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# Tuning PI Controllers for Integrator/Dead Time Processes

Chien and Fruehauf proposed an internal model control (IMC) approach to selecting the tuning constants for a PI controller in a process consisting of a pure integrator and a dead time. The only tuning parameter to be specified was the closed loop time constant. This paper points out that the IMC approach can lead to poor control unless care is taken in selecting the closed loop time constant. An alternative approach that avoids this problem is suggested. The proposed method uses classical frequency response methods. We show that there is a minimum reset time for the integrator/dead time process below which reasonable closed loop damping coefficients cannot be achieved. We also show that, for a given reset time, there is an optimum controller gain that minimizes the maximum closed loop log modulus. Equations for the calculation of the optimum reset time and the optimum gain of a PI controller for the control of an integrator/dead time process are developed.



## Introduction

During the course of some recent plantwide control studies, we were trying to tune a composition control loop in a fairly high-purity distillation column. These columns exhibit very large time constants for small changes around the set point (Fuentes and Luyben, 1983), and the time constants are so large that the response of the process looks almost like the response of a pure integrator. Therefore, Chien and Fruehauf (1990) suggested that an integrator/dead time model could be used for controller tuning. They proposed an internal model control (IMC) approach to finding the settings for a PI controller. The original discussion of PI tuning parameters based on IMC was presented by Rivera et al. (1986).

The process transfer function is assumed to be

$$G_{M(s)} = K_p e^{-Ds}/s \tag{1}$$

where $K_p$ is the slope of the ramp change in the controlled variable for a step change in the manipulated variable (percent/minute) and $D$ is the dead time (typically from a composition analyzer).

In the Chien and Fruehauf procedure, a closed loop time constant $\tau_{CL}$ is specified. Then the reset time $\tau_I$ and the gain $K_c$ of a PI controller are calculated from the following equations:

$$K_c = \frac{2\tau_{CL} + D}{(\tau_{CL} + D)^2} \tag{2}$$

$$\tau_I = 2\tau_{CL} + D \tag{3}$$

For example, suppose the process parameters are $K_p$ = 0.0506%/min and $D$ = 6 min. If a closed loop time constant of 16 min is specified, the PI tuning parameters are $K_c$ = 1.55 and $\tau_I$ = 38 min. If a closed loop time constant of 6 min is specified, the PI tuning parameters are $K_c$ = 2.47 and $\tau_I$ = 18 min.

However, if we use classical frequency response methods to calculate the maximum closed loop log modulus $L_{CL}^{max}$ (Luyben, 1990) for these two sets of tuning constants, the following results are obtained.

$$\tau_{CL} = 16 \text{ min}, \quad L_{CL}^{max} = +2.9 \text{ dB}$$

$$\tau_{CL} = 6 \text{ min}, \quad L_{CL}^{max} = +9.0 \text{ dB}$$

The system with the closed loop time constant of 6 min would be very oscillatory. We will use this numerical example throughout this paper.

During our simulation studies, we first found the ultimate gain $K_u$ and ultimate frequency $\omega_u$ from the relay-feedback method (ATV in Luyben, 1990). Then we attempted to use the Ziegler–Nichols settings. For the numerical example consider previously, $\omega_u$ = 0.25 rad/min and $K_u$ = 6.1, giving Ziegler–Nicholas setting $\tau_{ZN}$ = 21 min and $K_{ZN}$ = 2.8. The resulting response was found to be too oscillatory. This time-domain response was confirmed by calculating the maximum closed loop log modulus with this setting: +9.4 dB.

Next we tried the IMC approach of Chien and Fruehauf. When a closed loop time constant of 16 min was specified, the response was good. When a closed loop time constant of 6 min was specified, the response was very oscillatory as expected because these settings are about equivalent to the Ziegler–Nichols settings.

This example illustrates that the IMC approach requires some trial and error in order to specify a closed loop time constant that will give a reasonable closed loop damping coefficient. The purpose of this paper is to propose a method that yields the best settings attainable for a specified degree of closed loop damping. The basic approach is to specify a desired maximum closed loop log modulus and then to calculate the smallest reset time for which this maximum closed loop log modulus is achieveable when the optimum controller gain is used. The resulting controller settings will give the smallest closed loop time constant (or the largest closed loop resonant frequency) that is possible.

