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DYNAMICS AND EQUILIBRIUM IN A STRUCTURAL MODEL OF WIDE-BODY COMMERCIAL AIRCRAFT MARKETS

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SUMMARY

This paper develops and estimates a dynamic equilibrium model of the market for new and used commercial aircraft. The model is estimated by maximum simulated likelihood using data on wide-body aircraft owners and prices for transactions occurring 1978-1997. The importance of explicitly modeling dynamics and equilibrium in new and used markets for durable goods is illustrated in two counterfactual experiments. Estimates of the structural model are used to show that implementing an investment tax credit not only increases demand for new wide-body aircraft by the airlines that receive the tax credit, but also increases the number of new wide-body aircraft owned by airlines not directly affected by the policy. Further, the model indicates that a policy which improves the efficiency of secondary markets for used wide-body aircraft will also stimulate demand for new wide-body aircraft. Copyright © 2010 John Wiley & Sons, Ltd.

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Supporting information may be found in the online version of this article.

1. INTRODUCTION

Wide-body commercial aircraft were first introduced by commercial airlines in the early 1970s. Over the next few decades, as new wide-body aircraft continued to be manufactured, secondary markets for used aircraft developed. The potential importance of interactions between markets for new and used durable goods is well documented. I extend this literature by developing a dynamic structural model that integrates equilibria in markets for new and used durable goods and explicitly models consumers' multi-unit ownership decisions.

The expected future deterioration of an aircraft impacts its value today. Additionally, because there are secondary markets for used aircraft, the expected future distribution of all consumers' preferences for new and used aircraft impact the current value of an aircraft. The model developed in this paper explicitly models the dynamics and equilibria that characterize aircraft markets. In particular, in the model, forward-looking airlines simultaneously choose aircraft fleets that maximize the expected discounted present value of their profits, and airlines' simultaneous choices determine an equilibrium in new and used aircraft locations and prices.

Due to the revenue and trade surpluses generated by the sale of new wide-body aircraft, wide-body aircraft markets are considered strategically important to the USA. Therefore lawmakers often design and alter policies to stimulate wide-body aircraft markets. The structural model developed in this paper is used to show that policies that are directed at one segment of the wide-body aircraft market, e.g. the US market for new wide-body aircraft, will likely impact other new and used wide-body aircraft markets. Using the estimated model, I find that a 10% tax credit to US

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airlines on purchases of new wide-body aircraft induces a 20% increase in new wide-body aircraft purchases by large US airlines, while small US airlines increase their purchases of new wide-bodies by 9%. In addition, the tax credit alters equilibrium valuations of new and used aircraft in current and future periods. Therefore, even airlines outside of the USA (that are not directly affected by the tax credit) increase purchases of new aircraft by approximately 4%. The new market tax credit also has a modest effect on used markets. In particular, US airlines purchase fewer older aircraft than they would have otherwise, while purchases of used aircraft by airlines outside of the USA increase slightly.

In a second counterfactual experiment, I show that improving the efficiency of used markets may increase sales in new markets. Specifically, due to the fact that airlines are forward-looking and consider the future resale value of aircraft they purchase, cutting transaction costs on purchases of *used* wide-body aircraft in half increases *new* aircraft sales by approximately 7%.

The remainder of the paper is organized as follows. The next section provides a review of related papers from the literature on durable goods and dynamic structural models. Section 3 provides a brief history of the market for wide-body commercial aircraft. Section 4 describes the data. Section 5 details the structural model including the equilibrium concept. Section 6 discusses behavioral implications of certain functional form assumptions, which facilitate model solution and estimation. Model solution is discussed in Section 7. Section 8 describes the error distributions and identification of the structural model. Estimation is discussed in Section 9, and estimation results are discussed in Section 10. The estimated model is used to evaluate (1) the introduction of an investment tax credit and (2) a reduction in secondary market frictions, in Section 11, and finally, Section 12 concludes.

2. LITERATURE

This paper contributes to the literature on dynamics and demand in markets for new and used durable goods. There is a rich theoretical literature on consumers' intertemporal ownership decisions in primary and secondary markets for durable goods, and empirical economists have explored many avenues to minimize computational burden while still considering some of the important intertemporal and intratemporal features of durable goods markets. However, to my knowledge this is the first attempt to estimate a fully integrated dynamic equilibrium model of markets for new and used durable goods.

Several recent empirical models of demand for durable goods employ static discrete-choice frameworks (See, for example, Berry *et al.*, 1995; Goldberg, 1995; Petrin, 2002). Advances in this literature allow an econometrician to estimate very flexibly parameterized static models of demand. However, static models typically ignore features that are important to durable goods markets, like intertemporal substitution by consumers and secondary markets for used goods.

A few empirical studies attempted to capture intertemporal substitution by consumers indirectly by modeling the impact of used-good markets on new-good markets in static discrete-choice frameworks. For example, Manski (1983) and Berkovec (1985) estimate demand for new automobiles while controlling for the effect of changes in stocks of used automobiles. Manski and Berkovec's models capture some of the intertemporal substitution that characterize markets for durable goods, and are relatively easy to estimate. However, neither paper models the intertemporal links between individual consumer purchase decisions or the relationship between new and used good market equilibria that are the focus of this work.

Hendel and Lizzeri (1999) and Porter and Sattler (1999) develop theoretical models that characterize the intertemporal relationship between demand in new and used durable good markets.

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Both papers generate predictions about consumers' behavior using dynamic models, and then provide empirical evidence that support the predictions of their models. Each paper develops an alternative theory to the well-known 'lemons' phenomenon in used durable goods markets (Akerlof, 1970). Specifically, Hendel and Lizzeri show that differences in prices and quantities sold of different brands of automobiles in used markets is driven by heterogeneity in the durability of automobiles, while Porter and Sattler contend that heterogeneity in consumer tastes for 'newness' drives the observed patterns of trade in used automobile markets.

Rust (1985, 1986) developed a single-good, single-agent theoretical framework that emphasized intertemporal trade-offs in consumers' decisions to purchase and replace durable goods. In an empirical application of this framework, Rust (1987) modeled an agent's decision of whether or not to replace a bus engine with a new one in each period as a discrete-time, discrete-choice, optimal stopping problem. Variations of Rust's framework have since been used in several empirical papers, including Kennet's (1994) study of the replacement of aircraft engines, and Cho's 2004 examination of the repair and replacement of mainframe computers.

The modeling approach of this paper is most similar to recently developed dynamic structural models of demand for durable goods (e.g., Gowrisankaran and Rysman, 2009; Carranza, 2006; Engers *et al.*, 2009), as well as the calibrated dynamic equilibrium model of Chen *et al.* (2008). Both Carranza (2006) and Gowrisankaran and Rysman (2009) model the trade-off between buying a durable good today and delaying a purchase for one or more periods to allow the good to undergo technological improvements. Both of these papers show that modeling the dynamics of consumer behavior in durable-goods markets is important, even in the absence of prominent secondary markets for used goods. Engers *et al.* (2009) develop a dynamic structural model of the market for used cars. Similarly to Rust (1987) and others, Engers *et al.* use a single-agent, single-asset framework that permits car owners to trade in the used car they currently own for a different new or used car in every period. The authors estimate the extent to which the physical deterioration of cars and heterogeneity in consumer preferences for newness cause the market value of cars to fall with age. Like Hendel and Lizzeri (1999) and Porter and Sattler (1999), Engers *et al.* find little evidence of a lemons effect in the market for used cars. Finally, Chen *et al.* (2008) develop a dynamic equilibrium model of the market for cars. The authors model the dynamic programming problems of both consumers and manufacturers. Due to computational complexity, rather than estimating the model, the model's parameters are calibrated to aggregate data on new and used car transactions. The authors find that used cars sold in secondary markets provide car owners with reasonable substitutes for new cars, and therefore the existence of a efficient secondary market for used cars reduces new car sales and manufacturer profits. Like this paper, recent approaches explicitly model intertemporal substitution by consumers in a discrete-time, discrete-choice framework. Also similarly to this work, Engers *et al.* and Chen *et al.* model secondary markets for used goods.[1] However, unlike individual consumers of automobiles, cameras, DVD players, etc., airlines often purchase and/or sell several goods in each period. Therefore, modeling demand for new and used commercial aircraft involves the additional complication of multi-unit ownership.[2]

[1] Esteban and Shum (2007) and Benkard (2004) develop dynamic models that explicitly consider the impact of used product markets on the strategies employed by manufacturers of durable goods. In contrast to this research, all of the dynamics in these works are on the supply side of the model.

[2] Hendel (1999) considers the importance of multi-unit demand in a static model of businesses' demand for personal computers. The author shows that considering the purchase decisions of multi-unit owners to be independent of one another can systematically bias parameter estimates.

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3. THE MARKET FOR WIDE-BODY COMMERCIAL AIRCRAFT

In 1970 the 500-passenger 747 aircraft was introduced to air travelers by Boeing and Pan American Airlines. Although other major carriers were initially skeptical of the 'bigger-is-better' notion, eventually most of the major airlines adopted wide-body aircraft to service international and transcontinental routes. The success of the Boeing 747 produced enough momentum to inspire the development of two additional wide-body aircraft: the Douglas DC-10 and Lockheed L-1011. The Douglas DC-10 was first flown by American Airlines in 1971, and the Lockheed L-1011 was introduced by Eastern Airlines in 1972. Both aircraft were capable of carrying approximately 300 passengers and had a range at full capacity of 2500–3000 miles. From 1978 through 1997, four manufacturers (Boeing, Airbus, Lockheed-Martin, and McDonnell-Douglas) introduced nearly 3700 wide-body aircraft, and by 1997 more than 500 passenger, charter, and freight airlines had at one time or another owned one or more wide-body aircraft. In 1997, sales of new wide-body commercial aircraft accounted for approximately 30% of commercial aircraft (206) and 60% of aircraft manufacturer revenue ($36 billion). During the 1990s commercial aircraft were one of the USA's largest net exports, accounting for an average of $25–30 billion in trade surpluses.

As wide-body aircraft age they are often sold one or more times in secondary markets. In 1978, used wide-body aircraft accounted for approximately 50% of sales; by 1997 that number had grown to 85%. Most of the growth in used aircraft sales is due to the fact that, while new aircraft continued to be manufactured, very few used aircraft were taken out of service.[3] Indeed, the fact that approximately 13% of aircraft 25 years old or older were sold to new owners in 1997 (52 aircraft), while only 6% of used aircraft under the age of 24 were sold in the same year (210 aircraft), illustrates that the oldest wide-body aircraft in existence in 1997 were still valuable enough to be sold in used markets.

Finally, in a given year, an average of 94–95% of used wide-body aircraft are *not* sold in secondary markets. The relatively large number of used aircraft held by their current owner in each period indicates that either the degree of substitutability across different vintages of aircraft is high and/or there are significant frictions in used aircraft markets (e.g., costs associated with transactions, learning, or fleet adjustment).

The next section describes the data used to estimate the structural model, as well as some facts about primary and secondary markets for wide-body aircraft that motivate the model's specification.

4. DATA

Aircraft transaction data were provided by Back Aviation Solutions (BAS), and aircraft prices and appraisals were provided by Avmark Incorporated (AI).[4]

4.1. Transaction Data and Aircraft Mobility

The transaction data include detailed information about every aircraft that was registered to fly in the past 60 years at several times in their life cycle. Most importantly for this study, these data

[3] Wide-body aircraft are extremely durable goods. Back Aviation Solutions estimates the useful life of a wide-body aircraft to be 25–35 years. By 1997, fewer than 50 wide-body aircraft (less than 2%) had been taken out of service, and of the nearly 50 wide-body aircraft that were no longer active, 30 were destroyed in accidents.

[4] I thank Todd Pulvino at the Kellogg Graduate School of Management for providing me with the aircraft price and appraisal data.

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contain information on each change in aircraft operator or owner, including the identities of the buyers and sellers. I limit the sample to the just over 3500 wide-body commercial aircraft used from 1978 to 1997. Wide-body aircraft are the focus of this paper because (1) limiting the number of transactions was necessary to make the model tractable, (2) wide-body aircraft are arguably less substitutable with other classes of aircraft (i.e., because of the size and range of wide-body aircraft, the services provided by wide-body aircraft cannot be easily replicated by, for example, multiple narrow-body aircraft), and (3) markets for wide-body commercial aircraft have been the focus of other work in the literature (see, for example, Benkard, 2004).

In the model developed below, airlines select a fleet of aircraft once a year. The BAS data identifies the day aircraft transactions take place. However, it has been well documented in the literature on discrete-time dynamic decision problems that the computational burden of solving the dynamic model increases exponentially as the number of time periods increase. Therefore, airlines' periodic purchase/sale decisions are aggregated such that there is at most one change in an aircraft's ownership each year. One-year decision periods are short enough to capture the majority of ownership changes, yet tractable from a model solution and estimation standpoint. Admittedly, to the extent to which aircraft sales occur more frequently than once a year, defining year-long decision periods may discard important information, and as a result the estimated model may underpredict changes in aircraft ownership. However, it is not clear that some extremely short or seasonal ownership spells observed in the data represent changes in aircraft ownership. For example, some sell and buy/lease-back transactions may serve as exchanges of tax benefits, a form of refinancing, and/or temporary outsourcing of resources by aircraft owners.

