Chapter 10 Homework Solutions Fall 2016

1.

a.

$$G(s)=\frac{1}{s(s+2)(s+4)};\ G(j\omega)=\frac{1}{-6\omega^2+i(8\omega-\omega^3)}$$

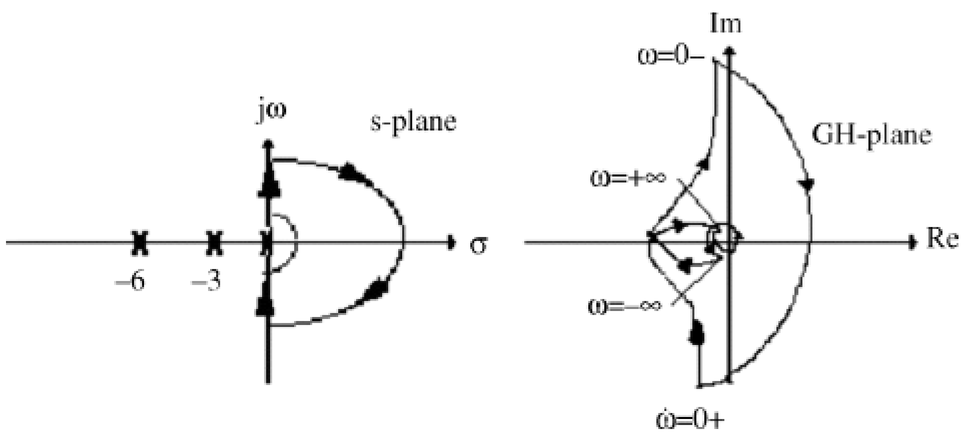
$$M(\omega)=\frac{1}{\sqrt{(8\omega-\omega^3)^2+(6\omega^2)^2}};\ \Phi(\omega)=-\left(\pi+\arctan\left[\frac{8-\omega^2}{-6\omega}\right]\right)$$

b.

$$G(s)=\frac{(s+5)}{(s+2)(s+4)};\ G(j\omega)=\frac{(\omega^2+40)-i(\omega^2+22)\omega}{\omega^4+20\omega^2+64}$$

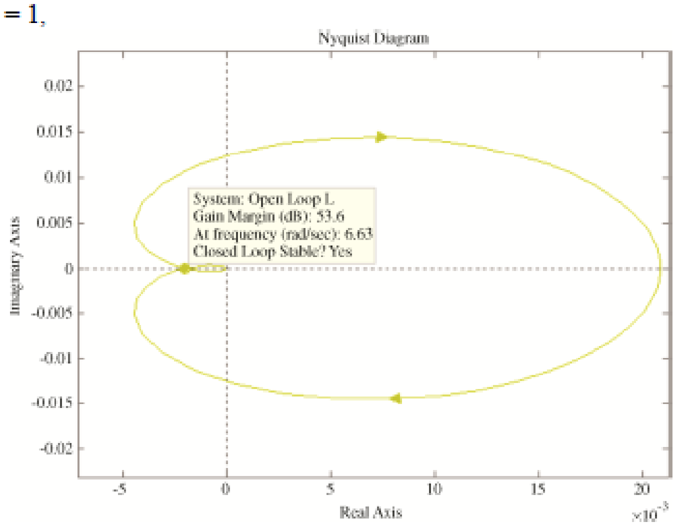
$$M(\omega)=\frac{\sqrt{(\omega^2+40)^2+\omega^2(\omega^2+22)^2}}{\omega^4+20\omega^2+64};\ \Phi(\omega)=\arctan\left(\frac{-[\omega^2+22]\omega}{\omega^2+40}\right)$$

5.



10.

