Chapter 9 Problem 21 Solution

Given

$$G = K\frac{1}{s(s+1)(s+3)}$$

Uncompensated

Percent Overshoot is given as 4.32%. This yields a damping factor of 0.707.

Using that information, we know that

$$s = -0.707\omega_n \pm j\omega_n\sqrt{1-(0.707)^\wedge 2} = (-.707 \pm 0.707j)\omega_n$$

Plugging that into 1+KG = 0 for the uncompensated transfer function:

$$1 + K\frac{1}{s(s+1)(s+3)} = 0$$

we can determine that **K = 1.11** and ω_n = 0.589. Knowing that, the uncompensated system has a settling time of 4/(0.707*0.589) = **9.6 seconds**. The steady state error constant is Kv = 1.11/3 = 0.37.

Compensated

According to the problem statement, we want the new value of the settling time to be **2.86 seconds** while maintaining the same percent overshoot. The time constant is 2.86/4 and the reciprocal of the time constant is 1.4. Thus, s = -1.4+1.4j is a point that we can use in designing the compensated system.

We will reduce the settling time by using **lead compensator**. In addition to reducing the settling time, we are hoping to reduce the steady state error by at least a factor of two. Thus, Kv must get bigger by a factor of two or more. The compensator that we use will be:

$$Gc = Kc\frac{s+1}{s+p}$$

The choice of z = 1 is arbitrary, and was made to simplify the computation. If the result is not satisfactory, we would need to guess another z and solve again. p will need to be large enough to reduce the settling time, and Kc will need to be larger to increase the path gain Kc/p and thereby reduce the steady state error.

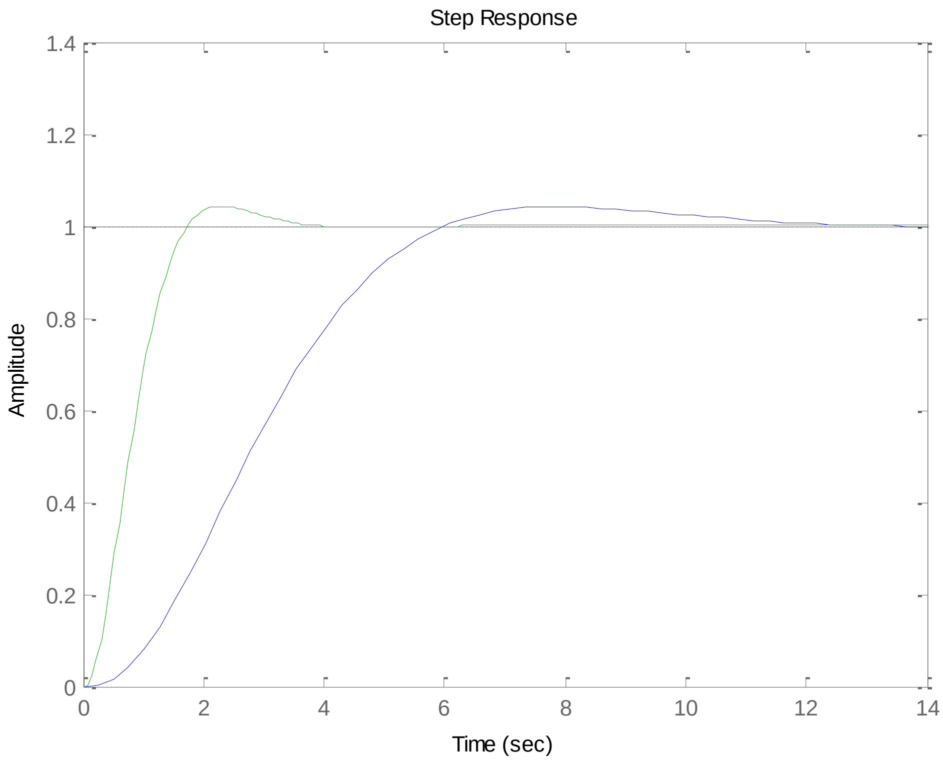
Using the known value for s in the equation

1+Gc*G = 0

There are two equations for two unknowns. We get p = 22.42 and the product Kc*K = 88.68.

The new steady state constant Kv = 88.68/(3*22.42) = 1.319, which results in a steady state error that is more than three times smaller than the uncompensated system.

Response of each system to a step input:



Both systems have a zero steady-state error to a step input and a constant error to a ramp input. The new system will has a smaller steady-state error to a ramp input.

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% Chapter 9 Problem 21
%
% Uncompensated
hold on

Ga_num = [0 0 0 1.11];
Ga_den = [1 4 3 0];
transferA = tf(Ga_num,Ga_den+Ga_num)
step(transferA)

% Compensated
Gb_num = [0 0 0 88.68];
Gb_den = conv([1 3 0],[1 22.42]);
transferB = tf(Gb_num,Gb_den+Gb_num)
step(transferB)
```