For the period 1978–1997 an average of approximately one-third of aircraft were leased.[5] In this paper, I treat any change in aircraft operatorship as a change in aircraft ownership. According to BAS, many leases of aircraft are an exchange of tax benefits from airlines that have more capital depreciation write-offs than they need, to banks or other financiers. When a transaction occurs where a bank purchases an aircraft and leases it to an airline, I consider the airline to have purchased the aircraft at the observed price. When one airline leases an aircraft to another airline and there is no observed price, I treat the leasing airline as the owner of the aircraft and impute the missing price (the imputation method is described in the next subsection).

The data provided by BAS also include several physical characteristics of each aircraft. In addition to the general attributes including an aircraft's make and model, the data also include more detailed attributes, such as aircraft length and seating capacity. Prior to making a purchase, airlines carefully consider the technical specifications of an aircraft, and airlines monitor each aircraft's performance as it ages. Therefore, it is reasonable to assume that the observed attributes of aircraft influence aircraft values. However, because increasing differentiation across aircraft increases computational burden, I am unable to fully exploit the observed differences across aircraft. Alternatively, I aggregate the 3500-plus aircraft into 20 types, which are defined by an aircraft's model and vintage.

[5] There is a general perception that aircraft are leased more often than are other durable goods. If leases do, in fact, serve as an exchange of tax benefits, it would make sense that airlines, which have been notoriously unprofitable over the past 20 years, would participate in an unusually large number of leases. Still, the proportion of aircraft leased during the sample period studied here does not differ significantly from the proportion of automobiles leased in recent years (20–30% since 2000; http://www.cars.com/go/advice/Story.jsp?section=lease&subject=buy_lease&story=buyLease). While leases have been largely ignored by empirical studies of automobile markets (see, for example, Berry *et al.*, 1995; Goldberg, 1995), it seems unreasonable to ignore them here where it would not be uncommon for an airline to operate 30 aircraft of similar models and vintages, while owning 20 of them and leasing the other 10.

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Table I gives counts of each model of aircraft observed in the sample. The first wide-body aircraft to enter the market, the Boeing 747, has the greatest number of aircraft in the sample with 1078, while the most recently introduced wide-body aircraft, the Boeing 777, has the fewest with only 104.

The structural model assumes airlines' valuations of aircraft are influenced by differences in aircraft characteristics. Table II shows that there are, in fact, significant differences in some important aircraft characteristics across the aircraft in the sample. The mean length of wide-body commercial aircraft was approximately 59 m. The Boeing 747 was the longest model of aircraft, at over 70 m, while some versions of the Airbus A310 were just under 50 m long. The average aircraft in the sample had a seating capacity of about 410 passengers. The Boeing 747 carried up to 600 passengers, while the Boeing 767 had a maximum seating capacity of only 280 passengers. The mean fuel capacity of wide-body aircraft was 128,000 gallons, and the average range of flight when at full capacity was approximately 5900 miles.

Figure 1 describes differences in aircraft life cycles. Each bar in Figure 1 measures the proportion of each make of aircraft in a particular age category that were sold in secondary markets. The data indicate that aircraft tend to be sold more frequently as they age. For example, only 1.5% of Boeing aircraft in their fifth year after manufacture are sold, while 6.4% of Boeing aircraft in their 15th year of existence are sold. Airbus aircraft were sold the *most* frequently for 13 of the 19 age categories, while Boeing aircraft were sold the *least* frequently for 10 of the first 11 age categories. Aircraft may be sold frequently because they physically degrade quickly and therefore more rapidly move out of the range of desired usefulness to their current owner, and/or frequently traded aircraft may be easy to adapt to several tasks (i.e., have low transaction costs). To attempt to measure the relative importance of these factors, the structural model allows the value of different types of aircraft to be influenced differently by aging and transaction costs.

The parameterization of the structural model also allows different types of airlines to have varying preferences for different types of aircraft. Figure 2 shows the proportion of aircraft sold

Table I. Aircraft

Model	N
Boeing 747	1078
Boeing 767	690
Boeing 777	104
Airbus 300/310	682
Airbus 330/340	191
Douglas DC-10	371
Douglas MD-11	174
Lockheed L-1011	236
Total	3526

Table II. Observed aircraft attributes

Attribute	Mean	Standard deviation	Minimum	Maximum
Length (m)	59.44	8.33	46.7	70.7
Seating capacity	410.93	98.98	253	550
Fuel capacity (1000 gallons)	128.05	57.65	44.01	219.62
Payload range (1000 miles)	5.87	2.01	1.04	9.21

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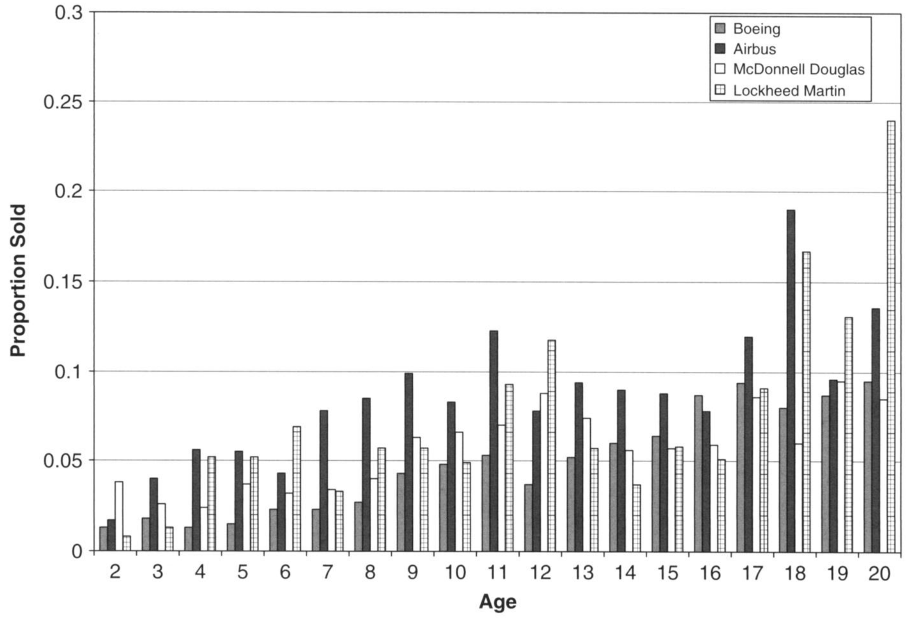

Figure 1. Proportion of aircraft sold in secondary markets

to secondary markets that are sold to passenger airlines. Older aircraft are generally less likely to be purchased by passenger airlines and more likely to be purchased by a freight or charter airline, which indicates that passenger airlines may have a relatively higher preference for aircraft quality. Whether some aircraft deteriorate more quickly than others, or perhaps some aircraft are more easily converted to serve the purposes of freight or non-passenger airlines than others, it is evident in Figure 2 that some types of aircraft are sold to freight or non-passenger airlines earlier in their life cycle than others. For example, of Boeing aircraft sold in secondary markets, approximately 84% of the newest age range and 70% of the oldest age range were sold to passenger airlines, while approximately 87% of the newest age range and 41% of the oldest age range of Airbus aircraft sold were sold to passenger airlines.

4.2. Price Data

The price data include transaction prices for 1165 deliveries and sales for the period 1978–1993. Prior to 1993, airlines in the USA were required by law to report the price of any new or used aircraft they purchased or sold. The prices were reported to the Department of Transportation and the Federal Aviation Association, and later compiled by AI. I normalize aircraft prices to 1982–1984 chained US dollars.[6]

In estimation, I assume all observed aircraft prices are measured with error and impute 'real' equilibrium prices using aircraft model-vintage time-specific cell means. 42 of 300 type-vintage–time combinations had no observed prices in the data. The missing cell means were

[6] A description of the construction of 1982–1984 chained US dollars is available at www.bls.gov.

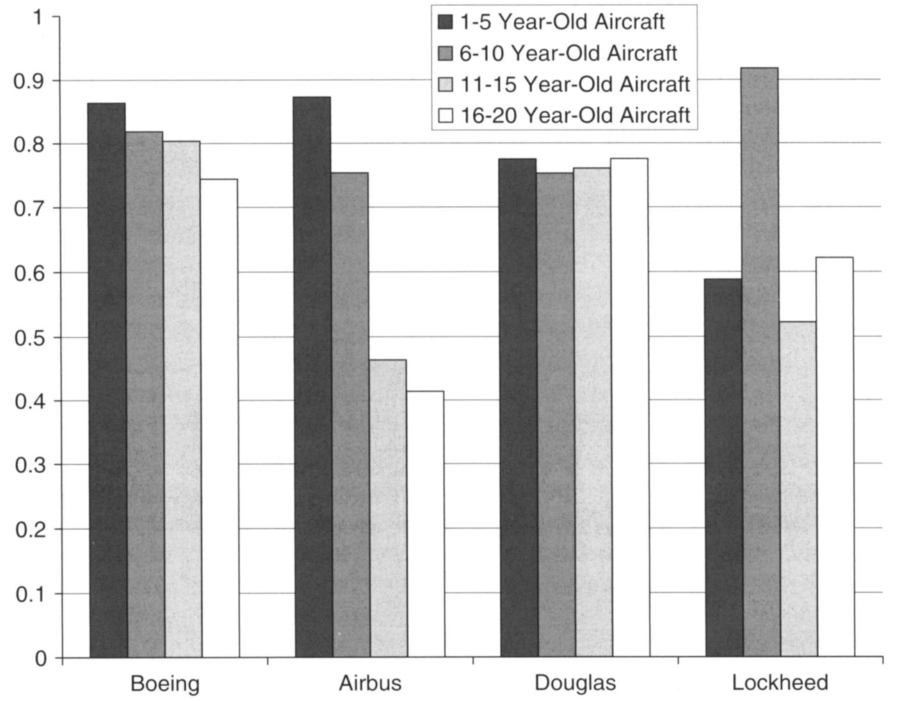

Figure 2. Proportion of aircraft Sold to passenger airlines

imputed using a regression of log price model, year, and vintage fixed effects. I aggregate prices for a few reasons. First, as was mentioned above, to facilitate the solution and estimation of the structural model wide-body aircraft are grouped into 20 model/vintage combinations, and the structural model's equilibrium require that aircraft of a given type are sold at a uniform price. Therefore, a uniform set of prices can better rationalize outcomes of the structural model. Second, I observe over 3000 changes in aircraft ownership, but only 1165 prices. So even if it was feasible to solve and estimate the model without grouping aircraft into types, I would have to impute prices for approximately two-thirds of the observed transactions.

Figure 3 illustrates the depreciation pattern of a representative wide-body commercial aircraft's value in used markets over the first 20 years of its life cycle (after netting out model and vintage fixed effects). Similarly to other durable goods, wide-body aircraft depreciation exhibits a convex decline. Price as a quadratic function of aircraft age fitted through points on the graph illustrates this convexity (the dotted line included in Figure 3).

5. STRUCTURAL MODEL

The structural model outlined here is designed to capture the most important elements of airlines' fleet management decisions. In each period the equilibrium prices and locations of all new and used aircraft are determined by airlines' preferences for available aircraft. The

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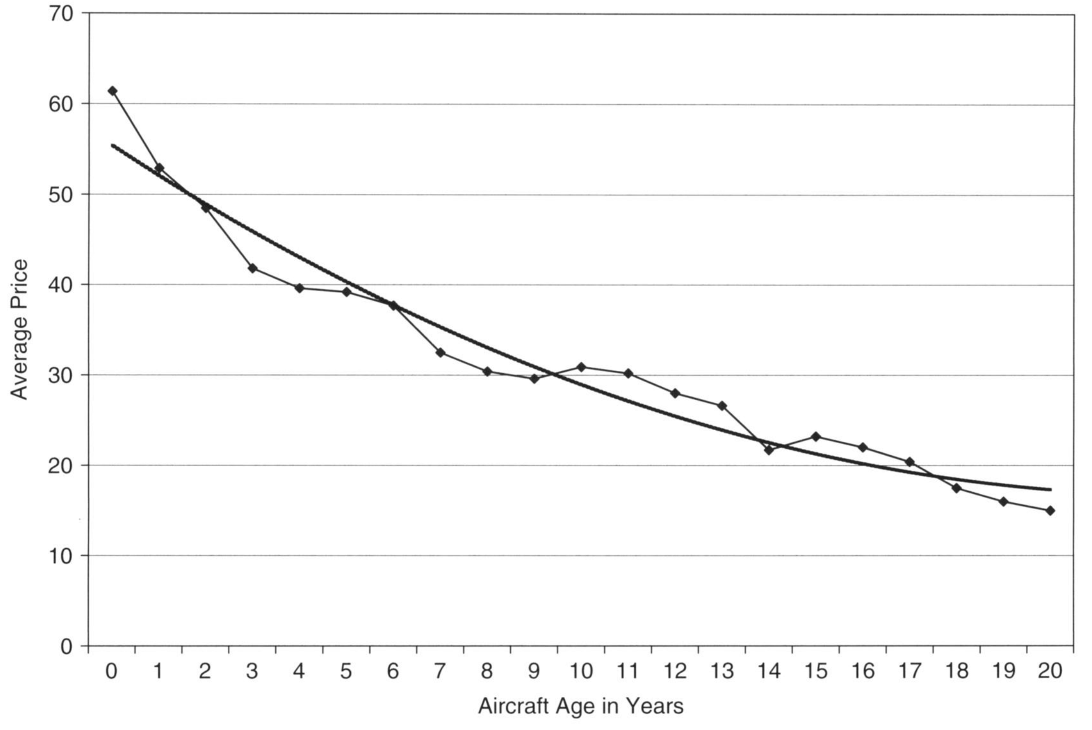

Figure 3. Average wide-body aircraft prices in millions of chained 1982–1984 US dollars. This figure is available in color online at wileyonlinelibrary.com/journal/jae

model's equilibrium requires either new aircraft prices or quantities to be exogenously determined. To make the model's description easier, I assume new aircraft quantities are exogenously determined and allow all aircraft prices to be determined by the equilibrium choices of airlines. However, in the counterfactual experiments, I pin down new aircraft prices and allow the quantity of new aircraft to be determined by airlines' equilibrium ownership choices.

