Structural versus Reduced-Form Estimation of Optimal Stopping Problems

Bill Provencher

In this paper I examine several statistical, interpretive, and policy implications of reduced-form (probit or logit) estimation of optimal stopping problems. The discussion proceeds in the context of an examination of the timber harvest decision of nonindustrial private forest (NIPF) owners. For a large class of optimal stopping problems a reduced-form model which closely approximates the statistical performance of its structural counterpart is readily found. Still, failure to properly interpret the relationship between the reduced-form model and the underlying optimal stopping problem invites flawed econometric analysis and inappropriate interpretation of reduced-form coefficients.

Key words: optimal stopping problems, reduced-form estimation.

Many problems in agricultural and natural resources economics are best characterized as Markovian optimal stopping problems (OSPs) in which the decision maker must choose when to start or stop a particular activity, and all information relevant to the decision is contained in current state variables. Examples in agricultural economics include decisions concerning crop harvest, exiting from farming, and participation in optional farm programs. Examples in resource economics include decisions about abandoning a mine or oil field, exiting a commercial fishery, harvesting a timber stand, and developing a wetlands or other wild place. For most of these problems the economic theory is fairly well developed, and economists have good insights to the optimal solution of the decision problem. Given the structural parameters of the problem, economists are able to advise decision makers about the effects of different courses of action.

Consider instead the quite different problem of modeling the behavior of individuals who supposedly solve an OSP. The task at hand is no longer a normative one, but a positive one; available data must be analyzed to explain behavior in a dynamic setting. One approach is reduced-form estimation in which the stopping decision is cast as the dependent variable in a logit or probit regression and the relevant state variables serve as regressors. This approach requires specifying the functional form of the decision rule derived from solution of the underlying OSP and using available data to estimate the decision rule.

Unfortunately, reduced-form estimation tends to obscure the dynamic context of estimation, and so it is easy to misspecify the model, misinterpret coefficient estimates, or otherwise analyze the data incorrectly. Moreover, reduced-form estimation reveals no information about the structural processes underlying the harvest decision. For instance, in examinations of timber harvesting it provides no information about forest owner price expectations. As a consequence, the reduced-form model is subject to the Lucas Critique—it is vulnerable to faulty predictions of the effect of policy changes because such changes affect the underlying structural process in ways that cannot be discerned in the reduced-form model. The reduced-form parameters are themselves dependent on the underlying structure of the decision problem. When the decision environment changes, these parameters change as a result, but how they change is not known a priori.

An alternative to the apparent interpretive and predictive weakness of reduced-form estimation is direct estimation of the underlying structural decision model. In recent years structural estimation of dynamic decision problems has been used to examine such diverse prob-

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lems as patent renewal (Pakes), bus engine replacement (Rust 1987), dairy cow replacement (Miranda and Schnitkey), and timber harvesting (Provencher 1995a, 1995b). In all of these efforts, variability in the data is captured by an additional state variable that is contemporaneously observed by the decision maker but not observed by the analyst. Unfortunately, the computational cost of structural estimation can be imposing. For even modest-sized problems, considerable effort by an experienced programmer is required to write an algorithm that will successfully estimate the model. Typically, compromises must be struck to obtain a computationally feasible model of behavior. As a result, estimable structural models remain, at best, very simplified representations of the dynamic decision processes being considered. In this light, any claim for structural modeling that rests solely on the Lucas Critique is not immediately compelling. Rust and Pakes observe, "The 'Lucas Critique', which is so often cited by structural modellers as the justification for using structural as opposed to reduced-form models to predict the impact of policy changes, becomes significantly less potent if one admits that the structural model itself is only a crude approximation to reality" (p. 4). This notwithstanding, the authors argue for structural estimation on two grounds. First, initial comparisons indicate that structural models of dynamic decision processes do outperform their reduced-form counterparts in out-of-sample forecasting (see, for instance, Lumsdaine, Stock, and Wise). And second, the exercise of carefully estimating a dynamic decision process often leads to a fuller understanding of the model and the data. This perspective is consistent with the critique of reduced-form estimation as often obscuring the dynamic context of estimation.

The purpose of this paper is to examine the statistical, interpretive, and policy implications of reduced-form estimation of OSPs. The discussion proceeds in the context of an examination of the timber harvest decision of nonindustrial private forest (NIPF) owners. There exists considerable literature on the subject, including the empirical studies of Binkley, Boyd, Dennis, and Hyberg and Holthausen. All of these studies used reduced-form logit or probit regression to examine the timber harvest decision, and thus they serve as a point of reference for examining the issues involved in reduced-form estimation. Attention focuses on the role of timber price in the harvest decisions of NIPF owners, which is a perplexing issue in the recent empirical studies. Various points are illustrated using data generated via Monte Carlo simulation of a harvest problem solved by NIPF owners.

Estimation of an Optimal Stopping Problem

Consider an activity for which there is a payoff each period the activity is continued and a different payoff when the activity is stopped. Let $i = 0$ denote the decision to continue the activity, and let $i = 1$ denote the decision to stop. The amount of the payoff in a period depends on the state variables $\mathbf{x}$, which evolve over time according to the probability distribution function $f(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$, the decision-specific random shock ε^i, and perhaps other variables, denoted by the vector $\mathbf{y}$, that are invariant over time. The random shocks are contemporaneously observed by the decision maker but never observed by anyone else. For simplicity we assume these shocks are additive and identically and independently distributed over time. Letting $R^i(\mathbf{x}_t, \mathbf{y}) + \varepsilon_t^i$ denote the decision-specific payoff at time t, the relevant decision problem can be stated as

$$(1) \quad V(\mathbf{x}_t, \mathbf{y}) = \max\{R^0(\mathbf{x}_t, \mathbf{y}) + \varepsilon_t^0 + \theta E[V(\mathbf{x}_{t+1}, \mathbf{y})]\}, R^1(\mathbf{x}_t, \mathbf{y}) + \varepsilon_t^1\}$$

where θ is a discount factor.

