E_t(\varepsilon_s^c \mid V_s^S < V_s^L)] \\ & + \text{prob}(V_s^S > V_s^L)[W_s^m + E_t(V_{s+1}) + E_t(\varepsilon_s^m \mid V_s^S > V_s^L)]. \end{aligned}$$

With $V_s^S - V_s^L = a_s + \varepsilon_s$, as shown in eq. (5) in the text, the two prob values above are just normal probabilities evaluated at a_s. The expected values of the error terms also solve out, for

$$E_t(\varepsilon_s^c \mid V_s^S < V_s^L) = -[\text{cov}(\varepsilon_s^c, \varepsilon_s)/\sigma_\varepsilon] f(a_s/\sigma_\varepsilon)/\text{prob}(V_s^S < V_s^L),$$

$$E_t(\varepsilon_s^m \mid V_s^S > V_s^L) = [\text{cov}(\varepsilon_s^m, \varepsilon_s)/\sigma_\varepsilon] f(a_s/\sigma_\varepsilon)/\text{prob}(V_s^S > V_s^L).$$

Hence,

$$\text{prob}(V_s^S < V_s^L) E_t(\varepsilon_s^c \mid V_s^S < V_s^L) + \text{prob}(V_s^S > V_s^L) E_t(\varepsilon_s^m \mid V_s^S > V_s^L) = \sigma_\varepsilon f(a_s/\sigma_\varepsilon),$$

since $[\text{cov}(\varepsilon_s^m, \varepsilon_s) - \text{cov}(\varepsilon_s^c, \varepsilon_s)] = \sigma_\varepsilon^2$. Thus the equation above can be recursively solved forward for successive future values of $E_t(V_s)$ until $s = T$, at which point $V_{T+1} = 0$.

The reenlistment decision is thus based on the linear index function shown in (5), which is a function of a nonstochastic component a_t and an error term. The latter is a difference between the military and civilian errors, as shown in (6), and the former is a weighted sum of current and future compensation differences plus a remainder, as shown in (7). Each of the future compensation differences is weighted by the discount rate and by a term r_τ, which is the probability that the individual will not have left by time τ. These probabilities will be critical in the discussion of different estimation procedures below. The remainder is a sum of expected values of truncated error terms, since the individual always picks the maximum of V_t^L and V_t^S and hence has an above-zero expected value of ε_t.

Estimation of the model in this form is not difficult. It is a probit model in which the parameters β and σ_ε enter the right-hand side of (7) nonlinearly, both explicitly as well as implicitly in the r_τ. The r_τ must be computed by backwards recursion, but the recursion formula is just (9) with (7) substituted in for future a_k. No difficult calculations are involved in computing the values of all r_τ in this way. Standard errors can be obtained from either analytic or numerical derivatives of (7) and (9).

The representation of the expected value of future income as a weighted sum of the incomes at different leaving dates, with the weights related to the probabilities of leaving at those dates, can be generalized to more complex dynamic choice models with a larger state space. Indeed, Hotz and Miller (1994, proposition 1) have proven the existence in a general dynamic choice model of a representation of the expected value function which explicitly contains the conditional choice probabilities of future sequences of states.

We add a vector of observable variables X (education, race, etc.) into the model by allowing such variables to affect unobserved elements of military and civilian compensation (essentially, relative "tastes" for the military). That is, the effects of the X vector, which we denote by δ, may be interpreted as reflecting implicit valuations of nonmonetary characteristics of military and civilian environments. The model in (4)–(9) is unchanged except for (7), which becomes:

$$\begin{aligned} a_t &= W_t^m - W_t^c + X\delta \\ &\quad + \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_\tau (W_\tau^m - W_\tau^c + X\delta) + \sigma_\varepsilon \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_{\tau-1} f_\tau. \end{aligned} \tag{10}$$

To simplify the estimation slightly, we set $\psi = 1/\sigma_\varepsilon$ and $\tilde{\delta} = \delta/\sigma_\varepsilon$ and divide (10) through by σ_ε, obtaining

$$\begin{aligned} a_t/\sigma_\varepsilon &= \psi(W_t^m - W_t^c) + X\tilde{\delta} \\ &\quad + \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_\tau [\psi(W_\tau^m - W_\tau^c) + X\tilde{\delta}] + \sum_{\tau=t+1}^{T} \beta^{\tau-t} r_{\tau-1} f_\tau, \end{aligned} \tag{11}$$

which is equivalent since probit estimates are based only on a_t/σ_ε. Thus we estimate a coefficient on the military-civilian compensation difference directly.

We also introduce two slightly complicating factors for realism. First, we allow the civilian wage profile to depend upon the time spent in the military, since it is generally thought that military service is not completely substitutable for civilian work. We will recognize this dependence by denoting civilian wages by W^c_{st}, where s is the individual's last period in the military. Second, since departure from the military is difficult at periods other than the end points of fixed terms, we shall assume instead that the decision points occur only every few years, depending on the length of term (we provide exact details of these lengths in our data discussion below). We shall therefore assume that there are n discrete decision points, which occur at times t_i, $i = 1, 2, \ldots, n$.

