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Author(s): Füsun Gönül and Meng Ze Shi
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Optimal Mailing of Catalogs: A New Methodology Using Estimable Structural Dynamic Programming Models

Füsun Gönül • Meng Ze Shi
Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213-3890

We investigate the key determinants of the optimal direct mail policy in a dynamic environment where customers maximize utility and the direct mailer maximizes profits. We measure the sensitivity of the customers to receiving a catalog in the mail, while controlling for customer characteristics such as elapsed time in responses and number of purchases. We apply our model to a database from a national cataloger that markets nonseasonal products. We summarize the results of our model that are valid for these types of products. We find that the dynamic model significantly outperforms its single-period counterpart. We find that it is not optimal to mail to individuals at low recency levels because they are likely to buy anyway. It is better to save the mailing dollars for customers at higher recency levels. We find that it is optimal to mail to customers who have purchased only a small or a medium number of times to induce them to continue to buy from this catalog and not switch to others. It is not necessary to mail often to customers who have purchased many times before from the company unless they have high recency values. We find that under the optimal mailing policy the cataloguer enjoys higher profits than under the current mailing policy.
(*Structural Models; Principle of Optimality; Catalogs; Mailing Policy*)

1. Introduction

According to a recent survey by *Catalog Age* one of the most critical concerns of the direct mail industry executives is how to tailor the mailing policy in order to save distribution costs and to increase response rates (Hayes 1992). Currently most direct mail organizations send mail periodically, a pattern adopted from magazines and newspapers. Such a uniform mailing pattern creates unwanted mail, wastes resources, and results in misdirected and ineffective marketing effort. We develop a model and investigate how the mailing policy can be improved by using feedback from customer responses. Our first contribution to the literature is the development, estimation, and testing of an *analytical* model that deals with mailing policies. The second contribution of our paper is to model *customers' purchase behavior* as a function of the current and future mailing decisions in addition to elapsed time since the last response and number of purchases. By solving the model we arrive at a new policy that is both utility-maximizing for the customer and profit-maximizing for the firm over an infinite time horizon.

The estimation technique we introduce is within the framework of estimable structural dynamic programming models that have been introduced to econometrics in the last decade. Although it is beyond the scope of our paper a good survey on estimable structural models can be found in Rust (1994). More recently, the method has been applied in marketing by Gonul and Srinivasan (1996). To the best of our knowledge, the existing estimable structural models examine optimization from the customer's side alone. We model the firm's side as well and integrate the customer and the company in a stochastic framework.

Our work is related to two earlier studies in the direct mail literature: Bult and Wansbeek's (1995) response

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model and Bitran and Mondschein's (1996) optimal mailing of catalogs model. Bult and Wansbeek (1995) determine an optimal mailing list by estimating a binary consumer choice model (yes / no responses to direct mail). From the estimates they obtain the optimal proportion of households that yields maximum profit in a single-period environment. A dynamic environment for the firm is adopted in Bitran and Mondschein (1996) who derive heuristics for a cataloger for when to mail and how often to mail. However, they do not explicitly model the customers' response behavior.

In our model we assume a dynamic environment for both the firm and the customer, estimate customer responses, and obtain the optimal mailing policy for different segments of the customer base. We apply our model to a data set from a national cataloger that sells household items. (The nature of the products and the identity of the cataloger are kept confidential at the request of the company.) Although there are similar methods in the literature to compute the mailing decision, our study mainly differs by modeling customers who *maximize their utility functions* over a *time horizon*.

A single-period model where customers do not optimize over the planning horizon is sharply rejected in favor of our dynamic model. The test result is consistent with the conventional wisdom that a dynamic choice model performs better than a single-period one. The direct marketers also benefit from a long-term approach over a short-term approach. Since response rates are typically low and initial losses are relatively high, benefits accrue only after long periods of time for direct marketers (Direct Marketing White Paper 1990).

In the rest of the paper, §2 presents the estimable dynamic programming model, the algorithm to solve for the optimal mailing policy, and the generalizability of the methodology to other applications. Section 3 describes the data used in this application and presents the estimation results. Section 4 discusses the implications of the results and §5 concludes with a summary and extensions for future research.

2. The Model

2.1. The Repeated Interaction Between the Cataloguer and Customers

In each time period the direct mailer decides whether or not to send a catalog to each of its customers on the house list. In each time period a customer decides whether or not to respond in the form of a purchase. The direct mailing and the customer response interaction is repeated every period in this manner. (The figure in Appendix A illustrates.)

The two catalog companies we dealt with in the process of this research market nonseasonal products. Since the items in the catalog are nonseasonal, the catalogs mailed out in different periods are similar in content. The executives from both companies state that only the cover of the catalog is changed from one mailing to the next.[1] Therefore, in our model we treat catalogs as alike. In §5 we discuss how the model can be extended to encompass variation across mailings in the case of seasonal products.

Although a new catalog may serve as a reminder advertisement and boost sales, it is not absolutely necessary in generating an order. Customers may order from the previous catalogs without having received the latest catalog. Our models allow for a nonzero probability of buying without the current catalog. Although the customer *knows how the firm makes the mailing decision* and the contents of the catalog, upon receiving a catalog his or her response rate is still expected to be boosted. In this sense our context is similar to the prior literature on the optimal mailing of catalogs where the products do not change seasonally (Bult and Wansbeek 1995, Bitran and Mondschein 1996).

The direct mailer's objective is to maximize its expected lifetime profits from each customer. The direct mailer is assumed to know how each customer responds to the mail except a random component in the utility function. The customer's objective is to maximize long-term utility while making purchase and nonpurchase decisions in each period. When a customer makes a decision she or he not only considers the current utility of buying and not buying, but also the impact of her or his current action on the future decisions. The customer is assumed to know how the direct mailer makes the mailing decision. Specifically, we assume that the customer knows the cost of mail and the direct mailer's

[1] The vice president of marketing of the catalog company we obtained the data from assured us that the contents do not vary substantially from one mailing to the next. We obtained a few back issues of the catalog to verify this for ourselves.

decision rule in a simplified form. Since the customers know that their current responses will affect the direct marketer's future mailing policy towards them, they behave optimally.

