Estimating the Impact of Consumer Expectations of Coupons on Purchase Behavior: A Dynamic Structural Model

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Abstract

We examine the basic premise that consumers may anticipate future promotions and adjust their purchase behavior accordingly. We develop a structural model of households who make purchase decisions to minimize their expenditure over a finite period. The model allows for future expectations of promotions to enter the purchase decision. Households with adequate inventory of the product may face a trade-off of buying in the current period with a coupon or defer the purchase until next period, given their expectations of future promotions. Thus, we provide a framework for examining the impact of consumer expectations on choice behavior.

The target audiences for our paper are (a) empirical researchers who intend to make structural models part of their applied research agenda; and (b) managers who value and seek to understand the impact of consumers' coupon expectations on current purchase behavior. Our research objective is to provide an empirical framework to examine whether and to what extent consumers anticipate future coupon promotions and adjust purchase behavior. The central premise of our approach is that a rational consumer minimizes the present discounted value of the cost of a purchase where cost in a single period consists of purchase price, inventory holding cost, gains from coupons, and potential stockout cost. We aim to test whether our hypotheses regarding the various elements of the cost structure are supported and that whether consumers take into account future discounted cost when making current purchase decisions.

The research methodology we adopt is relatively new in econometrics and known as the estimable stochastic structural dynamic programming method. The methodology amounts to incorporating a maximum likelihood routine embedded in a dynamic programming problem. The dynamic programming problem is solved several times within a maximum likelihood iteration for each value of the state space elements and for each value of the parameters in the parameter set. The state space in our model consists of purchase and nonpurchase alternatives in each time period, coupon availability and no coupon availability in each time period, level of inventory in each time period for each household, and consumption rate of each household.

We use scanner panel data on purchases in the disposable diaper product category and promotions. We estimate the inventory holding and stockout costs, brand-specific value of coupons, and consumers' expectations of future coupons. The key insights and lessons learned can be summarized as follows: (1) Our results are consistent with the notion that consumers hold beliefs about future coupons, and that such beliefs affect the purchase decision. We find that the dynamic optimization model performs significantly better than a single-period optimization model and a naive benchmark model. (2) We find a high and significant stockout cost, consistent with the essential nature of the product category. Our estimate of the holding cost yields a reasonable annualized percentage value when converted to the cost of capital. We find that consumer valuation of coupons differ markedly across brands. (3) Our empirical evidence supports the notion that consumers hold beliefs about future coupon availability. We also find that the expectations about future coupons, estimated endogenously, differ depending upon whether or not a coupon was available in the current period. Thus, the proposed model structure yields rich managerial insights and facilitates several "what if" scenarios.

A possible limitation of our model, and estimable structural models in general, is the computational cost. While it is possible to conceptually extend the state space to accommodate variations across households and add a richer parameter structure, each addition multiplies the size of the state space and the computation time. For this reason, we have kept the state space as tight as possible and refrained from additions that would otherwise enable us to incorporate heterogeneity in consumer decisions. For example, we assumed that consumers are similar other than reflected by their purchase behavior. We built a category purchase incidence model rather than a brand choice model. We refrained from including unobserved heterogeneity in the parameters. We chose to opt out of modeling autocorrelation and other time-dependent error term patterns in the likelihood function. Thus, we have made an effort to build a structural model that reasonably reflects consumer purchase behavior without requiring expensive computation. Currently, there are developments in econometrics to approximate the computation of the valuation functions without sacrificing much accuracy. When these methods are well developed we expect that structural models will become more commonplace in marketing.

(*Consumer Expectations; Econometrics; Estimable Stochastic Dynamic Programming Models; Promotions; Structural Models*)

0732-2399/96/1503/0262$01.25

1. Introduction

Consumer promotion is one of the key marketing tools to gain market share in several frequently purchased consumer product categories. As promotion intensifies and the frequency of the promotion becomes higher, a loyal and rational consumer may stockpile the promoted brand for future consumption. While there is evidence in favor of stockpiling, (Blattberg, Eppen and Lieberman 1981; Neslin, Henderson and Quelch 1985), there is also research that finds that advance purchases are either marginal (Neslin and Shoemaker 1989) or insignificant (Davis, Inman and McAlister 1992). On a related line of research, Krishna (1990) raises the interesting possibility that expectations of promotions influence purchase decisions even if they are inaccurate.

Over time, households may form expectations about future availability of coupons in a product category and may adjust their purchase behavior accordingly. While a household takes such expectations into account when buying in a product category, the researcher's knowledge is limited to the purchase history. We propose a framework that allows us to combine purchase history data with a model of consumer expectations and glean important insights about the impact of expectations of promotions on purchase probability.

The central idea behind our approach rests on the notion that a rational household maximizes benefits (utility) derived from a product category over a finite period of time. On each week, the decision to buy in a product category is affected by the following factors: current inventory level of the product, the cost of holding the inventory, if any, the cost of stockout of the product in the event of inadequate inventory, and category attractiveness in terms of coupon promotions. Suppose the household has sufficient quantity of the product to avoid stockout until the next purchase occasion. In spite of that, if the household purchases in the category, two possible explanations can be offered: (1) the stockout cost is sufficiently high that even a small probability of a stockout (due to unexpected high consumption) encourages the household to buy now; or (2) the household exploits any temporary price reduction and stockpiles the product.[1] The first explanation is clear and does not need further elaboration. Let us examine the second explanation carefully. Note that prices remain relatively stable (particularly in the category studied here) and temporary price reductions are often caused by promotional activities such as couponing. If a household chooses to buy (and stockpile) instead of waiting until the next period, *then the household expects the prices to be higher on the next purchase occasion*. Possibly, the coupon may expire and the household assigns a low probability to coupon availability on the next purchase occasion. Given the expectations about future coupons, inventory holding cost, and stockout cost, the household optimally decides to purchase or not. The same idea extends to any problem where the household makes dynamic purchase decisions over a time horizon recognizing the interdependencies and trade-offs between inventory accumulation and coupon expectations.

The main thrust of our paper is to achieve direct empirical estimation of the optimal purchase behavior in the presence of uncertain expectations over future coupon offers. The household knows holding and stockout costs, the distribution of the random cost components, and expectations on future coupons. The household costs are subject to a stochastic component drawn from a distribution in each period. The error term is akin to the random component in the (indirect) utility of reduced-form choice models such as logit or probit. Given the cost realization, the household makes the optimal purchase decision. The researcher does not observe the costs, the random component, and households' expectations, but estimates the associated parameters conditional on observed purchase behavior. Note that there is a random error in the optimal decision unobserved by the researcher, and hence, the household's decision is optimal only in a probabilistic sense. In each period, the probability of either of the two outcomes purchase or nonpurchase, is estimated. The expression for the prob-

[1] A third possibility, variety seeking, is not studied here. Since our analysis is on the disposable diaper product category, the impact of such a behavior is likely to be minimal. Also, the development of the model does not consider purchase of multiple units in the product category consistent with the empirical application. Purchase data in the diaper category suggest that on 99% of the occasions, consumers buy only one unit of the product. Our model can be directly extended to include purchases of multiple units.

ability of the observed decision in each period incorporates the expected optimal decision from that period until the end of the time horizon consistent with the dynamic programming approach.

The random component in our model represents the difference between the actual behavior and the optimal behavior suggested by the model. The stochastic structure provides for positive probability of being along the optimal path while yielding nontrivial probability of deviating from the optimal strategy. The extent to which consumer behavior is consistent with the structural model is determined by the fit of the model to purchase data. The random term enables estimation and testability of the model.

