Chapter 9 Solutions

Problems 1, 6, 16, 26, 39

1.

Uncompensated system: Search along the $\zeta = 0.5$ line and find the operating point is at -1.5356 ± j2.6598 with K = 73.09. Hence, $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 = 16.3\%$; $T_s = \dfrac{4}{1.5356} = 2.6$ seconds; $K_p = \dfrac{73.09}{30} = 2.44$. A higher-order pole is located at -10.9285.

Compensated: Add a pole at the origin and a zero at -0.1 to form a PI controller. Search along the $\zeta = 0.5$ line and find the operating point is at -1.5072 ± j2.6106 with K = 72.23. Hence, the estimated performance specifications for the compensated system are: $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 = 16.3\%$; $T_s = \dfrac{4}{1.5072} = 2.65$ seconds; $K_p = \infty$. Higher-order poles are located at -0.0728 and -10.9125. The compensated system should be simulated to ensure effective pole/zero cancellation.

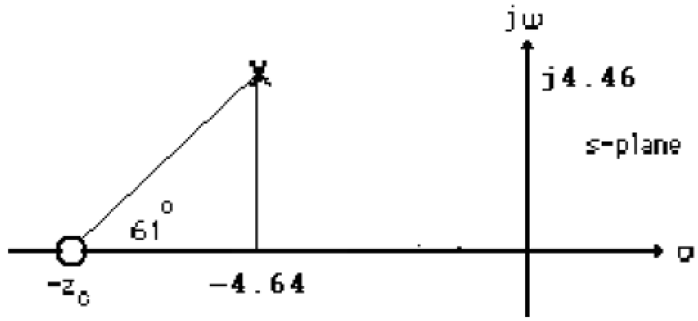
6.

Uncompensated: Searching along the 135° line ($\zeta = 0.707$), find the operating point at -2.32 + j2.32 with K = 4.6045. Hence, $K_p = \dfrac{4.6045}{30} = 0.153$; $T_s = \dfrac{4}{2.32} = 1.724$ seconds; $T_p = \dfrac{\pi}{2.32} = 1.354$ seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 = 4.33\%$;

$\omega_n = \sqrt{2.32^2 + 2.32^2} = 3.28$ rad/s; higher-order pole at -5.366.

Compensated: To reduce the settling time by a factor of 2, the closed-loop poles should be – 4.64 ± j4.64. The summation of angles to this point is 119° . Hence, the contribution of the compensating zero should be 180° -119° =61° . Using the geometry shown below,

$\dfrac{4.64}{z_c - 4.64} = \tan(61°)$. Or, $z_c = 7.21$.



After adding the compensator zero, the gain at -4.64+j4.64 is K = 4.77. Hence,

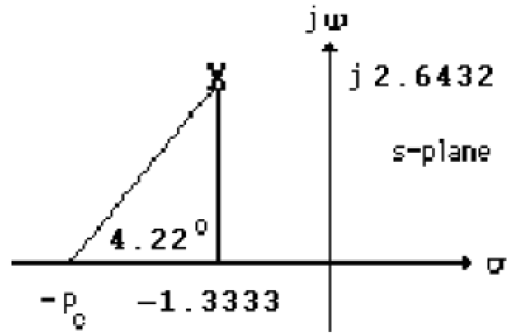
$K_p = \dfrac{4.77x6x7.21}{2x3x5} = 6.88$. $T_s = \dfrac{4}{4.64} = 0.86$ second; $T_p = \dfrac{\pi}{4.64} = 0.677$ second;

%OS = $e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ x100 = 4.33%; $\omega_n = \sqrt{4.64^2 + 4.64^2} = 6.56$ rad/s; higher-order pole at -5.49. The problem with the design is that there is steady-state error, and no effective pole/zero cancellation. The design should be simulated to be sure the transient requirements are met.