With these complications the model becomes

$$V^L_{t_i} = W^c_{t_i-1,t_i} + \sum_{\tau=t_i+1}^{T} \beta^{\tau-t_i} W^c_{t_i-1,\tau} + \varepsilon^c_{t_i}, \tag{12}$$

$$V^S_{t_i} = \sum_{\tau=t_i}^{t_{i+1}-1} \beta^{\tau-t_i} W^m_\tau + \beta^{t_{i+1}-t_i} E_{t_i}(V_{t_{i+1}}) + \varepsilon^m_{t_i}, \tag{13}$$

$$V_{t_i+1} = \max(V^L_{t_i+1}, V^S_{t_i+1}). \tag{14}$$

Adding the X vector, normalizing by σ_ε, and solving forward to T, we obtain the model:

$$\left.\begin{aligned} S_{t_i} &= 1 \qquad \text{if } S^*_{t_i} \geq 0 \quad \text{(stay)}, \\ S_{t_i} &= 0 \qquad \text{if } S^*_{t_i} < 0 \quad \text{(leave)}, \end{aligned}\right\} \tag{15}$$

$$\begin{aligned} S^*_{t_i} &= V^S_{t_i} - V^L_{t_i} \\ &= a_{t_i} + \varepsilon_{t_i}, \end{aligned} \tag{16}$$

where

$$a_{t_i}/\sigma_\varepsilon = \tilde{a}_{t_i} + \sum_{k=i+1}^{n} \beta^{(t_k-t_i)} r_k \tilde{a}_{t_k} + \sum_{k=i+1}^{n} \beta^{(t_k-t_i)} r_{k-1} f_{t_k}, \tag{17}$$

$$\begin{aligned}\tilde{a}_{t_i} = &\sum_{\tau=t_i}^{t_{i+1}-1} \beta^{\tau-t_i}[\psi(W_\tau^m - W_{t_i-1,\tau}^c) + X\tilde{\delta}] \\ &+ \sum_{\tau=t_{i+1}}^{T} \beta^{\tau-t_i}[\psi(W_{t_{i+1}-1,\tau}^c - W_{t_i-1,\tau}^c)],\end{aligned} \tag{18}$$

$$\varepsilon_{t_i} = \varepsilon_{t_i}^m - \varepsilon_{t_i}^c, \tag{19}$$

$$f_{t_k} = f(a_{t_k}/\sigma_\varepsilon), \tag{20}$$

$$r_{t_k} = \prod_{j=i+1}^{k} F(a_{t_j}/\sigma_\varepsilon). \tag{21}$$

The major change in the model is shown in (18), whose second component shows that the reenlistment decision will depend on the present value of the change in the lifetime civilian earnings stream from delaying departure from the military.

Finally, we modify the model to account for observing repeated decisions by a panel of military enlistees. In our data, we will observe two decision points, those at the end of the first and second terms in the army. Treating the successive observations on the same individuals as independent would likely lead to dynamic selection bias, for those who leave the military are likely to have systematically different draws for the error term ε than those who stay (e.g., different relative tastes for military vs. civilian life). Thus those observed at later decision points may have different retention rates simply because they are an increasingly self-selected portion of the original population.

We incorporate such unobserved heterogeneity in the simplest fashion possible by introducing a conventional individual random effect that differs across individuals but which is constant over time. We introduce two such effects, one for the military and one for the civilian sector, and we simply replace W_t^m by $W_t^m + \gamma^m$ and W_t^c by $W_t^c + \gamma^c$. Thus we interpret the random effects in a fashion exactly analogous to the vector X—indeed, $X\delta$ can now be interpreted to be the mean of $\gamma^m - \gamma^c$—namely, as representing the difference in implicit valuations of unobserved and nonmonetary characteristics and tastes for the two sectors. We also assume that the individual knows his two values of the effects. The implicit compensation difference in each period becomes $(W_t^m - W_t^c + X\delta + \gamma)$ in all periods, where $\gamma = \gamma^m - \gamma^c$.[3]

When we add the two random effects to the two wages and rederive the form of the model, the result is identical to that in equations (15)–(21) except that $X\tilde{\delta}$ in (18) is replaced by $X\tilde{\delta} + (\gamma/\sigma_\varepsilon)$. Hence γ is involved in all terms of (17) and (18), including the r_k. To avoid confusion, we also replace ε_t in the model with the error term ν_t, since the error term in the model is now conditional on γ. Letting $\nu_t \sim N(0, \sigma_\nu^2)$, we therefore replace σ_ε by σ_ν as well. We retain the variable ε_t but redefine it in standard random-effects terms as[4]

$$\varepsilon_t = \gamma + \nu_t. \tag{22}$$

Since the revised model is conditional on γ, and γ is unobserved, it must be integrated out. Despite the nonlinear way in which it enters, standard quadrature techniques available for the panel probit model (e.g., Butler and Moffitt 1982) can be used. Rewriting the probit index function in (16) as

$$S_{t_i}^* = a_{t_i}(\gamma) + \nu_{t_i}, \tag{23}$$

to show the dependence of a_{t_i} on γ explicitly, the probability of observing two successive decisions is

$$\text{prob}(S_1, S_2) = \int_{-\infty}^{\infty} \text{prob}(S_1 | \gamma)\text{prob}(S_2 | \gamma) g(\gamma) d\gamma, \tag{24}$$

where g is the density function of γ and where $\text{prob}(S_j = 1 | \gamma) = F[a_j(\gamma)/\sigma_\nu]$ and $\text{prob}(S_j = 0 | \gamma) = F[-a_j(\gamma)/\sigma_\nu]$, $j = 1, 2$. For individuals who leave at the end of the first term, only $\text{prob}(S_1 | \gamma)$ enters the probit likelihood function. We assume that $\gamma \sim N(0, \sigma_\gamma^2)$.[5]

The probability in (24) can be approximated with quadrature methods relatively easily. As a practical matter, since quadrature approximations just involve evaluating the kernel of (24) at several different values of the

[3] We assume that the error term is orthogonal to the regressors; i.e., we assume a random rather than fixed effects model. Fixed effects cannot be consistently estimated in probit models with low numbers of observed time periods.

[4] As can be seen from (17) and (18), when $\tau = t_i$, the error term in (16), inclusive of γ, is simply $\varepsilon_{t_i} = \gamma + \nu_{t_i}$.

[5] A normalization is required for probit, as usual, which we accomplish by setting the variance of ν_t equal to 1. We need estimate only σ_γ, since σ_ε is calculable as the square root of $1 + \sigma_\gamma^2$. Note as well that the percent of the total variance explained by the random effect is $\rho = \sigma_\gamma^2/(1 + \sigma_\gamma^2)$.

integrating variable (γ), the added computational burden of the model when unobserved heterogeneity is allowed is essentially that required by having to evaluate the single-period dynamic model described above multiple times. Since the single-period model is not overly burdensome itself, its multiple evaluation is still well within the power of modern computational facilities.

