*Econometrica*, Vol. 61, No. 5 (September, 1993), 1173–1197

# SEQUENTIAL LABOR DECISIONS UNDER UNCERTAINTY: AN ESTIMABLE HOUSEHOLD MODEL OF WEST-AFRICAN FARMERS

BY MARCEL FAFCHAMPS[1]

This paper reports estimates of the structural parameters of a stochastic control model that describes the labor decisions of small farmers in Burkina Faso, West Africa. The focus of the estimation is on measuring flexibility in production and intertemporal substitutability in consumption. Full information maximum likelihood estimates of the primitive parameters of the model are computed even though optimal labor decision rules cannot be derived analytically. Vuong's non-nested model specification test shows that this method yields parameter estimates that are superior to those derived by assuming that farmers solve a deterministic control problem.

The low levels of agricultural labor effort commonly observed in the survey area are shown to be a consequence both of the low productivity of labor in archaic rainfed agriculture and of farmers' awareness that, in the absence of a labor market, overly ambitious production plans lead to seasonal manpower constraints. To meet rainfed farmers' concerns, agricultural research institutes and extension services should factor flexibility into their research agendas.

KEYWORDS: Africa, agricultural development, stochastic control, flexibility, maximum likelihood estimation, non-nested model specification test.

## INTRODUCTION

WEST AFRICAN FARMERS TRY TO BALANCE the benefits from production flexibility against the utility gains from leisure consumption smoothing. A correct understanding of this trade-off is essential in designing technological packages that fit the concerns and attitudes of these farmers. This paper estimates the structural parameters of a stochastic control model describing the labor decisions of small farmers in three regions of Burkina Faso, West Africa. The model formalizes how much labor farmers allocate to planting and weeding as a function of exogenous shocks to production. Because of nonlinearity, analytic expressions cannot be derived for the labor decisions. But by an iterative nested computer algorithm, full information maximum likelihood (FIML) estimates are computed for the primitive parameters of the structural model, that is, for the utility and production functions. Although very computer-intensive, the approach leads to meaningful estimates of flexibility in production and intertemporal substitutability in consumption.

The econometric implementation approach is inspired by that of Rust (1987, 1988) and Wolpin (1984, 1987), and by the ideas in Hartley (1983). There are,

[1] I wish to thank Peter Matlon, Alain de Janvry, Brian Wright, George Judge, Kenneth Judd, Thomas MaCurdy, Frank Wolak, Bernard Salanié, and the journal referees for their very useful comments on earlier drafts of this paper. I also benefited from comments by participants to seminars at Stanford, Chicago, Northwestern, and North Carolina State Universities. The data were kindly made available to me by the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT) which also supported part of the research.

however, a number of differences between those papers and this one: the model considered here has a finite horizon and is nonstationary; decisions are continuous; the state space is not discretized; and data are available only on the final state of nature, not on intermediate states. These differences explain why, even though this paper shares a common philosophy of econometric implementation with these authors, its actual implementation is specific.

The analysis attempts to reconcile the concern for possible manpower shortages, often expressed by African farmers, with the low levels of labor inputs that are observed on average (Cleave (1974), Fafchamps (1986)). The results show that farmers' concern for leisure smoothing is low, their labor supply elasticity is high, and those with a large planted acreage face manpower shortages at weeding time whenever rains are abundant. Consequently, low levels of agricultural labor use should not be attributed to some inherent "laziness" of Africans, but rather to two related factors: the low productivity of labor in archaic rainfed agriculture; and farmers' awareness that overly ambitious production plans are likely to lead to weeding manpower constraints in the absence of a labor market.

Furthermore, the results confirm farmers' claims that planting labor is a direct function of area cultivated *and* initial rainfall. They also show that farmers retain a significant amount of flexibility in their weeding decisions and can modulate their weeding effort depending upon the information available to them at the time. This kind of flexibility plays a critical role in reducing the cost of risk to farmers (Fafchamps (1989)). For this reason, technological innovations that increase productivity but reduce flexibility may fail to be adopted by farmers.

Econometric results are also contrasted with those given by the FIML implementation of an alternative, simpler model of farmers' behavior without uncertainty. A non-nested model specification test (Vuong (1989)) shows that the stochastic control model dramatically outperforms the deterministic control model, amply justifying the extra estimation cost. The test also confirms that uncertainty plays an essential role in the labor decisions of the surveyed households in all three agro-climatic zones.

The paper is organized as follows. In the first section, the data are presented and the technological and economic environment of Burkina Faso farmers briefly described. Next, the farmers' decision problem is discussed in Section 2. The estimated model and the estimation algorithm are presented in Sections 3 and 4, respectively. Results are discussed in Section 5. Actual village level data on rainfall are incorporated in the estimation process in Section 6.

## 1. THE SURVEYED HOUSEHOLDS

The data used here were collected between 1981 and 1983 in six villages of Burkina Faso by the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT). These three years were fairly representative of the weather pattern in the area. Two villages were chosen in each of three major agro-climatic zones of West Africa: the sahelian zone, with low rainfall (480 mm./year) and sandy soils; the sudanian zone, with slightly higher rainfall (724 mm./year) and shallow soils; and the north-guinean zone, with moderate rainfall (952 mm./year) and better soils (Matlon (1988)). A panel of approximately 25 households was followed in each village. Labor inputs were recorded regularly every ten days, together with other crop production data. Physical agricultural output was measured by plot at the end of each agricultural season. For estimation purposes, labor input data were aggregated by period. Total crop income was computed using information on unit weights and market prices gathered by a variety of ICRISAT surveys in the same villages. Villages were grouped by ecological area. The three years of data were combined in the estimation.

All the surveyed farmers are very poor (Table I). Their major factors of production are land and labor. Very little use is made of modern agricultural inputs and technology (Matlon and Fafchamps (1989)). Investments in land (e.g., clearing) or fertility (e.g., manuring, animal paddocking, crop rotation) are predetermined at the beginning of the agricultural season (Prudencio (1983, 1987)). After the rains have started, the only variable factor of production is labor.

For reasons pertaining to the egalitarian distribution of land and the lack of specialization in production (Binswanger and McIntire (1987)), a rural labor market is virtually absent in all the surveyed villages. Farming households rely almost exclusively on their own manpower (Matlon and Fafchamps (1989)). Some exchange of labor takes place between households, mostly on a reciprocal basis. Only in the north-guinean villages has some degree of farm differentiation developed due to the cultivation by some farmers of cotton as a cash crop. As a consequence, labor transactions there are a little more frequent, mostly at harvesting (Table I).

In all three regions, agriculture is exclusively rainfed and yields are subject to very large exogenous shocks. Rains are erratic over time and space. The absence of rain at key periods of the agricultural season frequently leads to crop failures. Famines are recurrent in the area, particularly in the sahelian and sudanian zones (Reardon, Matlon, and Delgado (1988)). Yields are reduced by weeds and pest infestation. Moreover, crop nutrients, being concentrated in the top shallow soil, are easily washed away by heavy convectional showers. Excessive rains may lead to waterlogging and fast weed growth. These shocks combine to generate a large degree of idiosyncratic, i.e. plot and farm specific, risk (Reardon, Delgado and Matlon (1992), Udry (1991), Carter (1991), Morduch (1991)).

Farmers deal with agricultural risk essentially in three ways: by opting for varieties, crops (e.g., millet), crop mixes (e.g., cowpea intercrops), and farming strategies (e.g., early planting) that are resistant to weather, pests and diseases; by diversifying their portfolio of varieties (Matlon (1980)), crops (Matlon and Fafchamps (1989)), intercrops, activities (e.g., livestock, seasonal migrations—Reardon, Delgado, and Matlon (1992)), and sources of support (Fafchamps (1991, 1992)); and by explicitly building flexibility into their farming practices. The focus is here on the third aspect.

