Clay Freeman

ME 415 : Feedback Control Theory

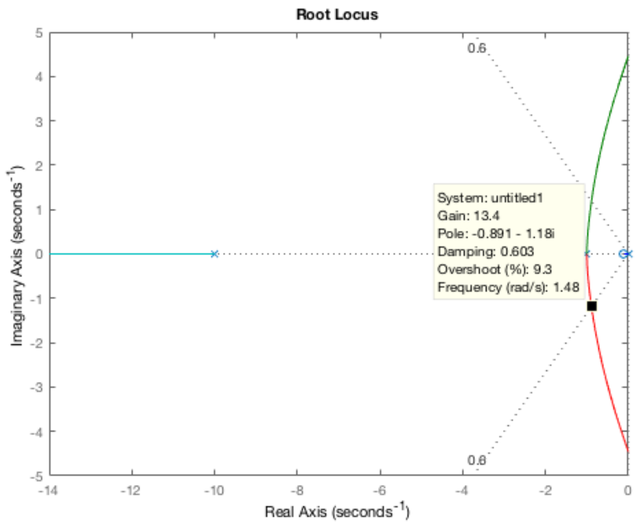
Dr. D. Justice - Fall 2018

Homework Assignment 9

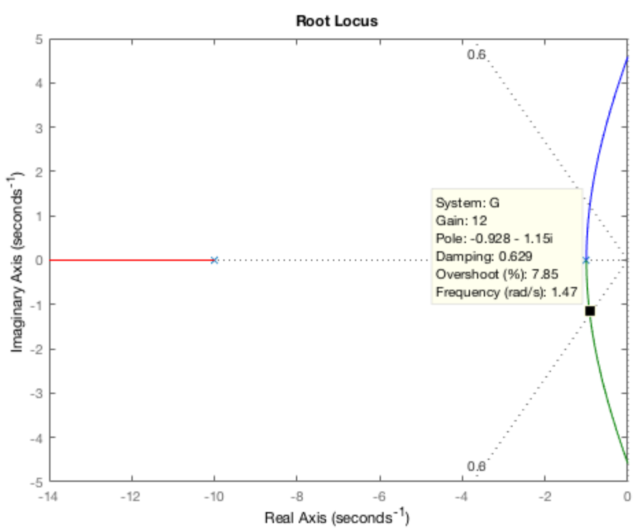
1, 6, 16, 27, 42

6 November 2018

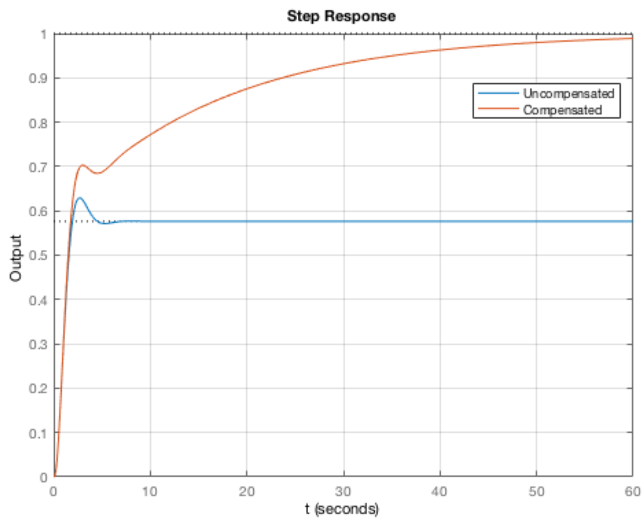
Problem 1



Root locus plot of uncompensated system generates dominant poles at $s_{1,2}= -0.891\pm j1.18$ with a gain of 13.4



