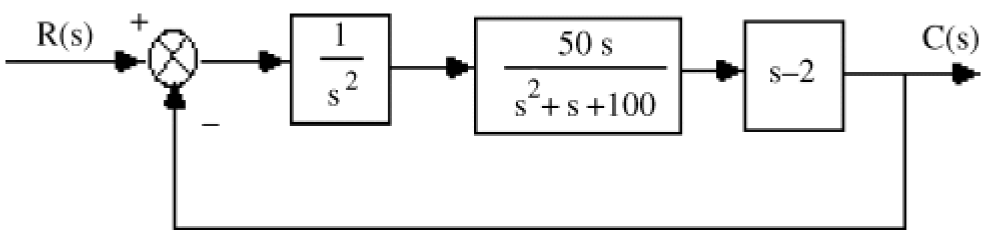
Chapter 5 Homework solutions

1.

a. Combine the inner feedback and the parallel pair.



Multiply the blocks in the forward path and apply the feedback formula to get,

$$T(s) = \frac{50(s-2)}{s^3+s^2+150s-100}$$

b.
Program:

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'G1(s)'
G1=tf(1,[1 0 0])
'G2(s)'
G2=tf(50,[1 1])
'G3(s)'
G3=tf(2,[1 0])
'G4(s)'
G4=tf([1 0],1)
'G5(s)'
G5=2
'Ge1(s)=G2(s)/(1+G2(s)G3(s))'
Ge1=G2/(1+G2*G3)
'Ge2(s)=G4(s)-G5(s)'
Ge2=G4-G5
'Ge3(s)=G1(s)Ge1(s)Ge2(s)'
Ge3=G1*Ge1*Ge2
'T(s)=Ge3(s)/(1+Ge3(s))'
T=feedback(Ge3,1);
T=minreal(T)
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11.

$T(s) = \dfrac{225}{s^2+15s+225}$. Therefore, $2\zeta\omega_n = 15$, and $\omega_n = 15$. Hence, $\zeta = 0.5$.

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 16.3\%$; $T_s = \dfrac{4}{\zeta\omega_n}$ =0.533; $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ =0.242.

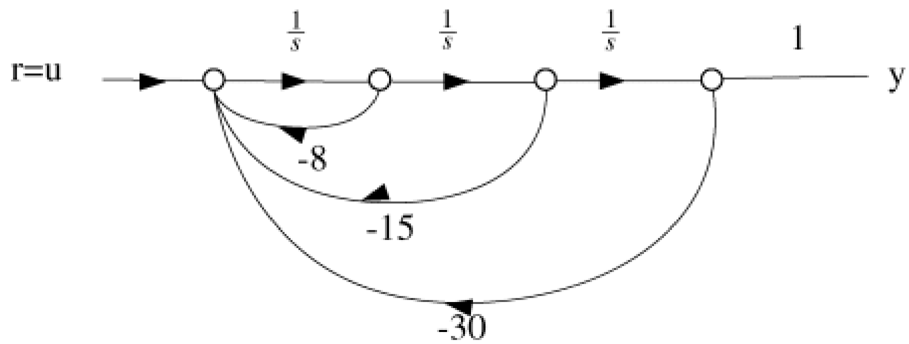
14.

Since $G(s)=\dfrac{K}{s(s+30)}$, $T(s)=\dfrac{G(s)}{1+G(s)}=\dfrac{K}{s^2+30s+K}$. Therefore, $2\zeta\omega_n = 30$. Thus, $\zeta = 15/\omega_n =$ 0.5912 (i.e. 10% overshoot). Hence, $\omega_n = 25.37 = \sqrt{K}$. Therefore K = 643.6.

34.

a. Phase Variable form

$$T(s)=\frac{30}{s^3+8s^2+15s+30}$$



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -30x_1 - 15x_2 - 8x_3 + 30r$$

$$y = x_1$$

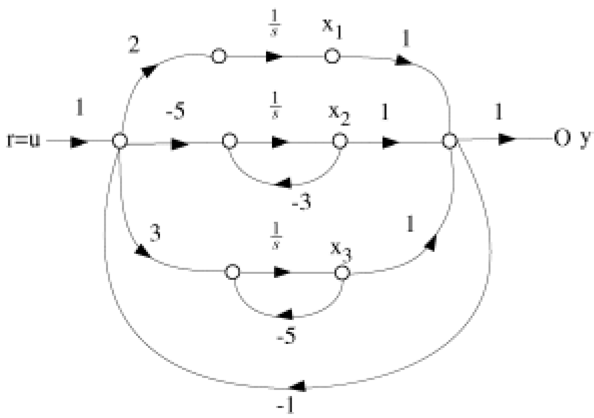
Or in matrix form

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -15 & -8 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

