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An Estimable Dynamic Stochastic Model of Fertility and Child Mortality

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This paper develops a finite-horizon dynamic stochastic model of discrete choice with respect to life-cycle fertility within an environment where infant survival is uncertain. The model yields implications for the number, timing, and spacing of children. A tractable estimation method is developed for the linear constraint–quadratic utility case that is intimately tied to the dynamic optimization problem, and the method is applied to Malaysian household data. Estimation is based on integrating the numerical solution of the dynamic programming model of behavior with a maximum likelihood procedure.

Introduction

Connections between child mortality and fertility are at the root of many explanations of the demographic transition and are important for population policy in less developed countries. In this paper, I present an estimable dynamic stochastic model of life-cycle fertility within an environment where infant survival is uncertain. The optimization model provides a unified treatment of replacement behavior (Ben-Porath 1976; Schultz 1976; Olsen 1980), that is, the reaction to a realized infant death; of generalized survival uncertainty, that is, the

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reaction to the mortality environment; and of spacing behavior. The model yields implications for the number, timing, and spacing of children in such an environment.[1] A tractable estimation method is developed for the linear (constraint)–quadratic (utility) case that is intimately tied to the dynamic optimization problem, and the method is applied to data from Malaysia. Although the econometric methodology is interesting in its own right, the paper also provides empirical estimates of important substantive phenomena in the context of a sequential fertility model.[2]

The finite-horizon dynamic programming problem that is assumed to be solved by the individual yields time-variant decision rules concerning the choice whether or not to have a child in each period. Analytical representation of those decision rules is not feasible. Estimation is based on integrating the numerical solution of the dynamic programming model with a maximum likelihood procedure. The complexity of the behavioral model is limited by the computational burden of the dynamic programming solution algorithm given the iterative nature of the estimation procedure. The methodology, however, is general to any finite-horizon dynamic stochastic model that can be numerically solved and would be applicable, for example, to job search models, labor supply models, and schooling choice models.[3]

The methodological approach is highly structured and follows the tradition of Heckman and MaCurdy (1980) in the analysis of life-cycle labor supply or of Sargent (1978) in macro time-series analysis. Because of the discipline this approach imposes on the researcher, it is generally the case (and there is no exception here) that the model is quite simple, even if analytical solutions are not required for estimation.[4] This characteristic has led to fundamental objections to the approach, not necessarily as a research agenda, but as a more immediate tool for addressing substantive issues.

[1] Spacing involves a fertility pattern in which births are not all consecutive in the context of discrete time. Timing relates to life-cycle stage. Births may occur early or late in the childbearing period, and they may be closely or widely spaced. The model presented here explicitly characterizes the birth sequence and thus solves for both aspects of fertility.

[2] Sequential fertility models have been studied by Heckman and Willis (1976), Hotz (1980), Moffitt (1980), and Vijverberg (1981). The model presented here is very similar to that of Heckman and Willis. None of these studies develop the econometric methodology necessary to provide estimates of "structural" parameters.

[3] Other very recent examples of the type of estimation method developed here are Miller's (1982) job-matching model and Pakes's (1983) patent model. Issues related to the estimation of dynamic programming models in job search contexts can be found in Flinn and Heckman (1982). Work by Hansen and Sargent (1981) is related.

[4] In a naive sense, all models are estimable save for computational expense. See Hartley (1981) who advocates a much greater effort in this direction.

The advantages and disadvantages of structural estimation as opposed to econometric specifications as "approximations" to optimizing models are well known. A practical issue arises, however, if the approaches lead to different substantive results but fit the data equally well. The structural model, if correctly specified, provides legitimate restrictions that permit more precise inference and a more parsimonious representation of complex relationships; but if the model is incorrectly specified, all inferences may be contaminated regardless of the offending assumptions. Relationships estimated as "approximations to theory," since they do not explicitly follow from theory, contain implicit and unknown assumptions but are in a sense less restrictive and may provide more accurate, though cruder, estimates of particular responses of interest. The choice of estimates is likely to be problem specific. Those who have strong reservations about the substantive implications of the structural model estimated here can view this as a prototype of potentially more fruitful models yet to be developed.

The paper is organized as follows. In the next section I specify a discrete-time, discrete-outcome dynamic stochastic fertility model with exogenous but uncertain infant mortality and income. I then describe the estimation strategy and present the data and results. A discussion section interprets the results and briefly addresses some of the broader methodological issues. The final section summarizes.

A Discrete-Time, Discrete-Outcome Dynamic Stochastic Model of Fertility

The household is assumed to possess an intertemporally separable utility function defined over a finite horizon having as arguments the number of surviving children and a single composite consumption good. There is no direct consumption value to the spacing or timing of births except insofar as the future is discounted. A period is defined to be that length of time within which a single birth may occur.[5] Births may occur only during the fertile stage, the length of which is known with certainty to be T periods.[6] There are $\tau - T$ subsequent periods of sterility. Contraception is assumed to be perfect during the fertile stage, and, conversely, conception can be timed without error. The choice is of a discrete nature, that is, to have a

[5] As in all discrete-time models the length of a period is somewhat arbitrary. In the case of fertility, however, biological considerations provide some guidance.