System 1: For K = 1,

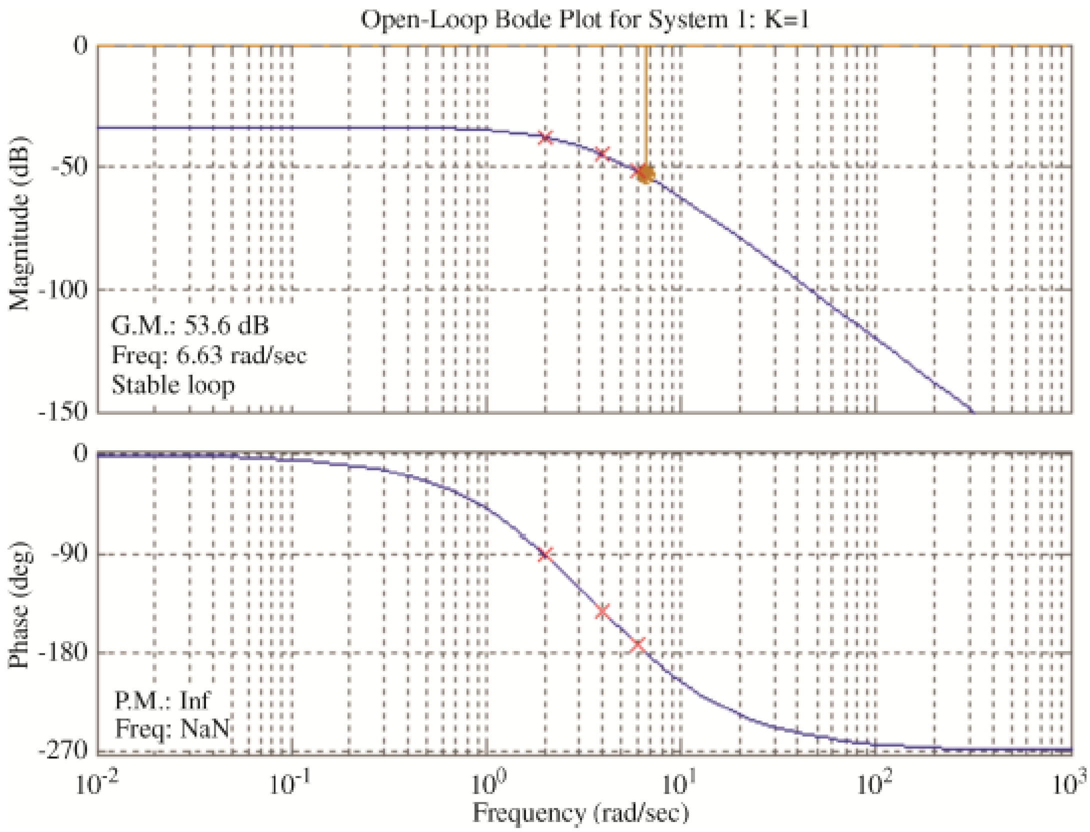


The Nyquist diagram intersects the real axis at -0.0021. Thus K can be increased to 478.63 before there are encirclements of -1. There are no poles encircles by the contour. Thus P = 0. Hence, Z = P - N, Z = 0 + 0 if K <478.63; Z = 0 –(-2) if K > 478.63. Therefore stability if 0 < K < 478.63.

11.

Note: All results for this problem are based upon a non-asymptotic frequency response.

System 1: Plotting Bode plots for K = 1 yields the following Bode plot,



K = 1000:

For K = 1, phase response is 180° at ω = 6.63 rad/s. Magnitude response is -53.6 dB at this frequency.

For K = 1000, magnitude curve is raised by 60 dB yielding + 6.4 dB at 6.63 rad/s. Thus, the gain margin is

- 6.4 dB.

Phase margin: Raising the magnitude curve by 60 dB yields 0 dB at 9.07 rad/s, where the phase curve is 200.3°. Hence, the phase margin is 180°-200.3° = - 20.3°.

K = 100:

For K = 1, phase response is 180° at ω = 6.63 rad/s. Magnitude response is -53.6 dB at this frequency. For K = 100, magnitude curve is raised by 40 dB yielding – 13.6 dB at 6.63 rad/s. Thus, the gain margin is 13.6 dB.

Phase margin: Raising the magnitude curve by 40 dB yields 0 dB at 2.54 rad/s, where the phase curve is 107.3°. Hence, the phase margin is 180°-107.3° = 72.7°.

K = 0.1:

For K = 1, phase response is 180° at ω = 6.63 rad/s. Magnitude response is -53.6 dB at this frequency. For K = 0.1, magnitude curve is lowered by 20 dB yielding – 73.6 dB at 6.63 rad/s. Thus, the gain margin is 73.6 dB..

23.

System 1: Using non-asymptotic frequency response plots, the zero dB crossing is at 9.7 rad/s at a phase of –163.2°. Therefore the phase margin is 180° – 163.2° = 16.8°. $|G(j\omega)|$ is down 7 dB at 14.75 rad/s. Therefore the bandwidth is 14.75 rad/s. Using Eq. (10.73), ζ = 0.15. Using Eq. (4.38), %OS = 62.09%. Eq. (10.55) yields T_s = 2.76 s, and Eq. (10.56) yields T_p = 0.329 s.

