Econometrica, Vol. 74, No. 4 (July, 2006), 1013–1062

MINIMUM WAGE EFFECTS ON LABOR MARKET OUTCOMES UNDER SEARCH, MATCHING, AND ENDOGENOUS CONTACT RATES

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Building upon a continuous-time model of search with Nash bargaining in a stationary environment, we analyze the effect of changes in minimum wages on labor market outcomes and welfare. Although minimum wage increases may or may not lead to increases in unemployment in our model, they can be welfare-improving to labor market participants on both the supply and demand sides of the labor market. We discuss identification of the model using Current Population Survey data on accepted wages and unemployment durations, and show that by incorporating a limited amount of information from the demand side of the market it is possible to obtain credible and precise estimates of all primitive parameters. We show that the optimal minimum wage in 1996 depends critically on whether or not contact rates can be considered to be exogenous and we note that the limited variation in minimum wages makes testing this assumption problematic.

KEYWORDS: Minimum wages, matching models, Nash bargaining, matching function.

1. INTRODUCTION

DETERMINING THE EQUILIBRIUM EFFECTS of minimum wage changes on labor market outcomes is a challenging modeling and estimation problem; arriving at policy recommendations is a task even more daunting. Faced with the inherent difficulties of modeling equilibrium labor market events given the limited amount of data to which researchers have access, much recent research has been performed outside of an explicit behavioral framework, with researchers pursuing the more limited objective of carefully describing the observed effects of recent minimum wage changes using quasi-experimental methods (see Card and Krueger (1995) for a summary of these studies and a comprehensive, critical survey of most of the previous research done in this area). In our view, these recent studies have been particularly useful in indicating that the "textbook" competitive model of the labor market, which has been used as

[1]This research was partially supported by the W. E. Upjohn Institute for Employment Research and the C. V. Starr Center for Applied Economics at NYU. I have received many useful comments from participants in seminars held at North Carolina, Georgetown, Johns Hopkins, Columbia, the New York Fed, NYU, LSE, Yale, Bocconi, the BLS, and Western Ontario, and in conferences, including the Econometric Society Meetings (New York, 1999), the Institute for Research on Poverty Summer Workshop (Madison, 1999), the Society for Economic Dynamics (Sardegna, 1999), the Goldwater Labor Markets Conference (2004), and the European Econometric Society Meetings (1999 and 2004). I am especially grateful to Joop Abbring, Jim Albrecht, Matt Dey, Luca Flabbi, Martin Gervais, Sarah Hamersma, Winfried Koeniger, Ricardo Lagos, James Mabli, Julien Pratt, Holger Sieg, a co-editor, and three anonymous referees for constructive comments and suggestions. I remain solely responsible for all errors, omissions, and interpretations.

an interpretive framework for the bulk of the empirical work performed using aggregated time series data, may have serious deficiencies in accounting for minimum wage effects on labor market outcomes when confronted with disaggregated information.

Although the quasi-experimental results have raised a number of interesting challenges to orthodox theory, few cogent models have been advanced that are consistent with the results that have been found. Some of the explanations for these empirical findings (e.g., lack of significant employment losses, impacts on the wage distribution above the minimum) do not seem to be testable given our current data resources. In fact, it appears difficult to operationalize many of the explanations proposed even for the more modest purpose of empirical implementation.

It has long been recognized that for the imposition of minimum wages to have beneficial effects on the welfare distribution of workers and searchers, firms must have some degree of monopsony power (see, for example, Manning (1995)). Search frictions and the existence of match-specific capital are capable of providing this. The existence of both seems to be in accord with common sense and empirical findings (on the existence of match-specific capital, see Miller (1984) and Flinn (1986), for example). Below we formulate a model that includes match-specific capital, job search, and worker–firm bargaining over match-specific rents that is capable of accounting for many of the empirical observations cited by Card and Krueger (1995, Chapter 7) in their survey of minimum wage research based on individual level data.[2]

In the standard monopsony model of minimum wage effects, minimum wages lead to higher wages and employment levels, but lead to lower levels of firm welfare. By adding some general equilibrium elements to the model, it is possible for binding minimum wages to lead to increases in the welfare of firms, as well. We utilize a matching function formulation of the worker–firm contact rate to accomplish this (for a thorough summary of this approach, see Pissarides (2000)). From Hosios (1990), we know that the efficiency of equilibrium in such a framework requires that the elasticity of the matching function with respect to the size of the set of searchers be equal to the workers' share of the match surplus. In a world in which workers receive a low share of the surplus in comparison with their associated matching function elasticity, increases in the minimum wage can be viewed as a way to increase their "effective" bargaining power, leading to increased search activity on their part and an equilibrium outcome closer to what is obtained when the Hosios condition is satisfied.

[2]As Kennan (1995) points out in his informative review of Card and Krueger, there exist a number of studies of minimum wage impacts within local labor markets that utilize disaggregated data, some of which were conducted at the beginning of the last century. Although the local nature of the labor market makes generalization somewhat difficult, the fact that these minimum wage changes were substantial in many cases alleviates the problem of conducting policy analysis outside of the narrow range of historical federal minimum wage rates.

One goal of our empirical research is to obtain estimates of worker bargaining power and the matching function elasticity so that optimal—in the sense of efficient—minimum wages can be determined in a general equilibrium setting. This is the real point of departure of our analysis from the macroeconomics literature on labor market efficiency. In those analyses, it is at least implicitly assumed that the social planner has control of the division of the rents in any given match. Here we assume that this is not the case and that the workers' bargaining power is a primitive parameter determined outside of the model that is not necessarily equal to the match function elasticity. The minimum wage is a crude, blunt tool potentially available to policy makers that can increase aggregate welfare in some situations. By estimating the primitive parameters of the model, we are able to determine what optimal minimum wages should have been given the labor market environment and standard welfare criteria.

The model we develop is a logical descendent of the econometric model of search of Flinn and Heckman (1982) (hereafter referred to as FH) and the econometric model of minimum wage effects on the distribution of wages and the probability of employment estimated by Meyer and Wise (1983a, 1983b). FH formulated an equilibrium continuous-time search model in which searchers encountered potential employers according to a Poisson process; upon meeting, the potential value of the contact was determined by a draw from a fixed distribution G. In their analysis, the match value was arbitrarily assumed to be divided evenly between workers and firms. The setup of the basic model proposed here is similar, with the important difference that rents are divided using an explicit Nash-bargaining criteria. The bargaining power parameter that appears in this formulation is of key importance in determining the welfare consequences of imposing, or increasing, a minimum wage.[3]

Meyer and Wise (1983a, 1983b) estimated a model of minimum wage effects using individual-level data, which allowed them to infer what the wage distribution and employment level would have been in the absence of a minimum wage. Their model was both original and suggestive. Although it has been criticized by a number of researchers (e.g., Card and Krueger (1995, pp. 232–236) and Dickens, Machin, and Manning (1998)), it remains one of the better econometric attempts to identify minimum wage effects using individual-level data

[3]We think of the bargaining power parameter as being a sort of summary statistic of the labor market "position" of a particular group. For example, the match value distribution for low-skilled workers may be stochastically dominated by the match value distribution for high-skilled workers, but, in addition, low-skilled workers may be at a disadvantage due to their having little bargaining power. Their low bargaining power may derive from there being many substitutes for them in the production process or due to the relative number of low-skilled workers to the number of positions for this type of worker. Although we will conduct policy experiments that fix the value of the bargaining power parameter at our point estimate of it, it should be borne in mind that the validity of doing so is questionable, particularly when considering large policy (i.e., minimum wage) changes.

in the literature (another example is Heckman and Sedlacek (1981)). From our perspective, the main weakness of their model is the arbitrary specification of the manner in which a minimum wage "distorts" the preexisting wage distribution. In our model, optimizing behavior by searchers and firms determines the nature of this "distortion," and it is roughly consistent both with the Meyer and Wise specification and with the empirical evidence cited in Card and Krueger.

In this research, we attempt to integrate theory and measurement to analyze minimum wage effects on the labor market. There are a number of papers in the literature that consider the possibility of welfare-enhancing minimum wage rates, typically through the alleviation of some market inefficiency. For example, Drazen (1986) considers a case in which adverse selection is alleviated by moving from the equilibrium competitive wage to a higher minimum wage, which induces an improvement in the average quality of the work force that more than offsets the increase in average labor costs. Lang (1987) views the potentially beneficial effects of a minimum wage in the context of a signaling model. By increasing the wage unskilled labor earns, it lowers the incentive of low-ability individuals to mimic the signals of high-ability workers, thus allowing a reduction in expenditures on wasteful signaling investments. Rebitzer and Taylor (1996) make a case for minimum wages using a Shapiro and Stiglitz (1984) efficiency wage model, where the setting of a minimum wage above the "no shirking" wage results in short-term, and sometimes long-term, employment gains. Swinnerton (1996) uses an Albrecht and Axell (1984) equilibrium search framework with heterogeneous firms to argue for the possibility of Pareto-improving minimum wages as a result of the elimination of inferior firms from the market. A recent addition to this literature is van den Berg (2003), in which minimum wages are used as equilibrium selection devices when there exist multiple equilibria in the economy (and a minimum wage can be used to exclude the possibility of a Pareto-dominated equilibrium).[4] As applied theory exercises, the objective of these papers is to point out the various ways in which minimum wages can lead to welfare improvements in highly stylized environments with very particular types of informational constraints and production technologies. The environment considered in this paper, by contrast, is extremely simple and is solely generated by search frictions and differences in bargaining ability. Within this model, we are able to define conditions under which minimum wages are welfare-enhancing, and by using estimates of model parameters, we are directly able to determine whether these conditions are satisfied at particular minimum wage levels. In addition, we are able to arrive at estimates of the optimal level of the minimum wage under certain explicit welfare criteria.

There do exist a number of empirical studies that examine the role of minimum wages within equilibrium labor market models different than the one

[4]This model generally implies that the equilibrium wage distribution would not have a mass point at the minimum wage. A mass point is clearly indicated in the data used in our application.

used here. Eckstein and Wolpin (1990) generalize and estimate the equilibrium search model of Albrecht and Axell (1984). In these models, firms are heterogeneous in terms of profitability, so that a minimum wage effectively serves as an equilibrium selection device in that it selects less profitable (and hence lower wage) firms out of the market. By doing so, the wage distribution is shifted to the right, but the cost is a lower rate of job finding due to the exit of firms. Using their estimates, Eckstein and Wolpin find an optimal minimum wage of zero, although they qualify this result because they feel that the model does not adequately fit the data.

van den Berg and Ridder (1998) modify the Burdett and Mortensen (1998) equilibrium search model and estimate it using data from the Netherlands. The equilibrium in these papers relies critically on the existence of on-the-job search, which is lacking in the equilibrium model estimated here (primarily due to data limitations) and in the Albrecht and Axell (1984) and Eckstein and Wolpin (1990) models. Their extensions involve adding firm heterogeneity in innate profitability, permanent productivity differences across individuals, and a legislated minimum wage. The imposition of an economy-wide minimum wage can result in certain sets of individuals becoming permanently unemployable, and also selects certain low productivity firms out of the market, as in Eckstein and Wolpin (1990). Based on their empirical analysis the authors argue that the relatively high minimum wage rate in the Netherlands has resulted in a potentially large amount of structural (i.e., permanent) unemployment. Although they present no quantitative measures of the welfare effect of minimum wage changes, they suggest that the minimum wage is probably too high.

The implications of these models are similar to that of the model developed here in the sense that unemployment is generally increased by the imposition of a minimum wage. In Eckstein and Wolpin (1990) and van den Berg and Ridder (1998), unemployment cannot decrease as a result of an increase in the minimum wage. In the model considered here, with endogenous contact rates and labor market participation decisions, it is not necessarily the case that unemployment increases when the minimum wage does. Moreover, even when it does, it is possible for the steady state employment rate to increase due to inflows to the labor market (which occur whenever the value of unemployed search increases as a result of the minimum wage change). Thus the impact of the minimum wage on the steady state distribution of the population by labor market status is a bit more general in the setup considered here.

The biggest difference between those models and the current one, however, is in the form of the steady state wage distribution. The Eckstein and Wolpin (1990) model produces a mass point at the minimum wage, as does the model developed here, although their framework also implies a discrete wage distribution, something only problematically matched to individual-level wage observations. Although we do find evidence that wages are concentrated on only a few values (see Table I in Section 5), for reasons given below we believe that this phenomenon is more likely to be the result of response error than discreteness in the underlying equilibrium wage distribution.

As is the case in Burdett and Mortensen (1998), the van den Berg and Ridder (1998) model implies an absolutely continuous equilibrium wage distribution. In particular, there exists no mass point at the minimum wage. The authors present an extensive justification for the reasonableness of this implication, at least within the context of the labor market data they use to estimate the model. We argue below that, in the labor market we examine, the mass point at the minimum wage is real and significant; that the mass shifts in a way consistent with the structure of our model when the minimum wage is increased; and that, as a result, it is necessary to estimate a model consistent with this observation if we are to perform informative policy exercises.

The contributions of the paper are primarily econometric and empirical. We first develop a "restricted" Nash-bargaining procedure that maps a continuous underlying productivity distribution into a mixed continuous-discrete accepted wage distribution, with the sole mass point located at the minimum wage. Rather than treating offer arrival rates as exogenously determined, we graft onto this model a matching function, the quantitative properties of which are important for assessing the welfare effects of changing the minimum wage. Using cross-sectional information from the supply side of the market and information on labor's share of revenues from the demand side, we derive consistent estimators for all of the primitive parameters of the model. We demonstrate how repeated cross-sectional information from the labor market under more than one minimum wage can be used to identify parameters of the matching function without access to direct information on vacancies. The estimates of primitive parameters we obtain are credible, and with them we are able to perform a relatively complete characterization of the effects of the minimum wage on labor market and welfare outcomes in the population we consider.

The plan of the paper is as follows. In Section 2, we develop a bargaining model in a continuous-time search environment, both in the absence and in the presence of a binding minimum wage, in which contact rates between searching individuals and firms are endogenously determined. Section 3 contains a discussion of the impact of minimum wage changes on labor market equilibrium outcomes and welfare. In Section 4, we present the econometric framework within which the model is estimated and we devote a great deal of attention to identification issues. We present the estimation results in Section 5, and in Section 6, these estimates are used to perform welfare experiments that involve the selection of an optimal minimum wage. A conclusion is offered in Section 7.

2. LABOR MARKET SEARCH WITH BARGAINING

In this section, we describe the behavioral model of labor market search with matching and bargaining. The model is formulated in continuous time and assumes stationarity of the labor market environment. In the first subsection, we derive the decision rules for terminating search and for dividing the match value between worker and firm given the presence of a binding minimum wage and exogenous contact rates between searchers and firms. In the second subsection, we explicitly bring in labor market participation decisions of individuals and the vacancy creation decisions of firms. Taken together, the addition of these decisions allows us to endogenously determine the rate of contact between both sides of the market.