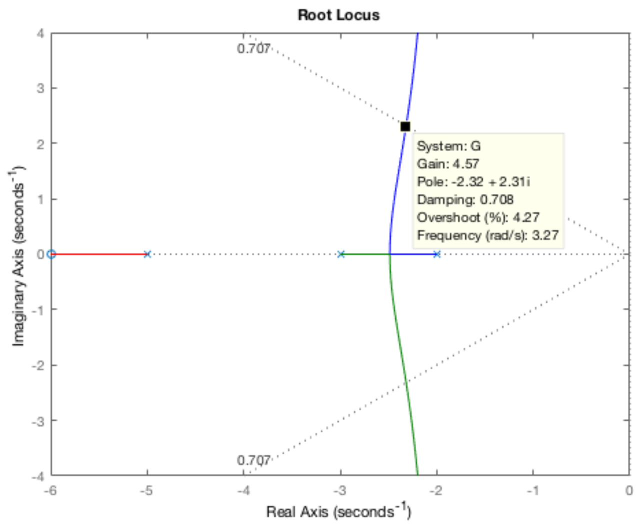
Root locus plot of compensated system. Adding a zero at -0.1 creates pole zero cancellation that moves the dominant pole pair to $-0.928 \pm j1.15$ with a corresponding gain of 12. Since the higher order pole stays at s=-10, the approximation is valid.



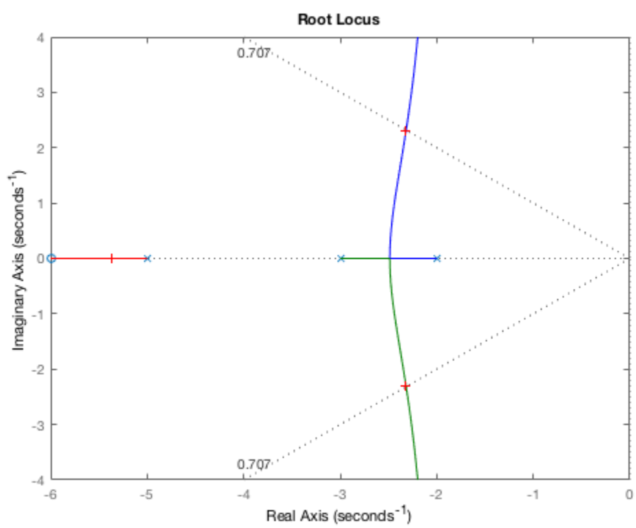
The step response for the compensated and uncompensated systems with the PI controller at a zero steady state error. PI controller for step response is $G(s) = \frac{K(s+0.1)}{s(s+1)^2(s+10)}$

```
G = zpk([], [-1 -1 -10],1);
rlocus(G);
sgrid(0.6, 0);
axis([-14 0 -5 5])

Gc = zpk([-0.1],[0],1);
rlocus(Gc*G);
sgrid(0.6,0);
axis([-14 0 -5 5])

syms t
t = 0:0.0001:60;
G2 = zpk([],[-1 -1 -10],13.6);
G2c = zpk([-0.1], [0 -1 -1 -10],13.4);
step(feedback(G2,1),t);
hold on
step(feedback(G2c,1),t);
grid
xlabel t
ylabel Output
legend('Uncompensated','Compensated')
```

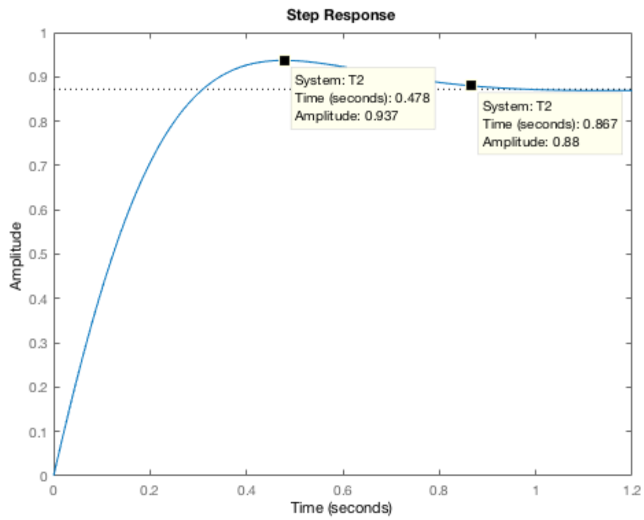
Prob 6



Settling time for uncompensated system is 1.72 seconds.



