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Reviewed work(s):
Source: *Journal of Labor Economics*, Vol. 7, No. 3 (Jul., 1989), pp. 348-369
Published by: The University of Chicago Press on behalf of the Society of Labor Economists and the NORC at the University of Chicago
Stable URL: http://www.jstor.org/stable/2535293
Accessed: 23/11/2011 13:22

Estimating a Simultaneous Search Model

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The primary goal of this article is to specify and estimate a structural simultaneous search model and then determine the empirical importance of simultaneous search. The results indicate that new labor force entrants search simultaneously. A secondary goal is to identify and estimate job offer arrival rates and wage offer rejection probabilities separately. The results indicate that a significant portion of unemployment spells are caused by slow arrival rates, but policies intended to speed arrival rates would increase the average length of unemployment spells.

I. Introduction

Over the last few years there has been a resurgence of interest in simultaneous search as first discussed by Stigler (1961). Burdett and Judd (1983) show that nonsequential search is a necessary condition for search equilibria in models where all agents face positive search costs. Wilde (1977) and Stern (1989) examine welfare issues specific to nonsequential search models. Gal, Landsberger, and Levykson (1981) and Morgan (1983) have analyzed general properties of nonsequential search models.

The primary goal of this article is to specify and estimate a structural simultaneous search model and then determine the empirical importance of simultaneous search. A theoretical model, similar to the models in Gal, Landsberger, and Levykson (1981) and Morgan (1983), is presented in which each individual chooses both a reservation wage and how many applications to send simultaneously. A necessary first-order condition is

I would like to thank Ken Wolpin, Zvi Eckstein, Dale Mortensen, Jon Skinner, Jonathan Eaton, Don Andrews, and participants at workshops at the University of Virginia and Yale University for helpful comments. All remaining errors are mine.

[*Journal of Labor Economics,* 1989, vol. 7, no. 3]

0734-306X/89/0703-0003$01.50

derived for the optimal number of job applications. The number of job applications affects the distribution of unemployment spells in a way similar to a search intensity parameter in a continuous time model. Unlike a search intensity parameter in a continuous time model, however, it also skews the distribution of accepted wage offers to the right. This identifies the number of potential offers an individual can receive per period. A discrete time model is used to emphasize the need for individuals to choose among jobs being offered simultaneously.[1] The model is estimated using maximum-likelihood estimation. The results indicate that new labor force entrants do search simultaneously. Furthermore, there are reasonable search technologies that can explain their behavior.

A secondary goal of this article is to identify and estimate job offer arrival rates and wage offer rejection probabilities separately. Most of the search literature has focused on wage offer rejection to explain unemployment spells. Stern (1989), however, shows that search behavior can be generated in a model where an agent always accepts a wage he is offered. Previous empirical papers have found that a significant portion of unemployment spells are caused by slow offer arrival rates.[2] The results of this study are consistent with those previous estimates.

The remainder of the article is divided into five sections. Section II presents the theoretical model. Section III describes the data used to estimate the model. The model is estimated using data on unemployment spells and accepted and rejected wage offers found in the 1981 National Longitudinal Survey (NLS) Youth Cohort. The maximum-likelihood estimation procedure is explained in Section IV. Section V presents estimation results and test statistics. The last section contains concluding remarks.

II. The Model

In this section, a model of job search is described and analyzed. Each worker chooses a reservation wage and the number of applications to send simultaneously. The necessary first-order condition for the optimal number of applications to send provides a test of the model in Section IV.

There are a large number of firms with potential job openings. Each risk-neutral unemployed worker applies to m firms per period. Applications may take the form of written applications, personal visits, or any other attempt to find a job. The cost of applying to m firms is $C(m)$ where $C(0) = 0$, $C'(m) > 0$, and $C''(m) \geq 0$. A job offer consists of a vector of characteristics whose value can be summarized in terms of a single quantity

[1] A more realistic model would have each individual be able to recall offers for a limited period of time. This would allow him to collect offers and choose among a group of offers. Such a model is very difficult to analyze. See Landsberger and Peled (1977) for a discussion of a model closer to this idea.

[2] These include Mortensen and Neumann (1984), Chang (1984), Wolpin (1987), and Eckstein and Wolpin (1987).

that will be called the wage. If a worker receives any acceptable offers, he accepts the best one and then remains in that job forever.[3] If he receives no acceptable offer, then he follows the same search strategy next period. This is an optimal strategy because the search environment is stationary. The probability of receiving a positive wage offer is $(1 - \gamma)$, and the cumulative distribution function of positive wage offers is $F(w)$. Each worker knows the distribution of positive wage offers and the rejection probability. Let the distribution of nonnegative wage offers be $G(w)$:

$$G(w) = \begin{cases} \gamma + (1-\gamma)F(w) & \text{if} \quad w > 0, \\ \gamma & \text{if} \quad w = 0. \end{cases} \tag{2.1}$$

The function $G(w)$ is the distribution of wage offers when rejection by the firm is treated as a zero wage offer. It is assumed that workers know γ and $F(w)$ but have no prior information about which firms are offering high wage offers (as in Salop [1973]). Thus, each worker randomly chooses among firms to which he applies.

A worker who is offered positive wages must decide whether to accept any of them. He either will accept the highest offer or will reject it and continue searching. Let ξ be a worker's reservation wage when he faces a wage offer distribution $G(w)$ and cost function $C(m)$. A worker's problem is to choose m and ξ. The number of applications sent is allowed to vary continuously. A noninteger value of m should be regarded as an approximation of the integer solution adjacent to it.