TABLE I

BRIEF DESCRIPTION OF THE SURVEYED BURKINABE VILLAGES

| Villages: | Sahelian Zone Woure | Sahelian Zone Silgey | Sudanian Zone Kolbila | Sudanian Zone Ouonon | North-Guinean Zone Koho | North-Guinean Zone Sayero |
|---|---|---|---|---|---|---|
| Rainfall (mm) | 480 | | 724 | | 952 | |
| Household size (5 years old and above) | 7.64 | 8.29 | 10.76 | 9.45 | 13.15 | 8.73 |
| Cultivated acreage (ha) | 6.27 | 6.54 | 5.74 | 4.27 | 6.01 | 5.83 |
| Area/head (ha) | 0.82 | 0.79 | 0.53 | 0.45 | 0.46 | 0.67 |
| Area/worker: Gini coef. | 0.23 | n.a. | 0.16 | 0.17 | 0.27 | 0.24 |
| Total agricultural labor per head: (in days of 6 hrs/year) | 41 | 44 | 55 | 44 | 50 | 47 |
| Household labor (6 hrs days/ha) | | | | | | |
| Early planting | 3 | 3 | 11 | 7 | 17 | 11 |
| Late plant/1st weeding | 9 | 9 | 26 | 21 | 25 | 15 |
| Weeding | 28 | 32 | 40 | 51 | 27 | 16 |
| Harvesting | 10 | 11 | 26 | 19 | 40 | 29 |
| Total | 50 | 55 | 102 | 97 | 109 | 71 |
| Non-household labor (% total labor) | | | | | | |
| Early planting | 0.5% | 0.9% | 5.0% | 1.3% | 9.6% | 7.8% |
| Late plant/1st weeding | 0.7% | 0.3% | 3.9% | 0.1% | 20.4% | 16.0% |
| Weeding | 3.5% | 2.7% | 16.0% | 4.1% | 22.9% | 20.6% |
| Harvesting | 5.4% | 8.0% | 15.9% | 6.3% | 29.4% | 21.9% |
| Livestock heads (in cattle equivalent) | 13.9 | 12.4 | 2.5 | 9.7 | 16.1 | 5.2 |
| Agricultural income (in US dollars/year) | | | | | | |
| Total | 461 | 363 | 553 | 352 | 805 | 917 |
| Per head | 60 | 44 | 51 | 37 | 61 | 105 |
| Technology | | | | | | |
| hh equipped with ANTRAC | 10% | 8% | 12% | 17% | 22% | 16% |
| chem. fert. per ha (kg) | 0.0 | 0.3 | 11.1 | 6.4 | 36.6 | 29.5 |
| organ. fert. per ha (kg) | 232 | 172 | 360 | 444 | 1532 | 1075 |
| area fert. previous year | 41% | 16% | 49% | 35% | 28% | 30% |
| area fallow prev. year | 91% | 88% | 69% | 91% | 89% | 80% |
| Animal traction use (in hrs per ha; users only) | | | | | | |
| Early planting | 1.3 | 0.9 | 6.0 | 2.1 | 7.3 | 6.2 |
| Late plant/1st weeding | 4.0 | 4.3 | 6.6 | 5.1 | 6.8 | 4.0 |
| Weeding | 4.3 | 10.7 | 2.1 | 4.7 | 18.1 | 2.7 |
| Harvesting | 0.7 | 0.8 | 5.3 | 0.6 | 3.6 | 2.3 |

*Source*: ICRISAT household surveys; Matlon (1988), Matlon and Fafchamps (1989).

## 2. THE HOUSEHOLDS' DECISION PROCESS

The optimization problem facing surveyed farmers is characterized as follows. Farming households try to control crop growth, represented by the state variable $y_t$. Control over plants requires a large number of discrete decisions. In the end, however, all those discrete decisions translate into a single *costly* and *continuous* aggregate, labor. Assuming technical efficiency among labor's (discrete) uses, the farmers' unique control variable is the amount of time they spend working in their fields $l_t$ at each point of the agricultural season.

The crop growth process depends not only on labor but also on various exogenous shocks that affect plants: planting must wait for early rains; the need for replanting depends on crop establishment; weeding requirements depend on weed infestation; and harvesting is a function of yield. This process is not evenly distributed over time, which means that the transition probability $p_t(y_{t+1}|y_t, l_t)$ is nonstationary.

Information about external shocks is revealed progressively over the cropping season. Because more information is better than less (Chavas, Kristjanson, and Matlon (1991)), it is in the interest of farmers to opt for flexible farming strategies, i.e. strategies that allow the postponement of action until shocks have been observed (Antle (1988)). But postponement bears a cost: the expected yield loss resulting from delays in planting, weeding, and harvesting. A production process is defined as more flexible if penalties are low for delaying action until more information is available; inversely, it is not flexible if waiting results in major output losses (Epstein (1978)). When labor is the only decision variable, flexibility is directly proportional to the degree of substitutability in production between labor effort at different points of the cropping season.

Because of the absence of a labor market, the cost of labor is the marginal utility of foregone leisure. It depends on the level of effort. High labor inputs in crop farming conflict with the time required for rest, meals, domestic chores, and social activities (Cleave (1974)) as well as livestock care (Delgado (1979)). As a consequence, farming households may wish to smooth their labor inputs over the year as well as over states of nature. To focus on this particular process, a household's utility is defined over crop income and leisure at various times of the cropping period (Epstein (1975)). Households maximize their utility subject to the crop growth process.

So defined, the decision process captures the essential trade-off between the desire to smooth labor inputs and the necessity to control the stochastic growth process of crops. The trade-off is made more acute with enhanced flexibility since flexible farming strategies call for delaying labor inputs, and varying them as a function of the state of nature.

## 3. THE ESTIMATED MODEL

Provided that utility and crop growth satisfy suitable convexity and boundedness conditions (Stokey and Lucas (1989)), the household's optimization problem described in the previous section has a unique solution. Furthermore, if one were willing to assume a linear crop growth process and a quadratic utility function, optimal labor decision rules could be derived analytically.[2] But assuming that crop growth is linear in labor amounts to imposing perfect substitutability between labor at different times of the cropping season. This is unacceptable given that a major objective of this research is to estimate flexibility in production and thus substitutability between labor uses across time.

FIML estimates of the utility and production parameters can be obtained for more general functional forms. To do so, one must rely on computer intensive iterative procedures like the one described in the following pages and in the Appendix. For such procedure to be manageable in terms of computer time and programming complexity, the estimated model must be kept simple. The approach, however, is applicable to the estimation of a wide variety of static and dynamic models without closed form solution. Given adequate computer power and programming modules, more ambitious models could be estimated.

Simplicity is achieved by making the following assumptions. Within each region, households are assumed to share identical preferences and production technology. The only household characteristic that enters the model is land assets. Year-to-year investment and saving decisions are ignored. Labor decisions are aggregated into three distinct periods—planting and replanting, weeding, and harvesting. Harvesting labor is assumed proportional to yields and consequently dropped from the analysis. Age/sex task specificity is ignored. Returns to scale in production are assumed constant. No effort is made to correct for the (imperfect) panel nature of the data.

Nested Constant Elasticity of Substitution (CES) functional forms are chosen for utility and production because they satisfy the three following requirements: they capture imperfect flexibility in production and the desire for labor smoothing in their substitution elasticities; being sparse in parameters, they reduce the size of the estimated parameter vector and minimize computer time; and they lead to an invertible system of Euler equations provided that production shocks are independent over time.

The estimated model is:

$$\text{(1)} \qquad \underset{l_1}{\text{Max}}\, E_{\theta_1,\theta_2}\, \underset{l_2}{\text{Max}}\, E_{\theta_2} \frac{1}{(1-R)} \times \left\{\delta\left(\gamma(1-l_1)^{\sigma} + (1-\gamma)(1-l_2)^{\sigma}\right)^{\rho/\sigma} + (1-\delta)\, y_3^{\rho}\right\}^{(1-R)/\rho}$$

subject to the stochastic laws of motion,

$$\text{(2)} \qquad y_1 = Ae^{\theta_0},$$

$$\text{(3)} \qquad y_2 = \left(\alpha y_1^{r_1} + (1-\alpha) l_1^{r_1}\right)^{1/r_1} e^{\theta_1},$$

$$\text{(4)} \qquad y_3 = b\left(\beta y_2^{r_2} + (1-\beta) l_2^{r_2}\right)^{1/r_2} e^{\theta_2}.$$

[2] Closed form solutions can also be derived for some extensions of the linear-quadratic model, e.g., Turnovsky (1976).