b. Parallel form

$$G(s) = \frac{30}{s(s+3)(s+5)} = \frac{2}{s} - \frac{5}{s+3} + \frac{3}{s+5}$$



The state equations are:

$$\dot{x}_1 = 2(u - x_1 - x_2 - x_3) = -2x_1 - 2x_2 - 2x_3 + 2u$$

$$\dot{x}_2 = -5(u - x_1 - x_2 - x_3) - 3x_2 = 5x_1 + 2x_2 + 5x_3 - 5u$$

$$\dot{x}_3 = 3(u - x_1 - x_2 - x_3) - 5x_3 = -3x_1 - 3x_2 - 8x_3 + 3u$$

$$y = x_1 + x_2 + x_3$$

In matrix form:

$$\dot{x} = \begin{bmatrix} -2 & -2 & -2 \\ 5 & 2 & 5 \\ -3 & -3 & -8 \end{bmatrix} x + \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x$$

#43

$$\dot{\mathbf{z}} = \mathbf{P}^{-1}\mathbf{APz} + \mathbf{P}^{-1}\mathbf{B}u$$
$$y = \mathbf{CPz}$$

$$\mathbf{P}^{-1} = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}; \ \therefore \ \mathbf{P} = \begin{bmatrix} -0.2085 & -0.3029 & -0.1661 \\ 0.0228 & -0.1075 & -0.0912 \\ 0.0130 & 0.0814 & -0.0521 \end{bmatrix}$$

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{bmatrix} 18.5961 & 25.4756 & 5.6156 \\ -12.9023 & -28.8893 & -8.3909 \\ -0.5733 & 11.4169 & 5.2932 \end{bmatrix}; \ \mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} -58 \\ 63 \\ -12 \end{bmatrix}; \ \mathbf{CP} = \begin{bmatrix} 1.5668 & 3.0423 & 2.7329 \end{bmatrix}$$

72.

a.

$T(s) = \dfrac{25}{s^2+s+25}$; from which, $2\zeta\omega_n = 1$ and $\omega_n = 5$. Hence, $\zeta = 0.1$. Therefore,

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 72.92\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 8$.

b.

$T(s) = \dfrac{25K_1}{s^2+(1+25K_2)s+25K_1}$; from which, $2\zeta\omega_n = 1+25K_2$ and $\omega_n = 5\sqrt{K_1}$.

Hence, $\zeta = \dfrac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = \dfrac{-\ln 0.2}{\sqrt{\pi^2 + \ln^2 0.2}} = 0.456$

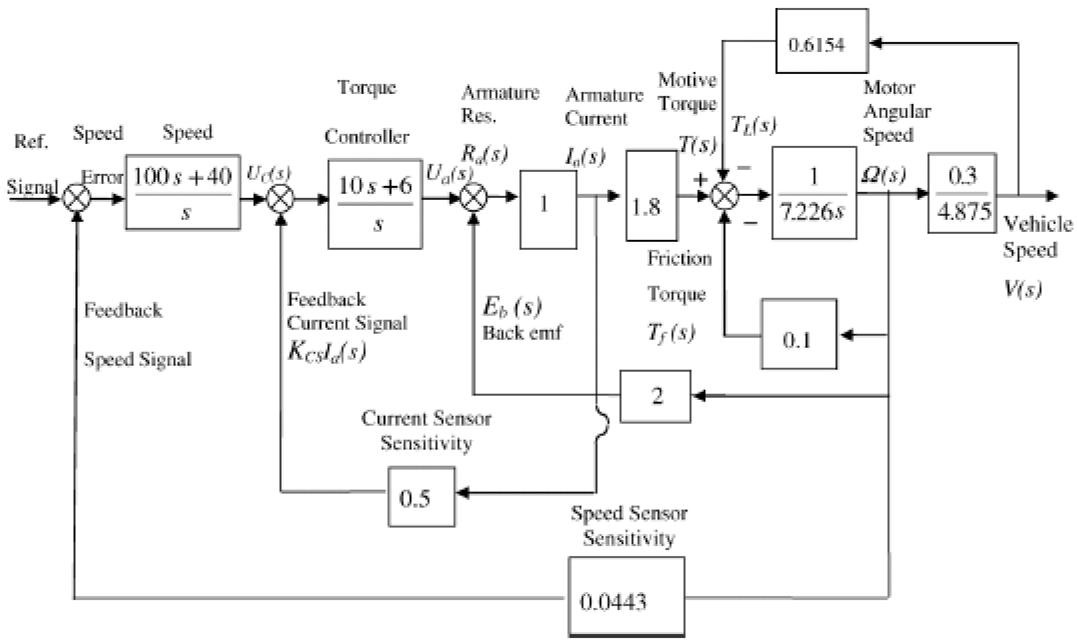
Also, $T_s = \dfrac{4}{\zeta\omega_n} = 0.25$, Thus, $\zeta\omega_n = 16$; from which $K_2 = \dfrac{32}{25} = 1.28$ and