The problem in equation (1) reflects Bellman's principal of optimality; in particular, the expected value of not stopping the activity in period t implicitly recognizes that optimal decisions are made in the future. This problem can be solved via the recursive methods of dynamic programming. The solution is an optimal decision rule $i(\mathbf{x}_t, \mathbf{y}, \varepsilon_t; \Gamma)$, where ε_t denotes the difference $\varepsilon_t^1 - \varepsilon_t^0$ and Γ is the set of structural parameters associated with the decision problem.

This basic problem can be adapted to the timber harvest decision on even-aged timber stands held in nonindustrial private forests (NIPFs)—those forests held by landowners not directly involved in the manufacture of forest products. The issue of timber harvests on NIPFs has received a considerable amount of attention in the forest economics literature because nationally these forests comprise about 75% of commercial timberland, and there is concern among forestry professionals that these forests are not well managed. Several studies have estimated reduced-form, stand-level models of harvest behavior on NIPFs (Binkley, Boyd, Dennis, Hyberg and Holthausen).

Formally, let s_t denote the age of the timber stand at time t, and let $u(s_t)$ denote a money measure of periodic utility from the standing forest; following the previous literature, the utility received by NIPF owners from the standing forest depends on the age of the timber stand.[1] Also, let p_t and $v(s_t)$ denote the price and volume of timber, and let λ denote the market value of bare forestland—the salvage value associated with the decision to harvest the timber stand.[2] It follows that here $\mathbf{x_t}$ is comprised of the state variables p_t and s_t, and $R^0(\mathbf{x_t}, \mathbf{y}) = u(s_t)$, $R^1(\mathbf{x_t}, \mathbf{y}) = u(s_t) + p_t v(s_t) + \lambda$. This model is used to facilitate the comparison between reduced-form and structural estimation.[3]

Now suppose there exist observations over time $t = 1, ..., T$, and across individuals (or firms) $j = 1, ..., J$, of decisions i_{jt} and variables $\mathbf{x_{jt}}$ and $\mathbf{y_j}$. There exist two divergent approaches to analyzing the data. These are discussed in turn.

Reduced-Form Estimation

Probit and logit regression can be rationalized as attempts to directly estimate the decision rule $i(\mathbf{x_{jt}}, \mathbf{y_j}, \varepsilon_{jt}; \Gamma)$. One might suppose, for instance, that the decision rule is characterized by a latent variable i^*_{jt}, the value of which is linear in the original variables:

$$(2) \qquad i^*_{jt} = \beta_x \mathbf{x_{jt}} + \beta_y \mathbf{y_j} + \varepsilon_{jt}$$

where β_x and β_y are conformable vectors of parameters. The decision i_{jt} relates to the latent variable i^*_{jt} as follows:

$$(3) \qquad i_{jt} = \begin{cases} 1 \text{ if } i^*_{jt} > 0 \\ 0 \text{ otherwise} \end{cases}$$

and so letting $g(\varepsilon \mid \sigma_\varepsilon)$ denote the probability distribution function of ε, where σ_ε is a set of parameters defining the distribution, the probability of observing termination of the activity by individual j at time t is given by

$$(4) \quad \text{prob}(i_{jt} = 1) = \text{prob}(\beta_x \mathbf{x_{jt}} + \beta_y \mathbf{y_j} + \varepsilon_{jt} > 0)$$
$$= \text{prob}(\varepsilon_{jt} > -\beta_x \mathbf{x_{jt}} - \beta_y \mathbf{y_j})$$
$$= \int_{\varepsilon > A} g(\varepsilon \mid \sigma_\varepsilon)$$

where $A = -\beta_x \mathbf{x_{jt}} - \beta_y \mathbf{y_j}$. Now define $\Gamma^r = (\beta_x, \beta_y, \sigma_\varepsilon)$, and let $\text{prob}(i_{jt} \mid \mathbf{x_{jt}}, \mathbf{y_j}, \Gamma^r)$ denote the probability of observed decision i_{jt}. Then if the random shock is independently and identically distributed over time and across individuals, the associated likelihood function is simply[4]

$$(5) \quad L(\Gamma^r) = \prod_{j=1}^{J} \prod_{t=1}^{T} \text{prob}(i_{jt} \mid \mathbf{x_{jt}}, \mathbf{y_j}, \Gamma^r) = \prod_{j=1}^{J} L_j(\Gamma^r).$$

Structural Estimation

Consider instead direct estimation of the structural problem (1). The probability of harvest by individual j at time t is defined by

$$(6) \quad \text{prob}(i_{jt} = 1) = \text{prob}(R^1(\mathbf{x_{jt}}, \mathbf{y_j})$$
$$- [R^0(\mathbf{x_{jt}}, \mathbf{y_j}) + \theta E\{V(\mathbf{x_{j,t+1}}, \mathbf{y_j})\}] + \varepsilon_{jt} > 0)$$
$$= \text{prob}(\varepsilon_{jt} > R^0(\mathbf{x_{jt}}, \mathbf{y_j}) + \theta E\{V(\mathbf{x_{j,t+1}}, \mathbf{y_j})\}$$
$$- R^1(\mathbf{x_{jt}}, \mathbf{y_j})) = \int_{\varepsilon > A} g(\varepsilon \mid \sigma_\varepsilon)$$

where now $A = R^0(\mathbf{x_{jt}}, \mathbf{y_j}) + \theta E\{V(\mathbf{x_{j,t+1}}, \mathbf{y_j})\} - R^1(\mathbf{x_{jt}}, \mathbf{y_j})$, and once again $g(\varepsilon \mid \sigma_\varepsilon)$ is the probability density function of ε. The likelihood function is structurally identical to equation (5), but the parameter vector to be estimated is the vector of structural parameters Γ, which includes the discount rate, parameters of the payoff functions $R^i(\cdot)$, parameters of the distribution function $f(\cdot)$, and the distribution parameters σ_ε. Formally, the likelihood function is[5]