The two decision variables are $l_1$, planting labor, and $l_2$, weeding labor. $A$ is predetermined cultivated acreage. All variables are divided by the number of working people in each household. Consequently, $1-l_1$ and $1-l_2$ stand for leisure at planting and at weeding time, respectively.

Information about intrinsic soil quality and external factors affecting plant growth is revealed to the household in three stages. The information revelation process is pictured here as various exogenous "shocks" that the farming household is able to observe over the cropping season. An initial shock $\theta_0$ takes place at planting time. It is observed by households when deciding how much labor to allocate to planting and replanting. A second shock $\theta_1$ takes place after planting. It determines weed and plant growth. Households decide how much weeding to perform after observing $\theta_1$. The third shock $\theta_2$ takes place after weeding, and determines final output $y_3$. Except for average rainfall in each village and each year, no direct information on any of these exogenous contributions to crop growth was collected by survey enumerators.

The state of the crop growth process is represented by $y_1$ and $y_2$ at planting and weeding time, respectively. Again, this information was available to decision makers, but was not collected by survey enumerators. $y_1$ and $y_2$ have been normalized so that they capture *all* the contribution to plant growth made by past exogenous shocks, either directly or via their correlation with later shocks. Thus $y_1$ and $y_2$ summarize all the information available to the household at planting and weeding time, respectively. Consequently, the normalized shocks $\theta$'s can be treated as independently distributed. They are also assumed to be jointly normal with zero mean and variances $\sigma_i$. They differ across households and between years.

The parameters of equations (1)–(4) summarize the characteristics of the household's decision problem. The elasticity of substitution in utility between leisure in period one and in period two is $1/(1-\sigma)$; it measures the desire for labor smoothing. The elasticity of substitution between income and leisure is $1/(1-\rho)$. Together, $\sigma$ and $\rho$ measure flexibility in consumption. $1/(1-r_1)$ is the elasticity of substitution in production between land $A$ and early rainfall $\theta_0$ on one hand, and planting labor $l_1$ on the other. $1/(1-r_2)$ is the elasticity of substitution between crop growth at the end of the first period, $y_2$, and weeding labor $l_2$. Flexibility in production is thus measured by $r_1$ and $r_2$. $\gamma$, $\delta$, $\alpha$, and $\beta$ are "share" parameters. $b$ is a scaling parameter. $R$ measures the global concavity of the utility function and is a measure of risk aversion (Kihlstrom and Mirman (1974, 1981), Hall (1988)).

$l_1$, $l_2$, $A$, and $y_3$ are the only observed variables available for the estimation of the structural parameters of equations (1)–(4). The structural model contains 10 parameters, 3 priors about the variance of exogenous shocks, and 3 sample variances. Given that everything enters the normalized model in a nonlinear fashion, one may be under the impression that all parameters can in principle be estimated. But one must look for possible nonlinear dependence between parameters. In the absence of analytic equations to estimate, there is no straightforward way of doing so on the basis of the structural model itself.

Consequently, one must rely partly on intuition and partly on indications of possible underidentification suggested by difficulties arising during the likelihood maximization process.

Even when parameters are not jointly underidentified, quasi-underidentification may still arise whenever combinations of parameters lead to values of the likelihood function that are very similar. This latter situation creates two types of difficulties. The first is that numerical hill-climbing algorithms are very sensitive to the existence of nearly flat spots or ridges, particularly when they occur in the neighborhood of the maximum. The second is that, even if the numerical difficulty could be surmounted, the precision of the estimated parameters would be very low.[3]

The first underidentification problem concerns the "share" parameters of $l_1$ and $l_2$ in utility and production. To see why they cannot be estimated separately, suppose that $l_1$ is consistently lower than $l_2$. On the basis of this fact alone, there is no way to decide whether this results either from low labor productivity in the first period or from a strong preference for first period leisure. Since there is no strong reason to believe that the utility of leisure is different at any point in time, the most appropriate way of eliminating underidentification is to assume that leisure contributes equally to utility in period 1 and 2 and thus to set $\gamma$ to one half.

The two other identification difficulties that plague equations (1)–(4) have to do with risk and risk aversion. In both cases, numerical experimentation indicates that the observed data alone cannot serve as a basis for accurate joint estimation. The first of the two concerns households' priors about the variance of exogenous shocks. Given the structure of the model, it is not possible to infer households' priors while at the same time deriving maximum likelihood (ML) estimates of the sample variances. This can be seen by the following thought experiment. Suppose that households believed variances to be large. The model predicts that they would modify their behavior accordingly. But ML estimates of the variances are inferred from observed behavior. Consequently, any guess about households' priors will lead to different sample variances. One may still hope that the value of the likelihood function at various guesses about priors would be sufficiently different to enable one to select among them. Numerical results reveal that this is not the case.

In this paper, the solution to this dilemma has been to assume that households form rational expectations about the distribution of exogenous shocks. This is equivalent to the restriction that ML estimates of the variances be equal to households' priors. This assumption is plausible because the economic and technological environment in which the surveyed households operate is fairly stable and the three years during which the data were collected are representative of the distribution of exogenous shocks in the three surveyed areas. Consequently, households had had enough time to learn the true variance of the shocks, of which estimated sample variances are consistent estimators.

[3] The effects of quasi-underidentification are very similar to multicollinearity in linear models.

The last underidentification problem has to do with risk aversion. Theoretical models of decision making under uncertainty often suggest risk and risk aversion to be the mirror of each other. For instance, a highly risk averse individual facing little risk often behaves in a way that is similar to a mildly risk averse person facing a lot of risk (Diamond and Stiglitz (1974)). Consequently, one would not expect to be able to estimate risk *and* risk aversion on the basis of a single set of observed decisions. To disentangle them, one would need either an independent evaluation of risk or multiple decisions involving outcomes whose different risk characteristics can be unambiguously assessed.[4] Numerical experimentation suggests that the multiple labor decisions that households face in equations (1)–(4) are too intermingled to allow risk aversion to be estimated with any degree of accuracy on that single narrow base.[5] The parameter $R$, therefore, was arbitrarily set equal to one half, a value corresponding to a moderate level of risk aversion.

To improve the estimation of the contribution of labor to output, the very small quantities of nonhousehold labor recorded in the surveys was added to production labor as a predetermined variable. Because of moral hazard problems, nonhousehold labor was assumed 30 percent less productive than household labor. The minor changes that this implies to the estimated model are not shown here to make the presentation clearer.

## 4. THE ESTIMATION ALGORITHM

Full Information Maximum Likelihood (FIML) is the estimation approach adopted here. The estimation algorithm used to compute FIML estimates or the parameters of equations (1)–(4) is similar in spirit to that of Wolpin (1984, 1987) and Rust (1987, 1988), although different in its details. In particular, no discretization of the labor decisions and output is used here. Opting for a model of continuous choice is motivated by the objective of the estimation process: to derive estimates of the elasticities of substitution in utility and production. Discretization would lose the precious information that is required to identify substitution parameters with any degree of accuracy.

The algorithm starts by making a guess about the priors and the structural parameters of the model—i.e., the 8 free parameters of equations (1)–(4). Given this guess, the algorithm calculates the likelihood of observing the data. To do so, it first inverts, for each observation, the system of Euler equations derived from equations (1)–(4), that is, it computes the values of $\theta_0$ and $\theta_1$ that would have made the observed decisions $l_1$ and $l_2$ optimal. Having found $\theta_0$ and $\theta_1$, it then computes $\theta_2$ using the realized crop output, $y_3$. In a second step, it computes the contribution of each observation to the likelihood of the parameter vector guess, using the inferred random shocks $\theta_0$, $\theta_1$, and $\theta_2$, and the computed value of the Jacobian at these shocks. Finally, the likelihood

[4] For instance, if in the above model one had, in addition to labor, decisions about crop portfolio or investment in different assets, one may be able to estimate both risk and risk aversion.

[5] Intertemporal substitution is estimated separately.

values for each observation are combined into the likelihood of observing the entire sample. A hill climbing algorithm then searches various parameter vectors until it finds the one that maximizes the value of the likelihood function. Priors are then updated and set equal to ML sample variances. The whole process is repeated until sample variances and priors are equal. Those steps are briefly described in the following pages. Technical details are presented in an appendix.