$\omega_n = 35.09$. Hence, $K_1 = 49.25$.

(Note for problem 79, you may use Kss = .0443 as shown in solution instead of .0433. Also, Mason's Gain Rule is an acceptable method of solution instead of block diagram reduction rules.)

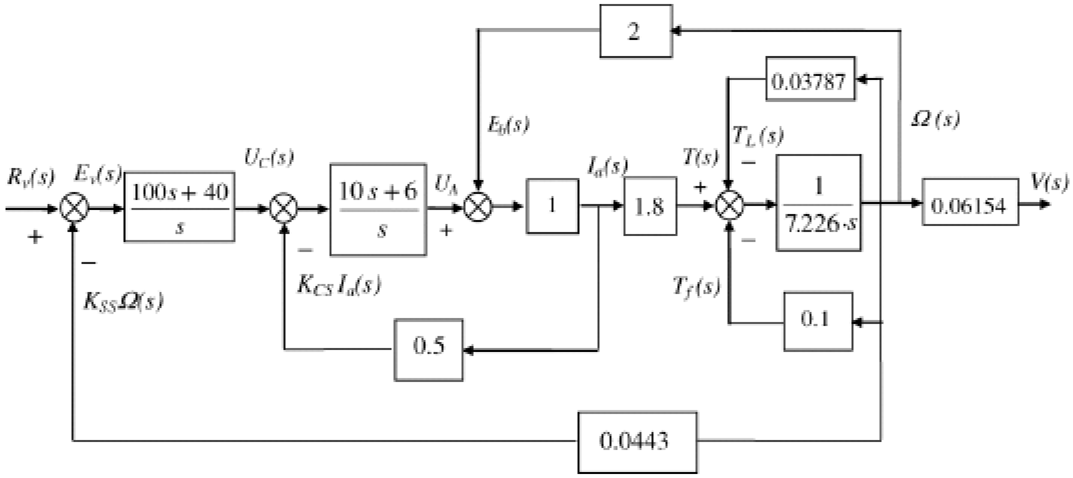
79.

a. Substituting all values and transfer functions into the respective blocks of the system (Figure 4), we get:

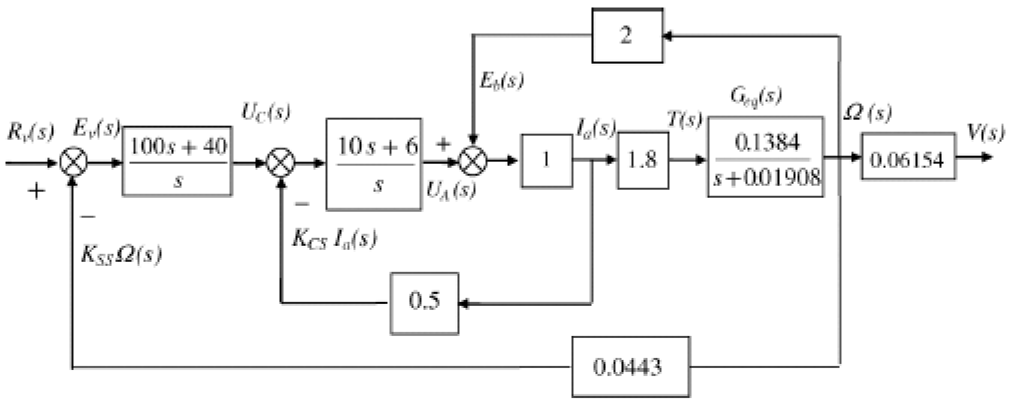


Moving the last pick-off point to the left past the $\frac{r}{i_{tot}} = \frac{0.3}{4.875} = 0.06154$ block and changing the position of the back-emf feedback pick-off point, so that it becomes an outer loop, we obtain the block-diagram shown below. In that diagram the $\frac{1}{7.226s}$ block (representing the total inertia) has two parallel feedback blocks. Reducing these two blocks into one, we have the following equivalent feedback transfer function:

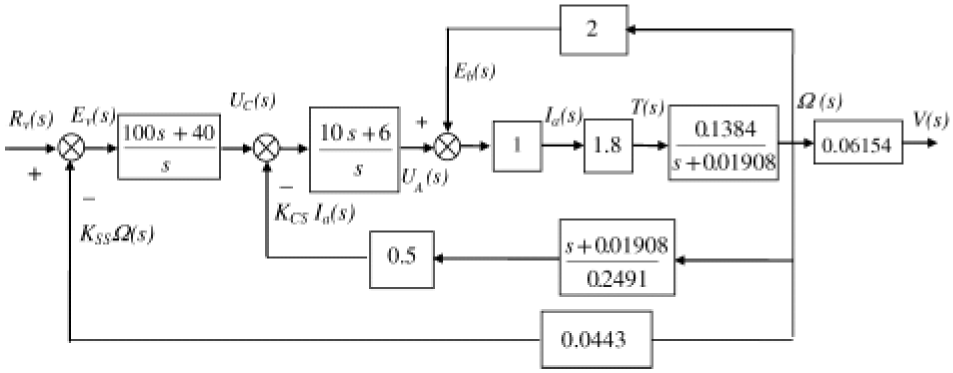
$$G_{eq}(s) = \frac{\Omega(s)}{T(s)} = \frac{\frac{1}{7.226s}}{1+\frac{0.13787}{7.226s}} = \frac{0.1384}{s+0.01908}$$



Replacing that feedback loop with its equivalent transfer function, $G_{eq}(s)$, we have:

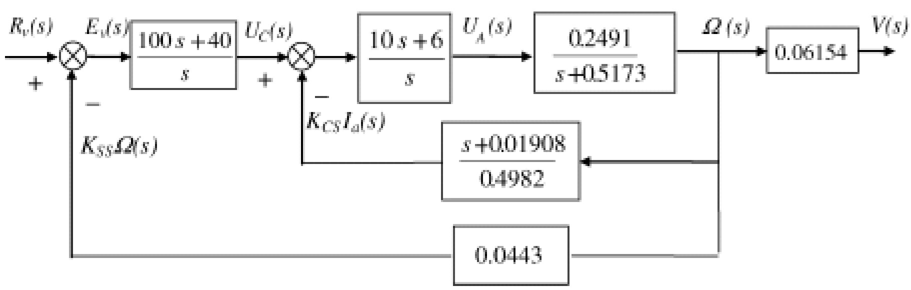


Moving the armature current pick-off point to the right past the $\frac{T(s)}{I_a(s)}$ and $G_{eq}(s)$ blocks, the above block-diagram becomes as shown below.



The latter, in turn, can be reduced to that shown next as the cascaded blocks in the feedback to the torque controller are replaced by the single block: $\frac{K_{CS}I_a(s)}{\Omega(s)} = \frac{s+0.01908}{0.4982}$ and the inner feedback loop is replaced by its equivalent transfer function:

$$\frac{\Omega(s)}{U_A(s)} = \frac{\frac{0.2491}{s+0.01908}}{1+\frac{0.2491}{s+0.01908}\times 2} = \frac{0.2491}{s+0.5173}$$



Thus: $$\frac{\Omega(s)}{U_C(s)}=\frac{\left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)}{1+\left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)\left(\frac{s+0.01908}{0.4982}\right)}=\frac{0.2491\ (10s+6)}{s(s+0.5173)\ +\ 0.5\ \ (10s+6)\ \ (s+0.01908)}$$