Analytical formulation of consumer promotions, as they influence purchase behavior, dates back to the work by Blattberg, Buesing, Peacock and Sen (1978). They developed a dynamic cost minimization model as a theoretical basis to identify the deal-prone segment by demographic variables. Blattberg, Eppen and Lieberman (1981) modeled promotions as an outcome of inventory transfer from retailers to households because of the relatively lower inventory holding costs of households. Golabi (1985) proposed a theoretical model of expected cost minimization given uncertainty in ordering prices, and solves for the critical price levels to determine optimal inventory policies. Krishna (1992) employed Golabi's model and simulated scenarios where prices and coupon drops differ over time to compare purchase probabilities before and after promotions. Two differences between these models and our work are noteworthy. First, we incorporate expectations of future prices (as affected by coupons) in the purchase decision. Second, we subject the model to empirical test to endogenously estimate these expectations. Meyer and Assuncao (1990) conducted experiments to explore whether consumers follow optimal purchase strategies when prices were random. They found systematic bias (when compared to the optimal strategy) in the behavior of consumers. They offered a prospect-theory based explanation for their findings. Assuncao and Meyer (1993) extended the work on optimal ordering policy to the case where consumption was a variable. Helsen and Schmittlein (1992) further analyzed the implications of Golabi's model by including uncertainties in the length and depth of promotions. None of these studies incorporate consumers' expectations. Our attempt to endogenously do so in a structural framework is the key departure from these studies.

We develop and discuss the details of the proposed structural model in the next section. For additional information on estimable structural dynamic stochastic optimization models, see Gönül (1989) and Eckstein and Wolpin (1989). (For the remainder of the paper, we shorten the term, *"an estimable structural dynamic stochastic optimization model,"* to *"a structural model."*)

If consumers anticipate coupons and adjust their purchase behavior accordingly, then couponing may be a costly promotional activity. A firm may take into account such a behavior on the part of consumers while designing its promotions. For example, a firm may reduce coupon promotions. Alternatively, a firm may create higher variance in coupon dropping intervals to disrupt consumers' ability to anticipate coupon offerings correctly. The ability of consumers to exploit coupon offerings is affected by the inventory holding cost and shortage cost. For example, if the inventory holding cost is high, a consumer may not exploit an available coupon even with the expectation of low probability of a coupon in the next period. In this context, the *managerial* contributions of our paper are as follows:

- We provide a framework to examine whether and to what extent consumers anticipate future coupon promotions and adjust their purchase behavior accordingly.
- Our method directly yields estimates of holding cost and stockout cost.

Structural models are relatively new and are gaining favor on the ground that they are built on better theoretical underpinnings than reduced-form models. To our knowledge, our work represents the first effort in incorporating coupon expectations in a dynamic framework. Such expectations are important in a number of marketing contexts of both nondurable and durable purchase decisions. (For example, future expectations of price and quality determine the time of purchase of durables such as personal computers.) The *methodological* contribution of our effort is as follows:

- We develop a methodological framework to incorporate expectation about future coupons and examine its implications on the purchase decision.

We use scanner panel data on disposable diaper purchases provided by the A. C. Nielsen Company. We estimate the model in two steps embedded in each other. First, we solve the stochastic dynamic programming problem using Bellman's (1957) backward recursion algorithm, conditional on a set of parameter values. Then, we derive the purchase probability from the model in each time period and evaluate the likelihood function of observed purchase and nonpurchases. We iterate the two steps with different parameter values until we obtain convergence. Upon estimation, we validate the model on a holdout sample.

Our main findings based on the analysis of the disposable diaper product category are as follows:

1. We find that the dynamic optimization model performs significantly better than the single-period optimization model and a naive benchmark model. In other words, our results appear to support the main hypothesis that households take into account the future when making purchase decisions. The parameter estimates are in the expected direction and facilitate several "what if" scenarios.

2. We find a high and significant stockout cost, consistent with the essential nature of the product category. The inventory cost is significant and interestingly, the annualized rate appears meaningful in the context of the cost of capital. Thus, evidence presented in this work is consistent with the notion that consumers take inventory into consideration while making a purchase/nonpurchase decision. Our findings also indicate that consumer valuation of coupon differs significantly across brands.

3. The findings support the notion that consumers hold beliefs about future coupon availability. In addition, their expectations are different depending upon whether or not they have a coupon available in the current period. Their expectation of coupon in the next period when there is a coupon available in the current period is lower than the expectation when there is no coupon available in the current period. Thus, we find support that consumers' expectations are better characterized by a first-order Markov process than a zero-order one.

2. Dynamic Stochastic Optimization Model of Purchase

We consider a household that aims to maximize the benefits derived from purchases in a product category over the entire time horizon. We adopt the widely used random utility framework of McFadden (1974) with the modification that the objective function is the cost (or disutility) associated with the buy/no buy decision. Since the decision in each period is to buy or not, the problem can be viewed as analogous to minimizing the total expected expenditure given consumption of the product.

2.1 The Objective Function

Consider a household that minimizes the present discounted value of total expenditure over a finite time-horizon. Then the objective function can be stated as

$$\min \sum_{t=1}^{T} \delta^{t-1} \mathcal{C}(B_t), \tag{1}$$

where T is the length of the decision horizon, $\mathcal{C}(B_t)$ is the cost in period t, B_t is the buying decision at time t ($B_t = 0, 1$), and δ is the discount factor. We estimate the model on household purchases in the disposable diaper product category. We measure the purchases in box units and time in weeks. Since children are typically toilet trained at the age of three or younger, we consider a time horizon of 156 weeks. While we specify the model for 156 weeks, the survey period is only 52 weeks. However, we know the actual age of child of each household and use the information to set the likelihood expressions for the relevant period. We provide additional details on the likelihood function at the end of §2.2.

We postulate that the cost function consists of deterministic and stochastic parts:

$$\mathcal{C}(B_t) = \bar{\mathcal{C}}(B_t) + \epsilon_{B_t}t \tag{2}$$

where the unobserved components of not buying and buying are denoted by ϵ_{0t} and ϵ_{1t}, respectively. The random components allow for a gap between observed and depicted behavior, and enable us to construct probability statements to estimate the model. The household knows the distribution of random terms; and each period's cost is affected by the draws from that distribution. Then the household makes the optimal decision by choosing the minimum of the two costs, namely, cost of buying and cost of not buying in the product category. The researcher does not observe the costs but can estimate the probability of choosing the minimum cost option in each period, given data and distributional assumptions on the random terms.

A household's decision to buy in the product category is determined by the price of brands in the product category and category attractiveness due to coupon promotions. We model category attractiveness as the weighted sum of the coupon availability variables. Since the prices of the brands are highly comparable in a given week throughout the survey period, we let the prices remain invariant across brands. Further, they remain relatively stable during the observation period as well. Therefore, we do not employ the weighting scheme for prices. Since storing the product involves holding costs, the household incurs storage costs on the accumulated inventory. When the household decides not to buy the product in the current period, it does not incur any current expenditure on the purchase of the product and foregoes an opportunity to use a coupon when available. However, the household runs the risk of suffering from potential stockouts.

The two alternatives and their consequences can be compactly written in the following equation for the deterministic cost function in period t:

$$\bar{\mathscr{C}}(B_t) = \alpha(1 - B_t) + \eta_1 I_t^+ + \eta_2 (I_t^+)^2 + \left(p_t - \sum_{j=1}^{J} w_{jt}(X_{jt}\beta_j + C_{jt}\gamma_j)\right) B_t, \quad (3)$$

where α is the cost of a potential stockout, η_1 and η_2 are the linear and quadratic holding costs per box, respectively, I_t^+ is positive inventory, if any, in period t, p_t is the unit price, w_{jt} are time-varying preference weights for each brand ($j = 1, 2, \ldots, J$) computed separately for each household from past purchases up to and including $t - 1$, X_{jt} are brand-specific promotional variables such as display and feature, β_j is the coefficient vector measuring the impact of promotion, $C_{jt}(=0, 1)$ denotes whether or not a coupon is available for brand j in week t, and γ_j is the perceived value to the consumer for the coupon which may be different from the face value. Note that $w_{jt} \in [0, 1]$ and $\Sigma_{j=1}^{J} w_{jt} = 1$.