Let $V(\xi, m)$ be the value of applying to m firms and having a reservation wage of ξ. The value $V(\xi, m)$ is equal to the expected value of an acceptable offer times the probability of receiving one, plus the value of search next period times the probability of not receiving an acceptable offer, minus search costs:

$$V(\xi, m) = \beta\left\{\int_{\xi}^{\infty} wmg(w)G(w)^{m-1}dw + G(\xi)^m V^*\right\} - C(m), \tag{2.2}$$

where β is the 1-period discount factor and V^* is the value of search next period. The density of the highest wage offer is $\partial G(w)^m/\partial w = mg(w)G(w)^{m-1}$. Wage payments begin the period after a job is accepted, and application costs are paid immediately. If the worker is in a steady state, that is, $G(w)$ and $C(m)$ do not change over time and the worker's time horizon is sufficiently far away, then the optimal reservation wage is set equal to V^* and $V(\xi, m) = V^*$. Equation (2.2) becomes

$$\xi = \beta\left\{\int_{\xi}^{\infty} wmg(w)G(w)^{m-1}dw + G(\xi)^m \xi\right\} - C(m), \tag{2.3}$$

[3] The infinite tenure assumption can be altered by reinterpreting $C(m)$. This is discussed in more detail at the end of Section IV.

which can be transformed through integration by parts to

$$h_1(\xi, m) = \xi - \beta\left\{\int_{\xi}^{\infty} (1 - G(w)^m)dw + \xi\right\} + C(m) = 0. \quad (2.4)$$

This is the necessary first-order condition for the optimal reservation wage given m. It is a standard reservation wage equation generalized for multiple applications. The function $h_1(\xi, m)$ is the difference between the reservation wage and the value of search. At the optimal strategy, this difference should be zero.

The necessary first-order condition for m is

$$h_2(\xi, m) = \beta\int_{\xi}^{\infty} G(w)^m \ln G(w)dw + C'(m) = 0. \quad (2.5)$$

It is an implicit equation for the optimal level of applications conditional on ξ. An elegant way to derive it is to differentiate $h_1(\xi, m)$ totally and to set $d\xi/dm$ equal to zero. This provides the desired result because the optimal reservation wage is equal to the value of search. The function $h_2(\xi, m)$ is the difference between the marginal benefit and marginal cost of an extra application. A worker's optimal strategy, (ξ, m), is the simultaneous solution to equations (2.4) and (2.5). A sufficient condition for a unique optimum is that $C''(m) \geq 0$.[4]

Comparative statics for the reservation wage yield $d\xi/d\gamma < 0$, $d\xi/d\beta > 0$, $d\xi/d\mu > 0$, and $d\xi/dC' < 0$, where μ is the mean of $F(w)$ and C' is the marginal cost of an application. The intuition for these results is that ξ is the optimal value of search; it decreases as γ and C' increase and increases as β and μ increase. The comparative static results for m are less clear. Although it can be shown that $dm/d\beta > 0$ and $dm/dC' < 0$, the signs of $dm/d\gamma$ and $dm/d\mu$ are ambiguous.

III. The Data

The data used to estimate the model were collected from the 1981 National Longitudinal Survey (NLS) Youth Cohort. For each individual, a

[4] To prove this, first show that the $\operatorname{argmax}_{\xi} V(\xi, m)$ has a unique solution for fixed m. This is a standard search problem. Then show that equation (2.5), which is the first-order condition for maximization of $V(\xi(m), m)$ where $\xi(m)$ solves eq. (2.4), has a unique solution. This is shown by noting that

$$\frac{dh_2(\xi(m), m)}{dm} = \beta\left\{\int_{\xi}^{\infty} G(w)^m[\ln G(w)]^2 dw - G(\xi)^m \ln G(\xi)\right\} + C''(m) > 0;$$

$h_2(\xi, m)$ can cross zero at most once.

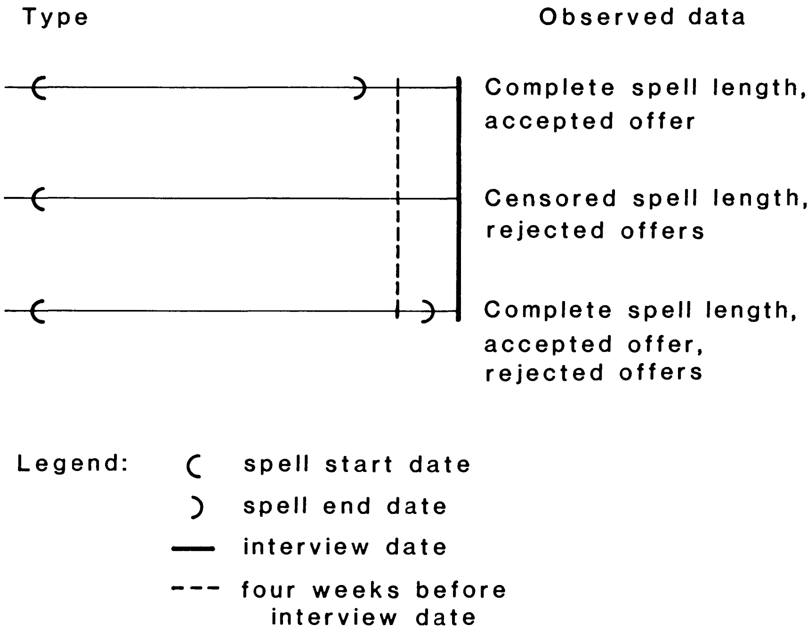

FIG. 1.—Data types

single spell of unemployment was observed. Some spells were censored at the interview date. If the spell was complete, an accepted wage was observed. If either the spell was censored or it had ended in the 4 weeks preceding the interview date, then some rejected wage offers were observed. Further, other exogenous characteristics (e.g., sex, education) were observed for each individual. The three types of data that occurred are depicted in figure 1.

In order to be selected into the sample an individual had to satisfy the following criteria:

a) The individual was not interviewed in the summer.
b) The individual's "main activity" at the time of the interview was not "attending school."
c) The individual was at least 18 years old.
d) The individual was either searching for a job at the time of the interview or, while employed, had found a job less than 2 years before the interview.
e) The individual was not married.
f) No relevant information was missing for the individual. This included information on wage offers, accepted wages, spell lengths, and exogenous variables used to stratify the individuals.