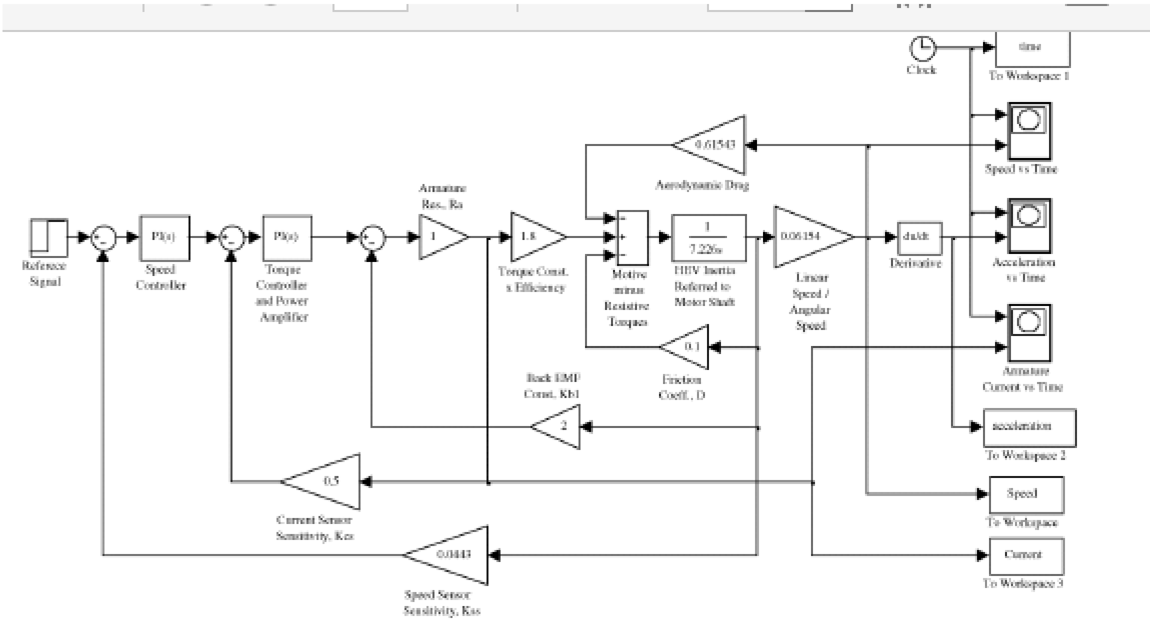
Finally $$\frac{\Omega(s)}{R_v(s)}=\frac{\left(\frac{100s+40}{s}\right)\left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)}\right)}{1+0.0443\left(\frac{100s+40}{s}\right)\left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)}\right)}$$ or

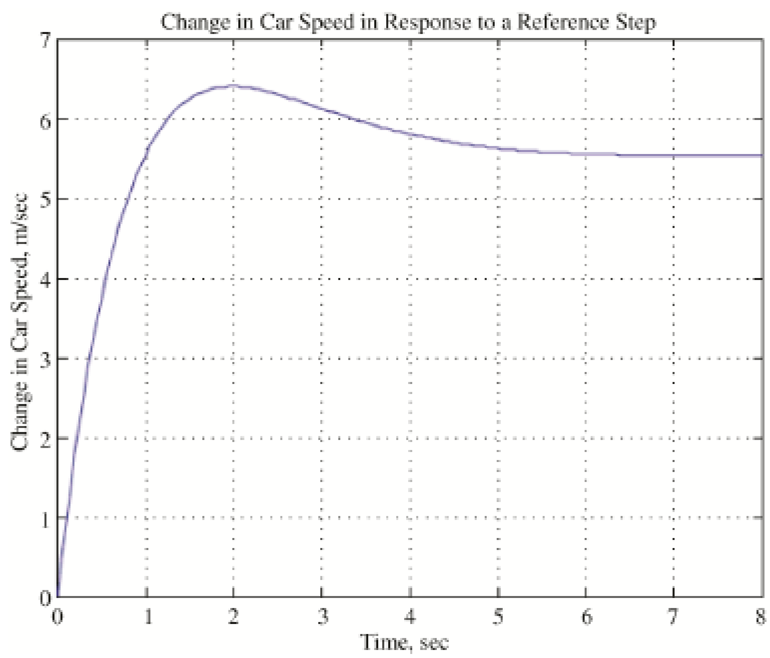
$$\frac{\Omega(s)}{R_v(s)}=\frac{249.1\ (s+0.4)\ (s+0.6)}{s\left(6s^2+3.613s+0.0572\right)+11.035\ \left(s^2+s+0.24\right)}$$
$$=\frac{249.1\ (s+0.4)\ (s+0.6)}{6s^3+14.644s^2+11.09s+2.65}$$