We use the purchase history of households to compute brand preference weights and dynamically update the household-specific preference weights with each purchase. Essentially, these weights are fraction of past purchases of brands by a given household. (Gupta 1991, adopts a similar approach in estimating an interpurchase time model of whether or not to buy in a product category.) For the first purchase, we let the weights to be equal but update them for subsequent purchases. For the solution of the dynamic programming problem beyond the survey period, we use the latest preference weights available for each household. For comparison, we also estimate a model where we let the preference weights be equal across brands ($w_{jt} = 1/J$) for all households. We generate weekly brand-specific coupon availabilities from two methods: 1) We use an outside sample of households to determine coupon availability. 2) We obtain coupon availability from the estimation sample using a method developed by Gönül and Smith (1996). We provide additional details in §3.1. We estimate the structural model for each of the four combinations of preference weights (household-specific or equal) and coupon availability (outside sample or estimation sample.)

One of the key costs of not purchasing in any period is the cost of a potential stockout. The standard inventory flow equation relies on the average consumption of the product in each period. That is, current inventory. (I_t) is previous week's inventory augmented by current purchase and reduced by current consumption. Thus,

$$I_t = I_{t-1} + B_t - K_t \quad (4)$$

where I_{t-1} is beginning inventory and K_t is consumption during period t. To simplify the analysis, we treat K as a constant and set it equal to the average weekly consumption. We measure inventory at discrete levels of one-tenth of a box. Figure 1 shows all possible levels of inventory given I_0. However, actual consumption of the product may vary from period to period due to factors such as sickness of the child. Hence, inventory at any time period determined from the average consumption may only be a limited surrogate to reflect stockout. If there is a purchase in a given period, we assume sufficient inventory would be available and, hence, the stockout cost would be zero. However, if a household does not purchase in the product category, the risk of stockout arises as measured by the intercept. Note that in view of the essential nature of the product category, the term measures any perceived valuation of disutility from stockout in addition to transaction cost of obtaining a box of diapers in the event of a stockout. We the

Figure 1 **Possible Values of I_t**

$$I_t = I_{t-1} + B_t - K$$

$$I_0 = I_0$$

$$I_1 = \{I_0 - K, I_0 + 1 - K\}$$

$$I_2 = \{I_1 - K, I_1 + 1 - K\} = \{I_0 - 2K, I_0 + 1 - 2K, I_0 + 2 - 2K\}$$

$$I_3 = \{I_2 - K, I_2 + 1 - K\} = \{I_0 - 3K, I_0 + 1 - 3K, I_0 + 2 - 3K, I_0 + 3 - 3K\}$$

■

■

■

$$I_{t-1} = \{t \text{ distinct possibilities}\}$$

intercept α as a measure of the stockout cost and formulate it as a function of average consumption per week. That is,

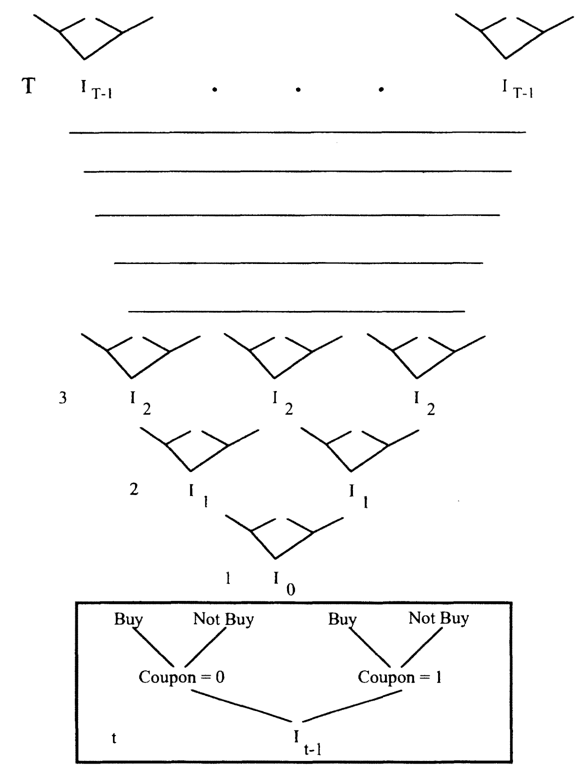
$$\alpha = \alpha_0 + \alpha_1 \cdot K. \tag{5}$$

We expect α_0 to be positive, given the essential nature of the product category. We also expect α_1 to be positive, since households who use diapers more than others will face a higher stockout cost due to their greater reliance on the product. However, even light users are expected to face a high stockout cost given the nature of the product category. Therefore, we expect, $\alpha_0 \gg \alpha_1$. The household is assumed to incur linear and quadratic holding costs, measured by coefficients η_1 and η_2, proportional to the magnitude of positive inventory. The quadratic holding cost may arise if there is any space constraint (given the bulkiness of the product) or more importantly, if there is any category-specific budget allocation (households may not want to spend more than a certain amount in a given category).

Both the value of promotional variables (X_{jt}) and availability of coupons (C_{jt}) are subtracted from cost, weighted by their respective preference for brands. In our model, there are no brand-specific intercepts. Therefore, there may be a concern that the brand-specific valuations of coupons may be confounded by the brand preferences. It is important to note that the brand purchase and coupon availability are separately captured by different variables. Thus, coupon valuation parameters are likely to be free from any bias due to intrinsic preferences. Moreover, the time-varying, household-specific weights act as a surrogate for intrinsic preferences. In that sense, we directly control for such brand preferences while estimating coupon valuations.

The valuation of a coupon accounts for a subjective assessment of the coupon benefit. For example, a perception of a smart shopper may yield a positive overvaluation of the coupon while the transaction cost involved in coupon redemption may lower the coupon value. In the literature, it is argued that psychological benefits may arise (Shimp and Kavas 1984). In addition,

Figure 2 **The State Space Diagram**



Note: Number of elements in state space $= \frac{T(T+1)}{2} \times 2 \times 2$ where T = length of time horizon.

Gönül and Srinivasan (1993) show that a coupon is a more effective tool than a comparable price reduction in achieving higher sales. More recently, Srinivasan, Leone and Mulhern (1996) attribute the incremental sales from previous nonusers to the advertising exposure effect when coupons appear in free standing inserts. In summary, we let the model capture differential impact of promotion across brands but reiterate that the decision under study pertains to buying or not in the product category.

Now, we can put together the deterministic and random components of the cost function by using Equations (2), (3), and (5),

$$\begin{aligned}\mathscr{C}(B_t) &= \bar{\mathscr{C}}(B_t) + \epsilon_{B_t}t \\ &= (\alpha_0 + \alpha_1 K)\cdot(1 - B_t) + \eta_1 I_t^+ + \eta_2 (I_t^+)^2 \\ &\quad + \left(p_t - \sum_{j=1}^{J} w_{jt}(X_{jt}\beta_j + C_{jt}\gamma_j)\right)B_t + \epsilon_{B_t}t. \end{aligned} \tag{6}$$

A *single period* model, that abstracts from future periods in the finite horizon, uses the function above to determine the household's decision rule. That is, the decision to buy in any period t is taken if and only if $\mathscr{C}(B_t) = 1) < \mathscr{C}(B_t) = 0)$. The corresponding purchase probability is,

$$\begin{aligned}\Pr(B_t = 1) &= \Pr(\mathscr{C}(B_t = 1) < \mathscr{C}(B_t = 0)) \\ &= \Pr\Bigg((\epsilon_{1t} - \epsilon_{0t}) < [\alpha_0 + \alpha_1 K \\ &\qquad + \eta_1 (I_{t-1} - K)^+ + \eta_2((I_{t-1} - K)^+)^2] \\ &\qquad - [\eta_1(I_{t-1} + 1 - K)^+] \\ &\qquad - \eta_2((I_{t-1} + 1 - K)^+)^2 \\ &\qquad - \left(p_t - \sum_{j=1}^{J} w_{jt}(X_{jt}\beta_j + C_{jt}\gamma_j)\right)\Bigg), \end{aligned} \tag{7}$$

and its complement is, $\Pr(B_t = 0) = 1 - \Pr(B_t = 1)$. Incidentally, a single error term can be defined since the difference of the two epsilons is sufficient to estimate the model. Note that even though for a given inventory level the purchase probability is constant in the single-period model, there will be variation in the probabilities from week to week since inventory varies across periods.