*Computing the Shocks*

The first step of the algorithm is to find, for a given guess about the parameter vector and for each observation separately, the value of $\theta_0$ and $\theta_1$ that would have led the households to apply the observed levels of labor $l_1$ and $l_2$. Given the recursive structure of the model, the optimal labor decisions must be of the form:

$$l_{j1} = g_1(A, \theta_0), \tag{5}$$

$$l_2 = g_2(A, \theta_0, l_1, \theta_1), \tag{6}$$

$$y_3 = g_3(A, \theta_0, l_1, \theta_1, l_2, \theta_2), \tag{7}$$

where $g_1(\cdot)$ is the optimal $l_1$ decision rule, $g_2(\cdot)$ the optimal $l_2$ decision rule, and $g_3(\cdot)$ is obtained by inserting equations (2) and (3) into equation (4).

Because of the choice of functional form and the assumption that shocks are independently distributed, the relationship between any of the shocks and its associated observed variable (labor or output) is one-to-one. This is true for all parameter vectors, although the sign of the relationship may change (Fafchamps (1989)). Therefore, if one knew equations (5) to (7), it would be possible to invert them and solve them recursively for the shocks:

$$\theta_0 = g_1^{-1}(A, l_1), \tag{8}$$

$$\theta_1 = g_2^{-1}(A, \theta_0, l_1, l_2), \tag{9}$$

$$\theta_2 = g_3^{-1}(A, \theta_0, l_1, \theta_1, l_2, y_3). \tag{10}$$

Although a closed-form expression cannot be derived for functions $g_1$ and $g_2$, inverting the decision rules can still be achieved using the Euler equations of the model:

$$E_{\theta_1}\left[E_{\theta_2}\left[U_{y_3}\frac{\partial y_3}{\partial y_2}\frac{\partial y_2}{\partial l_1} - U_{l_1}\right]\right] = 0, \tag{11}$$

$$E_{\theta_2}\left[U_{y_3}\frac{\partial y_3}{\partial l_2} - U_{l_2}\right] = 0. \tag{12}$$

Indeed, by the implicit function theorem, they define the policy functions $g_1$ and $g_2$ *as well as* $g_1^{-1}$ and $g_2^{-1}$. Consequently, $\theta_0$ can be computed by finding the value that sets equation (11) to zero when $l_1$ is equal to its observed value.

Similarly, $\theta_1$ is computed from equation (12). Finding $\theta_0$ and $\theta_1$ thus boils down to finding recursively the zeroes of equations (11) and (12). Once $\theta_0$ and $\theta_1$ are known, $\theta_2$ can be inferred directly from equation (4).

*The Likelihood Function*

After the $\theta$'s have been found for each observation, the likelihood function can be computed as:

$$(13) \qquad h_{l_1, l_2, y_3}(l_1, l_2, y_3) = |J| h_{\theta_0, \theta_1, \theta_2}(\theta_0, \theta_1, \theta_2),$$

where the joint distribution of the $\theta$'s is denoted $h_{\theta_0, \theta_1, \theta_2}(\theta_0, \theta_1, \theta_2)$, and $|J|$ is the absolute value of the determinant of the Jacobian matrix. It is formed by the partial derivatives of $g_1$, $g_2$, and $g_3$ with respect to the $\theta$'s. Because of recursivity, the Jacobian matrix is lower triangular. Therefore, its determinant is the product of its diagonal terms:

$$|J| = \left| \frac{d\theta_0}{dl_1} \frac{d\theta_1}{dl_2} \frac{d\theta_2}{dy_3} \right|.$$

The value of the Jacobian determinant can be computed for each observation by noting that the Jacobian matrix can be obtained by totally differentiating the Euler equations (11) and (12). $d\theta_2/dy_3$ is simply $-1/y_3$. To simplify notation, one can rewrite (11) as $m^1(l_1, l_2, \theta_0) = 0$ and (12) as $m^2(l_2, \theta_1) = 0$. Then:

$$\frac{d\theta_1}{dl_2} = \frac{-E_{\theta_2} m^2_{l_2}}{E_{\theta_2} m^2_{\theta_1}},$$

$$\frac{d\theta_0}{dl_1} = \frac{-E_{\theta_1} E_{\theta_2}\left[ m^1_{l_1} + m^1_{l_2} \dfrac{dl_2}{dl_1} \right]}{E_{\theta_1} E_{\theta_2}\left[ m^1_{\theta_0} + m^1_{l_2} \dfrac{dl_2}{d\theta_0} \right]}$$

$$= \frac{E_{\theta_1} E_{\theta_2}\left[ m^1_{l_1} - m^1_{l_2} \dfrac{E_{\theta_2} m^2_{l_1}}{E_{\theta_2} m^2_{l_2}} \right]}{E_{\theta_1} E_{\theta_2}\left[ m^1_{\theta_0} - m^2_{l_2} \dfrac{E_{\theta_2} m^2_{\theta_0}}{E_{\theta_2} m^2_{l_2}} \right]}.$$

By assumption, the random shocks are independent and jointly normal. The concentrated log-likelihood function is thus (Berndt et al. (1974), Judge et al.

(1985)):

$$l(\lambda | A, l_1, l_2, y_3) = -\frac{3N}{2}(1 + \log(2\Pi)) - \frac{N}{2}\sum_{i=1}^{3}\log\left(\frac{1}{N}\sum_{j=1}^{N}\theta_{ij}^2\right) + \sum_{j=1}^{N}|J_j|, \tag{14}$$

where $\lambda$ is the parameter vector to be estimated. $N$ stands for the number of observations and $|J_j|$ is the absolute value of the determinant of the Jacobian matrix evaluated at observation $j$.

Equation (14) can be computed for any (reasonable) guess about the parameter vector $\lambda$. Finding the maximum likelihood estimate of $\lambda$ is then a matter of searching possible parameter vectors until the global maximum of the likelihood function has been reached. The imposition of the rational expectations restrictions is discussed in the Appendix.

## 5. THE RESULTS

### *Presentation of the Results*

The FIML estimates of the structural parameters are presented in Table II for the three regions. Parameter estimates were restricted to fall within a meaningful range: share parameters were to fall between 0 and 1, and elasticities of substitution were restricted to be positive (i.e. $\sigma, \rho, r_1, r_2 < 1$). All parameter estimates, except one, fell within their assigned range. $\sigma$ in the sudanian agro-climatic zone is at its upper bound.

Parameter estimates are fairly uniform across regions. They indicate that households are less concerned with the smoothing of their labor inputs over the season than is sometimes assumed. Indeed, $\sigma$ is not significantly different from 1 in all regions, suggesting an infinite substitutability across periods in the consumption of leisure. The moderately high elasticity of substitution between (crop) income and leisure consumption (3.68, 2.65, and 3.04 respectively in the sahelian, sudanian, and north-guinean zones) confirms that households are responsive to changes in the marginal returns to their labor and are willing to sacrifice leisure for more income whenever possible.