Admittedly, though it is designed to capture the dynamics and equilibria that typically characterize durable goods markets, due to the complexity of solving and estimating a dynamic structural model the model still contains several potentially meaningful abstractions from reality. For example, the behavior of manufacturers and scrappers, who determine how many aircraft enter and exit the market in each period, is assumed exogenous. Also, solving the model requires that the evolution of state variables is either (1) deterministic and known to airlines or (2) completely random. Therefore, airlines not only know where each used aircraft was in the previous period, but also how many of each type of new aircraft will enter the market in the current and future periods. Further, outside of idiosyncratic airline–aircraft-specific shocks, airlines know exactly how the observed and unobserved attributes of aircraft change over time.

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5.1. The Scrapper

Similarly to Rust (1985) and others, I assume that there is a scrapper that is willing to acquire and scrap any number of aircraft at a price of $\underline{P}_j$. The estimated model assumes $\underline{P}_j = 0$ for all j, regardless of the aircraft's type or level of deterioration.[7]

5.2. Manufacturers

Let there be an arbitrarily large but finite number of discrete time periods indexed $t = 1, 2, \ldots, T$. At each time t, there are J_t types of aircraft, where types are indexed $j = 1, \ldots, J_t$. All aircraft of the same type (i.e., model and vintage) are identical. The number of distinct types J_t may differ for different t. I assume that the number of each type of new aircraft supplied to the market at time t is exogenously determined and equal to those observed in the data. However, which airline ends up owning each new aircraft and at what price are endogenously determined. That is, once new aircraft enter the market, they are allocated to airlines and their prices are determined in the same efficient auction that determines the prices and locations of used aircraft.

5.3. Airlines

The supply of available aircraft at time t is comprised of used aircraft owned by airlines in the previous period and new aircraft supplied to the market by manufacturers. All available aircraft are then acquired airlines or the scrapper. If acquired by an airline, aircraft are again available in the following period. If acquired by the scrapper, aircraft are destroyed and leave the market forever.

There are I airlines, indexed $i = 1, \ldots, I$, and the scrapper, indexed $i = 0$, in the market at each time t. Airline choices at time t are summarized by a vector $q_{it} = (q_{i1t}, \ldots, q_{iJ_t t})$, where q_{ijt} is the quantity of type j aircraft owned by airline i at time t. Airline i may alter q_{it} by buying aircraft, selling aircraft, and/or scrapping aircraft. There are a finite number of type j available at time t, $\overline{q}_{jt}$. Therefore, the set of feasible airline choices are all possible permutations of the vector $(q_{i1t}, \ldots, q_{iJ_t t})$ such that $0 \leq q_{ijt} \leq \overline{q}_{jt}$ for each i and j, and $\sum_i q_{ijt} \leq \overline{q}_{jt}$ for all j.

5.2.1 Profit Flow Equations

The profit flow generated by airline i at time t is a function of the state determined by the vectors q_{t-1} and ξ_t. The vector $q_{t-1} = (q_{11t-1}, \ldots, q_{1J_{t-1}t-1}, \ldots, q_{I1t-1}, \ldots, q_{IJ_{t-1}t-1}, \overline{q}_{1t}, \ldots, \overline{q}_{J_t t})$ encompasses all airlines' fleet choices in the previous period and the total number of each type of aircraft produced by manufacturers. $\xi_t = (\xi_{11t}, \ldots, \xi_{1J_t t}, \ldots, \xi_{I1t}, \ldots, \xi_{IJ_t t})$ is a vector of both deterministic and random state variables that is observed to airlines but is unobserved to the econometrician. In practice, each ξ_{ijt} is decomposed into two additively separable components. A detailed description of the joint distribution of the components of ξ_t is given in Section 8. Agents in the model know the current realization of ξ_t, but know only the distribution of future realizations $\xi_{t+1}, \xi_{t+2}, \ldots$. Other airlines' observed and unobserved state variables affect airline i's profit indirectly through aircraft prices $P_t(q_{t-1}, \xi_t) = (P_{1t}, \ldots, P_{J_t t})$. That is, prices are determined

[7] Assuming a price of zero for scrap aircraft does not appear to be a significant deviation from reality. New and used aircraft are generally sold for millions of dollars, while scrappers can typically acquire unwanted aircraft for less than \$100,000.

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by equilibrium, which is determined by state vectors q_{t-1} and ξ_t. Airline i's profit flow at time t is also influenced by the attributes of the individual aircraft they own, X_t. Given all of the factors above, airline i's profit flow at time t can be summarized as

$$\pi[q_{it}, s_{it}] = \pi[q_{it}, q_{it-1}, X_t, P_t(q_{t-1}, \xi_t), \xi_{it}] \tag{1}$$

where q_{it} is airline i's fleet choice in the current period, and $s_{it} \equiv [q_{it-1}, X_t, P_t(q_{t-1}, \xi_t), \xi_{it}]$ describes airline i's state. The parametric specification of the profit flow equation (1) used to estimate the model is detailed in Section 6.

The evolution of each element of the state vector s_{it} is as follows. q_{t-1}, which summarizes the location of each aircraft when they enter period t (before they are reallocated), is determined by airlines' ownership choices in period $t-1$. Occasionally, aircraft do have accidents. Therefore, a stock flow specification $s_t = q_{t-1}$—crash_{t-1} would seem appropriate. However, crashes are extremely rare (about one a year on average) and difficult to separately identify in the data from other scrappage. Scrappage is endogenous in the model. Therefore, crashes would be characterized in the model by a large negative shock to an aircraft at time t, resulting in the aircraft being sold to the scrapper. $X_t = [X_{1t}, \ldots, X_{jt}, \ldots, X_{Jt}]$ is limited to variables that are deterministic and have known paths (e.g., seating capacity and age). Therefore, the complete path of each element of X_t is known to each airline at $t = 0$. As is discussed in greater detail in the estimation section, all components of ξ_t are either deterministic and known to airlines or idiosyncratic. That is, mean time effects, though unobserved to the econometrician, are anticipated by airlines, but there are random shocks around those trends for each airline. More elaborate specifications of the error distribution are desirable and theoretically feasible, but such specifications of ξ_t would make the already difficult model solution and estimation problem described below prohibitively time consuming.

It is worth noting that unlike in previous work (see, for example, Rust, 1987) where the depreciation process for the durable was treated as random, through, for example, mileage accumulation, the depreciation of aircraft is treated as a deterministic function of aircraft age. Previous work treats the purchase/replacement decisions over each durable as independently determined, while I treat airlines' ownership choices over all of the aircraft in their fleets as jointly determined. In addition, I allow every aircraft in existence in a given period to be a potential part of each airline's fleet. Therefore, allowing for potential heterogeneity in the depreciation of aircraft of the same model and vintage is infeasible, because even if the distribution of the depreciation process is discretized or simulation methods are used the model would still require airlines to track the depreciation status of thousands of aircraft in each period.

5.2.2 Value Function

At time t, each airline chooses a fleet of aircraft to maximize the sum of profit at time t and discounted expected future profits. That is, airline i chooses the quantity vector

$$q_{it}^* = \arg\max_{q_{it}} \left\{ \pi[q_{it}, s_{it}] + E_t \left(\max_{Q} \sum_{r=t+1}^{\infty} \beta^{r-t} \pi[q_{ir}, s_{ir}] \right) \right\} \tag{2}$$

and obtains

$$V[s_{it}] = \max_{q_{it}} \left\{ \pi[q_{it}, s_{it}] + E_t \left(\max_{Q} \sum_{r=t+1}^{\infty} \beta^{r-t} \pi[q_{ir}, s_{ir}] \right) \right\}. \tag{3}$$

$0 < \beta < 1$ is a common discount factor and Q summarizes the infinite sequence of an airline's future decisions $Q = \{q^*_{it+1}, q^*_{it+2}, \ldots\}$, where each choice q^*_{it} depends entirely on the state, s_{it}, which depends on the entire history of market equilibria. The expectation operator E_t is taken over the joint distribution of future unobserved state vectors, ξ. Equation (3) is an airline's value function, which can be more conveniently expressed in the well-known Bellman equation form

$$V[s_{it}] = \max_{q_{it}} \{\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]\}. \tag{4}$$

Equation (4) is appropriately read from left to right as the value of i's optimal fleet choice at time t equals its profit at time t plus its discounted expected value of its value at time $t+1$.

5.4. Equilibrium

Equilibrium is defined by an aircraft allocation where no airline would be better off changing its fleet by buying, selling or trading one or more aircraft, and a vector of aircraft prices that satisfy standard market clearing conditions. Specifically, there is a mapping between airlines' choices and aircraft prices, and equilibrium defines a fixed point to the mapping such that airline choices maximize value functions given observed aircraft prices, and aircraft prices rationalize observed airline choices. Formally, equilibrium is defined as follows:

Definition 1. *At each time t, aircraft prices* $P^*_{1t}, \ldots, P^*_{jt}, \ldots, P^*_{J_t t}$ *and airline quantity choices* $q^*_t = (q^*_{11t}, \ldots, q^*_{ijt}, \ldots, q^*_{IJ_t t})$ *characterize an equilibrium if*

$$q^*_{it} = \arg\max_{q_{it}} \{\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]\} \quad for\ all\ i \tag{5}$$

and the market clearing conditions

$$\begin{aligned} &if\ P^*_{jt} > 0 \quad then \quad \sum_i q^*_{ijt} = \overline{q}_{jt} \quad or \\ &if\ P^*_{jt} = 0 \quad then \quad \sum_i q^*_{ijt} \leq \overline{q}_{jt} \end{aligned} \tag{6}$$

are satisfied for all aircraft types j.

If the last equation $\sum_i q^*_{ijt} \leq \overline{q}_{jt}$ is satisfied with strict inequality, then the excess aircraft are acquired by the scrapper and destroyed.

6. FUNCTIONAL FORM ASSUMPTIONS AND BEHAVIORAL IMPLICATIONS

The specific functional form airline profit flow used to estimate the structural model is given by

$$\pi[q_{it}, q_{it-1}, X_t, \xi_{it}, P_t(q_{t-1}, \xi_t); \theta_{ij}] = \sum_{j=1}^{J_t} \{q_{ijt}[X_{jt}\gamma_i + \xi_{ijt}] - 1(q_{ijt} > q_{ijt-1})\lambda_j[q_{ijt} - q_{ijt-1}] - c[q_{ijt} - q_{ijt-1}]^2 \tag{7}$$

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$$-[q_{ijt} - q_{ijt-1}]P_{jt}\} - \delta_i \left[\sum_{j=1}^{J_t} q_{ijt}\right]^2.$$

Recall that X_{jt} is a vector of observed characteristics for type j aircraft at time t, and ξ_{ijt} is a random state variable that shifts airline i's preference for aircraft j in period t. λ_j measures transaction costs associated with, for example, reconfiguring and learning to operate an aircraft, or forgone warranties and guarantees. The indicator function 1(.) equals one if its argument is satisfied and zero otherwise. Therefore, when airlines increase the number of type j aircraft they own, they incur transaction costs equal to λ_j times the magnitude of the increase. The I-dimensional vector γ_i measures the intensity of airline i's preferences for various observed attributes of aircraft. The final two features of equation (7) are particularly important. The I-dimensional quadratic airline fleet size term, δ_i, allows for declining marginal returns to aircraft, and helps determine the total size of each airline's fleet. The other important term is the quadratic fleet adjustment cost term, c. This term is quadratic individually for each type of aircraft, which explains why airlines have more than one type of aircraft in their fleets.

