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# DISCRIMINATION AND SKILL DIFFERENCES IN AN EQUILIBRIUM SEARCH MODEL\*

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We analyze an equilibrium search model with three sources for wage and unemployment differentials among workers with the same (observed) human capital but different appearance (race): unobserved productivity, search intensities, and discrimination due to an appearance-based employer disutility factor. We show that the structural parameters are identified using labor market survey data. Estimation results for a black and white high school graduate sample imply: black productivity is 3.3% lower than white productivity; the employer's disutility factor is 31% of the white's productivity level; and 56% of firms have a disutility factor toward blacks.

#### 1. Introduction

Substantial evidence exists on large wage differentials across workers with the same observable productive characteristics (human capital, experience, etc.) but with different appearance (race, gender, etc.). In addition, wage differentials are often accompanied by unemployment rate and job duration differentials. For example, among young male high school graduates, the average hourly wage for blacks is about 15% lower than the equivalent for whites, and the unemployment rate of blacks is twice that of whites (16% versus 8%).<sup>2</sup>

<sup>2</sup> See Table 1 on p. 1325. The empirical literature on the economics of discrimination is vast and largely emphasizes the earnings gaps as surveyed by Cain (1986) and Altonji and Blank (1999). In the case of racial differences, Donohue and Heckman (1991) emphasized the convergence pattern of the black—white wage gap between 1960 and 1980. Chay and Lee (1997), and the literature cited there, indicated that convergence stopped in the 1980s with the mean wage gap remaining between 15 and 20%. Most recent discrimination research has focused on better controlling for productivity differences between blacks and whites. Neal and Johnson (1996) found that a large portion of the black—white wage gap can be explained by differences in skill as measured by AFQT scores. However, Chay and Lee found a substantial gap remains even under their attempt to control for unmeasured productivity differences. The extent of discrimination and the determinants of the observed wage gap

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Two common explanations for these phenomena are unobserved productivity differences and discrimination. The main difficulty is to empirically distinguish between these explanations. Standard reduced form wage regressions cannot separately identify unobserved productivity and discrimination effects (Eckstein and Wolpin, 1999). The standard wage regression estimates the level of discrimination by the difference of the conditional mean wage given observed skills. One can argue, however, that the observed conditional mean difference is not due to discrimination but to unobserved productivity differences.

Using readily available data on wages and unemployment durations of workers, the only way to distinguish between the effects of unobserved productivity and discrimination is by using a theory of discrimination to analyze the empirical implications regarding the wage distribution.<sup>3</sup> The contribution of this article is in providing a way of using a common theory of discrimination to identify and estimate both the unobserved productivity and the discrimination parameters.<sup>4</sup>

In this article, we analyze an equilibrium labor market search model that contains both discrimination and skill differences among workers of different appearance.<sup>5</sup> We follow Becker's (1957) theory of discrimination by assuming that there exists a positive fraction of firms/managers that have a disutility taste parameter toward workers with a certain appearance, called type B (e.g., black) workers. The presence of search friction in the model allows firms to have monopsony power, and, therefore, any discriminatory behavior can survive in equilibrium. Type B workers may also have a lower-productivity (skill) level than the other type of workers, type A(e.g., white). Workers search for jobs while unemployed and while they are working. Job offer rates are lower, and job destruction rates are higher for disliked workers (type B). Employers maximize utility (a function of profits and potential disutility) by choosing a wage for workers depending on their skill and appearance. In equilibrium, the utility from each type of worker is equalized across firms with the same attitude toward workers. The steady-state (Nash) equilibrium earnings distributions and unemployment rates are solved endogenously for workers of different appearance and skill.<sup>7</sup>

remains an unresolved issue as illustrated by the symposium on discrimination in the spring 1998 issue of *The Journal of Economic Perspectives*.

- <sup>3</sup> Alternatively, the availability of firm data on productivity matched with worker data on wages might help to differentiate between these two explanations (see, e.g., Hellerstein and Neumark, 1995). Yet, without theory, it is impossible to estimate both the unobserved productivity differences and discrimination.
- <sup>4</sup> Within a search—matching—(Nash) bargaining model, Eckstein and Wolpin (1999) and Flinn (1999) suggested using the bargaining power parameter as a measure of discrimination. We compare the approach in these papers to ours at the end of the introduction.
  - <sup>5</sup> The model is based on the Mortensen (1990) equilibrium search model.
- <sup>6</sup> Note that, without search friction and with (CRS) linear technology, the equilibrium wage will be equal to the marginal product, and firms with a positive disutility taste parameter will be inactive. Heckman (1998) makes the point that as long as employers have income to spend on taste indulgences, then the Becker equilibrium need not disappear in the long run. That is, only if the supply of firms is perfectly elastic or if there are enough nonpredjudiced employers to hire all the blacks will there be no discrimination.
- <sup>7</sup> The model is most closely related to Black (1995). In Black's equilibrium search model, a fraction of firms refuse to hire some workers on the basis of appearance. This leads to a lower reservation wage

Using the analytical solutions for these distributions, we show that the disutility taste parameter, the fraction of firms with this parameter, and the skill differential can be identified using standard labor market survey data on wages and unemployment. This is possible because unobserved productivity differences and discrimination affect the equilibrium earnings distributions in distinctly different ways. Since production is assumed to be linear and separable in skills, the two types of firms offer the same wage distribution for type A workers. When there is no discrimination (i.e., no disutility), the wage offer distributions for type B workers are also the same for both firm types (albeit different from the type A distribution in the presence of productivity differences). The presence of discrimination via the disutility of a fraction of firms implies that both types of firms offer lower wages to type B workers. That is, both types of firms discriminate against type B workers. However, now the wage offer distributions of the two firm types are distinct. Firms with a disutility offer low wages and firms without disutility offer higher wages with no overlap in the support of the two wage offer distributions. In sum, productivity differences affect the conditional mean difference between the wage offer and earnings distributions of the two types of workers, whereas discrimination affects the wage offer and earnings distributions at low wage levels. We exploit this implication of the model to identify the parameters related to discrimination and skill differences from standard available data.

To demonstrate the empirical relevance of the model, we use data on black and white male high school graduates from the National Longitudinal Survey of Youth (NLSY). The data consist of workers' unemployment durations, job durations, and wages. The unemployment and wage differentials in the sample are consistent with evidence from many other sources. Matching first moments from these data with predicted moments from the model, we estimate the structural parameters. The estimated parameters fit the mean unemployment and wage differentials observed in the NLSY sample and are consistent with the model. Both discrimination and skill differences play a role in explaining the black—white wage and unemployment differentials. We estimate that the productivity level of blacks is 3.3% lower than that of whites. Furthermore, we estimate that 56% of the firms have disutility from employing blacks, and their disutility factor is 31% of the white productivity level.

Our approach can be compared to that of Eckstein and Wolpin (1999) and Flinn (1999). Both studies used a search–matching–(Nash) bargaining model (Eckstein and Wolpin, 1995) to estimate the level of labor market discrimination toward blacks. In this model, the bargaining parameter is a free parameter that is assumed to represent the degree of labor market discrimination. Eckstein and Wolpin (1995) discussed in general the identification of the bargaining power parameter

for those workers and hence a lower mean wage. Our model deviates from Black's in two important ways. First, the disutility from hiring type B workers is allowed to be low enough such that all employers hire type B workers albeit at a possibly lower rate, and second, workers are allowed to search both on and off the job.

<sup>8</sup> Bowlus et al. (2001) also estimate a wage posting search model, similar to our model presented here, for both black and white workers using data from the NLSY. In the absence of modeling discrimination, they find search friction differences across blacks and whites, especially differences in the job destruction rate, and explain a large portion of the black—white wage differential.

in the model using standard labor market data and concluded that identification is not robust to simple functional form modifications of the model. Eckstein and Wolpin (1999) restricted the model such that the bargaining power parameter split the worker–firm match productivity proportionally. In this simplified version of the model, they showed that identification of the bargaining power parameter depends on strong assumptions regarding the equality of unobserved productivity differences across blacks and whites. Flinn (1999) also imposed conditions on the search–matching–(Nash) bargaining model that guarantee identification of the bargaining power parameter. Here, our model of discrimination is based on employer tastes toward blacks that affect wage-posting strategies and, hence, the equilibrium wage distribution. This framework allows us to identify the discrimination parameter in the presence of unobserved productivity differences between blacks and whites.

In the next section we describe the model, and Section 3 discusses its properties and their relation to the data. In Section 4, we discuss identification and the data, and show the estimation results. Since the estimates imply that discrimination is a factor, we conduct an analysis of equal pay policies in Section 5. In comparing worker types of equal productivity, we find that equal pay policies do not necessarily eliminate the wage differential. If discriminatory hiring practices are in place, equal pay policies may reduce but cannot eliminate the wage differential between type A and type B workers. However, if the equal pay policy is supported by equal offer rates and employment rates for the different worker types, wages and unemployment will be equalized. This result is in contrast to the result by Coate and Loury (1993), who use a version of the statistical discrimination theory studied initially by Arrow (1973). In their model, affirmative-action policies may imply that equally productive workers are perceived by employers to be unequally productive.

# 2. THE MODEL

There are M workers divided into two types:  $(1-\theta)M$  are type A and  $\theta M$  are type B. The worker types differ by appearance as well as productivity. Type A(B) workers have productivity level  $P_A(P_B)$ , where  $P_A \geq P_B$  since we assume that type A workers may have a higher skill level. Firms are managed by owners, and they maximize utility that depends on profits and the owner/manager preferences over the types (A and B) of workers. A fraction  $\gamma_d$  of the owners/managers have a linear disutility d when employing a type B worker (labeled as disutility firms) and  $1-\gamma_d$  firms do not have a disutility (labeled as nondisutility firms). The number of firms is normalized to 1.  $\theta$  and  $\gamma_d$  are exogenously given.

<sup>&</sup>lt;sup>9</sup> There is a large theoretical literature on discrimination. Coate and Loury (1993) provided a nice survey of this literature and the results on affirmative-action policies.

 $<sup>^{10}</sup>$  It is, of course, possible to also model reverse discrimination where some firms have a preference for type B workers and/or a dislike for type A workers. This extension of the model works against explaining why type B workers face lower wages and higher unemployment levels than type A workers, and thus complicates the model without providing further insight into explaining these observed differences.

The arrival rates of offers from the firm types vary across worker type and state of employment. Arrival rates, or matching functions, are assumed to be exogenously given as in Mortensen (1990) and most search theory. However, we assume that the arrival rates of offers to type B workers from disutility firms are lower than the type A arrival rates from both firm types and the type B rates from nondisutility firms. It seems natural to assume that, if a firm/manager does not like a particular type of worker, the effort made by her to meet such workers and the effort made by these workers to meet such firms would be lower. As a result the arrival rates are lower.  $^{11}$ 

The arrival rate of offers to unemployed (employed) type A workers from both firm types is  $\lambda_0$  ( $\lambda_1$ ), and it is assumed that workers search more intensively while unemployed than while employed,  $\lambda_0 > \lambda_1$ . The difference in arrival rates between type A and type B workers due to disutility is governed by a proportional factor  $k, 0 \le k \le 1$ . If k = 0, disutility firms do not search for and, therefore, do not hire type B workers. If k = 1, the search intensities for type B workers by disutility and nondisutility firms are the same. If d = 0 (or  $\gamma_d = 0$ ) we set k = 1, resulting in a model with pure productivity differences. In general, the arrival rate of offers to unemployed (employed) type B workers by nondisutility firms is  $\lambda_0$  ( $\lambda_1$ ) and by disutility firms is  $k\lambda_0$  ( $k\lambda_1$ ). The exogenous job destruction rate,  $\delta$ , is assumed to differ across worker types such that  $\delta_A \le \delta_B$ .

2.1. Firms. Managers maximize utility (U) by setting wages for type A and B workers taking the reservation wages and wage offer distributions as given. Firms are allowed to only post one wage offer for each worker type.<sup>13</sup> That is, wage offers can be conditional on worker type but not on the state or current wage of a

<sup>&</sup>lt;sup>11</sup> The literature includes many specifications for arrival rates (matching functions). It is often assumed that the arrival rate is a function of the number of vacancies, unemployed workers, search effort by firms, and search effort by workers. Robin and Roux (1997) endogenized the arrival rates in the Mortensen (1990) model by making it a function of effort. In their model arrival rates in equilibrium are positively correlated with marginal profits. The critical assumption that the arrival rates for type B workers at disutility firms are lower than that for type A workers is consistent with their result that effort by firms and individuals is positively related to profits and wages. It is likely that, if the arrival rates would have depended on effort, then some of our results would have depended on the functional form chosen for the matching function making the analysis much more complicated.