In a *multi-period, dynamic* framework consumers take into account the future when making current purchase decisions and minimize the present discounted value of the expected expenditure as given below:

$$E\left\{\sum_{\tau=t}^{T} \delta^{\tau-t}\mathscr{C}(B_t)\right\}. \tag{8}$$

The notation is consistent with the discussion following Equation (1). We now propose that households are uncertain about future coupon offers in order to incorporate the role of coupon expectations on current purchase decisions in the dynamic model. (We cannot link the future to the present in this way in the single-period framework.) We formally capture coupon expectations through a first-order heuristic:

$$\begin{aligned}&\Pr(C_{t+1} = 1 \mid C_t = 1) = q_1, \quad 0 \le q_1 \le 1, \quad \text{and,} \\ &\Pr(C_{t+1} = 1 \mid C_t = 0) = q_0, \quad 0 \le q_0 \le 1. \end{aligned} \tag{9}$$

We let a household's expectations of coupons in the next period depend on an differ by the availability of a coupon in the current period. If $q_1 = q_0$ then expectations obey a zero-order heuristic rather than a first-order. Although the current valuation of coupons can be brand-specific (captured by the γ_j parameters, as in Equation (3)), the expectations are not brand-specific. The consumers are assumed to form expectations about coupon availability per se and not about brand-specific coupon availability. The simplifying assumption is necessary to keep the state space manageable. Our model is built on buy/no buy decisions, and therefore, the state space does not span possible choices among the three brands. (See Figure 2 and the accompanying description towards the end of this section regarding the size of the state space.) Hence, the subscript j is not used in the expectation terms. From Equation (9), the expectation of coupon in the future period, conditional on coupon in the current period is,

$$E(C_{t+1} \mid C_t) = q = \begin{cases} q_1 & \text{if } C_t = 1, \\ q_0 & \text{if } C_t = 0. \end{cases} \tag{10}$$

Thus, q_0 and q_1 can be viewed as conditional expectations of coupon availability. We explain how we generate coupon availability for the category in §3.1.

Theoretically, the consumer should be forecasting price and promotion variables, such as display and feature, in addition to coupon. In our product category, prices remain relatively constant and promotional activity is virtually zero. (Please see §3 for details on the data.) However, if we were to incorporate variations in price and other promotional variables, we would assume that consumers forecast these variables with some pre-specified forecasting rule (potentially based on the patterns they have observed in the current periods and the recent past). We could then substitute the forecasted values in the future periods instead of actual values. To avoid notational clutter, we use p_t and X_{jt} instead of notation denoting their forecasted values.

In a discrete-time structural model, the binary decision has to be examined at each discrete and equal interval. We assume that weekly intervals are reasonable time partitions for examining the purchase decision in the diaper product category. We can model the decision to buy or not in the category conditional on store visit where the interpurchase intervals are approximated to the nearest week. Therefore, we adopt the weekly decision formulation. When computational difficulties do not arise, the model can be easily generalized for a finer partition of the time interval.

When a household decides not to visit a store, the implicit decision is not to buy in the category. When a household visits a store, it may decide not to buy in the category. Therefore, we can generalize the model to distinguish non-purchase decisions based on whether the household visited the store or not. Due to state space consideration, we do not estimate this general model.

We now turn to the solution of the full-fledged dynamic programming model. We adopt the backward recursion method to solve the dynamic optimization problem depicted in Equation (8). According to Bellman's *principle of optimality*, whatever the initial state and decision, the remaining decisions must be optimal with regard to the state resulting from the first decision (Bellman 1957). We let the expenditure function for each period (F_t) be the optimum (minimized) present value of the sum of the current and future costs. That is, the expenditure function at time t is,

$$F_t(I_{t-1}(B_{t-1}), C_{t-1}) = \min_{B_t} \{\mathcal{C}(B_t) + \delta E[F_{t+1}(I_t(B_t), C_t)]\} \qquad (11)$$

where past inventory as a function of past purchase (B_{t-1}) and past coupon availability (C_{t-1}) enter as state variables in the expenditure function arguments. *The household buys if and only if the expenditure of buying is less than the expenditure of not buying.* The solution in the finite horizon model is nonstationary that changes every period.

We show in Appendix A the method to compute the expenditure functions for the last two periods. In a similar vein, expenditure functions for previous periods (t = T-2, . . . , 1) are generated recursively. The expenditure functions for each possible level of coupon and entering inventory are calculated to find the optimal solution to the dynamic programming problem in each time period. Since coupon has two possibilities in each time period, and entering inventory has t possibilities, a total of $T \times (T + 1) \times 2$ calculations are required to find the optimal solution. Figure 2 demonstrates the state space that is spanned by all possible levels of coupon availability and inventory accumulation. Next we show that the expected value of future expenditure function has a closed-form expression:

PROPOSITION. *The expected value of the expenditure function is:*

$$E[F_t(I_{t-1}(B_{t-1}), C_{t-1})] = A_{0t} \Pr(B_t = 0) + E(A_{1t} | C_{t-1}) \Pr(B_t = 1) - \phi(A_{0t} - E(A_{1t} | C_{t-1})) \qquad (12)$$

where $\phi(\cdot)$ *stands for the standard normal probability density function* (p.d.f.), *and* A_{0t} *and* A_{1t} *are the nonstochastic components of not buying and buying. For* $t = T$,

$$A_{0T} = \alpha_0 + \alpha_1 K + \eta(I_{T-1} - K)^+,$$

$$A_{1T} = \eta(I_{T-1} + 1 - K)^+ + \left(p_T - \sum_{j=1}^{J} w_{jt}(X_{jT}\beta_j + C_{jT}\gamma_j)\right),$$

and for $t < T$,

$$A_{0t} = \alpha_0 + \alpha_1 K + \eta_1(I_{t-1} - K)^+ + \eta_2((I_{t-1} - K)^+)^2 + \delta E[F_{t+1}(I_t(B_t = 0), C_t)],$$

$$A_{1t} = \eta_1(I_{t-1} + 1 - K)^+ + \eta_2((I_{t-1} + 1 - K)^+)^2 + \left(p_t - \sum_{j=1}^{J} w_{jt}(X_{jt}\beta_j + C_{jt}\gamma_j)\right) + \delta E[F_{t+1}(I_t(B_t = 1), C_t)].$$

The term $E(A_{1t}|C_{t-1})$ in Equation (12) *suggests that $E(C_t|C_{t-1})$ is to be replaced by $q = \{q_0$ or $q_1\}$ depending on the value of C_{t-1} as shown in Equation* (10).

PROOF. See Appendix A.

2.2 The Likelihood Function

While the household solves its expenditure minimization problem, the researcher estimates the likelihood function that consists of the product of purchase and nonpurchase probabilities, and obtains the maximum likelihood estimates of the parameters of the model. From the above proposition and from the symmetry of the normal p.d.f., we obtain

$$\Pr(B_t = 0) = \Phi(A_{1t} - A_{0t}), \qquad (13)$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution. It follows that the purchase probability is

$$\Pr(B_t = 1) = \Phi(A_{0t} - A_{1t}). \qquad (14)$$

The expressions for the probabilities are interpreted as follows. If the (nonstochastic part of the) expenditure of buying is greater than the (nonstochastic part of the) expenditure of not buying (that is, if $A_{1t} > A_{0t}$), then households are less likely to buy. If A_{1t} is less than A_{0t} then they are more likely to buy. If A_{1t} equals A_{0t} then the households are indifferent between buying and not buying. Note that the functions A_{1t} and A_{0t} include the expenditure function from $t + 1$ to the end of the time horizon and, thus, incorporate the inherent dynamics of the decision process.