[6] The model would be significantly more complicated to estimate if T were itself a random variable.

child or not.[7] There is a fixed cost of bearing a live child whether or not the child survives the first period. There is also a technologically determined fixed cost of maintaining a live child for the first period of its life.[8] Infant mortality and household income are exogenous, and each follows a stochastic process known to the household.[9]

The decision problem for household i at time t is given by

$$\max E_t \sum_{k=0}^{\tau-t} \delta^k U^i_{t+k}(M^i_{t+k}, X^i_{t+k}), \tag{1}$$

with respect to n^i_t, where $n^i_t = 1$ indicates a birth at t and $n^i_t = 0$ indicates no birth; X^i_{t+k} is the level of goods consumption in period $t + k$ and M^i_{t+k} is the stock of surviving children. The utility function is assumed to be quadratic, namely, at any period t:

$$U^i_t(M^i_t, X^i_t) = (\alpha_1 + \xi^i_t)M^i_t - \alpha_2 M^{i^2}_t + \beta_1 X^i_t - \beta_2 X^{i^2}_t + \gamma M^i_t X^i_t. \tag{2}$$

The taste parameter ξ^i_t is assumed to be random with zero mean and finite variance.[10] The law of motion for M_t (without the i superscript) is

$$\begin{aligned} M_t &= M_{t-1} + n_t - d_t, \quad t = 1, \ldots, T, \\ M_t &= M_T, \qquad\qquad\quad\; t = T + 1, \ldots, \tau, \end{aligned} \tag{3}$$

with initial condition

$$M_0 = 0. \tag{4}$$

The variable d_t is unity if a death occurs during period t of a child born at t and is zero otherwise. A child is assumed to survive forever, that is, longer than the parents, if it survives its first period.[11] In equation (1), E_t is the expectations operator conditional on the infor-

[7] Introducing contraceptive risk would change the nature of the problem somewhat. The choice would become whether or not to contracept. Observations either on contraceptive behavior or on births could be used in estimation. Since my primary focus is on infant survival, accommodating additional forms of uncertainty seemed overly ambitious.

[8] It is, of course, more natural to assume that the maintenance cost of a child is positive at older ages, as has been assumed in other theoretical life-cycle fertility models (e.g., Hotz 1980). However, doing so expands the dimensionality of the state space from one that is equal to the number of surviving children in any particular period (i.e., T possible states in period T) to one in which the size depends on potential sequences of children. If the maintenance cost changed each period of a child's life, e.g., there would be 2^T possible state values to consider at period T.

[9] I ignore the possibility that parental expenditures can influence infant mortality in order to avoid modeling a second decision variable.

[10] This particular placement of ξ^i_t in the utility function is purposeful as will become apparent when the estimation method is presented.

[11] In the application below and in many other LDC contexts a substantial component of child mortality occurs within the first few years of life.

mation set as of t and δ is the discount factor. Period 1 is the first period in which a woman makes an independent decision about fertility.[12]

Child deaths are generated as follows:

$$\begin{aligned} d_t &= 0 \text{ iff } \mathbf{G}_t\pi_1 + u_t \geqslant 0 \\ d_t &= 1 \text{ iff } \mathbf{G}_t\pi_1 + u_t < 0, \end{aligned} \tag{5}$$

where the $\mathbf{G}_t$ vector may contain time-varying variables, for example, calendar time, or possibly household characteristics. The random component u_t has zero mean and finite variance.

The budget constraint is assumed to be satisfied period by period:

$$Y_t = X_t + b(n_t - d_t) + cn_t, \tag{6}$$

with Y_t the household income at t, c the fixed cost of a birth, and b the maintenance cost of a surviving child. Income at time t has exogenous determinants H_t, for example, age, and is given by

$$Y_t = \mathbf{H}_t\pi_2 + v_t, \tag{7}$$

where v_t has zero mean and finite variance.

It is important to specify precisely the content of the information set. As of period t the household is assumed to know the number of surviving children up to period t, M_{t-1}. The systematic components of income and infant mortality are also assumed known at each t for all $t = 1, \ldots, \tau$, that is, $\mathbf{G}_1\gamma_1, \ldots, \mathbf{G}_T\pi_1, \mathbf{H}_1\pi_2, \ldots, \mathbf{H}_\tau\pi_2$ is known at the initial period. Past realizations of income are obviously known, but neither the current realization (v_t) nor future realizations ($v_{t+1}, \ldots, v_\tau$) are assumed known.[13] Similarly, past deaths are known, but obviously not the fate of the current (to be born) child or future children. The household is also assumed to know its current (and past) tastes, ξ_t^i, though not those it will have in the future. Finally, the joint distribution (conditional at t) of the random income, mortality, and taste components is assumed known for all $t = 1, \ldots, \tau$ at the initial period.

The maximization problem given by (1)–(7) can be solved using the principle of optimality (Bellman 1957). There is only a single (discrete) control variable (n_t) and a single state variable (M_{t-1}). The solution is obtained by backward recursion. Decision rules, which entail comparisons between expected utility levels with and without a birth, are not time invariant given the finite decision horizon (T) and are difficult to represent analytically.

[12] Period 1 does not necessarily correspond to the date of marriage, since the decision not to marry may be a childbearing decision.

[13] Thus, the fertility decision at t is assumed to be made prior to the realization of income, which captures the fact that the decision to have a child precedes the birth by the gestation lag.

Some features of the model can be explored analytically within a two (decision) period framework though in a longer-horizon model they can most efficiently be exhibited through simulation.[14] The most important features are these:[15] (1) It is capable of generating a wide variety of birth sequences, that is, timing and spacing patterns. Given the assumption that the household cannot save or dissave, a rising income profile may generate spaces (and delay) as it counteracts the desire to have children as fast as possible.[16] (2) The model is capable of generating a wide variety of replacement patterns from full replacement of any child death to zero replacement. Replacement patterns and spacing are highly interdependent phenomena. (3) The model is not likely to generate higher fertility in high mortality environments, that is, where the infant survival probability is low. Essentially, since a child can die only in its first period, it will be optimal to follow a sequential replacement strategy unless children are desired at the end of the fertile stage where replacement is physiologically not possible.

Method of Estimation

To focus on the central estimation problem, assume that the income and survival rate functions (eqq. [5] and [7]) are known to the researcher.[17] Also, for ease of presentation, assume that ξ_t^i, u_t^i, and v_t^i are identically independently distributed in both the household (i) and period (t) dimensions and are stochastically independent. These assumptions can all be relaxed in principle to permit generalized vector time-series representations in the sense that the estimation procedure outlined below would still be valid. However, the mathematical presentation that follows would need to be suitably modified.

Now, with these assumptions and given, in particular, the quadratic

[14] See Wolpin (1982) for a more complete discussion of the analytics of the solution and for a presentation of simulation results.