<sup>&</sup>lt;sup>12</sup> Allowing the job destruction rate to differ helps in our empirical exercise to capture the unemployment rate differences across blacks and whites. It is also consistent with the evidence found in Bowlus et al. (2001). We do not allow the job destruction rate to vary by worker and firm type as this unnecessarily complicates the model by introducing the need for firm-specific reservation wage rules, particularly while employed. The restriction  $\delta_A \leq \delta_B$  is not only in line with the observed data, but also follows from the assumption that  $P_A \geq P_B$ . In a matching framework where job destruction is endogenous, e.g., Mortensen and Pissarides (1994), lower productivity levels lead to higher rates of destruction.

<sup>&</sup>lt;sup>13</sup> As in most models of discrimination, we allow firms to set wages conditional on appearance. In the presence of productivity differences, this would not constitute illegal behavior although the wage differential may in equilibrium be wider than the productivity differential due to search and discrimination factors. In Section 5, we consider the case where productivity levels are equal, and firms are required by law to make equal wage offers. We show that such legislation need not eliminate wage differentials across appearance.

worker. Utility is additive in worker type. For nondisutility managers  $U_n$  is equal to the firm's profit function.

(1) 
$$U_n(w_A, w_B) = (P_A - w_A)l_n^A(w_A) + (P_B - w_B)l_n^B(w_B)$$

where  $w_i$  is the wage offered to type i workers by the firm and  $l_n^i(w_i)$  is the steady-state labor stock of type i workers for a nondisutility firm offering wage  $w_i$  (i = A, B). For disutility managers,  $U_d$  is equal to profits minus the disutility (d > 0) they receive from employing type B workers:

(2) 
$$U_d(w_A, w_B) = (P_A - w_A)l_d^A(w_A) + (P_B - d - w_B)l_d^B(w_B)$$

We assume d is small enough such that disutility firms receive positive net utility from employing type B workers.

2.2. Workers. Workers maximize utility over an infinite horizon in continuous time by adopting a reservation wage strategy that is state dependent. The reservation wage while employed is the current wage, w. That is, the optimal strategy of an employed worker is to accept any outside wage offer greater than the current wage. The reservation wage while unemployed is solved by equating the value of unemployment with the value of being employed at the reservation wage. The value of being unemployed depends on the value of nonmarket time and the value of future possible states. These include receiving and accepting an offer from a nondisutility or disutility firm. If no offer is received or if an offer is rejected, one continues to receive the value of being unemployed. Thus, the value of unemployment for a type A worker,  $V_U^A$ , is given by

(3) 
$$(1 + \beta dt)V_U^A = b dt + \lambda_0 (1 - \gamma_d) dt E_w^n \max (V_E^A(w), V_U^A)$$
$$+ \lambda_0 \gamma_d dt E_w^d \max (V_E^A(w), V_u^A) + (1 - \lambda_0 dt)V_U^A$$

where  $\beta$  is the rate of time preference, and b is the common value of nonmarket time. <sup>14</sup> It is the sum of the value of nonmarket time, the probability of getting a job offer from a nondisutility firm and the expected value of that offer, the probability and the expected value of getting an offer from a disutility firm, and the probability and value of remaining unemployed.

Likewise the value of being employed is a function of the current wage and possible transitions such as accepting an outside offer and moving to another firm or having a job destroyed and moving to unemployment. The value of being employed at wage w (independent of firm type) for a type A worker,  $V_E^A(w)$ , is given by

 $<sup>^{14}</sup>$  To keep the focus on productivity differences and discrimination factors, we assume that the value of nonmarket time b is the same for type A and B workers.

(4) 
$$(1 + \beta dt)V_E^A(w) = w dt + \lambda_1 (1 - \gamma_d) dt E_{w'}^n \max \left( V_E^A(w'), V_E^A(w) \right)$$

$$+ \lambda_1 \gamma_d dt E_{w'}^d \max \left( V_E^A(w'), V_E^A(w) \right) + \delta_A dt V_U^A$$

$$+ (1 - (\lambda_1 + \delta_A) dt) V_E^A(w)$$

It is the sum of the current wage, the probabilities and expected values of job offers from nondisutility and disutility firms, the probability and value of losing a job, and the probability and value of remaining employed at wage w.

The value functions for type B workers differ from Equations (3) and (4) because of arrival and job destruction rate differences and possible wage offer differences. They are given by

(5) 
$$(1 + \beta dt)V_U^B = b dt + \lambda_0 (1 - \gamma_d) dt E_w^n \max \left(V_E^B(w), V_U^B\right)$$

$$+ k\lambda_0 \gamma_d dt E_w^d \max \left(V_E^B(w), V_U^B\right)$$

$$+ (1 - (\lambda_0 (1 - \gamma_d) + k\lambda_0 \gamma_d) dt)V_U^B$$

and

(6) 
$$(1 + \beta dt)V_E^B(w) = w dt + \lambda_1 (1 - \gamma_d) dt E_{w'}^n \max \left( V_E^B(w'), V_E^B(w) \right)$$

$$+ k\lambda_1 \gamma_d dt E_{w'}^d \max \left( V_E^B(w'), V_E^B(w) \right) + \delta_B dt V_U^B$$

$$+ (1 - (\lambda_1 (1 - \gamma_d) + k\lambda_1 \gamma_d + \delta_B) dt) V_E^B(w)$$

Because expectations are taken over wage offers, the worker's value functions are a function of the wage offer distributions of the firm types. Let  $F_n^i(w)$  ( $F_d^i(w)$ ) be the endogenously determined wage offer cumulative distribution function (cdf) of nondisutility (disutility) firms for type i (i = A, B) workers. These need not be the same, and in general are not, across the worker types. The reservation wage while unemployed for a worker of type i is the value of  $r_i$  that solves the equation,  $V_E^i(r_i) = V_U^i$ . For the value functions given here,  $r_A$  and  $r_B$  are given by (see Mortensen and Neumann (1988))

(7) 
$$r_A = b + \int_{r_A}^{\infty} \frac{(\lambda_0 - \lambda_1) ((1 - \gamma_d) (1 - F_n^A(w)) + \gamma_d (1 - F_d^A(w)))}{\beta + \delta_A + \lambda_1 ((1 - \gamma_d) (1 - F_n^A(w)) + \gamma_d (1 - F_d^A(w)))} dw$$

and

(8) 
$$r_B = b + \int_{r_B}^{\infty} \frac{(\lambda_0 - \lambda_1)(1 - \gamma_d)(1 - F_n^B(w)) + k(\lambda_0 - \lambda_1)\gamma_d(1 - F_d^B(w))}{\beta + \delta_B + \lambda_1(1 - \gamma_d)(1 - F_n^B(w)) + k\lambda_1\gamma_d(1 - F_d^B(w))} dw$$

For the sake of simplicity, and without loss of generality, we assume that  $\beta$  is equal to zero.<sup>15</sup>

<sup>&</sup>lt;sup>15</sup> In this regard we follow Mortensen (1990) and Burdett and Mortensen (1998). For very small values of  $\beta$ , Equations (7) and (8) give the wealth maximizing reservation wage. Thus Equations (7) and (8) with  $\beta = 0$  give the limit of the reservation wage as the discount factor goes to zero. We note that at  $\beta = 0$ , there are other optimizing strategies, but these are not of interest.

- 2.3. Equilibrium. We use the following standard equilibrium conditions (e.g., Mortensen, 1990) to solve for the steady-state equilibrium wage offer distribution and labor supply:
  - (a) The reservation wages of the two worker types are utility maximizing given their respective wage offer distributions.
  - (b) The flows of workers in and out of each state are equal.
  - (c)  $U_n$  is equalized across nondisutility firms and, given the reservation wage strategies of both worker types and the wage offer strategies of the disutility firms,  $U_n$  is maximized.
  - (d)  $U_d$  is equalized across disutility firms and, given the reservation wage strategies of both worker types and the wage offer strategies of the nondisutility firms,  $U_d$  is maximized.

Because the utility functions are additive in worker types and firms are setting type-specific wage offers, the equilibrium can be solved as if the workers were in separate markets. That is, one can solve for the steady-state flows and equilibrium wage offer distributions for type A and B workers separately.

In steady state, the flows of workers in and out of unemployment and employment at each firm must be equal. Section A.1 in the Appendix provides the equations for the flow conditions. Together these equations imply expressions for the labor stocks, unemployment levels, and earnings distributions in terms of the wage offer distributions of the firms. We turn our attention now to the solution of the equilibrium wage offer distributions for type A and B workers. The separability of the production function and the supply of labor enables us to analytically solve the equilibrium wage distribution for each type of worker independently.

2.4. Wage Distribution: Type A Workers. Because the utility from type A workers is equal across the firm types, the wage offer distributions for type A workers are the same, i.e.,  $F_n^A(w_A) = F_d^A(w_A) = F^A(w_A)$ . The equilibrium wage offer distribution is the same as in Mortensen's (1990) model with homogeneous firms and workers. This equilibrium wage offer distribution, as shown by Mortensen, is unique and is given by

(9) 
$$F^{A}(w_{A}) = \frac{1 + \kappa_{1A}}{\kappa_{1A}} - \left(\frac{1 + \kappa_{1A}}{\kappa_{1A}}\right) \left(\frac{P_{A} - w_{A}}{P_{A} - r_{A}}\right)^{\frac{1}{2}} \qquad r_{A} \leq w_{A} \leq wh_{A}$$

where  $\kappa_{0i} = \lambda_0/\delta_i$  and  $\kappa_{1i} = \lambda_1/\delta_i$  (i = A, B) and  $wh_A$  is the highest wage paid to type A workers. The reservation wage,  $r_A$ , is solved for by substituting  $F^A(w_A)$  into Equation (7) and  $wh_A$  from  $F^A(wh_A) = 1$ . The resulting expressions are

(10) 
$$r_A = \frac{(1 + \kappa_{1A})^2 b + (\kappa_{0A} - \kappa_{1A}) \kappa_1 P_A}{(1 + \kappa_{1A})^2 + (\kappa_{0A} - \kappa_{1A}) \kappa_{1A}}$$

and

(11) 
$$wh_A = P_A - \left(\frac{1}{1 + \kappa_{1A}}\right)^2 (P_A - r_A)$$

The earnings distribution,  $G^A(w_A)$ , can be solved from Equation (A.6) and is given by

(12) 
$$G^{A}(w_{A}) = \frac{1}{\kappa_{1A}} \left[ \left( \frac{P_{A} - r_{A}}{P_{A} - w_{A}} \right)^{\left(\frac{1}{2}\right)} - 1 \right] \qquad r_{A} \leq w_{A} \leq w h_{A}$$

2.5. Wage Distribution: Type B Workers. To solve for the type B wage distribution, we follow Mortensen (1990) and show that for all  $0 \le k \le 1$ , the distribution is a mixture of two distinct distributions in which disutility firms offer low wages and nondisutility firms offer higher wages. Formally we state this in Proposition 1.

PROPOSITION 1. If  $0 \le k \le 1$ , then there exists an equilibrium that satisfies

(13) 
$$l_d^B(w_B) = \frac{k\kappa_{0B}(1 + \kappa_{1B}^k)\theta M}{\left(1 + \kappa_{0B}^k\right)\left(1 + k\kappa_{1B}\gamma_d\left(1 - F_d^B(w_B)\right) + \kappa_{1B}(1 - \gamma_d)\right)^2}$$

$$r_B \le w_B \le wh_d$$

$$(14) \quad l_n^B(w_B) = \frac{\kappa_{0B} (1 + \kappa_{1B}^k) \theta M}{\left(1 + \kappa_{0B}^k\right) \left(1 + \kappa_{1B}(1 - \gamma_d) \left(1 - F_n^B(w_B)\right)\right)^2} \quad w h_d \le w_B \le w h_B$$

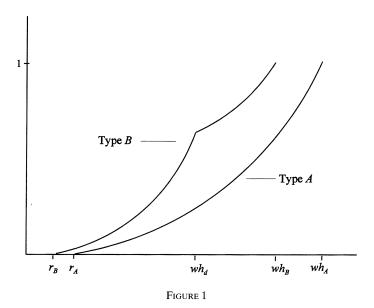
(15)
$$F^{B}(w_{B}) = \frac{\frac{1 + \kappa_{1B}^{k}}{k\kappa_{1B}} - \left(\frac{1 + \kappa_{1B}^{k}}{k\kappa_{1B}}\right) \left(\frac{P_{B} - d - w_{B}}{P_{B} - d - r_{B}}\right)^{\frac{1}{2}}}{\frac{1 + \kappa_{1B}}{\kappa_{1B}} - \left(\frac{1 + \kappa_{1B}(1 - \gamma_{d})}{\kappa_{1B}}\right) \left(\frac{P_{B} - w_{B}}{P_{B} - wh_{d}}\right)^{\frac{1}{2}}} \quad wh_{d} \leq w_{B} \leq wh_{B}$$

and

(16)
$$G^{B}(w_{B}) = \frac{\kappa_{0B}}{\kappa_{1B}\kappa_{0B}^{k}} \left[ \left( \frac{P_{B} - d - r_{B}}{P_{B} - d - w_{B}} \right)^{\frac{1}{2}} - 1 \right] \quad r_{B} \leq w_{B} \leq wh_{d}$$

$$\frac{\kappa_{0B}}{\kappa_{1B}\kappa_{0B}^{k}} \left[ \frac{\left( 1 + \kappa_{1B}^{k} \right)}{(1 + \kappa_{1B}(1 - \gamma_{d}))} \left( \frac{P_{B} - wh_{d}}{P_{B} - w_{B}} \right)^{\frac{1}{2}} - 1 \right] \quad wh_{d} \leq w_{B} \leq wh_{B}$$

 $^{16}$  The assumption of a proportional reduction in the offer arrival rates is more restrictive than needed for this proposition to hold. A sufficient and more general condition for segmentation is  $k_1 \le k_0$  where  $k_1$  ( $k_0$ ) is the disutility firm's reduction in the type B worker's offer arrival rate while employed (unemployed). Van den Berg (1998) discusses conditions under which the equilibrium with both firm types active is unique. That is, there does not exist another equilibrium with only one of the firm types active. Here we assume that the parameter values are such that this is the case. Essentially this implies a restriction on the value of nonmarket time, b, such that it is low enough to guarantee that  $r_B$  is less than  $P_B - d$ .