Ultimately the estimation algorithm makes use of equilibrium conditions defined in terms of the marginal value of aircraft. Therefore, it is useful to define the marginal value of aircraft j to airline i at time t as the value of the airline fleet defined by q_{it} minus the value of the airline fleet defined by q_{it} less one aircraft j. That is,

$$\begin{aligned} M_j(q_{it}) \equiv \pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}] - \\ \pi[q_{it} - e_j, s_{it}] + \beta E_t V[s_{it+1} - e_j] \end{aligned} \tag{8}$$

where e_j is a J_t-dimensional vector with a one in the jth position and zeros elsewhere. Given the functional form of the profit function, the marginal value of aircraft j to airline i at time t is given by

$$\begin{aligned} M_j(q_{it}) = X_{jt}\gamma_i + \xi_{ijt} - 1(q_{ijt} > q_{ijt-1})\lambda_j - \\ [2q_{ijt} - 2q_{ijt-1} - 1]c - P_{jt} - \left[2\left(\sum_{j=1}^{J_t} q_{ijt}\right) - 1\right]\delta_i \\ + \beta E_t\{V[s_{it+1}] - V[s_{it+1} - e_j]\}. \end{aligned} \tag{9}$$

In the remainder of this section I describe some of the implications of the model, including Theorem 1, which expresses airlines' equilibrium fleet choices given in Definition 1 in terms of marginal values of aircraft. Lemmas 1 and 2 establish a discrete-choice analog to the global concavity of the value function. Because both current and future profit flows depend on q_{ijt}, and q_{ijt} indirectly influences future choices $q_{i1t+1}, \ldots, q_{ijt+1}, \ldots, q_{iJt+1}$, it is not obvious that the marginal value of an aircraft is decreasing in q_{ijt}. Lemma 1 shows that, given the functional form of airline profit flow equations, airlines do, in fact, have decreasing marginal values of aircraft.

Lemma 1 *Given* $c > 0$, $\lambda_j > 0$, $\delta_i > 0$, *and* $0 < \beta < 1$, $\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]$ *is finite at* $\xi_t = 0$, *and* $M_j(q_{it}) > M_j(q_{it} + e_j)$ *for all* j, q_{it}, s_{it} *and* ξ_t.

Proof. Proofs of all propositions are available online as supporting information.

Next, again given functional form, Lemma 2 establishes stronger conditions on the marginal value of aircraft. Specifically, Lemma 2 states that the marginal contribution of aircraft j to airline i's value function at time t at the vector q_{it} is greater than the marginal contribution of aircraft j of a swap that adds an additional aircraft j and subtracts an aircraft of another type, which is greater than the marginal contribution of aircraft j when an additional aircraft j is added to i's fleet.

Lemma 2 *Given* $c > 0$, $\lambda_j > 0$, $\delta_i > 0$, *and* $0 < \beta < 1$, $M_j(q_{it}) > M_j(q_{it} + e_j - e_{j'}) > M_j(q_{it} + e_j)$ *for all* j, j', q_{it}, s_{it} *and* ξ_t.

Lemmas 1 and 2 ensure that for each airline, for a given price vector P_t, there is a unique value function maximizing vector of aircraft q^*_{it}. Given Lemmas 1 and 2, Theorem 1 states the conditions that each airline's optimal fleet choice, q^*_{it}, must satisfy in terms of marginal values.

Theorem 1 *Given prices,* P_t, *for all* i

$$q^*_{it} = \arg\max_{q_{it}} \{\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]\},$$

if and only if

$$M_j(q^*_{it}) > 0 \ \forall j \tag{10}$$

$$M_j(q^*_{it} + e_j) < 0 \ \forall j \tag{11}$$

and

$$M_j(q^*_{it}) - M_{j'}(q^*_{it} + e_{j'} - e_j) > 0 \ \forall j, j' \text{ and } j \neq j' \tag{12}$$

As was stated above, Theorem 1 implies that if an airline's value function does not increase by choosing any of the vectors of aircraft that are adjacent to the vector of aircraft it currently holds (i.e., a single aircraft purchase, a single aircraft sale, or a swap involving any two aircraft), the airline cannot increase its value function by choosing any alternative airline fleet. The conditions of Theorem 1 are used to simplify computation in two areas of this paper. First, the inequalities greatly reduce the number of aircraft allocations that must be searched over when solving for equilibrium at each iteration of the model solution algorithm. Without the results of Theorem 1, all possible combinations of quantity vectors would have to be evaluated to find the optimal fleet choice of each airline, which requires computing $\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]$ an extremely large number of times, and since the model must be solved several times to estimate its parameters, searching for equilibrium by performing a full grid search would be computationally prohibitive. Second, since the inequalities in Theorem 1 can be expressed in terms of the errors of the model, and therefore the probability of observing a particular outcome can be computed by integrating

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over the joint distribution of those errors, the inequalities of Theorem 1 are also used to construct the likelihood function used to estimate the model.

As discussed above, the specification of the model facilitates both model solution and estimation. However, as is often the case, restrictions imposed by functional form assumptions have costs. The theoretical properties of the model provided by Theorem 1 are true if and only if λ_j, c, and δ_i are positive, and the discount factor, β, is between 0 and 1. The conditions on δ_i and β, decreasing marginal returns and discounted expected future values, are typical in economic models. Positive switching and adjustment costs are also common in theoretical models, but the assumptions may be particularly restrictive in this context. The interpretation of c as an adjustment cost and the assumption that $c > 0$ is particularly problematic if one believes that an aircraft's contribution to airline profit is positively affected by ownership of other aircraft of the same type. Southwest Airlines, for example, owns only one type of aircraft, the Boeing 737 (the 737 is not a wide-body aircraft). Southwest is well known to be a successful and efficient airline, and therefore one might suspect that owning many of one type of aircraft is a particularly efficient strategy. However, the strategies of large airlines in my sample apparently differ from Southwest's strategy. For example, in 1997, Delta owned nine aircraft types (model/vintage), United owned 15 aircraft types, American owned nine aircraft types, and Northwest owned 10 aircraft types. Now, one could argue that this fact partially explains why Southwest was profitable while major airlines struggled, but the fact remains that the structural model must explain the ownership patterns exhibited by Delta, United, etc. Ideally, I would more flexibly parameterize potential scope effects and allow the data to sort out the specific shape of airlines' profit flow equations. However, allowing such flexibility compromises the global properties of the value function posited in Theorem 1, and render the model estimation of the model using the proposed framework impossible. As a practical matter, if airlines' usefulness for aircraft of the same type are significantly positively correlated, one might expect c to be negative for the above specification. In practice, I assume c is positive to establish the theoretical properties below, and I use the theoretical properties in model solution and estimation. However, I do not restrict c above zero in estimation. The estimate of c stayed a small but significant amount above zero, which indicates that the estimated model is consistent with its theoretical properties.

7. MODEL SOLUTION

The solution to each airline's dynamic decision problem relies on equilibrium aircraft prices, which in turn rely on solutions to all other airlines' dynamic decision problems. The model solution described here is comprised of an algorithm for finding an equilibrium aircraft allocation and prices nested within an algorithm that simulates the solution to each airline's dynamic decision problem. That is, taking the most recently calculated expected future value functions, an equilibrium aircraft allocation and set of prices are determined using an efficient auction mechanism. Then, using an algorithm similar to the one developed in Pakes and McGuire (2001), the value functions calculated at equilibrium are used to update the simulated expected future values functions. It may seem redundant to calculate equilibrium prices given that I have them in the data. However, the equilibrium prices observed at time $t + 1$, $t + 2$, etc. follow from a single realization of the elements of the state space that are random to airlines at time t. Calculation of expected future values at time t relies on integration over *all possible realizations* of the model's random shocks at points in the future, and each possible realization of the random components may lead to

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a different equilibrium aircraft allocation and set of prices. Therefore simulating the model's solution requires calculating equilibrium allocations *and* prices at a large number of simulation draws of the sequence of possible random states.

Dynamic decision problems are often solved backwards recursively by iterating on the value function equations:

$$V[s_{it}] = \max_{q_{it}} \{\pi[q_{it}, s_{it}] + \beta E_t V[s_{it+1}]\}. \tag{13}$$

Equation (13) implies that to find its best choice in period t airline i must compute the expected value of its best choice in period $t+1$ for all feasible s_{it+1}, which depends on q_{it}. The last term on the right-hand side of equation (13) is given by

$$E_t V[s_{it+1}] = \int \cdots \int \max_{Q} \sum_{r=t+1}^{\infty} \beta^{r-(t+1)} \pi[q_{ir}, s_{ir}, \xi] \mathrm{d}F(\xi) \tag{14}$$

where $F(\xi)$ is the joint density of the error vector ξ. Computing equation (14) at each possible s_{it+1} is computationally burdensome because: (1) the right-hand side of equation (14) is in general a multidimensional integral, which, depending on error distributions, makes computing $E_t V[s_{it+1}]$ difficult or impossible; and (2) $E_t V[s_{it+1}]$ must be computed for every feasible state vector s_{it}, and for any reasonable specification of the model, the number of possible permutations of s_{it} is extremely large. The problem is made even more difficult because $V[s_{it+1}]$ depends on $E_t V[s_{it+2}]$, and $V[s_{it+2}]$ depends on $E_t V[s_{it+3}]$, and so on. Therefore, the computational burden of solving a dynamic model backwards recursively grows exponentially in the number of decision periods. Given these factors, solving the model using standard backward solution methods is not feasible.

Papers by Rust (1997) and Pakes and McGuire (2001) propose using simulation to reduce the computational complexity of solving the dynamic model. The basic idea behind these simulation algorithms is to iterate between guessing future value functions, computing value functions at simulation draws of the errors of the model, treating resulting outcomes as possible realizations of the future that can be averaged in to update guesses of future value functions. The simulation algorithm used to solve the model has two important computational advantages over traditional backward solution methods. First, rather than computing the complex integrals that comprise $E_t V[s_{it+1}]$, the $E_t V[s_{it+1}]$ terms are simulated. Second, given an airline's current guess of $E_t V[s_{it+1}]$ and a draw of the random components of the model, say ξ_t^r, the algorithm solves for only a single sequence of states in periods $t = 1, 2, \ldots$. Solving the model at only a single sequence of states allows each iteration of the algorithm to be performed very quickly, and choices that are more likely to occur given the parameters of the model will occur with greater frequency. Therefore, the approximations of $E_t V[s_{it+1}]$ will be more accurate at 'important' states. However, the computational benefits of the stochastic algorithm are not without costs. The model often must be solved several thousand times to get a reasonable approximation of the $E_t V[s_{it+1}]$ terms, for $t = 1, 2, \ldots$ Also, the stochastic algorithm is generally less precise than backward solution methods that use numerical integration. However, the precision of the approximation of the $E_t V[s_{it+1}]$ terms increases with the number of times the model is solved and, because the algorithm focuses on only a fraction of the total number of possible states, the number of times the model must be solved to obtain an accurate approximation of the $E_t V[s_{it+1}]$ terms is not necessarily related to the size of the state space.

The model solution algorithm used to solve airlines' fleet management problems is implemented as follows. Iterations of the algorithm are indexed by r, and a threshold parameter $\varepsilon > 0$ determines

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when the algorithm has converged. Starting with $r = 1$, set the initial guess of each $E_tV[s_{it+1}]$ term equal to $\widehat{E_tV[s_{it+1}]}^0 = \frac{\pi[q_{it+1}, s_{it+1}, 0]}{1-\beta}$.[8]

(1) Draw error vectors $\xi_1^r, \ldots, \xi_T^r$.

(2) Given $\widehat{E_tV[s_{it+1}]}^{r-1}$ and $\xi_1^r, \ldots, \xi_T^r$, calculate equilibrium aircraft allocations and prices for $t = 2, \ldots, T$. Specifically, an efficient mechanism determines an aircraft allocation and set of prices such that

$$q_{it}^r = \arg\max_{q_{it}} \{\pi[q_{it}, s_{it}^r, \xi_t^r] + \beta \widehat{E_tV[s_{it+1}^r]}^{r-1}\}$$

for all i, with associated value functions:

$$V[s_{it}^r] = \max_{q_{it}} \{\pi[q_{it}, s_{it}^r, \xi_t^r] + \beta \widehat{E_tV[s_{it+1}^r]}^{r-1}\}$$

and the market clearing conditions:

$$\begin{array}{lll} \text{if} P_{jt}^r > 0 & \text{then} & \sum_i q_{ijt}^r = \overline{q}_{jt}^r \quad \text{or} \\ \text{if} P_{jt}^r = 0 & \text{then} & \sum_i q_{ijt}^r \leq \overline{q}_{jt}^r \end{array} \qquad (15)$$

are satisfied for all aircraft types j.

(3) The value functions at the equilibrium outcomes of iteration r are then used to update the approximations of the $E_tV[s_{it+1}]$ terms. Specifically:

$$\widehat{E_{t-1}V[s_{it}^r]}^r = \frac{(n(s_{it}^r) - 1) \times \widehat{E_{t-1}V[s_{it}^r]}^{r-1} + V[s_{it}^r]}{n(s_{it}^r)}$$

where the state s_{it}^r depends on the vector q_{t-1}^r, and $n(s_{it}^r)$ is the number of times an outcome q_{t-1}^r has occurred up to and including iteration r. If, on the other hand, a vector q_{t-1} (and associated s_{it}) does not occur, the approximations of $\widehat{E_{t-1}V[s_{it}]}^r$ do not change. That is:

$$\widehat{E_{t-1}V[s_{it}]}^r = \widehat{E_{t-1}V[s_{it}]}^{r-1}$$

at all s_{it} except the s_{it}^r that occur at iteration r.