16.

a. From 20.5% overshoot evaluate $\zeta = 0.45$. Also, since $\zeta\omega_n = \dfrac{4}{T_s} = \dfrac{4}{3}$, $\omega_n = 2.963$. The compensated dominant poles are located at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ = - 1.3333 ± j2.6432. Assuming the compensator zero at -0.02, the contribution of open-loop poles and the compensator zero to the design point, - 1.3333 ± j2.6432 is -175.78°. Hence, the compensator pole must contribute 175.78° - 180° = -4.22°. Using the following geometry, $\dfrac{2.6432}{p_c - 1.3333} = \tan 4.22°$, or p_c = 37.16

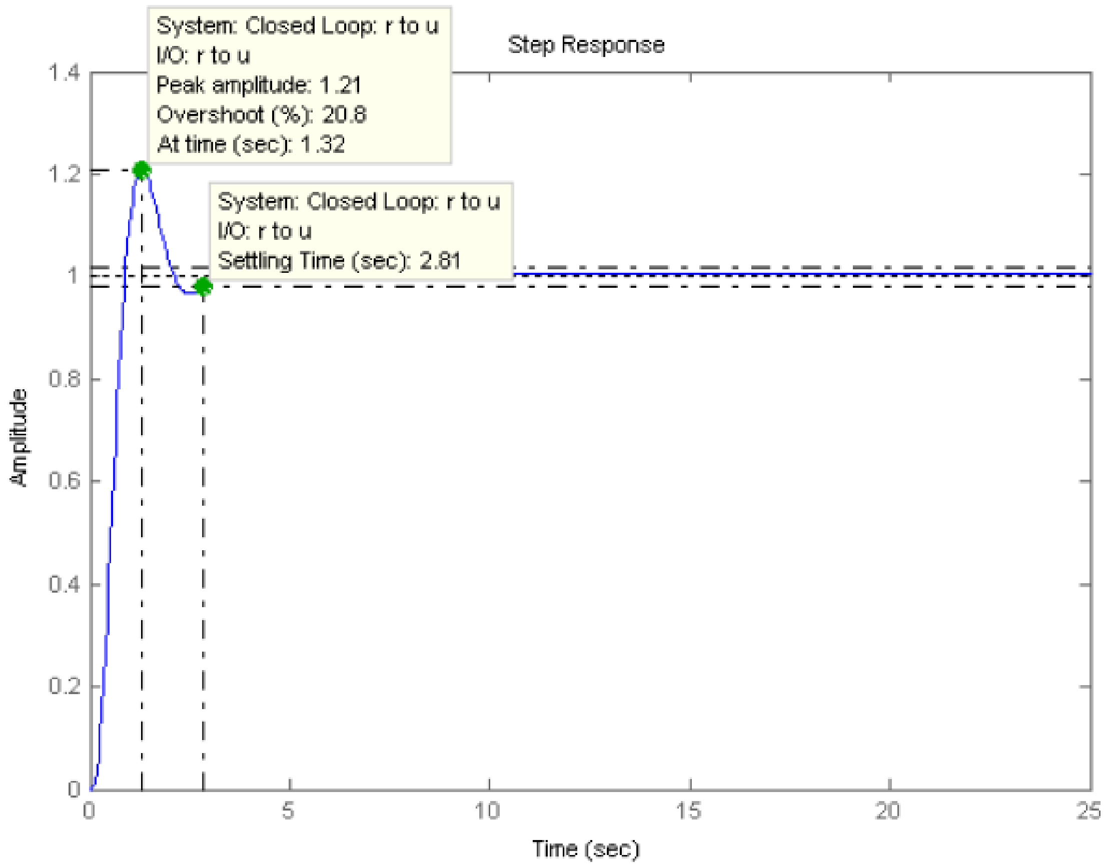
Adding the pole to the system, K = 4401.52 at the design point..



b. Searching along the real axis segments of the root locus for $K = 4401.52$, we find higher-order poles at -0.0202, -13.46, and -37.02. There is pole/zero cancellation at -0.02. Also, the poles at ,

-13.46, and -37.02 are at least 5 times the design point's real part. Thus, the second-order approximation is valid.

c.