EARNINGS DISTRIBUTIONS OF TYPE A AND B WORKERS

where  $wh_B$  is the highest wage offered to type B workers;  $wh_d$  is the highest (lowest) wage offered to type B workers by disutility (nondisutility) firms;  $\kappa_{iB}^k = \kappa_{iB}(1 - \gamma_d + k\kappa_{iB}\gamma_d \ (i=0,1)$ ; and  $F^B(w_B)$  is the market wage offer distribution, the fraction of all firms paying  $w_B$  or less to type B workers  $(F^B(w_B) = (1 - \gamma_d)F_n^B(w_B) + \gamma_d F_d^B(w_B))$ .

PROOF. See Section A.2 in the Appendix.

Figure 1 presents the earnings distributions for type A and B workers. The presence of search friction implies that both distributions are nondegenerate. This is important as most discrimination models produce degenerate distributions for both groups of workers yielding no means of identifying the sources of observed wage differentials. Here the shapes and locations of the type A and B distributions are different, due to both discrimination (d and  $\gamma_d$ ) and the productivity differential, but in different ways. The productivity differential affects the conditional mean, whereas the presence of discrimination primarily affects the lower end of the wage distribution. These differences enable us to identify the parameters associated with discrimination and the productivity differential.<sup>17</sup>

The mixture of two distinct distributions for type B workers depicted in Figure 1 stems from our assumption of two firm types. If firm productivity (or disutility) would have had a distribution among firms, one would expect a smoother earnings

<sup>&</sup>lt;sup>17</sup> In Section 4 we discuss the way in which the discrimination and productivity parameters can be identified from the equilibrium wage distributions. We also discuss the robustness of identification to extending the model to include firm heterogeneity.

distributions. In most equilibrium models, however, the wage equilibrium is one point and with two firm types it would be two points. Here we have the analogous case of a nondegenerate earnings distribution composed of one and two distributions, respectively. In both settings, these are, in general, simplifying assumptions that, when taken to the data, are often modified and augmented so as to provide a better fit.

# 3. EQUILIBRIUM PROPERTIES

3.1. Wage Differentials. Wage differentials between type A and B workers, as well as unemployment rate and duration differences, can be generated in this model through three main mechanisms: productivity differences, search intensity differences, and discrimination. In this section, we present several propositions describing the main features of the equilibrium in Section 2. The first key result is that the type A earnings distribution stochastically dominates the type B distribution.

PROPOSITION 2. If  $P_B \leq P_A$ ,  $\delta_A \leq \delta_B$ , and  $0 \leq k \leq 1$ , then  $r_B \leq r_A$ ;  $G^A(w) \leq G^B(w)$  for all w; and  $E_o^A(w_A) > E_o^B(w_B)$  where  $E_o^i(w_i)$  is the mean wage offer for type i workers (i = A, B).

PROOF. See Section A.2 in the Appendix.

The relationship between the earnings distributions is depicted in Figure 1. The lower wages for type B workers stem from their lower productivity level, lower arrival rates, higher job destruction rate, and the influence of disutility firms on the shape of the wage offer distribution. The latter implies that type B workers have lower wages even if their job arrival and destruction rates and productivity levels are the same as type A workers. These rate differences do imply an added effect of hindering the movement of type B workers up their wage offer distribution relative to type A workers. This, in turn, drives the earnings distributions farther apart.

Because discrimination is a common explanation for wage differentials, Proposition 3 establishes the effects of the disutility parameter, d, and the fraction of disutility firms,  $\gamma_d$ , on the mean wage differential.

PROPOSITION 3. The ratio of mean earnings,  $E^B(w_B)/E^A(w_A)$ , is negatively related to d and  $\gamma_d$  where  $E^i(w_i)$  is the mean earnings level for type i workers (i = A, B).

Proof. See Section A.2 in the Appendix.

Thus, the ratio of mean earnings and, hence, the mean wage differential are functions of the disutility parameter, unlike in Black's (1995) model where only the fraction of disutility firms matters and the competitive framework where no differential emerges only segregated firms. Here complete segregation, i.e., firms with only type A workers, occurs only if k equals zero. As long as k is greater than zero, the labor stocks of all firms are composed of both type A and B workers. If k

is less than one, then the fraction of type B workers at disutility firms is less than  $\theta$ , the population proportion, and the fraction at nondisutility firms is greater than  $\theta$ . Of course, the productivity differential also affects the wage differential. A widening of this differential widens the wage differential. In addition Proposition 4 characterizes the ordering of the productivity and wage differentials.

PROPOSITION 4. If only 
$$P_A$$
 and  $P_B$  differ, then  $E^B(w_B)/E^A(w_A) > P_B/P_A$ .

PROOF. See Section A.2 in the Appendix.

This is a general point that says that search selection for jobs implies that mean wage differentials are lower than the difference in productivity.

3.2. Unemployment Differentials. The different job offer arrival and job destruction rates to type A and B workers generate differences in unemployment rates and average unemployment and job durations. If the search intensities for the worker types differ, the effective arrival rates of job offers are different across the worker types. The arrival rate of offers while unemployed for type A workers is  $\lambda_0$ , whereas it is  $\lambda_0(1-\gamma_d)+k\lambda_0\gamma_d$  for type B workers. Since all wage offers are accepted during unemployed search and the arrival processes are Poisson, unemployment durations are exponential with means  $1/\lambda_0$  and  $1/(\lambda_0(1-\gamma_d)+k\lambda_0\gamma_d)$  for type A and type B workers, respectively. If  $0 \le k < 1$ , the mean duration of unemployment is higher for type B workers. Unemployment rates,  $ue_A$  and  $ue_B$ , are found by solving Equations (A.4) and (A.5) for  $UE^A$  and  $UE^B$  and dividing by  $(1-\theta)M$  and  $\theta M$ , respectively. Given  $0 \le k \le 1$  and  $\delta_A \le \delta_B$ , a comparison of the two rates yields the following relationship:

(17) 
$$ue_B = \frac{\lambda_0(1 - \gamma_d) + k\lambda_0\gamma_d}{\delta_B + \lambda_0(1 - \gamma_d) + k\lambda_0\gamma_d} \ge \frac{\lambda_0}{\delta_A + \lambda_0} = ue_A$$

Job spell durations are also governed by exit rates. The average exit rates for type A and type B workers are, respectively, given by

(18) 
$$\int_{r_A}^{wh_A} \left( \delta_A + \lambda_1 \left( 1 - F^A(w_A) \right) \right) g^A(w_A) \, dw_A = \frac{\delta_A(1 + \kappa_{1A})}{\kappa_{1A}} \ln \left( 1 + \kappa_{1A} \right)$$

anc

(19) 
$$\int_{r_B}^{wh_B} \left( \delta_B + \lambda_1 (1 - \gamma_d) \left( 1 - F_n^B(w_B) \right) + k \lambda_1 \gamma_d \left( 1 - F_d^B(w_B) \right) \right) g^B(w_B) dw_B$$
$$= \frac{\delta_B \left( 1 + \kappa_{1B}^k \right)}{\kappa_{1B}^k} \ln \left( 1 + \kappa_{1B}^k \right)$$

 $^{18}$  In making these compositional comparisons, we assume that the placements of the firm in the type A and B wage offer distributions are the same. That is,  $F^A(w_A) = F^B(w_B)$  for all firms. Without this assumption, it is not possible to make comparisons across firm types with respect to labor stock composition. This assumption also implies that the wages paid to type A and B workers within a firm are positively correlated. That is, firms paying relatively high wages to type A workers will also pay relatively high wages to type B workers.

where  $g^i(w_i)$  is the probability density function of the earnings distribution,  $G^i(w_i)$ , for type i workers (i=A,B). The expression  $(1+x)^* \ln(1+x)/x$  is increasing in x. Therefore, if  $\delta_A = \delta_B$ , the average exit rate for type A workers is greater than that for type B workers since  $\kappa_{1A} \ge \kappa_{1B}^k$ . A higher average exit rate implies shorter average job spell durations for type A workers. However, if  $\delta_A \le \delta_B$ , then it is possible for the exit rate to be higher for type B workers, implying shorter job durations for them. Thus, the model is able to generate not only wage differentials, but also many of the duration and rate differentials often found in conjunction with wage differences. Note, however, that if there are no search intensity differences (k=1) and no job destruction differences  $(\delta_A = \delta_B)$ , the mean durations, average exit rates, and unemployment rates are equal.

3.3. Profit Differentials. Because of the presence of d, the total utility a disutility manager receives from hiring type B workers is lower than that received by nondisutility managers. This is true even if the arrival rates are the same (k = 1). Since the utility from hiring type A workers is the same across the firm types, this results in a lower level of utility overall for disutility managers. Profits for nondisutility firms are the same as the utility of a nondisutility manager posting wage offers  $(w_A, w_B)$ , and therefore, profits are equalized across nondisutility firms. Profits for disutility firms do not equal the utility of their managers because of the disutility. Across the firm types, profits are the same from type A workers, but not from type B workers. Proposition 5 characterizes the properties of the profit functions.

PROPOSITION 5. The profit function of disutility firms is nondecreasing in  $w_B$  for  $r_B \le w_B \le wh_d$ . Disutility firms earn lower profits than nondisutility firms.

PROOF. See Section A.2 in the Appendix.

This last result is a general finding in the discrimination literature. That is, disutility firms make lower profits. Here the equilibrium is sustainable because the presence of search friction results in monopsony power for all firm types.

# 4. IDENTIFICATION, DATA, AND ESTIMATION

In this section, we first show that we can differentiate between discrimination and unobserved productivity differences by identifying all of the underlying parameters using the NLSY data. We examine the data and show that they are

<sup>&</sup>lt;sup>19</sup> For further discussion on the calculation of job exit rates and mean job durations for various sampling schemes, see Ridder and Van den Berg (1998).

This relationship is easiest seen by comparing the utility of a disutility and a nondisutility manager offering wage  $wh_d$  to type B workers. The disutility manager has a lower per worker utility because of the presence of d and, if k < 1, a lower labor stock of type B workers as well.

 $<sup>^{21}</sup>$  If arrival rates were tied to total utility then this ordering may give some justification, besides prejudicial behavior, for disutility managers to offer lower arrival rates to type B workers.

consistent with the recent work on discrimination. Using moments from this data we provide estimates of the model's parameters.

The convexity of earnings distributions results in densities that are monotonically increasing (except at  $wh_d$  for type B workers). This result implies that the wage distribution from the model cannot fit observed wage distributions well. However, certain moments may be used to demonstrate the identification of the discrimination and productivity parameters and this is the approach we pursue here. We leave for future work modifying the model to empirically implement its full estimation.<sup>22</sup>

4.1. *Identification*. A necessary condition for being able to differentiate between unobserved productivity differences and discrimination in our model is that each affects the earnings distributions of type A and B workers differently.<sup>23</sup> To see this, it is helpful to examine the pure-productivity case (d=0) and the pure-discrimination case  $(P_A=P_B)$  separately. The earnings distributions for type A and B workers are shown in Figures 2 and 3 for the pure-productivity and pure-discrimination cases, respectively. Note that in the pure-productivity case the type B distribution is shifted to the left but it retains a similar shape as the type A distribution. This is not true in the pure-discrimination case. Here the type B earnings distribution is a mixture of two distributions (disutility and nondisutility) where the presence of disutility firms affects the lower range of wages. Note that, although the earnings distributions for type A workers are the same under the two cases, they substantially differ for type B workers.