Hence: $$\frac{V(s)}{R_v(s)}=0.06154\frac{\Omega(s)}{R_v(s)}=\frac{15.33\ (s+0.4)\ (s+0.6)}{6s^3+14.644s^2+11.09s+2.65}$$

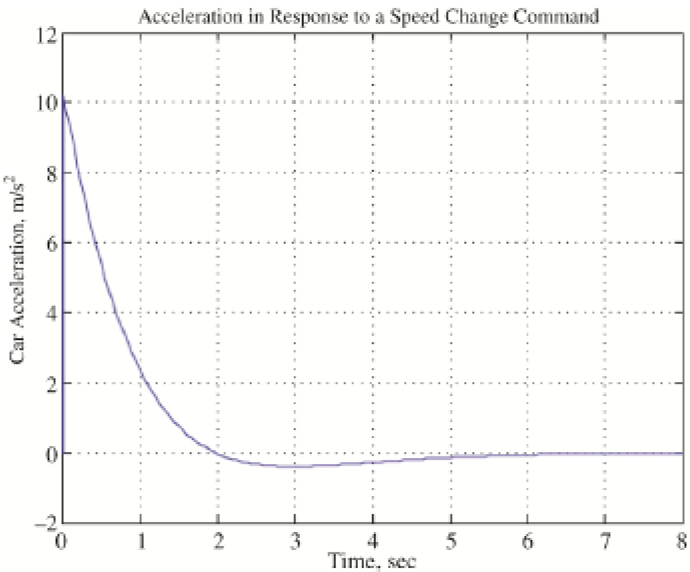
b. Simulink was used to model the HEV cascade control system. That model is shown below. The reference signal, $r_v(t)$, was set as a step input with a zero initial value, a step time = 0 seconds, and a final value equal to 4 volts [corresponding to the desired final car speed, $v(\infty)$ = 60 km/h, e.g. a desired final value of the change in car speed, $\Delta v\ (\infty)$ = 5.55 m/s]. The variables of interest [time, change in car speed, acceleration, and motor armature current] were output (in array format) to four "workspace" sinks, each of which was assigned the respective variable name. After the simulation ended, Matlab plot commands were utilized to obtain and edit the required three graphs. These graphs are shown below.

The simulations show that in response to such a speed reference command, car acceleration would go initially to a maximum value of 10.22 m/s^2 and the motor armature current would reach a maximum value of 666.7 A. That would require an electric motor drive rated around 80 kW or using both the electric motor and gas or diesel engine, when fast acceleration is required. Most practical HEV control systems, however, use current-limiting and acceleration-limiting devices or software programs.

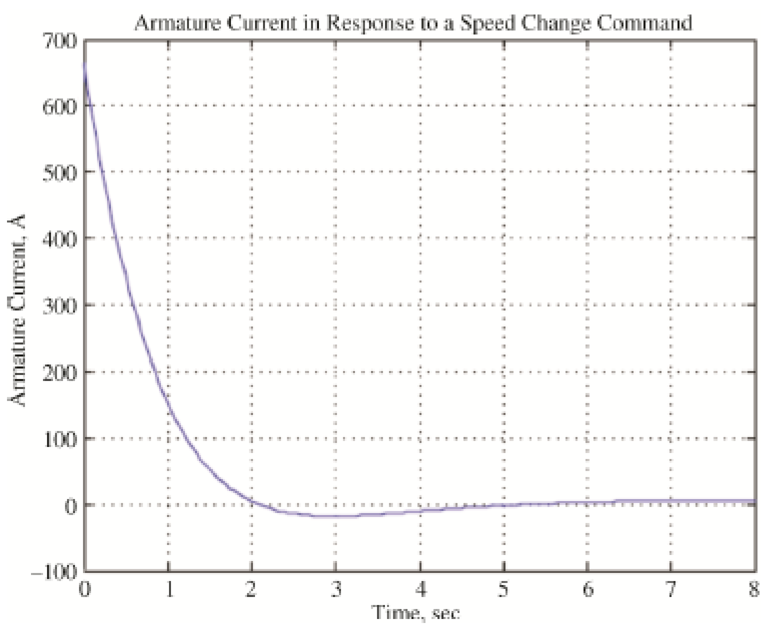




Change in car speed in response to a speed reference signal step of 4 volts



Car acceleration reponse to a speed reference signal step of 4 volts



Motor armature current reponse to a speed reference signal step of 4 volts