71.

a. With the speed controller configured as a proportional controller [$K_{I_{SC}} = 0$ and $G_{SC}(s) = K_{P_{SC}}$], the open-loop transfer function is:

$$G_{SC}(s)G_v(s) = \frac{0.11 \quad K_{P_{SC}}(\ s + 0.6)}{s \quad (s + 0.5173) \quad + \quad 5 \quad (s + 0.6) \quad \times \quad (s + 0.01908)}.$$

Expanding the denominator of this transfer function, gives: $D_G(s) = 6s^2 + 3.613\ s\ +\ 0.05724.$

Solving for the roots shows that there are two open-loop poles: − 0.5858 and − 0.0163. Thus, the open-loop transfer function may be re-written as:

$$G_{SC}(s)G_v(s) = \frac{0.11 \quad K_{P_{SC}}(\ s + 0.6)}{6\ \ s^2 +\ \ 3.613\ \ s\ \ +\ \ 0.05724} = \frac{K_1(\ s + 0.6)}{(\ s + 0.5858)(\ s + 0.016\ 3)} = -1 \quad (1)$$

In this equation: $K_1 = \dfrac{K_{P_{SC}} \times 0.11}{6}$ (2)

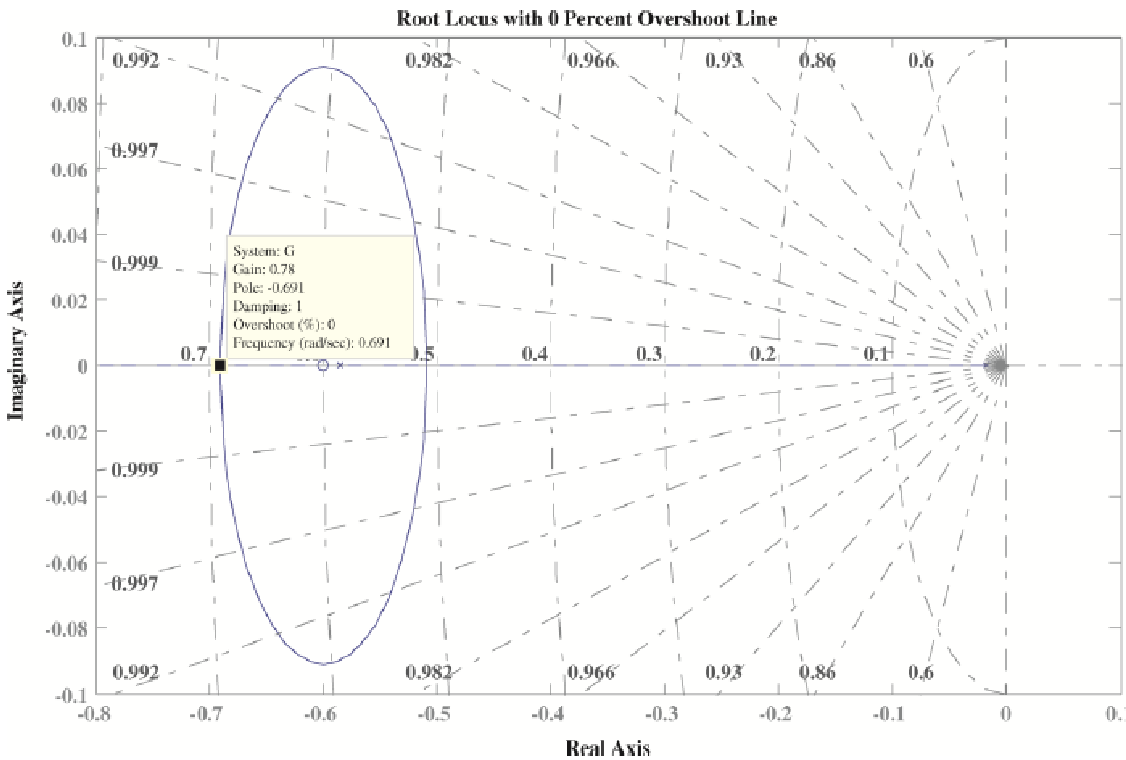
The following MATLAB M-file was written to plot the root locus for the system and to find the value of the proportional gain, K_1, at the breakaway or break-in points.