[15] The simulation results assume that the individual has unchanging tastes, i.e., $\xi_t^i = 0$ for all t.

[16] Heckman and Willis (1976) present a model that differs in the existence of contraceptive uncertainty rather than survival uncertainty. The spacing-timing result is conjectured there. Spacing could also arise if capital markets were perfect and the rate of interest exceeded the rate of time preference. That modeling strategy was not pursued since any full-solution estimation method would have required solving for optimal savings behavior along with optimal fertility behavior. Neither extreme savings assumption would seem appropriate in the Malaysian context.

[17] If they are estimated by the researcher, then efficient estimation of the entire structural model would require joint estimation of these relationships with the fertility decision rules. Procedurally, this is a straightforward task, though in the empirical application it is ignored for computational reasons. It would also permit a test of cross-equation restrictions and, in this sense, of the model.

form of the utility function, the decision to have a child at any arbitrary period t is determined by the sign of the following expression:

$$J_t = E_t(LU_t|n_t = 1, \xi_t = 0) - E_t(LU_t|n_t = 0, \xi_t = 0) + P_t\xi_t, \quad (8)$$

where LU_t is lifetime utility at t and P_t is the infant survival probability at t.[18] The decision rule given by (8) is deterministic for the household; that is, the household decides to have a child if J_t is positive and decides not to have a child if it is not. If the researcher does not know ξ_t, then from that perspective the fertility decision is random. Assuming that the conditional expectations expressions in equation (8) can be calculated in a numerical sense for given parameter values by the researcher, the estimation procedure is straightforward. Obviously, there exists a unique value of ξ_t for which J_t is exactly zero.[19] The critical value, say ξ_t^*, depends on the value of the parameters that are to be estimated as well as the data on income and mortality.[20]

If ξ_t is assumed to follow a normal distribution, with variance σ^2,

[18] Recall that $E_t\xi_t = \xi_t$; i.e., the household knows its draw prior to making its decision.

[19] The placement of the estimation error in α_1 was governed entirely by computational considerations. Any other parameters of the model could have been assumed random, α_2, β_2, γ, b, c, or δ. Furthermore, an estimation error could have been introduced into the income or survival probability functions, i.e., as variables unknown to the researcher but known to the household. Some of these other choices would have destroyed the simple form of eq. (8). There would be no guarantee of a unique critical value and all solutions would have to be located numerically.

[20] As an example, the decision rule at T is given by

$$\begin{aligned} J_T = P_T\Bigg[&\alpha_1\Bigg(1 + \sum_{t=T+1}^{T} \delta^{t-1}\Bigg) - \alpha_2(2M_{T-1} + 1)\Bigg(1 + \sum_{t=T+1}^{T} \delta^{t-1}\Bigg) \\ &- \beta_2(b^2 - 2bH_T\pi_2 + 2cb) \\ &+ \gamma\Bigg(-bM_{T-1} + H_T\pi_2 - c - b + \sum_{t=T+1}^{T} \delta^{t-1}H_t\pi_2\Bigg)\Bigg] \\ &+ c[-\beta_1 - \beta_2(c - H_T\pi_2 - \gamma M_{T-1})] \\ &+ P_T\xi_T. \end{aligned}$$

The following conditional expectations are necessary for the derivation:

$$\begin{aligned} E_{t-j}(X_t|n_t &= 1, d_t = 0) = H_t\pi_2 - c - b, \\ E_{t-j}(X_t|n_t &= 1, d_t = 1) = H_t\pi_2 - c, \\ E_{t-j}(X_t|n_t &= 0, d_t = 0) = H_t\pi_2, \\ E_{t-j}(X_t^2|n_t &= 1, d_t = 0) = (H_t\pi_2)^2 + \sigma_v^2 + c^2 + b^2 - 2(c + b)H_t\pi_2 + 2cb, \\ E_{t-j}(X_t^2|n_t &= 1, d_t = 1) = (H_t\pi_2)^2 + \sigma_v^2 + c^2 - 2cH_t\pi_2, \\ E_{t-j}(X_t^2|n_t &= 0, d_t = 0) = (H_t\pi_2)^2 + \sigma_v^2, \end{aligned}$$

for $j = 0, \ldots, t$. It should be noted that although $(H_t\pi_2)^2$ and σ_v^2 appear in expressions for $E_T(LU_T|n_T = 1)$ and $E_T(LU_T|n_T = 0)$, they do not appear in J_T. In the 1-period model, certainty equivalence obtains.

then the probability that household i is observed to have a child in period t is given by

$$\Pr(n_t^i = 1|M_{t-1}^i) = 1 - \Phi\left(\frac{\xi_t^{i*}}{\sigma}\right), \tag{9}$$

where Φ is the cumulative standard normal. Similarly,

$$\Pr(n_t^i = 0|M_{t-1}^i) = \Phi\left(\frac{\xi_t^{i*}}{\sigma}\right). \tag{10}$$

The probabilities are conditioned on M_{t-1}^i to emphasize the fact that they depend on past fertility choice. The probability of observing any sequence of birth decisions for a household is merely the product of probabilities such as (9) and (10).

Now, consider the problem of calculating the deterministic part of the decision rules (as given in [8]). To do so requires characterizing the dynamic programming solution explicitly. The exercise is carried out backward through period $T - 1$ only, as the remaining recursions follow by induction.

