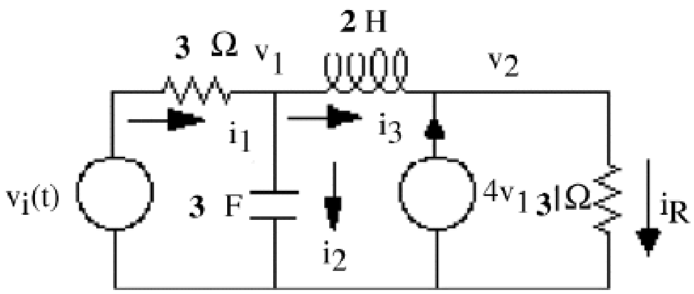
2.

Add branch currents and node voltages to the schematic and obtain,



Write the differential equation for each energy storage element.

$$\frac{dv_1}{dt} = \frac{1}{3} i_2$$

$$\frac{di_3}{dt} = \frac{1}{2} v_L$$

Therefore the state vector is $\mathbf{x} = \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

Now obtain v_L and i_2, in terms of the state variables,

$$v_L = v_1 - v_2 = v_1 - 3i_R = v_1 - 3(i_3 + 4v_1) = -11v_1 - 3i_3$$

$$i_2 = i_1 - i_3 = \frac{1}{3}(v_i - v_1) - i_3 = -\frac{1}{3}v_1 - i_3 + \frac{1}{3}v_i$$

Also, the output is

$$y = i_R = 4v_1 + i_3$$

Hence,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{1}{9} & -\frac{1}{3} \\ -\frac{11}{2} & -\frac{3}{2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{9} \\ \mathbf{0} \end{bmatrix} v_i$$

$$y = \begin{bmatrix} 4 & 1 \end{bmatrix} \mathbf{x}$$

5. The impedance equations are:

$$\left(s^2+2s+1\right)X_1(s)-sX_2(s)-(s+1)X_3(s)=0$$
$$-sX_1(s)+\left(2s^2+2s+1\right)X_2(s)-(s+1)X_3(s)=0$$
$$-(s+1)X_1(s)-(s+1)X_2(s)+\left(s^2+2s+2\right)X_3(s)=f(t)$$

Taking the inverse Laplace transform

$$\ddot{x}_1+2\dot{x}_1+x_1-\dot{x}_2-\dot{x}_3-x_3=0$$
$$-\dot{x}_1+2\ddot{x}_2+2\dot{x}_2+x_2-\dot{x}_3-x_3=0$$
$$-\dot{x}_1-x_1-\dot{x}_2-x_2+\ddot{x}_3+2\dot{x}_3+2x_3=f(t)$$

$$\ddot{x}_1=-2\dot{x}_1-x_1+\dot{x}_2+\dot{x}_3+x_3$$

$$\ddot{x}_2=\frac{1}{2}\dot{x}_1-\dot{x}_2+\frac{1}{2}x_2+\frac{1}{2}\dot{x}_3+\frac{1}{2}x_3=0$$
$$\ddot{x}_3=\dot{x}_1+x_1+\dot{x}_2+x_2-2\dot{x}_3-2x_3+f(t)$$

Define the state variables

$z_1=x_1$; $z_2=\dot{x}_1$; $z_3=x_2$; $z_4=\dot{x}_2$; $z_5=x_3$; $z_6=\dot{x}_3$

The equations are rewritten as

$\dot{z}_1=\dot{x}_1=z_2$

$\dot{z}_2=\ddot{x}_1=-2z_2-z_1+z_4+z_6+z_5$

$\dot{z}_3=\dot{x}_2=z_4$

$\dot{z}_4=\ddot{x}_2=\frac{z_1}{2}-z_4-\frac{z_3}{2}+\frac{z_6}{2}+\frac{z_5}{2}$