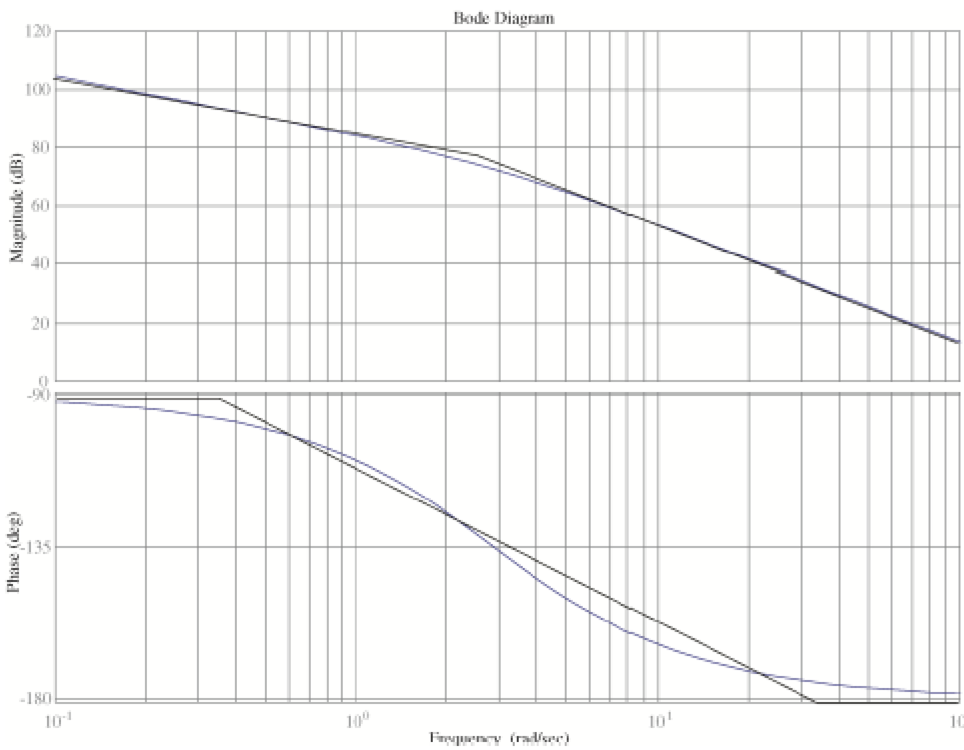
27.

The phase margin of the given system is 20°. Using Eq. (10.73), ζ = 0.176. Eq. (4.38) yields 57% overshoot. The system is Type 1 since the initial slope is - 20 dB/dec. Continuing the initial slope down to the 0 dB line yields K_v = 4. Thus, steady-state error for a unit step input is zero; steady state error for a unit ramp input is $\frac{1}{K_v}$ = 0.25; steady-state error for a parabolic input is infinite.

36.

The exact Bode plot and the asymptotic approximations are shown on the following figure. The magnitude asymptotes are obtained by noting that when $\omega \to 0$, $P(s) \approx \frac{16782}{s}$. So when $\omega = 0.1$, $|P(j0.1)| = 167820 = 104.5db$ and the slope of the line is -6db/oct. This means that $|P(j0.2)| = 104.5db - 6db = 98.5db$. So a line is drawn between these two points until $\omega = 2.89$ is reached. For higher frequencies the slope is -12db/oct, so the line is continued.

To plot the phase asymptote at very low frequencies the phase is -180° due to the integrator until 2.89/10=0.289rad/sec. At very high frequencies from 2.89*10=28.9 rad/sec and up the phase will be -270° due to the plant's pole contribution. A line is drawn between 0.289 and 28.9 rad/sec with -135° at 2.89 rad/sec.



Problem 40

The Bode plot is shown below. The phase response is 180° at $\omega = 3.87$ rad/s, where the gain is – 6.59 dB. Thus, the gain margin is 6.59 dB. Unity gain is at $\omega = 2.52$ rad/s, where the phase is -76° and at $\omega = 3.12$ rad/s, where the phase is -133.1°. Hence the phase margin is measured at $\omega = 3.12$ rad/s and is $180^\circ - 133.1^\circ = 46.9^\circ$.

The margin data displayed in the command window after the M-file was run was virtually identical:

Gm(dB)	Pm(deg.)	180 deg. freq.(r/s)	0 dB freq. (r/s)
6.589 dB	46.87°	3.874 r/s	3.121 r/s