Expected lifetime utility associated with having a child at $T - 1$ conditional on the information set at $T - 1$ is given by:

$$\begin{aligned} E_{T-1}(LU_{T-1}|n_{T-1} = 1) &= E_{T-1}(U_{T-1}|n_{T-1} = 1, \xi_{T-1} = 0) \\ &+ (P_{T-1} + M_{T-2})\xi_{T-1} + \delta P_{T-1}E_{T-1} \max [E_T(LU_T|n_T = 1, \\ &\quad M_{T-1} = M_{T-2} + 1), E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2} + 1)] \\ &+ \delta(1 - P_{T-1})E_{T-1} \max [E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2}), \\ &\quad E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2})]. \end{aligned} \tag{11}$$

Since ξ_T is not known to the household at $T - 1$, the terms inside the maximum operators are random. Defining $\xi_T^{**}(M_{T-1} = M_{T-2} + 1)$ to be the (unique) value of ξ_T that sets $J_T = 0$ conditional on having a child at $T - 1$ who survives ($M_{T-1} = M_{T-2} + 1$) and conditional on all other information as of $T - 1$, and likewise defining $\xi_T^{**}(M_{T-1} = M_{T-2})$ to be the value of ξ_T that sets $J_T = 0$ conditional either on having a child at $T - 1$ who does not survive or on not having a child at $T - 1$ ($M_{T-1} = M_{T-2}$), and conditional on all other information as of $T - 1$, equation (11) can be written as

$$\begin{aligned} &E_{T-1}(LU_{T-1}|n_{T-1} = 1) \\ &= E_{T-1}(U_{T-1}|n_{T-1} = 1, \xi_{T-1} = 0) + (P_{T-1} + M_{T-2})\xi_{T-1} \\ &\quad + \delta P_{T-1}\Big[\Big(\{E_{T-1}E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2} + 1, \xi_T = 0) \\ &\qquad + (P_T + M_{T-2} + 1)E_{T-1}[\xi_T|\xi_T \geq \xi_T^{**}(M_{T-1} = M_{T-2} + 1)]\} \end{aligned}$$

$$\begin{aligned}
&\times \{\mathrm{pr}[\xi_T \geqslant \xi_T^{**}(M_{T-1} = M_{T-2} + 1)]\}\Big) \\
&+ \Big(\{E_{T-1}E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2} + 1, \xi_T = 0) \\
&+ (M_{T-2} + 1)E_{T-1}[\xi_T|\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2} + 1)]\} \\
&\times \{\mathrm{pr}[\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2} + 1)]\}\Big)\Big] \qquad (12) \\
+ \delta(1 - P_{T-1})&\Big[\Big(\{E_{T-1}E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2}, \xi_T = 0) \\
&+ (P_T + M_{T-2})E_{T-1}[\xi_T|\xi_T \geqslant \xi_T^{**}(M_{T-1} = M_{T-2})]\} \\
&\times \{\mathrm{pr}[\xi_T|\xi_T \geqslant \xi_T^{**}(M_{T-1} = M_{T-2})]\}\Big) \\
&+ \Big(\{E_{T-1}E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2}, \xi_T = 0) \\
&+ M_{T-2}E_{T-1}[\xi_T|\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2})]\} \\
&\times \{\mathrm{pr}[\xi_T|\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2})]\}\Big)\Big].
\end{aligned}$$

In analogous fashion, the expected lifetime utility associated with not having a child at $T - 1$ is

$$\begin{aligned}
&E_{T-1}(LU_{T-1}|n_{T-1} = 0) \\
&\quad = E_{T-1}(U_{T-1}|n_{T-1} = 0, \xi_{T-1} = 0) + M_{T-2}\xi_{T-1} \\
&\qquad + \delta\Big[\Big(\{E_{T-1}E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2}, \xi_T = 0) \\
&\qquad\quad + (P_T + M_{T-2})E_{T-1}[\xi_T|\xi_T \geqslant \xi_T^{**}(M_{T-1} = M_{T-2})]\} \\
&\qquad\quad \times \{\mathrm{pr}[\xi_T \geqslant \xi_T^{**}(M_{T-1} = M_{T-2})]\}\Big) \qquad (13) \\
&\qquad\quad + \Big(\{E_{T-1}E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2}, \xi_T = 0) \\
&\qquad\quad + M_{T-2}E_{T-1}[\xi_T|\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2})]\} \\
&\qquad\quad \times \{\mathrm{pr}[\xi_T < \xi_T^{**}(M_{T-1} = M_{T-2})]\}\Big)\Big].
\end{aligned}$$

Under the normality assumption, if we recall that ξ_T is serially uncorrelated, conditional expectations are

$$\begin{aligned}
E(\xi_T|\xi_T \geqslant \xi_T^{**}) &= \frac{\sigma\phi(\xi_T^{**}/\sigma)}{1 - \Phi(\xi_T^{**}/\sigma)}, \\
E(\xi_T|\xi_T < \xi_T^{**}) &= \frac{-\sigma\phi(\xi_T^{**}/\sigma)}{\Phi(\xi_T^{**}/\sigma)}.
\end{aligned} \qquad (14)$$

The decision rule given by (8) for $t = T - 1$ can now be obtained from (12) and (13). The complete dynamic programming solutions can be obtained numerically by continuing to work backward in the fashion described above. Differencing (12) and (13) yields

$$
\begin{aligned}
J_{T-1} &= E_{T-1}(U_{T-1}|n_{T-1} = 1, \xi_{T-1} = 0) \\
&- E_{T-1}(U_{T-1}|n_{T-1} = 0, \xi_{T-1} = 0) + P_{T-1}\xi_{T-1} \\
&+ \delta P_{T-1}\Bigg(\{E_{T-1}E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2} + 1, \xi_T = 0)\} \\
&\times \left\{1 - \Phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2} + 1)}{\sigma}\right]\right\} \\
&+ \{E_{T-1}E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2} + 1, \xi_T = 0)\} \\
&\times \left\{\Phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2} + 1)}{\sigma}\right]\right\} \\
&- \{E_{T-1}E_T(LU_T|n_T = 1, M_{T-1} = M_{T-2}, \xi_T = 0)\} \\
&\times \left\{1 - \Phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2})}{\sigma}\right]\right\} \\
&- \{E_{T-1}E_T(LU_T|n_T = 0, M_{T-1} = M_{T-2}, \xi_T = 0)\} \\
&\times \left\{\Phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2})}{\sigma}\right]\right\} \\
&+ P_T\sigma\Bigg\{\phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2} + 1)}{\sigma}\right] \\
&- \phi\left[\frac{\xi_T^{**}(M_{T-1} = M_{T-2})}{\sigma}\right]\Bigg\}\Bigg).
\end{aligned}
\tag{15}
$$