## Properties of an Integrator/Dead Time Process with PI Control

Figure 1a gives a Nyquist plot of the process described by eq 1. Note that it starts (when $\omega$ = 0) at −90° because of the integrator. Also shown in Figure 1 are two plots of $G_{M(i\omega)}B_{(i\omega)}$ for a PI controller with two different values of reset (20 and 52.5 min). Now the plots start at −180° because of the double integrator. If the reset time is chosen to be too small, the $G_M B$ curve will be very close to or may even encircle the (−1,0) point no matter what value of gain is used. Therefore there is a minimum reset time below which stability and reasonable damping cannot be achieved.

The damping of a closed loop system is directly related to the minimum distance between the $G_M B$ curve and the (−1,0) point. The parameter that indicates this minimum approach is the maximum closed loop log modulus, i.e., the peak in the log modulus plot of the closed loop servo transfer function.

$$L_{CL} = 20 \log \left| \frac{G_M B}{1 + G_M B} \right| \tag{4}$$

It is important to also realize that even for reset times greater than the minimum there is a nonmonotonic relationship between the maximum closed loop log modulus and the controller gain $K_c$. This is illustrated in Figure 2. When the gain is small, the low-frequency portion of the curve is close to the (−1,0) point. This gives a large resonant peak at low frequency. When the gain is large, the high-frequency portion of the curve is close to the (−1,0) point. This gives a large resonant peak at high frequency.

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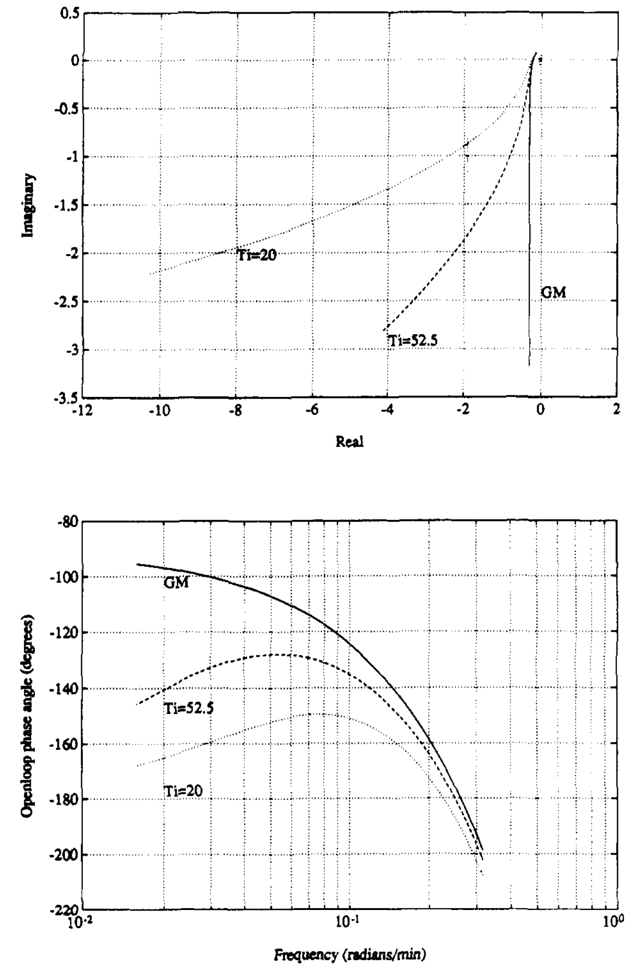


**Figure 1.** Nyquist (top) and open loop Bode (bottom) phase angle plots for open loop integrator/dead time process with PI control.

Thus there is an optimum controller gain that gives the smallest peak in the curve.

Another way to look at this system is to use the phase angle plot from an open loop Bode plot. The effect of changing reset time is shown in Figure 1b. The smaller the $\tau_I$, the lower the peak in the phase angle curve.