2.2. An Estimable Structural Dynamic Programming Model

The objective function of the customer is the expected present discounted utility over a time horizon:

$$E\left(\sum_{t=1}^{\infty} \delta_c^{t-1} u_{it} d_{it}\right) \tag{1}$$

where δ_c is the customer's discount factor, u_{it} is utility from purchase at period t, d_{it} is a binary variable denoting response.[2] Time period is a month to suit the nature of catalog mailings. Since a life time (measured in months) is quite long we treat the horizon as infinite. The time subscript (t) depends on the customer, and hence to be precise it should be t_i. However, since we present the model at the individual customer level we suppress the individual subscript i on t_i for brevity.

The response variable for customer i is defined as follows:

$$d_{it} = \begin{cases} 1 & \textit{if individual } i \textit{ makes a purchase at time } t, \\ 0 & \textit{otherwise.} \end{cases} \tag{2}$$

Before we present the specific structure of our model we discuss the status quo in the industry regarding the mailing decision to different segments of the customer base. We have gathered this information from three sources: (1) Direct marketing textbooks (Roberts and Berger 1989, Stone 1996); (2) Direct Marketing Educational Foundation seminars for professors; and (3) extensive conversations with the executives of two catalog companies, one of which gave us the data.

The scenario in a typical direct mail firm is as follows: Research analysts develop a customer response model. For example, they estimate a multiple regression or a logit/probit equation where the left-hand side is a discrete dependent variable for purchase/nonpurchase (such as d_{it}). The independent variables are typically composed of recency, frequency, and monetary value of the purchase amount (RFM).[3]

Upon estimation of the customer response model an expected profit function is set up where predicted purchase probability determines expected revenue. The direct marketing firm sends catalogs to customers for whom the expected profit is positive. If the company has to heed a mailing budget constraint then not every customer with positive expected profit can be sent mail. A decision is made on an appropriate threshold in order to stay within the budget, and only those customers with sufficiently large expected response rates are included in the next mailing list.

In our model we also define the state variables as recency (r_{it}), which is elapsed time since individual i's last purchase, and frequency (f_{it}), which is the number of times the individual has purchased from this catalog until period t. We discuss the inclusion of monetary value as an extension of our model in Appendix B.

Recency and frequency for individual i evolve according to a Markov transition process. Recency reduces to one upon response ($d_{it} = 1$), and increases by one otherwise ($d_{it} = 0$):

$$r_{i,t+1} = \begin{cases} 1 & \textit{if } d_{it} = 1, \\ r_{it} + 1 & \textit{if } d_{it} = 0. \end{cases} \tag{3a}$$

Frequency evolves as

$$f_{i,t+1} = \begin{cases} f_{it} + 1 & \textit{if } d_{it} = 1, \\ f_{it} & \textit{if } d_{it} = 0. \end{cases} \tag{3b}$$

For a customer buying for the first-time from this catalog, we set the recency and frequency to one. Henceforth, we collect the two state variables under the notation $S_{it} = \{r_{it}, f_{it}\}$ which stands for the state space. Figure 1 shows the state space diagram.

The mailing decision to individual i in period t is a binary variable. That is,

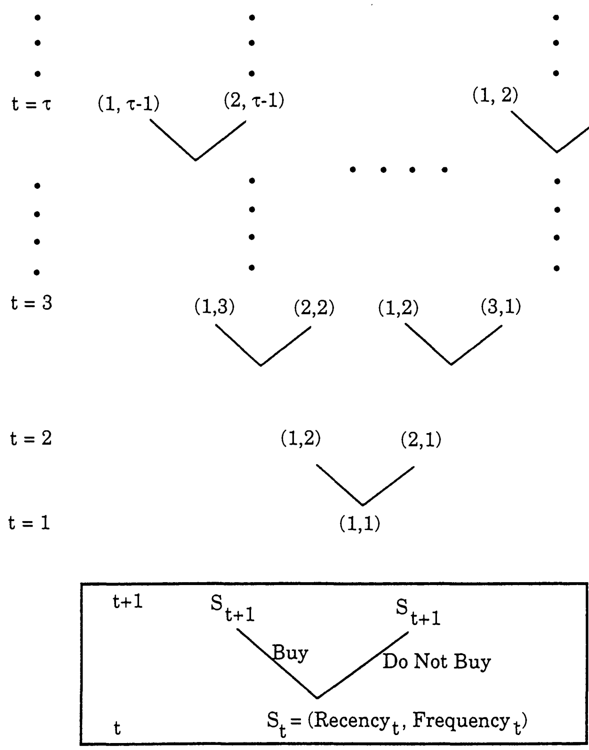
$$m_{it} = \begin{cases} 1 & \textit{if a catalog is mailed to customer } i \\ & \quad \textit{in period } t, \\ 0 & \textit{otherwise.} \end{cases} \tag{4}$$

[2] The discount factor is $1/(1 + \text{discount rate})$.

[3] Recency stands for elapsed time since the last purchase, frequency refers to the number of purchases in the past or proportion of purchases over a period of time, and monetary value is the amount spent so far or average amount per purchase so far.

Figure 1 The State Space Diagram

Note: Recency (r_t) and frequency (f_t) pairs achieve $(t-1)t/2+1$ distinct levels for each $t = 1 \cdots, T$. The state space elements satisfy the following condition: either ($r_t + f_t \le t+1$ and $f_t > 1$ and $r_t < t$) or ($r_t = t$ and $f_t = 1$).



The mailing variable links the customer's and the firm's optimization problems. We model the utility function to depend on whether or not a mail offer is received, recency, and frequency. That is,

$$u_{it} = \alpha + \beta_m m_{it} + \beta_{1r} r_{it} + \beta_{2r} r_{it}^2 + \beta_{1f} f_{it} + \beta_{2f} f_{it}^2 + \epsilon_{it} = \bar{u}_{it} + \epsilon_{it} \qquad (5)$$

where $\bar{u}_{it}$ represents the deterministic component of the utility function.

Since the sample consists of customers who have already bought from this particular catalog at least once, we expect them to derive positive utility from perusing the new catalog. In other words we expect the mailing coefficient to be positive. For frequently purchased products recency is expected to have a negative impact and frequency is expected to have a positive impact on the response rate. However, for other products the effects may be in the opposite direction. For example, for durable goods response probability may increase with recency (Roberts and Berger 1989). By assuming a nonlinear form we will be able to empirically determine the overall impact of both recency and frequency.