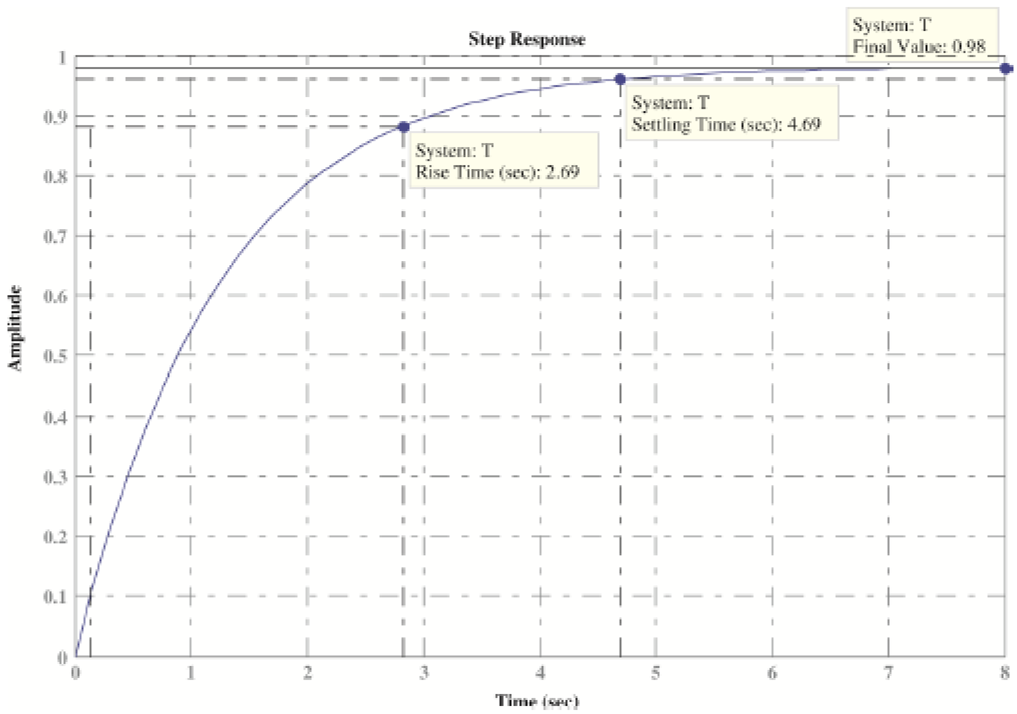
```
numg = [1 0.6];
deng = poly ([-0.0163 -0.5858]);
G = tf(numg, deng);
rlocus(G);
pos=(0);
z=-log(pos/100)/sqrt(pi^2+(log(pos/100))^2);
sgrid(z,0)
title(['Root Locus with ', num2str(pos) , ' Percent Overshoot
Line'])
[K1,p]=rlocfind(G);
pause
T=feedback(K1*G,1);        %T is the closed-loop TF of the system
T=minreal(T);
step(T);
axis ([0, 8, 0, 1]);
grid
```

The root locus shown below was obtained. Using MATLAB tools, the gain at the break-in point was found to be larger and, hence, would yield a faster closed-loop unit-step response. The following repeated real poles were found, which indicated that the step response is critically damped: p = -0.6910, - 0.6910. These poles corresponded to: $K_1 = 0.78$ (which corresponds to $K_{P_{SC}} = 42.54$). The closed-loop transfer function, T(s), was found to be:

$$T(s) = \frac{0.78\,s + 0.468}{s^2 + 1.382\,s + 0.4775}$$

Therefore, it was used to find the closed-loop transfer function of the system, to plot its unit-step response, c(t), shown below, and to find the rise-time, T_r, and settling time, T_s.