III. Comparison with Alternative Models

Our model is a special case of more general dynamic choice models with discrete choice variables (see Eckstein and Wolpin 1989; and Rust 1991; for surveys). Our case is a particularly simple one, for the choice of military reenlistment is a simple optimal stopping rule with only a small, finite number of alternatives (specifically, $T - t$ or less). It is the simplicity of the model and the low dimensionality of the state space that permit us the computational flexibility to introduce serial correlation in the error terms through the assumption of unobserved heterogeneity. Virtually all past dynamic choice models have ignored serial correlation for computational reasons.

In the literature on military reenlistment, the closest model to ours is that of Gotz and McCall (1984). Gotz and McCall modeled the stay/leave decisions of air force officers in a manner closely related to our model, assuming that the decision is based on relative military and civilian pay in the present and in the future. Gotz and McCall also permitted an individual random effect. However, computational difficulties in their model forced Gotz and McCall to estimate only a highly restricted version, one with no exogenous covariates and with the discount rate fixed a priori (in addition, no standard errors could be obtained). Our more flexible formulation of the problem, together with computer technological advances since 1984, makes estimation of the model much less difficult.

The historical approaches to the military retention decision in the literature are much less sophisticated. For example, much of the early literature assumed that the individual calculates the present value of staying in the military (V_t^S) only over some exogenous, fixed remaining term. For example, some of the early literature on the effects of reenlistment bonuses assumed as an approximation that the present value of relative compensation only 4 years into the future was relevant for the reenlistment decision.

Given the obvious arbitrariness of picking the horizon, a preferable model was developed, the ACOL model (e.g., Warner and Goldberg 1984), which optimized over that horizon. The ACOL model is the most well known model in the military retention literature, so we shall exposit it in some detail, and we shall estimate it for comparison with our dynamic model.

We will first demonstrate a variant of the ACOL model which we will call the TCOL (total cost of leaving) model that is closer to our dynamic retention model (DRM).[6] Denote V_{st}^S as the present value at t of staying in the military for s periods beyond t:

$$V_{st}^S = \sum_{\tau=t}^{t+s-1} \beta^{\tau-t} W_\tau^m + \sum_{\tau=t+s}^{T} \beta^{\tau-t} W_\tau^c, \tag{25}$$

and define $\bar{V}_t^S = \mathrm{Max}_s V_{st}^S$ $(s = 1, \ldots, T - t)$ as the maximum of these present values over all possible positive s. Then the index function for "staying in the military" in what we term the "TCOL" model is

$$S_t^* = \bar{V}_t^S - V_t^L + \varepsilon_t, \tag{26}$$

$$\left.\begin{aligned} S_t &= 1 \quad \text{if } S_t^* \geq 0 \quad \text{(stay)}, \\ S_t &= 0 \quad \text{if } S_t^* < 0 \quad \text{(leave)}. \end{aligned}\right\} \tag{27}$$

Thus the individual stays in the military if the maximal present value of staying over all possible positive s is greater than the present value of leaving in the current period (plus an error term).

This model is an approximation to our dynamic retention model, which treats uncertainty in a different fashion. When expanded, equation (26) can be seen to be identical to equation (7) if $r_\tau = 1$ for $\tau < \bar{s}$, where $\bar{s} = \mathrm{argmax}(V_{st}^S)$, and $r_\tau = 0$ for $\tau \geq \bar{s}$, and if the third term in (7) (the sum of truncated expected normals) is omitted. Thus the TCOL model assumes that the future leaving date ($\bar{s}$) is known with certainty, and thus the probability weights r_τ present in our dynamic retention model do not appear.

In more formal terms, the TCOL model replaces the expected value of the maximum of future V, which appears in our model, by the maximum of the expected value of future V. See Stern (1991) for a discussion of this point in the context of the model of Stock and Wise (1990). This has the unfortunate consequence that, because future leaving dates other than $t + \bar{s}$ are assigned probability zero, all changes in future W_τ^m for $\tau > t + \bar{s}$ have an identically zero effect on the current retention probability so long as those changes do not affect the value of $\bar{s}$. The TCOL model also embodies a form of time inconsistency inasmuch as the current decision is affected by unobservables and transitory shocks (ε_t) whose future existence is assumed to be ignored by the individual.

[6] The issues outlined here have been previously discussed in an exchange by Black, Moffitt, and Warner (1990*a*, 1990*b*) and Gotz (1990).

The advantage of the TCOL model is its computational simplicity, for, conditional on a value of β, $P_t = \bar{V}_t^S - V_t^L$ can be calculated prior to estimation and entered as a regressor in the stay-leave equation. If so desired, an optimal value of β can be determined by reestimating the model for different β values to determine which maximizes the value of the probit log likelihood. A vector of X variables can be added to (26) as well. However, as we have argued in the last section, the dynamic retention model in this case, which does not suffer from the undesirable properties of the TCOL model, is of such a simple form that computation is not overly burdensome in any case.[7]

A variant of the TCOL model has also been recently applied in the retirement literature by Stock and Wise (1990), who assume that individuals deciding whether or not to retire at a given time t consider only the maximum of the expected present values of remaining lifetime income over all possible future retirement dates. The details of the Stock and Wise model differ considerably from the simple models laid out here—utility differences rather than income differences are specified and a different error structure is assumed—but the basic dynamic behavioral assumption (of a probability-one optimal future leaving date) is the same as that in the TCOL model.[8]

