C9 Number 6 Solution by Author

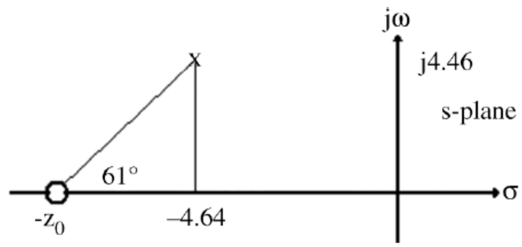
**6.**

Uncompensated: Searching along the 135° line ($\zeta = 0.707$), find the operating point at $-2.32 + j2.32$ with K = 4.6045. Hence, $K_p = \frac{4.6045}{30} = 0.153$; $T_s = \frac{4}{2.32} = 1.724$ seconds; $T_p = \frac{\pi}{2.32} = 1.354$ seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 = 4.33\%$;

$\omega_n = \sqrt{2.32^2 + 2.32^2} = 3.28$ rad/s; higher-order pole at -5.366.

Compensated: To reduce the settling time by a factor of 2, the closed-loop poles should be $-4.64 \pm j4.64$. The summation of angles to this point is 119° . Hence, the contribution of the compensating zero should be 180° -119° =61° . Using the geometry shown below,

$\frac{4.64}{z_c - 4.64} = \tan(61°)$. Or, $z_c = 7.21$.




After adding the compensator zero, the gain at -4.64+j4.64 is K = 4.77. Hence,

$K_p = \frac{4.77x6x7.21}{2x3x5} = 6.88$. $T_s = \frac{4}{4.64} = 0.86$ second; $T_p = \frac{\pi}{4.64} = 0.677$ second; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 = 4.33\%$; $\omega_n = \sqrt{4.64^2 + 4.64^2} = 6.56$ rad/s; higher-order pole at -5.49. The problem with the design is that there is steady-state error, and no effective pole/zero cancellation. The design should be simulated to be sure the transient requirements are met.