In the pure-productivity case (Figure 2), the distance between the two distributions is governed by the difference in  $P_A$  and  $P_B$ . In the pure-discrimination case (Figure 3), the distance between the two distributions is influenced by the disutility parameter d as it affects the reservation wage of type B workers, the highest wage paid to type B workers, and the lower portion of the type B earnings distribution, i.e., all wages paid by disutility firms. The point at which the two distributions intersect,  $wh_d$ , is also directly related to the fraction of disutility firms in the market,  $\gamma_d$ . And, finally, the curvature of the lower portion of the type B earnings distribution relative to the type A distribution is influenced by k, the reduction in search intensity for type B workers by disutility firms.

The differing shapes of the type B earnings distributions lead to predictions regarding trends in the wage differential as one moves up the earnings distribution. That is, as one examines higher percentiles of the earnings distribution, different

 $<sup>^{22}</sup>$  One possibility is to extend the model as in Mortensen (1990) by introducing firm heterogeneity in the form of additional firm types within a single market. Maximum likelihood estimation techniques have been developed for this version of the model by Bowlus et al. (1995, 2001). Extending the model to include firm heterogeneity in the form of additional firm types does not affect the main results of this article, including the properties presented in Section 3 as long as the degree of productivity heterogeneity is the same for type A and B workers. The latter is consistent with the notion that we are studying a single market.

<sup>&</sup>lt;sup>23</sup> We proceed by showing explicitly that the wage distribution is a function of the two parameters and that the derivatives of the wage distribution with respect to the parameters differ. This, together with the existence of many moments for the wage distribution, establishes the necessary conditions for identification. The estimation example below provides evidence that this result has empirical content.

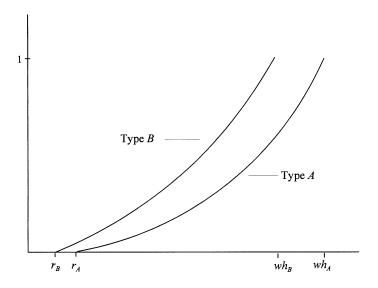
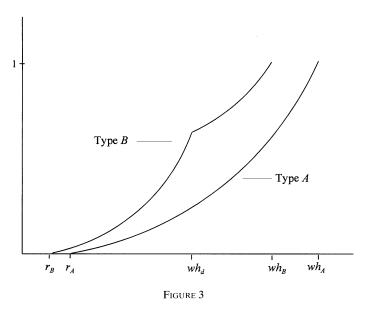


FIGURE 2
PURE-PRODUCTIVITY CASE



PURE-DISCRIMINATION CASE

predictions emerge as to the direction the differential is moving. These are outlined in Proposition 6.

PROPOSITION 6. Define  $w_i^a$  such that  $G^i(w_i^a) = \alpha$  (i = A, B). In the pure-productivity case with  $\delta_A = \delta_B$ , the  $\alpha$ -percentile wage ratio,  $w_A^{\alpha}/w_B^{\alpha}$ , is decreasing

in  $\alpha$ . In the pure-discrimination case with k=1 and  $\delta_A=\delta_B$ , the  $\alpha$ -percentile wage ratio is decreasing (increasing) in  $\alpha$  if  $\alpha < (>) G^B(wh_d)$ .

PROOF. See Section A.2 in the Appendix.

When both unobserved productivity differences and discrimination are present (Figure 1), it is not possible to sign the direction of the percentile wage ratio.

Because of these differential effects wages can be used to identify the productivity and discrimination parameters. They can also help with the identification of the arrival rate parameters. However, the main source of identification for the arrival rates comes from information on durations and transitions across states. Type A workers' durations and transitions are governed by, and therefore can identify, the arrival rate parameters  $\lambda_0$ ,  $\lambda_1$ , and  $\delta_A$ . The rates, durations, and transitions for type B workers are functions of their effective arrival rates  $\lambda_0(1 - \gamma_d) + k\lambda_0\gamma_d$ ,  $\lambda_1(1 - \gamma_d) + k\lambda_1\gamma_d$ , and  $\delta_B$ . Thus, differences in rates, durations, and transitions across the types identify the difference between  $\delta_A$  and  $\delta_B$  and the parameter combination  $(1 - \gamma_d(1 - k))$ . Since  $\gamma_d$  can be identified from wage data, k is then recoverable from a duration or transition difference across type A and B workers.

We have shown that unobserved productivity differences and discrimination affect the earnings distribution differently in our model and can therefore be identified. It should be noted that a necessary condition for identification is that there exist some firms with no disutility for type B workers, that is,  $\gamma_d < 1$ . Otherwise, if  $\gamma_d = 1$ , one cannot distinguish between two cases: (i) a market with unconstrained unobserved productivity differences and d = 0 and (ii) a market where d is positive and equal to the productivity difference in the first case. However, these extreme cases are of no particular interest.

An alternative specification is where type B workers are of two productivity types,  $P_B$  and  $P_B - d$ , whereas type A workers are only of one type. This case is not equivalent to ours, as the productivity differences now all reside with the workers and are not specific to any firm. It resembles the pure-productivity case with  $P_B$  replaced by the expected productivity of type B workers and, therefore, does not result in a mixture of distributions, as there is effectively only one firm type now. To retain the feature of a mixture distribution, it is necessary to assume that type B workers are only less productive at a certain fraction of firms,  $\gamma_d$ . In this case, worker productivity is firm specific. An explanation is then needed as to why type B workers are less productive at some firms than others, when type A workers are equally productive at all firms. One possibility is the claim of customer discrimination. In this case, the analysis would be the same as in our model. An alternative explanation is to assume segmented markets for type A and B workers with some low productivity firms only offering wages to type Bworkers. However, this leads to segregated firms and is not consistent with a single market endogenously producing wage differentials.

 $<sup>^{24}</sup>$  For the case of an extended model with discrete firm heterogeneity, identification is still possible. The main conditions are that all nondisutility firm types hire both type A and B workers, the unobserved productivity differences are the same for all firms, and the disutility parameters, including d,  $\gamma_d$ , and k, are the same for all disutility firms.

	· · · · · · · · · · · · · · · · · · ·		
	Whites	Blacks	
Unemployment rate—April 1985	0.077	0.157	
Unemployment duration in weeks	22.15	29.05	
Fraction completed job spells ending in unemployment	0.405	0.595	
Mean weekly earnings—April 1985	273.90	230.96	
Mean weekly offers	240.09	194.12	
Job spell durations in weeks	264.26	162.72	

Table 1

NLSY MOMENTS FOR MALE HIGH SCHOOL GRADUATES, 1985–1988

4.2. Data. For estimation, we use standard labor market wage, duration, and transition data from the NLSY covering the period between 1985 and 1988. To maintain a more homogeneous sample in line with our model, we restrict our sample to black and white males who graduated from high school between 1978 and 1984<sup>25</sup> and have not pursued further education nor served in the military over our sample period. Each respondent must have been interviewed in 1986 and must either be employed in a full-time job (≥35 hours per week) in the private sector in April 1985 or, if unemployed, have found a private sector full-time job before December 1988. For each respondent, we collect his state of employment in 1985, wage, job duration, transition following the job duration (either job-to-job or employment-to-unemployment), and any unemployment durations prior to or following the job duration. A complete description of the data and information collected is given in Section A.3 in the Appendix.

Table 1 contains a comparison of mean values for each of these components across blacks and whites.  $^{26}$  For this group of young black and white males, several differentials emerge. Black males have an unemployment rate that is twice that for white males with a mean unemployment duration that is five weeks longer. Their mean job duration is shorter and a higher fraction of their jobs end in unemployment. In addition, the mean weekly earnings for black males is 84% that of white males. Thus, within the framework of our model, we define white males as type A workers and black males as type B workers.

The data in the NLSY exhibit a black—white wage differential that is consistent with other studies (see footnote 2). Note that our sample is fairly homogeneous given the age and graduation restrictions and our direct control for gender and education. As stated in the introduction, standard reduced form wage regressions cannot separately identify unobserved productivity differences and discrimination effects. One approach in the reduced form literature has been to look for proxies for productivity differences in the data. With respect to the NLSY Neal and Johnson (1996) have, for example, studied the relationship between wages and AFQT test scores. They found that AFQT scores are a significant determinant of wages and explain 63% of the black—white wage differential after controlling for

<sup>&</sup>lt;sup>25</sup> GED recipients have been dropped.

<sup>&</sup>lt;sup>26</sup> Section A.3 in the Appendix contains a description of how these means are calculated.

Table 2
NLSY EARNINGS REGRESSIONS (DEPENDENT VARIABLE: LOG WEEKLY WAGE)

	(1)	(2)	(3)
Black	-0.1723	-0.1720	-0.0709
	(0.0453)	(0.0440)	(0.0500)
Age		0.0476	0.0477
		(0.0092)	(0.0092)
AFQT		, ,	0.0779
			(0.0195)
AFQT <sup>2</sup>			-0.0557
			(0.0156)
Constant	5.5427	4.4121	4.4299
	(0.0174)	(0.2202)	(0.2187)
$R_2$	3.2	8.6	13.1

Note: Standard errors are in parentheses.

education.<sup>27</sup> Table 2 presents analogous results for our sample. In the first column of Table 2 we report the overall black—white wage differential to be explained using log wages.<sup>28</sup> Age differences are controlled for in the second column. This does not lead to a reduction in the black—white wage differential for our sample. In the third column, we include AFQT scores and their square.<sup>29</sup> Even though the samples differ,<sup>30</sup> our results match closely those of Neal and Johnson. After controlling for AFQT scores, they found blacks still earn 7.2% less than whites, down from 19.6% with controls for age and education. In our sample, controlling for AFQT scores reduces the black—white wage differential from 17.2% to 7.1%. Treating the test scores as a measure of skills that translate into productivity differences through a production function and assuming workers are paid their marginal products produces an estimate of a 10.1% average unobserved productivity differential between white and black high school graduates.

It is not clear if the AFQT scores measure ability or other ethnic and cultural differences between individuals. However, even if we agree to the Neal and Johnson approach of using AFQT scores as a measure of pre-labor market factors affecting productivity, there is still a substantial wage gap that has to be accounted for. In the next section, we estimate the productivity differences jointly with discrimination without using the AFQT data. It is of interest then to compare our estimates to those from the regression estimates using AFQT scores.

<sup>&</sup>lt;sup>27</sup> AFQT scores explain 75% of the black–white wage differential in the absence of education controls. Since we control for education, we compare our findings to those in Neal and Johnson (1996) with education controls.

<sup>&</sup>lt;sup>28</sup> For the regressions in Table 2 we combine all weekly wage observations in our sample.

<sup>&</sup>lt;sup>29</sup> As in Neal and Johnson (1996), we use age-adjusted AFQT scores from the 1989 version. On average, blacks in our sample score 39.44 points lower on the AFQT than whites. Neal and Johnson reported nearly the same value for their sample at 39.25. The scores used in the regressions have been standardized to have a zero mean and a standard deviation of one.

<sup>&</sup>lt;sup>30</sup> Neal and Johnson (1996) used only those workers who graduated from high school before 1981, included all levels of final education, and examined wages from 1990 and 1991.

4.3. Estimation. Using the data described above we estimate the structural parameters of our model. The estimation is based on matching first moments of unemployment and wages to the moments predicted by the model.<sup>31</sup> This is a simple way to demonstrate the identification of the discrimination and productivity parameters and to check whether the model can match the observed wage and unemployment differentials.<sup>32</sup>

We estimate the model using moments in stages. This helps to demonstrate the identification claims above. We start with a model where there are no differences between blacks and whites and work toward the full model with both productivity differences and discrimination. Estimation of the parameters allows us to distinguish between the competing hypotheses for the observed wage differential. On the one hand, if in the full model d=0 and k=1 or if  $\gamma_d=0$ , then the differences observed in Table 1 between black and white males can be attributed to productivity differences only. On the other hand, if  $P_A=P_B$ , then discrimination is the only source for differences between blacks and whites.

To estimate the first version of the model, we calculate moments from the full sample combining blacks and whites in the sample. This gives us an overall mean unemployment duration (udur) of 23.65 weeks, an unemployment rate (ue) of 8.9%, 43.4% of job spells ending in unemployment (ju), and a mean earnings level ( $E_G(w)$ ) of \$268.03. To estimate  $\lambda_0$ ,  $\delta$ ,  $\lambda_1$ , and P we solve the following system of equations

$$udur = 23.65 = \frac{1}{\lambda_0}$$
$$ue = 0.089 = \frac{\delta}{\delta + \lambda_0}$$

<sup>32</sup> As mentioned previously, our model specification is not rich enough to estimate by maximum likelihood. However, because we are matching means, adding firm heterogeneity is unlikely to have much effect on our estimates. For example, Koning et al. (2000) showed that the mean of the earnings distributions can be written as follows

$$E(w) = \frac{E(P)\kappa_1 + r}{1 + \kappa_1} - \left(1 - \frac{1}{1 + \kappa_1}\right) \int_{P}^{\bar{P}} \left[1 - \Gamma(x) - \left(\frac{(1 + \kappa_1)(1 - \Gamma(x))}{1 - \kappa_1(1 - \Gamma(x))}\right)^2\right] dx$$

where the productivity distribution is given by  $\Gamma(P)$ . The first term is the homogeneous mean evaluated at the mean of the productivity distribution (see Equation (A.33)). The second term depends on the productivity distribution and  $\kappa_1$ , a measure of search friction. The latter term disappears as the level of search friction reduces and the competitive solution emerges. In our estimation results,  $\kappa_{1A}$  is found to be relatively large. Thus our productivity estimates can be interpreted as estimates of the average level of productivity in a market with firm heterogeneity. We thank Gerard van den Berg for pointing out this result to us.