Throughout we assume that there exists an invariant, technologically determined distribution of worker–firm productivity levels that is given by $G(\theta)$. When a potential employee and a firm meet, the productive value of the match, θ, is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash-bargaining framework. The searcher's instantaneous discount rate is given by $\rho > 0$. The rate of (exogenous) termination of employment contracts is $\eta \geq 0$. While unemployed individuals search, their instantaneous utility is given by b, which can assume positive or negative values. For simplicity, and due to the nature of the data we utilize, we assume that employed individuals do not receive alternative offers of employment, i.e., there is no on-the-job search. The extension of the model to this case is discussed at the conclusion of this section, and its implications for the policy analysis we conduct are considered in the conclusion of the paper.

2.1. *Labor Market Decisions with Exogenous Contact Rates*

We assume that the only factor of production is labor and that total output of the firm is simply the sum of the productivity levels of all of its employees. Then if the firm "passes" on the applicant, that is, does not make an employment offer, its "disagreement" outcome is zero (it earns no revenue, but makes no wage payment).[5] While the firm is employing someone with a match value of θ, its instantaneous profit from that employee is given by $\theta - w$, where w is the instantaneous wage paid. We begin by assuming that the arrival rate of contacts (from the point of view of searchers) is equal to λ, a predetermined "primitive" parameter. The applicant's disagreement value is the value of continued search, which we denote by V_n. For any given value of V_n, there exists a corresponding critical "match" value $\theta^* = \rho V_n$, which has the property that all matches with values at least as great as θ^* will result in employment, whereas all those matches of lower value will not. For any $\theta \geq \theta^*$, the Nash-bargained wage is given by

$$(1) \qquad w(\theta, V_n) = \arg\max_{w} [V_e(w) - V_n]^{\alpha} \left[\frac{\theta - w}{\rho + \eta} \right]^{1-\alpha},$$

[5]The zero outside option assumption made in this section is justified under the analysis presented in the following subsection. A free entry assumption assures that the ex ante value of an unfilled vacancy is zero.

where $V_e(w)$ is the value of being employed at wage w and α is the bargaining power parameter, which takes values in $(0, 1)$. Without loss of generality it has been assumed that the firm shares the employee's effective rate of discount, $\rho + \eta$. Thus $(\theta - w)/(\rho + \eta)$ is the present value to the employer of a match θ when the employee's wage is w. It is well known (see, e.g., Pissarides (2000)) that in this case the value of a job with a wage of w is

$$V_e(w) = \frac{w + \eta V_n}{\rho + \eta},$$

the value of unemployed search is

$$\rho V_n = b + \frac{\alpha\lambda}{\rho + \eta}\int_{\rho V_n}[\theta - \rho V_n]\,dG(\theta),$$

and the equilibrium wage contract is

$$w(\theta, V_n) = \alpha\theta + (1 - \alpha)\theta^*. \tag{2}$$

We see from (2) that the accepted wages are an affine mapping from the left-truncated match distribution $G(\theta|\theta \geq \theta^*)$ into $F(w|w \geq w^*)$, where the lowest accepted wage is equal to the lowest accepted match value, or $w^* = \theta^*$. If, for example, the match distribution is continuous on $[0, \infty)$, then the wage distribution is continuous as well on the semi-open interval $[w^*, \infty)$.

Now consider the case in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage, m, is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation the firm can adjust so as to "undo" the minimum wage payment requirement.

In terms of wage payments, the minimum wage acts solely as a side constraint on the Nash-bargaining problem. The negotiation problem is identical to the one given in (1) except that the set of feasible wages is now restricted to $[m, \infty)$. Recall that the ex post value of the match from the point of view of the firm is proportional to $(\theta - w)$. Firms cannot earn positive profits on matches that have a value less than m. Because $m > \theta^*$ (i.e., we only consider the case of binding minimum wages), an immediate implication of the imposition of the minimum wage is that fewer contacts will result in jobs—the standard employment effect.[6]

The effect on the solution is relatively intuitive. Under the "constrained" Nash-bargaining problem, there will exist a value of search, which we denote

[6]This implication critically depends on the assumption of a fixed contact rate. We will return to this issue in the following subsection.

$V_n(m)$. If we ignore the minimum wage constraint in determining the wage payment given a match value of θ and the search value $V_n(m)$, we get

$$w(\theta, V_n(m)) = \alpha\theta + (1-\alpha)\rho V_n(m). \tag{3}$$

Under this division of the match value, the worker would receive a wage of m when $\theta = \hat{\theta}$, where

$$\hat{\theta}(m, V_n(m)) = \frac{m - (1-\alpha)\rho V_n(m)}{\alpha}.$$

Then if $\hat{\theta} \leq m$, all "feasible" matches would generate wage offers at least as large as m. When $\hat{\theta} > m$, this is not the case. When θ belongs to the set $[m, \hat{\theta})$, the offer according to (3) is less than m. However, when confronted with the choice of giving some of its surplus to the worker versus a return of zero, the firm pays the wage of m for all $\theta \in [m, \hat{\theta})$. Wages for acceptable θ outside of this set are determined according to (3).

The steady state value of unemployed search given the minimum wage constraint is

$$\rho V_n(m) = b + \frac{\lambda}{\rho+\eta}\left\{\int_m^{\hat{\theta}(m,V_n(m))} [m - \rho V_n(m)]\, dG(\theta) + \alpha \int_{\hat{\theta}(m,V_n(m))} [\theta - \rho V_n(m)]\, dG(\theta)\right\}, \tag{4}$$

where the first term in braces is the surplus value associated with finding an acceptable match $\theta \in [m, \hat{\theta}]$ with an associated wage payment of m and the second term is the surplus associated with finding a match value $\theta > \hat{\theta}$ for which the minimum wage constraint is not binding.

We shall often refer to the value $\rho V_n(m)$ as the "implicit" reservation wage. A binding minimum wage results in a (positive-valued) wedge between the minimal acceptable wage imposed by the policy maker m and the wage offer that a searcher would be willing to take, $\rho V_n(m)$. Conditional on the value of a binding minimum wage m, the equilibrium wage distribution is described by

$$p(w; m) = \begin{cases} \frac{\alpha^{-1} g(\tilde{\theta}(w, V_n(m)))}{\tilde{G}(m)}, & w > m, \\ \frac{\tilde{G}(m) - \tilde{G}(\hat{\theta}(m, V_n(m)))}{\tilde{G}(m)}, & w = m, \\ 0, & w < m. \end{cases} \tag{5}$$

The minimum wage side constraint produces an equilibrium wage distribution that has a mass point at m and has wages continuously distributed on the interval (m, ∞) if match values are themselves continuously distributed.

2.2. *Endogenous Contact Rates*

As in Pissarides (2000), we adopt a highly stylized model of the decision to participate in the labor market. We continue to assume that agents are (ex ante) identical when entering the market and we assume that individuals in the population are heterogeneous in their value of remaining outside of the market. In this case, outside options include schooling, leisure, etc., and because a large proportion of the 16–24 year old population that is the focus of our analysis does not participate in the market, this is an important consideration. From a pragmatic perspective, assuming heterogeneity of this form makes the modeling task more straightforward because differences in the value of participation are independent of labor market outcomes conditional on participation, enabling us to analyze the participation decision and labor market choices independently.[7]

Let the (normalized) value of not participating in the labor market for an individual be given by ρV_o, and assume that in the population this random variable has a parametric distribution $Q(\rho V_o; \zeta)$, where ζ is a finite-dimensional parameter vector. If an agent enters the labor market he or she enters as an unemployed searcher, the normalized value of which is $\rho V_n(m)$ given minimum wage m. Thus all agents in the population with a value of V_o less than or equal to $V_n(m)$ will enter the market under m. It follows that the participation rate is given by $Q(\rho V_n(m); \zeta)$ in the steady state.

We adopt the standard setup (in the macroeconomics literature) for modeling firms' decisions to create vacancies. Let there exist a constant returns to scale matching technology

$$M(\tilde{u}, v) = vq(k),$$

where $k \equiv \tilde{u}/v$, $\tilde{u}$ is the size (measure) of the set of unemployed searchers, and v is the size of the set of vacancies. The contact rate per vacancy is given by $M(\tilde{u}, v)/v = q(k)$, and the contact rate per unemployed agent is $M(\tilde{u}, v)/\tilde{u} = q(k)/k$. We will make a parametric assumption regarding q and write $q(k; \omega)$, where ω is a finite-dimensional parameter vector.

Assuming that there exists a population of potential (firm) entrants with an outside option value of zero, firms create vacancies until the point that ex-

[7]One alternative (or additional) source of heterogeneity could be permanent individual-specific differences in productivity. Let an individual have permanent productivity d, so that their total productivity at a firm is $\theta + d$, where the match value is θ. Let the distribution of d in the population be given by $R(d)$. With search frictions, employers will be able to capture some of the returns to the worker's value of d, thus complicating the solution to the bargaining problem. In addition, changing the values of primitive parameters will affect the distribution of d in the labor market in equilibrium, thus requiring joint estimation of the participation decision and labor market processes. Such an extension may be of questionable value given the extremely limited information content of the Current Population Survey data we use.

pected profits are zero. Let the cost of creating a vacancy be given by $\psi > 0$. Then the expected value of creating a vacancy is given by

$$\rho V_v = -\psi + q(k;\, \omega)\tilde{G}(r)(J - V_v),$$

where r denotes the acceptance match value (equal to the maximum of $\{\rho V_n(m), m\}$), $q(k;\, \omega)\tilde{G}(r)$ is the rate at which a firm fills a vacancy, J is the expected value of a filled vacancy (where the expectation is taken with respect to the distribution of acceptable matches $\theta \geq r$), and V_v is the value of a vacancy. Under the free entry assumption (FEC), $V_v = 0$ and we have

$$0 = -\psi + q(k;\, \omega)\tilde{G}(r)J. \tag{6}$$

Then (6) can be used to solve for an equilibrium number of vacancies given the expected value of a filled vacancy J and the size of the set of unemployed searchers. In the steady state, the probability that a labor market participant is unemployed is given by

$$u = \frac{\eta}{\eta + \tilde{G}(r)q(k;\, \omega)/k}.$$

If we denote the size of the set of labor market participants by l, then the size of the set of unemployed searchers (relative to the entire population) is

$$\begin{aligned} \tilde{u} &= lu \\ &= \frac{\eta l}{\eta + \tilde{G}(r)q(\tilde{u}/v;\, \omega)/(\tilde{u}/v)}. \end{aligned} \tag{7}$$

With endogenous contact rates, a labor market equilibrium in the presence of a minimum wage (that may or may not be binding) is characterized by the quadruplet $(l, u, v, \rho V_n(m))$, which is solely a function of the primitive parameters $(\rho, b, \eta, \alpha, G, \zeta, q, \psi)$. An equilibrium, if one exists, can be constructed by first fixing a value of λ. Let $x \equiv \rho V_n(m)$. Then given λ, (4) determines $x(\lambda)$. The participation rate is then determined as $l(\lambda) = Q(x(\lambda);\, \zeta)$. From (7) we have $\tilde{u}(\lambda) = l(\lambda)\eta/(\eta + \lambda\tilde{G}(\max\{x(\lambda), m\}))$. Finally, $\tilde{u}(\lambda)$ and $J(\lambda)$ are used with (6) to determine $v(\lambda)$. We define $T(\lambda) = q(\frac{l(\lambda)u(\lambda)}{v(\lambda)};\, \omega)/(\frac{l(\lambda)u(\lambda)}{v(\lambda)})$. There exists a unique equilibrium if and only if there exists a unique value λ^* such that $\lambda^* = T(\lambda^*)$. In general, without further restrictions on the parameter space and the functional forms of q, G, and Q, there may exist no or multiple equilibria. Although we have found a number of cases of nonexistence (when m assumes large values), when an equilibrium existed we found it to be unique in the sense that $\lim_{n\to\infty} T^n(\underline{\lambda}^0) = \lim_{n\to\infty} T^n(\overline{\lambda}^0) = \lambda^* \in [\underline{\lambda}^0, \overline{\lambda}^0]$, where $\underline{\lambda}^0$ and $\overline{\lambda}^0$ are "small" and "large" starting values of λ in the successive approximation sequence.

2.3. *On-the-Job Search*

Given the nature of the data available to us and our focus on identification issues, we do not consider on-the-job (OTJ) search in this paper. However, the present framework is extended to include OTJ search in a companion paper (Flinn and Mabli (2005)). In the conclusion, we will indicate how the policy implications derived from estimates of the model without OTJ search are altered when it is allowed. In this subsection, we simply describe the issues that arise when performing this extension.

When one allows OTJ search (as in Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006)), the bargaining environment becomes considerably more complex, because an employed worker can bargain with two potential employers simultaneously (where one is his or her current match). When an employee currently at a job with match value θ and wage rate w meets a new potential employer where his or her match value is θ', we posit a Bertrand-type game in which the employers bid for the services of the employee until the one with the dominated match drops out.[8] Thus, employment decisions are always efficient, and these occasional outbreaks of "bidding wars" serve to transfer significant portions of the match surplus to workers over the course of an employment spell (see Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc, Postel-Vinay, and Robin (2006)). Estimates from these models with OTJ search and renegotiation indicate that low levels of bargaining power, as measured by α, are consistent with observed cross-sectional wage distributions and patterns of wage growth over employment spells.

Allowing OTJ search in the matching function setup involves some modeling choices regarding how unemployed and employed searchers are to be treated in determining the size of the searching population. For example, Pissarides (2000, Chapter 4) assumes that the number of active searchers is simply equal to the sum of the number of employed and unemployed agents. Flinn and Mabli (2005) generalize this by assuming that the effective number of searchers is given by $U + \nu E$, where ν is considered a primitive search technology parameter. This assumption generates differences in the contact rates between potential employers and employed and unemployed searchers that are consistent with partial equilibrium estimates of arrival rates for these two groups of labor market participants. The value of the parameter ν plays an important role in conducting policy experiments that involve the minimum wage, the nature of which we will briefly describe in the conclusion.

[8]When the match at the potential employer exceeds the current match, the wage paid at θ' is determined using the value of a wage payment equal to θ as the worker's outside option. Even when $\theta' < \theta$, so that the worker does not change jobs, he or she may "renegotiate" his or her wage at the current job as long as $\theta' > w$.