The very low elasticity of substitution between $y_1$ and $l_1$ (respectively .187, .129, and .123 in the sahelian, sudanian, and north-guinean zones) is consistent with casual observations and statements by farmers; planting labor is dictated by the area cultivated and early rainfall. On the other hand, the elasticity of substitution between $y_2$ and $l_2$ is significantly larger (.946, .608, and .546, respectively, in the sahelian, sudanian, and north-guinean zones). Farmers thus retain a fair amount of flexibility at weeding time and are able to tailor their weeding effort to the state of the crops. This is particularly true in the sahelian zone where the agricultural season is very short. If rains are poor and weed growth is slow, farmers compensate somewhat the expected reduction in yields

TABLE II

PARAMETER ESTIMATES FOR THE STOCHASTIC CONTROL MODEL (ASYMPTOTIC STANDARD ERRORS IN BRACKETS)

| Agroclimatic zone: | 1/Without rainfall | | | 2/With rainfall | | |
|---|---|---|---|---|---|---|
| Utility Parameters | Sahel | Sudan | Guinea | Sahel | Sudan | Guinea |
| $\delta$ | 0.76839 | 0.61736 | 0.83781 | 0.76839 | 0.61736 | 0.83781 |
| | (0.14935) | (0.08808) | (0.01081) | (0.09491) | (0.05472) | (0.01177) |
| $\sigma$ | 0.95534 | 0.99900 | 0.99682 | 0.96005 | 0.99900 | 0.99682 |
| | (0.10359) | (0.14948) | (0.21069) | (0.07040) | (0.10635) | (0.16900) |
| $\rho$ | 0.72795 | 0.62293 | 0.67108 | 0.72808 | 0.62293 | 0.67108 |
| | (0.32243) | (0.07575) | (0.06206) | (0.21331) | (0.05680) | (0.03523) |
| $\gamma$ (fixed) | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 |
| $R$ (fixed) | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 |
| Production parameters | | | | | | |
| $\alpha$ | 0.99900 | 0.93319 | 0.99577 | 0.99900 | 0.93319 | 0.99577 |
| | (0.00394) | (0.05113) | (0.00759) | (0.00182) | (0.09415) | (0.01693) |
| $r_1$ | −4.32815 | −6.74636 | −7.10693 | −4.32793 | −6.74635 | −7.10693 |
| | (1.66934) | (3.52009) | (2.37784) | (0.69972) | (4.82045) | (4.92342) |
| $\beta$ | 0.46377 | 0.68210 | 0.80801 | 0.46382 | 0.68210 | 0.80801 |
| | (0.37732) | (0.09751) | (0.03014) | (0.22714) | (0.01488) | (0.07045) |
| $r_2$ | −0.05717 | −0.64499 | −0.83248 | −0.05716 | −0.64499 | −0.83248 |
| | (0.84235) | (0.09201) | (0.22805) | (0.50421) | (0.00559) | (0.39352) |
| $b$ | 2.95781 | 3.07160 | 5.31969 | 2.95839 | 3.07160 | 5.31969 |
| | (0.92893) | (0.33642) | (0.99437) | (0.54780) | (0.31723) | (0.86633) |
| Rainfall parameters | | | | | | |
| $a_1$ | | | | 0.00661 | 0.00312 | 0.00740 |
| | | | | (0.00488) | (0.00207) | (0.00088) |
| $a_2$ | | | | 0.01799 | −0.00096 | 0.00244 |
| | | | | (0.00250) | (0.00073) | (0.00070) |
| $a_3$ | | | | −0.00268 | −0.00171 | −0.00598 |
| | | | | (0.00167) | (0.00139) | (0.00293) |
| Priors | | | | | | |
| $\theta_1$ | 1.16023 | 0.53693 | 0.71099 | 1.16023 | 0.53693 | 0.71099 |
| $\theta_2$ | 0.62301 | 0.60846 | 0.89703 | 0.62301 | 0.60846 | 0.89703 |
| Sample standard deviations | | | | | | |
| $\theta_0$ | 0.86582 | 0.66068 | 0.67076 | 0.84865 | 0.64910 | 0.48378 |
| $\theta_1$ | 1.15954 | 0.53666 | 0.71112 | 0.84956 | 0.53330 | 0.65161 |
| $\theta_2$ | 0.62278 | 0.60850 | 0.89700 | 0.61038 | 0.60498 | 0.87085 |
| Log-lik | −94.7837 | −32.7988 | −90.2818 | −45.9368 | −28.8412 | −42.4585 |
| Nber observ. | 143 | 149 | 139 | 143 | 149 | 139 |
| Lik-ratio test | | | | 97.6938 | 7.9153 | 95.6467 |

$\chi^2(3)$:0.005 = 12.83820; $\chi^2(3)$:0.05 = 7.81473 by more careful weeding. If rains are abundant and weed growth fast, farmers increase their weeding effort but not proportionally to weed infestation because of the increased opportunity cost of labor. As a consequence of less than optimal weeding, yields do not achieve their full potential. The difference in elasticities of substitution, however, means that the penalty for imperfect weeding is less stringent than the penalty for untimely planting. This is in line with observed farmer behavior in the area (e.g., Balcet and Candler (1982) in Northern Nigeria).

Standard errors of the parameters were approximated using the outer cross-product of the gradients as an estimate of the information matrix. If $l_n$ is the contribution of observation $n$ to the log-likelihood function, and $\lambda$ is the parameter vector, it can be shown that (Judge et al. (1985, p. 178)):

$$\sqrt{N}(\hat{\lambda}-\lambda) \xrightarrow{d} N\left[0, \lim(I(\lambda)/N)^{-1}\right]$$

where $I(\lambda)$, the information matrix, can be replaced by the consistent estimator

$$\left[\frac{1}{N}\sum_{n=1}^{N}\frac{\partial l_n}{\partial \lambda}\frac{\partial l_n}{\partial \lambda'}\right]^{-1}$$

(Berndt et al. (1974)).

The results suggest that some individual parameters, even though not strictly underidentified, cannot be identified very precisely. This is true in particular of the point estimates of the elasticities of substitution in consumption and production. The reason is that various combinations of parameters generate similar household behavior. Given the limited number of data points, the estimation process is not able to identify with precision which set of parameters actually generated the data. To the extent that one is primarily interested in simulating household behavior, however, this is not a serious handicap.

It is also interesting to compare the sample moments of land, labor, and income with their simulated moments (Table III). These were obtained by simulating household behavior using the estimated parameters and the actual land distribution and integrating moments numerically using Gauss-Hermite quadrature (Judd (1991)). Simulated means approximate fairly well the sample means for labor, but overestimate the average crop income (Table III). Similarly, the simulated and the sample covariance matrices are fairly close as far as labor and land are concerned, but the simulated model greatly overestimates the variance and covariance of crop output. That the sample and simulated moments are different is not entirely surprising, since the estimation philosophy was maximum likelihood, not the method of moments. This discrepancy, however, points in the direction of a possible model specification error, particularly regarding the assumption of uncorrelated exogenous shocks. In the model presented here, correlated shocks cannot be accommodated by the FIML approach because they destroy the invertibility of the decision rules. This difficulty may be avoided if one were to estimate parameters via the Method of Simulated Moments suggested by Pakes and Pollard (1989) and McFadden

TABLE III

SAMPLE MOMENTS AND SIMULATED MOMENTS

A. Mean
Agro-climatic zone:

| | Sahel | | Sudan | | Guinea | |
|---|---|---|---|---|---|---|
| | Sample | Simulated | Sample | Simulated | Sample | Simulated |
| $l_1$ | 0.218 | 0.226 | 0.353 | 0.383 | 0.275 | 0.308 |
| $l_2$ | 0.468 | 0.498 | 0.403 | 0.438 | 0.235 | 0.269 |
| $y_3$ | 1.881 | 2.959 | 1.494 | 1.928 | 2.769 | 4.640 |

B. Variance-covariance matrix
Agro-climatic zone:

| | Sahel | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Sample | | | | Simulated | | | |
| $l_1$ | 0.015 | | | | 0.021 | | | |
| $l_2$ | 0.003 | 0.042 | | | 0.018 | 0.065 | | |
| $y_3$ | −0.016 | 0.093 | 1.097 | | −0.670 | −1.473 | 15.553 | |
| land | 0.008 | 0.025 | 0.155 | 0.092 | 0.018 | 0.016 | 0.168 | 0.119 |
| | Sudan | | | | | | | |
| | Sample | | | | Simulated | | | |
| $l_1$ | 0.025 | | | | 0.045 | | | |
| $l_2$ | 0.011 | 0.022 | | | 0.028 | 0.038 | | |
| $y_3$ | 0.052 | 0.029 | 0.688 | | −0.739 | −0.844 | 4.361 | |
| land | 0.013 | 0.018 | 0.065 | 0.175 | 0.024 | 0.015 | 0.110 | 0.050 |
| | Guinea | | | | | | | |
| | Sample | | | | Simulated | | | |
| $l_1$ | 0.020 | | | | 0.028 | | | |
| $l_2$ | 0.013 | 0.020 | | | 0.017 | 0.030 | | |
| $y_3$ | 0.050 | 0.082 | 6.044 | | −1.431 | −1.246 | 64.560 | |
| land | 0.020 | 0.021 | 0.566 | 0.112 | 0.034 | 0.021 | 0.485 | 0.138 |

(1989). Attempting to estimate the model by that method is left for further research.