One way to find the minimum reset time would be to specify what the maximum peak in the phase angle plot should be and then calculate the corresponding $\tau_I^{min}$. This can be done analytically for this simple process.

$$G_M B = \frac{K_p e^{-Ds}}{s} \frac{K_c(\tau_I s + 1)}{\tau_I s} \quad (5)$$

$$\arg(G_M B) = -\pi - \omega D + \arctan(\omega \tau_I) \quad (6)$$

The peak in the phase angle curve will occur where the derivative with respect to frequency is zero.

$$\frac{d}{d\omega}[\arg(G_M B)] = -D + \frac{\tau_I}{1 + \omega_p^2 \tau_I^2} = 0 \quad (7)$$

where $\omega_p$ = frequency where the peak in the phase angle curve occurs. Solving eq 7 for $\omega_p$ gives

$$\omega_p = \frac{1}{\tau_I}\left(\frac{\tau_I - D}{D}\right)^{1/2} \quad (8)$$

Substituting back into eq 6 gives

$$[\arg(G_M B)]_{@\,peak} = -\pi - \frac{D}{\tau_I}\left(\frac{\tau_I - D}{D}\right)^{1/2} + \arctan\left(\frac{\tau_I - D}{D}\right)^{1/2} \quad (9)$$

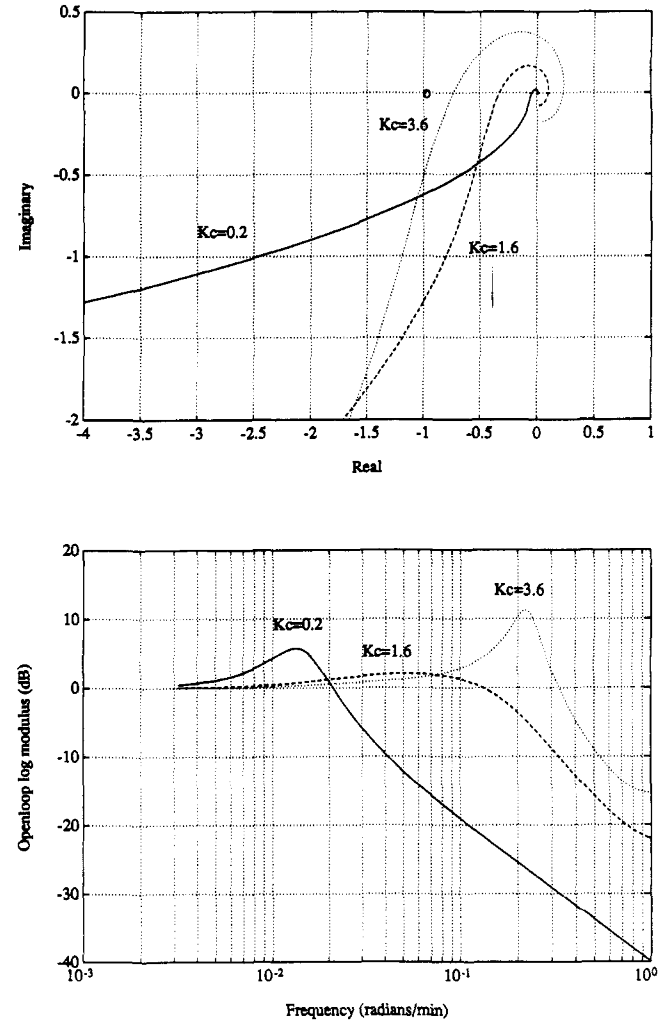


**Figure 2.** Nyquist (top) and closed loop Bode (bottom) log modulus plots for integrater/dead time process with PI control with different controller gains for $\tau_I$ = 52.5 min.

Note that this equation can be rearranged so that the only variable is the $\tau_I/D$ ratio.

$$[\arg(G_M B)]_{@\,peak} = -\pi - \frac{D}{\tau_I}\left(\frac{\tau_I}{D} - 1\right)^{1/2} + \arctan\left(\frac{\tau_I}{D} - 1\right)^{1/2} \quad (10)$$

Now the question is what maximum phase angle do we need to get a closed loop system that displays a reasonable amount of damping. A look at a Nichols chart shows that to have a $G_M B$ curve that is tangent to the +2-dB log modulus curve the open loop phase angle must rise up to at least −128°. Therefore we specify that the peak phase angle should be 2.23 rad. Solving for the required $\tau_I/D$ ratio gives

$$\tau_I/D = 8.75 \quad (11)$$

Thus we can calculate the optimum reset time once the dead time is known.