Table 1
Sample Sizes

Group	Accepted Offers	Rejected Offers	Spells
1. Male DROPOUTS	221	31	300
2. Female DROPOUTS	103	21	118
3. Male GRADUATES	220	26	280
4. Female GRADUATES	222	23	268
5. Male COLLEGE	97	7	104
6. Female COLLEGE	160	12	168

g) The individual was receiving no unemployment insurance (UI) benefits. Since UI benefits affect the cost of search, a different reservation wage and m would have to be estimated for each individual in the sample if UI recipients were included.

The inclusion of the first two criteria implies that no one in the sample was attending school during a period when school was in session. The inclusion of the third criterion excludes individuals who were relatively likely to return to school. The selection criteria tend to bias the sample toward individuals with low wage offers and long unemployment spells. Thus, this is not a representative sample. There is, however, no obvious selection bias of the estimated parameters caused by the selection criteria.

Spell lengths were computed by subtracting spell starting dates from spell ending dates. Wage offers were adjusted for state and federal tax rates and regional price variation and were represented as daily wages.[5] Appropriate tests were used to check for coding and other errors.

The data were stratified on the basis of sex and level of education. For both men and women, individuals were classified as either high school dropouts (DROPOUTS), high school graduates (GRADUATES), or high school graduates with some college education (COLLEGE). Thus, there are six separate estimations performed. The sample size for each group of individuals is reported in table 1. The number of observed rejected and accepted wage offers and spells of unemployment are listed. It is clear that the results for those with no college education will have more validity than those for individuals with some college education. Means and variances for accepted and rejected offers and completed spells are displayed in table 2. It is clear from table 2 that there is either unobserved heterogeneity within each group or individuals accept job offers based on other factors besides the wage offer.

[5] There were some wage offers that seemed unusually low when translated into daily wages. This is probably because the related jobs were only part-time. Unfortunately, in the survey there was no way to differentiate between offers for part-time and full-time jobs. All jobs were considered full time. This is equivalent to considering the value of leisure to be zero.

Table 2
Means and Variables of the Data

	Male DROPOUTS	Female DROPOUTS	Male GRADUATES	Female GRADUATES	Male COLLEGE	Female COLLEGE
Mean accepted offer:	29.46	23.61	31.81	24.04	39.29	32.33
SD	11.84	8.86	13.59	7.79	16.11	12.12
Minimum	1.62	.50	6.49	.56	13.48	.51
Mean rejected offer:	23.97	23.61	25.96	22.64	18.52	14.60
SD	8.73	5.11	8.49	8.35	11.64	11.91
Maximum	47.79	35.55	51.78	34.25	29.12	26.33
Mean complete spell:	128.66	133.52	92.14	113.14	90.90	86.62
SD	119.42	115.13	89.19	102.17	86.07	80.33

Table 3
Regression Results for Hazard Rates*

Group†	Mean‡	Constant	T§	T^2	$F^{\|\|}$	D-W
1	3.97	8.05	−.513	.0116	8.22	2.02
		(1.12)	(.127)	(.0030)		
2	3.98	5.80	−.174	.00316	1.05	1.31
		(1.45)	(.242)	(.0056)		
3	5.48	6.93	−.0280	.00158	2.24	1.99
		(1.65)	(.025)	(.0018)		
4	4.96	6.25	−.1267	.00236	.40	1.59
		(1.57)	(.176)	(.0042)		
5	6.38	7.76	−.0997	.00121	.31	1.68
		(2.40)	(.269)	(.0064)		
6	7.97	7.32	−.0235	.00203	.23	1.63
		(3.42)	(.392)	(.0092)		
6#	7.16	8.08	−.128	.00340	.05	1.61
		(2.98)	(.441)	(.014)		

NOTE.—D-W is the Durbin-Watson statistic. Standard errors are in parentheses.
* The first 40 hazard rates are used and are multiplied by 100.
† Group numbers should be taken from table 1.
‡ Mean is the mean hazard rate.
§ T is the duration.
‖ F has an F distribution with 2 degrees of freedom in the numerator and 37 degrees of freedom in the denominator. 5.3 is the 1% significance level.
Based on only the first 30 hazard rates. This was done because of the sparsity of data for the thirtieth to fortieth hazard rates.

The economic model in Section II assumes that there is no duration dependence in the model. The inclusion of duration dependence would make the econometric model too costly to estimate. To determine the validity of the duration independence assumption, the first 40 hazard rates were regressed on a constant, time, and time squared. The results are listed in table 3. It was found that only male high school dropouts exhibited any duration dependence. Thus, except for the effect of unobserved heterogeneity on the measure of duration dependence, duration independence is a valid assumption.[6] However, it should be noted that there is no control.

Hazard rates vary across groups. Male and female high school dropouts have hazard rates of less than 4% resulting in an average spell of unemployment of longer than 24 weeks. Meanwhile, men and women with some college education have hazard rates higher than 6% resulting in an average spell length of less than 16 weeks. The discrepancy between the high average spell lengths reported in table 2 and the moderate spell lengths implied by the hazard rates are due mostly to a few very long spells. This may indicate that there is duration dependence for spells after 40 weeks.

IV. Estimation

The first step in developing the estimation procedure is determining the likelihood function. Let X be a vector of individual-specific characteristics

[6] See Flinn and Heckman (1982) or Lancaster (1979) for a discussion of unobserved heterogeneity and how to adjust for it.

(e.g., race or local unemployment rate). Assume that there are n different values of X that can occur.[7] Call a type i person one that has characteristics X_i, $i = 1, 2, \ldots, n$. Even though continuous variables may be included in X, they still must be stratified into groups so that estimation will be computationally feasible.