We normalize the utility of not buying from the catalog in a single period to zero. The random term (ϵ_{it}) is assumed to be from a standard normal distribution with zero mean and unit variance. The normality assumption enables closed-form solution of the expected value functions (see Equation (9) ahead). The unit variance ($\sigma_\epsilon = 1$) assumption is needed because in a two-alternative model the parameters are identified up to a scaling constant.

The maximization of equation (1) is accomplished by the choice of the optimal sequence of control variables $\{d_{it}\}$. We invoke the *principle of optimality* which states that regardless of the initial state and decision, the remaining decisions must be optimal with regard to the state resulting from the first decision (Bellman 1957). The sum of the current utility, u_{it}, and the expected discounted future value from period $t+1$ onwards, $EV_{i,t+1}(S_{i,t+1})$, yields the value function, $V_{it}(S_{it})$ in period t. That is,

$$V_{it}(S_{it}) = \begin{cases} \bar{u}_{it} + \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 1) + \epsilon_{it} & \textit{if } d_{it} = 1, \\ \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 0) & \textit{if } d_{it} = 0, \end{cases} \qquad (6)$$

where the state variables evolve according to equation (3). That is, $(S_{i,t+1} \mid d_{it} = 1)$ implies that recency is one and frequency is incremented by one from its value in the previous period, due to the purchase in the current period. Likewise $(S_{i,t+1} \mid d_{it} = 0)$ implies that recency is incremented by one from its value in the previous period and that frequency remains the same as its previous value.

We assume the customer knows the parameters of the value function and the parameters of the distribution of the random term. The customer makes a draw from the distribution in each period and decides whether or not to purchase in that period. That is, $d_{it}^* = 1$ if and only if

$$\bar{u}_{it} + \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 1) + \epsilon_{it} > \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 0)$$

where (*) denotes optimality. The researcher does not observe the random draw but can estimate the probability of buying or not buying from the catalog conditional on data. Thus, the response (purchase) probability is:

$$\begin{aligned}
&\mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it})\\
&\quad = \mathrm{Prob}(\bar{u}_{it} + \delta_c EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 1) + \epsilon_{it}\\
&\quad > \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 0))\\
&\quad = \Phi[\bar{u}_{it} + \delta_c(EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 1)\\
&\qquad - EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 0))]
\end{aligned} \tag{7}$$

where Φ is the standard normal cumulative distribution function. The non-response probability is the complement of this expression:

$$\mathrm{Prob}_{it}\,(d_{it} = 0 \mid S_{it}, m_{it}) = 1 - \mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it}).$$

The interpretation of the response probability is intuitive: If the (deterministic part of the) value function from buying exceeds the (deterministic part of the) value from not buying, then the customer is likely to purchase from the catalog. Otherwise, the customer is less likely to respond. If the two parts are equal then the customer is indifferent between responding (buying) and not responding (not buying).

A closed-form expression for the expected valuation function is necessary in order to compute the response and nonresponse probabilities. We use the rule of finding the expected value of a maximum when there are two possibilities, that states that

$$\begin{aligned}
E(\max(a, b)) &= E(a \mid a > b)\,\Pr(a > b)\\
&\quad + E(b \mid b > a)\,\Pr(b > a)
\end{aligned}$$

where a and b correspond to the values of responding and not responding as shown in Equation (6) (see, for example, Karlin and Taylor 1975, Ch. 1). Then,

$$\begin{aligned}
&EV_{it}(S_{it} \mid d_{i,t-1}) = \mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it})\\
&\quad \times E(\bar{u}_{it} + \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it} = 1) + \epsilon_{it} \mid d_{it} = 1)\\
&\quad + (1 - \mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it}))\\
&\quad \times \delta_c EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 0).
\end{aligned} \tag{8}$$

We simplify this equation in two ways. First, we note that the conditional expectation of a deterministic term is the term itself, for example, $E(\delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it}) = \delta_c EV_{i,t+1}(S_{i,t+1} \mid d_{it})$. Second, we employ Mills' ratio for the conditional expectation of the error term. Mills' ratio for normally distributed random variables suggests that $E(\epsilon \mid \epsilon > A)\Pr(\epsilon > A) = \sigma_\epsilon \phi(A / \sigma_\epsilon)$ for any real A, where ϕ is the standard normal probability density function (Kendall et al. 1987). We use the assumption that $\sigma_\epsilon = 1$. Upon substitution and rearranging terms we obtain:

$$\begin{aligned}
EV_{it}(S_{it}) &= \mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it})\\
&\quad \times [\bar{u}_{it} + \delta_c EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 1)]\\
&\quad + \mathrm{Prob}_{it}\,(d_{it} = 0 \mid S_{it}, m_{it})\\
&\quad \times \delta_c EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 0)\\
&\quad + \phi[\delta_c(EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 0)\\
&\quad - EV_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 1)) - \bar{u}_{it}].
\end{aligned} \tag{9}$$

The expected valuation function is evaluated recursively using the algorithm presented ahead in §2.4.

2.3. Profit Function and Optimal Mailing Policy

In this section, we model the firm's profit maximization problem. The profit of the direct mailer from individual i in state S_{it} in the current period is,

$$\pi_{it}(S_{it}, m_{it}) = R \times \mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it}) - c \times m_{it} \tag{10}$$

where R is net revenue (monetary value of the purchase net of production costs) from customer i and c is the unit mailing cost.

We define $P_{it}(S_{it})$ as the maximal expected profit that customer i in state S_{it} contributes in the time interval (t, ∞):

$$P_{it}(S_{it}) = \sum_{j=t}^{\infty} \delta_f^{j-t} \pi_{ij}(S_{ij}, m_{ij}^*(S_{ij})) \tag{11}$$

where δ_f is the firm's discount factor (which we set to 0.998 to yield ten percent discount rate per annum) and (*) represents optimality. The equation can be rewritten as,

$$\begin{aligned}
P_{it}(S_{it}) &= \max_{m_{it}} \{\pi_{it}(S_{it}, m_{it})\\
&\quad + \delta_f[\mathrm{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it}) P_{i,t+1}\,(S_{i,t+1} \mid d_{it} = 1)\\
&\quad + \mathrm{Prob}_{it}\,(d_{it} = 0 \mid S_{it}, m_{it}) P_{i,t+1}(S_{i,t+1} \mid d_{it} = 0)]
\end{aligned} \tag{12}$$

using the optimality principle (Ross 1983). In the equation future recency and frequency are updated according to current response behavior, as depicted in Equation (3).