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[1] In general, utility from the forest stand also depends on characteristics of the forest owner, such as age and education. Here these other variables are suppressed to simplify the discussion.

[2] Because the minimum rotation age for clearcut timber stands is usually at least fifteen to twenty years, for realistic values of the discount rate it is reasonable to exclude current timber price as a determinant of salvage value.

[3] The specification of R^1 implies that the harvest occurs at the end of the period, after utility for the period has been gained. This is done solely to facilitate the exposition later in the discussion and has no impact on the basic results derived.

[4] If the disturbance ε is not independently and identically distributed over time and across observations, then the likelihood function is not a simple product of probabilities, and the calculation of likelihood values is a more difficult exercise for which many econometric packages are not equipped. See, for instance, Lerman and Manski.

[5] Footnote 4 applies to this case as well.

$$(7)\quad L(\Gamma) = \prod_{j=1}^{J} \prod_{t=1}^{T} \text{prob}(i_{jt} | \mathbf{x}_{jt}, \mathbf{y}_j; \Gamma) = \prod_{j=1}^{J} L_j(\Gamma).$$

Note, however, that here estimation is far more complicated than for the reduced-form model, because the search for the parameter vector that maximizes the likelihood function involves solving the decision problem (1) each time new parameter values are evaluated in the search; this is apparent by the presence of $E\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\}$ in the limit of integration in equation (6). Maximum likelihood estimation thus involves the nesting of an "inner" dynamic programming algorithm within an "outer" hill-climbing algorithm (Rust 1994).

Issue One: Specification of Reduced-Form Models

In reduced-form estimation it is standard practice to represent the latent variable i^*_{jt} as linear in the original variables, as in equation (2). This approach was taken, for instance, in the empirical studies of timber harvesting on NIPFs.[6] A comparison of the first lines of equations (4) and (6) reveals that if the structural model is the "true" decision model, then the latent variable in the reduced-form model implicitly denotes the expected net gain from stopping the activity, where the expectation is taken over the future value of the activity. It follows that the "standard" reduced-form model, in which the latent variable is linear in the original variables, is appropriate only if the expected net gain from stopping the activity is itself linear in the original variables. Formally, we require

$$(8)\quad \beta_x \mathbf{x}_{jt} + \beta_y \mathbf{y}_j \equiv -A \equiv R^1(\mathbf{x}_{jt}, \mathbf{y}_j) - [R^0(\mathbf{x}_{jt}, \mathbf{y}_j) + \theta E\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\}]$$

where A is the limit of integration in equations (4) and (6). For the simple model of NIPF owner behavior, equation (8) can be restated as

$$(9)\quad \beta_0 + \beta_1 s_{jt} + \beta_2 p_t \equiv -A \equiv p_t v(s_t) + \lambda - \theta E\{V(s_{j,t+1}, p_{t+1})\}$$

which states that the expected net gain from harvesting is linear in timber price and stand age. Generally this is not true. Consider, for instance, the case where the price expectations of NIPF owners take the naïve form examined by Brazee and Mendelsohn; in particular, NIPF owners suppose that observed prices are random fluctuations around a fixed mean price. In this case $E\{V(s_{j,t+1}, p_{t+1})\}$ is not a function of the current price p_t, and so clearly equation (9) cannot hold so long as the volume function $v(s_t)$ is anything but a constant. Standard probit estimation omits a relevant dependent variable [total revenue, $p_t v(s_t)$], and includes an irrelevant dependent variable (p_t). Still another problem is the implicit attempt to represent the discounted expected value function $\theta E\{V(s_{j,t+1})\}$ as a linear function of s_{jt}.

To illustrate the potential for misspecification of reduced-form models, consider a simple model of Loblolly Pine harvesting based on the quarterly model examined in Haight and Holmes. The forest owner harvests if the net gain of harvesting is positive. The price of bare forestland is \$550/acre, and volume growth is defined by $v(s) = -16.54 + 1.029s - 0.00522s^2$, where volume is measured in thousand board feet (mbf) per acre. Trees are harvested for sawtimber only, and the minimum stand age for harvest is thirty years. The amenity value of the standing forest is $u(s) = \alpha \ln v(s)$, where $\alpha = 50$. For this value of α, the annual amenity value of a standing forest of age fifty is \$154 per acre per year, about 28% of the value of bare forestland.[7] The standard deviation of the random shock ε is \$55, or 10% of the value of bare forestland. Sawtimber prices are random deviations about a fixed mean of \$125/mbf, with a standard deviation of \$12.5/mbf. Except for modifications noted when they occur, this is the model of harvesting used throughout the paper.[8]

Figure 1 presents the right-hand side of equation (9) for this model, that is, the expected net gain from harvesting timber, $-A$. Recalling that for this model the price next period is not a function of the price this period, from equation (9) we have $-(\partial A/\partial p_t) = v(s_t)$, which is not a constant β_2. This is reflected in figure 1; the slope of the surface in the price dimension is steeper at high volumes (high stand ages) than

[6] Here and after, mention of the empirical studies of timber harvesting on NIPFs refers to the studies of Binkley, Boyd, Dennis, and Hyberg and Holthausen.