Let V be the log of the value of a job offer. As in Section II, let $F(v \mid X)$ be the distribution of V and $\gamma(X)$ be the rejection probability.[8] Define

$$G(v) = \gamma + (1 - \gamma)F(v).$$

The individual's wage may vary from the value of the job. This occurs because there may be two types of measurement error in the wage-offer data. First, there may be errors in reporting wages and hours or in imputing tax rates or price levels. Second, the wage-offer data is being used as a measure of the expected value of job offers. There may be errors because the value of fringe benefits, the layoff probability, the human capital value (experience), working conditions, and many other facets of the job are not being observed. For the remainder of this article, any deviation between the value of a job and its wage will be referred to as measurement error.

A way to handle the measurement error is to model the unobserved part of the value of a job explicitly. Let W be the log wage offer attached to a job of log value V. Define Z by $W = V + Z$, where Z has density $f_z(z)$ and Z and V are independent. Let m be the number of applications sent out per period. A period is the amount of time that passes between the time when applications are sent and when offers either must be accepted or rejected. It is assumed that a period is 1 week. As is discussed later, the only parameter sensitive to this assumption is γ.

Because the probability of receiving no acceptable offer in a particular period is $G(\xi)^m$, the probability of observing a completed spell of unemployment of length s is

$$G(\xi)^{m(s-1)}(1 - G(\xi)^m), \tag{4.1}$$

where ξ is the observed individual's log reservation wage. The density of an accepted log wage offer is

$$f_w(w) = \int_{-\infty}^{w-\xi} [mg(w - z)G(w - z)^{m-1} f_Z(z)/(1 - G(\xi)^m)]\,dz. \tag{4.2}$$

This is the density function for the best of m alternative offers (some of which may be rejections), $v_1, v_2, \ldots, v_m$, conditional on at least one of

[7] For example, if X_1 is race (white or not white), X_2 is sex, and X_3 is local unemployment rate (high, medium, or low), then $n = 2 \times 2 \times 3 = 12$.

[8] It is assumed that, for each individual, X is constant over the relevant period. For the rest of this article, X will be suppressed from the notation.

them being acceptable. Therefore, the contribution to the likelihood function of an individual with a completed spell s and an accepted log wage offer w^a is

$$G(\xi)^{m(s-1)} \int_{-\infty}^{w^a-\xi} mg(w^a - z)G(w^a - z)^{m-1} dF_Z(z). \qquad (4.3)$$

Rejected offers need not be greater than other offers to be observed. Thus, the contribution of an individual with incomplete spell of length d and rejected offers, w_j^r, $j = 1, 2, \ldots, k$, is

$$G(\xi)^{md} \prod_{j=1}^{k} \left[\int_{w_j^r-\xi}^{\infty} f(w^r - z) dF_Z(z)/F(\xi) \right].^9 \qquad (4.4)$$

Each term in the product is the density of V_j conditional on its being unacceptable.

The estimation procedure used here is similar to Kiefer and Neumann (1979) and Wolpin (1987) in that it uses parametric maximum-likelihood estimation to estimate the parameters of a structural search model in discrete time. It is dissimilar to Flinn and Heckman (1982) in that the estimate of the reservation wage is dependent on the functional form of the wage offer and error distributions. The procedure proposed in this article has the advantage of allowing for measurement error in observed job values and for using standard asymptotic theory for testing hypotheses. It has the disadvantage of being sensitive to functional form specification.

It is worthwhile to provide some intuition for how each parameter is identified. Assume that $m = 1$. Given data on accepted and rejected wage offers, the parameters of the distribution function $F(w)$ and the reservation wage ξ could be identified. As m increases, the density function for accepted wage offers skews to the right as in figure 2. This occurs because $G(w)^m$ decreases in m. Thus, the difference in skewness of the density function for observed wage offers to the left and right of the reservation wage allows m to be identified. While it is true that estimates of m will be sensitive to functional form assumptions made about the distribution of wage offers F, m can be identified under a very general specification of F and that specification can be tested. Note that since m is identified by the wage data, the introduction of observed durations of spells allows γ, the rejection

[9] Note that this term in the likelihood function is conditioned on having observed a positive wage (the density function is $f(w)$ rather than $g(w)$). This is because the frequency of rejected offers being reported in the sample does not correspond to the frequency of receiving rejected offers. Individuals may not be reporting all offers received and may have a different concept of offers than that used in this article. Thus, the rejected offers provide no information about γ, the rejection probability.

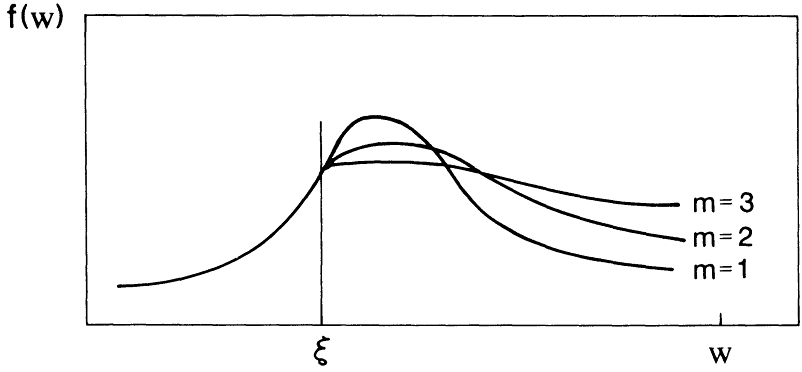

FIG. 2.—The effect of applications on the density of accepted wages

probability, to be identified. Furthermore, since γ is the only parameter identified by the duration data, it is the only parameter sensitive to the assumption that a period is a week.

The parameters remaining to be identified are the discount factor β and the parameters of the cost function $C(m)$. Since neither β nor search costs are observed, these terms can be identified only by the restriction implied by the first-order conditions, equations (2.4) and (2.5).[10] Given ξ, m, and $G(w)$, two extra parameters can be identified. For example, if $C(m) = cm$, then both c and β can be identified. If, however, $C(m)$ is not linear, then all of the estimates will be inconsistent. Therefore, I specify the cost function as $C(m) = c_1 m + c_2 m^2$ and graph combinations of β, c_1, and c_2 that satisfy the first-order conditions.