2.4. An Algorithm to Solve for the Firm's Profit Function and Customer's Value Function

We propose an algorithm to simultaneously solve for the firm's profit maximization and the customers' utility maximization problems. In the infinite horizon a stationary solution only depends on the value of state variables and is independent of time. Stationary solutions are limits of time-dependent solutions. For a given level of the state variables S_i the profit function is $P_i(S_i) = \lim_{t\to\infty} P_{it}(S_i)$ and the value function is $EV_i(S_i) = \lim_{t\to\infty} EV_{it}(S_i)$.

We use the method of successive approximations to calculate the stationary solution (Bertsekas 1976, Ross 1983). Since the one-period utility function and the profit function are both bounded the solutions converge after a finite number of iterations. The convergence criterion is defined by setting a tolerance level η which is a small positive number. Once the improvement is smaller than the tolerance level, the program stops and the stationary solution is approximated by the current solution. The solution contains a profit function $P_i(S_i)$, a value function $EV_i(S_i)$, and a mailing policy $m_i^*(S_i)$.

PROPOSITION. *There exists a unique stationary solution for the profit function and value function. The solution is computed by the algorithm presented below.*

PROOF. See Appendix A.

Algorithm

Step 0: Initialization: Let the profit function $P_{i,t+1}(S_{i,t+1})$ and value function $EV_{i,t+1}(S_{i,t+1})$ be zero for all state variables. Set a tolerance level $\eta > 0$.

Step 1: Use Equation (7) to compute $\text{Prob}_{it}\ (d_{it} = 1 \mid S_{it}, m_{it})$, the response probability of customer i in state S_{it}, for both values of the mailing policy, $m_{it} = 0, 1$.

Step 2: Use Equation (12) to calculate the maximal expected profit from customer i, $P_{it}(S_{it})$, and the corresponding optimal mailing decision $m_{it}^*(S_{it})$.

Step 3: Use Equation (9) to compute the expected future value of customer i in state $S_{it}(EV_{it}(S_{it}))$ by employing the optimal mailing policy calculated in *Step 2*.

Step 4: Stopping Criterion: Let $d_1 = EV_{i,t+1} - EV_{it}$ and $d_2 = P_{i,t+1} - P_{it}$.
If $d_1'd_1 + d_2'd_2 < \eta$, then stop. Otherwise let $EV_{i,t+1} = EV_{it}$ and $P_{i,t+1} = P_{it}$, and go to Step 1.

Thus, we obtain the optimal mailing policy that is both profit maximizing and utility maximizing over the infinite time horizon. The computation is lengthy. One evaluation of the dynamic programming problem until convergence is reached takes nearly one hour on a 60 megahertz PC with a Pentium chip.

2.5. Generalizability of the Model

We propose an *as if* model of customers' response behavior to direct mail over a time horizon and take into account the effect of current and future mailing decisions. Below we review the main assumptions of the model and discuss the generalizability of our methodology to other situations.

A. Impact of Constraints for Mailing Budget and Inventory on Customer Response Function. A typical direct mail merchant operates in a constrained environment as we allude to in §2.2. In each period a firm allocates a certain budget to mailing expenses, determined by the purchase orders from the previous period, the inventory costs, and other factors. Catalogs are sent to those customers whose expected profits exceed a certain threshold determined by the budget constraints. Thus, some customers may not receive mail in certain periods, if the budget constraint restricts the firm from sending mail to them. Bitran and Mondschein (1996) depict such a constrained profit maximization solution for the firm. They develop heuristics instead of solving the entire dynamic programming problem, because the dimension of the state space is large and the mathematical problem is complex.

However, our main focus is on the customer response model. We explicitly model customers' rational beliefs about the firm's mailing decisions. Consequently, our approach requires that the customer solves the firm's problem as well. The model we have so far only requires the customer to know the average purchase amount, industry margin, and the cost of mail in order to solve for the mailing decisions. We assume these pieces of information are relatively easy for the customers to obtain. However, the inventory cost, budget constraint, size of the house list, and other private information are not typically available to a customer. We suggest that customers solve the future mailing *only with the available information*. Therefore, even if the firm solves a more complicated constrained optimization model, the customers still only solve the simpler dynamic programming model we propose.

We acknowledge that firms typically make their mailing decisions in a constrained operating environment. Since we include both the consumers' and the firm's optimization concerns we refrain from detailed modeling of the constraints the firm faces in order to keep the model structure manageable. A more complete model would combine our customer-specific approach with one that incorporates firm-specific considerations such as inventory control, size of the house list, mailing budget, and new customer acquisition.

B. Incomplete Information. We recognize that customers do not have complete information about the firm's accounts as discussed above. An alternative to our model is to estimate the customers' belief on the possibility of receiving the next catalog. When a customer recognizes that the firm's mailing policy is the outcome of a constrained optimization model, she or he may assign a probability q_{it} to be on the next mailing list and a probability $1 - q_{it}$ to be left out. Thus, we define:

$$q_{it}(S_{it}) = \text{Prob}(P_{it}(S_{it}, m_{it}) > P_t^0) \tag{13}$$

where P_t^0 is the threshold ($P_t^0 > 0$) that is determined by the budget constraints (unknown to the customer). Since the customer does not have access to complete information about the firm's house list, the customer only attaches a probability that she or he will meet the threshold.

The mailing decision is now random from the perspective of the customer. The notation is changed from m_{it} to its expected value q_{it} in the formulations. The current period profit function in Equation (10) can now be modified as an expected value:

$$\begin{aligned} &E_m(\pi_{it}(S_{it}, m_{it})) \\ &\quad = (R \times \text{Prob}_{it}(d_{it} = 1 \mid S_{it}, m_{it} = 1) - c) \times q_{it}(S_{it}) \\ &\qquad + R \times \text{Prob}_{it}(d_{it} = 1 \mid S_{it}, m_{it} = 0) \\ &\qquad \times (1 - q_{it}(S_{it})) \end{aligned} \tag{14}$$

where E_m indicates the expectation over the mailing variable. The expected future value of customer i in state S_{it} becomes

$$\begin{aligned} E_m(EV_{it}(S_{it})) &= (EV_{it}(S_{it}) \mid m_{it} = 1) q_{it}(S_{it}) \\ &\quad + (EV_{it}(S_{it}) \mid m_{it} = 0)(1 - q_{it}(S_{it})). \end{aligned}$$

The new parameters in the model (q_{it}) can be jointly estimated with the rest of the model parameters. Ideally a separate q can be estimated for each state (recency and frequency pair). However, since this would result in too many parameters a parsimonious form can be used such as $q_{it} = 1/(1 + \exp(-(a + br_{it} + cf_{it})))$. Such a functional form also ensures that the probability stays in the unit interval.