The likelihood function for a given household i is the product of purchase and nonpurchase probabilities for the observed data. That is, the individual likelihood function for household i is

$$L_i = \prod_{m=m_{i0}}^{M_i} \{\Pr(B_m = 1)\}^{B_m}\{\Pr(B_m = 0)\}^{1-B_m} \qquad (15)$$

where the probabilities are determined by formulae from the previous section, m_{i0} is the first observed period, and M_i is the last observed period for household i. They are determined by the age of the child in weeks. The observation interval for a given household i, $[m_{i0}, M_i]$, is within the time horizon $[1, T]$. Thus, while we solve for the expected future expenditure functions for each period in the entire horizon, we only use the expected future expenditure functions for the observed periods in the data and substitute them in the likelihood function. For example, suppose we have data on diaper purchases of a household over a 52-week period spanning the age of the child from 16 to 68 weeks. Though we solve the model from weeks 1 to 156, we select the probabilities for weeks 16 to 68. If another household's observation period pertains to weeks 32 to 84, we select the appropriate buy and no buy probabilities for these weeks to develop the log-likelihood expression. The sample log-likelihood function is $\ln L = \Sigma_i \ln L_i$ over all households in the estimation sample.

2.3 Discussion of Model Assumptions

We propose an "as if" model of purchase behavior over a finite horizon and take into account future expectations of coupons as well as current inventory levels. Before we estimate the model, we discuss the validity of the assumptions of the model.

A. Solving the Dynamic Model over a Finite Horizon. The model proposes that consumer purchase behavior is determined by solving a finite horizon problem. Such a behavior may be viewed with justifiable skepticism. However, as we discussed in the Introduction, the error term allows the consumer to deviate from the optimal strategy with a nontrivial probability on each purchase occasion. It is important to note that consumers may employ simplifying heuristics that nicely approximate the optimal solution. For instance, the well-known (s, S) rule in the inventory control theory suggests that the optimal solution is to order up to S if inventory falls below s (Love 1979). Though not studied here, consumers may engage in such heuristic decisions that mimic the optimal solution to the dynamic programming problem.

B. Long Time Horizon. In the subsequent estimation, we consider a time horizon of 156 weeks. While it is reasonable to expect that consumers take the future into consideration, this time horizon may seem quite long. However, note that the discount factor provides a natural way of taking this issue into account. On any purchase occasion, distant time periods are less important than immediate time periods. Further, we can allow for differences in the discount factor across households. Thus, we can account for the possibility that some (or most) of the households may consider effectively the next few periods only.

C. Holding Cost. A valid question may be raised as to whether or not consumers really take into account inventory cost for repeatedly purchased consumer goods. Moreover, the relative bulkiness of the product may cause storage problems. In addition, we infer inventory levels based on average consumption and actual purchases. Therefore, the variable is potentially measured with error. Our attempt to model inventory cost has to be interpreted in light of these issues.

D. Consumer Expectations. Given the complexity of estimation due to state space limitations, we offer a simple first-order Markov process to capture expectations of future promotions. While the formulation is more general than a zero-order process, consumers may hold more complex expectations. Our model can be generalized to accommodate higher-order expectations. However, the estimation becomes difficult due to state space considerations.

3. Data, Results, and Tests of the Model

3.1 Data

Our sample consists of households from the scanner panel data of the A. C. Nielsen Company who regularly bought disposable diapers between mid-1986 and mid-1987. We focus on purchases from the three leading brands that comprise more than 90% of the market. The composition of the sample is such that there are 66 households with average consumption (K) of 0.5 boxes per week, 34 with 0.6, 21 with 0.7 boxes, and 17 with 0.8 boxes per week. There are a total of 138 households. We reserve 40 weeks of the observation year for estimation and the remaining 12 weeks for validation. The estimation sample consists of 1963 weeks where purchase occurs and 2056 weeks where no purchase occurs. We begin the estimation with the first purchase of each household.

The purchase frequency varies across the three national brands as follows: Brand 1 is bought on about 28% of the occasions, Brand 2 on about 19%, and Brand 3 on about 53%. The price of the three brands are highly comparable and remain relatively stable over the observation period. We assume with minimal loss of information that the pre-promotion price of a box of diapers remains the same. (We let $p_t = p$.) For the time period under consideration, there is hardly any display, store coupon, or feature promotion in the category (less than 1%). Hence, any bias arising from the impact of such variables on purchase is likely to be insignificant. (We let $X_{jt} = 0$.) Significant differences arise due to coupon, however. Thus, we focus on expectation of future coupon availability as the main source of price uncertainty.

For each household in a given week, we need to know whether or not a household has a coupon available for one or more brands. We approximate the availability proportion from the data in two different ways. In the first method, we use a calibration sample of households (outside the estimation sample) to infer availability. In the second method, we employ a procedure suggested by Gönül and Smith (1996) to endogenously derive coupon availability. (Note that we do not need coupon availability beyond the survey period. The coupon expectations are sufficient to calculate future cost outside the survey period.) We offer details of the two methods below.

All the households in our total sample have used a coupon at least once during the observation period. More than one-half (57%) of the purchases are made with manufacturer coupons. The brand-specific values are 49% (Brand A), 58% (Brand B) and 61% (Brand C). We develop an outside sample by randomly drawing one-half of the households for each level of consumption (K). In the resulting outside sample, 59% of the purchases are made with coupons which is comparable with the estimation sample. The values for the three brands are 48%, 55% and 64%. From the outside sample, we calculate the *weekly* proportion of purchases made with a coupon for each brand. Then, we make a random draw from a uniform distribution on [0, 1]. If the value is below the weekly percentage for a given brand, we assume a coupon is available for that brand in that week; otherwise, a coupon is not available. We repeat this process for each household in the estimation sample for a given brand in a given week. Thus, weekly coupon availabilities for each brand are determined exogenously from an outside sample.

We also employ the Gönül and Smith (1996) method to infer coupon availability within the estimation sample.

The essential details of the procedure are summarized in Appendix B. From the estimates, we obtain a mean availability rate of 0.57 across all brands over the sample period. We calculate the brand-specific weekly availability probabilities and repeat the process described above.

Recall that we assume that coupon availability expectations do not vary by brand in order to manage the state space. Therefore, we need to generate category-level coupon availability which we do as follows: We multiply the brand-specific coupon availability (calculated by the two methods described above) by the household's preference weights, and sum the resulting values across the three brands. Using the resulting probability, we make a random draw from a uniform distribution to determine category availability. We repeat the procedure for the case of equal weights. We also obtain category availability by using weekly brand-specific availability probabilities (instead of the binary values generated from those probabilities), adjusted for preference weights. We observe that parameter estimates remain robust when category availability is determined in this manner. Note that if brand-specific expectations are specified, there is no need for approximating category availability.

While brand-specific preference weights might be appropriate for the current purchase decision, equal preference weights might be appropriate in obtaining the future expectations as described above. Thus, a case can be made for such a hybrid model. We estimate the hybrid model as well.

Before we present the results, we summarize the key assumptions of our analysis so that our findings can be interpreted accordingly.

(a) We consider a time horizon of 156 weeks.

(b) The consumption rate for a given household is constant.

(c) Initial inventory is set to the consumption rate. We test the parameters for robustness for other values.

(d) Current coupon availability is determined separately for each brand. However, future coupon availability is determined for the category.

(e) Coupon expectations follow a first-order process. To minimize computational burden, they are restricted to be the same across brands.

(f) In our estimation, prices remain stable and hence are not forecasted by the consumers.

3.2 Estimation Procedure

The estimation method we adopt is based on a recursive numerical solution of the dynamic programming problem embedded in a maximum likelihood procedure. First, given a set of initial parameter values, we solve for future expenditure functions in *each* period for *each* possible level of coupon and inventory. Second, we evaluate the likelihood function by substituting the appropriate probabilities derived in §2. We repeat the two steps with a new set of parameter values. We iterate in this fashion until the log-likelihood function converges. We use the numerical optimization software offered by the GAUSS package for personal computers. We re-estimate our models with different initial values and obtain convergence to the reported point in all cases.

3.3 Model Estimates and Goodness of Fit

The parameter estimates for the models are given in Tables 1 and 2. The models in Table 1 are estimated both with household-specific and with equal weights (1/3) for brand preferences where coupon availability is obtained from an outside sample. The models in Table 2 are estimated using coupon availabilities determined by the Gönül and Smith (1996) method. Note that since the data itself differs with the two different ways of measuring coupon availability, the likelihood values are not comparable.