Three different kinds of tests are performed on the model. The first tests if there is nonsequential search, that is, $m > 1$. This is of importance because of the issues raised in Section I. This hypothesis can be tested using a Wald statistic, $\psi = (m - 1)^2/\mathrm{var}(\hat{m})$. Since m is restricted to be at least unity, the distribution of the test statistic under the null hypothesis is not χ^2.[11] Instead, it should have a mass point at $m = 1$ with probability equal to the probability that the estimate of m is equal to unity. To the right of unity the asymptotic distribution should be approximately χ_1^2. Therefore, the critical value for a test with power α is that value η such that $\mathrm{pr}[\psi > \eta] = 2\alpha$.

The second kind of tests are specification tests for wage-offer distribution functional forms. Given a distribution of log job values $F(v)$, the distribution of log accepted wage offers can be written as

[10] These have to be adjusted in the appropriate way because ξ is now the log reservation wage.

[11] The number of applications m is restricted to be at least unity parameterizing m in terms of a deep parameter θ_9 as $m = 1 + \theta_9^2$.

$$F_w(w) = \int_{w-t}^{w-\xi} \frac{G(w-z)^m - G(\xi)^m}{1 - G(\xi)^m} f_Z(z)dz, \tag{4.5}$$

where t is the upper boundary of the support of V. Let W be a log accepted random wage offer and $Y = F_w(W)$. Then Y has a uniform [0, 1] distribution under the null hypothesis. If the parameters of F_w are known and log accepted wage offers are ordered in ascending order, then

$$D = \max_{W_i} \left| F_w(W_i) - \frac{i}{n} \right| \tag{4.6}$$

has a Kolmogorov-Smirnov distribution. Unfortunately, the parameters of F_w are estimated rather than known. Therefore, the Kolmogorov-Smirnov statistic is biased downward.[12] Nevertheless, the Kolmogorov-Smirnov test statistic can be used informally to test the assumption. Furthermore, graphs of the transformed wage offers y against the uniform distribution help to see where the specified distribution fits poorly.

The last kind of test is a test of the theory presented in Section II. Graphs of combinations of β, c_1, and c_2 (where $C(m) = c_1m + c_2m^2$) that solve the first-order conditions, equations (2.4) and (2.5), are presented. It is not necessary to compute the distribution of these combinations because there are reasonable values of these three parameters conditional on the other estimates that solve the first-order conditions.

V. Results

Before reporting any results, it is necessary to specify the functional forms of the relevant distributions. Job value offers are assumed to have a Singh-Maddala distribution (see Singh and Maddala 1976) truncated from above so that the distribution of log positive job values is

$$F(v) = \left\{1 - \left[1 + \left(\frac{e^v}{b}\right)^a\right]^{-q}\right\} \Big/ \left\{1 - \left[1 + \left(\frac{e^t}{b}\right)^a\right]^{-q}\right\}, \tag{5.1}$$

where t is the upper truncation point and a, b, and q are the parameters of the Singh-Maddala distribution.[13] The Singh-Maddala distribution is a generalization of both the Sech-2 distribution (see Fisk 1961) and the Weibull distribution. It is used because of its flexibility and because of its success in estimating income distributions in McDonald (1984).[14]

[12] See Durbin (1973) for a further discussion and a solution to a particular example of this problem.

[13] This will be called a Singh-Maddala distribution even though it is e^V that has a Singh-Maddala distribution.

[14] Estimation was also attempted with wage offers and measurement errors having a lognormal distribution. The estimators never converged to a point with a negative definite Hessian. Thus they are not reported.

Measurement error is assumed to have a Singh-Maddala distribution normalized so that the mean of the multiplicative error is unity. This is done, following McDonald (1984), by setting b_z in the distribution function

$$F_Z(z) = 1 - \left[1 + \left(\frac{e^z}{b_z}\right)^{a_z}\right]^{-q_z} \tag{5.2}$$

equal to

$$b_z = a_z\Gamma(q_z)/\Gamma\left(\frac{1}{a_z}\right)\Gamma\left(q_z - \frac{1}{a_z}\right). \tag{5.3}$$

Estimates are listed in table 4. The parameters estimated are q, a, b, t, γ, q_z, a_z, m, and ξ. The parameters q, a, b, and t are the parameters of the distribution of positive job values in equation (5.1). Wage offers enter the likelihood function as log daily wages. The parameter γ is the rejection probability. The parameters q_z and a_z are the parameters of the measurement error distribution in equation (5.2). The parameter m is the number of applications sent per period, and ξ is the reservation wage.[15]

Standard errors are listed under estimates for each parameter except for the estimates of q_z for female GRADUATES and COLLEGE. For these two groups, it was found that no matter how large q_z was set, $d \log L/dq_z$ was positive. As $q_z \to \infty$ in equations (5.2) and (5.3), the Singh-Maddala distribution approaches an extreme value distribution. Therefore, for these two groups the mixing distribution was an extreme value distribution.[16]

Estimates of the means and standard deviations for the distribution of the value of a job and the distribution of measurement error were computed numerically using the estimates of q, a, b, q_z, and a_z listed in table 4. The results are listed in table 5, part A. Means and variances both rise with education and are higher for males than for females. None of these results are unexpected.

[15] There are restrictions placed on some of the parameters. The distribution parameter q is restricted to be positive by using the transformation $q = \theta_1^2$. The distribution parameters a and b are restricted to be positive in the same way: $a = \theta_2^2$ and $b = \theta_3^2$. The cost parameter c is restricted to be positive by using the transformation $c = \exp\{\theta_5\}$. The rejection probability γ is restricted to fall between zero and one by using the transformation $\gamma = \exp\{\theta_6\}/(1 + \exp\{\theta_6\})$. The measurement error parameter a_z is restricted to be positive: $a_z = \theta_7^2$. The measurement error parameter q_z is restricted to be greater than the reciprocal of a_z so that the mixing distribution will have nice properties: $q_z = (1 + \theta_8^2)/a_z$. The number of applications m is restricted to be at least unity: $m = 1 + \theta_9^2$.