With regard to the arrival and destruction rate parameters, we note, first, that they are not estimated using wage data and, second, that they are not in general sensitive to specifications of the productivity distribution (Bowlus et al., 2001).

<sup>&</sup>lt;sup>31</sup> In a cross-country study, Ridder and van den Berg (1998) also used moments (from aggregate data) to estimate the Mortensen (1990) model and showed that for many countries the results are similar to those found using maximum likelihood on panel data.

$$ju = 0.434 = \frac{\lambda_1}{(\delta + \lambda_1) \ln \left(1 + \frac{\lambda_1}{\delta}\right)}$$
$$E_G(w) = 268.03 = \frac{\lambda_1 P + \delta r}{\delta + \lambda_1}$$

In this case,  $\lambda_0$  is identified from the mean unemployment duration,  $\delta$  from the unemployment rate, and  $\lambda_1$  from the fraction of job spells ending in unemployment. Alternatively, we could use the mean job duration for whites to estimate  $\lambda_1$ . However, it is difficult to find a parameter set that matches all of the remaining moments if we use job spell durations.<sup>33</sup> With respect to the moment for mean earnings that contains the reservation wage, we simplify the model by assuming that a minimum wage is effective. The main reason is that, without measurement error in wages, the lowest observed wages are the consistent estimators for the reservation wages. This assumption is not necessarily compatible with  $r_A \ge r_B$  and, therefore, we assume that the minimum wage is the effective lowest wage for both blacks and whites and modify the model accordingly. The only difference from the model above is that the minimum wage, w, is now the lowest wage offered by firms and is known a priori.<sup>34</sup> We set the weekly minimum wage at \$134.00.<sup>35</sup> Thus with r replaced with \$134.00, P is identified from the mean earnings level. In relation to the full model, we impose the following restrictions on the parameters:  $P_A = P_B = P > w$ , d = 0, k = 1,  $\lambda_0 > 0$ ,  $\lambda_1 > 0$ , and  $\delta_A = \delta_B = \delta > 0$ . The results of this estimation are presented in the first column of Table 3. In the second column, we relax the assumption that the productivity levels are equal. In this case, we estimate  $P_A$  and  $P_B$  using the mean earnings for whites and blacks, respectively, given in Table 1 with the restriction that  $P_A \ge P_B > w$ . All other parameters are estimated using the same moments and restrictions as in the first column. Here we see that the large mean wage differential between blacks and whites is reproduced in the productivity differential with  $P_A$  significantly different from  $P_B$ . By Proposition 4, we have that the mean wage ratio is larger than the productivity ratio. Hence, the black-white mean wage differential is smaller than the productivity differential. As noted above, one way to capture the observed differences in unemployment rates found in Table 1 between blacks and whites is to allow the job destruction rate to vary by race. The third column presents results allowing both the productivity level and the job destruction rate to vary by race. In order to estimate  $\delta_A$  and  $\delta_B$ , we use the white and black unemployment rates, respectively, in Table 1 and impose that  $\delta_B \geq \delta_A > 0$ . Since the fraction of spells ending in unemployment is also a function of the job destruction rate, we use the

<sup>&</sup>lt;sup>33</sup> Bowlus et al. (2001) also found the model had a difficult time matching the mean of the job duration distribution in conjunction with the other relationships in the data.

 $<sup>^{34}</sup>$  Under the binding minimum wage assumption,  $r_A$  and  $r_B$ , and therefore b, are not identified from the data. Given we observe wages at or near the minimum, we assume b is low enough such that the unique equilibrium is one where  $\underline{w}$  is binding for both type A and B workers, i.e.,  $r_B < r_A < \underline{w}$ .

<sup>&</sup>lt;sup>35</sup> The federal minimum wage in the United States in 1985 was \$3.35. Assuming a full-time work week of 40 hours gives us a minimum weekly wage of \$134. See Section A.3 in the Appendix for a discussion regarding the treatment of observed wages below the minimum.

Table 3						
PARAMETER ESTIMATES USING MOMENTS FROM NLSY						

Parameter	(1)	(2)	(3)	(4)	(5)	(6)
$\lambda_0$	0.0423	0.0423	0.0423	0.0423	0.0451	0.0451
	(0.0033)	(0.0033)	(0.0033)			
$\lambda_1$	0.0261	0.0261	0.0281	0.0281	0.0300	0.0300
	(0.0061)	(0.0061)	(0.0088)			
$\delta_A$	0.0041	0.0041	0.0035	0.0035	0.0038	0.0038
	(0.0006)	(0.0006)	(0.0007)			
$\delta_B$	0.0041	0.0041	0.0079	0.0079	0.0064	0.0064
	(0.0006)	(0.0006)	(0.0016)			
$\underline{w}$	134.00	134.00	134.00	134.00	134.00	134.00
$\overline{P}_A$	289.28	296.08	291.45	291.45	291.45	291.45
	(5.67)	(7.06)	(6.83)			
$P_B$	289.28	246.33	258.12	281.70	281.70	291.45
	(5.67)	(7.53)	(8.34)	•		
d	0	0	0	90.18	90.18	83.21
γd	0	0	0	0.4278	0.5637	0.6732
k	1	1	1	1	0.5787	0.6472

Note: Standard errors are in parentheses.

fraction reported in Table 1 for white workers to estimate  $\lambda_1$  in order to maintain a set of moments that is consistent with the parameter estimates. The third column contains the pure productivity case that generates results similar to those found by Bowlus et al. (2001).<sup>36</sup> The results reveal that it is important to allow for variation in the job destruction rate across race, as  $\delta_B$  is significantly different from  $\delta_A$  in the third column and from the common value in the second column.

We now turn our attention toward the addition of discrimination. The first estimation we do restricts k=1. That is, disutility firms are not allowed to search less intensively for black workers. This specification frees up two parameters, d and  $\gamma_d$ . We estimate these two parameters by including in our set of moments the mean wage offer for the blacks (Equation (A.32)) from Table 1 and the ratio of black to white median earnings of 0.87. We impose the following additional restrictions:  $P_B > d + w$ ,  $d \ge 0$ , and  $0 \le \gamma_d \le 1$ . The results are presented in the fourth column of Table 3.<sup>37</sup> Here we find that the productivity level of blacks

 $<sup>^{36}</sup>$  Bowlus et al. (2001) also allow  $\lambda_0$  and  $\lambda_1$  to vary by race although they find the difference in  $\delta$  to be the most significant of the three in terms of the black–white wage differential.

<sup>&</sup>lt;sup>37</sup> We do not report standard errors for the discrimination model. The calculation of standard errors in this case is complicated by our use of percentile wage ratios in our set of moments. We tried to use bootstrapping to recover the covariance matrix for the moments and discovered that our estimates are not robust to small changes in the mean of the black earnings distribution. For slightly lower values, a parameter combination does not exist that can reproduce the observed means, and slightly higher values imply parameter combinations that contradict model assumptions, in particular the productivity level for blacks is greater than that for whites whereas the disutility level is quite large. This stems from the very specific form of the wage distribution in the model and the noted mismatch between this distribution and that observed in the data. Our intention here is to demonstrate identification of these parameters. For robust parameter estimates, one would need to modify the model by adding heterogeneity in firm productivity levels.

increases such that it is much closer to the level for whites. The disutility level, d, is 31% of the white productivity level, and 42.8% of the firms have a disutility toward blacks. Thus, with the addition of discrimination, the importance of productivity differences in explaining the mean wage differential between blacks and whites is reduced dramatically. Note that none of the specifications up to this point can explain the observed differences in unemployment durations. To resolve this issue and derive estimates for the fully specified model, we allow k to be less than 1 and estimate  $\lambda_0$  using the mean unemployment duration of whites and the parameter combination  $(1 - \gamma_d(1 - k))$  using the black mean unemployment duration. Finally, given an estimate of  $\gamma_d$ , we can recover k from this estimate of  $(1 - \gamma_d(1 - k))$ . The following additional restriction is imposed:  $0 \le k \le 1$ . These results are shown in the fifth column.

The parameter estimates in the fifth column indicate that productivity, discrimination, and search intensity differences play significant roles in the determination of the wage differential. In terms of search differences, note that with k < 1, the job destruction rate for blacks need not be as large in order to explain the differences in the unemployment rate although it is still larger than that for whites. Turning to wages, the productivity level of blacks is still 96.7% that of whites. We find that 56% of the firms are disutility firms with a disutility level (d) that is 31% of the white productivity level. Disutility firms are also found to substantially reduce their search effort for black workers with k at 0.58. An implication of the model is that, although disutility firms make up more than 50% of the market, they employ only 14% of black workers  $(G^B(wh_d) = 0.141)^{.38}$  The reduction in search by disutility firms and the arrival rates while employed are both high enough that most blacks are either never employed by disutility firms or are able to move out of disutility firms and into higher paying nondisutility firms. However, because blacks have a much higher job destruction rate than whites, they spend more time in unemployment.

Finally in the sixth column of Table 3, for comparison purposes, we present estimates assuming a model with pure discrimination. That is, we restrict the productivity levels to be equal by setting  $P_B$  equal to  $P_A$ . We then estimate d and  $\gamma_d$  using the mean earnings and mean wage offer levels for blacks. Here the fraction of disutility firms increases whereas the disutility level decreases.

Given that fewer moments are used to estimate the pure discrimination (column 6) and productivity specifications (column 3), we can use the remaining moments to assess the fit of the models. As already noted, the lack of a search reduction parameter in the pure-productivity case leads to the prediction of blacks

 $^{38}$  It is actually possible to estimate  $wh_d$  using either the formula given in Equation (A.19) or using the empirical cumulative distribution function of earnings for black workers. In our case, both methods give similar results: \$153.23 by the first method and \$155.65 by the second. An implication of the model is that all black workers observed to have a wage below (above)  $wh_d$  must work for a disutility (nondisutility) firm. We note that this is a statement about the firm type and not about worker characteristics and thus to investigate such a statement one would need to have information on firm characteristics. Although we do have some information that is likely employer related – industrial sector, hours, and occupation, we unfortunately observe very few black workers (less than 20) with wages below  $wh_d$ . Thus small sample sizes prevent us from pursuing this interesting implication of the model.

and whites having the same mean unemployment duration, which is inconsistent with the data.<sup>39</sup> In assessing the model, one can also look at moments that were not used for estimation. For example, both models with discrimination predict the 75th percentile wage ratio to be around 0.91. The pure-productivity model, however, predicts the 75th ratio to be 0.86, which is closer to that in the data of 0.84.<sup>40</sup> Alternatively one could compare the model results to those from standard wage regressions. Our estimates regarding the unobserved productivity differential range from 3.3% in the model with both discrimination and productivity differences to 11% in the model with only productivity differences. The latter is close to the 10% estimate calculated using the regressions with AFQT scores. These assessments tend to support the notion that productivity differences account for the wage differential, but leave as an open question the reasons behind the unemployment rate differential. Of course, formal testing of the models to determine which is a better approximation of reality requires more elaborate econometric methods.

# 5. EQUAL PAY POLICIES

As we have seen, even if worker types are equally productive, the presence of disutility firms in the labor market generates wage differentials if firms are free to set wage offers conditional on appearance. This differential is widened if, in addition, the disutility firms search less intensively for type B workers. In this section, we examine the effects of imposing equal pay restrictions on the firms. That is, each firm must post and pay only one wage. To focus and simplify, we do so within the pure-discrimination model ( $P_A = P_B = P$ ). There are two cases. The first case is where the offer arrival rates are equal across firms (k = 1). The existence of disutility firms does not justify this case as possible without policy intervention, such as antidiscriminatory hiring legislation. The second case is where search intensities are different (k < 1). In each case, we restrict the job destruction rates to be equal ( $\delta = \delta_A = \delta_B$ ) and define  $\kappa_i = \lambda_i/\delta$  (i = 0, 1).

5.1. Equal Pay when Offer Rates Are Equal. Because the arrival rates are equal, the labor supply behavior of the worker types is the same. The only difference between the firm types is their expected utility per worker: P for a nondisutility firm and  $P(1-\theta)+(P-d)\theta=P-\theta d$  for a disutility firm. Like the wage offer distribution for type B workers in Section 2, the equilibrium wage offer distribution (for both type A and type B workers), F(w), is segmented, with the disutility firms offering lower wages than the nondisutility firms.