[7] Except as noted, the general nature of the results presented below were the same for a wide range of parameter values.

[8] Solutions to this and related decision problems were obtained from a dynamic programming algorithm in which the value function is approximated as a Chebyshev polynomial. The algorithm is available from the author.

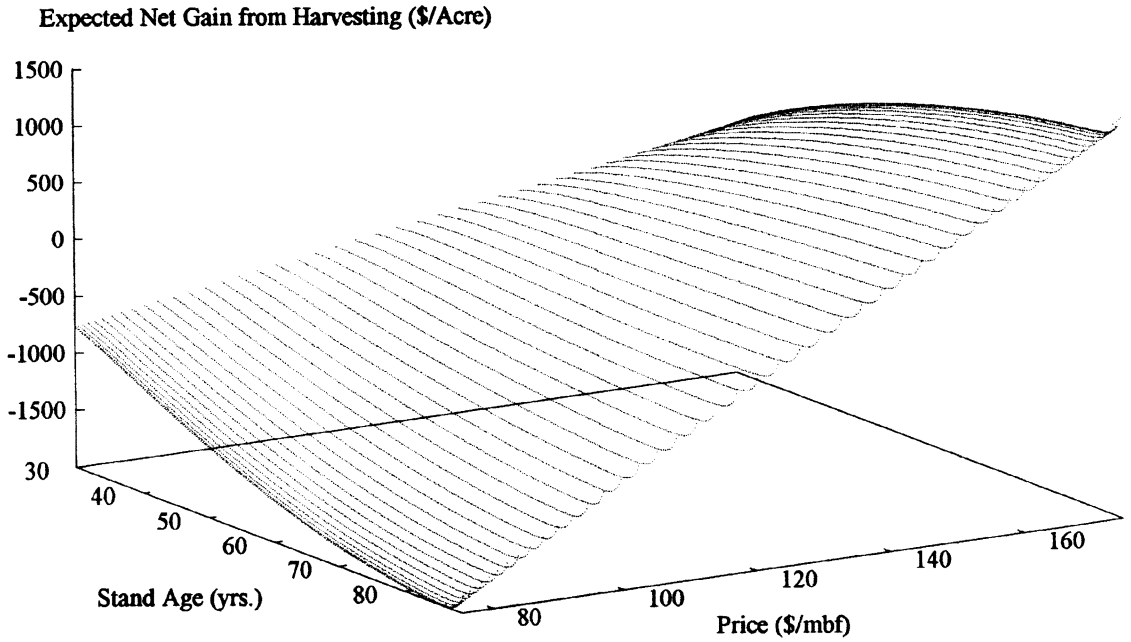

Figure 1. Expected net gain from harvesting

at low volumes (low stand ages). Moreover, figure 1 reveals that at low prices the effect of stand age on the expected net gain of harvest is negative, while at high prices the effect of stand age is positive. The explanation for this result is the following. At low prices the forest owner is always better off postponing harvest, because there is a high probability of a higher price next period. The payoff associated with this future price increase is greater for older (high-volume) stands, thus the negative effect of stand age on the expected net gain from immediate harvest. On the other hand, at a high price the net gain from harvesting is greater for an old stand than for a young stand, because the potential for substantial future growth in the young stand mitigates the failure to take advantage of the high price by harvesting. The important point to be made is that, as with timber price, the effect of stand age on the expected net gain of harvesting cannot be captured by a single coefficient.

The resolution of such specification problems is immediately apparent once it is understood that the latent variable i^*_{jt} is a measure of the expected net gain from stopping the activity. In estimation, the right-hand side of equation (2) should reflect what is known of the structure of the expected net gain from stopping the activity. This may require nothing more than including higher-order terms intended both to represent the payoffs $R^0(\mathbf{x}_{jt}, \mathbf{y}_j)$ and $R^1(\mathbf{x}_{jt}, \mathbf{y}_j)$, and to approximate the expected value function $\theta E\{V(\mathbf{x}_{j,t+1})\}$; or it may require something more complicated, such as relaxing the assumption that the disturbance in equation (2) is serially uncorrelated.[9] Indeed, if in structural estimation the expected value function is itself approximated via polynomial interpolation [see, for instance, Judd (chap. 12, p. 8) and Rust (1994, p. 86), who discuss the use of Chebyshev polynomial approximations of the value function as a means for quick solution of dynamic programming problems] it should be possible to closely approximate the statistical performance of the structural model simply by adding higher-order terms.

That a reduced-form model can be found which closely describes the observable behavior associated with an OSP is demonstrated by comparing the probability of timber harvest de-

[9] Suppose, for instance, that the random shock in the structural model is reasonably modeled as a first-order autoregressive process. Such would be the case, for instance, if one conceived of the shock as a harvest premium that persists over time. In this case, the disturbance in the reduced-form model also should be modeled as a first-order autoregressive process, and estimation of both the structural and reduced-form models is a considerably more difficult enterprise (see footnotes 4 and 5).