(4) If for the state vector $s = (s_{11}, \ldots, s_{it}, \ldots, s_{IT})$, $\left\| \widehat{E_tV[s_{it+1}]}^r - \widehat{E_tV[s_{it+1}]}^{r-1} \right\| < \varepsilon \forall i$ and t, stop. If not, set $r = r + 1$ and return to step (1).

Generally, the number iterations of the above algorithm necessary to obtain convergence ranged from 1000 to 5000. In practice I set $\varepsilon = \frac{1}{100} \times \left\| \widehat{E_tV[s_{it+1}]}^1 - \widehat{E_tV[s_{it+1}]}^0 \right\|$. This threshold is low enough that increasing the threshold does not have a meaningful effect on the model solution results or parameter estimates. Occasionally, the algorithm did not stop for a couple of days. Therefore, a second stopping mechanism was added that stopped the algorithm when a new state point was not added within 100 iterations.

[8] In practice, an $\widehat{E_tV[s_{it+1}]}^0$ is not calculated until it is needed to evaluate an airline's optimization problem. Some state points do not occur, and therefore do not necessitate the calculation and storage of $\widehat{E_tV[s_{it+1}]}^0$ terms.

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Nested within the above algorithm, in step (3), the equilibrium of the model must be computed. The auction algorithm used to find equilibrium aircraft allocations and prices is equivalent to the following algorithm. I describe a faster algorithm that more quickly arrives at the same equilibrium in detail in the Appendix, available online as supporting information. In each period, start with a finite number of each type of aircraft, $\overline{q}_{jt}$. Order aircraft from $j = 1, \ldots, J$, and set the initial price of each aircraft equal to 0. Starting with iteration $k = 1$:

1. Given the current vector of prices P_t^k, allow airlines to choose their highest-valued airline fleets. Due to Lemmas 1 and 2 above, this can be accomplished by allowing each airline to choose aircraft one at a time by highest marginal value.
2. If $\sum_{i=1}^{I} q_{ijt}^k = \overline{q}_{jt}$, or $P_{jt}^k = 0$ and $\sum_{i=1}^{I} q_{ijt}^k < \overline{q}_{jt}$, for all j, then go to (4). Otherwise, for all j, starting with $j = 1$, increase P_{jt}^k until $\sum_{i=1}^{I} q_{ijt}^k = \overline{q}_{jt}$. Alternatively, if $\sum_{i=1}^{I} q_{ijt}^k < \overline{q}_{jt}$, leave $P_{jt}^k = P_{jt}^{k-1} = 0$.
3. P_t^k is the new vector of prices. Set $k = k + 1$ and go to (1).
4. P_t^k is the vector of equilibrium prices and q_t^k is the vector of equilibrium quantities.

An existence proof for the above equilibrium is available in Smith (2005).

8. ERROR DISTRIBUTIONS AND IDENTIFICATION

Each error ξ_{ijt} is decomposed into two additively separable components, i.e.,

$$\xi_{ijt} = \eta_{it} + \varepsilon_{ijt} \tag{16}$$

where η_{it} is an unobserved airline-specific shock to productivity that can be attributed to, for example, the economy, weather, or the threat of terror and ε_{ijt} is a idiosyncratic shock to airline–aircraft matches at time t. In practice, I assume that there are three classes of airlines: small US airlines (e.g., TWA and Pan Am), large US airlines (e.g., Delta and United), and all other airlines (e.g., international carriers and US freight and charter airlines) and that all airlines within each class have identical preferences. Time-specific shocks to the demand for air travel differ across classes of airlines because, for example, freight airlines may be impacted by the introduction of alternative shipping mechanisms that do not affect passenger airlines. The η_{it} are specified as the sum of a linear time trend and a dummy variable that shifts returns after the Tax Reform Act of 1986 occurred:

$$\eta_{it} = \alpha_i t + \mu D_{1986,t} \tag{17}$$

where α_i is an airline-type specific parameter that captures the fact that different groups of airlines may have different growth rates in the segments of their fleets allocated to wide-bodied aircraft, and μ is a parameter that captures differences in airline investment before and after the Tax Reform Act of 1986. ε_{ijt} is an idiosyncratic shock to the productivity of a particular airline–aircraft match in time t. I assume

$$\varepsilon_{ijt} \sim \text{i.i.d.} N(0, \sigma_\varepsilon)$$

across airlines, aircraft and time.

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Next, I provide a brief discussion of how the parameters of the structural model are identified by the data. Recall that the vector of parameters to be estimated is

$$\theta = (\gamma_1, \ldots, \gamma_I, \delta_1, \ldots, \delta_I, \lambda_1, \ldots, \lambda_J, \alpha_1, \ldots, \alpha_I, \mu, c, \sigma_\varepsilon^2)$$

and the data used to estimate the parameters is given by

$$\{q_{111}, \ldots, q_{ijt}, \ldots, q_{IJT}; X_{11}, \ldots, X_{jt}, \ldots, X_{JT};$$
$$P_{11}, \ldots, P_{jt}, \ldots, P_{JT}; 1, \ldots, t, \ldots, T\}.$$

I estimate 27 γ parameters (three airline classes times nine aircraft types (model/vintage combinations)). The γ parameters are identified by variation in the types of aircraft different classes of airlines choose to own, as well as variation in prices across across aircraft types. The transaction cost parameters, λ_j, are identified by variation in the frequency of sale of different types of aircraft. Only four λ parameters are estimated (one for each aircraft make: Boeing, Airbus, McDonnell-Douglas, and Lockheed Martin). The scale parameters, δ_i, are identified by variation in mean number of aircraft owned across the different types of airlines. Adjustment costs, c, are identified by variation in the magnitude of changes in the composition of airline fleets. Recalling the decomposition of the errors of the model given by equations (16) and (17), the α_i are identified by variation in the size of an airline fleet over time, while the parameter μ is identified by differences in the size of airline fleets before and after the Tax Reform Act of 1986. Finally, because it is assumed that the airlines' relative preferences for the different types of aircraft do not change over time, the discount factor β is weakly identified by changes over time in investment patterns. However, I do not attempt to identify β, but instead assume it is constant and equal to 0.9.

9. ESTIMATION

The parameters of the structural model are estimated by simulated maximum likelihood (SML). Construction of the likelihood function follows from the equilibrium conditions of the model and the distributions assumed for the errors, and the high-dimensional integrals of the likelihood function are simulated using (SML). A key assumption in the construction of the likelihood function is that the prices we observe in the data are a sufficient statistic for all other airlines' fleet choices (i.e., observed prices are equilibrium prices), and therefore, conditional on observed prices, an individual airline's equilibrium fleet choices can be made independently.

9.1. The Likelihood Function

Estimation imposes the equilibrium defined by Definition 1 and Theorem 1 on the observed price and quantity data. Given the functional form assumptions for $\pi(q_{it}, s_{it}, \varepsilon_{it})$, airline profit flow is additively separable in the errors ε_{ijt}, i.e., $\pi(q_{it}, s_{it}, \varepsilon_{it}) = \overline{\pi}(q_{it}, s_{it}) + \sum_j q_{ijt}\varepsilon_{ijt}$. It follows that the marginal value of aircraft j to airline i's fleet, given by equation (9), is also additively separable in ε_{ijt}:

$$M_j(q_{it}) = \overline{M}_j(q_{it}) + \varepsilon_{ijt}.$$

To facilitate the construction of the outcome probabilities that comprise the likelihood function, I define

$$\Delta^{+}_{ijt} \equiv -\overline{M}_j(q_{it} + e_j) \tag{18}$$

$$\Delta^{-}_{ijt} \equiv -\overline{M}_j(q_{it}) \tag{19}$$

$$\Delta_{ij'jt} \equiv \overline{M}_j(q_{it}) - \overline{M}_{j'}(q_{it} + e_{j'} - e_j). \tag{20}$$

Using the newly defined Δ terms, the conditions given by equations (10), (11), and (12) of Theorem 1 can be rewritten in terms of ε_{ijt} as

$$\varepsilon_{ijt} > \Delta^{-}_{ijt} \tag{21}$$

$$\varepsilon_{ijt} < \Delta^{+}_{ijt} \tag{22}$$

$$\varepsilon_{ijt} < \Delta_{ijj't} + \varepsilon_{ij't} \quad \text{for all } i, j, j' \text{ and } t. \tag{23}$$

Combining the conditions given by equations (21), (22), and (23), the upper and lower bounds of each ε_{ijt} are given by

$$\begin{aligned} \overline{b}_{ijt} &= \min(\Delta^{+}_{ijt}, \varepsilon_{i1t} + \Delta_{ij1t}, \ldots, \varepsilon_{ij-1t} + \Delta_{ijj-1t}, \\ &\quad \varepsilon_{ij+1t} + \Delta_{ijj+1t}, \ldots, \varepsilon_{iJt} + \Delta_{ijJt}) \text{ and} \\ \underline{b}_{ijt} &= \max(\Delta^{-}_{ijt}, \varepsilon_{i1t} - \Delta_{i1jt}, \ldots, \varepsilon_{ij-1t} - \Delta_{ij-1jt}, \\ &\quad \varepsilon_{ij+1t} - \Delta_{ij+1jjt}, \ldots, \varepsilon_{iJt} - \Delta_{iJjt}). \end{aligned} \tag{24}$$

The probability that a vector of airline quantity choices $q^*_{it} = (q^*_{i1t}, \ldots, q^*_{iJt})$ are optimal for airline i at time t can be expressed in terms of ε_{ijt}, $\overline{b}_{ijt}$, and $\underline{b}_{ijt}$ as

$$Q_{it} = \Pr[\underline{b}_{ijt}(q^*_{it}) < \varepsilon_{ijt} < \overline{b}_{ijt}(q^*_{it}) \quad \forall j]$$

which, given that the ε_{ijt} are i.i.d.$N(0, \sigma^2_\varepsilon)$, can be written

$$Q_{it} = \int \cdots \int \prod_{j=1}^{J} 1(\underline{b}_{ijt}(q^*_{it}) < \varepsilon_{ijt} < \overline{b}_{ijt}(q^*_{it})) \frac{1}{\sigma^2_\varepsilon} f\left(\frac{\varepsilon_{ijt}}{\sigma_\varepsilon}\right) \mathrm{d}\varepsilon_{ijt}, \tag{25}$$

where $(\cdot)$ is an indicator variable that equals one if its argument is satisfied and zero otherwise, and f is the standard normal probability density function. Finally, since the errors ε_{ijt} are independent across airlines, the log-likelihood of observing a sequence of equilibrium aircraft allocations in periods $t = 1, \ldots, T$ can be calculated:

$$L(\theta) = \sum_{t=1}^{T} \sum_{i=1}^{I} \ln[Q_{it}] \tag{26}$$

where $\theta = (\gamma, \delta, \lambda, \alpha, \mu, c, \sigma_\varepsilon)$ is the vector of parameters to be estimated.

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9.2. Simulated Maximum Likelihood

I construct an importance sampling simulator that focuses its attention on the regions of the error distribution that are consistent with conditions given by equations (21), (22) and (23), and is continuous in the parameters of the model. The simulator developed in this research is similar to the well-known GHK simulator (Geweke, 1991; Hajivassiliou and McFadden, 1990; Keane, 1994). Details of how the Q_{it} are simulated are given below, and properties of the simulator are discussed in the Appendix, available online as supporting information.

The SML estimator replaces Q_{it} with the simulated probability $\widehat{Q}_{it}$, to get

$$\widehat{L}(\theta) = \sum_{t=1}^{T}\sum_{i=1}^{I}\ln[\widehat{Q}_{it}]. \tag{27}$$

Because $E[\ln(x)] \neq \ln[E(x)]$, SML estimators are consistent but biased for a finite number of simulation draws. However, Borsch-Supan and Hajivassiliou (1993) show that SML estimators that use importance simulation techniques, such as the well-known GHK simulator or the Stern (1994) simulator, perform well relative to unbiased method of simulated moments estimators in Monte Carlo experiments.