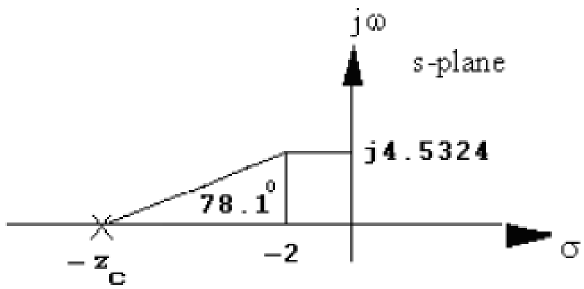
From the plot, T_S = 2.81 seconds, and %OS = 20.8%. Thus, the requirements are met.

26.

a. The desired operating point is found from the desired specifications. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{2} = 2$ and

$\omega_n = \frac{2}{\zeta} = \frac{2}{0.4037} = 4.954$. Thus, $\text{Im} = \omega_n\sqrt{1-\zeta^2} = 4.954\sqrt{1-0.4037^2} = 4.5324$. Hence the design point is –2 +j4.5324. Now, add a pole at the origin to increase system type and drive error to zero for step inputs.

Now design a PD controller. The angular contribution to the design point of the system poles and pole at the origin is 101.9^0. Thus, the compensator zero must contribute $180^0 - 101.9^0 = 78.1^0$. Using the geometry below,



$\frac{4.5324}{z_c - 2} = \tan(78.1^0)$. Hence, $z_c = 2.955$. The compensated open-loop transfer function with PD compensation is $\frac{K(s+2.955)}{s(s+4)(s+6)(s+10)}$. Adding the compensator zero to the system and evaluating the gain for this at the point –2 + j4.5324 yields K = 294.51 with a higher-order pole at -2.66 and -13.34.

PI design: Use $G_{PI}(s) = \frac{(s+0.01)}{s}$. Hence, the equivalent open-loop transfer function is

$$G_e(s) = \frac{K(s+2.955)(s+0.01)}{s^2(s+4)(s+6)(s+10)}$$ with K = 294.75.

b.

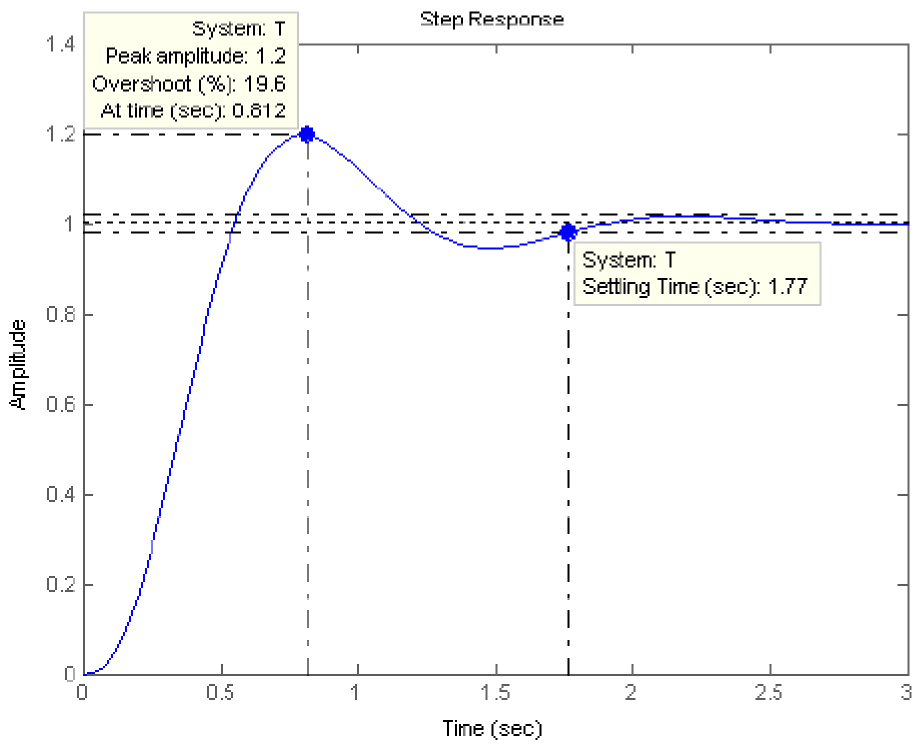
Program (Step Response):

```
numg=[-2.995 -0.01];
deng=[0  0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
step(T)
```