However, the model usually estimated in the military retention literature is not the TCOL model but instead the ACOL (annualized cost of leaving) model (Warner and Goldberg 1984; Black et al. 1990*a*; Smith, Sylwester, and Villa 1991). In this model, the value of staying in the military for s periods beyond t is assumed to contain the unobserved, individual-specific component γ which we discussed in the last section:

$$\tilde{V}_{st}^S = \sum_{\tau=t}^{t+s-1} \beta^{\tau-t}(W_\tau^m + \gamma) + \sum_{\tau=t+s}^{T} \beta^{\tau-t} W_\tau^c. \tag{28}$$

[7] Hotz and Miller (1994) and Hotz et al. (1993) have proposed an alternative approach to reducing the computational difficulty of estimating models of this type which requires only initial consistent estimates of the r_k. They use the same representation of the future expected value function as we do, as noted earlier, in order to make the r_k explicit. This can be thought of as the first step in an iterative procedure in which maximum likelihood (ML) estimates of the full model are achieved. As we noted previously, however, our model is sufficiently simple to compute that we can obtain the fully efficient ML estimates. A rather different approach that also reduces computational burden has been proposed by Manski (1993). See Eckstein and Wolpin (1989), pp. 590–95, for a detailed discussion of both these approaches.

[8] Lumsdaine, Stock, and Wise (1992) compare their model with a simpler version of the dynamic retention model estimated in Daula and Moffitt (1991). They find that the two models do not differ greatly in fit or in predicted effects of certain types of changes in retirement plans.

In this case, since γ is unobserved, the maximum of (28) with respect to s cannot be computed a priori and used as a regressor. However, since

$$\tilde{V}_{st}^{S} - V_{st}^{L} > 0 \Rightarrow \gamma > -\frac{\sum_{\tau=t}^{t+s-1} \beta^{\tau-t}(W_{\tau}^{m} - W_{\tau}^{c})}{\sum_{\tau=t}^{t+s-1} \beta^{\tau-t}}, \tag{29}$$

one may define the ACOL variable

$$A_t = \max_s [(V_{st}^{S} - V_{t}^{L}) / \sum_{\tau=t}^{t+s-1} \beta^{\tau-t}], \tag{30}$$

and insert this into the retention probit instead of P_t (V_{st}^{S} in [30] is identical to [25], i.e., without γ). The variable A_t is simply the maximum annualized, or annuitized, income flow (over periods remaining in the military) that the individual could receive if he were to consider all possible leaving dates s. Equation (30), like the P_t variable in the TCOL model, can be computed prior to estimation and hence simplifies the problem considerably. Like the TCOL model, the ACOL model also ignores the future random disturbances ε when computing a maximum over future possible leaving dates.

Unfortunately, the ACOL model has the difficulty that the insertion of the max condition only after equation (30) is arrived at is not legitimate, for the value of s that maximizes (30) will not maximize the present value of lifetime income. This can be seen for the case when $\gamma = 0$, when (25) applies. The maximum of (25), which we denoted $\bar{s}$ previously, will not in general equal the s which maximizes (30). Thus the ACOL and TCOL models do not generate the same optimal leaving date and, since it is presumably the present value of the lifetime income stream that the individual maximizes, the TCOL model is to be preferred to the ACOL model.[9] In addition, the ACOL model has the same knife-edge property as the TCOL model—namely, the lack of responsiveness to changes in future compensation that occur after the maximal leaving date and that do not alter it—as well as the same time-inconsistency property previously discussed for the TCOL model.

The presence of observed as well as unobserved heterogeneity creates difficulties in the TCOL and ACOL models as well. For example, if observed heterogeneity (as in the $X\delta$ term in our DRM model) affects the per-period flow of relative compensation, it will affect the calculation of

[9] This ignores γ, however. But our DRM model treats that variable correctly—γ is included for periods when in the military and not thereafter in that model.

the optimal leaving date as well (i.e., it should appear just as γ does in [28]). An internally consistent treatment should therefore require that iteration over δ include its effect on the optimal leaving date. Unfortunately, this greatly reduces the computational advantage of the models because the P_t or A_t variables can no longer be calculated prior to estimation. As a consequence, in the military retention literature, the optimal leaving date and the variables P_t and A_t have been calculated ignoring $X\delta$. Instead, $X\delta$ is simply entered into the retention probit in linear and additive form. The same problem arises for unobserved heterogeneity, as represented by γ, as just noted; proper treatment of it, as in (28), requires that it affect the optimal leaving date in both the TCOL and ACOL models.[10]

We will estimate both the TCOL and ACOL models for comparison to our dynamic retention model. To maintain comparability with the military retention literature, we will calculate optimal horizons and the variables P_t and A_t ignoring γ and $X\delta$, and we will add both linearly to the retention probit.

IV. Data and Results

As we have noted previously, our study is an examination of the probability of reenlistment at the end of the first and second terms of army enlistees. Our data are drawn from a sample of men who enlisted in the infantry between fiscal year 1974, shortly after the beginning of the all-volunteer force, and fiscal year 1984. The data track the individuals on an annual basis until 1987 or separation from the army, whichever occurs first. We select a random sample of enlistees who successfully completed their first term of service. The data set contains 2,528 observations on personnel who completed their first terms, and 257 of those were observed again at the end of their second terms.

The first row of table 1 shows the means of the dependent variable (the retention rate) in our data set. Of the 2,528 observations eligible for end-of-first-term reenlistment, 33% chose to reenlist. Of the 257 observations eligible for second-term reenlistment by the end of our observation period, 63% chose to reenlist at the end of their second terms. The higher reenlistment rate at the second term could be partly the result of dynamic selection bias—those with low retention rates may have left at the first term and hence would not be present in the second-term sample. Our inclusion of the heterogeneity term γ is intended to capture this effect.

The independent variables are drawn from a data base assembled and provided to us by Smith et al. (1991). Military pay profiles are estimated for each individual from military pay schedules by years of service and

[10] In the ACOL model, a linear representation of γ and $X\delta$ is more consistent since (29) implies that they should enter in that form. Nevertheless, the misplacement of the max condition in that model will affect those parameter estimates.