3. THE WELFARE IMPACT OF A MINIMUM WAGE

We begin by describing a fairly general framework within which to consider welfare impacts. Because the model is stationary in nature, we will focus our attention on the long run welfare impact; this is consistent with the approach taken by Hosios (1990). At any point in time, all agents who are (potentially, at least) on the supply side of the market will be nonparticipants, others will be unemployed, and the remainder will be employed. On the demand side of the market, some set of agents will not have created vacancies, others will currently be carrying an unfilled vacancy, and others will have a filled vacancy. We will assume that the minimum wage is the only policy instrument available to a social planner and that the planner's objective can be expressed as a Benthamite social welfare function, defined as

$$S(m,a) = a_o T_o(m)\overline{V_o}(m) + a_n T_n(m) V_n(m) + a_e T_e(m)\overline{V_e}(m) + a_f T_f(m)\overline{J}(m), \tag{8}$$

where $T_o(m)$ is the size of the set of individuals who are out of the labor force under minimum wage m and $\overline{V_o}(m)$ is their average welfare level, $T_n(m)$ is the proportion of the population unemployed (all unemployed individuals have the same welfare level $V_n(m)$), and $T_e(m)$ is the proportion of the population who are employed in the steady state with $\overline{V_e}(m)$ denoting their average welfare level. On the demand side of the market, we have ignored the welfare contribution of firms that have chosen not to create a vacancy and those that currently hold one, because the free entry condition implies that the welfare of firms (or potential firms) in either of these classes is zero. Thus the only term from the demand side that enters is associated with firms with a filled vacancy. The size of this set is denoted by $T_f(m)$ and the average value of $J(m)$ in this set is $\overline{J}(m)$.

First consider the size of the set of individuals who do not participate in the market. Given an equilibrium outcome of $\rho V_n(m)$, we have

$$T_o(m) = 1 - Q(\rho V_n(m)) \equiv \tilde{Q}(\rho V_n(m)).$$

Because we assume that ρV_o is continuously distributed in the population, T_o is a monotonically decreasing function of $V_n(m)$. Thus any minimum wage change that results in an increasing value of search leads to greater labor force participation.

The size of the set of searching participants is given by the participation rate multiplied by the steady state probability of unemployment (given participation). Thus

$$T_n(m) = Q(\rho V_n(m)) \times \frac{\eta}{\eta + \lambda(m)\tilde{G}(r)},$$

where $r = \max\{\rho V_n(m), m\}$ and where $\lambda(m)$ signifies that the contact rate (from the searching individual's perspective) depends on the equilibrium participation and vacancy creation decisions that are a function of m (when m is binding).

The size of the set of employed individuals is defined in a similar way:

$$T_e(m) = Q(\rho V_n(m)) \times \frac{\lambda(m)\tilde{G}(r)}{\eta + \lambda(m)\tilde{G}(r)}.$$

In the literature it is common to assume that each firm creates at most one vacancy, which will be filled or unfilled at a given moment in time. In this case, the set of firm owners who have a filled vacancy must be equal to the size of the set of employed searchers, or

$$T_f(m) = T_e(m).$$

We will consider two welfare criteria in what follows. The first is total welfare ($TW(m)$), where $a_o^0 = a_u^0 = a_e^0 = a_f^0 = 1$. The second is participants' welfare ($PW(m)$), in which $a_u^0 = a_e^0 = a_f^0 = 1$ and $a_o^0 = 0$. This is the criterion used by Hosios (1990) in his influential study of labor market efficiency. These criteria would be the same if we were to make an analogous free entry condition for the supply side of the market by normalizing the value of the nonparticipation state to equal zero for all population members. Because we do not want to impose such a condition, we will define a separate welfare measure that applies only to market participants (on both the supply and demand side). This measure will be the focus of our analysis in what follows, both because we can relate optimization of this social welfare function to the Hosios (1990) analysis and because our specification of welfare of nonparticipants on the supply side of the market is essentially arbitrary.

Within a relatively general bargaining framework (consistent with ours in the absence of a binding minimum wage) and with a constant returns to scale matching technology, Hosios (1990) showed that the maximization of aggregate welfare PW required that the effects of search externalities be appropriately recognized in the allocation of surplus shares to the unemployed searchers and vacancy creators. Thus the policy variable of his analysis was α.

Given that we will only be able to estimate extremely simple matching functions given the data available to us, consider the implications of this result when the matching technology is Cobb–Douglas. Then if the elasticity of the matching function with respect to $\tilde{u}$ is ω, the Hosios condition for efficiency is[9]

$$\alpha = \omega.$$

[9]Hosios considers a slightly more general decision problem because he allows both firms and searchers to choose a search intensity. Given the severe econometric identification problems we face, the estimation of a model that allows for the choice of intensity is not possible, even though it would clearly be desirable.

When this condition holds, there is no scope for a minimum wage to increase aggregate welfare as defined by $PW(m)$.

The discussion in Hosios (1990) treats α as a policy variable that can be set by the social planner when designing an optimal compensation scheme. Our perspective is decidedly different, in that we consider α to be set by market forces, preference characteristics, and the generic bargaining environment, all of which are considered to be beyond the purview of the social planner.[10] In the econometric analysis below, a substantial amount of effort is devoted to identification and estimation of α and ω based on Current Population Survey (CPS) data and information taken from the aggregate economy or a private-sector firm. Our estimates strongly suggest that $\alpha \neq \omega$. In this case, a minimum wage may improve welfare, as measured by $PW(m)$ or $TW(m)$, by increasing the "effective" bargaining power of searchers on the supply side and inducing them to increase their participation rate to approach what it would be under $\alpha = \omega$. This increased participation rate can have positive impacts on all market participants by increasing the size of the employed population (which is equal to the size of the set of filled vacancies).

From this perspective, the minimum wage can be viewed as an instrument to increase total welfare of labor market participants or the entire society. It is crude in the sense that its performance will not be as good as equating the values of α and ω. In its favor is the relative ease of implementation. Note that the minimum wage can properly be viewed as a part of a larger policy that entails truncating the wage distribution, whether from below or above. Thus, if it is found that vacancy creators get too low a share of the surplus relative to $(1 - \omega)$, an efficiency-enhancing wage truncation policy could result in wage caps instead of wage floors.

4. ECONOMETRIC ISSUES

The model as formulated relies heavily on stationarity assumptions, which enable us to use what is essentially a cross-sectional data set to estimate a dynamic model. The CPS point-in-time sample contains only information on the length of time individuals currently unemployed have been actively searching for a job and the hourly wage rate and/or gross weekly earnings and usual hours worked per week for those reporting that they are currently employed. No information is available on the length of time an employed individuals have worked for their current employer.

[10]Our axiomatic Nash-bargaining framework provides no explicit mechanism through which α is determined. One possibility for rectifying this situation is to employ a strategic approach to the bargaining problem as in Rubinstein (1982). In the case of an alternating offers game to determine the share of a pie allotted to the two parties, α was found to be a function of the discount rates of the two parties. If this discount rate is considered to be a part of the preference map of the individual parties, then it is not reasonable to consider it a policy variable.

The information in the CPS will not be useful for learning the parameters that characterize the vacancy creation process. That is, there will be an equilibrium value of the contact rate, from the perspective of searchers, that is given by $\lambda = q(k)/k$. Although we will be able to provide conditions under which $\tilde{u}$ is estimable from the CPS data if data on vacancies are not available we cannot determine k. We will show that, given an estimator for λ based on the CPS data and a consistent estimator for $\tilde{u}$, we will be able to estimate the steady state vacancy rate v if q contains no unknown parameters. In this case, estimating v given q or simply estimating λ are equivalent. For this reason, the bulk of our identification analysis is for the case of a given λ. At the end of this section we consider the estimation of parameters directly relevant to the vacancy creation process.

We will begin our analysis by laying out the information content of the CPS data for the purposes of estimating the labor market model developed above. We shall see that a major stumbling block to the estimation of such a model is the identification of the bargaining power parameter α. The difficulties of estimating the bargaining power parameter using event history data such as those employed here have been appreciated for quite some time.[11] A number of studies using aggregated time series data have obtained estimates of α, as well as the matching function in some cases.[12] The major limitation of these studies is the necessity of relying on a representative agent formulation of the economy. This feature is a particular liability when looking at policies with strong distributional impacts, such as the minimum wage. The recent availability of matched employer–employee data has led to the availability of information on firm performance measures as well as employee characteristics. Cahuc, Postel-Vinay, and Robin (2006) use French data of this type to estimate a bargaining model and utilize methods similar to that employed here to identify the bargaining power parameter for different classes of employees.[13] The key to their being

[11]In their discussion of an estimable equilibrium search model, FH impose the assumption that the match value (θ here) is split evenly between the firm and worker, but provide little justification for doing so (pp. 142–143). Kooreman and Kapteyn (1990) provide an enlightening discussion and example of the identication of a "sharing" parameter, much like our α, in the context of a cooperative household labor supply model. Eckstein and Wolpin (1995) provide a deeper discussion of the problem of identifying α when using only supply side data than do FH, and in the end impose the restriction $\alpha = 0.5$ when estimating their model. The same authors revisit this issue (Eckstein and Wolpin (1999)), and in that research use a clever restriction on the model to obtain an upper bound estimate of the black–white difference in the logarithm of the share parameter (akin to our α). As do FH and Eckstein and Wolpin (1995), they assume that wages are a constant proportion of the match value θ, which is not the case in the model estimated here.

[12]The case most comparable to ours is probably that of Yashiv (2003), in which he estimates the bargaining power parameter and a Cobb–Douglas matching function parameter using aggregate time series data from Israel. Macro time series studies can potentially avail themselves of information on vacancies, unemployment rates, wage distributions, and firm profitability.

[13]They use a different matching structure in which the match value is given by $\theta = A \times B$, where A is the fixed productivity characteristic value of the worker and B is the productivity character-

able to identify α is their access to firm profit information, which we shall find indispensable as well.

In most empirical implementations of behavioral search models, an allowance is made for measurement error in the wage data (but not in the duration measures). Allowing for measurement error may be indicated when the sample contains very low wage observations, which without measurement error would imply a perhaps incredibly small reservation wage. When allowing for on-the-job search, which we do not do here, measurement error is required for the model of expected wealth maximization to be consistent with direct job-to-job transitions in which the destination job offers a lower wage than the job that was left.

In the presence of a binding minimum wage, it is not possible to add "classical" measurement error to the model. If the reported wage distribution is assumed to be given by the convolution of the distribution of actual wages and some continuously distributed independent and identically distributed random variable, then even though the true wage distribution is of the mixed continuous-discrete type, the observed wage distribution will be absolutely continuous. Because this is inconsistent with even the most casual inspection of the data (see Figures 1(a) and 1(c), for example), classical measurement error cannot be introduced. Although one could consider allowing for "contamination" in the data, i.e., that some unknown proportion τ of wages is measured with classical error while the remainder is not, the estimation of τ would seem not to be possible without further arbitrary assumptions. As a result, we assume throughout that all observations are measured exactly. Although we have already noted that a large number of observations are "heaped" at focal points, our hope is that the regular pattern of the heaping process will not bias our estimates to any great degree.[14]

istic of the firm, and they allow for on-the-job search and contract negotiation as in Postel-Vinay and Robin (2002) and Dey and Flinn (2005). With this matching structure, they are able to recover estimates of A for the firms in their large sample. This specification of match heterogeneity is particularly useful with these types of data because it is able to generate systematic firm size differences, something which our independent and identically distributed match heterogeneity distribution cannot. Presumably a minimum wage could be introduced into their model as well, and it would have similar impacts to those produced by the models estimated by Eckstein and Wolpin (1990) and van den Berg and Ridder (1998) in that low quality firms would drop out of the market, a result beneficial in terms of the equilibrium wage distribution coming at the cost of a reduction in contact rates.

[14]Flinn and Mabli (2005) extend the framework developed here to encompass the case of on-the-job search. They introduce measurement error, but assume that only a proportion ϖ of observations is contaminated with classical measurement error, and go on to estimate the parameter ϖ and the parameters of the measurement error distribution. The cost of introducing measurement error is that misspecification of its nature typically results in inconsistent estimates of all model parameters. Recently Eckstein, Ge, and Petrongolo (2005) have looked at the question of whether wage observations less than m should be interpreted as measurement error or noncompliance with minimum wage laws.

The likelihood function based on the CPS data is constructed as follows. Each individual observation can be characterized by the pair of observations (t_i, w_i), $i = 1, \ldots, N$. The variable t_i is the length of the ongoing spell of unemployed search, which is positive if the sample member is unemployed at the time of the survey and is otherwise equal to zero. If the individual is employed, the hourly wage w_i is recorded. If the individual is paid an hourly wage, w_i corresponds to the hourly wage rate they report. If the individual is not paid on an hourly basis, the hourly rate is imputed by dividing the gross weekly wage by the usual hours of work per week. This imputation procedure is standard, but is particularly problematic in this application because of the likely undercount of individuals paid exactly the minimum wage. On the positive side, the vast majority of our sample members report an hourly wage rate so that the imputation procedure has to be utilized for less than 20 percent of the sample in any given year.

There are essentially three components of the likelihood function: one for the unemployed, one for the employed paid the minimum wage, and one for the employed paid more than the minimum. Let us first consider the contribution of a sample member who is unemployed with an ongoing unemployment spell of length t. Given being in the unemployed state, the likelihood of finding someone in an ongoing spell of length t in the steady state is given by $r_u(t) = \tilde{F}_u(t)/E(t)$, where $\tilde{F}_u$ is the population survivor function of unemployment durations and $E(t)$ is the population mean duration of unemployment spells. When the population unemployment spell duration distribution is of negative exponential form, it is well known that $r_u(t)$ is equal to the population density. Thus, given unemployment, the density associated with an ongoing spell length of t is

$$f_u(t|u) = \lambda\tilde{G}(m)\exp(-\lambda\tilde{G}(m)t),$$

where the acceptance match value of m presumes the existence of a binding minimum wage. Using standard ergodic results, the steady state probability of unemployment is given by

$$p(u) = \frac{\eta}{\eta + \lambda\tilde{G}(m)},$$

so that the joint likelihood of (t, u) is given by

$$f(t, u) = \frac{\eta\lambda\tilde{G}(m)\exp(-\lambda\tilde{G}(m)t)}{\eta + \lambda\tilde{G}(m)}.$$

Next we consider the likelihood contribution of an individual who is paid the minimum wage m. We will assume that whenever two or more population members are paid the minimum wage m, the minimum wage is binding for their population type. Therefore, the likelihood of being employed and being paid a wage of m is

$$p(w=m,e)=\frac{\lambda\left[\tilde{G}(m)-\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)\right]}{\eta+\lambda\tilde{G}(m)}.$$

The contribution of individuals paid more than the minimum wage is determined as follows. To be paid a wage $w>m$, it must be the case that the match value exceeds $(m-(1-\alpha)\rho V_n(m))/\alpha$. Thus the likelihood of being paid a wage w given employment and $w>m$ is

$$f(w|w>m,e)=\frac{\frac{1}{\alpha}g\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)}.$$

The probability that a sample member is paid a wage greater than m given that he or she is employed is

$$p(w>m|e)=\frac{\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}(m)},$$

so that the likelihood contribution for such an individual is given by

$$\begin{aligned}f(w,w>m,e)&=\frac{\frac{1}{\alpha}g\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)}\frac{\tilde{G}\left(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\tilde{G}(m)}\frac{\lambda\tilde{G}(m)}{\eta+\lambda\tilde{G}(m)}\\&=\frac{\frac{\lambda}{\alpha}g\left(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha}\right)}{\eta+\lambda\tilde{G}(m)}.\end{aligned}$$

The log likelihood for the parameters that describe the labor market environment of a given population can be written as

$$\begin{aligned}(9)\qquad \ln L&=N\left[\ln(\lambda)-\ln(\eta+\lambda\tilde{G}(m))\right]+N_u[\ln(\eta)+\tilde{G}(m)]\\&\quad-\lambda\tilde{G}(m)\sum_{i\in U}t_i+N_M\ln\left(\tilde{G}(m)-\tilde{G}\left(\frac{m-(1-\alpha)\theta^*}{\alpha}\right)\right)\\&\quad-N_H\ln(\alpha)+\sum_{i\in H}\ln\left(g\left(\frac{w_i-(1-\alpha)\theta^*}{\alpha}\right)\right),\end{aligned}$$

where N is the number of individuals in the sample, N_U is the number of unemployed, N_M is the number paid the minimum wage, N_H is the number paid more than the minimum wage, U is the set of indices of sample members who are unemployed, H is the set of indices of sample members who are paid more than the minimum wage, and $\theta^*\equiv\rho V_n(m)$ is the "implicit" reservation wage in this population. Recall that G will be assumed to belong to a parametric family, so that it can be characterized in terms of a finite-dimensional parameter vector τ. Then the parameters that appear "directly" in the log likelihood are λ, η, τ, α, and θ^*. The value θ^* is not a primitive parameter per se, but rather is a scalar that is determined endogenously and is a function of all the parameters of the model, including ρ and b. For the purposes of estimation, however, we shall treat it as a parameter of the model.