Now we want to find the value of the controller gain $K_c$ that gives the smallest value of the maximum closed loop log modulus. The maximum closed loop log modulus should be +2 dB at this gain if the design procedure has been followed correctly. It is easy to vary $K_c$ over a range of values up to near the ultimate gain and pick off the peak in the closed loop log modulus curve for each value of gain. By looking at several numerical cases we found that this optimum can be simply expressed as a function of the dead time $D$ and the process gain $K_p$.

$$K_c = 0.487/(K_pD) \quad (12)$$

This unique relationship occurs because we are shifting both the phase angle curves and the log modulus curves together along the frequency axis as we vary $D$. This relationship could probably be derived analytically if one could fight through the complex algebra that gives the peak in the closed loop log modulus plot.

## Proposed Controller Design Procedure

Now that we understand the system, we can summarize the design procedure.

(1) Determine $K_p$ and $D$. This can be done in several ways. If a step test is used, the dead time $D$ can be determined and $K_p$ can be calculated from the slope of the ramp output curve. If the relay-feedback method is used, both $D$ and $K_p$ can be calculated from the ultimate gain $K_u$ and ultimate frequency $\omega_u$.

$$[\arg (G_M)]_{\omega_u} = -\pi = -\omega_u D - \pi/2 \quad (13)$$

$$|G_M|_{\omega_u} = 1/K_u = K_p/\omega_u \quad (14)$$

Knowing the ultimate gain and ultimate frequency, eq 13 can be solved for $D$ and eq 14 can be solved for $K_p$.

(2) Knowing $D$, use eq 11 to find the minimum reset time $\tau_I$. For the numerical example considered earlier in this paper with $K_p = 0.0506\%/\text{min}$ and $D = 6$ min, the minimum reset is $\tau_I = 52.5$ min.

(3) Calculate the optimum controller gain from eq 12. For the numerical example, a peak of +2.06 dB occurs when $K_c = 1.6$. The peak is higher for any other value of controller gain, as illustrated in Figure 2b.

An alternative way to present the proposed tuning rule is to express the recommended gain and reset time in terms of the ultimate gain and ultimate frequency which can be coveniently obtained from a relay-feedback test. For the integrator/dead time process, we can solve eq 13 for $D$ in terms of $\omega_u$, express $\omega_u$ in terms of $P_u$, and substitute into eq 11 to obtain

$$\tau_I = 2.2P_u \quad (15)$$

This can be compared with the Ziegler-Nichols recommended setting: $\tau_I = P_u/1.2$.

We also can combine eqs 12-14 to get an expression for the controller gain.

$$K_c = K_u/3.22 \quad (16)$$

This can be compared with the Ziegler-Nichols recommended setting: $K_c = K_u/2.2$. Equations 15 and 16 are a convenient way to remember the proposed tuning rules.

Note that the frequency at which the peak in the closed loop log modulus occurs is the resonant frequency, and the closed loop time constant is the reciprocal of the resonant frequency. For the numerical example, the resonant frequency is 0.052 rad/min, giving a closed loop time constant of 19.2 min. This is the smallest closed loop time constant that can be attained in this system (for given $D$ and $K_p$) with a +2-dB maximum closed loop log modulus.

Our numerical examples also showed that this resonant frequency (and therefore the minimum closed loop time constant) can also be expressed as an explicit function of dead time $D$.

$$\omega_r = 1/(D\sqrt{10}) \quad (17)$$

$$\tau_{CL} = D\sqrt{10} \quad (18)$$

To provide a second numerical example, the process studied by Chien and Fruehauf (their example 1) is considered. Parameter values are $K_p = 0.2\%/\text{min}$ and $D = 7.4$ min. They chose a closed loop time constant of 8 min and, using the IMC procedure, found controller settings $K_c = 0.493\%/\%$ and $\tau_I = 23.4$ min. Note that these settings give a maximum closed loop log modulus of +8.2 dB, which implies that the closed loop process should be fairly oscillatory, although Figure 8 in their paper does not seem to show this.