Table 1
Means of the Variables

	First-Term Sample	Second-Term Sample
Dependent variable:		
Retention rate*	.332	.634
Independent variables:		
Military-civilian pay differential†	.582	.743
Present value of leaving military†	3.38	5.34
Annual cost of leaving military†	.498	.827
Initial enlistment length (years)	3.34	3.46
No. of dependents	.49	1.63
AFQT score	44.1	40.4
Race (1 = black)	.241	.342
Educational benefits	4.48	5.19
Pay-grade difference	−.62	−.128
Fiscal year 1983 dummy	.520	.833
No. of observations	2,528	257

* Fraction who stayed in military
† In tens of thousands of dollars. Measured at time of reenlistment using the pay schedule in effect at that time.

pay grade (or rank) and from estimates of the individual's promotion probabilities over his military career.[11] Military pay includes base pay, allowances for housing and subsistence, reenlistment bonuses, and a variety of other forms of special pay (e.g., parachutist pay, demolition pay, etc.). Except for special pays, military pay is based solely on the individual's rank and time in service. Pay increases almost as much with longevity as it does with promotions. For example, under the current pay table, a married soldier with 3 years of service who is promoted from corporal (pay grade E-4) to sergeant (pay grade E-5) receives an increase in monthly basic pay and allowances from $1,477.90 to $1,639.20. In contrast, moving from 3 to 4 years of service would have increased this individual's pay and allowances to $1,566.90. To predict the individual's rate of promotion to various ranks, we used the results of a waiting time model for promotion reported in Smith et al. (1991).

A service member becomes vested under the military retirement system when he or she completes 20 years of service. Prior to that point, the individual has no guaranteed retirement benefits. After vesting, an individual becomes eligible to receive a monthly annuity equal to .025 times years of service times basic pay at exit (or for members of our sample who entered the military between fiscal year 1981 and fiscal year 1985, the

[11] We thus treat promotion probabilities as fixed and ignore individual uncertainty regarding promotion.

average of the basic pay over the soldier's three highest years of earnings).[12] The annuity is fully indexed for inflation. For each career path in our dynamic program, we calculate the retirement annuity for which the individual is eligible and include it in our estimate of the civilian income he would receive following his departure from the military.

Civilian earnings are estimated from Internal Revenue Service data on the postservice civilian earnings of veterans, and a separate pay stream is estimated for veterans with different numbers of years of military service, as discussed previously in the model specification section above. See Smith et al. (1991) for details.

We assume that an enlistee has decision points every 4 years after reenlistment up until his twentieth year of service (when, as noted above, he becomes vested for military retirement benefits); 4 years is the modal length of reenlistment term in the army. After that point we assume annual decision points up to 29 years of service and that all individuals who have not left by their twenty-ninth year of service do so in their thirtieth year. Service beyond the thirtieth year is essentially not permitted by the army, and existing departure rates are heavily concentrated in the region between the twentieth and twenty-ninth years of service (in this period an enlistee need give only 60 days notice); it is for this reason that we assume annual decision points in this period. The actual number of decision points varies from individual to individual in the sample because individuals come up for their first-term reenlistment decision with different numbers of years of service.[13]

For the X vector we include a number of variables available in the administrative data base: length of initial enlistment term, number of dependents, Armed Forces Qualifying Test (AFQT) score (at enlistment), a race dummy, and entitlement to educational benefits.[14] We also include a dummy variable for whether reenlistment occurred after fiscal year 1983, because reenlistment rates dropped sharply after that date for reasons not related to those in our model.[15] Finally, we specify a variable equal to the difference between the individual's pay grade and the average pay grade for enlistees with the same number of years of service in order to capture some differ-

[12] Congress modified the military retirement system in 1985 to provide a reduced benefit for individuals who entered the service after fiscal year 1985. Our sample does not contain anyone who entered after 1984, however.

[13] The different decision intervals over the career may generate heteroskedasticity in the transitory disturbances. However, we ignore this possibility in our estimation.

[14] The educational level at enlistment was also tested, but its coefficient was insignificant.

[15] In 1983 there was a one-time attempt by the army to lower reenlistment rates by informal means.

ences across individuals in tastes for the military (i.e., those correlated with relative success in the military).[16]

Means of the independent variables are shown in table 1. There are sharp differences in the means between the first-term and second-term samples. The former have lower military-civilian pay differentials, lower ACOL values, fewer dependents, lower AFQT scores and educational benefits, and are more likely to be white. Although it is tempting to draw immediate inferences from these differences regarding retention effects, the potential for self-selection from unobserved heterogeneity makes such inferences hazardous.

Results

Table 2 shows the results of estimating several different models.[17] The first column shows the results for our basic dynamic retention model. The coefficient on the pay difference is positive and significant, indicating that a higher military-civilian pay difference encourages retention in the military. The magnitude of the coefficient implies that a 10% increase in the military-civilian pay difference in all years would increase retention rates in the first term by 4.6% and in the second term (conditional on having reenlisted at the first term) by 5.3%. A 10% increase in military pay alone, holding constant civilian pay, would increase the first-term retention rate by 22% and the conditional second-term retention rate by 13% (these are necessarily larger than the relative pay elasticities because the magnitude of the pay change is greater). These elasticities fall within the range estimated in prior studies in the literature.[18]

The estimate of β is .905. This implies a real discount rate of .10 and is somewhat lower than estimates used by the army, which are around .14. The other variables show that retention is more likely the greater the initial

[16] We use army-wide figures for mean pay grade rather than means from our own sample. Since this pay-grade variable is potentially endogenous, we also estimate the model without it. We should also note that we hold the two time-varying variables in our model, pay grade and number of dependents, fixed. The former is more plausible than the latter, for we interpret the pay-grade difference as proxying tastes, which should be fixed.