Figures 1 and 2 present the relationship between labor and shocks in the Sudanese agro-climatic zone (figures for other regions are similar). They show that households increase their labor effort whenever the crop growth process is hit by a positive shock, but they are restricted in doing so by their total available manpower. The graphs also confirm that, at planting time, effective manpower is reduced because planting can only be performed immediately following a rain shower. Planting is thus hindered when early rains falls several days apart. Consequently, households face labor shortages at planting *when rains are scattered* and at weeding *when rains are plentiful* (Norman (1972), Eicher and Baker (1982), Delgado (1979)). This explains why some fields go unplanted when early rains are insufficient and unweeded when rains are excessive. On the other hand, in bad years, the marginal productivity of labor in crop production falls so low that households prefer to reallocate their time to other pursuits. This observation reconciles the concern for possible manpower shortages often

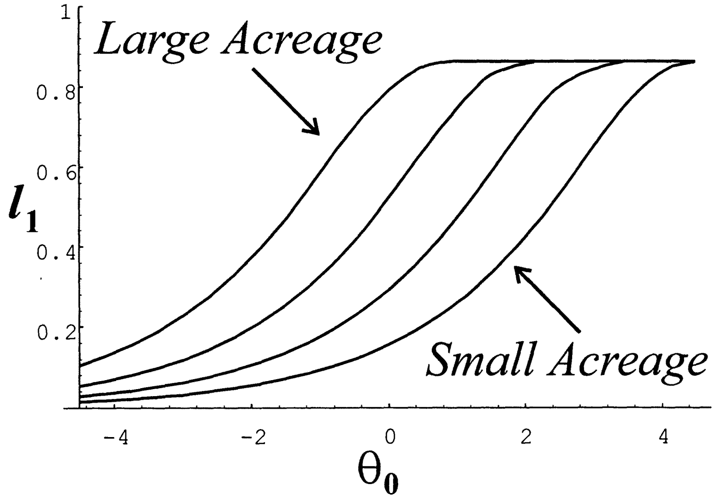

FIGURE 1.—Planting labor as a function of land and $\theta_0$.

expressed by African farmers with the low levels of labor inputs that are observed on average (Cleave (1974), Fafchamps (1986)).

The average shadow prices of labor and land were simulated using the estimated parameters. The results (Table IV) reproduce very closely the intuitive ranking of the three agro-climatic zones: high labor and land productivity in the well-watered north-guinean zone; low labor and medium land productivity in the overcrowded sudanian zone (Mossi plateau); medium labor and low land productivity in the dry, extensive farming, sahelian zone (Prudencio (1983, 1987)). Expected marginal returns to labor are also of the same order of

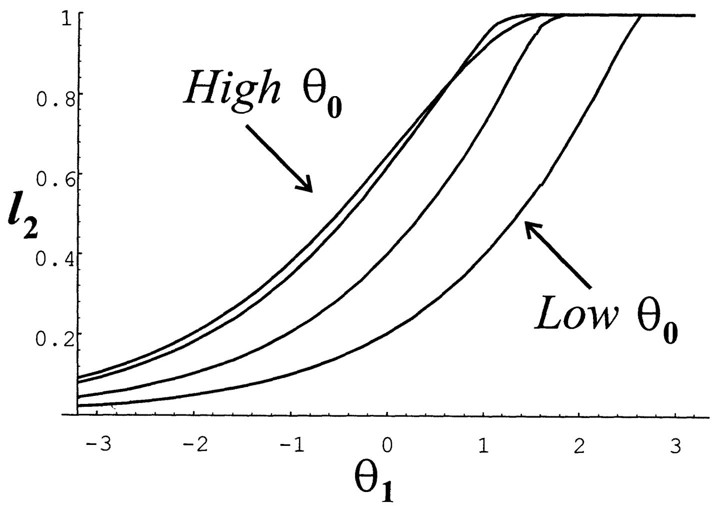

FIGURE 2.—Weeding labor as a function of $\theta_0$ and $\theta_1$.

TABLE IV

EXPECTED MARGINAL VALUE PRODUCT OF LABOR AND LAND (IN CFAF)
(340 CFAF ≈ 1 US dollar in 1982)

| | Agro-climatic zone: Sahel | Sudan | Guinea |
|---|---|---|---|
| $E(MP_{l_1})$ | 552 | 253 | 587 |
| $E(MP_{l_2})$ | 390 | 192 | 631 |
| $E(MP_A)$ | 6998 | 13463 | 17412 |

magnitude as the wage ratε recorded in (a very small number of) hired labor contracts in the area.

### *A Non-nested Model Specification Test*

To test whether the FIML implementation of the model was worthwhile, a non-nested model specification test was performed. A competition was organized between the FIML model described so far, and an alternative, simpler model, the deterministic control (DC) model. The DC model is essentially the same as the stochastic control (SC) model, except that households are assumed to make their labor decisions under certainty. Most applied econometric work is based on deterministic models—ideally obtained by solving a structural model (e.g., a profit or cost function), more often by writing its reduced form directly—to which residuals are later added, as an afterthought.[6] The DC model thus can be thought of as an application of that estimation philosophy to this decision problem. The result of the competition between the DC and the SC models throws light on the magnitude of the bias that results from ignoring the role that uncertainty and the gradual revelation of information play in individuals' decisions.

Formally, the DC optimization problem is

$$\max_{l_1, l_2(l_1,\theta_1)} \delta\big(\gamma(t-l_1)^{\sigma} + (1-\gamma)(t-l_2)^{\sigma}\big)^{\rho/\sigma} + (1-\delta)y_3^{\rho}$$

subject to

$$y_2 = a\big(\alpha A^{r_1} + (1-\alpha)l_1^{r_1}\big)^{1/r_1},$$

$$y_3 = b\big(\beta y_2^{r_2} + (1-\beta)l_2^{r_2}\big)^{1/r_2}.$$

Closed-form solutions for the labor decision rules do not exist, but the model can be solved numerically. The numerical functions defining the optimal choices of labor inputs are defined as $l_1^* = g_1(A;\lambda)$ and $l_2^* = g_2(A;\lambda)$.

[6] There are important exceptions, like estimation based on the Generalized Method of Moments (Hansen and Singleton (1982)).

The stochastic structure of the model is then defined by adding random error terms to those functions. The system to be estimated is

$$l_1 = g_1(A;\lambda)e^{\varepsilon_1}, \tag{15}$$

$$l_2 = g_2(A;\lambda)e^{\varepsilon_2}, \tag{16}$$

$$y_3 = f(A, l_1^*, l_2^*;\lambda)e^{\varepsilon_3}, \tag{17}$$

where $f(\cdot)$ is shorthand for the nested CES production function. The residuals $\varepsilon_i$ are assumed to be jointly normal with mean zero and variance-covariance matrix $\Phi$. Independence across residuals is not assumed.

FIML estimation of the DC parameters can then proceed in exactly the same fashion as for the SC model. The results are presented in Table V. Standard errors are estimated as before, using the outer cross-product of the gradients. Quasi-underidentification is more severe in this case, as shown by the high standard errors of some of the parameters. Probably for the same reason, several parameter estimates are at their upper or lower bound.

Vuong's (1989) test for non-nested models can now be computed to compare the two models. The contribution of each observation to the log-likelihood function is $l_n^s$ for the SC model and $l_n^d$ for the DC model. $L^s(\hat{\lambda}^s)$ and $L^d(\hat{\lambda}^d)$ are the values of the log-likelihood function respectively for the SC and the DC model, each evaluated at its FIML parameter vector $\hat{\lambda}^{s,d}$. The difference between the two log-likelihoods is $LR(\hat{\lambda}^s, \hat{\lambda}^d)$.