As could be seen from the graph, these times are:

$$T_r = 2.69 \text{ sec and } T_s = 4.69 \text{ sec}$$

b. When integral action was added (with $K_{I_{SC}}/K_{P_{SC}} = 0.4$), the transfer function of the speed controller became: $G_{SC}(s) = K_{P_{SC}} + \frac{K_{I_{SC}}}{s} = \frac{K_{P_{SC}}(s+0.4)}{s}$ and the open-loop transfer function obtained was:

$$G_{SC}(s)G_v(s) = \frac{0.11K_{P_{SC}}(s+0.6)(s+0.4)}{s(6s^2+3.613\ s\ +\ 0.05724)} = \frac{K_1(s+0.6)(s+0.4)}{s(s+0.5858)(s+0.016\ 3)} = -1$$

Where $K_1 = \frac{0.11K_{P_{SC}}}{6}$ or $K_{P_{SC}} = \frac{6K_1}{0.11} = 54.5455 \times K_1$

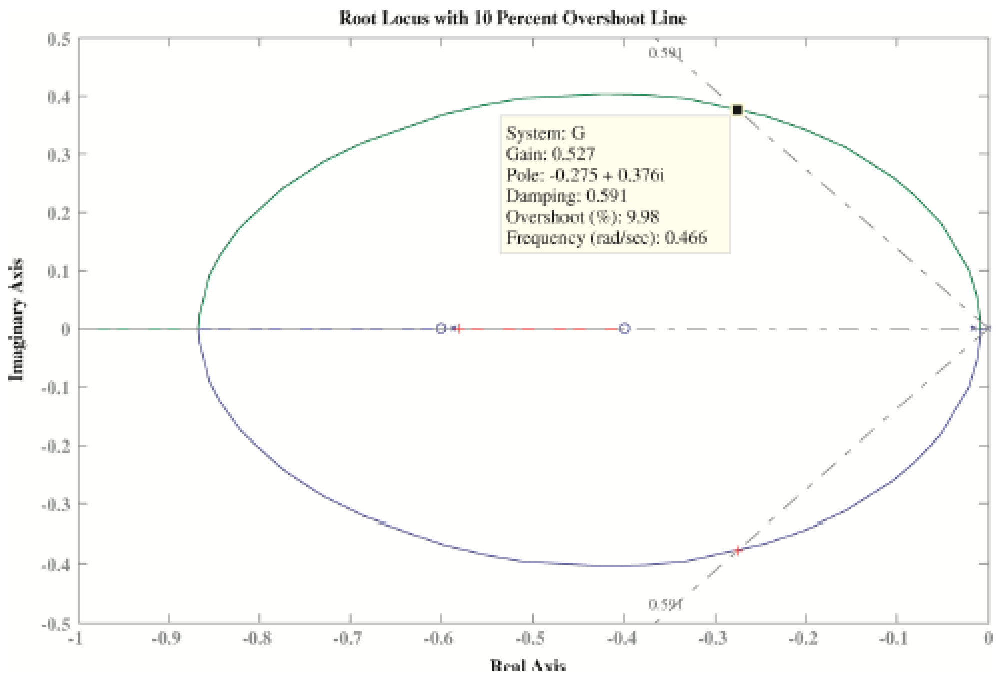
The following MATLAB M-file was written to plot the root locus for the system and to find the gain, K_1, which could result in a closed-loop unit-step response with 10% overshoot.

