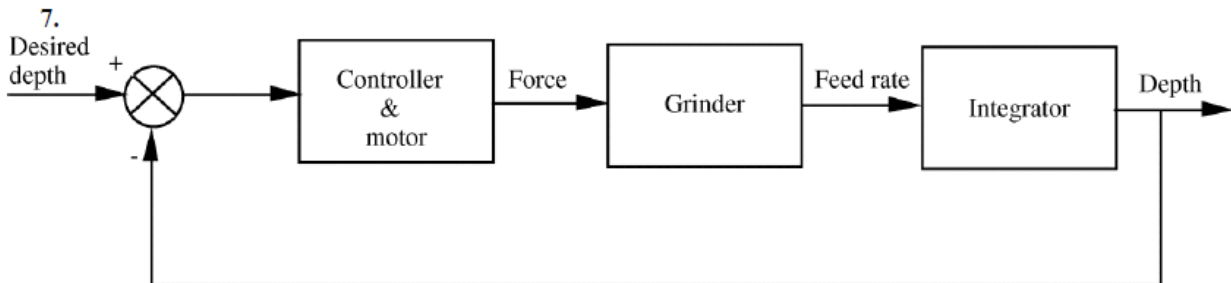


## Chapter 1 Solutions

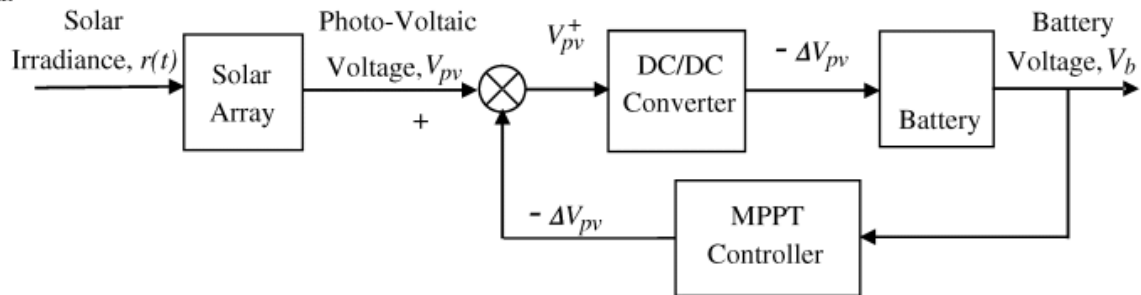
1. Five turns yields 50 v. Therefore  $K = \frac{50 \text{ volts}}{5 \times 2\pi \text{ rad}} = 1.59$

On problem #1, block diagram is just a gain block with a gain of  $K = 1.59$ .