Vuong's non-nested model specification test $V$ is equal to $N^{-1/2}LR(\hat{\lambda}^s, \hat{\lambda}^d)/\hat{\omega}$, where $\hat{\omega} \equiv (1/N)\Sigma_{n=1}^N[l_n^2 - l_n^d]^2 - (1/N)\Sigma_{n=1}^N(l_n^2 - l_n^d)]^2$. Under the null hypothesis that the two models equally fit the data, $V$ is distributed as a standard normal variable. If the SC model is better than the DC model, $V \to \infty$; and if the certainty model is better, $V \to -\infty$.

The computed values for $V$ are all very large and show with a very high degree of confidence that the SC model dominates the DC model. They confirm that uncertainty plays an essential role in the labor decisions of the surveyed households in all three agro-climatic zones. They also suggest that large parameter biases can result from ignoring that observed decisions are influenced by information revealed gradually.

## 6. ADDING RAINFALL TO THE MODEL

Although individual production shocks were not observed, one of the major exogenous contributions to crop growth, rainfall, was measured at the village level. Following Chavas, Kristjanson, and Matlon (1991), aggregate rainfall information was incorporated in the model to increase estimation efficiency.

New residuals were defined as follows. $P_0$, $P_1$, and $P_2$ stand for rainfall observed, respectively, before and during early planting; during late planting and early weeding; and during late weeding and before harvesting. $p_i$ stands for

TABLE V

PARAMETER ESTIMATES FOR THE DETERMINISTIC CONTROL MODEL (ASYMPTOTIC STANDARD ERRORS IN BRACKETS)

| Agroclimatic zone: | 1/Without rainfall | | | 2/With rainfall | | |
|---|---|---|---|---|---|---|
| Utility parameters: | Sahel | Sudan | Guinea | Sahel | Sudan | Guinea |
| $\delta$ | 0.69297 | 0.75669 | 0.06863 | 0.70989 | 0.75693 | 0.27926 |
| | (0.18555) | (0.00859) | (0.00161) | (0.14790) | (0.00867) | (1.04136) |
| $\sigma$ | −0.24076 | −7.36277 | −1.66334 | 0.12165 | −7.55473 | −26.80488 |
| | (1.28134) | (8.52014) | (3.13628) | (3.48669) | (7.06746) | (92.71520) |
| $\rho$ | 0.84600 | 0.99000 | 0.96851 | 0.83298 | 0.99000 | 0.84049 |
| | (0.65505) | (0.02108) | (0.03244) | (0.22449) | (0.03611) | (0.66920) |
| $\gamma$ (fixed) | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 |
| Production parameters: | | | | | | |
| $\alpha$ | 0.70058 | 0.19046 | 0.99000 | 0.74005 | 0.18838 | 0.95670 |
| | (1.11933) | (0.03084) | (0.00060) | (0.34962) | (0.03449) | (0.26272) |
| $r_1$ | 0.33708 | 0.87905 | 0.76301 | 0.05824 | 0.87281 | −0.01732 |
| | (3.02636) | (0.28959) | (0.21817) | (0.29998) | (0.50169) | (3.37792) |
| $\beta$ | 0.62260 | 0.21084 | 0.98978 | 0.59379 | 0.20492 | 0.97028 |
| | (0.46962) | (0.23658) | (0.00059) | (0.26024) | (0.18315) | (0.21561) |
| $r_2$ | −0.77522 | −5.38658 | 0.99000 | −0.57477 | −5.62192 | 0.83334 |
| | (3.98861) | (5.80891) | (0.02225) | (2.89840) | (5.05749) | (0.83261) |
| $b$ | 3.09682 | 3.34891 | 3.42099 | 3.12573 | 3.34794 | 3.63016 |
| | (1.89598) | (0.15541) | (0.18243) | (0.98732) | (0.16282) | (1.09012) |
| Rainfall parameters: | | | | | | |
| $a_1$ | | | | 0.00837 | 0.00214 | 0.00563 |
| | | | | (0.00276) | (0.00101) | (0.00070) |
| $a_2$ | | | | 0.00576 | −0.00047 | 0.00077 |
| | | | | (0.00082) | (0.00044) | (0.00044) |
| $a_3$ | | | | 0.00472 | 0.00135 | 0.00043 |
| | | | | (0.00095) | (0.00113) | (0.00159) |
| Log-lik | −296.116 | −238.147 | −323.899 | −266.232 | −233.892 | −283.3751 |
| Nber observ. | 143 | 149 | 139 | 143 | 149 | 139 |
| Lik-ratio test: | | | | 59.76644 | 8.51054 | 81.04864 |
| $\chi^2(3)$:0.005 = 12.83820; $\chi^2(3)$: 0.05 = 7.81473. | | | | | | |
| Vuong's test | | | | | | |
| $\omega$ | 1.46974 | 4.24097 | 4.66031 | 1.61707 | 3.76930 | 4.27467 |
| $V$ | 11.45530 | 3.68269 | 4.26280 | 11.39231 | 4.45729 | 4.82279 |

the deviation of $P_i$ from its sample mean.[7] The shocks to production $\theta_i$ are now assumed to be the sum of two independent components: a shock common to all farmers and proportional to rainfall $r_i$; and an idiosyncratic shock $\psi_i$. The new

[7] This normalization was adopted only for convenience. Adding or subtracting any constant to the error term is equivalent to a multiplicative constant. Reestimating the model with a multiplicative constant would certainly force ML estimates for $\alpha$, $\beta$, and $b$ to compensate for it. The transformation is trivial and compensated parameters can easily be computed algebraically. Normalizing rainfall by dividing it by its sample mean simply ensures that parameter estimates derived from the model without rainfall are good starting points for the model with rainfall.

residuals $\psi$'s are thus defined as

$$\psi_i = \theta_i - a_i p_i$$

for $i = \{0, 1, 2\}$, where the $\theta_i$ are computed as before from the Euler equations. The $a_i$'s are new parameters to be estimated. The relationship between the $\theta$'s and the $\psi$'s is linear. Therefore, the likelihood function of the revised model is obtained simply by replacing the $\theta$'s by the $\psi$'s in equation (14).

Both the SC and the DC models were rerun with the above change. Results are presented in Tables I and IV. The likelihood ratio tests reported in the tables show that village level rainfall data significantly improve the performance of all models, but do not induce any change in the parameter estimates for the SC model. Repeating Vuong's test confirms that the stochastic control model still dramatically outperforms the deterministic control model. This suggests that, because of the importance of ideosyncratic shocks in the area, incorporating rainfall as an explanatory variable in the estimation of the production function (Chavas, Kristjanson, and Matlon (1991)) is insufficient to eliminate the simultaneity bias.

## CONCLUSIONS

Advances in computers have made it possible to estimate the structural parameters of a variety of dynamic models. In this paper, a finite horizon, stochastic control model of farmers' behavior was estimated econometrically using a nested algorithm in the tradition of Rust and Wolpin. The results were shown to be dramatically superior to those obtained by assuming that farmers solve a deterministic control problem. Incorporating observation about average rainfall improved the estimation, but was insufficient to eliminate the large gap between the stochastic and the deterministic control models. This suggests that, whenever data are generated as the solution to a stochastic control problem, large gains in efficiency can be expected by taking that fact adequately into account in the estimation process.

The model also provides an accurate although stylized account of rainfed farming in the African semi-arid tropics. Estimation results agree with farmers' perceptions about the dependency of planting labor on early rainfall and about the occurrence of manpower shortages. They confirm the importance of flexibility in production and provide a possible explanation of why weeding is often "insufficient" (from the point of view of agronomists) on large farms or in good years. They are in agreement with general perceptions about labor and land productivity in various agro-climatic zones of West Africa. They strongly suggest that surveyed farmers are far from lazy. On the contrary, farmers are shown to be willing to work long and unevenly distributed hours provided that the returns to their efforts are sufficient.

The policy recommendations that emerge from this analysis concern mostly the design and promotion of improved technological packages. In particular, a better use of manpower resources in the semi-arid tropics requires alternative technologies that improve farmers' control over the plant growth process, whether through full or partial irrigation or through water harvesting techniques like tied ridging (Ohm, Sanders, and Nagy (1986)). Rainfed farming, nevertheless, will continue to play a major role in agricultural production in the foreseeable future, and consequently crop growth will remain partly outside the control of West African farmers. Therefore, agricultural research institutes, particularly those working in the semi-arid tropics, should factor flexibility into their research agenda and develop new approaches to rainfed agriculture that emphasize and enhance flexibility, instead of reducing it. Similarly, extension services advising rainfed farmers should avoid promoting fixed packages of inputs and so-called optimal dates for performing given farming operations. Rather, they should help farmers increase their ability to correct adverse states of nature and take advantage of good ones.