The log-likelihood values for the single-period versions of the models, in each case, are worse than that of the dynamic model. Although in each case the dynamic model outperforms the single-period model in terms of the log-likelihood value, a formal likelihood ratio test cannot be performed because the two models are not nested in each other. They are merely different versions of our specification that differ by the value of the discount factor. Thus, we turn to cruder measures of overall fit as provided by the Akaike Information Criterion (AIC) and the Schwarz' Bayesian Information Criterion (BIC). The two measures are: AIC $= -2 \ln L + 2$ (Number of parameters); and BIC $= -2 \ln L +$ (Number of parameters)$*$ ln (Number of observations). For details on these criteria see Ramaswamy et al. (1993). Note that the preference is for a smaller value for either criteria. Both the AIC and the BIC criteria favor the dynamic models over the single-period models (see Tables 1 and 2). Neither AIC nor BIC is capable of rendering signif-

Table 1 Models with Coupon Availability Determined from an Outside Sample Parameter Estimates with *t*-statistics in Parentheses

	Models with Time-Varying Household-Specific Weights for Brand Preferences		Models with Equal Weights for Brand Preferences	
Parameter	Dynamic Model (A)	Single-Period Model (B)	Dynamic Model (C)	Single-Period Model (D)
Potential Stockout Cost ($\alpha_0 + \alpha_1 K$)				
α_0	9.0981***	9.2136***	9.3642***	9.2431***
	(111.7162)	(73.4031)	(102.0332)	(77.7854)
α_1	2.0200***	1.9222***	1.8642***	1.9259***
	(18.2420)	(9.8471)	(10.3642)	(10.4602)
Holding Cost				
η_1	0.0244***	0.2454	0.0155***	0.2385
	(5.2710)	(0.0843)	(7.0589)	(0.4279)
η_2	−0.0005	0.0000	−0.0003	−0.0004
	(−0.1560)	(0.5512)	(−0.1535)	(−0.2199)
Net Value of Coupon				
γ_1	0.1177***	0.2320***	0.2644***	0.2954***
	(21.0237)	(16.5830)	(17.9792)	(17.7427)
γ_2	0.2173***	0.2832***	0.1549***	0.2951***
	(20.8562)	(16.4851)	(18.0651)	(17.4780)
γ_3	0.7406***	0.8196***	0.8364***	0.8470***
	(5.3340)	(4.4973)	(3.2718)	(5.3002)
Coupon Expectations				
q_0	0.9989***	N.A.	0.9936***	N.A.
	(11.8797)		(12.0457)	
q_1	0.3804***	N.A.	0.3076***	N.A.
	(10.2759)		(10.0750)	
Log-Likelihood	−2616.5430	−2653.2081	−2596.2195	−2654.5932
AIC	5251.23	5320.35	5210.63	5323.17
BIC	5307.78	5364.51	5267.13	5367.33

*** Significant at 0.001 level.
N.A. Not applicable.

icance levels since they are not statistics but mere indices to rank models.

An appropriate null or baseline model is one where the purchase probability on each week is restricted to the sample percentage of purchases. This specification is achieved by assigning the appropriate value to the intercept term and setting the remaining parameters to zero. The likelihood values for the null models are far worse than the corresponding single-period and dynamic models. Therefore, not surprisingly, they are not favored by the criteria discussed above.

Another simple benchmark model is one without any coupon-related variables. We estimate such a model with just the stockout and holding cost parameters, and find that the log-likelihood value and goodness of fit criteria are worse than even our single-period models ($\ln L = -2737.8084$, AIC = 5483.52, BIC = 5508.82). This model gives relatively flat predictions of purchase behavior over time since it does not use weekly coupon availability data.

We now turn to the holdout period to make further goodness of fit comparisons. We calculate the log-

Table 2 Models with Coupon Availability Determined by the Gönül and Smith Method Parameter Estimates with *t*-statistics in Parentheses

	Models with Time-Varying Household-Specific Weights for Brand Preferences		Models with Equal Weights for Brand Preferences	
Parameter	Dynamic Model (E)	Single-Period Model (F)	Dynamic Model (G)	Single-Period Model (H)
Potential Stockout Cost ($\alpha_0 + \alpha_1 K$)				
α_0	9.4025***	9.3123***	9.5582***	9.2492***
	(90.9325)	(74.7657)	(90.0524)	(73.9838)
α_1	1.8331***	1.9347***	1.6689***	2.0907***
	(11.0409)	(9.9106)	(8.5574)	(10.6849)
Holding Cost				
η_1	0.0212***	0.2536	0.0196***	0.2021
	(3.8684)	(1.2831)	(2.9400)	(0.6096)
η_2	−0.0001	−0.0003	−0.0006	0.0004
	(−0.7024)	(−0.1931)	(−0.2208)	(0.1604)
Net Value of Coupon				
γ_1	0.1139***	0.2308***	0.3494***	0.4044***
	(22.7774)	(19.9667)	(16.6587)	(20.7848)
γ_2	0.2413***	0.2327***	0.3149***	0.3964***
	(10.9180)	(7.3527)	(22.0133)	(20.6293)
γ_3	0.7474***	0.8620***	0.8926***	0.9516***
	(7.9137)	(19.9404)	(4.3545)	(8.1501)
Coupon Expectations				
q_0	0.9292***	N.A.	0.9963***	N.A.
	(20.9697)		(27.5118)	
q_1	0.3868***	N.A.	0.3947***	N.A.
	(15.7367)		(22.0134)	
Log-Likelihood	−2621.8983	−2651.4571	−2628.6290	−2648.7429
AIC	5261.67	5316.74	5275.34	5311.51
BIC	5318.49	5361.01	5331.95	5355.58

*** Significant at 0.001 level.
N.A. Not applicable.

likelihood value on the holdout sample and compare hit rates using the parameter estimates obtained from the estimation sample. We present the results in Table 3. We find that the dynamic models outperform the benchmark and the single-period models in all cases.

3.4 Parameter Estimates

The parameter estimates are quite comparable across the four specifications, and therefore, we interpret the results for the case where the coupon availability is determined exogenously and where the brand preference weights are household-specific (Model A in Table 1).

We find that households attach a positive and significant cost to a potential stockout ($\hat{\alpha} = \hat{\alpha}_0 + \hat{\alpha}_1 K = \$9.0981 + 2.0200 \cdot K$, where $K = 0.5, 0.6, 0.7, 0.8$). For households that use diapers more than others (K increases), the stockout cost is higher. Both coefficients are significant. The holding cost is over about two cents for each box of inventory in each week and is significant ($\hat{\eta}_1 = 0.0244$). The quadratic effect of inventory is negative but insignificant.

Consumers seem to attach a different value to coupons dependent on the brand that issues them, although most coupons in the data have nearly the same face

Table 3 Performance in the Holdout Period

Model	Log-Likelihood	Hit-Rate
Simple Benchmark (*)	−936.9687	57.16%
Coupon Availability Obtained from an Outside Sample		
Household-Specific Weights		
Dynamic Model (A)	−759.2811	73.89%
Single-Period Model (B)	−796.9440	70.01%
Equal Weights		
Dynamic Model (C)	−754.1742	74.12%
Single-Period Model (D)	−799.2193	69.02%
Coupon Availability Obtained by the Gönül and Smith Method		
Household-Specific Weights		
Dynamic Model (E)	−767.2561	71.47%
Single-Period Model (F)	−800.1944	69.08%
Equal Weights		
Dynamic Model (G)	−776.3270	70.67%
Single-Period Model (H)	−798.7061	68.67%

(*) The benchmark model yields the following parameters estimates (and *t*-statistics):

$\alpha_0 = 9.3259^{***}$	$\alpha_1 = 1.8835^{***}$	$\eta_1 = 0.2832^{***}$	$\eta_2 = 0.0002$
(77.3323)	(9.8094)	(3.3854)	(0.3003)

value. For example, we find that the brands with lower market share (Brands 1 and 2) have a lower net coupon value than the market leader (Brand 3), but the second-highest brand has the lowest net coupon value.