4.1. *Model Identification*

Before we begin, let us review the identification results of FH, which are particularly relevant here. They demonstrated that the basic search model with only accepted wage and duration information is fundamentally underidentified, even in the absence of bargaining (i.e., $\alpha = 1$). In particular, they showed that a parametric assumption on G was required, and that only certain distributions with support on a subset of $\mathbb{R}_+$ were identified even under parametric assumptions. In the case we consider here, that of parametric G with support given by $\mathbb{R}_+$, all of this class of distributions satisfy what they term the "recoverability condition." For simplicity, we will assume that θ is lognormally distributed in the population. The lognormal is a common distributional assumption in the analysis of wage and earnings data, and it also is a recoverable distribution. The identification issues and results we present are broadly applicable to other parametric, recoverable distributions with support on $\mathbb{R}_+$.

FH also demonstrated that the parameters (ρ, b) were not individually identified, essentially because they both enter the likelihood function only through the critical value θ^*. Because of this, we can treat θ^* as a parameter to be estimated without loss of generality. When all parameter estimates have been obtained, the functional equation that determines θ^* can be used with point estimates of all identified parameters to "back out" the locus of estimates of (ρ, b). This is the procedure followed here. Thus we do not discuss identification of ρ and b explicitly below, but instead consider identification of θ^* along with the other true primitive parameters of the model.

Let us begin with the easiest problem, that of estimating the rate parameters η and h_U. After some manipulation of first order conditions, we get

$$\hat{h}_U = N_U / \sum_{i \in U} t_i,$$

$$\hat{\eta} = \frac{N_U^2 / \sum_{i \in U} t_i}{N_E}.$$

The estimators of η and h_U are independent of the specification of G. Of course, the specification of G will be critical in "decomposing" h_U into the products of the terms λ and $\tilde{G}(m)$.

Under the assumption that θ is distributed as a lognormal, the cumulative distribution function $G(\theta)$ is given by $\Phi(\frac{\ln(\theta)-c}{d})$, where Φ is the standard normal cumulative distribution function. The lognormal is a log location–scale distribution, in that the cumulative distribution function of $\ln(\theta)$ is characterized by the location parameter c and the scale parameter d. Given knowledge of c and d, we will be able to determine $\tilde{G}(m)$, thus enabling identification of λ given h_u. As a result, we need now only consider identification of (c, d, α, θ^*).

Flinn (2005, Chapter 5) shows that if the distribution G is not a location–scale distribution, then the parameters (c, d, α, θ^*) are identified based on the log likelihood given in (9). There are no location–scale distributions with support $\mathbb{R}_+$ that are recoverable in the sense of FH.[15] However, if we consider distributions with support $\mathbb{R}$, there exist a number of such distributions, most notably the normal. Thus, under an assumption of normality, the model is not identified. It is only the nonlinearity of the mapping between θ and the location and scale parameters c and d that enables identification. As a result, identification, while theoretically possible, is tenuous in practice. In Monte Carlo experiments we have found that in samples of the size used in the estimation reported below, the log likelihood is often monotone in α, with estimated values of α approaching 1, the theoretical upper bound on the parameter. With extremely large sample sizes, on the order of 250,000, estimates faithfully reproduced the population values with little variation across replications. No matter what the sample size, however, the correlation between parameter estimates was extremely high. The conclusion we drew from these exercises was that estimation of model parameters using only supply side data, while theoretically possible, can only be implemented with a sample of size not available to most researchers.

4.2. *Use of Demand Side Information to Identify and Estimate α*

In the end it is not terribly surprising that identification of an equilibrium model using data from only one side of the market is a nearly impossible task. In this section, we explore identification strategies that utilize both the CPS data and some information readily available from the financial statements of publicly held corporations.

The datum we utilize from the firm side of the market is labor's share of revenue at some "representative" firm j, which we denote by $\pi_W^j(m)$. Now

[15]The only location–scale distribution on $\mathbb{R}_+$ is the negative exponential, with cumulative distribution function

$$1 - \exp(-(x - c)/d).$$

However, with access only to censored observations, i.e., accepted wages greater than c, the parameter c is not estimable.

from the model,

$$\pi_W^j(m) = \frac{\sum_{i \in E^j} w(\theta_i; m, \rho V_n(m))}{\sum_{i \in E^j} \theta_i},$$

where E^j is the set of employees of firm j at some arbitrary point in time. Dividing both the numerator and the denominator by the number of employees at the firm, we get the ratio of average wages to average productivity:

$$\pi_W^j(m) = \frac{\overline{w}^j}{\overline{\theta}^j}.$$

If the number of employees at firm j is large enough for the law of large numbers to apply, then

$$\begin{aligned}(10)\qquad \pi_W^j(m) \doteq \Bigg(& mp(w = m) \\ &+ \int_{(m-(1-\alpha)\rho V_n(m))/\alpha} (\alpha\theta + (1-\alpha)\rho V_n(m))\, dG(\theta)/\tilde{G}(m)\Bigg) \\ &\times \left(\int_m \theta\, dG(\theta)/\tilde{G}(m)\right)^{-1}.\end{aligned}$$

If there exists no particular firm j with enough employees for the law of large numbers to be applicable, it must be the case that the average wage share of revenues across all firms in the market, $\sum_j \pi_W^j(m)/K$, assumes a value arbitrarily close to the right-hand side of (10) for a sufficiently large population of firms.

We use (10) to form an estimator of α jointly with γ, where γ denotes all other model parameters. In particular, we recognize that the wage share of firm revenues is a function of all of the primitive parameters of the model, including α. The idea is to form a concentrated likelihood function in which an estimator for α, written as a function of the parameter vector Γ and an observed value of $\pi_W^j(m)$, is substituted into the "unconditional" likelihood function. From (10) we see that any $\alpha \in (0, 1)$ and $\gamma \in \Gamma$ imply a unique value of the wage share of revenues, which we will now call simply π, thus

$$\pi = \pi(\alpha; \gamma, m).$$

The function π has the features (for all m)

$$\lim_{\alpha \to 1} \pi(\alpha; \gamma, m) = 1,$$

$$\lim_{\alpha \to 0} \pi(\alpha; \gamma, m) = \frac{m}{E(\theta | \theta \geq m)} \in (0, 1],$$

and

$$\frac{\partial \pi(\alpha;\gamma,m)}{\partial \alpha} = \frac{\int_{(m-(1-\alpha)\rho V_n(m))/\alpha} (\theta - \rho V_n(m))\, dG(\theta)}{\int_m \theta\, dG(\theta)} \in (0,1).$$

Then $\pi(\alpha;\gamma,m):[0,1] \to [\frac{m}{E(\theta|\theta\geq m)}, 1]$. Furthermore, π is monotone in α. Let the observed value of π be denoted by $\tilde{\pi}$. Then for $\tilde{\pi} \in (\frac{m}{E(\theta|\theta\geq m)}, 1)$ there exists a unique value $\alpha^*(\tilde{\pi};\gamma,m)$ such that

$$\tilde{\pi} - \pi(\alpha^*(\tilde{\pi},\gamma,m);\gamma,m) = 0.$$

The condition required for an interior solution for α to exist is that

$$\begin{aligned} &\tilde{\pi} > \frac{m}{E(\theta|\theta \geq m)} \\ &\Rightarrow \quad E(\theta|\theta\geq m) > \frac{m}{\tilde{\pi}}. \end{aligned}$$

This last condition, which involves $E(\theta|\theta \geq m)$, imposes a restriction on the parameter space Γ, because a subset of the parameters belonging to γ is those that characterize the distribution of θ, and a subset of these parameters appears in the conditional expectation function $E(\theta|\theta\geq m)$. We have another condition that the estimated parameters must implicitly satisfy, of course, which is that $\rho V_n(m) < m$. We impose this condition directly by working with the parameterization

$$\tilde{\gamma} \equiv (\rho V_n(m), \lambda, \eta, G).$$

Under a lognormality assumption regarding G, the distribution of θ is completely characterized by the two parameters (μ, σ^2), so that the parameter space for $\tilde{\gamma}$ is given by

$$\tilde{\Gamma}(m) = (-\infty, m) \times \mathbb{R}_+^2 \times \mathbb{R} \times \mathbb{R}_+,$$

because $\mu \in \mathbb{R}$ and $\sigma^2 \in \mathbb{R}_+$. When we impose the additional condition that $E(\theta|\theta \geq m) > m/\tilde{\pi}$, under the lognormality assumption this implies that

$$\text{(11)} \qquad \exp(\mu + 0.5\sigma^2)\frac{\tilde{\Phi}(z(m)-\sigma)}{\tilde{\Phi}(z(m))} > \frac{m}{\tilde{\pi}},$$

where $z(m) \equiv (\ln(m) - \mu)/\sigma$.[16] Given values of m and $\tilde{\pi}$, let the set of values of (μ, σ^2) that satisfy (11) be denoted by $\Omega(m, \tilde{\pi})$. Finally, define the parameter space

$$\Gamma'(m, \tilde{\pi}) = \Gamma(m) \cap \Omega(m, \tilde{\pi}).$$

[16]See Johnson and Kotz (1970, p. 129).

We form a concentrated likelihood function by substituting our estimator for α into the likelihood (9), which gives us

$$\tilde{L}(\tilde{\gamma}; \tilde{\pi}, m) = L(\tilde{\gamma}, \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)).$$

PROPOSITION 1: *Assume that* $\tilde{\gamma}^0 \in \operatorname{int}(\Gamma'(m, \tilde{\pi}))$ *and* $\alpha^0 \in (0,1)$, *where* $(\tilde{\gamma}^0, \alpha^0)$ *are the true population parameter values and* $\operatorname{int}(X)$ *denotes the interior points of set* X. *Then the maximum likelihood estimators*

$$\hat{\gamma} = \arg \max_{\tilde{\gamma} \in \Gamma'(m, \tilde{\pi})} \ln \tilde{L}(\tilde{\gamma}; m, \tilde{\pi})$$

and

$$\hat{\alpha} = \alpha^*(\tilde{\pi}, \hat{\gamma}, m)$$

converge in probability to $\tilde{\gamma}^0$ *and* α^0, *respectively. The asymptotic distribution of* $\sqrt{N}(\hat{\gamma} - \tilde{\gamma}^0)$ *is normal with mean* 0 *and covariance matrix* $\Sigma_{\hat{\gamma}}$, *which is of full rank. The asymptotic distribution of* $\sqrt{N}(\hat{\alpha} - \alpha^0)$ *is normal with mean* 0 *and variance* $\sigma^2_{\hat{\alpha}}(\Sigma_{\hat{\gamma}})$.

PROOF: The unconcentrated likelihood function $L(\tilde{\gamma}, \alpha)$ satisfies standard regularity conditions for $\tilde{\gamma}^0 \in \operatorname{int}(\tilde{\Gamma}(m))$ and $\alpha^0 \in (0,1)$. In particular, the support of the distribution of the data is not a function of $\tilde{\gamma}^0$ and α^0, although it is a function of the known constant m. The likelihood function is continuously differentiable in both $\tilde{\gamma}$ and α. The implicit function $\alpha^*(\tilde{\pi}, \tilde{\gamma}, m)$ is continuously differentiable in $\tilde{\gamma}$, so the function $\ln L(\tilde{\gamma}, \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)) \equiv \ln \tilde{L}(\tilde{\gamma}; \tilde{\pi}, m)$ is continuously differentiable in γ on the space $\Gamma'(m, \tilde{\pi})$. Continuous differentiability of α^* is sufficient for $\operatorname{plim}_{\tilde{\gamma} \to \tilde{\gamma}^0} \alpha^*(\tilde{\pi}, \tilde{\gamma}, m) = \alpha^0$, and differentiability of $L(\tilde{\gamma}, \alpha)$ in both arguments is sufficient for $\operatorname{plim}_{\alpha \to \alpha^0} \operatorname{plim}_{N \to \infty} \arg\max_{\hat{\gamma} \in \tilde{\Gamma}(m)} \ln L(\tilde{\gamma}, \alpha) = \gamma^0$. Then $\operatorname{plim}_{N \to \infty} \alpha^*(\tilde{\pi}, \hat{\gamma}, m)$.

Whereas the concentrated log likelihood function $\ln \tilde{L}(\gamma; \tilde{\pi}, m)$ satisfies all regularity conditions for asymptotic normality, the asymptotic distribution of $\sqrt{N}(\hat{\gamma} - \tilde{\gamma}^0)$ is $N(0, \Sigma_{\hat{\gamma}})$, where

$$\Sigma_{\hat{\gamma}} = \left(\left. \frac{\partial^2 \ln \tilde{L}(\tilde{\gamma}; \tilde{\pi}, m)}{\partial \tilde{\gamma}\, \partial \tilde{\gamma}'} \right|_{\tilde{\gamma} = \hat{\gamma}} \right)^{-1}.$$

The asymptotic covariance matrix of $\hat{\alpha}$ is

$$\operatorname{Var}(\hat{\alpha}) = \left. \frac{\partial \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)}{\partial \tilde{\gamma}} \right|'_{\tilde{\gamma} = \hat{\gamma}} \Sigma_{\hat{\gamma}} \left. \frac{\partial \alpha^*(\tilde{\pi}, \tilde{\gamma}, m)}{\partial \tilde{\gamma}} \right|_{\tilde{\gamma} = \hat{\gamma}}$$

by applying the delta method. *Q.E.D.*

As we shall see below, the use of wage share information from a firm or firms in conjunction with the CPS data allows us to obtain precise and credible estimates of all key model parameters. We do face a problem with this information when we attempt to estimate the model for subpopulations, however. Because we do not have wage and productivity information disaggregated by subpopulation, we are forced to assume that this ratio is the same across all subpopulation members.[17] This is a strong assumption that probably largely accounts for the similarities we observe in the estimates of α across subpopulations.