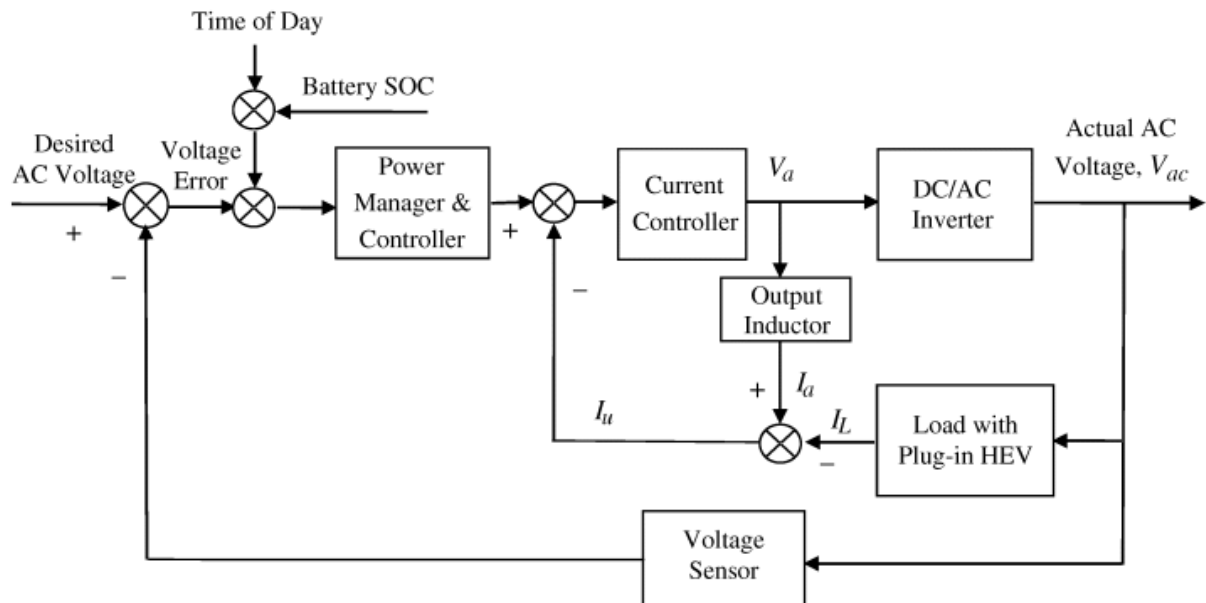


16.

a.



b.



b. Assume a particular solution of

$$x_p = A \sin 3t + B \cos 3t$$

Substitute into the differential equation and obtain

$$(18A - B) \cos(3t) - (A + 18B) \sin(3t) = 5 \sin(3t)$$

Therefore,  $18A - B = 0$  and  $-(A + 18B) = 5$ . Solving for A and B we obtain

$$x_p = (-1/65) \sin 3t + (-18/65) \cos 3t$$

The characteristic polynomial is

$$M^2 + 6M + 8 = (M + 4)(M + 2)$$

Thus, the total solution is

$$x = C e^{-4t} + D e^{-2t} + \left( -\frac{18}{65} \cos(3t) - \frac{1}{65} \sin(3t) \right)$$

Solving for the arbitrary constants,  $x(0) = C + D - \frac{18}{65} = 0$ .

Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{3}{65} \cos(3t) + \frac{54}{65} \sin(3t) - 4C e^{-4t} - 2D e^{-2t}$$

Solving for the arbitrary constants,  $\dot{x}(0) = -\frac{3}{65} - 4C - 2D = 0$ , or  $C = -\frac{3}{10}$  and  $D = \frac{15}{26}$ .

The final solution is

$$x = -\frac{18}{65} \cos(3t) - \frac{1}{65} \sin(3t) - \frac{3}{10} e^{-4t} + \frac{15}{26} e^{-2t}$$

#21

b. Assume a particular solution of

$$x_p = Ce^{-2t} + Dt + E$$

Substitute into the differential equation and obtain

$$C e^{-2t} + D t + 2D + E = 5e^{-2t} + t$$

Equating like coefficients,  $C = 5$ ,  $D = 1$ , and  $2D + E = 0$ .

From which,  $C = 5$ ,  $D = 1$ , and  $E = -2$ .

The characteristic polynomial is

$$M^2 + 2M + 1 = (M + 1)^2$$

Thus, the total solution is

$$x(t) = A e^{-t} + B e^{-t} t + 5e^{-2t} + t - 2$$

Solving for the arbitrary constants,  $x(0) = A + 5 - 2 = 2$  Therefore,  $A = -1$ . Also, the derivative of the solution is

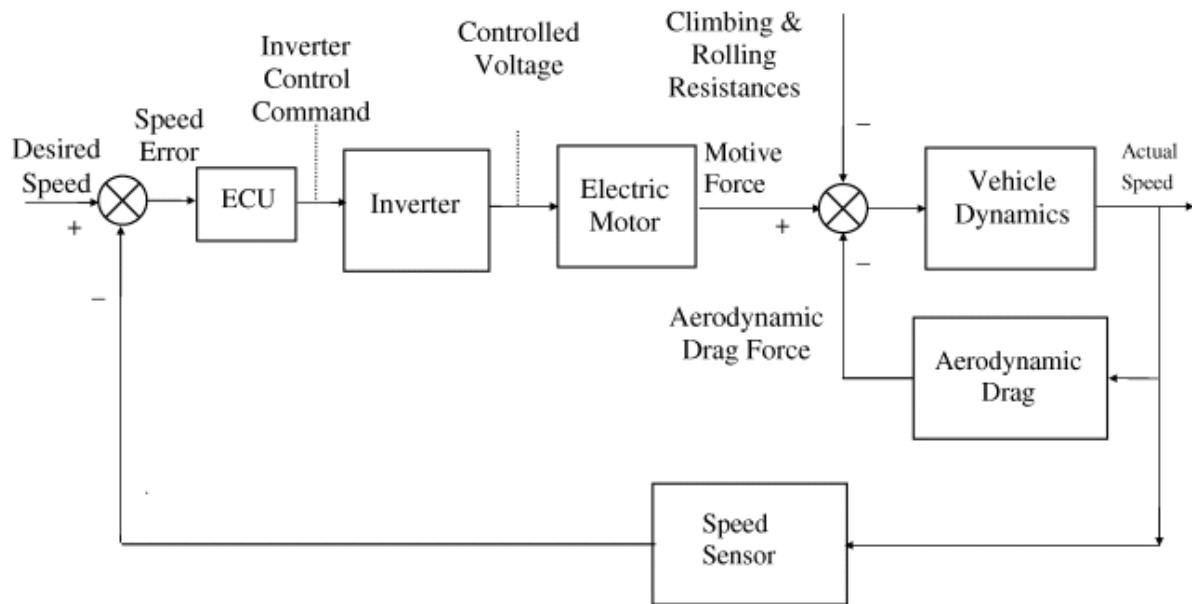
$$\frac{dx}{dt} = (-A + B)e^{-t} - Bte^{-t} - 10e^{-2t} + 1$$