rived from various reduced-form models to that derived from a "true" structural model of the harvest decision. To this end, Monte Carlo simulation was used to generate observations for 10,000 forest stands for which the harvest problem described above—the problem generating figure 1—is solved. In the current context, one can think of the "true" parameters for a particular reduced-form model as those to which maximum likelihood estimates converge as the sample size approaches infinity. The large sample generated via Monte Carlo simulation was intended to locate these parameters within narrow confidence intervals. Kenneth White's econometric package SHAZAM was used to fit two probit models to the data. The first model corresponds to the standard probit model implied by the left-hand side of equation (9). The second model is a theoretically more attractive model, hereafter called the "alternative" probit model. The interaction term $p_t v(s_{jt})$—total revenue—is added, and the price term is dropped because price does not affect the harvest decision apart from total revenue. Also added to better approximate the expected value function $\theta E\{V(s_{j,t+1})\}$ is the quadratic term s_t^2. So for the alternative probit model the latent variable is $i_{jt}^* = \beta_0 + \beta_1 p_t v(s_{jt}) + \beta_2 s_{jt} + \beta_3 s_{jt}^2 + \varepsilon_{jt}$, where $\beta_1 = 1$.[10]

Results for the exercise are summarized in figures 2 and 3, which map the difference between the probability of harvest for a probit model—the estimated standard model in figure 2 and the estimated alternative model in figure 3—and the probability of harvest for the underlying structural model. The orientations of the figures were chosen to best display the geometry of the surfaces, so they are slightly different, though the vertical scales of the figures are identical. In both figures, stand age extends only to age fifty-five because all stands are harvested by this age. As expected, the alternative probit model more closely approximates the structural model than does the standard probit model. For the alternative model, figure 3 shows that there is a narrow "ridge" in the price range \$140–\$146, and extending from stand age thirty to about stand age forty-eight, where the probability of harvest is 0.08–0.22 greater than for the structural model. For almost all other price and stand age combinations the probability difference is near zero. The probability of the observed price actually falling on the "ridge" in figure 3 is approximately 0.08–0.10, suggesting that, overall, the probability that the reduced-form and structural models yield different decisions is less than 2%. This is supported by the comparison in table 1 of how often the models yield the same predictions of outcomes in a Monte Carlo simulation. The simulation was identical to that used to estimate the probit models in the first place: 10,000 forest stands were observed from age thirty to harvest. The predicted outcome is to harvest if the probability of harvest is greater than 0.5; otherwise the predicted outcome is to postpone harvest. Note that although the structural model underlies the simulation, due to the random shock it may "predict" harvest when no harvest is observed, and vice versa.

Two basic conclusions can be drawn from table 1. First, the standard probit model makes wrong predictions about 3.6% more often than either the structural model or the alternative probit model. The standard probit model makes incorrect predictions a total of 5,244 times; on 3,573 occasions it fails to predict a harvest when a harvest occurs, and on 1,671 occasions it predicts a harvest when no harvest occurs. By comparison, the structural model makes wrong predictions on 5,061 occasions, and the alternative probit model makes wrong predictions on 5,081 occasions. The second, more significant conclusion is that the standard probit model does a good job of matching the structural model, but the alternative probit model does even better. For 169,627 of 170,320 total observations (99.6% of total observations) the structural and alternative probit models yield the same predictions. The structural and alternative probit models both predict harvest on 8,092 occasions; by comparison, the structural and standard probit models both predict harvest on 7,579 occasions.

Issue Two: Misinterpretation of Reduced-Form Models

The failure to properly interpret the relationship between a reduced-form model and the underlying OSP generating the data may lead to inappropriate econometric analysis and improper interpretation of coefficient estimates. A notable source of this failure is the probability distribution function $f(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$; its role in the OSP is obviously crucial but nonetheless often overlooked in reduced-form estimation. This

[10] It seems quite possible—even likely—that in reality stand volume is not observed at each point in time, whereas stand age is observed. In this case, the interaction term $p_t v(s_{jt})$ can be replaced by one or several interaction terms involving p_t and s_{jt}. I thank a reviewer for pointing this out.

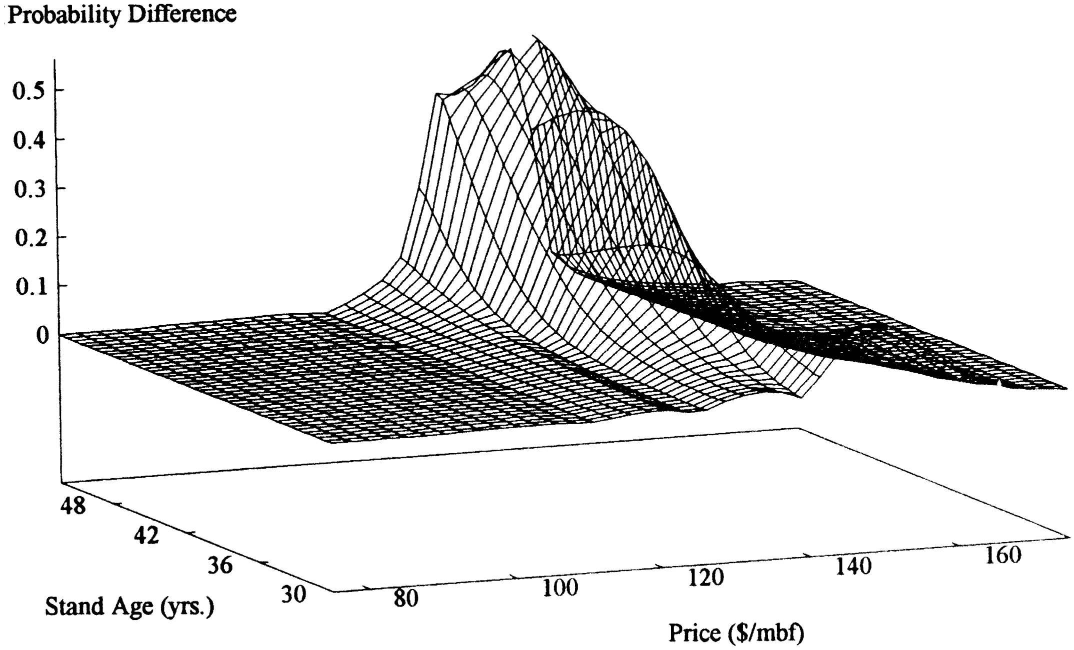

Figure 2. Difference in the probability of harvest: standard probit model versus structural model