4.3. *Estimation of Demand Side Parameters*

In this section, we discuss the identification and estimation of demand side parameters, which in the case of this highly stylized model consists of estimating the parameters ψ, the cost of a vacancy, and q, the matching function.[18] We show below that to be able to perform policy experiments, we also must have a consistent estimator of the distribution of "outside options," that is, the parameter ζ that indexes the distribution function Q. Identification and estimation of the parameters ζ, q, and ψ build on the consistent estimators of the parameters we have already considered, namely λ, η, G, b, and α. We begin the section by discussing identification of these parameters when only cross-sectional CPS information is available. The second subsection contains a discussion of identification when the CPS information from the months spanning the change in minimum wage rates is utilized.

4.3.1. *Cross-Sectional Information Only*

We begin by considering identification of the parameter vector ζ that is assumed to characterize the outside option distribution Q. The labor force participation rate is determined within the model by

$$l = Q(\rho V_n(m); \zeta).$$

Clearly, knowledge of the value of $\rho V_n(m)$ and one equilibrium value of l enables us to uniquely determine at most a scalar-valued ζ. Sufficient conditions

[17]Another alternative would be to collect aggregate wage share information by firm and to estimate the subpopulation-specific share using variability in subpopulation employment rates across firms. The use of such a procedure raises questions regarding the source of the variability in subpopulation employment rates across firms or industrial sectors and also faces the problem that few firms report wages as a separate item in their Consolidated Statement of Income.

[18]An early version of Ahn, Arcidiacono, and Wessels (2005) was the first to contain estimates of a general equilibrium model of search with minimum wages that included the matching function setup. Identification issues are much different in the two approaches, because the model that they estimate is static and access to data containing a large number of equilibrium outcomes, each from a state-level labor market, is assumed.

for identification of the scalar ζ are continuous differentiability of the cumulative distribution function Q, both in its argument and in the parameter ζ, and monotonicity of $Q(x;\zeta)$ in ζ for all values x in the support of Q. In this case, we can invert Q to obtain

$$\zeta = Q_\zeta^{-1}(\rho V_n(m); l)$$

for all $\rho V_n(m) \in \text{supp}(Q)$ and $l \in (0,1)$.

From supply side information, we are able to obtain a consistent estimator of $\rho V_n(m)$: $\widehat{\rho V_n(m)}$. The CPS is a representative sample of U.S. households, so from the CPS data we are able to obtain an unbiased and consistent estimator of the participation rate of our target population $\hat{l}$. Whereas Q is assumed to be continuously differentiable in both arguments over its support and the parameter space, Q_ζ^{-1} is continuously differentiable in both arguments as well. Then the estimator

$$\hat{\zeta} = Q^{-1}(\widehat{\rho V_n(m)}, \hat{l})$$

is consistent by the invariance property of maximum likelihood estimators.

Next consider identification of the matching function q. We assume that q is known up to a finite-dimensional parameter vector ω. Then we have

$$\lambda = q\left(\frac{ul}{v}; \omega\right)\frac{v}{ul}. \tag{12}$$

Assuming that λ, ul, and v are known, then from a single steady state equilibrium at most a scalar ω can be uniquely determined. Recalling that $k \equiv ul/v$, sufficient conditions for the identification of the scalar ω are continuous differentiability of q in k and ω, and monotonicity of $q(k;\omega)$ in ω for all $k \in \mathbb{R}_+$.[19] Then we can define the inverse

$$\omega = q_\omega^{-1}(\lambda k, k),$$

where q_ω^{-1} is continuously differentiable in both of its arguments.

From the augmented CPS data we can consistently estimate the contact rate parameter λ. The proportion of the population who are unemployed at a point in time, ul, can be consistently estimated by the CPS proportion of sample members in that state. Then if v is known, or at least a consistent estimator for the vacancy rate is available, consistent estimators for λk and k are available, meaning that the estimator $\hat{\omega} = q_\omega^{-1}(\widehat{\lambda k}, \hat{k})$ is consistent as well.

[19]One of the most common functional form choices for the matching function is the Cobb–Douglas, for which $q(k;\omega) = k^\omega$, with $\omega \in (0,1)$. In this case, the monotonicity property is satisfied for all $k \neq 1$.

Knowledge of the function q is sufficient to identify the cost of vacancy parameter ψ. Recall that the free entry condition can be written

$$\psi = q(k;\omega)\tilde{G}(r)J(\rho V_n(m), m, G, \rho, \eta, \alpha), \tag{13}$$

where J is the expected value of a filled job. Because the primitive parameters of the model include G and because J is determined by the model structure, then knowledge of the arguments of these three functions is sufficient to determine ψ. After assuming that ρ is known a priori, we have provided sufficient conditions for the existence of consistent estimators of all of the other arguments of the functions. Under these conditions, augmented CPS data are sufficient to estimate ψ consistently.

Unfortunately, information on v cannot be assumed to be (directly) available. Empirical studies of the matching function, particularly those conducted using U.S. data, typically use a vacancy index constructed from help wanted advertising (see Petrongolo and Pissarides (2001) for a survey of these studies). Without information on v, the parameter ω cannot be identified, so that q is unknown and ψ cannot be determined from (13). As a result, the model can only be identified using augmented CPS information if ω is known a priori. Rather than assume a given value of the parameter(s) that characterizes a particular function, such as the Cobb–Douglas, in the empirical work conducted below we assume a constant returns to scale (CRS) function that has no unknown parameters. In this case, q is known (completely) due to the functional form assumption.

In the case of a known q, it is rather straightforward to identify the vacancy flow cost parameter ψ. In terms of the matching function M, (12) can be written $\lambda\tilde{u} = M(\tilde{u}, v)$. Because $\tilde{u}$ and v are both productive factors in producing matches, M is monotone in each (holding the other factor fixed). With knowledge of λ and $\tilde{u}$, there then exists a unique value of v such that (12) is satisfied. Knowledge of M and consistent estimators of λ and $\tilde{u}$ enable us to form a consistent estimator of the vacancy rate $\hat{v}$. Substituting $\hat{v}$ into (13) along with consistent estimators of the other unknown parameters produces a consistent estimator of ψ.

In summary, using cross-sectional information from the CPS and labor share data, we can recover all of the parameters required to perform policy evaluation under the assumption of endogenous contact rates only if we assume that the matching function contains no unknown parameters. This stringent assumption is required due to the absence of credible measures of vacancy rates.

4.3.2. *Multiple CPS Cross Sections*

We now consider identification issues when we use the CPS Outgoing Rotation Group (CPS-ORG) samples from two months, $t < t'$, in which we assume that a steady state equilibrium (approximately) holds in each and where the minimum wages are different. Let the earlier minimum wage be less than the later one, and denote the minimum wage in period t by m_t. All primitive parameters are assumed to be identical in the two months.

We limit our attention to the case in which the matching function is Cobb–Douglas, so that

$$M(ul, v; \omega) = (ul)^{\omega} v^{1-\omega} \qquad (\omega \in (0,1))$$
$$\Rightarrow \quad q\left(\frac{ul}{v}; \omega\right) = \left(\frac{ul}{v}\right)^{\omega}.$$

Pragmatically speaking, this is not a severe restriction, because M is assumed to be CRS and because we know that it will be difficult to precisely estimate more than one parameter that characterizes M.

We know that the two different minimum wages will, in general, deliver different equilibrium outcomes. In terms of the contact rate parameters from the perspective of unemployed searchers, we have

$$\lambda_s = \left(\frac{(ul)_s}{v_s}\right)^{\omega-1}$$
$$\Rightarrow \quad v_s = \lambda_s^{1/(1-\omega)}(ul)_s \qquad (s = t, t').$$

Substituting this expression for v_s into (13) at time s, we have

$$\psi = \lambda_s^{-\omega/(1-\omega)} \tilde{G}(m_s) J_s, \tag{14}$$

where J_s is the equilibrium value of a filled vacancy under m_s. Whereas the primitive parameter ψ is fixed over time, we have

$$\lambda_t^{-\omega/(1-\omega)} \tilde{G}(m_t) J_t = \lambda_{t'}^{-\omega/(1-\omega)} \tilde{G}(m_{t'}) J_{t'}$$

at the two sets of equilibrium values. From this we have

$$\omega = \frac{A_{t,t'}}{1 + A_{t,t'}},$$

where

$$A_{t,t'} \equiv \frac{\ln \tilde{G}(m_{t'}) - \ln \tilde{G}(m_t) + \ln J_{t'} - \ln J_t}{\ln \lambda_{t'} - \ln \lambda_t}. \tag{15}$$

Knowledge of all components of $A_{t,t'}$ is sufficient for identification of the Cobb–Douglas parameter ω. Given knowledge of ω, ψ is determined using the (14) with either the period t or t' equilibrium values.

From the augmented CPS data, consistent estimators for J_s, λ_s, and $\tilde{G}(m_s)$, $s = t, t'$, are available. There is one complication in immediately asserting that $\hat{\omega} \equiv \hat{A}_{t,t'}/(1 + \hat{A}_{t,t'})$ is a consistent estimator of ω, which is that positivity of $\hat{A}_{t,t'}$ is not assured. When $\hat{A}_{t,t'} < 0$, the estimator $\hat{\omega} \notin (0, 1)$, which is the relevant parameter space. We will say that a valid estimator of ω exists under the following condition.

CONDITION C—Coherency: Let $\hat{\lambda}_s$, $\hat{J}_s$, and $\widehat{\tilde{G}(m_s)}$ be consistent estimators of the rate of arrival of contacts, the value of expected value of a filled job, and the probability of drawing a match value greater than the minimum wage, respectively, in the equilibrium associated with minimum wage m_s. Then two estimated equilibria are *coherent* if

$$\hat{\lambda}_t^{-\omega/(1-\omega)} \widehat{\tilde{G}(m_t)} \hat{J}_t = \hat{\lambda}_{t'}^{-\omega/(1-\omega)} \widehat{\tilde{G}(m_{t'})} \hat{J}_{t'}$$

for some $\omega \in (0, 1)$.

This condition is minimal in the sense that there are other restrictions on parameters that are implied by the model that are not imposed or checked in forming estimators of the demand side parameters. Model estimates may violate Condition C for a variety of reasons. Perhaps most obviously, even given the correct specification of all other model components, we may have misspecified the matching function M. Another problem is that the model structure forces us to consider equilibria not too temporally distant. Anticipation of the impending minimum wage change after time t could have led the economy out of a steady state equilibrium at time t. At the same time, the economy may not have reached its new steady state equilibrium at the time of the next measurement, t'. As we will see below, the greatest impediments to the satisfaction of Condition C are the small changes in the minimum wage rates and the small proportion of individuals directly impacted.

Given satisfaction of Condition C, $\hat{\omega}$ is a consistent estimator of ω. It follows immediately that a consistent estimator of ψ is given by

$$\hat{\psi} = \hat{\lambda}_s^{-\omega/(1-\omega)} \widehat{\tilde{G}(m_s)} \hat{J}_s \qquad (s = t \text{ or } t').$$

To summarize the results of this section, we have shown that when we have data from two CPS surveys in which data outcomes are generated from steady state equilibrium behavior associated with two values of the minimum wage, a Cobb–Douglas matching function parameter and the cost of a vacancy can be consistently estimated if estimates of a function of other primitive parameters at the two equilibrium values satisfy a condition that restricts the estimator of the Cobb–Douglas parameter to the unit interval. Although we can never expect the observations in the two CPS surveys to be exactly generated by two distinct steady state equilibria, the hope is that the approximation may be good enough to yield a credible estimate of the matching function parameter.

5. EMPIRICAL RESULTS

The data used contain information on 16–24 year olds (inclusive) from the Outgoing Rotation Group monthly Current Population Survey samples. We focused attention on this age group because minimum wage workers are disproportionately young; our feeling was that if minimum wage effects on welfare were determined to be small for this group of participants, they are likely to be even more insignificant for older individuals. The fact that the sample is restricted to this age group should be borne in mind when interpreting the parameter estimates and particularly when the welfare exercise results are reported.

The months used in the analysis are September 1996, February and August 1997, and January 1998. As we made clear in the previous section, identification of matching function parameters clearly depends on our having access to CPS data at more than two points in time. In September 1996 we assume that the market was in a steady state equilibrium at the minimum wage \$4.25, which was due to expire at the end of the month. When moving to the new steady state equilibrium associated with the minimum wage of \$4.75, there will be some transitional dynamics. To avoid capturing these effects, we will perform "pooled" estimation exercises using data from 5 months after the change, when we hope the market has nearly reached its new steady state equilibrium. We cannot allow for even more separation in time between the "before and after" observations because (a) the minimum wage changes once again on September 1, 1997 and (b) the model specification allows for no changes in the price level (the change in the Consumer Price Index between September 1996 and February 1997 was about 1.1 percent).

To examine the sensitivity of our estimates to the particular months chosen, we reestimate the model when the subsequent minimum wage change occurs at the beginning of September 1997. Unfortunately, the month before the change is August, a period when many students find temporary employment. This phenomenon is particularly at odds with the steady state equilibrium basis of our econometric model. The postchange month used in the pooled estimation exercise is January 1998, which also may exhibit some transitory labor market behavior in conjunction with the holiday period. For these reasons, we will not devote as much attention to the estimates generated using these months. Nevertheless, it is encouraging to note that the estimates of most primitive parameters are stable across these two periods of minimum wage change.

Some statistics that describe the data are presented in Table I. Importantly for the prima facie validity of our model, in all four months there exists a mass point at the minimum wage, with the probabilities given by 0.053, 0.063, 0.047, and 0.075. In terms of mass point rank in each month, these are the fourth,

TABLE I
DESCRIPTIVE STATISTICS CPS-ORG INDIVIDUALS AGED 16–24
(STANDARD DEVIATIONS IN PARENTHESES)

Characteristic	Month 9/96	2/97	8/97	1/98
N	3,199	3,236	3,117	3,150
N_o/N	0.365	0.369	0.307	0.375
$N_u/(N_u+N_e)$	0.097	0.119	0.082	0.104
Female	0.505	0.522	0.518	0.510
Black and Hispanic	0.252	0.248	0.274	0.254
t_u	3.411	2.889	2.612	2.515
	(4.973)	(3.230)	(1.006)	(3.576)
w	6.932[a]	7.016	7.254	7.513
	(3.480[a])	(3.874)	(4.361)	(4.178)
$\chi[w=4.25\|e]$	0.053	0.022	0.013	0.002
$\chi[w=4.50\|e]$	0.020	0.012	0.006	0.002
$\chi[w=4.75\|e]$	0.024	0.063	0.047	0.007
$\chi[w=5.00\|e]$	0.125	0.118	0.101	0.053
$\chi[w=5.15\|e]$	0.002	0.002	0.010	0.075
$\chi[w=5.25\|e]$	0.023	0.039	0.029	0.050
$\chi[w=5.50\|e]$	0.035	0.046	0.056	0.054
$\chi[w=6.00\|e]$	0.083	0.092	0.092	0.092
$\chi[w=7.00\|e]$	0.062	0.061	0.069	0.060
$\chi[w=8.00\|e]$	0.047	0.046	0.052	0.046
$\chi[w=9.00\|e]$	0.019	0.027	0.023	0.031
$\chi[w=10.00\|e]$	0.031	0.028	0.024	0.032
Total at these values	0.524	0.556	0.522	0.505
$\chi[w<m\|e]$	0.054	0.052	0.054	0.041

[a]One extremely large wage observation was deleted when computing the descriptive statistics for this month.

third, sixth, and second, respectively. The larger mass point sizes in each month correspond to natural reporting focal points. We also note that unemployment increases after each minimum wage change (comparing September 1996 to February 1997 and August 1997 to February 1998). Average wages also increased following minimum wage changes, but one should bear in mind that these are nominal amounts and that the composition of the workforce tends to change between summer and nonsummer months.