I simulate Q_{it} as follows. The simulator is an 'importance' simulator in that it uses the law of total probability and strategically orders the errors of the model to maximize the amount of time the algorithm spends in the regions of the error distributions the outcomes observed in the data are likely to occur. Therefore, the algorithm is able to simulate Q_{it} accurately using a relatively small number of simulation draws. Ordering aircraft in some way $((1),\ldots,(j),\ldots,(J))$ the probability

$$Q_{it} = \Pr[\underline{b}_{ijt}(q^*_{it}) < \varepsilon_{ijt} < \overline{b}_{ijt}(q^*_{it}) \quad \forall j]$$

can be rewritten

$$Q_{it} = \prod_{(j)=1} \Pr[\underline{b}^*_{i(j)t} < \varepsilon_{i(j)t} < \overline{b}^*_{i(j)t}|\varepsilon_{i(1)t},\ldots,\varepsilon_{i(j-1)t}] \tag{28}$$

where

$$\begin{aligned}\overline{b}^*_{i(j)t} = \min(&\Delta^+_{i(j)t}, \varepsilon_{i(1)t} + \Delta_{i(j)(1)t},\ldots,\varepsilon_{i(j-1)t} + \Delta_{i(j)(j-1)t}\\ &\Delta^+_{i(j+1)t} + \Delta_{i(j)(j+1)t},\ldots,\Delta^+_{i(J)t} + \Delta_{i(j)(J)t})\end{aligned} \tag{29}$$

and

$$\begin{aligned}\underline{b}^*_{i(j)t} = \min(&\Delta^-_{i(j)t}, \varepsilon_{i(1)t} - \Delta_{i(1)(j)t},\ldots,\varepsilon_{i(j-1)t} - \Delta_{i(j-1)(j)t},\\ &\Delta^-_{i(j+1)t} - \Delta_{ij+1jt},\ldots,\Delta^-_{i(J)t} - \Delta_{iJjt}).\end{aligned}$$

The simulation algorithm constructs the probability Q_{it} as suggested by equation (28) by sequentially drawing errors $\varepsilon_{i(j)t}$, where the draw $\varepsilon^r_{i(j)t}$ is conditioned on the draws $\varepsilon^r_{i(1)t},\ldots,\varepsilon^r_{i(j-1)t}$. Specifically, the algorithm is implemented as follows:

Order aircraft $\{j : \overline{q}_{jt} > 0\}$ *according to some criterion (the criterion I use is described in the Appendix, available online as supporting information). Let* (j) *be the* jth *element of the ordered set. Let* $J^* = \#\{j : \overline{q}_{jt} > 0\}$, *and initialize* $Q^r_{it} = 1$.

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(1) *For each* $(j) \leq J^*$,

(a) *Let*

$$\overline{b}^*_{i(j)t} = \min \begin{pmatrix} \Delta^+_{i(j)t}, \varepsilon^r_{i(1)t} + \Delta_{i(j)(1)t}, \ldots, \varepsilon^r_{i(j-1)t} + \Delta_{i(j)(j-1)t}, \\ \overline{\varepsilon}_{i(j+1)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) + \Delta_{i(j)(j+1)t}, \ldots, \\ \overline{\varepsilon}_{i(J)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) + \Delta_{i(j)(J)t} \end{pmatrix} \tag{30}$$

be an upper bound for $\varepsilon_{i(j)t}$, *where*

$$\overline{\varepsilon}_{i(j+1)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) = \min(\Delta^+_{i(j+1)t}, \varepsilon^r_{i(1)t} + \Delta_{i(j+1)(1)t}, \ldots, \varepsilon^r_{i(j-1)t} + \Delta_{i(j+1)(j-1)t}).$$

Also let

$$\underline{b}^*_{i(j)t} = \max \begin{pmatrix} \Delta^-_{i(j)t}, \varepsilon^r_{i(1)t} - \Delta_{i(1)(j)t}, \ldots, \varepsilon^r_{i(j-1)t} - \Delta_{i(j-1)(j)t}, \\ \underline{\varepsilon}_{i(j+1)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) - \Delta_{ij+1jt}, \ldots, \\ \underline{\varepsilon}_{i(J)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) - \Delta_{iJjt} \end{pmatrix}$$

be a lower bound for $\varepsilon_{i(j)t}$, *where*

$$\underline{\varepsilon}_{i(j+1)t}(\varepsilon^r_{i(1)t}, \ldots, \varepsilon^r_{i(j-1)t}) = \max(\Delta^-_{i(j+1)t}, \varepsilon^r_{i(1)t} - \Delta_{i(1)(j+1)t}, \ldots, \varepsilon^r_{i(j-1)t} - \Delta_{i(j-1)(j+1)t}).$$

(b) *Update*

$$Q^r_{it} = Q^r_{it} \times \left[\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right) \right]$$

(c) *Draw* $\varepsilon^r_{i(j)t}$ *conditional on* $\underline{b}_{i(j)t} \leq \varepsilon^r_{i(j)t} \leq \overline{b}_{i(j)t}$ *as*

$$\varepsilon^r_{i(j)t} = \sigma_\varepsilon \Phi^{-1} \left\{ \left[\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right) \right] u^r + \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right) \right\}$$

where $u^r \sim U(0, 1)$.[9]

(2) *if* $(j) = J^*$, Q^r_{it} *is the simulator, otherwise set* $(j) = (j) + 1$ *and return to* (1).

10. RESULTS AND MODEL FIT

This section presents the parameter estimates of the structural model and illustrates the quality of the estimated model's fit to the data.

[9] For the first couple of guesses of the parameters of the model, both $\overline{b}_{i(j)t}$ and $\underline{b}_{i(j)t}$ may be in the extreme tail of the distribution. In cases where both $\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right)$ and $\Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)$ are very close to zero or one, the estimation algorithm may crash. In such cases, I use L'Hopital's rule to derive the alternative simulator $\varepsilon^r_{i(j)t} = \phi^{-1}\left\{\left[\phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right] u^r + \phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right\}$.

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10.1. Parameter Estimates

Estimation results are given in Tables III and IV. The first set of parameter estimates, presented in Table III, are parameters that are common across the different types of airlines, including parameters measuring industry-wide time trends and market frictions. Table IV presents parameters that determine an airline's preferred scale of operation and fleet composition. As is typical in nonlinear structural models, almost all of the parameter estimates differ from zero at a high level of statistical significance.[10]

Recall that μ is a parameter that multiplies a dummy variable that is equal to one between 1986 and 1997. The estimate of μ is negative, but does not differ from zero at the 5% significance level. The insignificant estimate of μ does not necessarily imply that the Tax Reform Act of 1986 had no effect on airline investment. In fact, Smith (2009) suggests that the Tax Reform Act of 1986 did significantly dampen aircraft sales, especially in used markets. However, that study used a shorter panel of aircraft transactions that were likely more significantly impacted by the Tax Reform. In addition, the counterfactual experiment presented in the next section suggests that a tax reform similar to the Tax Reform Act of 1986 would, in fact, impact airline purchasing decisions.

There is significant heterogeneity across airlines' ownership choices, and the structural error ε captures any differences across airlines that are not captured by the parameters of the model. The point estimate of the standard deviation of the error, σ_ε, is 2.6158, which is equivalent to approximately \$13.68 million. The relatively large estimate of σ_ε indicates that the structural model has left a significant portion of the variation in airlines ownership behavior unexplained, which is not surprising given the modeling assumption that over 300 airlines can be grouped into just three types, and the ε are left to explain differences in behavior across airlines within each group.

The estimates of the transaction cost parameters, λ, are similar across the different makes of aircraft. The magnitude of the λ indicate that when an airline purchases the average 10-year-old wide-body aircraft for approximately \$30 million, the cost of reconfiguring the aircraft, retraining and/or recertifying pilots to fly the aircraft, etc. is approximately \$17.4 million. Estimates of transaction costs are admittedly large. As a point of comparison, Chen *et al.* (2008) find that transaction costs incurred by used car sellers are approximately \$2200 (approximately 24% of the

Table III. Structural estimates: industry and cost

μ: Tax reform	−0.0300 (0.0159)
c: Adjustment cost	0.0153 (0.003)
σ_ε: Error standard deviation	2.6158 (0.0677)
λ_1: Boeing transaction cost	0.9271 (0.0628)
λ_2: Airbus transaction cost	0.9347 (0.0284)
λ_3: McDonnell-Douglas transaction cost	0.8721 (0.1946)
λ_4: Lockheed-Martin transaction cost	0.9350 (0.0506)

Note: Standard errors are in parentheses.

[10] See, for example, Rust (1987) or Brien *et al.* (2006) for explanations of this fact.

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Table IV. Structural estimates: airline preferences

Large U.S. passenger airline:		Small U.S. passenger airline:		Other airline:	
Trend (α):	0.5725 (3.37e-04)	Trend (α):	0.0003 (7.29e-04)	Trend (α):	0.3574 (2.80e-04)
Rate of decreasing returns (δ):	0.0381 (3.92e-05)	Rate of decreasing returns (δ):	0.0545 (0.0013)	Rate of decreasing returns (δ):	1.36e-03 (9.29e-05)
Aircraft manufactured 1978-84 (ϕ_1):	0.0114 (0.0676)	Aircraft manufactured 1978-84 (ϕ_1):	0.0*	Aircraft manufactured 1978-84 (ϕ_1):	0.5145 (0.0096)
Aircraft manufactured 1985-91 (ϕ_2):	0.2957 (0.0708)	Aircraft manufactured 1985-91 (ϕ_2):	0.0*	Aircraft manufactured 1985-91 (ϕ_2):	0.9790 (0.0232)
Aircraft manufactured 1992-97 (ϕ_3):	0.4589 (0.0191)	Aircraft manufactured 1992-97 (ϕ_3):	0.0*	Aircraft manufactured 1992-97 (ϕ_3):	1.6952 (0.2239)
Boeing 747 (γ_1):	2.7624 (0.0912)	Boeing 747 (γ_1):	5.5771 (0.085)	Boeing 747 (γ_1):	3.2920 (0.0215)
Boeing 767 (γ_2):	2.5976 (0.2002)	Boeing 767 (γ_2):	2.1588 (0.2018)	Boeing 767 (γ_2):	0.7262 (0.0278)
Airbus 300/310 (γ_3):	2.4279 (0.0789)	Airbus 300/310 (γ_3):	4.9917 (0.0604)	Airbus 300/310 (γ_3):	6.3361 (0.2986)
Douglas DC-10 (γ_4):	3.1661 (0.0515)	Douglas DC-10 (γ_4):	5.1058 (0.2326)	Douglas DC-10 (γ_4):	3.4345 (0.0857)
Douglas MD-11 (γ_5):	2.5103 (0.1601)	Douglas MD-11 (γ_5):	3.7742 (0.1732)	Douglas MD-11 (γ_5):	3.4929 (0.2267)
Lockheed L-1011 (γ_6):	0.9050 (0.0111)	Lockheed L-1011 (γ_6):	4.5240 (0.5171)	Lockheed L-1011 (γ_6):	2.9021 (0.0299)

Note: Standard errors are in parentheses.
* These parameters were not estimated.

value of the average used car in their sample). The difference is undoubtedly a result of the fact that in a given year approximately 19% of used cars are sold, while only approximately 5% of used aircraft are sold. There are practical reasons to believe that transactions costs of acquiring wide-body aircraft are larger than those associated with cars. Airline pilots must be certified on each type of aircraft they wish to fly, and therefore the actual and opportunity costs of retraining and/or recertifying pilots are significant. However, the definition of transactions as changes in ownership that last a year is likely also partially responsible for the large estimate of transaction costs.

Adjustment costs, c, multiply the sum of squared differences from one period to the next in the number of aircraft of each type an airline owns. The point estimate of c is small relative to the average market value of a wide-body aircraft. However, since c enters an airline's profit flow quadratically, the incremental cost adding or subtracting aircraft increases with the magnitude of changes to airline fleets. For example, if the marginal value to an airline of the addition of a single aircraft is \$100 million, the marginal value of the tenth such aircraft is \$76 million after adjustment costs are incurred.

Variation in airlines' scale of operation and relative preference for different types of aircraft likely reflect differences in profit-maximizing strategies across airlines in the past, present, and future. For example, some US airlines have established most of their routes to airports in Europe with strict noise and emissions standards, while others have focused on establishing routes to airports in Asia or the Middle East where noise and emissions standards are less strict. In addition, some airlines satisfy a certain demand by offering more relatively low-passenger routes, while other airlines satisfy the same demand with fewer high-passenger routes. Different route structures require different aircraft. Differences across airlines in strategy are captured by differences in parameter estimates across the three groups of airlines considered in this work (large US airlines, small US airlines, and 'other' airlines). The large US scheduled airlines are Delta, United, and American. The small US passenger airlines are TWA, Eastern, Northwest, Continental, Piedmont, and US Air. In equation (17), the interaction $\alpha_i t$ multiplies the total number of aircraft an airline operates in the current period. Therefore, the marginal value of each aircraft an airline operates is increased by $\alpha_i t$ at time t. Referring to Table IV, the estimates of 0.5725, 0.0003, and 0.3574 imply that, *ceteris paribus*, the average marginal contribution of aircraft increased approximately 57% a year for large US airlines, not at all for small US airlines, and approximately 36% a year for 'other' airlines. The effect of the α parameters may be mitigated by other factors, such as scale effects, but in general the estimates of α reflect the different growth rates of different types of airlines.

The parameter δ_i, which captures the rate of decreasing returns for each of the different types of airlines, enters the airlines' profit flow equation negatively and multiplies the square of the total number of aircraft the airline owns. The parameter estimates indicate that if aircraft were added one at a time to a large US airline's fleet, each successive aircraft is worth approximately 7% less than the previously added aircraft. The decline in the marginal value of aircraft is more severe for small US airlines at approximately 11%.

