Chapter 7 Solution

5.

System is type 0. $K_p = 1.4881$.

For 20 $u(t)$, $e(\infty) = \frac{20}{1+K_p} = 8.04$

For 60 $t\,u(t)$, $e(\infty) = \infty$

For $81t^2\,u(t)$, $e(\infty) = \infty$

8.

$e(\infty) = \frac{15}{1+K_p}$; $K_p = \frac{1020(13)(26)(33)}{(65)(75)(91)} = 25.65$. Therefore, $e(\infty) = 0.563$.

15.

Collapsing the inner loop and multiplying by $1000/s$ yields the equivalent forward-path transfer function as,

$$G_e(s) = \frac{10^5(s+2)}{s(s^2+1005s+2000)}$$

Hence, the system is Type 1.

19.

$$e(\infty) = \frac{30}{K_v} = \frac{30}{Ka/30} = \frac{900}{Ka} = 0.005.$$

Hence, $Ka = 180000$.

25.

$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\frac{6K}{58}} = 0.08$. Thus, K = 111.

31.

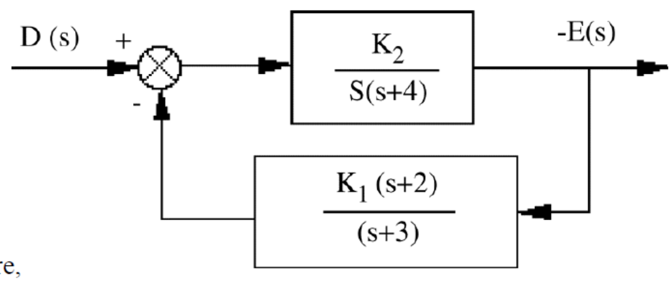
System Type = 1. $T(s) = \frac{G(s)}{1+G(s)} = \frac{K}{s^2+as+K}$. From G(s), $K_v = \frac{K}{a} = 110$. For 12% overshoot, $\zeta =$ 0.56. Therefore, $2\zeta\omega_n = a$, and $\omega_n{}^2 = K$. Hence, $a = 1.12\sqrt{K}$.

Also, $a = \frac{K}{110}$. Solving simultaneously,

$K = 1.52 \times 10^4$, and $a = 1.38 \times 10^2$.

39.

Error due only to disturbance: Rearranging the block diagram to show D(s) as the input,



Therefore,

$$-E(s) = D(s)\frac{\frac{K_2}{s(s+4)}}{1+\frac{K_1K_2(s+2)}{s(s+3)(s+4)}} = D(s)\frac{K_2(s+3)}{s(s+3)(s+4)+K_1K_2(s+2)}$$

For $D(s) = \frac{1}{s}$, $e_D(\infty) = \lim_{s\to 0} sE(s) = -\frac{3}{2K_1}$.

Error due only to input: $e_R(\infty) = \frac{1}{K_V} = \frac{1}{\frac{K_1K_2}{6}} = \frac{6}{K_1K_2}$.

Design:

$e_D(\infty) = -0.00001 = -\frac{3}{2K_1}$, or $K_1 = 150{,}000$.

$e_R(\infty) = 0.002 = \frac{6}{K_1K_2}$, or $K_2 = 0.02$

53.

From Eq. (7.70),

$$e(\infty) = 1 - \lim_{s \to 0}\left(\frac{\dfrac{K_1K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) - \lim_{s \to 0}\left(\frac{\dfrac{K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) = \frac{2-K_2}{2+K_1K_2}$$

Sensitivity to K_1:

$$S_{e:K_1} = \frac{K_1}{e}\frac{\delta e}{\delta K_1} = -\frac{K_1K_2}{2+K_1K_2} = -\frac{(100)(0.1)}{2+(100)(0.1)} = -0.833$$

Sensitivity to K_2:

$$S_{e:K_2} = \frac{K_2}{e}\frac{\delta e}{\delta K_2} = \frac{2K_2(1+K_1)}{(K_2-2)(2+K_1K_2)} = \frac{2(0.1)(1+100)}{(0.1-2)(2+(100)(0.1))} = -0.89$$

67.

a. When the speed controller is configured as a proportional controller, the forward-path transfer function of this system is:

$$G(s) = \frac{0.11\ (s+0.6)\times K_{P_{SC}}}{s\ (s+0.5173)\ +\ 5\ (s+0.6)\times(s+0.01908)} \quad (1)$$

For the steady-state error for a unit-step input, $r(t) = u(t)$, to be equal to 1%:

$$e_{step}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+\lim_{s\to 0}\left(\frac{0.11\ (s+0.6)\times K_{P_{sc}}}{s\ (s+0.5173)\ +\ 5\ (s+0.6)\times(s+0.01908)}\right)} = 0.01 \quad (2)$$

From equation (2), we get: $\frac{1}{1+\frac{0.11\times 0.6\times K_{P_{sc}}}{0+5\times 0.6\times 0.01908}} = 0.01$, which yields: $K_{P_{SC}} = 85.9$.

b. When the speed controller is configured as a proportional plus integral controller, the forward-path transfer function of the system becomes:

$$G(s) = \frac{0.11\ (s+0.6)\times(100s+K_{I_{sc}})}{s\left[s\ (s+0.5173)\ +\ 5\ (s+0.6)\times(s+0.01908)\ \right]} \quad (3)$$

For the steady-state error for a unit-ramp input, $r(t) = t\ u(t)$, to be equal to 2.5%:

$$e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} s\,G(s)} = \frac{1}{\lim_{s\to 0} s\left(\frac{0.11\ (s+0.6)\times(100s+K_{I_{sc}})}{s\left[s\ (s+0.5173)\ +\ 5\ (s+0.6)\times(s+0.01908)\right]}\right)} = 0.025 \quad (4)$$

From equation (4), we get: $\frac{1}{\frac{0.11\times 0.6\times K_{I_{sc}}}{0+5\times 0.6\times 0.01908}} = 0.025$, which yields: $K_{I_{SC}} = 34.7$.

c. We'll start by finding $G_1(s)$, the equivalent transfer function of the parallel combination, representing the torque and speed controllers, shown in Figure P7.35:

$$G_1(s) = \frac{13.53\ s}{(s+0.5)} + \frac{3\ (s+0.6)}{(s+0.5)}\left(\frac{100s+40}{s}\right) = \frac{313.53s^2+300s+72}{s(s+0.5)} \quad (5)$$

Given that the equivalent transfer function of the car is: $G_2(s) = \frac{6.13\times 10^{-3}}{s+0.01908}$, we apply equation 7.62* of the text taking into consideration that the disturbance here is a step with a magnitude equal to 83.7:

$$e(\infty) = -\frac{83.7}{\lim_{s\to 0}\frac{1}{G_2(s)}+\lim_{s\to 0} G_1(s)} = -\frac{83.7}{3.11+\infty} = 0$$