Chapter 2 Solutions 7th ed, Nise

#7b

```
syms s
'a'
G=(s^2+3*s+10)*(s+5)/[(s+3)*(s+4)*(s^2+2*s+100)];
pretty(G)
g=ilaplace(G);
pretty(g)
'b'
G=(s^3+4*s^2+2*s+6)/[(s+8)*(s^2+8*s+3)*(s^2+5*s+7)];
pretty(G)
g=ilaplace(G);
pretty(g)
```

Computer response:

```
b


           3      2
          s  + 4 s  + 2 s + 6
  ------------------------------------
           2               2
  (s + 8) (s  + 8 s + 3) (s  + 5 s + 7)

               /                      1/2         1/2    \
               |         1/2      4262 13     sinh(13    t) |
1199 exp(-4 t) | cosh(13    t) - ------------------------ |
               \                          15587          /
----------------------------------------------------------- -
                           417

                   /                              /  1/2    \ \
                   |                       1/2    | 3     t | |
```

15.

Program:

```
numg=[-5 -70];
deng=[0 -45 -55 (roots([1 7 110]))' (roots([1 6 95]))'];
[numg,deng]=zp2tf(numg',deng',1e4);
Gtf=tf(numg,deng)
G=zpk(Gtf)
[r,p,k]=residue(numg,deng)
```

Computer response:

```
Transfer function:
                         10000 s^2 + 750000 s + 3.5e006
--------------------------------------------------------------------------------
s^7 + 113 s^6 + 4022 s^5 + 58200 s^4 + 754275 s^3 + 4.324e006 s^2 + 2.586e007 s

Zero/pole/gain:
               10000 (s+70) (s+5)
-----------------------------------------------
s (s+55) (s+45) (s^2 + 6s + 95) (s^2 + 7s + 110)

r =

  -0.0018
   0.0066
   0.9513 + 0.0896i
   0.9513 - 0.0896i
  -1.0213 - 0.1349i
  -1.0213 + 0.1349i
   0.1353
p =

 -55.0000
 -45.0000
  -3.5000 + 9.8869i
  -3.5000 - 9.8869i
  -3.0000 + 9.2736i
  -3.0000 - 9.2736i
        0
k =

     []
```

#22

Equation for inverting amplifier: Vo/Vi = -Zf/Zi where Zf is the feedback impedance and Zi is the input impedance.

a.

$$Z_1(s) = 5x10^5 + \frac{1}{2x10^6 s}$$

$$Z_2(s) = 10^5 + \frac{1}{2x10^6 s}$$

Therefore,

$$-\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{5}\frac{(s+5)}{(s+1)}$$

b.

$$Z_1(s) = 10^5\left(\frac{5}{s}+1\right) = 10^5\,\frac{(s+5)}{s}$$

$$Z_2(s) = 10^5\left(1+\frac{5}{s+5}\right) = 10^5\,\frac{(s+10)}{(s+5)}$$

Therefore,

$$-\frac{Z_2(s)}{Z_1(s)} = -\frac{s(s+10)}{(s+5)^2}$$

25.

Writing the equations of motion,

$$(s^2+s+1)X_1(s) - (s+1)X_2(s) = F(s)$$
$$-(s+1)X_1(s) + (s^2+s+1)X_2(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{bmatrix}(s^2+s+1) & F(s)\\ -(s+1) & 0\end{bmatrix}}{\begin{bmatrix}(s^2+s+1) & -(s+1)\\ -(s+1) & (s^2+s+1)\end{bmatrix}} = \frac{(s+1)F(s)}{s^2(s^2+2s+2)}$$

From which,

$$\frac{X_2(s)}{F(s)} = \frac{(s+1)}{s^2(s^2+2s+2)}.$$

33.