 $<sup>^{39}</sup>$  It is possible to add a search reduction parameter, say  $\mu$ , for black workers that applies to all firms on the basis of blacks' lower productivity. However, the use of standard moments from wage and duration data only identifies  $\mu(1-\gamma_d)$  in the model with both discrimination and productivity differences. In the pure-productivity case, a reduction in search intensity of 0.76 is needed to match the mean unemployment duration differences.

<sup>&</sup>lt;sup>40</sup> In addition the percentile ratio decreases in the data from 0.87 at the median to 0.84 at the 75th percentile, which according to Proposition 6 is inconsistent with the pure-discrimination model.

(20) 
$$F(w) = \frac{\frac{1+\kappa_1}{\kappa_1} - \left(\frac{1+\kappa_1}{\kappa_1}\right) \left(\frac{P-\theta d-w}{P-\theta d-r}\right)^{\frac{1}{2}}}{\frac{1+\kappa_1}{\kappa_1} - \left(\frac{1+\kappa_1(1-\gamma_d)}{\kappa_1}\right) \left(\frac{P-w}{P-wh_d}\right)^{\frac{1}{2}}} \quad v \leq w \leq wh_d$$

where

(21) 
$$wh_d = P - \theta d - \left(\frac{1 + \kappa_1(1 - \gamma_d)}{1 + \kappa_1}\right)^2 (P - \theta d - r)$$

(22) 
$$wh = P - \left(\frac{1}{1 + \kappa_1(1 - \gamma_d)}\right)^2 (P - wh_d)$$

and

(23) 
$$r = \frac{(1+\kappa_1)^2 b + (\kappa_0 - \kappa_1)\kappa_1 (P - \theta d)}{(1+\kappa_1)^2 + (\kappa_0 - \kappa_1)\kappa_1} + \frac{(1+\kappa_1)^2 (\kappa_0 - \kappa_1)\kappa_1 (1-\gamma_d)^2 d}{((1+\kappa_1)^2 + (\kappa_0 - \kappa_1)\kappa_1)(1+\kappa_1 (1-\gamma_d))^2}$$

In this case, blacks and whites face the same wage offer distribution as well as the same arrival rates and therefore have the same reservation wage and mean wage offer and earnings. The wage differential is eliminated under equal pay.

To illustrate the effects of equal pay, we return to our empirical example in Section 4. In Table 3, we presented estimates from several different versions of the model with discrimination. Here we have assumed k=1,  $\delta_A=\delta_B$ , and  $P_A=P_B$ , a case not considered in Section 4. To account for this aspect, we impose these additional restrictions and estimate the pure-discrimination model without equal pay. This is similar to adding discrimination to the specification in the second column in Table 3. This addition results in the following differences from that set of estimates:  $P_B=296.08$ , d=70.70, and  $\gamma_d=0.8595$ . If we now impose equal pay on the firms, the mean earnings of whites falls from \$273.90 to \$268.06 and the mean earnings of blacks increases from \$230.96 to \$268.06.41 White workers are hurt by the equal pay legislation whereas black workers gain. Interestingly, both firm types are indifferent to the policy change. The lost utility due to the increase in wages they must now pay to black workers is exactly offset by utility gain from the decrease in wages to white workers.

5.2. Equal Pay when Offer Rates Are Not Equal. When disutility firms practice discriminatory hiring (k < 1), imposing an equal pay policy does not necessarily result in an elimination of the wage differential. Suppose we take an extreme

<sup>&</sup>lt;sup>41</sup> These calculations were done assuming  $\theta=0.14$ , the fraction of blacks in the NLSY sample. Increasing  $\theta$  lowers mean earnings and offers.

example of disutility firms refusing to hire type B workers (k=0).<sup>42</sup> Then, in the absence of equal pay legislation, the equilibrium is the same as in Section 2 with k set equal to 0. With disutility firms not offering wages to type B workers, the nondisutility firms now operate in the entire wage range for type B workers from  $r_B$  to  $wh_B$ . The resulting wage offer distribution for type B workers is

(24)
$$F^{B}(w_{B}) = \frac{1 + \kappa_{1}(1 - \gamma_{d})}{\kappa_{1}(1 - \gamma_{d})} - \left(\frac{1 + \kappa_{1}(1 - \gamma_{d})}{\kappa_{1}(1 - \gamma_{d})}\right) \left(\frac{P - w_{B}}{P - r_{B}}\right)^{\frac{1}{2}} \qquad r_{B} \leq w_{B} \leq wh_{B}$$

where

(25) 
$$r_B = \frac{(1 + \kappa_1 (1 - \gamma_d))^2 b + (\kappa_0 - \kappa_1) \kappa_1 (1 - \gamma_d)^2 P}{(1 + \kappa_1 (1 - \gamma_d))^2 + (\kappa_0 - \kappa_1) \kappa_1 (1 - \gamma_d)^2}$$

and

(26) 
$$wh_B = P - \left(\frac{1}{1 + \kappa_1(1 - \gamma_d)}\right)^2 (P - r_B)$$

In this case, the disutility parameter, d, does not enter into the wage offer distribution. All differences between type A and B workers are driven off the effective arrival rate differences.

Requiring the nondisutility firms to pay the same wage to both worker types results in the following: (i) the reservation wage of type B workers is still lower than that of type A workers due to the lower effective arrival rates of offers; (ii) it is possible that some nondisutility firms specialize in hiring an all type B work force by offering wages in the  $[r_B, r_A)$  range; (iii) disutility firms only offer wages above  $r_A$  since they wish to only attract type A workers; (iv) disutility and nondisutility firms allocate themselves along the type A wage range so as to equalize utility among hiring type A workers; and (v) nondisutility firms allocate themselves along the type B wage range (above  $r_A$ ) so as to equalize the utility generated from hiring type B workers.

Condition (ii) is only true if it is possible to generate as much utility in this range as that generated by offering  $r_A$  and attracting both type A and B workers. Simulations show this only occurs when  $\theta$  and  $\gamma_d$  are large. Since the wage differential remains even without this feature, we assume for simplicity that the conditions for wages in this range to be offered are not met. Because of (iv), type A workers face the same wage offer distribution as they do in the unequal pay case. Given the type A distribution and (v), the resulting wage offer distribution for type B workers is

 $<sup>^{42}</sup>$  The solution to the case of equal pay when 0 < k < 1 is currently unsolved. However, the example of k = 0 is sufficient to show equal pay policies do not always eliminate the wage differential within this framework.

(27) 
$$F^{B}(w) = \frac{1 + \kappa_{1}(1 - \gamma_{d})}{\kappa_{1}(1 - \gamma_{d})} - \frac{1 + \kappa_{1}(1 - \gamma_{d})}{\kappa_{1}(1 - \gamma_{d})} \left(\frac{P - w}{P - r_{A}}\right)^{\frac{1}{2}} \quad r_{A} \leq w \leq wh_{B}$$

where  $r_A$  is given by Equation (10) and  $wh_B$  equals

(28) 
$$wh_B = P - \left(\frac{1}{1 + \kappa_1(1 - \gamma_d)}\right)^2 (P - r_A)$$

It is straightforward to show that  $wh_B$  is less than  $wh_A$  (Equation (11)). Although the mean wage offer of type B workers has increased, due to the infeasibility of offering wages below  $r_A$ , it is still lower than the mean wage offer of type A workers.

Hence, equal pay policies reduce but do not eliminate the wage differential. In this case, type A workers are indifferent regarding equal wage policies, whereas type B workers prefer equal pay. Disutility firms are also indifferent, seeing the same utility from type A workers, but nondisutility firms reject the equal pay policy since they have to pay type B workers a higher wage and still attract the same labor stock.<sup>43</sup> It is possible in this case for equal pay to have no effect on the wage distribution. In the presence of a binding minimum wage, the lowest wage paid to both worker types under equal pay is still the minimum wage. Replacing  $r_A$  and  $r_B$  with  $\underline{w}$  in Equations (24) and (27) shows equal pay brings about no change in the type B wage offer distribution. Because we have a minimum wage in our empirical example in Section 4, imposing equal pay would not change the mean earnings of whites and blacks if disutility firms refused to hire blacks (k = 0).<sup>44</sup> Thus, the equal pay legislation would be completely ineffective.

Finally, so far the imposition of equal pay has not affected the unemployment rate gap between type A and B workers. If, in the example with k=0, we had not assumed conditions were such that the lowest wage paid to type B workers was  $r_A$ , but rather  $r_B(\langle r_A \rangle)$ , then the policy would have lowered the unemployment rate gap. The unemployment rate for type B workers would have remained unchanged, but the rate for type A workers would have increased to

(29) 
$$\frac{\delta}{\delta + \lambda_1 (1 - F(r_A))}$$

since  $F(r_A)$  is now greater than zero. That is, there are some wage offers made by nondisutility firms to type A workers that are rejected.

# 6. CONCLUSIONS

In this article we analytically solve for the equilibrium wage distribution of a search model where workers have different skills that are perfectly correlated with their appearance. Firms are heterogeneous with respect to their preferences

<sup>&</sup>lt;sup>43</sup> Note, again, that the separability of the production function is a key assumption regarding the segmentation of the labor market.

<sup>&</sup>lt;sup>44</sup> This minimum wage result is special to the k = 0 case.

regarding worker's appearance and, given the search friction, firms offer different wages to workers of potentially the same quality. As a result, the equilibrium earnings function includes the mean effects of skill differences, appearance differences (race or sex), and an "unobserved" variance that is due to search frictions.

We show that it is possible, using standard labor market survey data, to identify these three different effects. In particular, the disutility parameter, which affects a fraction of firms and is the reason for the existence of discrimination, is distinguishable from the unobserved productivity differential. This is because search friction yields nondegenerate earnings distributions and, importantly, the two sources have different effects on the earnings distributions of the discriminated workers. The presence of discriminatory behavior results in an earnings distribution that is a mixture of two distributions, where the wage differential is a function of the preference parameter and low wages are disproportionately affected. The preference parameter cannot be estimated without a complete specification of the relationship between the preferences of firms and their strategies regarding the wages of workers. We show the model fits well the main features of the data including earnings and unemployment gaps between blacks and whites. Furthermore, the estimated disutility level and fraction of disutility firms are plausible.

Estimating a statistical discrimination model (e.g., Sattinger, 1998, or Moro, 2000) for the labor market based on the same data is an important alternative to the Becker model estimated here. The questions are whether such a model is empirically distinguishable from a model of pure productivity differences, whether it can fit the observed wage and unemployment gaps between blacks and whites, and whether it can be tested against the Becker model. Such work would benefit the antidiscrimination policy debates.

### APPENDIX

A.1. Flow Conditions. Let  $UE^i$  be the steady-state number of type i unemployed workers and  $G^i(w_i)$  be the fraction of type i workers (i = A, B) earning  $w_i$  or less in steady state, that is,  $G^i(w_i)$  is the earnings cdf. Equation (A.1) equates the flow of type A workers into a firm (nondisutility or disutility) offering wage  $w_A$  to the flow out where  $l_n^A(w_A) = l_d^A(w_A) = l^A(w_A)$ .

(A.1) 
$$\lambda_0 U E^A + \lambda_1 G^A(w_A) ((1 - \theta) M - U E^A)$$
$$= \delta_A l^A(w_A) + \lambda_1 (1 - \gamma_d) (1 - F_n^A(w_A)) l^A(w_A)$$
$$+ \lambda_1 \gamma_d (1 - F_d^A(w_A)) l^A(w_A)$$

The equilibrium flow conditions that determine the labor stocks of type B workers in nondisutility and disutility firms are, respectively, given by

(A.2) 
$$\lambda_{0}UE^{B} + \lambda_{1}G^{B}(w_{B})(\theta M - UE^{B})$$

$$= \delta_{B}l_{n}^{B}(w_{B}) + \lambda_{1}(1 - \gamma_{d})(1 - F_{n}^{B}(w_{B}))l_{n}^{B}(w_{B})$$

$$+ k\lambda_{1}\gamma_{d}(1 - F_{d}^{B}(w_{B}))l_{n}^{B}(w_{B})$$

and

(A.3) 
$$k\lambda_{0}UE^{B} + k\lambda_{1}G^{B}(w_{B})(\theta M - UE^{B})$$
$$= \delta_{B}l_{d}^{B}(w_{B}) + \lambda_{1}(1 - \gamma_{d})(1 - F_{n}^{B}(w_{B}))l_{d}^{B}(w_{B})$$
$$+ k\lambda_{1}\gamma_{d}(1 - F_{d}^{B}(w_{B}))l_{d}^{B}(w_{B})$$

 $UE^A$ ,  $UE^B$ ,  $G^A(w_A)$ , and  $G^B(w_B)$  can also be solved from flow conditions. Equations (A.4) and (A.5) equate the flows out of and into unemployment for type A and B workers, respectively.