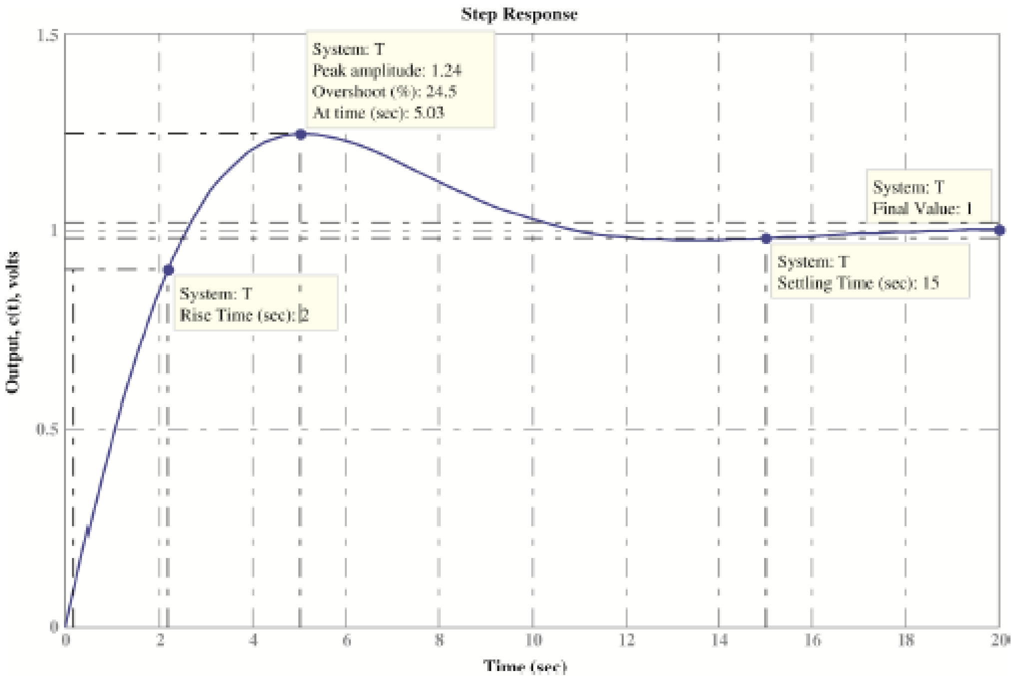
```
numg = poly ([-0.4 -0.6]);
deng = poly ([0 -0.0163 -0.5858]);
G = tf(numg, deng);
rlocus(G);
pos=(10);
z=-log(pos/100)/sqrt(pi^2+(log(pos/100))^2);
axis ([-1, 0, -0.5, 0.5]);
sgrid(z,0)
title(['Root Locus with ', num2str(pos) , ' Percent Overshoot
Line'])
[K1,p]=rlocfind(G);
pause
T=feedback(K1*G,1);        %T is the closed-loop TF of the system
T=minreal(T);
step(T);
axis ([0, 20, 0, 1.5]);
grid
```

The root locus shown below was obtained. Using MATLAB tools, the gain at the point selected on the locus (- 0.275 + j 0.376) was found to be $K_1 = 0.526$ (which corresponds to $K_{P_{SC}} = 28.7$). The corresponding closed-loop transfer function, $T(s)$, is:

$$T(s) = \frac{0.526\, s^2 \; + \; 0.526\, s \; + \; 0.1262}{s^3 \; + \; 1.128\, s^2 \; + \; 0.5355\, s + \; 0.1262}$$

T(s) has the closed-loop poles: p = – 0.580, – 0.275 ± j 0.376 and zeros at – 0.4 & – 0.6. Thus, the complex conjugate poles are not dominant, and hence, the output response, c(t), obtained using MATLAB, does not match that of a second-order underdamped system. Note also that the settling time, T_s = 15 sec, , the rise time, T_r = 2 sec, the peak time, T_p = 5.03 sec, and the overshoot is 24.5% (higher than the 10% corresponding to the dominant poles).





It should be mentioned that since we applied 1 volt-unit-step inputs (as compared to 4 volts in the Hybrid vehicle progressive problem in Chapter 5) in both parts (a) and (b) above, we should not be surprised that the final (steady-state) value of output voltage of the speed transducer was 1 volt, which corresponds to a change in car speed of only 5 km/hr.