Our evidence supports the notion that consumers hold expectations about future coupon offers. Moreover, we observe that their expectations differ conditional on current coupon availability. Both coupon expectation coefficients are significant ($\hat{q}_0 = 0.9989$, $\hat{q}_1 = 0.3804$). The consumers' expectation of future coupon activity in the category is lower if a coupon is available in the current week than if not. However, when there is no coupon available in the current period, consumers anticipate a coupon with near certainty in the next period. Such a high expectation is likely to cause consumers to defer purchase until the next period as long as they have sufficient inventory for the current period. The difference between the two probabilities is significant and, therefore, we reject the null hypothesis that coupon expectations are zero-order.

We also estimate the hybrid model discussed in §3.1. Most of the parameter estimates fall between the equal weights and brand-specific weights models. The only exceptions are marginally higher base stockout cost (α_0) and marginally lower coupon expectation when a coupon is available in the current period (q_1). For brevity, we do not report the complete details in the tables.

A noteworthy difference between the single-period and dynamic model estimates is the insignificance of the holding cost in the single-period models. A household with sufficient inventory may still buy if it expects future coupon availability to be low. The single-period model ignores such a trade-off and may incorrectly attribute such a behavior to the lack of significant holding cost. (In the naive benchmark model, holding cost is significant. Note that the magnitude is very large suggesting an unreasonable cost of capital of 140% underscoring the misspecification bias.) The substantive difference pertains to the inability of the single-period model to endogenously glean the information about consumers' expectations on future promotions and their influence on current purchase decisions.

4. Discussion, Managerial Implications, and Limitations

Overall, though more difficult to compute, the dynamic model is supported over the single-period model.

4.1 Stockout Cost

When a household does not buy in a given week, we raise the possibility of a stockout cost. Since the decision to buy or not is stochastic in each time period in our model, a decision not to buy influences the stockout cost of that time period as well as the total expected expenditure of all future periods. Thus, the stockout cost is not merely determined by a static outcome of a household's decision not to buy on a specific week.

We observe a highly significant and a relatively large stockout cost of over ten dollars ($\alpha_0 + \alpha_1 \cdot K$). The essential nature of the product category may explain the relatively high stockout cost suggested here. For products with readily available substitutes (for example, soda) the stockout cost may be much lower. We find that the stockout cost increases with increased consumption, though the magnitude is relatively modest. The stockout cost may also include transportation cost and value of time spent in obtaining the product. It must be noted, however, that the intercept term (α_0) captures the mean effect of omitted variables. Hence, care must be exercised in interpreting this term as a mere stockout cost.

4.2 Inventory Holding Cost

The findings of our research are consistent with the belief that consumers incur holding cost, and hence, are inherently averse to carrying excess inventory. The holding cost amounts to about 13% on the investment in inventory which is in the neighborhood of the short-term cost of capital and comparable to interest rate of credit cards. The reasonableness of this estimate provides a measure of face validity in our model. While we find no evidence for nonlinearity in the holding cost for diapers, this may not be the case for other product categories.

Though the holding cost is significant, we must caution that the inventory levels are inferred based on average consumption and hence the estimate may be potentially biased. To facilitate the inventory flow equation, the initial inventory (at time zero) must be set. We set that value equal to the average consumption of the household. (We modified the starting values significantly but found that the model estimates remained robust. The results reported here are for the case when $I_0 = K$.)

4.3 Valuation of Brand-specific Coupons

We observe that the subjective valuations of the coupons differ significantly from the face value of the coupon. The estimates obtained reflect the net outcome of the positive and negative effects of coupon discussed earlier. We find that valuation of coupons of the same face value sharply differs across the brands. Specifically, we find that for the leading brand (Brand 3) the perceived value is higher than the smaller brands. We note that for models with preference weights based on past purchases, the order of the value of the coupons differs from the order of the market shares.

4.4 Consumer Expectations of Coupons

The evidence we present is consistent with two hypotheses: first, consumers hold expectations about future coupon availability and second, expectations differ depending upon whether a coupon is available or not in the current period. Specifically, we find that the probability of coupon availability in the next period is higher when there is no coupon available in the current period than when it is available. These findings hold whether the coupon availability is determined exogenously from a calibration sample or derived endogenously based on the method proposed by Gönül and Smith (1996). The endogenous estimation of coupon expectations is consistent with the behavior suggested by Krishna (1990) that while expectations may be inaccurate, they influence purchase behavior.

We observe that the purchase probability declines with higher inventory levels. Further, at any level of inventory, the probability of purchase is higher when a coupon is available.

4.5 Generalizability

The main hurdle in developing richer structural models is the explosion of the state space causing serious estimation difficulty. The following extensions, described briefly, are relatively straightforward from a conceptual standpoint but enormously costly to estimate.

(a) Brand-specific expectations: The model directly generalizes to brand-specific expectations though it requires estimation of a brand-choice model at each discrete time interval.

(b) Heterogeneity: All parameters of interest, including the discount factor, can be specified to vary across households.

(c) Joint Purchase and Brand Choice Decision: We can easily formulate a two-stage model: purchase decision and conditional on purchase, brand choice decision (see, for example, Gupta 1988, Bucklin and Gupta 1992).

4.6 Strengths and Weaknesses of Structural Models

(i) Structural models yield insights that are not routinely available from reduced-form models. For example, current purchase incidence models cannot answer whether or not consumers adjust purchase behavior based on future expectations of coupons.

(ii) While structural models offer a better theoretical foundation, often they require simplifying assumptions to ensure estimability. These assumptions may undermine the merit of the structural models. For example, incorporating heterogeneity in these models is computationally burdensome while reduced-form models can be more easily generalized to incorporate such issues.

(iii) Structural models impose significant restrictions on consumer behavior when compared to reduced-form models, and therefore, are poor candidates if prediction is the key objective.

4.7 Limitations

We have estimated the model for two different coupon availability specifications, one exogenous and another endogenous. As information on coupon drops becomes increasingly available, this problem can be remedied to a large extent. In dynamic models, the measurement of inventory based on average consumption is prone to the error that the consumption rate may not be stable. Also, purchases in the product category we examine—disposable diapers—can quite possibly be made in retail outlets other than supermarkets. We have developed a framework "as if" consumers behave in the specified manner. All we can conclude is whether or not the results support such a behavior.

5. Conclusion

We develop, estimate, and test a dynamic stochastic optimization model of purchases in the disposable diapers category. The model provides an estimate of the stockout cost and a realistic assessment of the holding cost. More importantly, we are able to endogenously estimate consumers' expectations of future coupon availability and the impact of expectations on the current purchase decision. We find that a zero-order Markov expectation formulation where future coupon availability does not differ by current availability, is strongly rejected. The results validate the merit of the structural models to endogenously yield significant managerial insights unavailable from other models. In essence, the stochastic structural models of purchase can complement the simpler but eminently computable static choice models.

The key limitation of the stochastic structural model is not in the conceptual development of richer models but in the computational difficulty in estimation. However, two factors argue in favor of further development of the structural models in the future: first, the continuing rapid increase in computational facility; and second, new procedures developed by Hotz and Miller (1993) where it is shown that a random sampling of a small fraction of the state space is sufficient to estimate most models.

The logical next step is to investigate the optimal coupon scheduling by the firm given that consumers may anticipate coupon promotions and adjust their purchase behavior. This entails developing a structural model that incorporates the firm's decision as well. Another research direction is to systematically estimate consumer expectations, holding, and stockout costs across a number of product categories. We hope to address these issues in future research efforts.[2]

[2] Both authors contributed equally to the paper. We thank Rick Staelin, Brian Ratchford and two anonymous reviewers of the journal for several helpful comments and valuable suggestions. We also appreciate constructive comments from seminar participants at the University of Michigan and the University of Pennsylvania. We thank James Lattin, Robert Miller, Robert Shoemaker and Chester Spatt for their encouragement to pursue this line of research. The usual disclaimer applies.

Appendix A

Solution of the Dynamic Programming Problem

We start solving the dynamic programming problem from the last period.