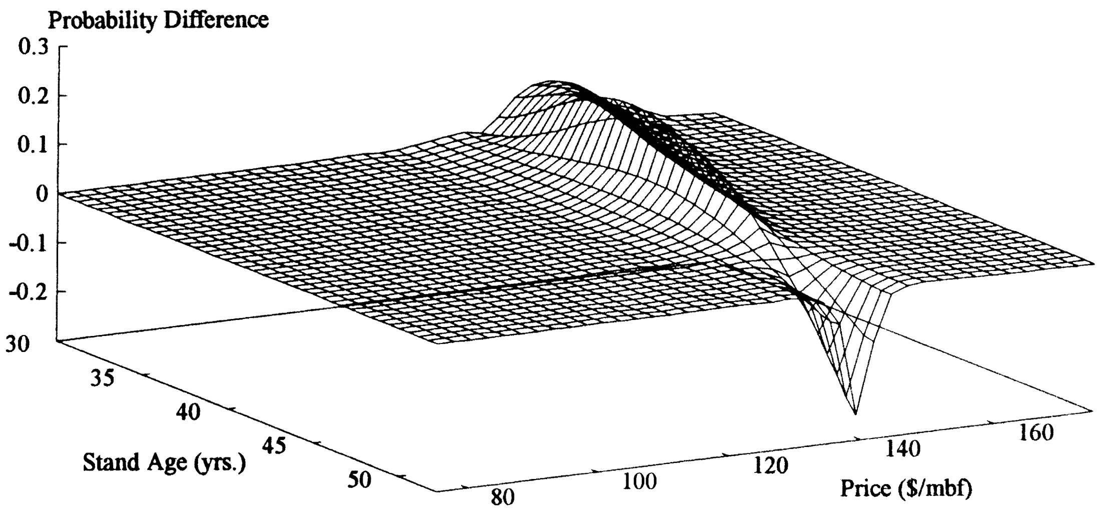

Figure 3. Difference in the probability of harvest: alternative probit model versus structural model

point is illustrated with two examples relevant to the issue of the effect of timber price on the harvest decisions of NIPF owners.

The empirical studies of the harvest decision on NIPFs yield mixed results concerning the effect of timber price on the harvest decision. Binkley and Boyd find a positive effect. Dennis finds a nonsignificant positive effect. Hyberg and Holthausen find a significant negative effect. All of these studies either used cross-sectional data only, in which observations from different timber price regions (such as the mountains of northwest Georgia and the coastal plain of southeast Georgia) were used in a

Table 1. Comparison of Actual and Predicted Outcomes for Structural and Reduced-Form Models

Harvest Predicted By:	Observed Result: Harvest	No Harvest	Total
All three models	6,185	1,344	7,529
Structural model only	68	56	124
Standard probit only	209	287	496
Alternative probit only	0	0	0
Structural and standard only	26	24	50
Structural and alternative only	325	238	563
Probit models only	7	16	23
None of the models	3,180	158,355	161,535
Total	10,000	160,320	170,320

single regression, or they used pooled cross-sectional and time-series data.

Estimating a reduced-form model using cross-sectional data, in which the probability distribution function $f(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$ varies across sets of observations, is problematical because the underlying OSPs will differ across the sets. Suppose, for instance, that data on timber harvesting are obtained from two distinct regions. In region 1 prices fluctuate randomly around a mean price of $75/mbf, with a standard deviation of $6.25/mbf, and in region 2 prices fluctuate randomly around a mean price of $150/mbf, with a standard deviation of $12.50/mbf. Now suppose that in each region 150 forest stands are observed from the minimum harvest age of thirty to the time of harvest. Figure 4 presents typical results generated by Monte Carlo simulation.[11] As expected, no-harvest observations in the price range $50–$90 are almost all from region 1, as are all the harvests in the price range $80–$100. Similarly, no-harvest observations in the price range $125–$190 are all from region 2, as are all the harvests in the price range $150–$190. Clearly, for each region price has a positive effect on the probability of harvest. It should be just as clear from the figure that if these data are combined in a single probit regression, the positive sign on price will be substantially "diluted." Indeed, when each region is treated separately in a regression using observations generated by Monte Carlo simulation of 300 timber stands, the sign on price is positive and significant, as expected. On the other hand, when the regions are combined in a single regression using observations generated via Monte Carlo simulation of 300 timber stands (150 from each region), the sign on price is negative and nonsignificant.[12] The upshot is that in reduced-form estimation it is imperative to fully differentiate among the OSPs solved by different agents.[13]

The price results in the Dennis and Hyberg and Holthausen studies may simply reflect the failure to distinguish among regional differences in the underlying OSPs. Although the Binkley and Boyd studies find that price has a positive effect on the harvest decision, these results are nonetheless confounded by this same problem. For instance, Boyd limited his empirical investigation to a cross-sectional study of the harvest decision in North Carolina in 1980. Sample variation in timber price arose from regional differences in the 1980 price. The author notes that using regional prices serves to avoid various estimation issues associated with time-series data. Yet the price expectations of NIPF owners in the mountains of North Carolina are different than those of NIPF owners on the coastal plain, where, historically, softwood timber prices are much higher; and it is just as plain that the rate of timber growth in the two

[11] The price of bare forestland is the same for the two regions. Of course, this is unrealistic, but for the point to be made here it is safely ignored. In experiments not presented here, bare land prices of $300 and $600 were used for regions 1 and 2, without a qualitative change in results.