Notice that all terms within the conditional expectations of lifetime utility that are independent of the fertility choice, that is, of whether $n_{T-1} = 1$ or 0, will not appear in J_{T-1}. These terms get differenced out, including the square of predicted income and the variance in income; only the conditional mean of income enters the decision rule.[21]

Putting the pieces together, a ξ_t^* is calculated for each t and for a given parameter vector (α_1, α_2, β_1, β_2, γ, c, b, δ) according to the

[21] Observe that in the previous footnote $(H_t\pi_2)^2$ and σ_{ν}^2 appear in all expectations regardless of the value of n_t or d_t, a carry-over of the certainty equivalence result in the static model.

dynamic programming solution outlined above. The likelihood taken over a sample of I women each of whom is observed for T_i periods is

$$\prod_{i=1}^{I} \prod_{t=1}^{T_i} \mathrm{pr}(n_t^i = \bar{n}_t^i | M_{t-1}^i), \tag{16}$$

where $\bar{n}_t^i$ is one if woman i has a birth at t and zero otherwise. Choosing alternative parameter values leads to alternative critical values (ξ_t^*) and therefore to different sample likelihood values. A derivative-free maximization routine is required in the absence of analytical derivatives. The computational burden lies in the fact that for each set of parameter values the dynamic programming algorithm must be used to calculate the new set of ξ_t^*'s for each individual.

As in standard probit analysis, σ is not identified even though it enters the decision rules explicitly; that is, ξ_t^* is a function of σ. Utility function parameters are thus only identified relative to each other, but cost parameters are identified in an absolute sense.[22] Heuristically, there should be more than sufficient information to estimate the parameters since all future-period expected incomes and survival probabilities enter each period's decision. Further, since ξ_t^* is serially uncorrelated, any set of periods may be used to calculate consistent maximum-likelihood estimates as the current choice is statistically independent of prior choices on which it is conditioned.

Although a generalized error structure for ξ_t is conceptually feasible, it is computationally impractical. A simple permanent-transitory scheme can, however, be estimated and fits neatly into the previous formulation. As is conventional, let $\xi_t^i = \nu^i + \eta_t^i$, where ν^i is a household-specific taste parameter known to the household in the initial period and η_t^i is a purely random taste component. In this case, contrary to the purely transitory taste scheme, since the decision rule at any period t is conditioned on previous fertility decisions as summarized in M_{t-1}, which is itself related to the unobserved permanent taste component, ν^i, estimates will not in general be consistent. However, if the decision is conditioned on the initial nonstochastic state $M_0 = 0$, that is, if the data contain complete life-cycle information, consistent estimates can be obtained.[23]

[22] This can be seen from (15). Increasing σ proportionately by λ and likewise the utility function parameters (α_1, α_2, β_1, β_2, γ) by λ leaves ξ_T^{**}/σ unchanged but increases all of the conditional expectations in (15) by λ. Thus, ξ_{T-1}^* increases proportionately by λ while ξ_{T-1}^*/σ is unchanged, which implies that the likelihood is unchanged.

[23] See Heckman (1979) for a discussion of the problem of initial conditions in discrete models and for suggested solutions. Notice too that there is true state dependence, although there is no single structural state dependence parameter. If identification of the structural parameters of the model is not problematic, then there is also no difficulty in disentangling true from spurious state dependence.

Suppose then that there are N types of households ($N < I$) with permanent taste components equal to $\nu^i, \ldots, \nu^N$ and comprising $q^1, \ldots, q^n$ proportions of the population, respectively. For each of the N possible values of ν^i one can compute ξ_t^* for each t exactly as was done in the single type case. One can therefore form the likelihood of observing any birth sequence given household type. Weighting a given sequence's likelihood for each type by the appropriate probabilities q^i, summing over all types, and producting over all individuals yields the sample likelihood. The distribution of the unobservable is then estimated along with other parameters, including possibly N.[24] The computational cost of introducing unobserved heterogeneity is approximately proportional to the number of household types for known N.

Data

The 1976 Malaysian Family Life Survey contains 1,262 households consisting of at least one ever-married woman under 50 years of age as of the survey date. The essential feature of the survey is that it contains a retrospective life history of each woman to age 15 or marriage, whichever is earlier, with complete information about births, infant deaths, and household income. A subsample was drawn consisting of 188 Malay women, all of whom were over 30, currently married, and married only once.[25] Household income is restricted to that of the husband.[26]

The length of the period was chosen to be 18 months as a compromise between the necessity to have only a single birth within a period and the computational benefit from having fewer periods in the life cycle.[27] The initial period was set at age 15 or age of marriage, whichever came first. The final decision period is assumed to occur after 20 periods or approximately at age 45.

In treating husband's income (earnings), the husband's life cycle was broken up into 18-month periods and matched to the woman's life cycle. Starting with the initial period (age) of the woman, hus-

[24] See Heckman and Singer (1982) for a discussion of nonparametric methods for estimating unobserved heterogeneity distributions in the context of continuous time duration models.

[25] Approximately 50 percent of the 1,262 women are Malay, the other 50 percent consisting of ethnic Chinese and Indians.

[26] Essentially, it is assumed that the woman does no market work or work at home for pay. Including the woman's income would have led to negative income effects since women who work for pay have fewer children. A weakness of the model is that it does not consider the female labor supply choice.

[27] Of the total of 3,086 periods, in only 30 periods (27 women) were there two births in 18 months. In those cases, a birth was moved to the next period in which there was no birth.

band's (average) income (possibly prospective if they were not then married) was computed for each 18-month period. In the data themselves husband's income is reported in an event-history form; namely, labor supply and wage rates are reported retrospectively at each moment (date) that either of them changes beginning at husband's age 15. A semilog (price-deflated) earnings function was estimated for each husband individually with a constant, period, and period squared as regressors, where period is simply age of husband suitably transformed to the woman's life cycle.[28] A predicted earnings profile was generated and forms the basis for the (forecasted) income observations used in the empirical analysis.[29] The woman is assumed to live for 10 periods (15 years) after her childbearing period, and husband's income for the period is assumed to be forecasted as constant and equal to predicted income in the twenty-first period. In the context of the model of the previous section, it is assumed that each woman knows the parameters of her husband's earnings functions beginning at the first decision period.