Writing the equations of motion,

$$(s^2+2s+1)\theta_1(s)-(s+1)\theta_2(s)=T(s)$$
$$-(s+1)\theta_1(s)+(2s+1)\theta_2(s)=0$$

Solving for $\theta_2(s)$

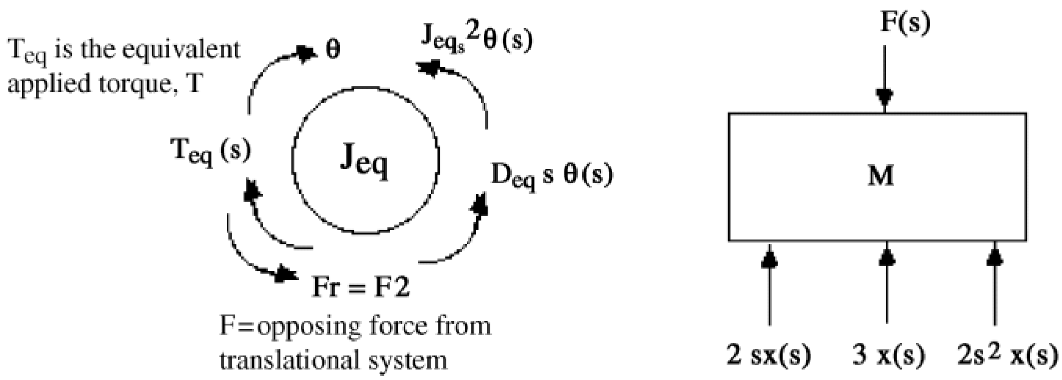
$$\theta_2(s)=\frac{\begin{vmatrix}(s^2+2s+1) & T(s)\\ -(s+1) & 0\end{vmatrix}}{\begin{vmatrix}(s^2+2s+1) & -(s+1)\\ -(s+1) & (2s+1)\end{vmatrix}}=\frac{T(s)}{2s(s+1)}$$

Hence,

$$\frac{\theta_2(s)}{T(s)}=\frac{1}{2s(s+1)}$$

43.

Draw a freebody diagram of the translational system and the rotating member connected to the translational system.



From the freebody diagram of the mass, $F(s) = (2s^2+2s+3)X(s)$. Summing torques on the rotating member,

$(J_{eq}s^2 + D_{eq}s)\theta(s) + F(s)2 = T_{eq}(s)$. Substituting F(s) above, $(J_{eq}s^2 + D_{eq}s)\theta(s) + (4s^2+4s+6)X(s) = T_{eq}(s)$. However, $\theta(s) = \dfrac{X(s)}{2}$. Substituting and simplifying,

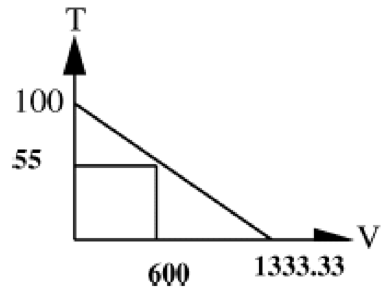
$$T_{eq}=\left[\left(\frac{J_{eq}}{2}+4\right)s^2+\left(\frac{D_{eq}}{2}+4\right)s+6\right]X(s)$$

But, $J_{eq} = 3+3(4)^2 = 51$, $D_{eq} = 1(2)^2 + 1 = 5$, and $T_{eq}(s) = 4T(s)$. Therefore,

$\left[\dfrac{59}{2}s^2+\dfrac{13}{2}s+6\right]X(s) = 4T(s)$. Finally, $\dfrac{X(s)}{T(s)}=\dfrac{8}{59s^2+13s+12}$.

47.

The following torque-speed curve can be drawn from the data given:



Therefore, $\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} = \frac{100}{12}$; $K_b = \frac{E_a}{\omega_{no-load}} = \frac{12}{1333.33}$. Also, $J_m = 7 + 105\left(\frac{1}{6}\right)^2 = 9.92$; $D_m =$

3. Thus,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\left(\frac{100}{12}\right)\frac{1}{9.92}}{s(s + \frac{1}{9.92}(3.075))} = \frac{0.84}{s(s+0.31)}$$. Since $\theta_L(s) = \frac{1}{6}\,\theta_m(s)$, $\frac{\theta_L(s)}{E_a(s)} = \frac{0.14}{s(s+0.31)}$.