[12] In repeated trials the coefficient on price in the single regression is in the neighborhood of –0.0007, and the t-statistic is in the neighborhood of –0.8.

[13] More generally, it is important to differentiate among OSPs in any estimation, reduced-form or structural. The point here is that in the absence of the discipline imposed by a structural model, the analyst is more likely to be complacent about the processes generating the data.

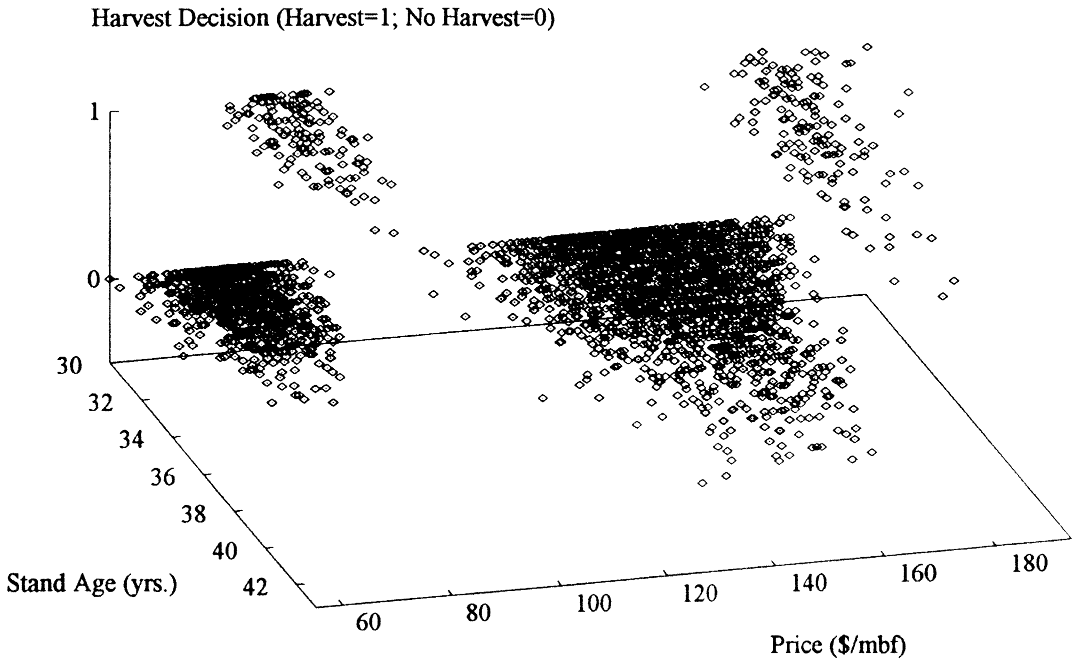

Figure 4. Plot of simulated harvest decision data for two regions distinguished by different price processes (see text)

regions is different, so we would expect the OSPs of the two regions to be different enough to warrant separate estimation. This is especially apparent when one considers the difficulty of interpreting the estimated coefficient on price in the Boyd study. This coefficient cannot be used to calculate, for instance, the change in the probability of harvesting in the mountains of North Carolina from a particular change in the observed price.

Even after correctly differentiating among OSPs, the particular form of the probability distribution $f(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$ may yield results in reduced-form estimation that are unexpected and perhaps even inexplicable if the relationship between the estimation and the underlying OSP generating the data is not understood. Suppose, for instance, that in reduced-form estimation the price coefficient is nonsignificant or negative, as in the Dennis and Hyberg and Holthausen studies. The authors of these studies offer a variety of explanations for this result, all of which may have some validity in general, but there is a simple alternative explanation concerning price expectations: NIPF owners may believe that prices follow a random walk. Simulation studies of even-aged timber harvesting indicate that when price expectations follow a random walk, price has either no effect (Reed and Clarke) or a negative effect (Haight and Holmes) on the harvest decision.[14] This is apparent from a simulation exercise involving 10,000 forest stands. In the exercise, price follows a random walk with a standard deviation of $12.5/mbf, and the decision problem is otherwise the same as the one described in the previous section. For the standard probit model—which is structurally most like the models estimated in the empirical studies of NIPF harvesting—the coefficient estimate on price was negative with a t-statistic of −13.2. In summary, this result is not due to an income effect associated with utility maximization, or to an errors-in-variables problem, or to multicollinearity, which are among the explanations offered by Dennis and Hyberg and by Holthausen for the failure to find that price has a significant positive effect on the harvest decision. Rather, it is a straightforward consequence of the particular probability distribution $f(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$ used in the OSP generating the data.

[14] The results from Reed and Clarke apply to a continuous time setting with timber price characterized by driftless Brownian motion (the continuous time equivalent of a random walk), and zero fixed costs. The result in Haight and Holmes applies to a model very similar to the one examined here; in particular, the price of bare forestland is fixed.