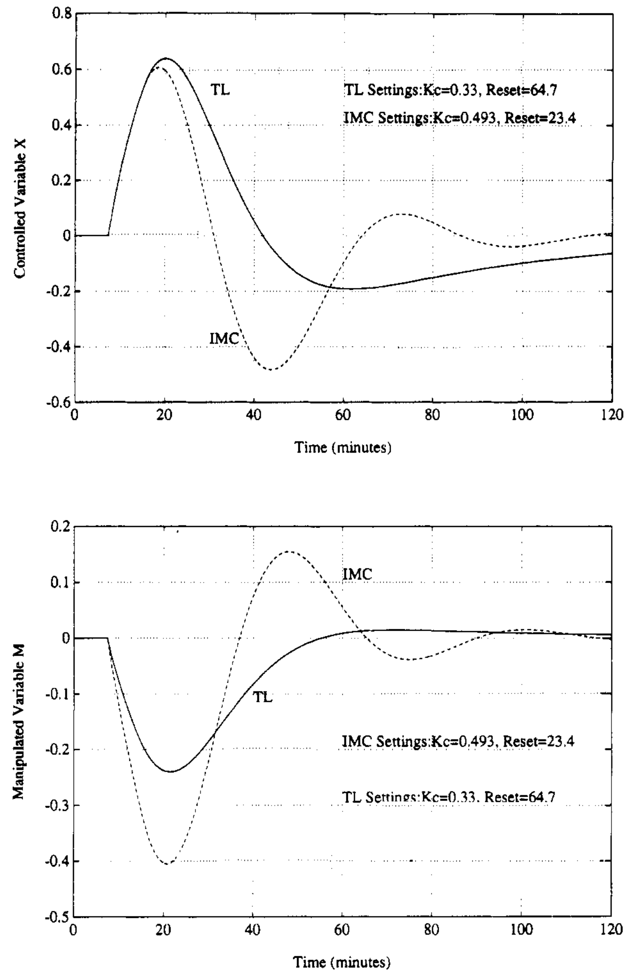


**Figure 3.** Load response of integrator/dead time process using IMC and TL settings.

The procedure proposed in this paper would give the following controller settings:

$$\tau_I = (8.75)(7.4) = 64.7 \text{ min}$$

$$K_c = \frac{0.487}{(7.4)(0.2)} = 0.33\%/\%$$

The resonance frequency is 0.0417, and the closed loop time constant is 24 min. Thus our procedure gives more conservative controller settings.

To illustrate the difference between the two settings, Figure 3 gives the results of a simulation of this dead time/integrator process. The disturbance is a unit step change in a load variable that enters the process through a 7.4-min dead time and a 10-min first-order lag. The solid curves show how the controlled and manipulated variables change using the settings recommended in this paper ("TL" settings). The dashed curves give results using the IMC procedure with the 8-min closed loop time constant.

It is clear that the TL settings are more conservative. The maximum positive deviation of the controlled variable is somewhat larger, but the overshoot is much less (the magnitude of the negative deviation is less than half that using the IMC settings). The variation in the manipulative variable is also much less. Note that the final steady-state value of the manipulative variable is zero, even with a unit step change in load. This is because the process contains an integrator.

Note that if less conservative settings are desired, the specified peak in the phase angle curve can be lowered in eq 10. The result will be a more underdamped closed loop system.

## Conclusion

A design procedure is proposed that permits the calculation of the tightest controller settings when a PI controller is applied to an integrator/dead time process. The basic criterion is a maximum closed loop log modulus of +2 dB. This is essentially equivalent to specifying a closed loop damping coefficient of about 0.4 and then finding the best settings that will minimize the closed loop time constant (maximize the closed loop resonant frequency).

Unlike the IMC approach in which a closed loop time constant must be assumed and then the results tested, the proposed procedure involves no trial and error. These settings have been tested on a wide variety of processes and have worked much better than the classical Ziegler–Nichols settings.

