Chapter 7

5.

System is type 0. $K_p = 1.488$.

$$\text{For } 30u(t),\ e(\infty) = \frac{30}{1+K_p} = 12.1$$

$$\text{For } 70tu(t),\ e(\infty) = \infty$$

$$\text{For } 81t^2u(t),\ e(\infty) = \infty$$

8.

$e(\infty) = \dfrac{15}{1+K_p}$; $K_p = \dfrac{1020(13)(26)(33)}{(65)(75)(91)} = 25.65$. Therefore, $e(\infty) = 0.563$.

15.

Collapsing the inner loop and multiplying by $1000/s$ yields the equivalent forward-path transfer function as,

$$G_e(s) = \frac{10^5(s+2)}{s(s^2+1005s+2000)}$$

Hence, the system is Type 1.

19.

$e(\infty) = \dfrac{40}{K_v} = \dfrac{40}{Ka/26} = \dfrac{1040}{Ka} = 0.006$. Hence, $Ka = 173333.33$.

25.

$e(\infty) = \dfrac{1}{1+K_p} = \dfrac{1}{1+\dfrac{6K}{58}} = 0.08$. Thus, $K = 111$.

30.

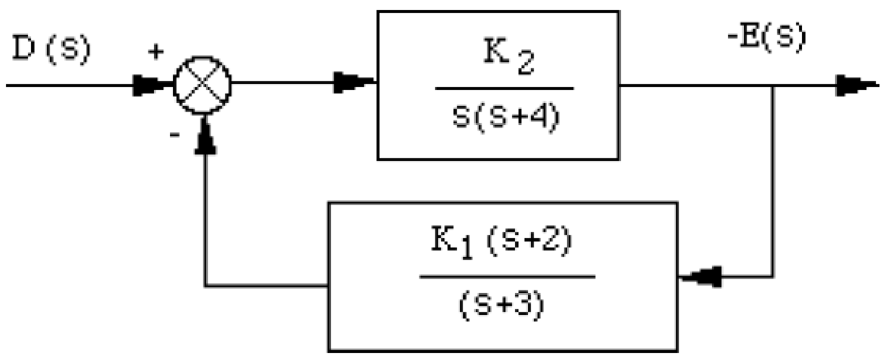
System Type = 1. $T(s) = \frac{G(s)}{1+G(s)} = \frac{K}{s^2+as+K}$. From G(s), $K_v = \frac{K}{a} = 110$. For 12% overshoot, ζ = 0.56. Therefore, $2\zeta\omega_n = a$, and $\omega_n^2 = K$. Hence, $a = 1.12\sqrt{K}$.

Also, $a = \frac{K}{110}$. Solving simultaneously,

$K = 1.52 \times 10^4$, and $a = 1.38 \times 10^2$.

39.

Error due only to disturbance: Rearranging the block diagram to show D(s) as the input,



Therefore,

$$-E(s) = D(s)\frac{\dfrac{K_2}{s(s+4)}}{1+\dfrac{K_1K_2(s+2)}{s(s+3)(s+4)}} = D(s)\frac{K_2(s+3)}{s(s+3)(s+4)+K_1K_2(s+2)}$$

For $D(s) = \frac{1}{s}$, $e_D(\infty) = \lim_{s\to 0} sE(s) = -\frac{3}{2K_1}$.

Error due only to input: $e_R(\infty) = \frac{1}{K_v} = \frac{1}{\frac{K_1K_2}{6}} = \frac{6}{K_1K_2}$.

Design:

$e_D(\infty) = -0.000012 = -\frac{3}{2K_1}$, or $K_1 = 125{,}000$.

$e_R(\infty) = 0.003 = \frac{6}{K_1K_2}$, or $K_2 = 0.016$

53.

From Eq. (7.70),

$$e(\infty) = 1-\lim_{s\to 0}\left(\frac{\dfrac{K_1K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) - \lim_{s\to 0} = \left(\frac{\dfrac{K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) = \frac{2\text{-}K_2}{2+K_1K_2}$$

Sensitivity to K_1:

$$S_{e:K_1} = \frac{K_1}{e}\frac{\delta e}{\delta K_1} = -\frac{K_1K_2}{2+K_1K_2} = -\frac{(100)(0.1)}{2+(100)(0.1)} = -0.833$$

Sensitivity to K_2:

$$S_{e:K_2} = \frac{K_2}{e}\frac{\delta e}{\delta K_2} = \frac{2K_2(1+K_1)}{(K_2-2)(2+K_1K_2)} = \frac{2(0.1)(1+100)}{(0.1-2)(2+(100)(0.1))} = -0.89$$

58.

a. For the inner loop, $G_{1e}(s) = \dfrac{K\dfrac{(s+0.01)}{s^2}}{1+K\dfrac{(s+0.01)}{s^2}} = \dfrac{K\,(s+0.01)}{s^2+Ks+0.01K}$, where $K = \dfrac{K_c}{J}$.

Form $G_e(s) = G_{1e}(s)\dfrac{(s+0.01)}{s^2} = K\dfrac{(s+0.01)^2}{s^2(s^2+Ks+0.01K)}$.

System is Type 2. Therefore, $e_{step} = 0$,

b. $e_{ramp} = 0$,

c. $e_{parabola} = \dfrac{1}{K_a} = \dfrac{1}{0.01} = 100$

d. $T(s) = \dfrac{G_e(s)}{1+G_e(s)} = \dfrac{K(s+0.01)^2}{s^4+Ks^3+1.01Ks^2+0.02Ks+10^{-4}K}$

s^4	1	$1.01K$	$10^{-4}K$	
s^3	K	$0.02K$	0	$0 < K$
s^2	$1.01K-0.02$	$10^{-4}K$	0	$0.0198 < K$
s^1	$\dfrac{0.0201\,K^2-0.0004\,K}{1.01\,K-0.02}$	0	0	$0.0199 < K$
s^0	$10^{-4}K$			$0 < K$

Thus, for stability $K = \dfrac{K_c}{J} > 0.0199$