Computer response:

```
Zero/pole/gain:


 294.75 (s+2.995) (s+0.01)
 -------------------------
  s^2 (s+4) (s+6) (s+10)
```

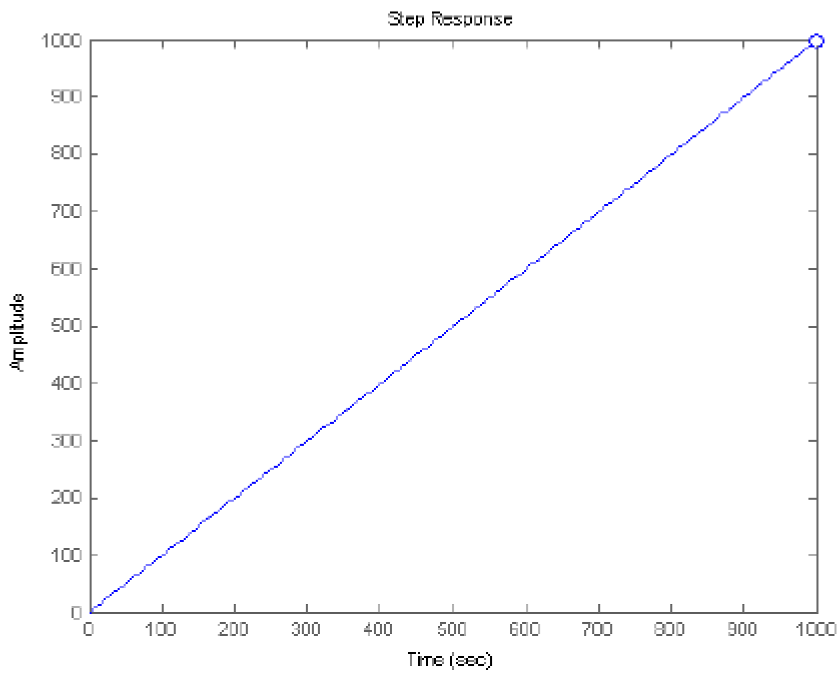


Program (Ramp Response):

```
numg=[-2.995 -0.01];
deng=[0 0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
Ta=tf([1],[1 0]);
step(T*Ta)
```

Computer response:

```
Zero/pole/gain:
294.75 (s+2.995) (s+0.01)
-------------------------
 s^2 (s+4) (s+6) (s+10)
```



39.

a. $T(s) = \frac{25}{s^2 + s + 25}$; Therefore, $\omega_n = 5$; $2\zeta\omega_n = 1$; $\zeta = 0.1$;

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 73\%$; $T_s = \frac{4}{\zeta\omega_n} = 8$ seconds.

b. From Figure P9.6(b), $T(s) = \frac{25K_1}{s^2 + (1 + 25K_f)s + 25K_1}$. Thus,

$\omega_n = \sqrt{25K_1}$; $2\zeta\omega_n = 1 + 25K_f$. For 25% overshoot, $\zeta = 0.404$. For $T_s = 0.2 = \frac{4}{\zeta\omega_n}$, $\zeta\omega_n = 20$.

Therefore $1 + 25K_f = 2\zeta\omega_n = 40$, or $K_f = 1.56$. Also, $\omega_n = \frac{20}{\zeta} = 49.5$.

Hence $K_1 = \frac{\omega_n^2}{25} = \frac{49.5^2}{25} = 98.01$.

c. Uncompensated: $G(s) = \frac{25}{s(s+1)}$; Therefore, $K_v = 25$, and $e(\infty) = \frac{1}{K_v} = 0.04$.

Compensated: $G(s) = \frac{25K_1}{s(s + 1 + 25K_f)}$; Therefore, $K_v = \frac{25 \text{ x } 98.01}{1+25 \text{ x } 1.56} = 61.26$, and

$e(\infty) = \frac{1}{K_v} = 0.0163$.