Parameters γ_1 through γ_3 capture each airline's propensity to operate different vintages of aircraft. Aircraft manufactured prior to 1978 are the comparison group. These parameters were set equal to 0 for small US airlines. (Attempts to estimate the parameters yielded parameter estimates that had extremely large standard errors, incorrect signs, and were non-monotonic. This is not surprising given that small US airlines purchased very few new aircraft during the sample period and kept many of the oldest generation of aircraft for many years.) As expected, parameter estimates indicate that newer aircraft are significantly more valuable than comparably equipped older aircraft to large US airlines. For example, if the marginal value of an aircraft in the oldest category is \$25 million to a large US airline, then *ceteris paribus* the marginal values of a comparably equipped aircraft manufactured from 1978 to 1984, from 1985 to 1991, and from 1992 to 1997 are \$25.2 million, \$33.6 million, and \$39.6 million, respectively. Even

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more striking is the premium airlines in other parts of the world place on newer aircraft. If the marginal value of an aircraft to 'other' airlines in the oldest category is \$25 million, then *ceteris paribus* the marginal values of a comparably equipped aircraft manufactured from 1978 to 1984, from 1985 to 1991, and from 1992 to 1997 are \$41.8 million, \$66.5 million, and \$136.2 million, respectively. The relatively high value 'other' airlines place on 'newness' may reflect the facts that (1) US airlines were the first to operate wide-body aircraft and therefore have retained more older aircraft, (2) European airlines are generally subject to more restrictive emission and noise restrictions, and (3) many airlines throughout the world are state-owned and therefore may not be strictly profit-maximizing.

Parameters γ_4 through γ_9 measure airlines' preference for different models of aircraft. Different types of airlines offer routes that are most efficiently operated by different models of aircraft. Comparing increases in marginal values for the same model of aircraft across the different types of airlines is difficult due to airlines' varying growth trends, scale effects, and preferences for different vintages of aircraft. However, it is easy to see that the parameter estimates are broadly consistent with the relative popularity of each model of aircraft to each type of airline. For example, the fact that the Boeing 747 aircraft have been owned and operated with great frequency worldwide is consistent with all airlines' having a relatively high mean valuation of these aircraft. Parameter estimates indicate that large US airlines receive a relatively high mean marginal value from operating Boeing 767 and McDonnell-Douglas aircraft, while small US airlines receive relatively high returns from DC-10, Airbus, and Lockheed L-1011 aircraft. The Douglas DC-10 and the Lockheed L-1011 have similar technical specifications, but parameter estimates indicate that the DC-10 was much more popular among the big three US carriers, Delta, United, and American, while the L-1011 was popular mainly among the small US carriers. The fact that large US and international carriers have grown at a faster rate than have small US carriers is consistent with the eventual demise of the Lockheed L-1011. Finally, 'other' airlines, many of whom are in Europe, receive relatively high returns from Airbus aircraft.

10.2. Model Fit

In this subsection, I discuss the fit of the equilibrium quantities generated by the model to those observed in the data. Although the null hypothesis that the equilibrium quantities generated by the structural model are equal to the quantities observed in the data is mostly rejected by statistical tests,[11] Figures 4 and 5 show that the quantities predicted by the model are qualitatively similar to the quantities observed in the data. Figure 4 shows the predicted and observed values of the total quantity of aircraft operated by each type of airline in each period. Because the trend in airline fleet growth is assumed to be linear, the model fails to capture sharp increases and decreases from year to year in the size of airline fleets. The model predicts a nearly flat growth trend for small US airlines. Therefore the model over-predicts the number of aircraft owned by small US airlines in the first few and last few periods of the sample, while the model under-predicts the number of aircraft owned by small US airlines in the middle years. In contrast, the growth trends of large US and 'other' airlines are smoother, and therefore the model fits the quantity choices of these airlines better.

[11] I performed chi-square goodness-of-fit tests comparing equilibrium aircraft quantities predicted by the model to those observed in the data. The model was rejected by the model in pairwise comparisons in approximately 85% of the tests.

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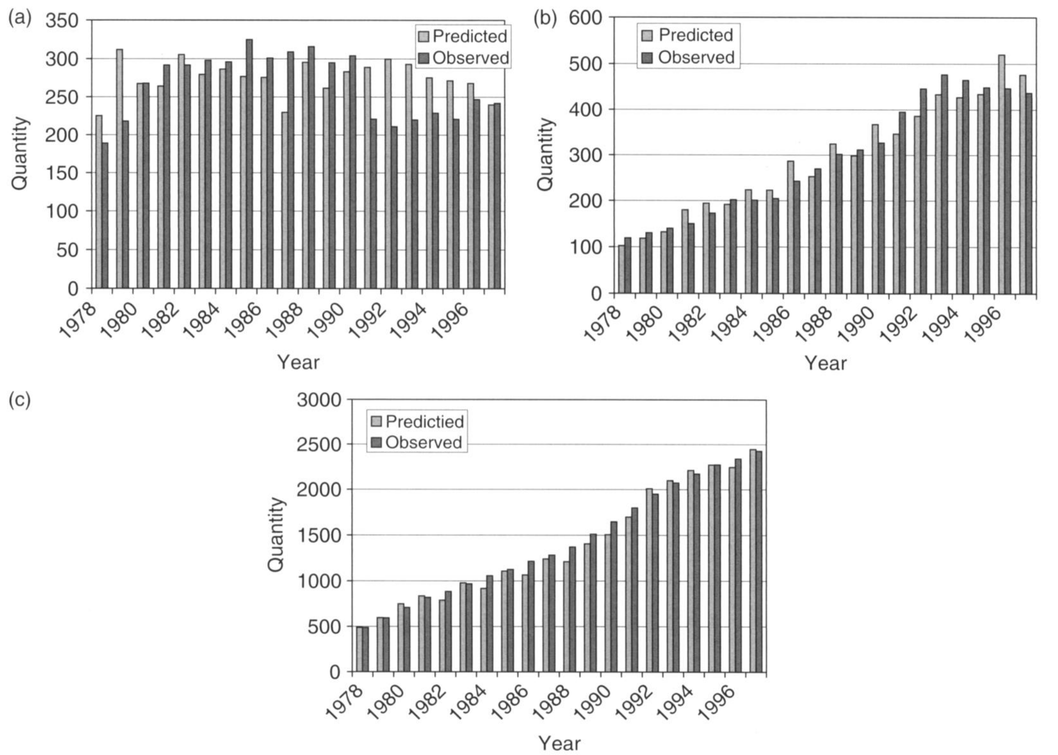

Figure 4. Quantity of wide-body jets owned by small US airlines. This figure is available in color online at wileyonlinelibrary.com/journal/jae

Since the model assumes that airline preferences for different aircraft are independent of airline growth trends, the model does not do a good job of matching increases in ownership for certain types of aircraft when they differ significantly from the overall growth in airline fleets. For example, Figure 5 shows the predicted and observed quantities owned in each time period of Boeing 747 aircraft. Although the model accurately replicates different types of airlines' mean preferences for 747 aircraft, the model does not predict changes in 747 aircraft ownership over time by either large or small US airlines particularly well. However, since the growth in 747 ownership is similar to their growth in total wide-body aircraft ownership, the model nearly matches the 747s owned by 'other' airlines.

11. COUNTERFACTUAL EXPERIMENTS

11.1. Tax Reform

From 1978 to 1986 airlines received a tax credit on purchases of new aircraft equal to 10% of the aircraft's purchase price. The investment tax credit was designed to stimulate investment in new durable assets. Additionally, since new and used aircraft are substitutes, one would expect a tax credit on new aircraft to cause used aircraft to be sold in used markets earlier in their life cycle.

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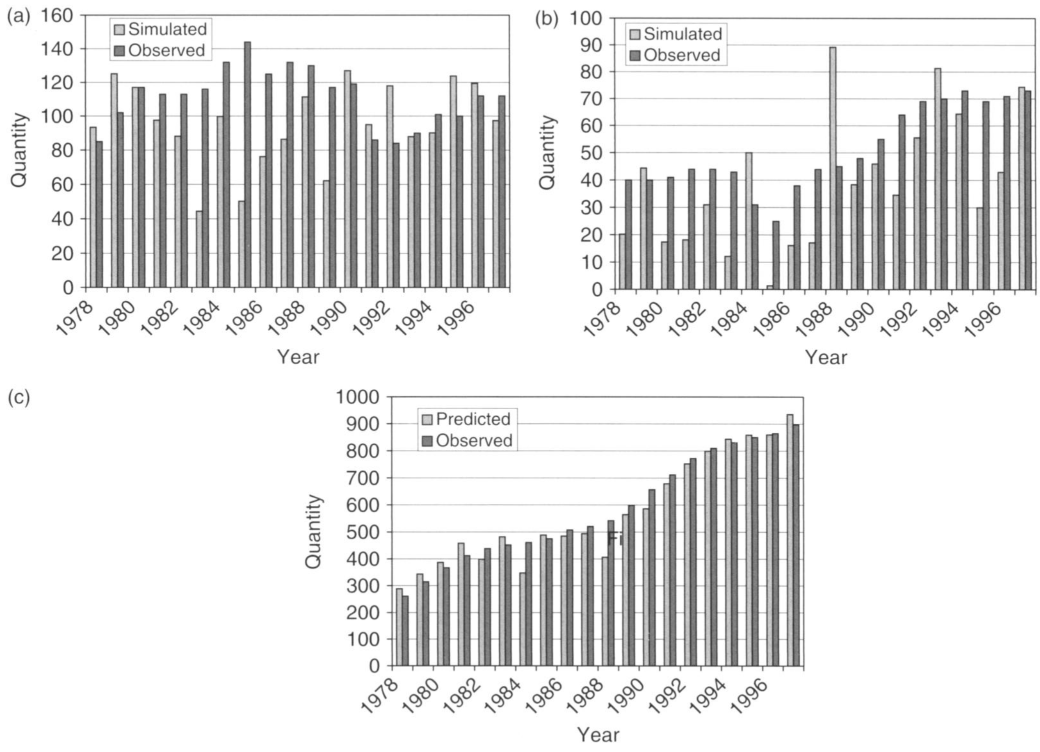

Figure 5. Quantity of Boeing 747 Aircraft owned by: (a) small US airlines; (b) large US airlines; (c) 'other' airlines. This figure is available in color online at wileyonlinelibrary.com/journal/jae

To assess the effects of instituting an investment tax credit similar to the one repealed by the Tax Reform Act of 1986, I use the estimated structural model to compare airline fleets five periods into the future with and without a 10% tax credit on purchases of new aircraft by US airlines.

In the description of the model, entry of new aircraft occurs according to an exogenous process that is known to airlines. Given a fixed number of aircraft, the efficient mechanism described above finds the minimum uniform price for each type (age/vintage combination) of aircraft such that the equilibrium conditions given by Definition 1 are satisfied. The mapping between aircraft prices and quantities determined by the allocation mechanism is one to one. Therefore, the same equilibrium can be generated by setting the prices of new aircraft and using the allocation mechanism to determine the location of all new and used aircraft, as well as the prices of used aircraft. This 'pinning down' of new aircraft prices rather than new aircraft quantities lends itself nicely to the policy experiments described here. For example, to evaluate the effect of an investment tax credit on aircraft ownership, I set 1998–2002 new aircraft prices equal to those observed in the data, and solve for equilibrium used aircraft prices, and new and used aircraft quantities. Then, I decrease the price of new aircraft 10%, and resolve for equilibrium.

Obviously, this exercise is not equivalent to implementing an investment tax credit, as the exercise imposes a 10% decrease in the price of new aircraft, while depending on the shape of

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Table V. Predicted quantities of aircraft owned by airlines 1998–2002 with and without a 10% investment tax credit

	Year	Year of manufacture					Total
		1969–1977	1978–1984	1985–1991	1992–1997	1998–2002	
Small US airlines							
No policy	1998	26.6	22.6	20.8	38	81.2	189.2
	1999	17.8	30.4	15	29.2	69.8	162.2
	2000	13.4	28	11.6	22.2	80.8	156
	2001	13.8	24	8.6	23.6	74	144
	2002	7.8	7.6	1.0	33.4	80.8	130.6
10% tax credit	1998	24	19.2	20.6	38	85.8	187.6
	1999	21	29.6	15.2	26.8	69.4	162
	2000	10.8	27.6	7.4	19	99	163.8
	2001	12	24	14.2	21.4	87.8	159.4
	2002	11	14.2	0.8	35	81.2	142.2
Large US airlines							
No policy	1998	7.8	34.4	37.4	268.8	76.4	424.8
	1999	13.4	14.6	46.6	260.4	103	438
	2000	15	15.8	61.4	263.8	93.4	449.4
	2001	27.4	15.6	51	251.2	117	462.2
	2002	11.8	38	47.2	237.2	123	457.2
10% tax credit	1998	9.6	38.2	34.8	265.6	74.2	422.4
	1999	9.2	13.8	36.6	259.8	114.4	433.8
	2000	14.4	13.6	51	255.6	116.2	450.8
	2001	16.6	17.2	35.4	250	151.4	470.6
	2002	8	39.8	39.6	212.4	160	459.8
'Other' airlines							
No policy	1998	617.4	223.8	130.4	380.8	287.8	1640.2
	1999	618	232.2	123.8	386	439.8	1799.8
	2000	620	231.4	94.8	378.6	551	1875.8
	2001	606	234.4	106	353.4	674.6	1974.4
	2002	626.4	226.8	113.6	352.6	767.6	2087
10% tax credit	1998	617	221.8	127.2	375.4	288.8	1630.2
	1999	618	232.8	128.2	383	445.6	1807.6
	2000	621.8	233	103.6	375	554	1887.4
	2001	617.4	232.4	111	331	711.4	2003.2
	2002	626.6	218.6	116	350.4	824	2135.6

demand, the market power of aircraft manufacturers, etc., the equilibrium price of new aircraft after the implementation of a 10% investment tax credit may decrease the equilibrium price of new aircraft by more or less than 10%. However, if the market for new commercial aircraft is relatively competitive (due to competition among aircraft manufacturers and competition from used aircraft markets), and manufacturers marginal cost curves are nearly flat, the exercise approximates the implementation of a 10% investment tax credit well.