The alternative model described in this section is in the same spirit as Gonul and Srinivasan (1996) where consumers are assumed to have beliefs about the coupon availability probability in a frequently purchased product category. However, neither model can generate an optimal policy for the firm (an optimal coupon distribution schedule or an optimal mailing policy) because the interaction between the customer and the firm is not modeled. This is so because when the mailing decision is made random from the customer's perspective, the interaction between the customer and the firm is lost. Such a model is essentially a one-sided customer choice model where the firm is a silent agent that occasionally sends catalogs or offers coupons. In contrast, in our model the optimal mailing decision and the customer's optimal choice are solved simultaneously. Therefore, we present the idea only to convey that incomplete information can be handled at the loss of some of the main contributions of our model.

C. Explosion of the State Space. The model presented in §2.2 classifies customers by recency (r_{it}) and frequency (f_{it}). We develop a possible extension of the model in Appendix B that incorporates monetary amount as well. In practice, managers may use several other characteristics to segment customers. For example, every year the *DMA Statistical Fact Book* presents response rate statistics by (a) purpose of campaign such as lead, traffic building, and others, (b) medium used such as print ad, direct mail, multimedia, and others, (c) target audience such as consumer or business-to-business, (d) demographic profiles, (e) creative techniques used such as two-color, four-color, humor, and others, and (f) incentives offered such as coupons, discount, free offer, and others.

In our data the available covariates are gender, privacy, and method of payment (American Express, Discover, etc.). Eighty-seven percent of the customers are female and 99% of all customers prefer privacy (that is, they do not prefer their names to be rented out to other direct mailers). Thus, there is not much variation in the gender and the privacy variables to meaningfully estimate their impact on purchase behavior. We also have experimented with the type of credit card in simpler models and found that the method of payment variable does not seem to matter in determining purchase probability.

Analytically, it is possible to include these and other factors in the response function. However, in practice, the dynamic programming model becomes computationally costly since the model has to be solved for each value of the state space. For example, if gender classification is used then the dynamic programming model would have to be solved once for males and once for females. In other words, there would be two trees as shown in Figure 1 since the number of states multiplies the number of times the problem should be solved.

The explosion of the state space is known as the curse of dimensionality in dynamic programming (Bellman 1957). As we mention in §2.4 one evaluation of the dynamic programming problem takes nearly one hour. Since we present an estimable model the computational cost is even higher than in a deterministic model. Because, each time the dynamic programming problem is solved for a given set of parameter values and the solution has to be iterated over the parameter set in order to maximize the likelihood function. Consequently, for computational reasons we refrain from expanding the state space and retain a carefully selected set of covariates that we include in the response function.

3. Estimation Issues

3.1. Data

Our data set consists of the purchase histories of 530 households from the house list of a national direct marketing firm. The items sold in the catalog are household products that are durable for one and a half or two years. The products are consumable all year round and the items displayed in the catalog do not have large seasonal fluctuations in sales.

We have the entire purchase history on each household starting with its initial order with the company and ending in December 31, 1994. The beginning date is different for each household. On average, households receive five or six catalogs a year. The contents of the catalog do not vary noticeably except the cover of the book that is changed from one mailing to the next. The average cost of one mailing is \$2.53. The average net revenue is \$27 which is the industry margin (30%) times the size of the average order (\$90) in the data. We summarize the remaining characteristics of the data in Table 1.

3.2. Likelihood Function

The likelihood function consists of products of response and non-response probabilities of customers over the observation period. The log-likelihood function for customer i in period t is:

$$L_{it} = d_{it} \times \ln(\text{Prob}_{it}\,(d_{it} = 1 \mid S_{it}, m_{it})) + (1 - d_{it}) \times \ln(\text{Prob}_{it}\,(d_{it} = 0 \mid S_{it}, m_{it})). \quad (15)$$

The sample log-likelihood function is $LL = \Sigma_{i=1}^{I} \Sigma_{t=b_i}^{B_i} L_{it}$ where I is the number of customers and the period $[b_i, B_i]$ is the interval during which individual i is observed. Since we have access to the purchase history of each customer starting with their first purchase, our models do not suffer from the initial conditions problem that often plagues longitudinal choice models (Heckman and MaCurdy 1980).

Table 1 Summary Statistics

Sample Characteristics	Mean	Std. Dev.	Min	Max
Number of observed months per household	33.42	22.58	2	111
Number of purchases per household	2.99	1.89	1	13
Number of catalogs received per year	5.79	2.32	1.76	11.14
Number of months between purchases	8.72	5.58	1	41

Note: Number of households in the sample is 530. Total number of months observed in 17,711.

The maximum likelihood estimation routine searches for the optimum over the parameter space by changing the parameter values. We iterate with different starting values until the log-likelihood function converges to the reported optimum. The completion of the estimation is lengthy as we discuss in §2.5.C. Practitioners in direct marketing often work with the assumption that there is an absorbing state of recency (r^*), where a customer's response probability changes little if there has been no response in the last n periods, $n \geq r^* - 1$. The cataloguer assumes that the absorbing state for recency is 60 months. To simplify computation we set the limit for frequency at 20 purchases. We vary r^* and f^*, however, the qualitative results remain robust.

3.3. Results of The Customer Response Function Estimation

Besides the dynamic model presented above, we estimate a single-period model where the customer does not value the future at all ($\delta_c = 0$). This model is a simple probit as can be seen by imposing the restriction on the discount factor in Equation (7). We estimate both the single-period and dynamic models with and without the impact of frequency for the sake of comparison.

The single-period and dynamic models are not nested in each other; they represent two versions of the same model with different values of the discount factor. Therefore, we cannot use the likelihood ratio test (a chi-squared test) that is valid for nested models. Instead we rank the models with the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The formula for AIC is $-2 \times (LL - p)$ where N stands for the number of observations and p for the number of parameters. The formula for BIC is $-2 \times LL + p \times \ln(N)$ (Akaike 1974). A lower value is preferred for either criterion.

We let the monthly discount factor be 0.998 so that customers are assumed to discount the future utility at ten percent per year. The value is reasonable and has been used in other structural models (Wolpin 1984, Gonul 1989, Hotz and Miller 1993).[4] The results from all models are presented in Table 2.