Before discussing model estimates, it is necessary to point out a few restrictions that were imposed on the sample information. For reasons cited above, classical measurement error in wages could not be introduced. That means that the model is not consistent with observed wages below the minimum wage. We dealt with this problem in two ways. The first method involved rounding all wages below the minimum up to the minimum wage, which adds approximately

0.05 of probability mass at m. Under the second method, observations with w less than m were simply deleted from the sample. We found that estimates of the primitive parameters did not vary much across the sample information modified in these two ways, so we report estimates from the first method only.

Estimates of primitive parameters, and particularly the bargaining power parameter α, were found to be highly sensitive to outliers in the wage data. As a result, we rounded all wage observations greater than \$30 per hour down to \$30. This affected only about 10 cases in any given month. No other modifications of duration or wage information were made.

In Section 4.2, we described an estimator that utilized the same CPS data that has been employed to this point and one additional piece of demand side information, the ratio of total wages paid to firm revenues. It goes without saying that our model is highly stylized and that in reality there is no single value of this ratio when looking across firms in the economy at any point in time. We have opted to use the value (or a reasonable approximation of it) from one large U.S. corporation, McDonald's. This firm was selected for a number of reasons. First, it is the largest private sector employer in the United States, so the law of large numbers results that we relied upon in Section 4.2 are appropriate. Second, this firm has a disproportionate number of employees from the age group in which we are interested, those between the ages of 16 and 24. Moreover, a large number of its employees are paid at or near the minimum wage. Third, McDonald's is one of the few large employers in this sector that reports sufficiently disaggregated information in its consolidated statement of income (CSI) to allow computation of the number in which we are interested.

To obtain a value for the wage share at McDonald's, we have used information from the company's 1996 CSI, which covers a period that includes the survey month of the CPS we are using (September 1996). For purposes of homogeneity, we have restricted attention to revenues and costs associated with restaurants owned by the company (i.e., we have excluded franchises). We computed our value of "net" revenue by subtracting "costs of food and packaging" plus "occupancy and other operating expenses" from "total sales by company-operated restaurants." The numerator of the ratio is given by "payroll and other employee benefits." We found the value of this ratio to be 0.576 in 1996. It is worthwhile to note that the ratio seems to be relatively stable over time. For example, using the same computational method, the ratio was equal to 0.568 in 1995 and 0.560 in 1994.

Table II contains estimates of the model with the profit information included. The first column of the top panel contains the estimates for the entire sample with those employed at a wage less than \$4.25 excluded. The estimated value of λ, 0.309, implies that offers arrive approximately every 3 months on average. The dismissal rate estimate of 0.031 implies that the average length of a job is about 30 months. We should bear in mind that these estimates apply to young participants, so the relative brevity of a job match is not surprising. The point estimates of the lognormal match distribution parameters μ and σ are 2.301

TABLE II

MODEL ESTIMATES WITH PROFIT INFORMATION SEPTEMBER 1996 AND AUGUST 1997 (STANDARD ERRORS IN PARENTHESES)

Parameter	Demographic Group				
	All	Males	Females	White Non-H	Black or H
			September 1996		
λ	0.309	0.278	0.356	0.328	0.300
	(0.023)	(0.027)	(0.039)	(0.030)	(0.040)
η	0.031	0.030	0.034	0.026	0.055
	(0.003)	(0.004)	(0.005)	(0.003)	(0.010)
μ	2.301	2.342	2.273	2.331	2.150
	(0.036)	(0.044)	(0.057)	(0.040)	(0.092)
σ	0.528	0.554	0.479	0.504	0.627
	(0.020)	(0.026)	(0.028)	(0.021)	(0.053)
$\rho V_n(m)$	3.093	3.313	2.798	2.901	3.572
	(0.146)	(0.173)	(0.273)	(0.202)	(0.165)
α	0.424	0.427	0.429	0.437	0.383
	(0.007)	(0.006)	(0.013)	(0.0122)	0.106
N	2,022	1,049	973	1,612	410
$\ln L$	−5,065.676	−2,739.338	−2,582.245	−3,972.758	−1,072.138
			August 1997		
λ	0.447	0.397	0.516	0.554	0.341
	(0.036)	(0.043)	(0.063)	(0.056)	(0.046)
η	0.034	0.032	0.037	0.033	0.047
	(0.004)	(0.005)	(0.006)	(0.005)	(0.008)
μ	2.218	2.318	2.104	2.252	2.167
	(0.043)	(0.047)	(0.078)	(0.047)	(0.089)
σ	0.622	0.594	0.646	0.617	0.581
	(0.024)	(0.028)	(0.040)	(0.026)	(0.048)
$\rho V_n(m)$	3.944	3.981	3.960	3.857	4.11
	(0.080)	(0.116)	(0.104)	(0.103)	(0.136)
α	0.375	0.388	0.355	0.387	0.336
	(0.006)	(0.006)	(0.007)	(0.005)	(0.009)
N	2,151	1,112	1,039	1,650	501
$\ln L$	−5,179.436	−2,772.805	−2,379.684	−3,960.016	−1,188.202

and 0.528, respectively, and their standard errors of 0.036 and 0.020 indicate that they are precisely estimated. The point estimate of the implicit reservation wage is 3.093 and its estimated standard error is 0.146.

Of most interest to us is the estimate of the bargaining power parameter. The point estimate of α is 0.424 and the estimated standard error of α is 0.007. With this level of precision, it is straightforward to reject the null hypothesis of symmetric Nash bargaining. The estimate clearly indicates that this class of worker is at a disadvantage with respect to extracting match surplus. Use of the profit data contains a large amount of information regarding the size of the pie to be divided and results in what we consider a reasonable estimate under our modeling assumptions.

There now exist a few other estimates of the bargaining power parameter obtained using disaggregated data. Cahuc, Postel-Vinay, and Robin (2006) obtain a much lower estimate of the bargaining power parameter, as do Dey and Flinn (2005). The main difference in the modeling approaches taken in those papers with the one followed here is in the allowance for on-the-job search. Both papers allow for bargaining between competing firms for an individual's services when a current employee of one firm meets another potential employer. With relatively frequent bidding wars, this mechanism suffices to transfer much of the match surplus to employees even with a low value of α.[20] Generally speaking, it appears that bargaining models that allow for on-the-job search and interfirm competition for labor services produce quite low estimates of the bargaining power parameter.[21] Models without on-the-job search do not have this transfer mechanism and must rely on larger values of α to fit observed wage distributions.

The other columns in the top panel of Table II demonstrate the relative stability of the α estimates across demographic groups. As was discussed in the Section 4.2, this near constancy is largely an artifact of our estimation procedure. Because productivity and wage information is not available by demographic group in the company's CSI, we have had to assume that the same ratio applied to all groups. This is the key piece of identifying information in determining $\hat{\alpha}$ and, as a result, near constancy of these estimates is almost guaranteed.[22]

Even if the group-specific values of $\hat{\alpha}$ are relatively similar, because of the high degree of precision with which they are estimated, there do exist statistically significant differences between them. For example, youths who are black or Hispanic are estimated to have a slightly lower level of bargaining power than others (0.383 versus 0.437). They also have a slightly lower estimated value of the mean and standard deviation of the logarithm of the match value. When we divide the sample along gender lines, we find that males are estimated to have a slightly higher degree of bargaining power than females, as well as having higher values of μ and σ.

[20] In fact, in Postel-Vinay and Robin (2002), employees are assumed to have *no* bargaining power, but due to the Bertrand game played between competing firms, the model is able to fit the data quite well.

[21] Yashiv (2003) is an exception. Using time series data from Israel, he finds a low estimate of α in a model without on-the-job search. The fact that his result is obtained using data gathered at a much higher level of aggregation and that α is identified from time series variation makes comparison with the other studies difficult.

[22] Cahuc, Postel-Vinay, and Robin (2006) are able to estimate group-specific values of α given their access to employer–employee matched data and their assumption that match values are the product of the idiosyncratic productivity values of an employer and an employee.

The estimates in the bottom panel of Table II are broadly consistent with those in the top panel. As noted previously, due to the lower aggregate unemployment rate during August 1997, $\hat{\lambda}$ is substantially higher using this sample. The estimate of $\rho V_n(4.75)$ is significantly greater than the estimate of $\rho V_n(4.25)$, which is due not only to "real" minimum wage impacts, but also to inflation. Inflation may also be contributing to the differences in the estimates of the match distribution parameters, particularly σ, in the top and bottom panels. Estimates of α are substantially lower in August 1997 than in September 1996. This is partially produced by the change in the shape of the estimated match distribution. It may also be due to composition effects in that the August sample includes more seasonal workers than the September one does, and it may be that this group of workers has less bargaining power than their less seasonal counterparts. We do not want to make too much of this point, because it is at odds with our assumption of a homogeneous population facing fixed primitive parameters. The differences we saw across demographic groups in the top panel are broadly mirrored in the estimates obtained using the August 1997 data.

In Table III we use the model to determine directly whether or not the minimum wage increase that occurred at the end of September 1996 was beneficial in the sense of our first welfare measure, the value of unemployed search. In this analysis, we estimate the model only for the total sample, and we begin by combining the September 1996 CPS-ORG sample with an analogous sample from the February 1997 survey. Given the sample design, these two surveys do not include the same individuals. In combining the surveys, we assigned wages as follows. For individuals who earned less than \$4.25 in September 1996, we assigned them a wage of \$4.25, the then current minimum. Similarly, we assigned all those who were making less than \$4.75 an hour in February 1997 a wage rate of \$4.75.

Before turning to the results, we must present a word of caution in terms of the interpretation of the estimates. Strictly speaking, the consistency of our estimator in this case depends on the search environment remaining stationary in the wake of the minimum wage change, so that all "primitive" parameters assume the same values in September 1996 and February 1997. There are a number of reasons why this assumption may be questionable; we shall consider a limited test of it below.

For our inferences to be valid requires constancy of environmental parameters. In the absence of endogenous contact rates and a labor market participation decision, no other assumptions are required to obtain valid estimates of primitive parameters and the value of search before and after the minimum wage change as long as the implicit reservation wage after the minimum wage change is no larger than the initial minimum wage rate. In terms of our application, we require $\rho V_n(4.75) < 4.25$. If this is the case, then even under the old minimum wage, match values in the interval $[4.25, 4.75)$ will be accepted because working at the wage associated with such a θ given $m = 4.25$ has a higher

TABLE III

MODEL ESTIMATES WITH PROFIT INFORMATION SEPTEMBER 1996 AND FEBRUARY 1997 (STANDARD ERRORS IN PARENTHESES; $N = 4{,}059$)

	Specification		
Parameter	A	B	C
$\lambda(4.25)$	0.355	0.356	0.353
	(0.018)	(0.018)	(0.022)
$\lambda(4.75)$	—	—	0.404
			(0.022)
η	0.039	0.039	0.039
	(0.003)	(0.003)	(0.003)
μ	2.253	2.244	2.243
	(0.028)	(0.029)	(0.029)
σ	0.575	0.578	0.579
	(0.015)	(0.016)	(0.016)
$\rho V_n(4.25)$	3.415	3.382	3.383
	(0.079)	(0.083)	(0.083)
$\rho V_n(4.75)$	—	3.510	3.511
		(0.096)	(0.096)
α	0.403	0.404	0.404
	(0.003)	(0.003)	(0.003)
$\ln L$	−10,118.373	−10,117.152	−10,117.124
LR test		2.442	0.056
p value		0.118	0.813

value than waiting to search under the new minimum wage. Point estimates from the model indicate that this restriction is not unreasonable.

By using information from a sample collected four to five months after the minimum wage change, it is hoped that we can ignore transitional dynamics in terms of the unemployment duration distribution. Whereas it is straightforward to adjust the likelihood function for transitional dynamics given exogenous λ and a fixed labor force, this is decidedly not the case under endogenous contact rates and participation decisions. Thus we have chosen a period long enough for us to hope that most transitional dynamics are absent and short enough to hope that price level changes can be ignored.

We estimated the model under three specifications. In specification A, it was assumed that all primitive parameters remained fixed between September and February, except for b, and that the rate of contacts between searchers and firms remained fixed. We say that b is allowed to change because we are also imposing constancy in the value of search across the two months, and because the minimum wage changed and all other parameters are fixed, unless b changes this cannot be the case under the model structure. Specification B allows the value of search to change between periods, but imposes constancy of the contact rate λ. Under specification C, both the contact rate and the value of search are allowed to change after imposition of the new minimum wage.

Comparisons between the estimates under the three specifications reveal few significant differences. Holding the contact rate fixed, the value of search does seem to have increased slightly between September and February, with the likelihood ratio statistic having a p value of 0.118 under the null of no change. Comparing columns B and C, there is absolutely no indication that the rate of contacts changed over the two months. On the basis of these results we cannot reject the hypothesis that λ is a primitive parameter, in the sense that it was seemingly invariant to at least this change in the minimum wage. We will come back to this point below, because assessing the policy impacts of the minimum wage will depend critically on estimates of α and the determination of λ.

We repeated the same exercise for the minimum wage change of September 1, 1997, and present the results in Table IV. We pooled monthly data from August 1997 and January 1998 in the estimation, and recoded all positive August wages less than \$4.75 to \$4.75 and all positive January wages less than \$5.15 to \$5.15. As mentioned above, results from this exercise are particularly troublesome to interpret due to seasonal employment in August and, to a lesser extent, in January. This is evidenced by the marked drop in the labor market

TABLE IV

MODEL ESTIMATES WITH PROFIT INFORMATION AUGUST 1997 AND JANUARY 1998 (STANDARD ERRORS IN PARENTHESES; $N = 4{,}111$)

	Specification		
Parameter	A	B	C
$\lambda(4.75)$	0.488	0.491	0.506
	(0.028)	(0.029)	(0.035)
$\lambda(5.15)$	—	—	0.476
			(0.033)
η	0.040	0.040	0.040
	(0.003)	(0.003)	(0.003)
μ	2.165	2.157	2.158
	(0.038)	(0.039)	(0.038)
σ	0.670	0.672	0.672
	(0.020)	(0.020)	(0.020)
$\rho V_n(4.75)$	4.056	4.033	4.032
	(0.055)	(0.059)	(0.059)
$\rho V_n(5.15)$	—	4.118	4.116
		(0.072)	(0.071)
α	0.370	0.371	0.371
	(0.005)	(0.006)	(0.005)
$\ln L$	−10,007.255	−10,006.487	−10,006.155
LR test		1.536	0.664
p value		0.215	0.415

participation rate between August and January, a drop that is not consistent with the small change in the value of search (which determines the participation rate within the model).