Table V displays the simulated number of aircraft owned from 1998 through 2002, disaggregated by vintage, with and without a 10% investment tax credit.[12] As expected, the tax credit induces

[12] As a side note, I mentioned in footnote 3 that very few wide-body aircraft had been taken out of service by the end of 1997 (fewer than 50). However, experts at Back Aviation Solutions predict that wide-body aircraft will start to leave commercial service when they are 25–35 years old. Therefore, several wide-body aircraft should be nearing retirement

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small increases in total aircraft ownership and relatively larger increases in ownership of the newest vintage of aircraft by US airlines. At the end of 5 years of the policy, the predicted ownership of the newest aircraft by large US airlines is approximately 20% higher than when there is no tax credit, and small US airlines increase ownership of the newest vintage of aircraft by approximately 9%. Interestingly, 'Other' airlines also increase their ownership of the newest vintage of aircraft by almost 4% when the policy is implemented, even though they did not receive the tax credit. This a good illustration of the second-order equilibrium effects captured by the dynamic equilibrium model. Specifically, an increase in demand for the newest vintage of aircraft in the market reduces their future prices in secondary markets, inducing 'Other' airlines to increase their consumption of these newer aircraft. US airlines substitute away from ownership of older aircraft when the tax credit is implemented to buy more new aircraft. In the first few periods of the policy 'Other' airlines equal or increase their consumption of used aircraft of all vintages. In the later periods of the policy, when second-order effects have had a chance to take effect, 'Other' airlines begin to decrease their holdings of most older vintages of aircraft, while continuing to increase purchases of new aircraft.

11.2. Facilitating Used Aircraft Sales

Explicitly modeling the interaction between new and used markets for wide-body aircraft allows me to evaluate the extent to which programs that facilitate the sale of used aircraft are likely to stimulate demand for new aircraft. In this example, I simulate the effect on new and used aircraft demand of cutting the transaction costs associated with purchasing used aircraft in half. The reduction of transaction costs may be thought of as, for example, the government (or aircraft manufacturers) brokering used aircraft sales, gathering and providing information about used aircraft to potential buyers, and/or providing training and certification to pilots for different types of used aircraft. Again, because I am interested in how the demand for new and used aircraft changes when transaction costs are reduced, as was done in the investment tax credit experiment, I fix new aircraft prices and solve for equilibrium used aircraft prices, and new and used aircraft quantities with and without the reduction in transaction costs.

Table VI displays the simulated number of aircraft owned with and without the reduction in transaction costs for used aircraft. The table shows that reducing frictions in the used market does in fact increase demand in the new market. The program induces a shift of older aircraft from 'Other' airlines to US airlines, which allows 'Other' airlines to own more of the newest vintage of aircraft, which they have the greatest preference for. After 5 years of the program demand for the newest vintage of aircraft by US airlines is virtually the same as it would have been in the absence of the program, while demand for new aircraft by 'Other' airlines has increased nearly 10%. Depending on the cost of facilitating used aircraft markets, such a program is economically appealing since it not only stimulates new aircraft demand but also facilitates the movement of used aircraft to their most efficient locations.

The previous literature has found mixed results as to whether a monopolist prefers a more efficient secondary market. In particular, Anderson and Ginsburgh (1994) find that a monopolist may want to reduce transaction costs locally, but if the monopolist can increase transaction costs enough to completely shut down used markets, she would prefer to do so. So perhaps increasing

age soon after 1997. Interestingly, the structural model predicts that nearly 100 wide-body aircraft will be scrapped 1998–2002.

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Table VI. Predicted quantities of aircraft owned by airlines 1998–2002: decreased transaction costs

	Year	Year of manufacture					Total
		1969–1977	1978–1984	1985–1991	1992–1997	1998–2002	
Small US airlines							
No policy	1998	26.6	22.6	20.8	38	81.2	189.2
	1999	17.8	30.4	15	29.2	69.8	162.2
	2000	13.4	28	11.6	22.2	80.8	156
	2001	13.8	24	8.6	23.6	74	144
	2002	7.8	7.6	1.0	33.4	80.8	130.6
Reduced transaction costs	1998	16	23.2	22	39	78.8	179
	1999	21.2	29.4	16.6	42.2	60.2	169.6
	2000	13	31.8	10.8	30.2	91.6	177.4
	2001	14.2	33.2	15.4	28	83.8	174.6
	2002	9.4	20.6	2.2	36.6	85.2	154
Large US airlines							
No policy	1998	7.8	34.4	37.4	268.8	76.4	424.8
	1999	13.4	14.6	46.6	260.4	103	438
	2000	15	15.8	61.4	263.8	93.4	449.4
	2001	27.4	15.6	51	251.2	117	462.2
	2002	11.8	38	47.2	237.2	123	457.2
Reduced transaction costs	1998	15.4	52	37.4	249.4	70	424.2
	1999	11.6	28.8	49.2	242.8	110.4	442.8
	2000	15.4	25.2	59.6	257	104.8	462
	2001	27	24.6	46.4	240	138	476
	2002	11.4	57	44.2	239.4	118	470
'Other' airlines							
No policy	1998	617.4	223.8	130.4	380.8	287.8	1640.2
	1999	618	232.2	123.8	386	439.8	1799.8
	2000	620	231.4	94.8	378.6	551	1875.8
	2001	606	234.4	106	353.4	674.6	1974.4
	2002	626.4	226.8	113.6	352.6	767.6	2087
Reduced transaction costs	1998	613.2	228.6	106.4	403.8	291.2	1646.8
	1999	609.6	243.2	102.6	394.4	450.8	1800.6
	2000	613	242	80.4	378.2	562.6	1876.2
	2001	599.6	240	87.6	353	705.8	1986
	2002	618.8	219.2	98.4	337.8	837.2	2111.4

transaction costs an extreme amount would increase the future demand for new aircraft even more than is observed in Table VI. (However, Anderson and Ginsburgh's results depend on a steady state where as many aircraft are being scrapped as are being purchased in each period, a steady state that the commercial aircraft market had not reached by the end of the sample period.) The results of this counterfactual experiment indicate that the manufactures would prefer more efficient secondary markets for used aircraft. This result is driven by the fact that airlines are forward-looking, and the expectation that a used aircraft will be more easily sold in the future increases the value of new aircraft today.

12. CONCLUSIONS

This paper is the first to estimate a fully integrated equilibrium model of primary and secondary markets for new and used durable goods. I develop a dynamic model of multi-unit ownership

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decisions by airlines to study the interaction between primary and secondary market equilibria for new and used wide-body aircraft.

Estimates of the structural model indicate that there are significant market frictions that induce airlines to keep aircraft for several years. In addition, significant differences in airline route development strategies across airlines are reflected by heterogeneity in airline preferences for the size and composition of their airline fleets.

The results of two counterfactual experiments illustrate the influence of equilibria in used markets on new markets and vice versa. A 10% tax credit to US airlines on the purchase of new wide-body aircraft induces an increase in demand for new aircraft by US airlines, as well as by 'Other' airlines that are *not* directly impacted by the tax credit. And a policy which reduces transactions costs in markets for used wide-body aircraft can also be used to stimulate demand for new wide-body aircraft.

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REFERENCES

Akerlof G. 1970. The market of 'lemons': quality uncertainty and the market mechanism. *Quarterly Journal of Economics* **84**: 488–500.

Anderson SP, Ginsburgh VA. 1994. Price discrimination via second-hand markets. *European Economic Review* **38**: 23–44.

Benkard CL. 2004. A dynamic analysis of the market for widebodied commercial aircraft. *Review of Economic Studies* **71**: 581–611.

Berkovec J. 1985. New car sales and used car stocks: a model of the automobile market. *RAND Journal of Economics* **16**: 195–214.

Berry S, Levinsohn J, Pakes A. 1995. Automobile prices in a market equilibrium. *Econometrica* **63**: 841–890.

Bertsekas DP, Castanon DA, Tsaknakis H. 1993. Reverse auction and the solution of asymmetric assignment problems. *SIAM Journal on Optimization* **3**: 268–299.

Borsch-Supan A, Hajivassiliou V. 1993. Smooth unbiased multivariate probability simulators for maximum simulated likelihood estimation of limited dependent variable models. *Journal of Econometrics* **58**: 347–368.

Brien M, Lillard L, Stern S. 2006. Cohabitation, marriage and divorce in a model of match quality. *International Economic Review* **47**: 451–494.

DOI: 10.1002/jae

Carranza JE. 2006. *Estimation of demand for differentiated durable goods*. Working paper, Department of Economics, University of Wisconsin.
Chen J, Esteban S, Shum M. 2008. *How much competition is a secondary market?* Working paper, Division of Humanities and Social Sciences, California Institute of Technology.
Cho S. 2004. *An empirical model of mainframe computer investment*. Mimeo, Department of Economics, Yale University.
Engers M, Hartmann M, Stern S. 2009. Are lemons really hot potatoes? *International Journal of Industrial Organization* **27**: 250–263.
Esteban S, Shum M. 2007. Measuring market power in durable goods markets: the case of sports cars. *RAND Journal of Econometrics* **38**: 332–354.
Geweke J. 1991. Efficient simulation from the multivariate normal and Student-*t* distributions subject to linear constraints. In *Computer Science and Statistics: Proceedings of the 23rd Symposium on the Interface* American Statistical Association, Alexandria, VA; 571–588.
Goldberg PK. 1995. Product differentiation and oligopoly in international markets: the case of the U.S. automobile industry. *Econometrica* **63**: 891–951.
Gowrisankaran G, Rysman M. 2009. *Dynamics of consumer demand for new durable goods*. Working paper, Department of Economics, Boston University.
Hajivassiliou V, McFadden D. 1990. *The method of simulated scores for the estimation of LDV models with an application to external debt crises*. Cowles Foundation Discussion Paper 967, Yale University.
Hendel I. 1999. Estimating multiple-discrete choice models: an application to computerization returns. *Review of Economic Studies* **66**: 423–446.
Hendel I, Lizzeri A. 1999. Adverse selection in durable goods markets. *American Economic Review* **89**: 1097–1115.
Keane M. 1994. A computationally practical simulation estimator for panel data. *Econometrica* **62**: 95–116.
Kennet DM. 1994. Structural model of aircraft engine maintenance. *Journal of Applied Econometrics* **9**: 351–368.
Manski C. 1983. Analysis of equilibrium automobile holdings in Israel with aggregate discrete choice models. *Transportation Research* **17B**: 373–389.
Pakes A, McGuire P. 2001. Stochastic approximation for dynamic models, symmetric Markov Perfect Equilibrium, and the 'curse' of dimensionality. *Econometrica* **69**: 1261–1281.
Petrin A. 2002. Quantifying the benefits of new products: the case of the minivan. *Journal of Political Economy* **110**: 705–729.
Porter R, Sattler P. 1999. *Patterns of trade in the market for used durables: theory and evidence*. NBER Working Paper No. 7149.
Rust J. 1985. Stationary equilibrium in a market for durable assets. *Econometrica* **53**: 783–805.
Rust J. 1986. When is it optimal to kill off the market for used durable goods? *Econometrica* **54**: 65–86.
Rust J. 1987. Optimal replacement of GMC bus engines: an empirical model of Harold Zurcher. *Econometrica* **55**: 999–1033.
Rust J. 1997. Using randomization to break the curse of dimensionality. *Econometrica* **65**: 487–516.
Smith LK. 2005. *A structural dynamic equilibrium model of the market for new and used commercial aircraft*. Dissertation, University of Virginia.
Smith LK. 2009. New market policy effects on used markets: theory and evidence. *BE Journal of Economic Analysis and Policy* **9**(1): Article 32.
Stern S. 1994. Two dynamic discrete choice estimation problems and simulation method solutions. *Review of Economics and Statistics* **76**: 695–702.
Swan P. 1970. Durability of consumption goods. *American Economic Review* **60**: 884–894.
Swan P. 1971. The durability of goods and regulation of monopoly. *Bell Journal of Economics* **2**: 347–357.
Waldman M. 1996. Durable goods pricing when quality matters. *Journal of Business* **69**: 489–510.
Wicksell K. 1934. *Lectures on Political Economy*, Vol. 1. Routledge & Kegan Paul: London.
Wykoff FC. 1973. A user cost approach to new automobile purchases. *Review of Economic Studies* **40**: 377–390.

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