[16] McDonald (1984) shows that when $q_z \to \infty$, the Singh-Maddala distribution approaches a Weibull distribution. Since my random variables are logs of his, and the log of Weibull random variable has an extreme value distribution, our results are consistent.

Table 4
Unrestricted Maximum Likelihood Estimates

	Male DROPOUTS	Female DROPOUTS	Male GRADUATES	Female GRADUATES	Male COLLEGE	Female COLLEGE
q^*	4.06	3.61	2.26	3.54	3.16	5.07
	(.154)	(.171)	(.089)	(.114)	(.180)	(.139)
a^*	33.8	35.9	34.8	34.0	23.6	34.40
	(.473)	(.541)	(.693)	(.395)	(.497)	(.342)
b^*	27.8	23.0	28.6	22.8	35.6	30.20
	(.320)	(.460)	(.398)	(.294)	(.761)	(.423)
t^*	5.15	4.98	5.83	5.28	6.42	5.63
	(.040)	(.045)	(.067)	(.037)	(.070)	(.030)
γ†	.909	.903	.789	.958	.835	.926
	(.023)	(.040)	(.126)	(.040)	(.037)	(.021)
q_z‡	351.0	416.2	1.10	∞	202.9	∞
	(11.94)	(21.8)	(.148)		(11.82)	
a_z‡	2.48	2.86	4.69	3.98	2.24	2.29
	(.084)	(.150)	(.256)	(.112)	(.130)	(.113)
m§	2.71	1.94	16.6	19.6	2.31	1.39
	(.136)	(.100)	(.748)	(.508)	(.160)	(.035)
ξ‖	28.09	23.19	30.83	23.47	36.60	29.58
	(.352)	(.494)	(.515)	(.324)	(.880)	(.416)
Log likelihood	−1,375.2	−630.7	−1,330.8	−1,358.5	−599.5	−980.4
No. of observations	221	103	220	222	97	160

NOTE.—Wage offers are represented in terms of daily wages. Standard errors are in parentheses.
* Singh-Maddala distribution terms.
† Rejection probability.
‡ Measurement error distribution terms.
§ Number of applications.
‖ Reservation wage.

Table 5
Auxiliary Statistics

	Male DROPOUTS	Female DROPOUTS	Male GRADUATES	Female GRADUATES	Male COLLEGE	Female COLLEGE
A. First two estimated moments for the Singh-Maddala distribution:*						
Mean	26.4	21.9	27.7	21.7	33.3	28.4
Variance	1.13	.709	1.36	.778	3.85	1.23
B. Variances for the wage-offer distribution and the measurement-error distribution ($ per day):				.78		
Value	1.13	.71	1.36	.12	3.85	1.23
Error	.18	.14	.16		.16	.21
C. Wald tests†	161.2	77.8	434.0	1,345.0	67.1	124.4

* The unit of measure is daily wages.
† The .5% critical value is 6.63, and the 2.5% critical value is 3.84.

The rejection probabilities are relatively high. Male GRADUATES and COLLEGE have the lowest rejection probabilities and female GRADUATES have the highest. All rejection probabilities are greater than .75.

It might be argued that the high estimates of the rejection probabilities depend on a period being 1 week long. If the market period is actually longer than assumed, then the rejection probability will be lower. However, there is little difference between frequent rejections and infrequent markets;[17] in both cases, the worker will be more concerned with finding a vacancy than with finding a high wage offer.

The long unemployment spells may be interpreted as a combination of search spells and nonparticipation spells. If an individual stopped searching temporarily in the middle of a spell, his rejection probability would be upwardly biased. However, there must have been a reason for him to stop searching temporarily. To the extent that he perceived that his chances of finding an acceptable temporary or permanent job were very small, there is still little difference in the implications of the different assumptions.

The truncation points correspond to maximum value offers in the range of \$145–\$614 per day. This is much higher than anyone in the sample earned. The inclusion of measurement error explains the discrepancy. The truncation points play a very important role in estimation because it is their inclusion that makes the Singh-Maddala distribution fit the data well.

The mixing distribution parameters are such that a significant but small part of the variation in the wage-offer data is due to measurement error. Table 5, part B, lists the variance of the measurement error and the variance of the value distribution.

The estimated number of applications sent per week varies from 1.39 for female COLLEGE to 19.6 for female GRADUATES. Only GRADUATES apply to more than three firms simultaneously. Nevertheless, all estimates of m are significantly greater than unity. The results of the Wald tests for the null hypothesis,

$$H_0: \quad m = 1,$$

$$H_A: \quad m > 1,$$

are reported in table 5, part C. The null hypothesis was rejected for every group. Thus, individuals search nonsequentially. The policy implications of this are discussed in the conclusions.

The next set of statistics test the Singh-Maddala distributional assumptions. Kolmogorov-Smirnov statistics are presented in table 6, and graphs of transformed wage offers are presented in figure 3. The truncated Singh-Maddala distribution fits the data reasonably well. While the null hypothesis

[17] For example, a job offer rate of .1 per week results in the same γ as .2 every 2 weeks.