Once again, the estimates vary little across specifications. Although the point estimate of the implicit reservation wage did increase, the p value associated with the test statistic is only 0.215. There is no indication that the contact rate λ changed between the two months. Thus the results we obtain are consistent across the two pooled samples. Whereas the value of search seems to have increased to some degree, the change is at best marginally significant. There is no indication that the parameter λ is endogenous, *at least over the range of values of m observed in this time period.*

We now move on to obtain estimates of the other parameters that characterize the model, namely, the flow cost of a vacancy, ψ, the Cobb–Douglas matching parameter (estimable in principle when we have access to observations of the steady state equilibrium outcomes under two minimum wages), and the parameter(s) that characterizes the distribution of outside (the labor market) values. In forming the estimates of the identifiable primitive parameters, we will begin with those that use data from a single cross section. In this case, we showed that a one-parameter outside option value distribution could be identified along with the cost of a vacancy, but the matching distribution could have no unknown parameters. We then utilize the pooled cross-sectional information to consider the estimation of the same parameters, with the addition of the single unknown parameter from the Cobb–Douglas matching function. All estimation results are contained in Table V, including standard errors. As a rule, standard errors are small in relation to the size of the point estimate. Exceptions are estimates of the flow vacancy cost across all three specifications and the estimated match elasticity under the Cobb–Douglas matching function specification. Given the relatively "indirect" manner in which these parameters are estimated, the imprecision of these particular estimates is not surprising. Because the policy experiments reported below are computed using the point estimates of these parameters, one should bear in mind the imprecision in some of these estimates when interpreting the results.

The first column of Table V contains estimates of the parameters ψ and ζ, which is the parameter of the negative exponential distribution of out of the labor force valuations. Whereas the data used are from the single month of September 1996, a known matching function must be assumed. We use the CRS form

$$M(ul, v) = v(1 - \exp(-ul/v)). \tag{16}$$

We will compute the vacancy rate v and then evaluate the elasticity of the matching function at the September 1996 equilibrium values of u, l, and v.

The estimated vacancy rate in September was 0.020, which should be compared with the unemployment rate (in the population) of 0.062. The matching

TABLE V

POINT ESTIMATES OF REMAINING PARAMETERS (STANDARD ERRORS[a] IN PARENTHESES)

Characteristic	Single CPS Samples 9/96	8/97	Pooled Sample 8/97 and 1/98
Vacancy rates			
v_t	0.020	0.030	0.024
	(0.002)	(0.004)	(0.006)
$v_{t'}$			0.026
			(0.006)
Flow vacancy cost			
ψ	128.960	105.025	2.460
	(13.853)	(12.589)	(1.016)
Match elasticity[b]	0.147	0.333	0.196
	(0.028)	(0.048)	(0.168)
Outside option			
Distribution parameter			
ζ^c	0.326	0.299	—
	(0.017)	(0.006)	

[a]Standard errors were computed by taking 5,000 draws from the asymptotic normal sampling distribution of the maximum likelihood estimates of parameters that describe rates and the distribution G, and using these draws to form estimates of α and all of the quantities reported in the table. The standard errors are the standard deviations of the quantities reported in the table in the sample of 5,000 draws.

[b]This was computed from the relevant matching function for the sample. Using only cross-sectional information, the matching function was $v(1 - \exp(-u/v))$. A CRS Cobb–Douglas matching function was used for the pooled data case, so that the elasticity is equal to the Cobb–Douglas parameter associated with the size of the set of unemployed searchers.

[c]The participation rate parameter was not estimated for the pooled data case, because the decline in the participation rate is inconsistent with the increase in the estimate of unemployed search under our assumption of a stable distribution of outside option values.

function elasticity, with respect to the unemployment rate, is only 0.147. The estimated flow cost of a vacancy is 128.960. Finally, the estimated parameter value of the negative exponential distribution of outside values is 0.326.

We repeated the same exercise using the August 1997 data, the main purpose being to assess the variability of the estimates of primitive parameters over time. The vacancy rate is estimated to be 0.030 in August, which is a value 50 percent greater than what was inferred 11 months earlier. This is an outcome variable, of course, so there is no presumption that it should be constant after a minimum wage change. The flow vacancy cost estimate declines a bit to 105.025. This is supposed to be a primitive parameter, so its value should not change over time. From the large standard error associated with each estimate and given that they are estimated from independently drawn observations, there is no basis for claiming that the estimates are significantly different.

The matching function elasticity is found to be substantially higher in August 1997 than in September 1996, moving from 0.147 to 0.333. It is slightly smaller than the estimated bargaining power parameter obtained using data from that month, the value of which was 0.375. These estimates imply that the Hosios condition was approximately satisfied; this will have implications for the welfare analysis conducted below. The estimate of the parameter that characterizes the distribution of outside values is little changed from the value obtained using the September 1996 data.

We then turned to estimation of the model parameterizations that were available to us using observations on outcomes associated with two distinct steady state equilibria. We first tried to implement the estimation procedure using the September 1996 and February 1997 pooled data. This was ultimately unsuccessful because the point estimates of the parameters did not satisfy Condition C (that is, the implied estimate of the Cobb–Douglas weight associated with unemployed search was outside the unit interval). We then attempted estimation using the pooled data from August 1997 and January 1998. In this case, Condition C was satisfied and we could obtain estimated values of parameters that were contained in the appropriate parameter space.

The estimated vacancy rates in August 1997 and January 1998 are very similar. Using the pooled information and a different matching function, the vacancy rate for August is estimated to be 0.024 as opposed to 0.030. The largest difference between the estimates of primitive parameters obtained from the single samples and the pooled sample is in the flow cost of a vacancy. That declines all of the way to 2.46, which is a huge drop from the single sample estimates of over 100. We should bear in mind, however, that the estimator of ψ is quite different in the two contexts, that $\hat{\psi}$ is very much a "backed out" parameter estimate, and that the matching functions are different.

With the pooled data we were able to estimate a one-parameter matching function. The elasticity of the this function with respect to the search input from the supply side of the market is the Cobb–Douglas parameter ω. We found $\hat{\omega} = 0.196$, which lies between the elasticities computed under the matching function used with the single month data. The similarities in the estimates is moderately reassuring. As noted above, the estimate of ω is quite imprecise.

We do not report an estimate of the parameter(s) of the distribution of the outside options using the pooled data. Because the estimated value of search increased between August and January, the model implies that the participation rate increased. Instead, it fell by six percent. This drop was in large part due to students returning to school from seasonal employment. We note that even if the participation rate had increased, a two-parameter distribution of outside options would have been necessary to fit the data, in general. Thus the distributional assumptions made of necessity will depend on the number of months that are pooled, which limits comparability of estimates across different sampling schemes.

It is of some interest to compare our estimates of the matching function elasticities with those obtained by linear regression estimation using aggregate time series and cross-sectional data on unemployment and vacancy rates.

Petrongolo and Pissarides (2001) provide a comprehensive survey of this literature, with empirical results collected in their Table 3. Our estimated values of the matching elasticity (with respect to ul) are at the low end of most of the estimates in the table. They seem to be closer to those obtained when the data used were from local labor markets. For example, Coles and Smith (1996) estimated a Cobb–Douglas matching function using unemployment and vacancy information from local labor markets in England and Wales in 1987, and found a matching elasticity of 0.3.

Our estimates can be expected to differ from those obtained by others for at least two reasons. First, we do not directly use information on vacancies. Vacancy data may be quite unreliable for a number of reasons. We are able to obtain estimates of the matching function without it by exploiting the model structure to infer vacancy levels at various points in time. Although this circumvents the need for noisy data, the validity of our estimates of v depends critically on the correctness of our model specification. Second, our population of interest is 16–24 year olds, and the matching process may be considerably different for this segment of the labor market. Whether this should result in low estimates of the matching elasticity cannot be analyzed within the current framework.

Before turning to the policy evaluation, we present a brief discussion of the ability of the model to fit the sample information. Figure 1 contains plots of hourly wage distributions for individuals 16–24 years of age in the Outgoing Rotation Groups of the Current Population Survey in September 1996 and February 1997 (by the sample design of the CPS, these are different individuals). The federal minimum wage was officially changed from \$4.25 to \$4.75 on September 30, 1996. Thus the data for September are from the month preceding the change and those from February 1997 were collected approximately five months after the change. The hourly wage distributions that appear in the top panels of Figure 1 exhibit a number of common features. In both cases, there are hourly wage rates below the mandated minimum wage—about 5 percent in each month. We have neglected this aspect of the wage distribution because we do not allow for measurement error and the model assumes compliance with the law. In addition, both distributions contain a large amount of probability mass concentrated on relatively few points. We have neglected all of these mass points in our econometric specification except for those at the two minimum wage values. The relationship between the size of the each mass point (except for that at m) and the probability immediately surrounding it suggests that integer wage values are serving as focal points with the "true" wage likely to be in the neighborhood of the focal point. This is our rationale for ignoring their existence in our econometric specification.

The bottom panels in Figure 1 contain plots of the simulated wage distributions that are generated from the estimates of primitive parameters taken from the first column of Table III. We see that the model predicts slightly more

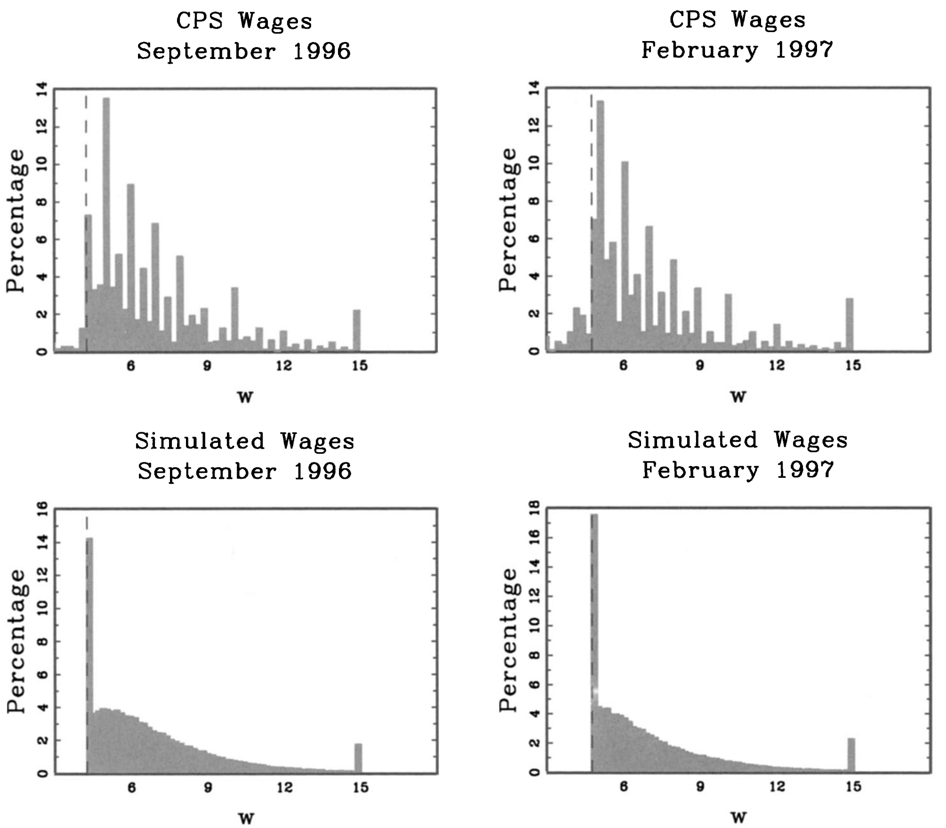

FIGURE 1.—Observed and simulated wage distributions for September 1996 and February 1997. The top panels are histograms of observed wages. The bottom panels are histograms constructed from 100,000 pseudo-random draws from the equilibrium wage offer distributions for the two months.

mass at the minimum wage than is observed.[23] For wages above the minimum, the behavioral model smooths out the masses at the focal points, as would a simple kernel estimator, and produces a more regular looking distribution. In terms of general shape, the distributions seem similar.

Figure 2 contains histograms of the durations of ongoing search spells at the time of the interview (the CPS collects information only on the search durations of respondents who report that they are currently looking for a job).[24]

[23] In the estimation phase we first round all minimum wages below the minimum wage up to the minimum wage. After this transformation, the proportion of the sample at the minimum wage is over 10 percent in both months, so the overprediction at the mass point is not substantial.

[24] For both the sample and the simulated duration distributions, we have rounded all durations above 12 months to 12 months for ease of graphical presentation.

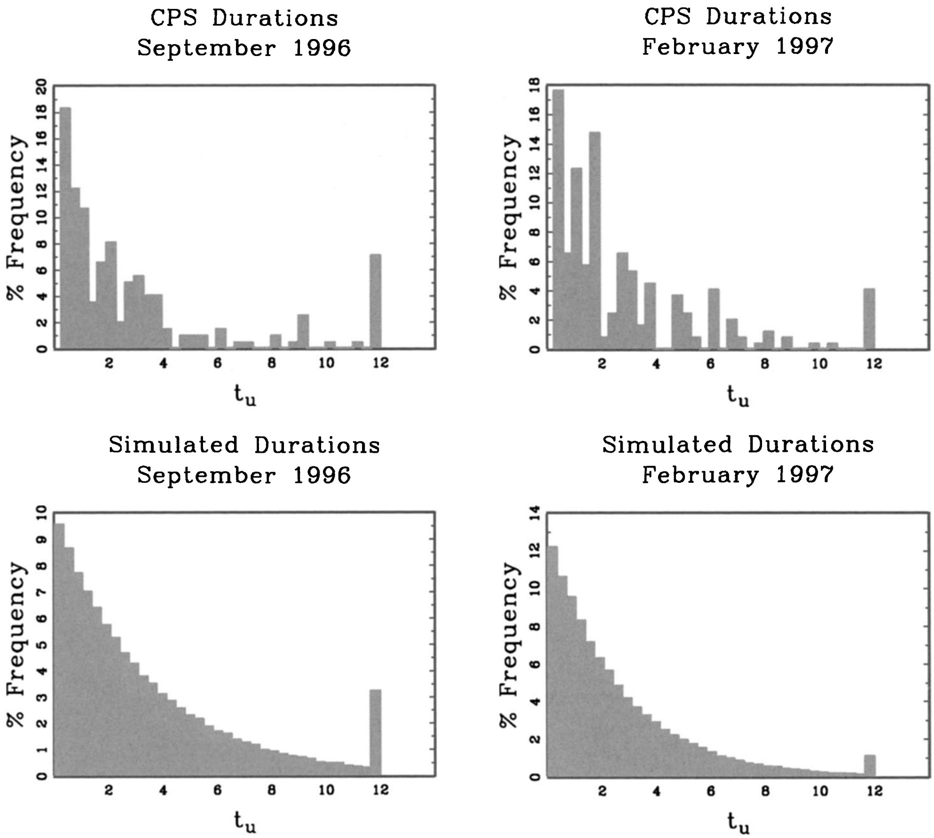

FIGURE 2.—Observed and simulated unemployment durations for September 1996 and February 1997. The top panels are histograms of the months of search in ongoing unemployment spells at the two survey dates. The bottom panels are histograms constructed from 100,000 pseudo-random draws from the equilibrium population unemployment duration distributions for the two months.