[17] We estimated the model with Fortran code on a UNISYS 5000, a super-mini at West Point operating under UNIX. The machine took about 450 CPU minutes per iteration, and it took approximately six or seven iterations to achieve convergence, on average. Various system constraints (e.g., inability to hold the data in memory) make these run times considerably higher than could be achieved on high-performance machines with adequate disk space that are currently available. We tried a variety of starting values and always obtained the same maximums.

[18] Baldwin and Daula (1985) estimated a relative first-term pay elasticity of .40, e.g., while Warner and Goldberg (1984), Hosek and Peterson (1985), and Smith et al. (1991) estimated first-term military pay elasticities of 1.0–2.0, 3.5, and 1.3, respectively.

Table 2
Estimates of Different Models

	DRM (1)	DRM (2)	TCOL	ACOL
Pay difference	.267*	.421*	...	...
	(.123)	(.097)		
Present value of leaving military	...	...	.207*	...
			(.038)	
Annualized cost of leaving military	...	...	...	1.458*
				(.260)
Discount rate (β):	.905*	...	.877[a]	.901[b]
	(.044)			
Retirement	...	.909*	...	...
		(.010)		
Nonretirement	...	.294	...	...
		(.418)		
Initial enlistment length	.034*	.039	.138*	.130*
	(.017)	(.038)	(.052)	(.052)
No. of dependents	.039*	.092*	.191*	.191*
	(.013)	(.051)	(.028)	(.027)
AFQT/10	.001	−.006	.008	.006
	(.003)	(.009)	(.013)	(.013)
Race	.091*	.224*	.411*	.410*
	(.031)	(.127)	(.063)	(.063)
Educational benefits	−.007*	−.014	−.026*	−.028*
	(.003)	(.011)	(.011)	(.011)
Pay-grade difference	.072*	.110*	.353*	.336*
	(.025)	(.051)	(.058)	(.056)
Fiscal year 1983	−.111*	−.328*	−.529*	−.555*
	(.029)	(.179)	(.064)	(.065)
Constant	−.346	.742*	−1.460*	−1.406*
	(.053)	(.351)	(.203)	(.196)
σ_γ	.135*	.201	...	...
	(.053)	(.156)		
ρ	...	...	.353*	.311*
			(.107)	(.113)
Log likelihood	−1,664.2	−1,658.2	−1,663.4	−1,663.0

NOTE.—Standard errors are in parentheses.
[a] Log likelihood maximized at interest rate of .14 (.897 = 1/1.14).
[b] Log likelihood maximized at interest rate of .11 (.901 = 1/1.11).
* Significant at the 10% level.

enlistment length, the greater the number of dependents (the military offers special benefits to families), for black enlistees, the greater the pay grade difference, and the lower the educational benefit (since educational benefits encourage enlistees to leave to take advantage of them).[19]

[19] When the potentially endogenous pay-grade-difference variable is omitted, the estimated parameters shift slightly. The parameter most affected is the standard error of γ, which rises by 20%. This is to be expected since the pay grade variable has persistence over time and therefore captures part of the variance of the individual

Column 2 of the table shows estimates of the model with a separate discount rate estimated for the retirement-income portion of the military-civilian pay difference and the nonretirement-income portion. This specification was tested to determine the role that retirement pay plays in identifying the discount rate, since past studies have found difficulty in identifying it. The results show that the discount rate on retirement pay is essentially the same as that in column 1 but that the discount rate on other compensation—mostly just future pay—is statistically insignificant. This is not surprising since the future civilian-military pay difference is highly collinear with its current value. We are able to identify the discount rate that applies to retirement pay because our data allow us to estimate retirement pay accurately and because our data contain substantial variation in retirement pay independent of current pay.[20]

The third and fourth columns of the table show estimates of the TCOL and ACOL models, respectively. For both models the coefficient on the appropriate pay difference variable is positive and significant. The estimate of β is slightly smaller in the TCOL model than in the DRM, but the estimate in the ACOL model is approximately the same. The estimates imply that a 10% increase in the TCOL variable (i.e., in relative military-civilian pay) would induce an 8.3% increase in the first-term retention rate and a 5.8% increase in the conditional second-term retention rate. The corresponding effects for the ACOL model are 8.3% and 6.1%. These elasticities, which are evaluated at the sample mean, are quite close to the corresponding relative pay elasticities for the DRM, as should be expected. The magnitudes of the other estimated coefficients are quite different in the TCOL and ACOL models than in the DRM partly because those variables enter differently. While in the TCOL and ACOL models the variables enter linearly as in conventional probit, in the DRM they enter as part of the pay difference in each year in the future and hence have a cumulative impact on current retention (see eq. [18]).[21]

Interestingly, the log-likelihood values for the TCOL and ACOL models are no worse than, and are in fact slightly greater than, those for the DRM. Evidently model fit is approximately the same regardless of which model

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random effect. However, no qualitative aspects of our results are affected by this change.

[20] In particular, two different retirement systems were in effect during our observation period (they were calculated differently before and after 1980); differences in promotion rates cause retirement pay to differ for personnel with the same near-term pay; and the value of the annuity received immediately after retirement varies with individual age.

[21] The parameter ρ at the bottom of the columns is the correlation coefficient between the error terms in the retention equations in the two periods estimated with bivariate probit. Bivariate probit is equivalent to probit random effects if there are only two waves in a panel.

Table 3
Goodness-of-Fit Statistics for Three Models

	DRM	TCOL	ACOL
Log likelihood	−1,664.2	−1,663.4	−1,663.0
Akaike Information Criterion*	1,675.2	1,674.4	1,674.0
Sum of squared residuals (first term)†	516.7	515.7	515.5
Sum of squared residuals (second term)†	54.48	54.72	54.75

* Minus log likelihood plus number of parameters estimated.
† Sum of squared deviations between reenlistment dummy and predicted reenlistment probability.

is used. Table 3, which shows additional measures of goodness-of-fit for the three models, shows a similar result.[22]

We also estimated a naive retention model with current retention a function only of the current pay difference and the other variables shown in the table. The pay coefficient was much larger (.729), but the model fit was much worse than any of the models shown in table 2 (log-likelihood value of −1,668.5). Hence we find that incorporating forward-looking behavior into the model improves fit considerably.