The survival probability was obtained from data on state-specific survival rates. Predicted survival probabilities are based on state-specific log odds regressions, which include a constant, time, and time squared. These predictions are assumed to be those used by households in making their fertility decisions. Less aggregated data would have been more appropriate but are not available, and the sample size is too small to do anything reasonable with the individual data.[30]

The data therefore consist of the actual periods of births and infant deaths for each woman, a predicted husband's income profile based on a semilog quadratic in time formulation, and a predicted survival probability profile based on a quadratic in time-logistic formulation. In each period, the probability of a birth depends in a complicated way on the number of surviving children to that time, the predicted future income path of the husband, and the predicted future survival probability path.

Estimation Results

The model was estimated for two samples, one using the birth outcomes of the first 10 periods of each woman's life cycle (1,880 periods)

[28] Earnings are deflated by the countrywide price index that prevailed at the beginning of each 18-month period. It is obtained from *National Accounts of West Malaysia, 1947–76*.

[29] Notice that I abstract from any serial correlation in the earnings function. New information acquired during the life cycle requires updating the future component of the dynamic programming decision rule each period, which greatly increases the computational burden.

[30] It was necessary to extrapolate outside the period covered by the state-level data. Also, the state-level data did not precisely conform to the data that were required. Details are provided in Wolpin (1982).

and the other using birth outcomes over each woman's entire life cycle (3,086 periods). The 10-period sample was used to test for unobserved heterogeneity since the composition of the sample changes as more periods are added. Because of computational cost limitations, only a model with two types of individuals was estimated. Since a likelihood ratio test clearly rejected the existence of unobserved heterogeneity, results from the 10-period sample are not reported.[31] It is assumed, therefore, that unobserved heterogeneity will not, if ignored, seriously contaminate estimates using the entire life cycle of each woman.[32] The estimates presented below therefore are based on the assumption that the random taste component of the model follows a serially independent stochastic process.

The model is formulated slightly more generally than in previous sections. In particular, the costs of birth in the first and second periods are permitted to differ from one another and from subsequent periods, and the cost profile is permitted to have a quadratic shape. It is unlikely a priori that income and survival probability profiles can by themselves trace the life-cycle fertility profile, as shown in table 1. Moreover, presumed biological constraints early and late in the life cycle associated with a reduced propensity to conceive can be formally expressed as a reduced contraceptive cost or an increased net cost of a birth. In addition, the woman's schooling is arbitrarily introduced as affecting the incremental utility of a surviving child.[33]

The structure of the model is, therefore, as follows:

$$LU = \sum_{t=1}^{T} \delta^{t-1}(\alpha_{1t}M_t - \alpha_2 M_t^2 + \beta_1 X_1 - \beta_2 X_t^2 + \gamma_1 M_t X_t + \gamma_2 M_t S),$$

$$Y_t = X_t + b(n_t - d_t) + (c_1 m_1 + c_2 m_2 + c_{30} + c_{31}t + c_{32}t^2)n_t,$$

where, in addition to previous definitions, m_1 is a dummy variable equal to unity if the period is the first and zero otherwise, m_2 is a

[31] Twice the difference in the log likelihood values of the two models was only .2, which falls far short of the critical $\chi^2(.05) = 5.99$. Of course, having only two types of individuals might not capture the true degree of heterogeneity in the sample.

[32] There is other evidence to support this view. First, the observed variance in the number of children born taken over all women and all periods is 1,394, the variance that would be obtained if the birth process was simply Bernoulli is 733, and the variance that would obtain if women had either a child each period or no children at all is 11,984, which suggests that extreme heterogeneity is not present. Second, there seems to be little discernible relationship between the number of surviving children and the observed frequency of children born in the subsequent period. The model predicts a negative relationship, and unobserved heterogeneity a positive relationship. However, both of these pieces of evidence abstract from observable heterogeneity determinants.

[33] Schooling could be modeled as affecting any or all cost or utility parameters. Since it is unlikely that these alternatives could be distinguished empirically, the simplest representation was adopted.

TABLE 1

Actual and Predicted Birth Probabilities

	Period									
	1	2	3	4	5	6	7	8	9	10
Actual	.138	.335	.489	.537	.511	.505	.511	.516	.516	.394
Predicted	.134	.330	.543	.538	.530	.518	.502	.483	.460	.434
	11	12	13	14	15	16	17	18	19	20
Actual	.410	.425	.321	.360	.310	.262	.200	.140	.140	.105
Predicted	.406	.382	.349	.316	.282	.249	.213	.178	.141	.112

dummy variable equal to unity if the period is the second and zero otherwise, and S is the woman's years of schooling. Thus $c_1 + c_{30} + c_{31} + c_{32}$ measures the birth cost in period 1, $c_2 + c_{30} + 2c_{31} + 4c_{32}$ the birth cost in period 2, and $c_{30} + c_{31}t + c_{32}t^2$ the birth costs in all periods after the second.

Table 2 presents the estimated parameters of the model.[34] Although individual parameters cannot be translated into the experiments of interest, they do provide a simple check on the plausibility of the estimates, and thus of the method. All of the signs in table 2 conform to strong priors; incremental or marginal utilities are positive and diminishing in surviving children and in the composite good, the cost of bearing a child is positive in all life-cycle periods as is the maintenance cost, and the discount factor translates into a rate-of-time preference of approximately .093. Magnitudes of the cost parameters are also not unreasonable, though at the high end. The cost of a live birth in period 1 (c_1) is M$22,280 (Malaysian dollars) and in period 2 (c_2) M$8,121. The cost of a birth is lowest in period 3, M$3,314, and rises at an increasing rate reaching M$25,407 by period 20. These costs are high given that the average level of husband's income was about M$12,000 and that the Malaysian dollar was worth about 33 percent of the U.S. dollar in 1960.[35] The child-maintenance cost, on the other hand, is relatively small, only M$1,470. Notice also

[34] Two maximization routines were used to obtain the final estimates. First, a search routine NMSIMP was used to scan broadly over the parameter space and then DFP was used to narrow in on the maximum. Both routines are part of the GQOPT package. Asymptotic standard errors are not reported since the hessian (at the maximum) was not negative definite. The model was also estimated with $c_{31} = c_{32} = 0$. The restriction was rejected by the appropriate likelihood ratio test. The χ^2 value for the likelihood ratio test was 204, which exceeds the critical $\chi^2_2(.05) = 5.99$.