Solving for the arbitrary constants,  $\dot{x}(0) = B - 8 = 1$ . Therefore,  $B = 9$ . The final solution is

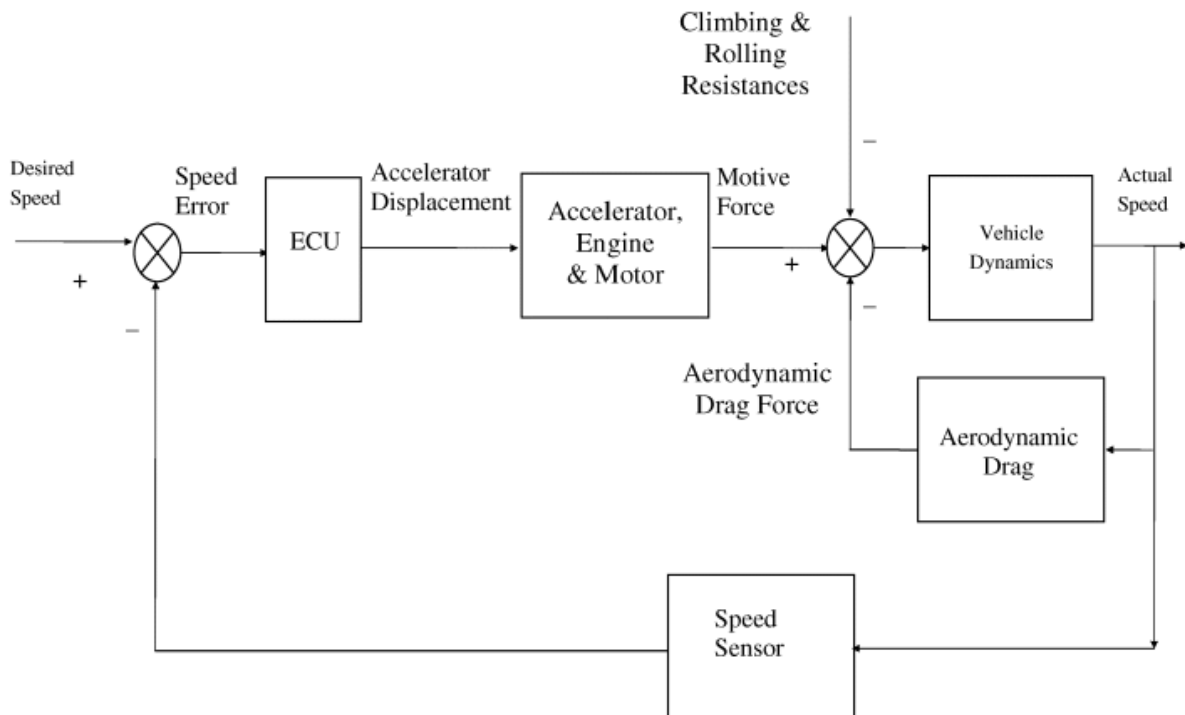
$$x(t) = -e^{-t} + 9te^{-t} + 5e^{-2t} + t - 2$$

23.

a.



b.



c.