Simulations

Figure 1 shows plots of simulated and actual retention rates up to the twenty-ninth year of service. The actual rates are taken from cross-sectional retention rates among infantry soldiers by years of service in fiscal year 1988 reported by the army, and hence are out of sample. The simulated rates from our DRM should not necessarily match the actual rates because the populations and time periods are different, and because the regressor variables took on different values in the different periods. However, on a priori grounds we should expect them to show the same patterns. Recall that our data go up only through the second term, roughly 6 or 7 years of service, so all simulations beyond that point are extrapolations of our model beyond our data. As the figure shows, the DRM tracks the upward pattern of retention rates fairly well through the nineteenth year of service. At the twentieth year of service, where retirement vesting occurs, the basic DRM predicts a considerably smaller drop than shown in the actual rates. However, the DRM with a separate retirement coefficient shows almost exactly the same drop as shown in the actual profile, a fairly impressive result given how far beyond the data is the DRM extrapolation. After the twentieth year of service, neither of the DRMs tracks retention rates particularly

[22] As noted previously, Lumsdaine et al. (1992) found a similar result for retirement models.

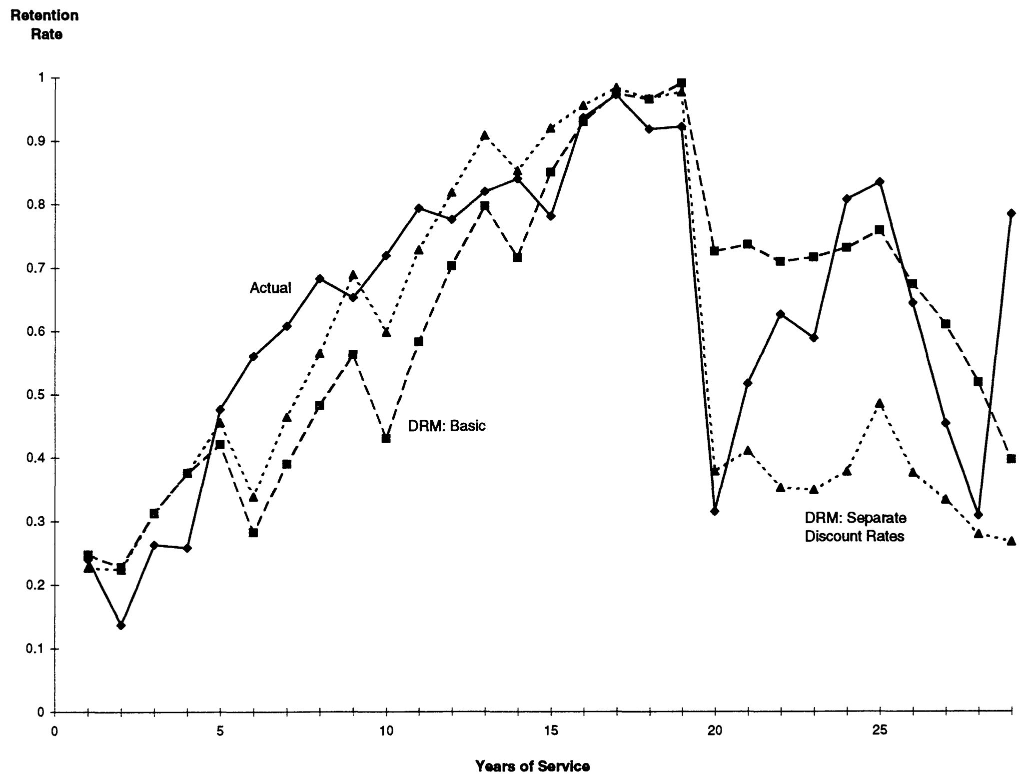

FIG. 1.—Simulated and actual retention rates, by years of service

well, although there are few enlistees remaining in the military in that range.

To illustrate the simulation capability of the model in a more relevant policy context, and to compare it to the TCOL and ACOL models, we simulate a rough version of the recent Voluntary Separation Incentive (VSI) program offered by the army in an attempt to reduce the size of its force. This program, available only for a few months in calendar 1992, offers those who do not reenlist certain supplementary payments from the military for a few years after their departure. The amount of the payment is equal to 2.5% times their years of service times their base pay, and hence is larger for those with more time in the army. The number of years for which the payment is guaranteed is equal to twice the number of years of service, again providing more of an inducement to leave to those with more time in the army.

Table 4 shows our simulations of the effect of VSI if it were offered in two different ways: (1) if it were offered immediately (i.e., at the current decision point in question) and (2) if it were offered at the next decision point. In both cases, the VSI is available only at those points, not if departure from the military takes place at any other decision point.[23] As the first two columns show, the DRM predicts that retention rates at the first term and at the second term would fall by approximately 3 and 8 percentage points, respectively, if VSI were offered at those decision points. The VSI improves the level of the "civilian" age-income profile and hence reduces retention rates in the army. The effect is larger at the second term because the magnitude of the VSI payment is larger then, as previously noted. If the VSI were offered at the next decision point (i.e., at the second term for first-termers and at the third term for second-termers), retention rates would rise at both decision points, as shown in the third row of the table. The

Table 4
Simulated Effects of VSI Program on Retention Rates

	DRM		TCOL		ACOL	
	First Term	Second Term	First Term	Second Term	First Term	Second Term
Baseline	.331	.650	.332	.636	.331	.636
If VSI offered immediately	.302	.568	.305	.561	.309	.564
If VSI offered at next point	.361	.676	.332	.636	.331	.640

[23] The actual VSI in effect restricts eligibility to those with at least 7 years of service. Also, it has to be applied for. We ignore these restrictions in order to make the across-model comparisons, including the first-term, reported in the table.