Period T: The expenditure function in the final period is merely the optimal cost in period T. More explicitly,

$$\begin{aligned} F_T(I_{T\text{-}1}(B_{T\text{-}1}), C_{T\text{-}1}) &= \min\{\mathcal{C}(B_T = 0), \mathcal{C}(B_T = 1)\} \\ &= \min\Big\{\alpha_0 + \alpha_1 K + \eta_1(I_{T\text{-}1} - K)^+ \\ &\quad + \eta_2((I_{T\text{-}1} - K)^+)^2 + \epsilon_{0T}, \\ &\quad \eta_1(I_{T\text{-}1} + 1 - K)^+ + \eta_2((I_{T\text{-}1} + 1 - K)^+)^2 \\ &\quad + \Big(p_T - \sum_{j=1}^{J} w_{jt}(X_{jT}\beta_j + C_{jT}\gamma_j)\Big) + \epsilon_{1T}\Big\} \end{aligned} \tag{A1}$$

Period T-1: The expenditure function in period T-1 is the minimum of the sum of the cost in the current period and the expected value of the discounted expenditure in period T. In algebraic terms,

$$\begin{aligned} &F_{T\text{-}1}(I_{T\text{-}1}(B_{T\text{-}2}), C_{T\text{-}1}) \\ &\quad = \min\{\mathcal{C}(B_{T\text{-}1} = 0) + \delta E[F_T(I_{T\text{-}1}(B_{T\text{-}1} = 0), C_{T\text{-}1})], \mathcal{C}(B_{T\text{-}1} = 1) \\ &\qquad + \delta E[F_T(I_{T\text{-}1}(B_{T\text{-}1} = 1), C_{T\text{-}1})]\}. \end{aligned} \tag{A2}$$

The first expected value term on the right-hand side can be rewritten as the sum of two possibilities weighted by their respective probabilities. That is,

$$\begin{aligned} &E[F_T(I_{T\text{-}1}(B_{T\text{-}1} = 0), C_{T\text{-}1})] \\ &\quad = E(\alpha_0 + \alpha_1 K + \eta_1(I_{T\text{-}1}(B_{T\text{-}1} = 0) - K)^+ \\ &\qquad + \eta_2((I_{T\text{-}1}(B_{T\text{-}1} = 0) - K)^+)^2 + \epsilon_{0T} \mid B_T = 0)\cdot \Pr(B_T = 0) \\ &\qquad + E(\eta_1(I_{T\text{-}1}(B_{T\text{-}1} = 0) + 1 - K)^+ \\ &\qquad + \eta_2((I_{T\text{-}1}(B_{T\text{-}1} = 0) + 1 - K)^+)^2 \\ &\qquad + p_T - \sum_{j=1}^{J} w_{jt}(X_{jT}\beta_j + \gamma_j E(C_T \mid C_{T\text{-}1})) + \epsilon_{1T} \mid B_T = 1)\cdot \Pr(B_T = 1). \end{aligned} \tag{A3}$$

A similar expression can be derived for the second expected value term $E[F_T(I_{T-1}(B_{T-1} = 1, C_{T-1})]$. The coupon expectation term $(E(C_T | C_{T-1}))$ in equation (A3) is replaced by q_0 or q_1 depending on the value of C_{T-1} as shown in the text. The expected value terms for the remaining periods $(T\text{-}2, \ldots, 1)$ are solved similarly by backward recursion.

PROOF OF THE PROPOSITION IN §2.1. Assume that the ϵ's are independently and identically distributed normal random variables. The normal distribution is convenient since it yields a closed-form solution for the conditional expectation terms. For details, see Kendall et al. (1987). We employ the rule of finding the expected value of a minimum when there are two possibilities (Karlin and Taylor 1975, Ch. 1). The rule states that

$$E(\min (a, b)) = E(a | a < b) \Pr(a < b) + E(b | b < a) \Pr(b < a) \qquad \text{(A4)}$$

where a and b are random variables. In our context, the two random variables are the costs of not buying and buying. Suppressing the arguments of the expenditure function for brevity and letting the difference of error terms be ν_t $(=\epsilon_{1t} - \epsilon_{0t})$,

$$\begin{aligned} E[F_t] &= E(\min(A_{0t} + \epsilon_{0t}, A_{1t} + \epsilon_{1t})) \\ &= E(A_{0t} | A_{0t} < A_{1t} + \nu_t) \Pr(A_{0t} < A_{1t} + \nu_t) \\ &\quad + E(A_{1t} + \nu_t < A_{0t} | A_{0t} > A_{1t} + \nu_t) \Pr(A_{0t} > A_{1t} + \nu_t) \\ &= A_{0t} \Pr(B_t = 0) + E(A_{1t} | C_{t-1}) \Pr(B_t = 1) \\ &\quad + E(\nu_t | \nu_t < A_{0t} - E(A_{1t} | C_{t-1})) \Pr(B_t = 1), \qquad \text{(A5)} \end{aligned}$$

where A_{0t} and A_{1t} are as defined in the Proposition in the text; $\Pr(B_t = 0)$ is the probability that the condition in the first conditional expectation argument is satisfied, that is, the expenditure of not buying is less than the expenditure of buying; and $\Pr(B_t = 1)$ is the probability that the condition in the second conditional expectation argument is satisfied, that is, the expenditure of buying is less than the expenditure of not buying. The last step is obtained by substituting for the probabilities of the conditioning events as the purchase probabilities. The probability of not buying is given by

$$\Pr(B_t = 0) = \Pr(A_{0t} < A_{1t} + \nu_t) = \Pr(\nu_t > A_{0t} - A_{1t}). \qquad \text{(A6)}$$

We use Mills' ratio for normally distributed random variables that states that $E(\nu_t | \nu_t < A) \Pr(\nu_t < A) = -\sigma_\nu \phi(A/\sigma_\nu)$ for any A, (for details, see Kendall et al. 1987). We normalize $\sigma_\nu = 1$ since the parameters in our model are identified only up to a scaling constant. Substituting corresponding terms, we obtain:

$$\begin{aligned} E[F_t] &= A_{0t} \Pr(B_t = 0) + E(A_{1t} | C_{t-1}) \Pr(B_t = 1) \\ &\quad - \phi(A_{0t} - E(A_{1t} | C_{t-1})). \quad \text{Q.E.D.} \qquad \text{(A7)} \end{aligned}$$

Appendix B

Gönül and Smith (1996) infer coupon availability probabilities from scanner panel data on brand choice and coupon redemption. The inference is based on modeling availability as an unobservable (0, 1) variable. A coupon for a specific brand is either available to the household or not. Let the (y_i, z_i) pair denote the brand and coupon choices on a purchase occasion i. If the brand chosen is j and the coupon decision is k, where $j = 1, 2, 3$, and $k = 0, 1$, then the probability of choosing the specific (j, k) pair is,

$$\Pr(y_i = j, z_i = k) = \sum_{n=1}^{8} \Pr(y_i = j, z_i = k | E_i = n) \cdot \Pr(E_i = n)$$

where E_i denotes the possible coupon environment for purchase occasion i. The environment can be one where all three brands' coupons are available, or only one is available but not the others, or none, etc. Hence, there are 8 possible environments for the binary coupon availability situations of the three brands.

The conditional probability in the above equation is a multinomial logit choice probability and the environment probabilities are simple parameters that yield the availability probabilities. The conditional multinomial logit choice probability has a different denominator depending on the environment. For example, in an environment where no coupons are available, the choice set consists of three alternatives only $\{(1, 0), (2, 0), (3, 0)\}$. Or, in an environment where only brand 1's coupons are available the choice set consists of $\{(1, 1), (1, 0), (2, 0), (3, 0)\}$. The explanatory variables in the model consist of traditional ones such as loyalty, price, demographic variables, as well as additional coupon-proneness variables. After estimating the model on the diaper data, we endogenously obtain estimates of coupon availability parameters.

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This paper was received July 7, 1994, and has been with the authors 324 days for 3 revisions; processed by Brian Ratchford.