[35] The quadratic utility specification does not rule out the possibility that, because of the magnitude of some of these cost parameters, consumption will be negative at some period for some individual.

TABLE 2

MAXIMUM LIKELIHOOD ESTIMATES

Utility Function Parameters	Cost Function Parameters
$\alpha_1 = 3.43 \times 10^{-2}$	$b = 1.47 \times 10^{3}$
$\alpha_2 = 2.94 \times 10^{-7}$	$c_1 = 1.77 \times 10^{4}$
$\beta_1 = 6.16 \times 10^{-5}$	$c_2 = 8.09 \times 10^{3}$
$\beta_2 = 1.07 \times 10^{-16}$	$c_{30} = 1.95 \times 10^{3}$
$\gamma_1 = 2.42 \times 10^{-7}$	$c_{31} = -2.86 \times 10^{2}$
$\gamma_2 = -5.42 \times 10^{-3}$	$c_{32} = 6.42 \times 10^{1}$
$\delta = 9.22 \times 10^{-1}$	

NOTE.—Log likelihood = − 1,920.3.

that the incremental utility of an additional surviving child rises with goods consumption ($\gamma_1 > 0$) and falls with mother's schooling ($\gamma_2 > 0$). However, quantitatively the nonlinear terms in the utility function other than schooling are quite small, and this feature is crucial to the substantive implications discussed below.

There are several ways to assess the fit of the estimated model. The log likelihood value as shown in table 2 is − 1,920.3. The model degenerates to a simple Bernoulli process for births if all of the parameters except α_1 are set to zero; α_1 must be estimated to retrieve the sample fraction of births. The log likelihood value for the pure chance model is −2,059.9. Twice the difference in the likelihood is 272.4, which is sufficient to reject the pure chance model at almost any significance level ($\chi^2[.01] = 26.2$ with 12 degrees of freedom).

Table 1 compares the actual and predicted birth probability profiles. Although they are quite similar, the reason lies in the specification of childbearing costs. If the five cost parameters (the c's) are collapsed to a single parameter, the predicted birth probability profile is essentially flat rather than conforming to the actual.[36] A more revealing comparison is shown in table 3. There the actual and predicted birth probabilities conditional on the number of surviving children are shown. Tests for the existence of a relationship by period in the actual data are shown by the first row of χ^2 statistics. The second row of statistics tests for the equality between predicted and actual. Notice first that the manner in which the actual birth probabilities vary with the number of surviving children differs across periods; where there is a statistically significant relationship (periods 8, 10, and 12), at least in one the relationship appears quadratic, the probability first rising, then falling. In only one period is there a discernible difference between the actual and the predicted probabilities, and there it seems to be the level that is misrepresented and not the

[36] Such a specification still does better than the pure chance model.

TABLE 3

Probability of a Birth Conditional on the Number of Surviving Children: Actual and Predicted

Number of Surviving Children	Period 3		4		5		6		7		8		9		10		11		12		13		14	
	A	P	A	P	A	P	A	P	A	P	A	P	A	P	A	P	A	P	A	P	A	P	A	P
0 or 1	.486	.464	.533	.539	.477	.523	.507	.507	.538	.481	.389	.472	.435	.391	...	...	.438	.375	...	...	...	...	...	...
1 or 2	.549	.454	.604	.622	.589	.540	.509	.528	.558	.500	.500	.471	.408	.408	.379	.400	.367	400	.292	.375	...	...	.133	.333
2 or 3			.556	.555	.544	.558	.514	.522	.495	.514	.529	.480	.470	.458	.375	.422	.275	.392	.209	.372	.256	.307	.241	.310
3 or 4					.555	.550	.532	.511	.429	.519	.505	.495	.570	.470	.451	.439	.377	.406	.320	.380	.225	.325	.289	.316
4 or 5							...	...	.500	.500	.543	.500	.594	.464	.442	.442	.430	.407	.574	.382	.220	.341	.369	.316
5 or 6									...	...	...	...	.519	.444	.415	.434	.435	.406	.574	.382	.323	.354	.400	.320
6 or 7											...	...	...	...	.550	.444	.552	.414	.459	.378	.379	.362	.340	.321
7 or 8													...	...	.440	.444	.533	.400	.350	.232	.483	.379	.333	.333
8 or 9															...	...	...	...	...	...	.500	.417	.474	.369
χ^2 (actual)	.813		1.30		2.95		1.52		4.77		16.7*		6.63		14.5*		11.1		28.9*		16.3		18.8	
χ^2 (difference)	2.20		1.34		2.11		.659		5.39		10.38		9.60		13.02		8.08		29.22*		12.21		15.57	

Note.—Cells with fewer than five observations are omitted from the total.
* Indicates statistically significant at the 5 percent level.

pattern. Thus, if there exists a relationship between the propensity to bear a child in any period and prior net fertility, it is complex and the estimated model seems to capture its essential features.

The estimated parameters may be used to explore the sensitivity of fertility over the life cycle to changes in the income profile, the survival probability profile, child deaths, and mother's schooling.