DRM predicts, as is plausible, that departure from the military would be postponed to take advantage of the VSI supplements at the later date.

The remaining columns in table 4 show the predictions of the TCOL and ACOL models of the same VSI programs. The effect of immediately-offered VSI on retention rates is approximately the same as that predicted by the DRM. In the TCOL and ACOL models, it will be recalled, retention behavior is assumed to be a function of a comparison of expected civilian earnings (if the individual leaves the military immediately) to expected compensation at a single optimal future leaving date. An immediately-offered VSI does not affect the optimal leaving date since that date is chosen over future dates, excluding the current one. But it does affect the civilian earnings profile and hence affects retention in a negative direction.

However, an offer of the VSI as of the next term has virtually no effect on retention rates in either the TCOL or ACOL models, in stark contrast to the predictions of the DRM model. The problem in the TCOL and ACOL models lies in their assumption that behavior is affected only by a single optimal future leaving date. In the absence of VSI, over 90% of optimal leaving dates are over 20 years of service because that is the point of retirement vesting. The VSI payments, if available at the leaving date only one decision point in the future, are not sufficient in size to move the optimal leaving date up to the VSI point for all but a handful of individuals (less than 1%). Hence the predicted effect of such a VSI is essentially zero, as shown in the table.

Figure 2 shows further evidence of the capability of our DRM model to simulate a flexible and plausible response to the VSI. The figure shows simulations of the effect of offering VSI at the end of the third term for those coming up for reenlistment at the end of the second term. The immediate positive effect of VSI on retention shown in table 4 is shown in figure 2 to occur around the ninth year of service, when most second terms end. Retention rates for the subsequent 5 or 6 years are lowered from what they would have been otherwise because the enlistees who have postponed departure from the military to take advantage of the VSI finally depart to take advantage of the more attractive package. Around 14 years of service, this effect has faded away. Neither the ACOL nor TCOL model is capable of providing such simulations in as easy or simple a fashion.[24]

V. Summary and Conclusions

In this article we have formulated and estimated a stochastic dynamic programming model for military reenlistment which makes differences in

[24] To construct the TCOL or ACOL forecasts comparable to fig. 2 would require recalculating optimal leaving dates at every date in the future (each of which requires rechecking all possible future leaving dates at each future date). This is much more cumbersome than in the DRM model, where the distribution of future retention rates requires a single run of the model.

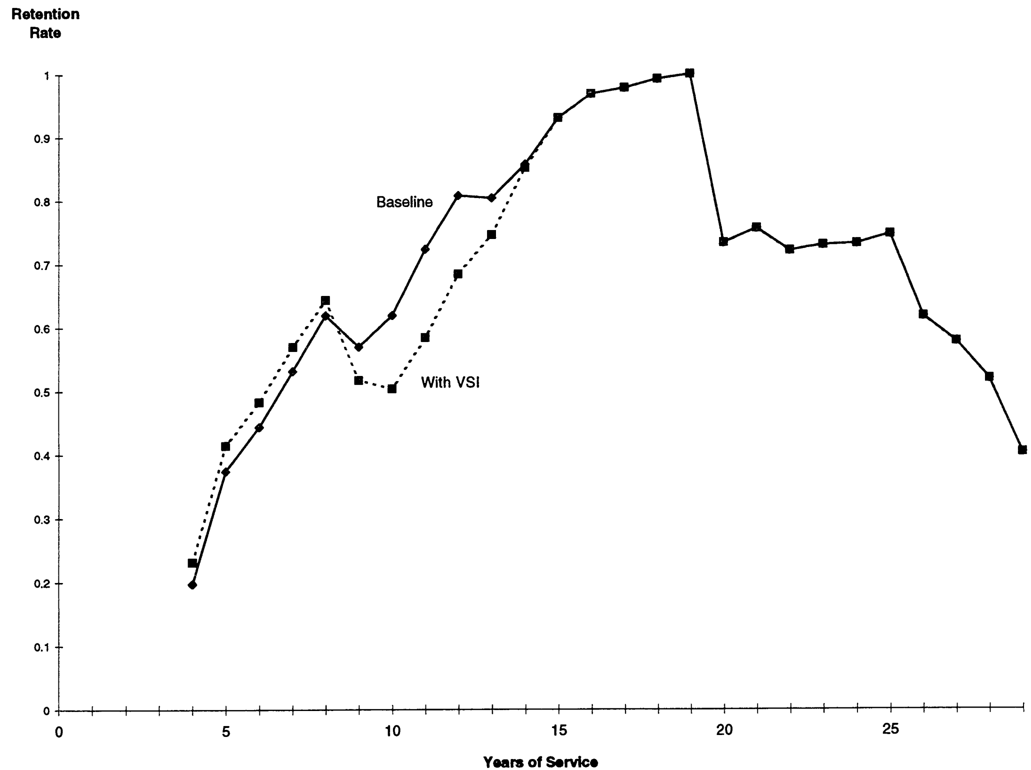

FIG. 2.—Retention rates with and without VSI

military-civilian lifetime earnings profiles offered at different reenlistment dates a central factor in decision making. We have shown that the model takes on a very simple form and is relatively easy to estimate even in the presence of unobserved heterogeneity, contrary to the implications of some past work in the area. Our results show that the military-civilian pay difference significantly affects military reenlistment, with potential military retirement benefits having the strongest influence. We also show that the model produces plausible simulated effects of changes in the compensation schedule, such as the Voluntary Separation Incentive program currently offered by the army. Other models, such as ACOL and what we term the TCOL model, produce implausible effects of the same program.

There are many important aspects of the military reenlistment decision we have not captured. Differences in the riskiness of the military and civilian earnings profiles are a prominent example. Nor have we attempted to model the relevance of military occupational training to the civilian labor market. In addition, the dynamic retention model we have used is still restrictive in its specification of serial correlation of unobservables and in its assumption of wealth rather than utility maximization. These and other topics provide avenues for future research.

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