A naive benchmark model with an intercept only, assumes that mailing, recency, and frequency have no effect on the purchase rate. Such a model yields a log-likelihood value of -5332.78, AIC = 10667.56, and BIC = 10695.34. A chi-squared comparison of the log-likelihood value of the naive model with any of our models indicates that the covariates in all our models are jointly significant. According to both AIC and BIC indices the dynamic models perform better than the single-period models. Moreover, the models with both recency and frequency perform better than the models with recency alone, in both the single-period and dynamic cases.

The parameter estimates from the best-fitting dynamic model indicate the following: A mailing generates a small but significant effect and increases the response probability, as expected. The impact of both recency and frequency are U-shaped: hence, we reject monotone declining or monotone increasing shapes for both variables in this product category.

3.4. Discussion

The purchase probability declines up to about two years of recency (23.60 months) and starts increasing afterwards (Figure 2a). Figure 2a may not clearly show the minimum point because the response probabilities over a wide range are very small and close to each other. The recency curve implies that customers tend to replace the products they bought within two years. It is comforting that the two-year average replacement rate for the product category as we were told by the cataloguer is verified by our estimates. The curve rises steeply at the end for two reasons: Response probability rises at high recency values and the absorbing recency state of 60 stands for all recencies greater than or equal to 60 (recall from §3.2). Thus, the probabilities at 60+ are artificially loaded on to recency of 60.

Cataloguers often use a monotonic measure of recency to predict the response rate. Instead, the U-shape indicates that people may buy after a prolonged

[4] Estimating the discount factor as a parameter is conceptually possible, however, proves difficult in practice. Our attempts in jointly estimating the discount factor with the rest of the parameters resulted in convergence problems. We also tried holding other parameters constant and only estimating the discount factor, however, this model did not give different results than the current one and converged at 0.998. As in the extant works cited above the standard practice in the literature is to assume a fixed value for the discount factor.

Table 2 Estimates from Single-Period and Dynamic Models

Parameters	Single-Period Models		Dynamic Models	
Intercept	−0.9311***	−0.3616***	−1.1373***	−1.0243***
	(−38.4645)	(−8.5862)	(−3.2153)	(−4.3907)
Mail	−0.0206	0.0062*	0.0215**	0.0104**
	(−0.6859)	(1.8677)	(1.9749)	(1.9852)
Recency	−0.0639***	−0.0648***	−0.0074***	−1.1240***
	(−18.6850)	(−19.7165)	(−5.1800)	(−14.7327)
Recency-squared	0.0011***	0.0010***	0.0011***	0.0237***
	(15.3023)	(15.7544)	(12.7846)	(30.9978)
Frequency		−0.3687***		−2.3144***
		(−15.1451)		(−7.5712)
Frequency-squared		0.0336***		0.1563***
		(12.5264)		(25.3031)
Log-Likelihood	−5,114.28	−4,977.01	−4,865.96	−4,773.17
AIC	10,236.57	9,966.03	9,739.92	9,558.35
BIC	10,267.70	10,012.72	9,771.05	9,605.03

Note: Significance at 0.01 level is denoted by (***) at 0.05 level by (**) and at 0.10 level by (*). The asymptotic normal statistics are placed in parentheses.

Figure 2 Empirical Findings

Figure 2a Recency vs. Response Probability

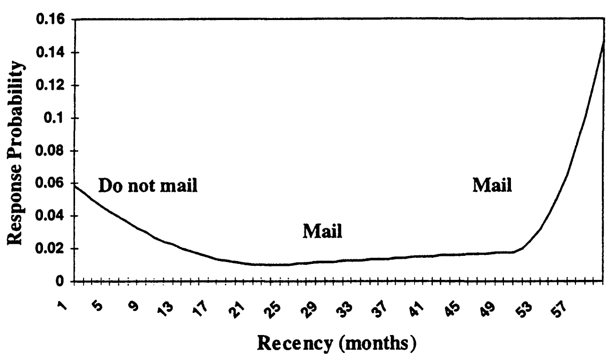
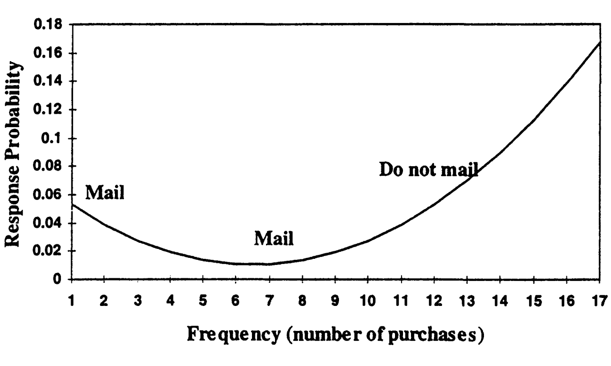


Figure 2b Frequency vs. Response Probability



recency. Thus, cataloguers who use monotonic measures possibly err when they eliminate customers with relatively high recency values from the mailing list. Note that this interpretation is valid for not-so-frequently purchased products like the ones we study (that are replaceable within a year or two). For more frequently purchased products deleting a high-recency customer from the list can be the correct decision, because a high recency value implies that the customer is probably buying elsewhere.

The impact of frequency is U-shaped as well, thus rejecting monotone increasing or decreasing impact of frequency on purchase probability (Figure 2b). The purchase probability declines until about seven purchases and starts increasing afterwards. This finding can be explained as follows. There is information uncertainty regarding the quality of products in a catalog. Unlike in a retail store customers cannot try the products in a catalog without a purchase. Thus, when a customer makes a small number of purchases from the catalog the customer is possibly in a trial mode and experimenting with the company. We find that such a customer is likely to sample more since the response probability is still relatively high. However, at medium levels of frequency the company is likely to lose the customer to competitors since the response probability declines. Once past the threshold of about seven purchases a customer appears to be a loyal shopper and unlikely to become a defector.

We note that the single-period model yields biased estimates of the impacts of recency and frequency. This model imposes the constraint that customers are myopic and therefore, is deprived of an inherent mechanism to explain the dynamics of the purchase process. As a result, the state variables, that are updated by each purchase/nonpurchase occasion, appear to get undue credit for the dynamics of the process. Thus, impacts of both recency and frequency are overestimated in a model that disregards customers' orientation for the future.