68.

a. From $a = \dfrac{F - F_w}{k_m \cdot m}$, we have: $F = F_w + k_m \cdot m \cdot a = F_{Ro} + F_L + F_{St} + +k_m \cdot m \cdot a$ (1)

Substituting for the motive force, F, and the resistances F_{Ro}, F_L, and F_{st} using the equations given in the problem, yields the equation:

$$F = \frac{P \cdot \eta_{tot}}{v} = f \cdot m \cdot g \cdot \cos\alpha + m \cdot g \cdot \sin\alpha + 0.5 \cdot \rho \cdot C_w \cdot A \cdot (v + v_{hw})^2 + k_m \cdot m \cdot a \quad (2)$$

b. Noting that constant acceleration is assumed, the average values for speed and acceleration are:

a_{av} = 20 (km/h)/ 4 s = 5 km/h.s = 5x1000/3600 m/s^2 = 1.389 m/s^2

v_{av} = 50 km/h = 50,000/3,600 m/s = 13.89 m/s

The motive force, F (in N), and power, P (in kW) can be found from eq. 2:

F_{av} = 0.011 x 1590 x 9.8 + 0.5 x 1.2 x 0.3 x 2 x 13.89^2 + 1.2 x 1590 x 1.389 = 2891 N

$P_{av} = F_{av} \cdot v / \eta_{tot}$ = 2891 x 13.89 / 0.9 = 44, 617 N.m/s = 44.62 kW

To maintain a speed of 60 km/h while climbing a hill with a gradient $\alpha = 5^{o}$, the car engine or motor needs to overcome the climbing resistance:

$$F_{St} = m \cdot g \cdot \sin\alpha = 1590 \cdot 9.8 \cdot \sin 5^{\circ} = 1358 \text{ N}$$

Thus, the additional power, P_{add}, the car needs after reaching 60 km/h to maintain its speed while climbing a hill with a gradient $\alpha = 5^{o}$ is:

$$P_{add} = F_{St} \cdot v / \eta = 1358 \text{ x } 60 \text{ x } 1000/(3{,}600 \text{ x } 0.9) = 25{,}149 \text{ W} = 25.15 \text{ kW}$$

c. Substituting for the car parameters into equation 2 yields:

$$F = 0.011 \text{ x } 1590 \text{ x } 9.8 + 0.5 \text{ x } 1.2 \text{ x } 0.3 \text{ x } 2\, v^2 + 1.2 \text{ x } 1590\, dv/dt$$

$$\text{or} \quad F(t) = 171.4 + 0.36\, v^2 + 1908\, dv/dt \quad (3)$$

To linearize this equation about v_o = 50 km/h = 13.89 m/s, we use the truncated taylor series:

$$v^2 - v_o^2 \approx \left.\frac{d(v^2)}{dv}\right|_{v=v_o} (v - v_o) = 2v_o \cdot (v - v_o) \quad (4)\text{, from which we obtain:}$$

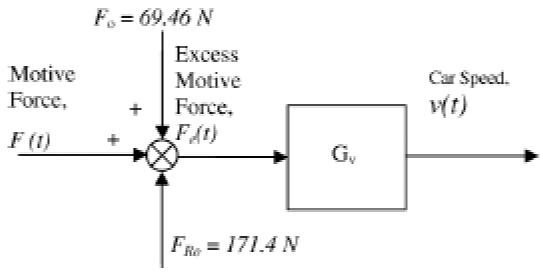
$$v^2 = 2v_o \cdot v - v_o^2 = 27.78 \cdot v - 13.89^2 \quad (5)$$

Substituting from equation (5) into (3) yields:

$$F(t) = 171.4 + 10\, v - 69.46 + 1908\, dv/dt \quad \text{or}$$

$$F_e(t) = F(t) - F_{Ro} + F_o = F(t) - 171.4 + 69.46 = 10\, v + 1908\, dv/dt \quad (6)$$

Equation (6) may be represented by the following block-diagram:



d. Taking the Laplace transform of the left and right-hand sides of equation (6) gives,

$$F_e(s) = 10\ V(s) + 1908\ sV(s) \quad (7)$$

Thus the transfer function, $G_v(s)$, relating car speed, $V(s)$ to the excess motive force, $F_e(s)$, when the car travels on a level road at speeds around v_o = 50 km/h = 13.89 m/s under windless conditions is:

$$G_v(s) = \frac{V(s)}{F_e(s)} = \frac{1}{10\ + 1908\ s} \quad (8)$$