## Nomenclature

$B$ = feedback controller transfer function
$D$ = dead time (min)
$G_M$ = process open loop transfer function
$K_c$ = controller gain (%/%)
$K_p$ = process gain (%/min)
$L_{CL}$ = closed loop log modulus (dB)
$L_{CL}^{max}$ = maximum closed loop log modulus (dB)

*Greek Letters*

$\tau_{CL}$ = closed loop time constant (min)
$\tau_I$ = reset time (min)
$\omega$ = frequency (rad/min)
$\omega_p$ = frequency at which phase angle peak occurs (rad/min)

## Literature Cited

Chien, I. L.; Fruehauf, P. S. Consider IMC Tuning to Improve Performance. *Chem. Eng. Prog.* **1990** (Oct.), 33–41.
Fuentes, C.; Luyben, W. L. Control of High Purity Distillation Columns. *Ind. Eng. Chem. Process Des. Dev.* **1983**, *22*, 361–366.
Luyben, W. L. *Process Modeling, Simulation and Control for Chemical Engineers*, 2nd ed.; McGraw-Hill: New York, 1990.
Rivera, D. E.; Morari, M.; Skogestad, S. Internal Model Control. 4. PID Controller Design. *Ind. Eng. Chem. Process Des. Dev.* **1986**, *25*, 252–265.

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# Solubility Characteristics of Tetrabromobisphenol-A Polycarbonate in Various Liquids

As a guide for preparing permselective membranes useful for gas separations, the relative solubility of the polymer tetrabromobisphenol-A polycarbonate (TBBA-PC) was observed at temperatures up to 100 °C in numerous organic liquids. Forty-five compounds were identified that dissolved at least 50 wt % polymer. Some cases of solution instability and limited solubility were observed. The reason for such behaviors is not known, but it is tentatively attributed to the formation of a weak "structure" through polymer–liquid interaction such as solvate formation and/or solvent-induced crystallization. The correlation of solvent ability with Hildebrand total solubility parameters was poor; the correlation with three-component Hansen parameters was much better. On the basis of observations with 33 solvents, the solubility parameter of TBBA-PC was estimated to be 20.8 $MPa^{1/2}$ (10.2 $(cal/cm^3)^{1/2}$).

## Introduction

The polymer poly[oxycarbonyloxy(2,6-dibromo-1,4-phenylene)(1-methylethylidene)(3,5-dibromo-1,4-phenylene)], commonly known as tetrabromobisphenol-A polycarbonate or TBBA-PC (Chemical Abstracts Service Registry No. 28774-93-8), is a promising candidate for permselective membrane compositions especially for the separation of gas mixtures, e.g., oxygen and nitrogen (Sanders et al., 1988, 1990a,b; Beck et al., 1990; Muruganandam et al., 1987). The polymer has a glass temperature in excess of about 260 °C (Muruganandam et al., 1987), and at the temperatures necessary for melt spinning of homogeneous membranes, significant thermal-induced degradation may result. However it is known that both homogeneous and asymmetric membranes can be readily prepared from many polymers, such as cellulose triacetate, polysulfone, and polyether sulfone, at significantly lower temperatures from suitable solutions in solvents or, optionally, in solvent–nonsolvent mixtures (Kesting, 1985). This has been found to be true also for TBBA-PC (Sanders et al., 1988, 1990a,b; Beck et al., 1990). Proper membrane formation from polymer solutions requires knowledge of the relative solubility of the particular polymer in various liquids. Until recently such information has been lacking for TBBA-PC. This paper will describe the solubility characteristics of TBBA-PC in over 100 organic liquids.

## Experimental Section

The polymer used was prepared from tetrabromobisphenol-A (Chemical Abstracts Service Registry No. 79-94-7) and phosgene in methylene chloride in the presence of pyridine. The polymer was end-blocked with *p-tert*-butylphenol and was precipitated in excess heptane. The resulting dried polymer was a low-density–high-bulk "fluff" with a fibrous consistency and a relatively high surface area. The inherent viscosity in methylene chloride at 25 °C was 0.412 dL/g. The liquids examined as potential solvents were from several sources, mainly from Aldrich

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