(A.4)  

$$\lambda_0 (1 - \gamma_d) (1 - F_a^A(r_A)) U E^A + \lambda_0 \gamma_d (1 - F_d^A(r_A)) U E^A = \delta_A ((1 - \theta) M - U E^A)$$

(A.5) 
$$\lambda_0 (1 - \gamma_d) (1 - F_n^B(r_B)) U E^B + k \lambda_0 \gamma_d (1 - F_d^B(r_B)) U E^B = \delta_B (\theta M - U E^B)$$

The flow conditions that relate the offer distributions and the earnings distributions are given in (A.6) and (A.7). The left-hand side of each equation gives the steady-state number of workers who receive acceptable wage offers below w from unemployment, and the right-hand side contains the number of workers with wages below w who exit to unemployment or to higher paying firms.

(A.6) 
$$\left[ \lambda_0 (1 - \gamma_d) \left( F_n^A(w_A) - F_n^A(r_A) \right) + \lambda_0 \gamma_d \left( F_d^A(w_A) - F_d^A(r_A) \right) \right] U E^A$$

$$= \delta_A G^A(w_A) ((1 - \theta) M - U E^A)$$

$$+ \left[ \lambda_1 (1 - \gamma_d) \left( 1 - F_n^A(w_A) \right) + \lambda_1 \gamma_d \left( 1 - F_d^A(w_A) \right) \right]$$

$$\times G^A(w_A) ((1 - \theta) M - U E^A)$$

$$(A.7) \quad \left[\lambda_0(1-\gamma_d)\left(F_n^B(w_B)-F_n^B(r_B)\right)+k\lambda_0\gamma_d\left(F_d^B(w_B)-F_d^B(r_B)\right)\right]UE^B$$

$$=\delta_BG^B(w_B)(\theta M-UE^B)$$

$$+\left[\lambda_1(1-\gamma_d)\left(1-F_n^B(w_B)\right)+k\lambda_1\gamma_d\left(1-F_d^B(w_B)\right)\right]$$

$$\times G^B(w_B)(\theta M-UE^B)$$

# A.2. Proofs.

PROOF OF PROPOSITION 1. Let  $w_B^n(w_B^d)$  be a utility maximizing wage for nondisutility (disutility) firms. Then by Equations (7), (8), and utility maximization,

(A.8) 
$$(P_B - w_B^n) l_n^B(w_B^n) \ge (P_B - w_B^d) l_n^B(w_B^d)$$

$$(P_B - d - w_B^d) l_d^B(w_B^d) \ge (P_B - d - w_B^n) l_d^B(w_B^n)$$

implying

(A.9) 
$$(P_B - w_B^n) l_n^B(w_B^n) - (P_B - d - w_B^n) l_d^B(w_B^n)$$

$$\geq (P_B - w_B^d) l_n^B(w_B^d) - (P_B - d - w_B^d) l_d^B(w_B^d) \qquad \forall w_B^n, w_B^d$$

Under the assumption of proportional arrival rates Equations (A.2) and (A.3) give us  $l_d^B(w_B) = k^* l_n^B(w_B)$ . Thus Equation (A.9) becomes

(A.10) 
$$(P_B - w_B^n) l_n^B(w_B^n) - (P_B - d - w_B^n) k l_n^B(w_B^n)$$

$$\geq (P_B - w_B^d) l_n^B(w_B^d) - (P_B - d - w_B^d) k l_n^B(w_B^d)$$

Define  $X(w_B^n)$  equal to the left-hand side of Equation (A.10)

(A.11) 
$$X(w_B^n) = (P_B - w_B^n) l_n^B(w_B^n) - (P_B - d - w_B^n) k l_n^B(w_B^n)$$

The derivative of  $X(w_R^n)$ 

(A.12) 
$$X'(w_B^n) = ((P_B - w_B^n)(1 - k) + kd)l_n^{'B}(w_B^n) - (1 - k)l_n^{B}(w_B^n) > 0$$

is strictly positive because by utility maximization we have  $(P_B - w_B^n)l_n^{B'}(w_B^n) = l_n^B(w_B^n)$ . Suppose the wage offer distribution is not segmented, i.e.,  $\exists \, w_B^d \in (\underline{w}_B^n, \bar{w}_B^n)$  where  $\underline{w}_B^n(\bar{w}_B^n)$  is the lower (upper) support of the nondisutility firm's set of utility-maximizing wage offers. Then by Equation (A.12)

(A.13) 
$$(P_B - \underline{w}_B^n) l_n^B (\underline{w}_B^n) - (P_B - d - \underline{w}_B^n) l_d^B (\underline{w}_B^n)$$

$$< (P_B - w_B^d) l_n^B (w_B^d) - (P_B - d - w_B^d) l_d^B (w_B^d) \qquad w_B^d > \underline{w}_B^n$$

which violates Equation (A.9). Thus the distribution is segmented with disutility firms offering wages in the lower range:

(A.14) 
$$F_{d}^{B}(w_{B}) = F_{n}^{B}(w_{B}) = 0 \qquad w_{B} \leq r_{B}$$

$$F_{d}^{B}(w_{B}) > 0; F_{n}^{B}(w_{B}) = 0 \qquad r_{B} < w_{B} \leq wh_{d}$$

$$F_{d}^{B}(w_{B}) = 1; F_{n}^{B}(w_{B}) > 0 \qquad wh_{d} \leq w_{B} \leq wh_{B}$$

$$F_{d}^{B}(w_{B}) = F_{n}^{B}(w_{B}) = 1 \qquad w_{B} \geq wh_{B}$$

where  $wh_B$  is the highest wage offered to type B workers, and  $wh_d$  is the highest (lowest) wage offered to type B workers by disutility (nondisutility) firms.

Substituting the conditions in Equation (A.14) into Equations (A.2), (A.3), (A.5), and (A.7) and solving for the labor stocksyields Equations (13) and (14).

The equalization of utility within manager type conditions is given by

(A.15) 
$$(P_B - d - r_B)l_d^B(r_B) = (P_B - d - w_B)l_d^B(w_B)$$

and

(A.16) 
$$(P_B - wh_d)l_n^B(wh_d) = (P_B - w_B)l_n^B(w_B)$$

Substituting in the labor stock functions from Equations (13) and (14) yields the wage offer distributions for disutility firms

(A.17) 
$$F_d^B(w_B) = \frac{1 + \kappa_{1B}^k}{k \kappa_{1B} \gamma_d} - \left(\frac{1 + \kappa_{1B}^k}{k \kappa_{1B} \gamma_d}\right) \left(\frac{P_B - d - w_B}{P_B - d - r_B}\right)^{\frac{1}{2}} \quad r_B \le w_B \le w h_d$$

and nondisutility firms

(A.18) 
$$F_n^B(w_B) = \frac{1 + \kappa_{1B}(1 - \gamma_d)}{\kappa_{1B}(1 - \gamma_d)} - \left(\frac{1 + \kappa_{1B}(1 - \gamma_d)}{\kappa_{1B}(1 - \gamma_d)}\right) \left(\frac{P_B - w_B}{P_B - wh_d}\right)^{\frac{1}{2}}$$
$$wh_d \le w_B \le wh_B$$

where  $wh_d$  is given by the solution to  $F_d^B(wh_d) = 1$ 

(A.19) 
$$wh_d = P_B - d - \left(\frac{1 + \kappa_{1B}(1 - \gamma_d)}{1 + \kappa_{1B}^k}\right)^2 (P_B - d - r_B)$$

and  $wh_B$  is given by the solution to  $F_n^B(wh_B) = 1$ 

(A.20) 
$$wh_B = P_B - \left(\frac{1}{1 + \kappa_{1B}(1 - \gamma_d)}\right)^2 (P_B - wh_d)$$

The reservation wage for type B workers is solved for by substituting the wage offer distribution expressions (A.17) and (A.18) into Equation (8) and is given by

(A.21) 
$$r_B = \frac{xb + yP_B}{x + y} - \frac{zd}{(x + y)(1 + \kappa_1 B(1 - \gamma_d))^2}$$

where  $x = (1 + \kappa_{1B}^k)^2$ ,  $y = \kappa_{1B}(\kappa_{0B} - \kappa_{1B})(1 - \gamma_d + k\gamma_d)^2$ , and  $z = \kappa_{1B}(\kappa_{0B} - \kappa_{1B})((1 - \gamma_d + k\gamma_d)^2(1 + \kappa_{1B}(1 - \gamma_d))^2 - (1 - \gamma_d)^2(1 + \kappa_{1B}^k)^2)$ . The market wage offer distribution, the fraction of all firms paying  $w_B$  or less to type B workers

 $(F^B(w_B) = (1 - \gamma_d)F_n^B(w_B) + \gamma_d F_d^B(w_B))$ , is then given by Equation (15) and the type *B* earnings distribution from Equation (A.7) is given by Equation (16).

PROOF OF Proposition 2. For  $r_B \le r_A$  the following must hold

(A.22) 
$$r_{B} = \frac{xb + yP_{B}}{x + y} - \frac{zd}{(x + y)(1 + \kappa_{1B}(1 - \gamma_{d}))^{2}}$$
$$< \frac{(1 + \kappa_{1A})^{2}b + (\kappa_{0A} - \kappa_{1A})\kappa_{1A}P_{A}}{(1 + \kappa_{1A})^{2} + (\kappa_{0A} - \kappa_{1A})\kappa_{1A}} = r_{A}$$

where x, y, and z are defined as in the proof of Proposition 1. Consider first the case when  $\delta_A = \delta_B = \delta$  such that  $\kappa_{iA} = \kappa_{iB} = \kappa_i$  (i = 0, 1). Then rearranging terms yields

(A.23) 
$$\frac{y(1+\kappa_1)^2(P_B-b) - x(\kappa_0-\kappa_1)\kappa_1(P_A-b) + y(\kappa_0-\kappa_1)\kappa_1(P_B-P_A)}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1} < \frac{zd}{(1+\kappa_1(1-\gamma_d))^2}$$

Given  $0 \le k \le 1$  and  $\lambda_0 > \lambda_1$ , z is positive; the denominator of the left-hand side is positive; and the first term in the left-hand side is negative because  $P_B \le P_A$  and

(A.24) 
$$x(\kappa_0 - \kappa_1)\kappa_1 - (1 + \kappa_1)^2 y$$

$$= (\kappa_0 - \kappa_1)\kappa_1(1 - (1 - \gamma_d(1 - k)))((1 + 2\kappa_1)$$

$$\times (1 - \gamma_d(1 - k)) + 1) > 0.$$

Thus, when  $\delta_A = \delta_B$ , the right-hand side is positive and the left-hand side is negative and the inequality holds. Now allow  $\delta_A < \delta_B$ , since  $\partial r_A/\partial \delta_A < 0$  if  $\lambda_0 > \lambda_1$ ; then  $\delta_A < \delta_B = \delta$  implies an increase in  $r_A$  such that the inequality still holds.