*Food Research Institute, Stanford University, Stanford, CA 94305-6084, U.S.A.*

*Manuscript received May, 1991; final revision received December, 1992.*

## TECHNICAL APPENDIX

### *Computing the Shocks*

Computing equation (11) for various values of $\theta_0$ requires the integration of its left-hand side with respect to $\theta_1$ and $\theta_2$. To do so, a Gauss-Hermite quadrature (Davis and Rabinowitz (1984)) was used with six nodes for each random shock. Experimentation with higher degree quadrature indicated no sizeable gain in precision over the six nodes approximation.

In evaluating the left-hand side of equation (11), the optimal choice of $l_2$ as a function of $l_1$ and past shocks must be computed. This was computed exactly at each $\theta_1$ node by a gradient method. This is costly in terms of computing. In principle, the search for a parameter vector could be significantly speeded up by building beforehand a Chebyshev approximation (Atkinson (1989)) to the $l_2$ policy function and using it in estimating the left-hand side of equation (11). But that method was found unreliable in some cases and had to be abandoned.

The search for the zero of equation (11) relied mostly on a Newton method, backed-up by a more robust subroutine in case of failure (Numerical Algorithms Group (1986), C05ADF). The value of $\theta_0$ in the last (successful) parameter iteration was used as initial guess. As the search for the FIML parameter vector converges, better initial guesses are inherited from previous iterations and the Newton method increases in speed and reliability.

Once the value of $\theta_0$ is found, $\theta_1$ can be calculated in a similar way by computing the zero of equation (12). Given $\theta_0$ and $\theta_1$, $\theta_2$ is computed algebraically from the production function. Because the algorithm spends most of its time in the subroutines evaluating the left hand sides of equations (11) and (12), significant gains in speed can be achieved by careful programming.

Some combinations of parameter vectors and land endowment are inconsistent with the observed labor decisions in the sense that there exists no value of $\theta_0$ or $\theta_1$ that would have led to the observed labor decisions. Strictly speaking, the likelihood of those parameter vectors is zero. To keep the search algorithm away from such parameter vectors, the failure to find a $\theta_0$ or $\theta_1$ triggers a penalty function, that is, it causes the subroutine that evaluates the likelihood function to return a large negative value. The choice of an appropriate penalty function is a difficult practical issue: too large a penalty creates discontinuities in the object function; too small a penalty is not sufficient to keep the algorithm away from bad parameter vectors. Furthermore, the shape of the penalty function influences the speed of convergence via its impact on the quasi-Newton updating of the Hessian matrix.

### *Maximizing the Likelihood Function*

A variety of search algorithms can be used to maximize the likelihood function. In this paper a combination of two methods was used: first, a "good" starting set of parameters was located with the help of a Nelder-Mead polytope algorithm (Nelder and Mead (1965), Gill, Murray, and Wright (1981)); once found, the search switched to a faster quasi-Newton method. The polytope algorithm was found useful whenever the initial guess about the parameter vector is inconsistent with some of the data points, i.e., whenever no value of $\theta_0$ or $\theta_1$ could be found that would have led to some of the observed labor decisions. The discontinuities created by the penalty functions necessitated the use of a very robust search algorithm until values of $\theta_0$ and $\theta_1$ could be found for each observation.

*Imposing Rational Expectations*

Let $\hat{\beta}$ be the vector of parameters to be estimated. Let $\hat{\sigma}$ be the vector of sample variances for $\theta_1$ and $\theta_2$. Let $\sigma_p$ be the vector of households' priors regarding the variance of the exogenous shocks $\theta_1$ and $\theta_2$. Finally, let $x$ stand for $\{l_1, l_2, y_3, A\}$. Then the likelihood function can be written as $L(\hat{\beta}, \hat{\sigma}, \sigma_p; x)$.

To avoid underidentification, rational expectations are imposed by forcing $\sigma_p = \hat{\sigma}$. Maximum likelihood estimation then boils down to the following constrained optimization problem:

$$\operatorname*{Max}_{\hat{\beta}, \hat{\sigma}, \sigma_p} L\left(\hat{\beta}, \hat{\sigma}, \sigma_p; x\right)$$

subject to

$$\sigma_p = \hat{\sigma}$$

or, more simply,

$$\operatorname*{Max}_{\hat{\beta}, \hat{\sigma}} L\left(\hat{\beta}, \hat{\sigma}, \hat{\sigma}; x\right).$$

Solving the above optimization problem could in principle be achieved in a variety of ways. The approach that immediately comes to mind is to iterate on structural parameters and priors at the same time. One would start with a guess about structural parameters and priors, derive the implied shocks, compute the likelihood value as well as the associated sample variances, and then use these sample variances as priors for the next iteration. This approach was tried but failed to converge. The reason is that changing priors from one iteration to the next affects the likelihood values and disorients the quasi-Newton likelihood maximization algorithm.

Consequently, a more cumbersome approach had to be relied on. In this approach, an initial guess about priors is made, say $\sigma_p^{(0)}$, and maximum likelihood estimates of the structural parameters and sample variances are computed iteratively *keeping these priors constant*. In other words, the following maximization problem is solved:

$$\operatorname*{Max}_{\hat{\beta}^{(0)}, \hat{\sigma}^{(0)}} L\left(\hat{\beta}, \hat{\sigma}, \sigma_p^{(0)}; x\right),$$

keeping $\sigma_p^{(0)}$ constant. Then a new guess about priors is formed, i.e. $\sigma_p^{(1)} = \hat{\sigma}^{(0)}$, and new maximum likelihood estimates are computed keeping the new priors constant, i.e.

$$\operatorname*{Max}_{\hat{\beta}^{(1)}, \hat{\sigma}^{(1)}} L\left(\hat{\beta}, \hat{\sigma}, \sigma_p^{(1)}; x\right),$$

keeping $\sigma_p^{(1)}$ constant.

The process is repeated until it converges, that is, until $\hat{\sigma}^{(n)} = \sigma_p^{(n)}$. At convergence one has found estimates of the structural parameters $\hat{\beta}$ and of the sample variances $\hat{\sigma}$ that maximize the likelihood function and at the same time satisfy the rational expectations constraint. One thus has found the solution to

$$\operatorname*{Max}_{\hat{\beta}, \hat{\sigma}} L\left(\hat{\beta}, \hat{\sigma}, \hat{\sigma}; x\right).$$

Although this simple but cumbersome process is not ensured to converge in general, it turned out to converge very rapidly for the purpose of this paper: priors and sample variances were found to agree up to the third digit after the third or fourth iteration. If this had not been the case, and if convergence had proven slow, a more efficient method for finding the zero of a function would have had to be used to set $\hat{\sigma}$ equal to $\sigma_p$ iteratively (Atkinson (1989), Judd (1991)).

*Simulation*

The simulation program solves the household's optimization problem using backward induction. Like the estimation algorithm, it relies on Gauss-Hermite quadrature and Newton optimization methods. The distribution of land is assumed to be log-normal.

Simulated moments are computed using Gauss-Hermite quadrature. The household's optimization problem is solved for a set of well chosen Gauss-Hermite nodes (Judd (1991)), and integration of the simulated means and variances is performed by summing up simulated values for labor and income using Gauss-Hermite weights. Experimentation showed that quadrature with eight nodes on each of the relevant exogenous variables ($\theta_0$, $\theta_1$, $\theta_2$, and $A$) gave at least fifth or sixth digit accuracy on the simulated moments.

*Computer Time*

The initial version of the estimation program was solved on a Cray X-MP. Improved programming now allows a single SC likelihood optimization to run overnight on a PC 386-25 equipped with a math coprocessor. Finding the maximum of the likelihood function is much faster in the DC case; searching time is at most an hour or two. In either case, by far the largest computer cost is not CPU time but programming.

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