$\dot{z}_5=\dot{x}_3=z_6$

$\dot{z}_6=\ddot{x}_3=z_2+z_1+z_4+z_3-2z_6-2z_5$

In matrix form

$$\dot{\mathbf{z}}=\begin{bmatrix}0&1&0&0&0&0\\-1&-2&0&1&1&1\\0&0&0&1&0&0\\\frac{1}{2}&0&-\frac{1}{2}&-1&\frac{1}{2}&\frac{1}{2}\\0&0&0&0&0&1\\1&1&1&1&-2&-2\end{bmatrix}\mathbf{z}+\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}f(t)$$
$$y=\begin{bmatrix}0&0&0&0&1&0\end{bmatrix}\mathbf{z}$$

9.

a. Using the standard form derived in the textbook,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -100 & -7 & -10 & -20 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$c = \begin{bmatrix} 100 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}$$

b. Using the standard form derived in the textbook,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -30 & -1 & -6 & -9 & -8 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$c = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}$$

15. Program:

```
'a'
A=[0 1 5 0;0 0 1 0;0 0 0 1;-7 -9 -2 -3];
B=[0;5;8;2];
C=[1 3 6 6];
D=0;
statespace=ss(A,B,C,D)
[num,den]=ss2tf(A,B,C,D);
G=tf(num,den)
'b'
A=[3 1 0 4 -2;-3 5 -5 2 -1;0 1 -1 2 8;-7 6 -3 -4 0;-6 0 4 -3 1];
B=[2;7;8;5;4];
C=[1 -2 -9 7 6];
D=0;
statespace=ss(A,B,C,D)
[num,den]=ss2tf(A,B,C,D);
G=tf(num,den)
```

Computer response:

```
a =
         x1   x2   x3   x4
    x1    0    1    5    0
    x2    0    0    1    0
    x3    0    0    0    1
    x4   -7   -9   -2   -3
```

```
b =
        u1
   x1   0
   x2   5
   x3   8
   x4   2


c =
        x1   x2   x3   x4
   y1   1    3    6    6

d =
        u1
   y1   0

Continuous-time model.

Transfer function:
75 s^3 - 96 s^2 - 2331 s - 210
------------------------------
s^4 + 3 s^3 + 2 s^2 + 44 s + 7
```

32.

a. The following basic equations characterize the relationships between the state, input, and output variables for the HEV common forward path of the figure:

$$u_a(t) = K_A \cdot u_c(t)$$

$$L_A \cdot \dot{I}_a + R_a \cdot I_a(t) = u_a(t) - e_b(t) = K_A u_c(t) - k_b \cdot \omega(t) \tag{1}$$

$$J_{tot} \cdot \dot{\omega} = T(t) - T_f(t) - T_c(t), \text{ where } J_{tot} = J_m + J_{veh} + J_w,$$

$$T(t) = k_t \cdot I_a(t),\ T_f(t) = k_f \cdot \omega(t)$$

b. Given that the state variables are the motor armature current, $I_a(t)$, and angular speed, $\omega(t)$, we re-write the above equations as:

$$\dot{I}_a = -\frac{R_a}{L_a} \cdot I_a(t) - \frac{k_b}{L_a} \cdot \omega(t) + \frac{K_A}{L_a} u_c(t) \tag{2}$$

$$\dot{\omega} = \frac{k_t}{J_{tot}} I_a(t) - \frac{k_f}{J_{tot}} \omega(t) - \frac{1}{J_{tot}} T_c(t) \tag{3}$$

In matrix form, the resulting state-space equations are:

$$\begin{bmatrix} \dot{I}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{k_b}{L_a} \\ \frac{k_t}{J_{tot}} & -\frac{k_f}{J_{tot}} \end{bmatrix} \begin{bmatrix} I_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{K_A}{L_a} & 0 \\ 0 & -\frac{1}{J_{tot}} \end{bmatrix} \begin{bmatrix} u_c \\ T_c \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} I_a \\ \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ \omega \end{bmatrix} \tag{5}$$