Replacement Effects

Table 4 shows the responsiveness of fertility to the occurrence of infant deaths. Each entry gives the expected number of children ever born (sums of unconditional probabilities) given that there are M_{t-1} surviving children as of period $t = 2, \ldots, 10$. Average (constant over time) income and survival probabilities and average schooling are assumed in the calculation. Replacement responses can easily be calculated by differencing any two columns for a given row. The last column averages the replacement effects within each row. As the figures reveal, replacement responses are small for the sample; an infant death induces an increase in the number of children ever born by at most 0.015.[37]

Income Effects

Income effects are also quite small. Elasticities tend to rise with the level of income with arc elasticities ranging from .013 for incomes between \$5,000 and \$10,000 to .06 for incomes between \$25,000 and \$50,000 (table available on request). Also, an increase in income tends to increase births more in earlier life-cycle periods and a rising income profile tends to delay births slightly.

Survival Probability Effects

Survival probability effects are not small. A reduction by 0.05 percentage points reduces the number of children ever born by about one-quarter (table available on request). As the survival probability falls, there is also a tendency to have children earlier. A rising survival probability profile tends to delay childbearing. Thus, women are more likely to bear children in periods when they are more likely to

[37] Olsen (1983) finds replacement effects for these survey data (but not the same sample) of about 30 percent. His methodology is "statistical" in nature.

TABLE 4

EXPECTED ADDITIONAL NUMBER OF BIRTHS BY NUMBER OF SURVIVING CHILDREN AND PERIOD

	Number of Surviving Children										
Period	0	1	2	3	4	5	6	7	8	9	Average Replacement
2	6.579	6.566									.013
3	6.229	6.214	6.200								.015
4	5.675	5.661	5.650	5.638							.012
5	5.130	5.000	4.987	4.974	4.964						.012
6	4.595	4.583	4.571	4.561	4.549	4.537					.012
7	3.868	3.856	3.844	3.832	3.819	3.810	3.798				.011
8	3.388	3.377	3.367	3.355	3.346	3.334	3.323	3.312			.011
9	2.937	2.928	2.917	2.907	2.896	2.884	2.876	2.865	2.854		.010
10	2.648	2.638	2.628	2.618	2.609	2.600	2.590	2.579	2.570	2.560	.010

survive and will bear more children overall in high survival environments.

Mother's Schooling Effect

Mother's schooling exerts a strong effect on fertility. Each additional year of schooling reduces the expected number of children by 0.35 evaluated at the means of the other variables.

Discussion

There is a straightforward interpretation of the estimation results. Essentially, the estimates imply that this sample of Malay women will follow a maximal fertility strategy in the following sense. If the cost of childbearing and the infant survival probability were constant over the life cycle, then each woman would either have a child in each period or have no children at all (subject to the randomly drawn taste parameter each period). The fact that childbearing costs are estimated to be rising over the life cycle limits the total number of children but not the pattern. This result is directly tied to the very negligible magnitudes of the second-order utility function parameters (α_2, β_2, γ_2). There is no replacement behavior because the marginal child, the last child for whom a "positive" net benefit is derived, does not change with child deaths. This would not be the case if the "marginal" utility of surviving children were falling substantially with additional surviving children. Moreover, systematic spacing (not based on random tastes) cannot arise given the utility function parameter estimates. Rising income profiles and survival probability profiles would otherwise generate spacing, but evidently not of patterns that would be consistent with observed fertility profiles. Individual variation in the level of fertility is related to the woman's education and to the infant mortality environment.

Two issues arise with respect to an assessment of this approach. First, would an astructural approach yield qualitatively similar results, and second, if it did, would the value of the present approach then be merely aesthetic, in the sense that estimation is tied to an optimization framework?

With respect to the second, in order to appreciate the economy of this methodology, one need only compare the number of parameters that would have to be estimated to achieve the same wealth of empirical implications using an astructural methodology. A rough calculation indicates that a minimum of 400 parameters would have to be estimated to obtain all of the responses that the 13-parameter structural model can characterize.[38] If the implicit restrictions of the structural model are correct, a great deal of statistical precision is gained. If incorrect, that precision is illusory. Unfortunately, the lack of analytical solutions for the decision rules seems to preclude a test of the restrictions.

To compare these results to a less structured approach, probit functions were estimated separately for each period of the woman's life cycle. To mirror the structural estimates qualitatively, explanatory variables consisted of the number of surviving children, the current period and 5- and 10-year-ahead expected income levels, the current period and 5- and 10-year-ahead survival probabilities, and mother's schooling. To summarize those results: income effects were negligible and mother's schooling "significantly" reduced fertility, as was found in the structural estimation results. Prior net fertility either had no "significant" effect or was positively related to the current decision. The latter result is not consistent with estimates from the structural model and suggests the existence of unobserved heterogeneity, a simple version of which was rejected in the structural estimation. Finally, the probit estimates imply that a general increase in the survival probability reduces, rather than increases, fertility, although as in the structural estimates the probability of a birth generally decreases as future survival probabilities increase. The probit estimates are thus somewhat different from the structural model estimates, and if the latter are a correct characterization of behavior, the former would also be quite misleading.

Conclusions

This paper is the first attempt specifically to model sequential life-cycle individual fertility behavior in an uncertain environment in a way that is conducive to estimation. More generally, it has demonstrated the empirical feasibility of estimating a dynamic programming model when the problem admits only of numerical solution. However, the theoretical formulation contained many important simplifying assumptions, and the sensitivity of the results to those assumptions is an important issue yet to be addressed. Significant computational issues must be resolved if extensions are to be pursued. Extensions of the basic framework to incorporate endogenous female

[38] In the first period, there are 20 future income variables and 19 future survival probabilities. In period 2, those numbers are 19 and 18, respectively, plus M_{T-1}. Following this logic for 20 periods yields the figure in the text.

labor supply, savings, and imperfect contraception would make up a rather full agenda for future research.

The particular problem addressed in this paper, that of the timing and spacing of children given significant infant mortality, has been of concern to many social science researchers. Specifically, the "replacement effect" estimated from this model, within a unified framework of the life-cycle fertility decision, is much smaller than that obtained using other methods. At this stage, in my view, these estimates should be taken no more or less seriously than those based on a looser theoretical tradition.

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