4. Optimal Mailing Policy

We emphasize that the results discussed below are only valid for this particular application. A field experiment or a theoretical approach is needed in order to generalize the results. We compute the optimal mailing policy from the equilibrium algorithm presented in §2.4 over the infinite horizon.

Our findings show that the optimal mailing policy varies by both recency and frequency (Figure 2). At low levels of recency it is not optimal to mail. It is better to save the mailing dollars for customers in the medium and high recency ranges in order to keep top-of-the-mind awareness and not lose them to competitors when they need to replace the product. For example, if a customer has recently made a purchase (for example, in the last 5 months) then the response probability is still high even without mail. However, if a customer has not bought in 15 months or longer then a catalog is necessary to activate the customer. If a customer has not bought in 60+ months a response is likely but it is still necessary to send mail in order to realize a purchase and bring down the recency to one.

Sending reminders in the mail to customers who have not purchased in a while is observed in the industry as well. For example, airlines mail out incentives for travel when customers start traveling less often (*Wall Street Journal* 1994). Bitran and Mondschein (1996) also arrive at the same result.

We find that it is optimal to mail to customers at low to medium ranges of frequency because they are in a trial mode. Such customers are likely to shift to other retailers. In contrast, customers who have bought a relatively high number of times have a higher probability of responding anyway. Thus, if a customer has bought many times before from the catalog it is not necessary to send mail unless they have relatively high recency values.

Bult and Wansbeek (1995) also find that customers who have bought many times in the past have higher response probabilities. We iterate that their model is not directly comparable because they do not control for the quadratic impact of frequency but leave it as linear. In addition, they model the firm and the customers in a single-period environment. Bitran and Mondschein (1996) model frequency as a binary variable for parsimony: single purchase and repeated purchase. Therefore, they do not explicitly focus on frequency but rather dwell on the impacts of recency and other factors on the mailing policy.

Note that our results differ from results obtained from conventional models where the firm is implicitly a single-period profit maximizer. In such models it is optimal not to mail to customers with low response probabilities because of low expected profits. However, if the firm is a dynamic profit-maximizer as assumed in our model then the firm may choose to retain such customers on the mailing list because of larger expected profits in the future periods.

We find that had the optimal mailing policy been used for the current sample of customers the expected profits would have been \$23,801 while the actual profits are \$20,524. Hence, the company could have increased profits by 16% if the optimal mailing policy had been used, holding everything else constant.

Incidentally, not to mail in any month is also the optimal solution if the cataloger is assumed to maximize profits in the current period alone, that is, if ($\delta_f = 0$). The result follows because the cataloger cannot break-even in one month if a mailing decision is undertaken. In a single period the response rate difference at which the direct mailer is indifferent between sending and not sending mail is: (average mailing cost ÷ average net revenue) = \$2.53 ÷ \$27 = 9.37%. Thus, in a single-period model profits with mail exceed profits without mail if and only if the difference in response rate with and without mail exceeds 9.37%. However, in the data the difference is less than 9.37%.

In contrast, in the case of dynamic profit maximization future expected responses ensure optimality of mailing in the current period despite single-period losses. A lot of small cataloguers go out of business because they cannot afford to wait to reap future profits (Direct Marketing White Paper 1990). If they adopt a profit maximization policy with a longer horizon they would possibly survive.

5. Conclusion

We investigate the key determinants of the optimal mailing policy in a dynamic environment where customers maximize utility and the direct mailer maximizes profits. We measure the sensitivity of the customers to receiving a catalog, while controlling for customer characteristics such as elapsed time in responses and frequency of purchase. We apply our model to a database from a national cataloger.

We iterate that the results discussed below are pertinent to the data set under consideration and that different results may emerge from other applications. The main findings from the catalog database we study are as follows: A model that assumes customers are short-sighted and only maximize utility in the current period is estimated and rejected. According to the dynamic model the impacts of recency and frequency on the response probability are both U-shaped. The optimal mailing policy should vary with both the individual's recency and frequency. It is not optimal to mail to individuals at low recency levels because they are likely to buy anyway. It is better to save the mailing dollars for customers at higher recency levels. It is optimal to mail to customers at low frequency levels to induce them to continue to buy. It is not necessary to mail often to customers who have purchased many times before from the company unless they have relatively high recency values.

Our model generalizes to other direct marketing situations as well. Using our model direct marketers can estimate response functions with purchase history data of customers and can generate the optimal mailing path for each customer segment. They can compare the optimal mailing policy with the current policy and improve their mailing policy to increase profits. They can use our model to determine which segments are the most profitable, what mailing policy is suitable for each segment, and how response rates can be increased by custom-tailoring the mailing policy.

Our model is not without limitations. In our data we have a catalog whose contents do not vary substantively across mailings. This may not be the case for other direct mail items. However, the model can be extended to apply to databases where contents of the mailings are sufficiently different to evoke different optimal response and mailing solutions. This can be done by modeling the (currently binary) mailing variable as multinomial, provided data are available.

We do not have data on competitive offers by retailers or mailings by other cataloguers to the individuals on the house list. Therefore, ours is a binary choice model. The model can be extended to multinomial choice situations where individuals choose among several outlets provided data are available. To the best of our knowledge currently no data collecting agency has such extensive information on catalog customers.

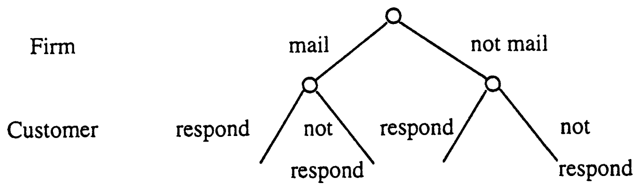
Our model focuses on retention of customers already on the house list but is silent on new customer acquisition from rental lists. Therefore, our model is more applicable to established cataloguers with house lists than to start-up companies that heavily rely on new customer acquisition.

The customer base in our data set is relatively homogeneous in terms of demographic profiles. The customers on the house list differ mainly by recency and frequency of their responses. We account for both types of heterogeneity in our model and find substantially different optimal measures for different segments. Other factors such as monetary value of the purchases can be handled by discretizing them and adding them as state variables. However, additional state variables substantially increase the computational cost. Hotz and Miller (1993) introduce a new method to approximate dynamic programming models and alleviate computational difficulty. As such approximations become more commonplace we can look forward to estimating structural models with unobserved heterogeneity.[5]

[5] We thank the Associate Editor and three referees of the journal for valuable comments. The usual disclaimer applies.