Table 6
Kolmogorov-Smirnov Statistics for Transformed Wage Distributions

	K-S Statistic	N	1% Significance Level
Male DROPOUTS	.174	216	.111
Female DROPOUTS	.164	102	.161
Male GRADUATES	.055	220	.110
Female GRADUATES	.102	220	.110
Male COLLEGE	.136	96	.166
Female COLLEGE	.173	158	.130

is rejected at the 1% significance level for three out of six groups, it fits the data very well for GRADUATES and male COLLEGE. This occurs because of the great flexibility that the Singh-Maddala distribution allows and because of the inclusion of the truncation point.[18]

The last set of tests measures the ability of the theory of Section II to explain the data. Figure 4 shows combinations of c_1 and c_2 that solve the first-order conditions given the annual discount factor. Also, average and marginal cost are graphed at levels of estimated applications. For example, for male DROPOUTS, when the annual discount factor is .80, $c_1 = 10.2$ and $c_2 = 3.59$, leading to a marginal application cost of \$29.67 and an average cost of \$19.94. Since these are reasonable parameter values, one could say that there are values of c_1, c_2, and β that are consistent with the observed search behavior of male DROPOUTS.

In fact, for most of the groups, there are values of β, c_1, and c_2 that seem reasonable. The problem group is male COLLEGE for whom average cost is negative for reasonable values of β. Some of the other groups have ranges of β where c_1 is negative. But c_2 is large enough so that marginal and average cost are positive. For these groups, the quadratic cost assumption may be a good assumption only locally.

Some of the cost numbers seem high. The most likely reason for this is that jobs accepted by young workers have a short expected tenure.[19] The cost terms must be interpreted as costs relative to the present value of wages from a job that lasts forever. Thus, if a job has a short expected tenure, the present value of its wages decreases, which increases the relative cost of finding the job.

One of the goals of the article is to help predict the effect of changes in government policy on search behavior. The results of three simulations for male GRADUATES are reported in table 7. The three experiments are to decrease the rejection probability γ by .1, to increase the cost of search (for a linear search technology) by 10%, and to offer unemployment com-

[18] In a smaller sample of female GRADUATES, the Singh-Maddala distribution without truncation fits the data poorly. Thus the truncation point seems to improve estimation considerably.

[19] See Topel (1984).

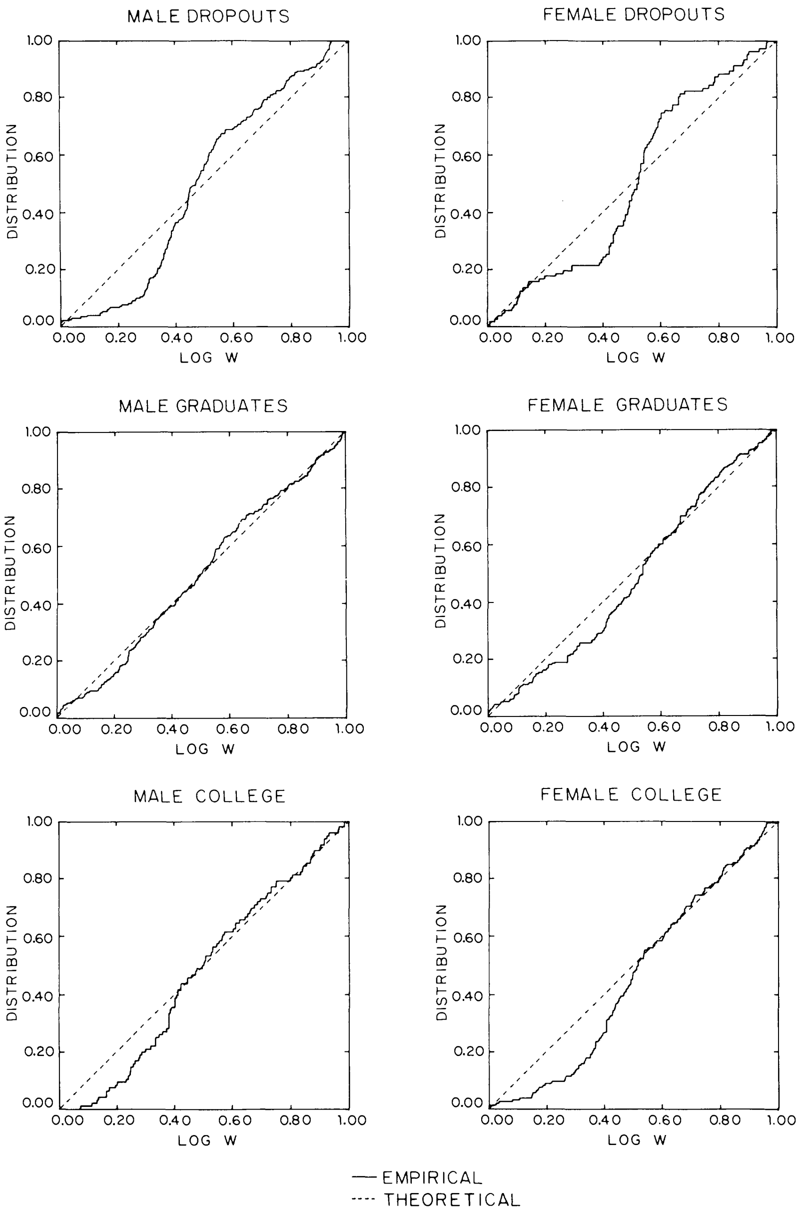

FIG. 3.—Transformed wage distributions

pensation equal to the cost of one application. For each experiment and the baseline, the reservation wage ξ, the number of applications m, the expected accepted wage $E(w|w > \xi)$, the probability of an offer being