For stochastic dominance, define  $w^{\alpha}$  such that  $G(w^{\alpha}) = \alpha$ . Then using the earnings distributions for type A and B workers given in Equations (12) and (16), we have

(A.25) 
$$w_A^{\alpha} = P_A - (P_A - r_A) \frac{1}{(1 + \alpha \kappa_{1A})^2} \qquad 0 \le \alpha \le 1$$

and

$$(A.26) w_{B}^{\alpha} = P_{B} - d - (P_{B} - d - r_{B}) \left(\frac{1}{1 + \alpha \kappa_{1B}^{k}}\right)^{2} \qquad 0 \leq \alpha \leq G^{B}(wh_{d}) P_{B} - (P_{B} - wh_{d}) \left(\frac{1 + \kappa_{1B}^{k}}{(1 + \kappa_{1B}(1 - \gamma_{d}))(1 + \alpha \kappa_{1B}^{k})}\right)^{2} \qquad G^{B}(wh_{d}) \leq \alpha \leq 1$$

It is straightforward to show

$$(A.27) w_B^{\alpha} < w_A^{\alpha} \quad \forall \ \alpha \in [0, 1]$$

if  $P_A = P_B = P$  and  $\delta_A = \delta_B = \delta$ . Since the derivative of  $w^{\alpha}$  with respect to P is positive and  $P_B \leq P_A$  and the derivative with respect to  $\delta$  is negative and  $\delta_A \leq \delta_B$ ,

$$(A.28) w_B^{\alpha} < w_A^{\alpha} \quad \forall \alpha \in [0, 1] \Longrightarrow G^A(w) \le G^B(w) \quad \forall w \in [r_B, wh_A]$$

Mean wage offers are given by the following expressions for type A and type B workers, respectively,

(A.29) 
$$E_o^A(w_A) = \gamma_d \int_{r_A}^{wh_A} w_A f_d^A(w_A) dw_A + (1 - \gamma_d) \int_{r_A}^{wh_A} w_A f_n^A(w_A) dw_A$$
$$= \int_{r_A}^{wh_A} w_A f^A(w_A) dw_A$$

and

(A.30) 
$$E_o^B(w_B) = \frac{k\gamma_d}{k\gamma_d + 1 - \gamma_d} \int_{r_B}^{wh_d} w_B f_d^B(w_B) dw_B + \frac{(1 - \gamma_d)}{k\gamma_d + 1 - \gamma_d} \int_{wh_d}^{wh_B} w_B f_n^B(w_B) dw_B$$

Equations (A.29) and (A.30) contain the expected wage offer from each firm type multiplied by the probability of an offer from that type. Solving Equations (A.29) and (A.30) yields the following

(A.31) 
$$E_o^A(w_A) = \frac{\kappa_{1A}(3 + 2\kappa_{1A})P_A + (3 + 3\kappa_{1A} + \kappa_{1A}^2)r_A}{3(1 + \kappa_{1A})^2}$$

and

(A.32) 
$$E_o^B(w_B) = \frac{\kappa_{1B}^k (3 + 2\kappa_{1B}^k) P_B + (3 + 3\kappa_{1B}^k + \kappa_{1B}^k)^2 r_B}{3(1 + \kappa_{1B}^k)^2} + \frac{k\kappa_{1B}\gamma_d d(2(1 + \kappa_{1B}(1 - \gamma_d)) + k\kappa_{1B}\gamma_d - 2(1 + \kappa_{1B}(1 - \gamma_d))^2 (1 + \kappa_{1B}^k)^2)}{3\kappa_{1B}^k (1 + \kappa_{1B}(1 - \gamma_d))^2 (1 + \kappa_{1B}^k)^2}$$

Since  $r_B < r_A$ ,  $\kappa_{1B}^k \le \kappa_1$ ,  $P_B \le P_A$ , and the expression in  $E_o^B$  containing d is negative,  $E_o^A(w_A) > E_o^B(w_B)$ .

PROOF OF PROPOSITION 3. The mean of the type A earnings distribution is

(A.33) 
$$E^{A}(w_{A}) = \int_{r_{A}}^{wh_{A}} w_{A} g^{A}(w_{A}) dw_{A} = \frac{P_{A} \kappa_{1A} + r_{A}}{1 + \kappa_{1A}}$$

Since this mean is not a function of d or  $\gamma_d$ , only the mean of the type B earnings distribution needs to be examined to determine what happens to the ratio of mean earnings. The type B mean is given by the following expression:

(A.34) 
$$E^{B}(w_{B}) = (1 - \gamma_{d}) \frac{1 + \kappa_{1B}^{k}}{1 - \gamma_{d} + k\gamma_{d}} \left[ \frac{\kappa_{1B}(1 - \gamma_{d})P_{B}}{(1 + \kappa_{1B}(1 - \gamma_{d}))^{2}} + \frac{r_{B}}{(1 + \kappa_{1B}^{k})^{2}} \right]$$
$$+ \gamma_{d} \frac{k}{(1 - \gamma_{d} + k\gamma_{d})(1 + \kappa_{1B}^{k})} \left[ \frac{k\kappa_{1B}\gamma_{d}(P_{B} - d)}{1 + \kappa_{1B}(1 - \gamma_{d})} + r_{B} \right]$$
$$+ \gamma_{d}(1 - \gamma_{d}) \frac{k\kappa_{1B}(1 + \kappa_{1B}^{k})(2 + 2\kappa_{1B}(1 - \gamma_{d}) + k\kappa_{1B}\gamma_{d})}{(1 - \gamma_{d} + k\gamma_{d})(1 + \kappa_{1B}^{k})^{2}(1 + \kappa_{1B}(1 - \gamma_{d}))^{2}}$$
$$\times (P_{B} - d)$$

From Equation (A.21),  $r_B$  is decreasing in d. Thus  $E^B(w_B)$  is decreasing in d, and the mean earnings ratio is decreasing in d. The derivative of  $E^B(w_B)$  with respect to  $\gamma_d$  is also negative. To see this, note that if  $\gamma_d = 0$ , then the expression in (A.34) reduces to that in (A.33). If  $\gamma_d = 1$ , (A.34) becomes

(A.35) 
$$E^{B}(w_{B}) = \frac{(P_{B} - d)k\kappa_{1B} + r_{B}}{1 + k\kappa_{1B}}$$

It is straightforward to show that (A.35) is smaller than (A.33). Equation (A.34), the expression for the mean earnings of type B workers, falls between (A.33) and (A.35) approaching (A.35) as  $\gamma_d$  increases. Thus the ratio of mean earnings is decreasing in  $\gamma_d$ .

PROOF OF PROPOSITION 4. Under the assumptions k = 1, d = 0, and  $\delta_A = \delta_B = \delta$ , mean earnings for type A and B workers are given by

(A.36) 
$$E^{i}(w_{i}) = \frac{P_{i}\kappa_{1} + r_{i}}{1 + \kappa_{1}} \quad i = A, B$$

where  $\kappa_j = \lambda_j / \delta$  (j = 0, 1) and  $r_i$  equals

(A.37) 
$$r_i = \frac{(1+\kappa_1)^2 b + (\kappa_0 - \kappa_1) \kappa_1 P_i}{(1+\kappa_1)^2 + (\kappa_0 - \kappa_1) \kappa_1} \quad i = A, B$$

Substituting  $r_i$  into Equation (A.36) we have

(A.38) 
$$\frac{E^B(w_B)}{E^A(w_A)} > \frac{P_B}{P_A}$$

if and only if  $P_A > P_B$ , which is true by assumption.

PROOF OF PROPOSITION 5. Since profits from type A workers are equalized across disutility firms, we only need to concern ourselves with the profits from

type B workers. For k = 0, disutility firms do not hire type B workers and thus profits do not vary with respect to  $w_B$ . For k > 0, we need to show

(A.39) 
$$(P_B - w_B)l_d^B(w_B) < (P_B - w_B')l_d^B(w_B') \ \forall \ w_B, w_B' \in [r_B, wh_d], w_B < w_B'$$

By utility equalization we have

(A.40) 
$$(P_B - d - w_B)l_d^B(w_B) = (P_B - d - w_B')l_d^B(w_B') \quad \forall w_B, w_B' \in [r_B, wh_d]$$

Solving for  $(P_B - w_B')l_d^B(w_B')$  and substituting into the profit condition yields

(A.41) 
$$d(l_d^B(w_B') - l_d^B(w_B)) > 0 \quad \forall w_B, w_B' \in (r_B, wh_d], w_B < w_B'$$

which is positive because  $l_d^B(w_B)$  is increasing in  $w_B$ .

To prove the second statement, we need only to compare the profits of the two firm types at  $wh_d$ , since profits are equalized across nondisutility firms and increasing in the wage for disutility firms. For profits to be greater for nondisutility firms, the following must hold

(A.42) 
$$(P_B - wh_d)l_n^B(wh_d) > (P_B - wh_d)l_d^B(wh_d)$$

It does because the labor stock is greater at nondisutility firms due to their higher offer arrival rates.

PROOF OF PROPOSITION 6. For the pure-productivity case with  $\delta_A = \delta_B = \delta$ , the  $\alpha$ -percentile wage ratio is given by

(A.43) 
$$\frac{w_B^{\alpha}}{w_A^{\alpha}} = \frac{P_B - (P_B - r_B)(1 + \alpha \kappa_1)^{-2}}{P_A - (P_A - r_B)(1 + \alpha \kappa_1)^{-2}}$$

The derivative of the  $\alpha$ -percentile wage ratio with respect to  $\alpha$  is

(A.44) 
$$\frac{\partial \frac{w_B^{\alpha}}{w_A^{\alpha}}}{\partial \alpha} = \frac{2(1 + \alpha \kappa_1) \kappa_1 (P_B r_A - P_A r_B)}{((1 + \alpha \kappa_1)^2 P_A - P_A + r_A)^2}$$

which is negative because  $P_B r_A - P_A r_B = (1 + \kappa_1)^2 b(P_B - P_A) < 0$  by assumption.

For the pure-discrimination case with k=1 and  $\delta_A=\delta_B=\delta$ , the  $\alpha$ -percentile wage ratio is given by

$$\frac{w_{B}^{\alpha}}{w_{A}^{\alpha}} = \frac{P - d - (P - d - r_{B})(1 + \alpha \kappa_{1})^{-2}}{P - (P - r_{A})(1 + \alpha \kappa_{1})^{-2}} \qquad 0 \le \alpha \le G^{B}(wh_{d})$$

$$\frac{w_{B}^{\alpha}}{w_{A}^{\alpha}} = \frac{P - (P - wh_{d})\left(\frac{1 + \kappa_{1}}{(1 + \kappa_{1}(1 - \gamma_{d}))(1 + \alpha \kappa_{1})}\right)^{2}}{P - (P - r_{A})(1 + \alpha \kappa_{1})^{-2}} \qquad G^{B}(wh_{d}) \le \alpha \le 1.$$

The derivative of the wage differential with respect to  $\alpha$  for  $\alpha < G^B(wh_d)$  is

(A.46) 
$$\frac{\partial \frac{w_B^{\alpha}}{w_A^{\alpha}}}{\partial \alpha} = \frac{2(1 + \alpha \kappa_1)\kappa_1((P - d)r_A - Pr_B)}{((1 + \alpha \kappa_1)^2 P - P + r_A)^2}$$

which is negative because  $(P-d)r_A - Pr_B < 0$ . The derivative of the wage differential with respect to  $\alpha$  for  $\alpha \ge G^B(wh_d)$  after substituting in for  $wh_d$  and simplifying is

(A.47) 
$$\frac{\partial \frac{w_B^{\alpha}}{w_A^{\alpha}}}{\partial \alpha} = \frac{2(1 + \alpha \kappa_1)\kappa_1 P\left(r_A - r_B + d\left(\left(\frac{1 + \kappa_1}{1 + \kappa_1(1 - \gamma_d)}\right)^2 - 1\right)\right)}{\left((1 + \alpha \kappa_1)^2 P - P + r_A\right)^2}$$

which is positive because  $r_A \ge r_B$  and  $\gamma_d \le 1$ .

A.3. Data Description. For the estimation, we used data from the NLSY cross-section sample, for whites and the cross section plus supplemental samples for blacks. To be in our sample, a respondent must be a black or white male, have been interviewed in 1986, graduated from high school after 1977 and before 1985 (GED recipients are dropped), not gone on to any further education before 1989, and not served in the military between 1985 and 1988. The respondent must either be employed in a full-time job (≥35 hours/week) in the private sector in April 1985 or, if unemployed, have found a private sector full-time job before December 1988.

For each respondent who meets these criteria, we collect the following: The state – unemployment or employment—they are in the first week of April 1985. If employed, we collect the wage on the current job and the duration of the job (we know the exact starting date so there is no left censoring). If the job ends prior to December 1988, 46 the transition to unemployment or another job is noted and, if the transition is to unemployment, the duration to the next full-time job is recorded. If the respondent is unemployed in April 1985, we record the wage on the first full-time job after April 1985, the job duration, the transition and the unemployment duration if available. All durations are in weeks and all wages are converted to weekly wages (all wages are in 1985 dollars). Because of problems

<sup>&</sup>lt;sup>45</sup> April 1985 was chosen to produce unemployment rates consistent with annual averages.

<sup>&</sup>lt;sup>46</sup> If the job does not end prior to December 1988, it is treated as censored.

with measurement error, we treat as missing all wage responses that do not fall within upper (95th percentile) and lower (5th percentile) bounds collected from the U.S. March Current Population Survey outgoing rotation groups. We also treat as missing any wage observations below the legislated minimum wage of \$3.35 \* 40 hours = \$134.00. The first restriction on wages tends to identify wage values that are less than \$100 and more than \$600. About 9% of the wage observations for both blacks and whites fall in this category. The minimum wage restriction further identifies those wages between \$100 and \$134. Only 2–3% of the wage observations fall in this range and the percentage is the same for blacks and whites. We do not remove those individuals with such wages from our sample, but rather only exclude them from the calculation of mean earnings and mean wage offers. In this way they are still allowed to influence nonwage related moments.

The means in Table 1 are calculated as follows: The unemployment rate is the fraction of respondents in the sample who do not have a job in the first week of April 1985; the mean unemployment duration is the Kaplan–Meier mean of the unemployment durations following the job spells; the fraction of completed spells ending in unemployment is the number of respondents employed in April 1985 that transition to unemployment before December 1988 divided by the total number of respondents employed in April with jobs that end prior to December 1988; the mean earnings is the mean of weekly wages from jobs that are ongoing in April 1985; the mean wage offer is the mean of the weekly wages of jobs that start after April 1985; and the mean job duration is the Kaplan–Meier mean of job spell durations ongoing in April 1985. Sample weights from 1986 are used in calculating these means.

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