Appendix. A Proof of the Proposition in Section 2.4

The model presented in our paper is essentially a stochastic game between the direct mailer and each customer. In each time period the direct mailer chooses whether or not to mail, and the customer decides whether or not to respond. An extensive form of the one-period game is given below. The same game repeats over time. Both sides expect rational behavior from each other.



Existence. A theorem in game theory states that in any stochastic game with a countable state space and finite number of actions, there exists a Markov-perfect equilibrium (Fudenberg and Tirole 1993).

The game presented in our model has a countable state space (all integers) and a finite number of actions (mail or not mail, respond or not respond). The history in each period t can be summarized by the state variables S_{it} (recency and frequency) for customer i. Since a customer's response probability only depends on the state and on the firm's mailing policy, the firm's payoff at time t only depends on the state and its own action. The state follows a Markov process and the transition probability is determined by the response probability. Then existence follows by invoking the theorem given in Fudenberg and Tirole.

Uniqueness. For each possible state $S_i = (r_i, f_i)$ and stationary policy $m_i^*(S_i)$, the firm's optimality Equation (12) becomes:

$$
\begin{aligned}
P_i(S_i) &= \pi_i(S_i, m_i^*(S_i)) \\
&\quad + \delta_f[\text{Prob}_i\ (d_i = 1 \mid S_i, m_i^*(S_i))P_i(1, f_i + 1) \\
&\quad + \text{Prob}_i\ (d_i = 0 \mid S_i, m_i^*(S_i))P_i(r_i + 1, f_i)].
\end{aligned}
$$

Since there is one equation for each possible state the uniqueness of the solution for both the profit function and the value function directly follows (Bertsekas 1976). Q.E.D.

Appendix B. Discrete Response and Purchase Amount Model

In this Appendix we develop a model where customers decide not only whether or not to buy from the catalog but also decide how much to spend on each purchase. We expand the state space in order to include the monetary amount variable: $S_{it} = \{r_{it}, f_{it}, M_{it}\}$. The recency and frequency variables are defined as before. The monetary amount variable is defined as the average amount spent so far on purchases.

Because the dollar amount is continuous in nature, we suggest discretizing the variable into several categories in order to implement the control variable as discrete-valued. In particular, let the purchase amount to be in one of the K brackets. The cutoff points for the brackets can be arbitrarily determined depending on the data. For example, if the purchase amount is \$0 then $k = 1$, if the purchase amount is less than \$25 then $k = 2$, if the purchase amount is higher than \$25 but less than \$50 then $k = 3$, and so on.

In each period t there is a utility u_{it0} from not buying from the catalog, and a utility u_{itk} from buying from the catalog and making a purchase amount in bracket k. The utility function for each alternative is:

$$
\begin{aligned}
u_{itk} &= \alpha_k + \beta_{Mk}M_{it} + \beta_{M-sq,k}M_{it}^2 + \beta_{mk}m_{it} \\
&\quad + \beta_{1rk}r_{it} + \beta_{2rk}r_{it}^2 + \beta_{1fk}f_{it} + \beta_{2fk}f_{it}^2 + \epsilon_{itk}, \\
u_{it0} &= 0 \text{ (normalized)}.
\end{aligned} \tag{A1}
$$

A reasonable expectation of the signs of the coefficients β_{Mk} and $\beta_{M-sq,k}$ would be positive and negative respectively, to reflect first increasing then decreasing utility as a function of the purchase amount. However, what follows holds regardless of the functional form of the utility function.

Individual i decides among $K + 1$ options in each period to maximize:

$$
E\left(\sum_{t=1}^{\infty} \delta_c^{t-1} \sum_{k=0}^{K} u_{itk} d_{itk}\right). \tag{A2}
$$

The d_{itk} variables are either 0 or 1 and $\Sigma_{k=0}^{K} d_{itk} = 1$ holds in each time period since the individual may choose one and only one alternative in period t.

The alternative-specific value functions satisfy the Bellman equation for all t:

$$
V_{itk}(S_{it}) = u_{itk} + \delta_c E\{V_{i,t+1}(S_{i,t+1} \mid d_{itk} = 1\} \tag{A3}
$$

where S_{it} is the state space at time t that summarizes all the available information to individual i. The alternative-specific value function explicitly links the choice at time t to the optimal time path from $t + 1$ onwards. The equation also indicates that future choices are made optimally for any given current period decision, consistent with the dynamic programming formulation.

Besides the increase in the state space another difference from the model in the text is the replacement of the normality assumption on the error terms with the Type I extreme value distribution assumption. In the literature this is done for convenience because the multivariate probit integrals are cumbersome to compute when the number of choices exceeds three or four.

Because of the Type I extreme value assumption on the error terms the expected value function that appears on the right-hand side of equation (A3) has the following closed-form expression (Hotz and Miller 1993, Rust 1994):

$$
E\{V_{it}(S_{it})\} = \gamma + \ln\left[\sum_{k=0}^{K} \exp(\overline{V_{itk}(S_{it})})\right] \tag{A4}
$$

where γ is Euler's constant and where an upper bar indicates the deterministic part of the value functions. The choice probabilities achieve the following convenient formulation:

$$
\text{Prob}_{itk}\ (d_{itk} = 1 \mid S_{it}, m_{it}) = \frac{\exp(\overline{V_{itk}(S_{it})})}{\Sigma_{k'=0}^{K} \exp(\overline{V_{itk'}(S_{it})})}. \tag{A5}
$$

Although the choice probability expression resembles a reduced-form multinomial logit choice probability, the structural dynamic programming model does not suffer from the independence from irrelevant alternatives (IIA) restriction that typically plagues the reduced-form logit models. (The IIA refers to the ratio of two choice probabilities being a function of only attributes of those two alternatives.) The dynamic programming structure circumvents the IIA restriction because it enables the incorporation of the influence of all possible alternatives on the current choice (Rust 1994).

In summary, we have developed a discrete response and purchase amount model that is estimable with individual-level data. In this model the tree in the state space diagram (Figure 1) would have multiple branches at each node instead of two branches to take care of each distinct monetary value. In addition the number of parameters is higher to accommodate the multiple utility functions. We refrain from estimating this model for computational reasons. As faster computers and more importantly, computers with larger work space memory, become available we can look forward to estimating such extensions.

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