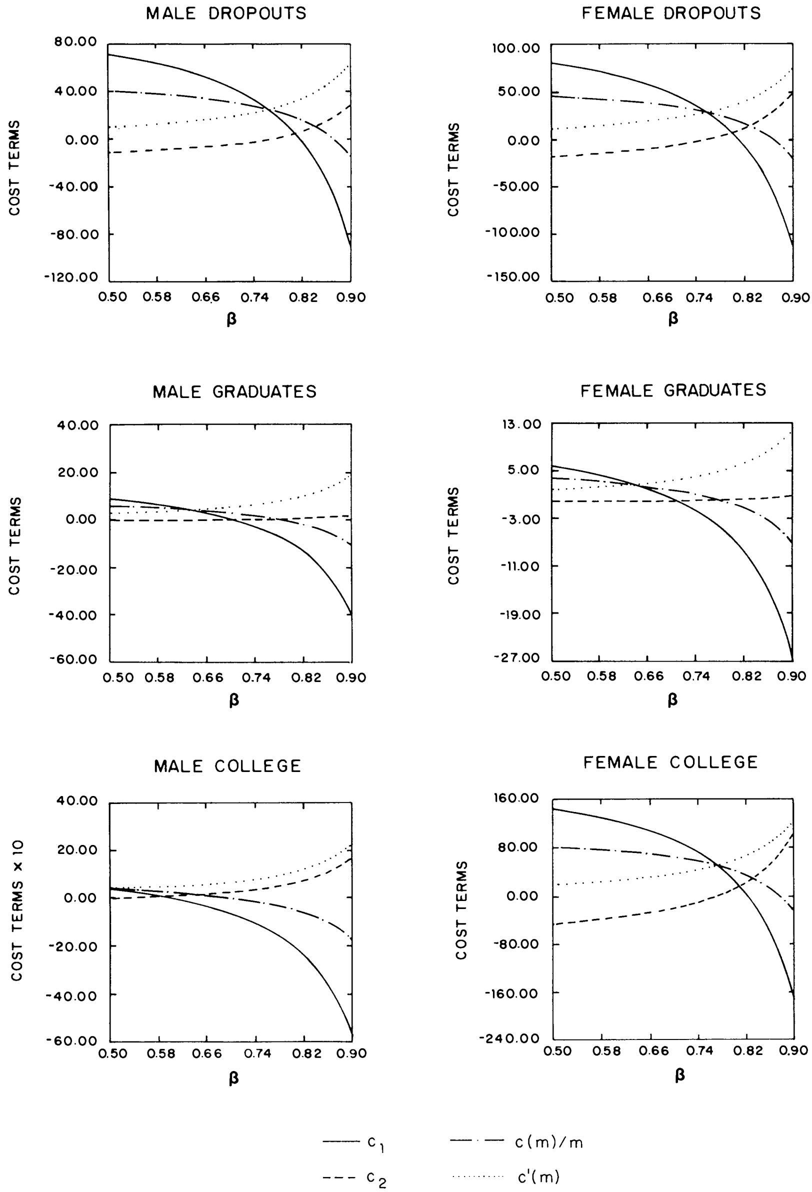

FIG. 4.—Combinations of cost and the discount factor that solve first-order conditions

unacceptable $F(\xi)$, the probability of not receiving an acceptable offer from an application $G(\xi)$, the probability of receiving no acceptable offer in a period $G(\xi)^m$, and the expected search duration $(1 - G(\xi)^m)^{-1}$ is

Table 7
Comparative Statics for Male GRADUATES

	Baseline	$\gamma - .1$	$c \times 1.1$	UI
ξ	30.83	31.03	30.78	30.86
m	16.6	16.6	16.4	16.1
$E(w \mid w > \xi)$	30.97	31.17	30.93	31.0
$F(\xi)$	.9605	.9752	.956	.964
$G(\xi)$	.99166	.99230	.99074	.99248
$G(\xi)^m$	.8702	.8794	.8587	.8853
$(1 - G(\xi)^m)^{-1}$	7.7	8.3	7.1	8.7

reported. From the baseline results, one can see that male GRADUATES accept 4% of the offers they receive, and only 20% of applications result in an offer. When the rejection probability decreases, the reservation wage increases and the number of applications does not significantly change. This leads to acceptance probabilities that are so much lower that the expected search duration increases by 8%. Thus, a government program that decreased rejection probabilities would improve male GRADUATES' well-being (as measured by the reservation wage) but increase the amount of unemployment among male GRADUATES through longer average search spells.

When costs increase by 10% the reservation wage falls slightly and applications decrease so that the expected search duration decreases by 8%. This is not equivalent to a reduction in unemployment compensation benefits because c is a cost per application. Unemployment compensation could be modeled by specifying $C(m) = c_0 + c_1 m$, where $-c_0$ is the unemployment compensation. When $-c_0 = c_1$, the reservation wage rises and applications fall so that the expected search duration increases by 13%.

VI. Conclusions

A simple model of job search was presented and estimated. The parameters of the model estimated were the rejection probability, the cost of an application, the parameters of the distribution of positive job values, the parameters of the distribution of measurement error, the reservation wage, and the number of applications sent per period. The estimates were sensitive to the functional form of the two distributional assumptions and the cost technology assumption. Fortunately, specification tests were available to evaluate the validity of the different assumptions. While the Singh-Maddala was rejected for DROPOUTS and female COLLEGE, it was accepted for GRADUATES and male COLLEGE. The quadratic search technology performed very well for all groups except male COLLEGE.

The high value of the rejection probability should lead us to think that much of search activity is spent looking for a vacancy as well as for a high wage. Mortensen and Neumann (1984), Chang (1984), and Wolpin (1987) have found similar results. This indicates that more research should be done on the generation of vacancies and the job-worker matching process. Some normative work has been done concerning optimal algorithms for matching jobs and workers.[20] Very little positive work has been done. The high rejection probabilities also must lead us to question why workers are not offered their marginal product.

The estimated number of applications sent per period is consistently greater than unity. Thus, the present interest in nonsequential search has empirical relevance at least for new entrants to the labor force. The high number of simultaneous applications submitted and the high rejection probability together imply that finding offers is an empirically important component of search. Yet policies that reduce rejection probabilities or costs of search would increase the average length of unemployment spells, at least for male high school graduates. This does not imply that such policies would be counterproductive, though, because the value of search and of resulting matches would be greater.

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[20] See, e.g., Gale and Shapley (1962), Crawford and Knoer (1981), and Roth (1984).

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