The stationary, homogeneous environment we assume implies that unemployment duration distribution should be negative exponential, as should all point-sampled ongoing spells of unemployed search. In terms of the histograms from the CPS samples, we see a fairly regular pattern of decreasing sample proportions with duration, although there are a number of exceptions that tend to occur at natural focal points.[25] Although the shapes of the unemployment spells in the data and from the simulations are not greatly at odds, we went a bit further and conducted some statistical tests for negative exponentiality in the

[25]The respondent reports search duration in weeks, which we have converted to months here because the econometric model utilizes that time period. Heaping in the weekly data is especially pronounced at 4, 8, 12, 36, and 52 weeks.

sample distributions. Some tests assumed a parametric alternative (the Weibull distribution) and others a nonparametric one. Without going into detail, all of the tests indicated rejection of negative exponentiality in the September data and marginal rejection of the null in the February data. Given that negative exponentiality would be rejected if there were any heterogeneity in the labor market environment, we view the rather modest statistical rejections that we obtained to not be disturbing enough to cast doubt on the usefulness of the model for carrying out some illustrative policy experiments.

6. POLICY EXPERIMENTS

In this section, we use the model estimates obtained under different model specifications to determine the welfare and unemployment impacts of the minimum wage. Specifically, we will plot the equilibrium steady state distribution of the population across labor market states as a function of the minimum wage; the size of the employed population will be equal to the size of the set of firms with filled vacancies. We will also plot the average steady state values associated with the states of out of the labor force (OLF), unemployment, employment, and having a filled vacancy. Finally, we will plot the values of the two aggregate welfare measures, $TW(m)$ and $PW(m)$, differentiated by whether they include the OLF subpopulation.

The results of the policy experiments we conduct depend critically on the assumptions we make regarding the model. We first consider the impact of minimum wages on labor market outcomes and welfare under the assumption that *contact rates are invariant with respect to changes in* m. We take the results of this exercise seriously because the empirical evidence we have presented is consistent with this assumption. From the results in Tables III and IV we concluded that there was no evidence that the contact rate λ differed before and after the two minimum wage changes that occurred in the late 1990s. This finding is consistent with λ being a constant, although it is also consistent with λ being determined endogenously. The small changes in the minimum wage and the relatively small number of matches directly impacted make finding powerful tests of the endogeneity of λ extremely difficult.

Most of our results are presented in graphical form. In Figure 3 we plot the size of the sets of individuals and firms that occupy the various labor market states and the average welfare level in the state as a function of the minimum wage *when* λ *is held fixed at the estimated value of* 0.309. The top panel contains the plots of the sizes of the various states. We see that the size of the employed population actually increases by a small amount, with increases in the minimum wage up to approximately \$8 per hour, after which it begins to decrease relatively rapidly. The size of the OLF population declines as the minimum wage increases up to the point at which the value of unemployed search begins to decrease. The unemployed population grows steadily in the minimum wage. Due to the participation margin, we see that for minimum wage changes

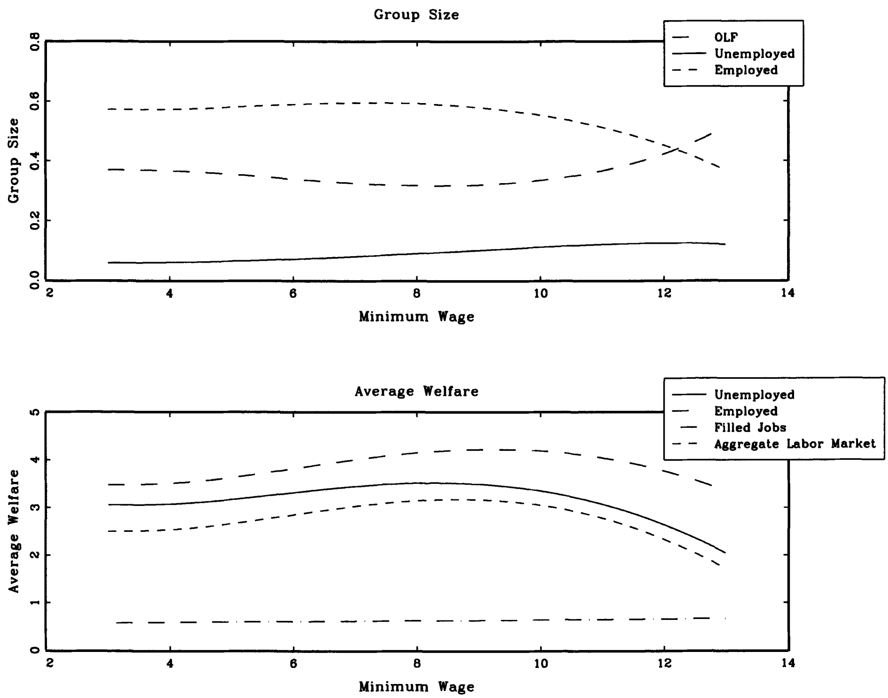

FIGURE 3.—Equilibrium population proportions and average welfare levels by labor market state as a function of the minimum wage given exogenous contact rates. The top panel plots the proportion of the population in each labor market state. The bottom panel plots the average welfare level of each group and the average for all labor market participants on both sides of the market.

up through approximately \$8 per hour both unemployment and employment register gains.

The bottom panel of Figure 3 plots the changes in the average value of occupying the state with changes in m. The average welfare values in the unemployed and employed states are single-peaked in m. The value of m that maximizes the value of the unemployed is \$8.29. The value of m that maximizes the welfare of labor market participants is \$8.66. The average welfare of firm owners with filled vacancies is monotonically increasing in m, through the truncation effect and the fact that the threat point of workers $(\rho V_n(m))$ is not rising at a fast enough rate to offset the positive selection impact of an increasing m.

In Figure 4 we plot the results from the endogenous contact rate case, and the conclusions we draw are strikingly different. From the top panel in Figure 4 we see that the size of the employed population declines rapidly in the minimum wage, as does the total number of employed and unemployed individuals. The decreases in the number of employed individuals is already sig-

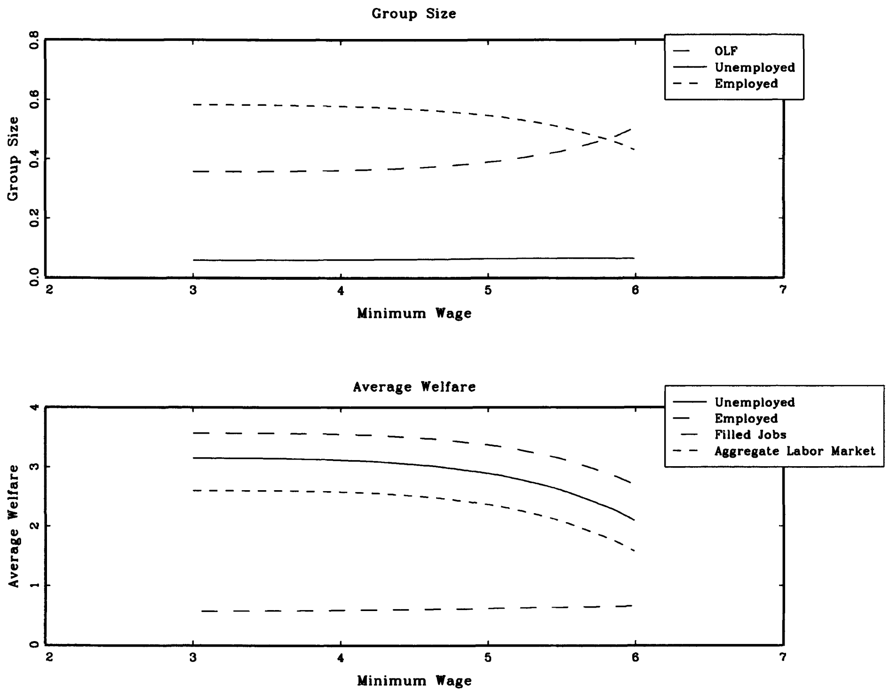

FIGURE 4.—Equilibrium population proportions and average welfare levels by labor market state as a function of the minimum wage given endogenous contact rates. The descriptions are the same as in Figure 3.

nificant even for an m of \$5. In the bottom panel of Figure 4 we see that the average welfare of employed individuals increases in the minimum wage up to \$3.33, at which point it begins to decrease in m. The average welfare of firms is monotonically increasing in m; this is the case due to the constant erosion in the worker's threat point as m increases in combination with the truncation effect. When we form the aggregate (labor market) welfare measure, the end result is a single-peaked function of m that has its maximum at \$3.36.

The results of the policy experiment are summarized in Table VI. Column 1 contains the baseline values in the reference month of September 1996. Under our model, whether with endogenous contact rates or not, the aggregate labor market welfare measure was 613.974. Column 2 contains the results from the model in which λ is fixed. As noted above, the aggregate labor market welfare-maximizing value of m is \$8.66 in this case, which is slightly more than double the baseline value. At this value of m there is significantly more participation in the labor market by individuals on the supply side; the OLF rate decreases 13 percent from the baseline value. There is a marked increase in the unemployment rate of labor market participants, on the order of 48 percent. Nonetheless, aggregate labor market welfare increases by 24.2 percent.

TABLE VI

POLICY EXPERIMENTS BASED ON ESTIMATES FROM SEPTEMBER 1996 CPS-ORG DATA (PROPORTIONATE CHANGE WITH RESPECT TO BASELINE)

		Given Optimal m	
Outcome	Baseline	Fixed λ	Endogenous λ
m	4.25	8.66	3.36
		(+1.038)	(−0.209)
OLF proportion	0.365	0.318	0.358
		(−0.129)	(−0.019)
Unemployment rate	0.096	0.142	0.092
		(+0.479)	(−0.042)
Aggregate labor	613.974	762.408	624.509
Market welfare		(+0.242)	(+0.017)

Under endogenous contact rates (the last column of Table VI), the optimal minimum wage is 20 percent less than the baseline value. At this value of m, the market participation rate is slightly greater than at the baseline and the unemployment rate among participants decreases by about 5 percent. Aggregate labor market welfare is less than 2 percent greater than in the baseline. Thus this large decrease in the minimum wage translates into relatively small effects on outcomes and welfare under this model.

Given such a large discrepancy in the policy implications that depend on the assumption made regarding the determination of contact rates, it is natural to ask which results should be given more credence (if either). The model with endogenous contact rates has the advantage of being a general equilibrium one, even if it is extremely stylized. The main disadvantage of that model, from an empirical perspective, is the necessity of utilizing a simple "black box" matching technology (with no unknown parameters) to close it. Because identification requirements severely limit the range of matching technologies we can consider, we cannot properly assess the sensitivity of model estimates and policy implications to variations in the specification of this function.

The exogenous contact rate specification undoubtedly has more of a partial equilibrium flavor. It seems reasonable to suppose that contact rates will vary with sufficiently large increases in the minimum wage, but of course the same can be said about the dismissal rate, bargaining power, and virtually all of the "primitive" parameters. Given the small changes in the minimum wage that were observed and the relatively small number of matches that were directly impacted by changes in m, identification of changes in parameters of an econometric model (be it "structural" or "reduced form") is extremely difficult. Our formal econometric tests for a change in the contact rate for searching individuals did not lead us to conclude the contact rate was endogenous *with respect to this small policy intervention*. This does not mean that it is valid to conclude that the contact rate would be unaffected by a sufficiently large change in the minimum wage. In particular, at the "optimal" minimum wage rate of \$8.66, it may well be the case that the contact rate will be different than the estimated value of λ for September 1996. However, given the range of policy choices in the data, we have no way to assess the accuracy of model predictions for such high minimum wage rates.

7. CONCLUSION

We have formulated simple partial and general equilibrium models of wage determination and labor market dynamics that carry implications broadly in accord with the empirical findings of other researchers who have worked with disaggregated data. By simply appending a side constraint to the standard Nash-bargaining problem, we were able to generate accepted wage distributions with a mass point at the minimum wage and a continuous density of wages to the right of m. The model is parsimoniously specified in that the equilibrium wage and unemployment spell distributions are characterized in terms of a small set of primitive parameters.

Often studies of minimum wage impacts on the labor market have focused exclusively on unemployment, employment, and participation rates. Besides looking at the effect of minimum wages on observable labor market outcomes such as these, we are able to assess their welfare effects as well. We look at the effect of minimum wages on the total welfare of labor market participants on both the supply and the demand sides. Using the matching function formulation along with Nash bargaining between workers and firms, the results of Hosios (1990) inform us that an efficient labor market equilibrium requires that the elasticity of the matching function with respect to the size of the set of searchers be equal to the share of the surplus they receive. Although macroeconomic models of search equilibrium have postulated the presence of a social planner that could ensure satisfaction of this condition, we do not. In the absence of the satisfaction of this condition, the minimum wage, or a maximum wage for that matter, can be viewed as a crude instrument that moves the labor market in the direction of an efficient allocation. Our estimates of the bargaining power parameter, although significantly less than 0.5, are significantly larger than our estimates of the match function elasticity and, as a result, yield an optimal minimum wage rate less than the then current value of \$4.25. Our empirical work did, however, cast some doubt on the applicability of the matching function framework in these data. In particular, we could not reject the null hypothesis that the contact rate parameter was invariant with respect to minimum wage changes. If we fix the contact rate at its estimated (equilibrium) value in September 1996, we find a welfare-maximizing minimum wage rate of over \$8 per hour. We are unable to say more regarding which estimate is more credible given the restrictions imposed by our modeling assumptions and the data available to us.

As was discussed above, an obvious extension of the current framework is to include on-the-job search. In the United States, over one-half of all job spells terminate with the direct transition into another job; the proportion is even higher for young labor market participants, so its exclusion may seriously call into question the policy analysis conducted here. However, Flinn and Mabli (2005) find that the OTJ search model produces even higher values of optimal minimum wages than were derived here, *both* in the exogenous and endogenous contact rate cases. The main reason for this result is that the estimated value of α is much lower in the OTJ case, with the estimated distribution of match values significantly shifted to the right. In this case, high minimum wages have little impact on the employment rate, even when the contact rate is endogenous. Thus it appears that the conclusion that minimum wages considerably higher than current levels could lead to general welfare improvements survives this generalization.

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Manuscript received May, 2003; final revision received February, 2006.

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