Issue Three: The Lucas Critique

The Lucas Critique of policy analysis with reduced-form models is well known, though perhaps not so obvious in the context of the estimation of discrete choice models. Such models are used to describe how policy instruments affect the probability of a particular action or decision, and here it is shown with a brief example that, as applied to reduced-form estimation of OSPs, such analysis is often ill-advised. Consider, for instance, the initial model of timber harvesting on NIPFs, where price fluctuates around a fixed mean of \$125, with standard deviation \$12.5. The coefficient on total revenue in the alternative probit model can be used to describe the effect on the probability of harvest of, say, a \$10/mbf decrease in the observed price. This is evident from figure 3 and table 1. But what if we are interested in the effect on the probability of harvest of the introduction of a permanent \$10/mbf yield tax? For the alternative probit model, the obvious and only way to model the tax is via the total revenue term $p_t \times v(s_t)$. A \$10 tax implies a \$10 reduction in the net price of timber, and so the predicted effect of the tax on the probability of harvest would be the same as that for a \$10 decrease in the price of timber. For the structural model, the obvious way to model the tax is also via the total revenue term, but now the optimal decision would reflect the decrease in the net price in the current period as well as all future periods. Table 2 compares results for the two models. For each price-age combination, the top number is the actual structurally derived probability of harvest with the yield tax, and the bottom number is the actual structurally derived probability of harvest without the yield tax. The middle number is the probability of harvest with the yield tax predicted by the alternative probit model. The actual effect of the yield tax is negligible, because it lowers the net price of timber in all periods. It has the same effect as lowering the mean price of timber from \$125 to \$115. Yet the reduced-form model must interpret the yield tax as a drop in the current price of timber only, which of course it is not, and so it badly underestimates the probability of harvest.

It deserves mention that using a structural model to analyze policy changes is, in general, a more difficult enterprise than indicated above. For instance, a change in the forestry tax structure would be expected to affect the price of bare forestland and perhaps the variability of prices. Nonetheless, a structural model at least provides points of entry—namely, the structural parameters—for analyzing the effects of long-term changes in the macroeconomic environment on microeconomic decisions.

Summary and Conclusions

Two points concerning the specification of reduced-form models of optimal stopping problems (OSPs) deserve emphasis. First, this specification has a structural interpretation. In particular, the latent variable of the reduced-form model denotes the expected net gain from stopping the activity, where the expectation is taken over the value of the activity in the future. By assuming a reduced-form structure in which the latent variable is linear in the original state variables, empirical studies of OSPs incidentally impose linearity on the underlying value function—generally an unlikely restriction. Second, so long as the value function is itself smooth, the statistical performance of a reduced-form model can be made arbitrarily close to that of a structural model by including higher-order terms intended to approximate the value function. This point was demonstrated via Monte Carlo simulation of a simple model of the harvest decision on nonindustrial private forests (NIPFs).

Still, the relationship between a reduced-form model and the underlying OSP should be clearly understood for two reasons. First, the failure to understand the process generating the data may lead to incorrect econometric analysis and misinterpretation of coefficients. Second, the Lucas Critique is otherwise easily overlooked, and policy analysis based on the model may lead to wrong conclusions. That reduced-form estimation should be explicitly related to the OSP generating the data may not be obvious in practice, and in any case it is a caveat usually unheeded. In the agriculture and resource economics literature it is commonplace to cast the decision of an OSP in a static framework. This eases the transition to reduced-form (logit or probit) estimation, but it serves to obscure the subtleties of decision making in a dynamic environment. It obscures the interpretation of the disturbance term as an unobserved state variable which may be correlated over time. It obscures the need to include higher-order terms in estimation to facilitate the approximation of the value function. And, as a final example, it conceals differences among OSPs arising from differences in the dynamic processes governing state variables.

Table 2. Actual and Predicted Probabilities of Harvest, With and Without the Addition of a $10/MBF Yield Tax

	Stand Age (yrs.)			
	35	40	45	50
Price ($/mbf)	Probability of Harvest			
130	0.0000	0.0003	0.0004	0.0004
	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0002	0.0003	0.0003
135	0.0000	0.0269	0.0588	0.0986
	0.0000	0.0000	0.0000	0.0000
	0.0066	0.0245	0.0497	0.0798
140	0.0960	0.3264	0.5711	0.7522
	0.0010	0.0023	0.0012	0.0001
	0.1007	0.3123	0.5383	0.7191
145	0.4538	0.8483	0.9728	0.9965
	0.0284	0.0844	0.0934	0.0413
	0.4645	0.8388	0.9672	0.9950
150	0.8583	0.9941	0.9999	1.0000
	0.2327	0.5338	0.6567	0.5902
	0.8643	0.9933	0.9999	1.0000

Note: For each price-age combination, the top number is the actual (structurally derived) probability of harvest with the yield tax, and the bottom number is the actual probability of harvest without the yield tax. The middle number is the probability of harvest with the yield tax predicted by the reduced-form model (see text).

Structural estimation of OSPs is not without problems. The most obvious is the difficulty of specifying a model that is computationally feasible. To date, most of the estimated structural models in the literature have been fairly simple. Yet continuing methodological advances should allow the estimation of more complicated models in the future. Even so, at least in the near term, complicated structural models that are accessible via reduced-form approximation will remain extremely difficult to estimate directly. There exists a clear tradeoff between estimating a simplified structural version of a complicated OSP, and approximating the relevant decision rule with reduced-form estimation. For now the better approach will remain a matter of circumstance and the intentions and judgment of the analyst. For instance, if the intent of the analyst is to characterize the decision process of a timber owner, perhaps to evaluate the effect on harvest behavior of structural changes in forest taxation, then reduced-form estimation clearly will not suffice. On the other hand, if the intent of the analyst is to predict the harvest response of timber owners to demand shocks, perhaps as part of an exercise to forecast annual participation in timber reforestation programs, then reduced-form estimation using historical data should be sufficient. Future attempts to estimate dynamic structural models will better illuminate the tradeoffs involved with the two estimation approaches.

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