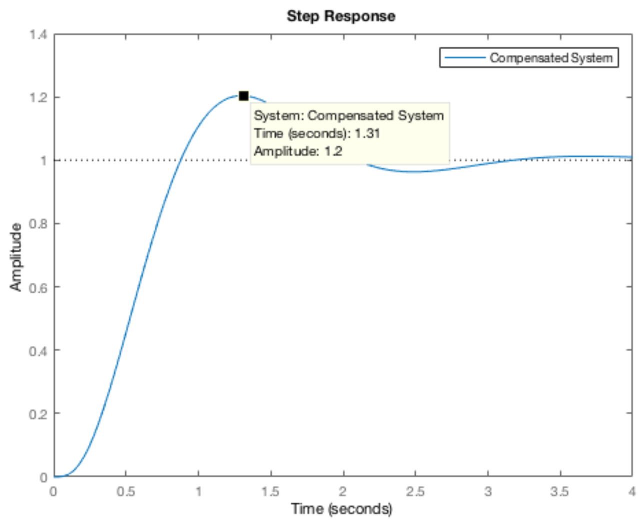
Plot generated from selecting a point using the command rlocfind(G) as pole input to the matlab script. New gain was found to be K = 4.6319 and the compensator used was (s + 7.2073)



Settling time was reduced to 0.86 seconds but the uncompensated steady state error was 0.94 while the compensated system generated a steady state error of 6.8151.

```
G = zpk([-6],[-2 -3 -5],1);
rlocus(G);
sgrid(1/sqrt(2), 0);
axis([-6 0 -4 4])
[gain, poles] = rlocfind(G);

t_settle = 4/-real(poles(2));
sigma_new = 8/t_settle;
s_new = -sigma_new +sigma_new*1i;

new_angle = ((s_new + 6) / ((s_new + 2)*(s_new + 3)*(s_new + 5)));
contrib = angle(new_angle)*(180/pi);

needed_angle = 180-contrib;
z_c = (sigma_new / tand(needed_angle)) + sigma_new

comp_gain = abs(((s_new + 2)*(s_new + 3)*(s_new + 5)) / ((s_new + 6)*(s_new +
z_c)));

close
G2 = zpk([-6 -z_c],[-2 -3 -5],comp_gain);
T2 = feedback(G2, 1);
step(T2)

ess_uncomp = 6*comp_gain / (2*3*5)
ess_comp = 6*comp_gain*z_c / (2*3*5)
```

Prob 16



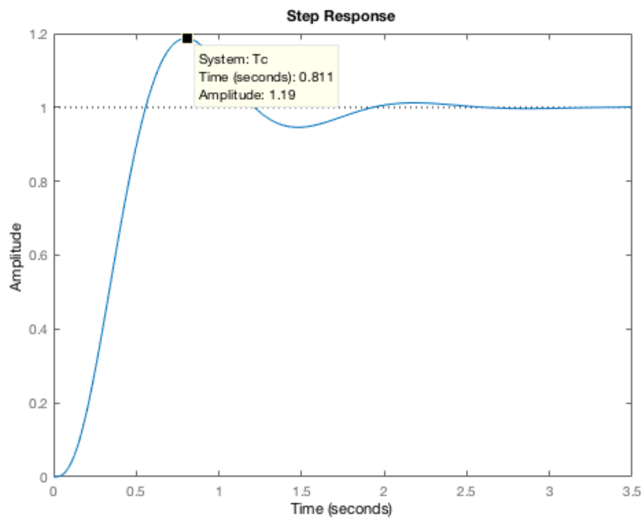
Compensated system transfer function:

$$G_c(s) = \frac{4220.2(s + 0.01)}{(s + 35.73)}$$

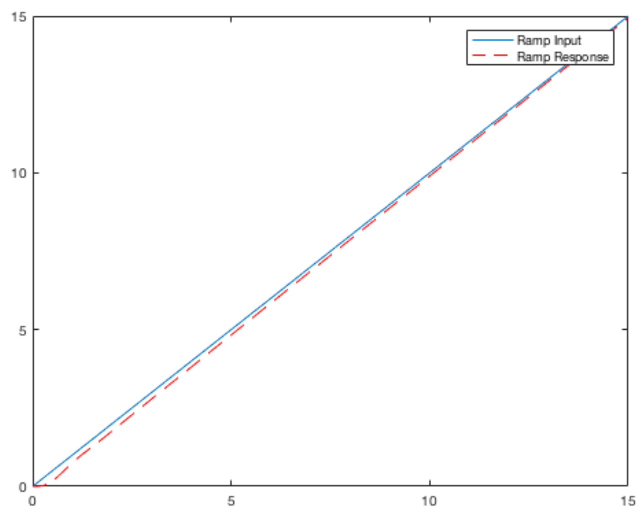
b.) The added closed loop pole at -0.01 is cancelled with the compensator zero at -0.01. The other higher order poles are more than 10 times further left in the s-plane so this contribution is negligible. The approximation is valid.

```
os_pcnt = 20.5;
t_settle = 3;
G = zpk([],[0 0 -4 -12],1);

zeta = (-log(os_pcnt/100)) / sqrt(pi^2 + log(os_pcnt/100)^2);
sigma = 4/t_settle;
w_d = sigma * tan(acos(zeta));
d_poles = (-sigma + w_d*1i);

% Let compensator have zero at 0.01
% G_c(s) = (K(s+0.01) / (s+p))
ang_con = 180 + (angle(d_poles + 0.01) - 2*angle(d_poles) - angle(d_poles +
4) - angle(d_poles + 12))*(180/pi);
p_comp = (w_d / tand(ang_con)) + sigma;

new_gain = -real(((d_poles + p_comp) * d_poles^2 * (d_poles + 4) * (d_poles +
12))/(d_poles + 0.01));

sprintf('G_c(s) = %1.1f (s + 0.01) / s + %1.2f',new_gain, p_comp)

Gc = zpk(-0.01, -p_comp, new_gain);
Tcomp = feedback(Gc*G, 1);
step(Tcomp)
legend('Compensated System')
```

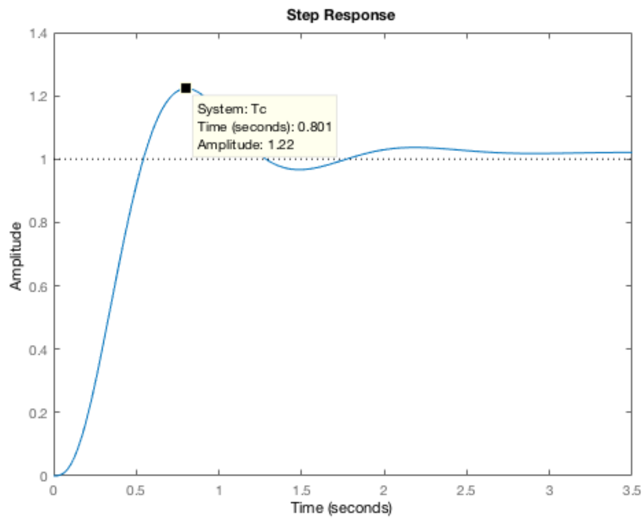
Prob 27



PD compensated step response gives settling time just short of 2 seconds. Calculated PD pole at -2.95 with gain of 294.57



Ramp response to input starts with some lag but converges to input signal within 15 seconds.



Final transfer function gain has 22% OS and settles within the 2 second design criteria.

$$G_c(s) = \frac{296.9\,(s + 0.1)(s + 2.95)}{s}$$

```
os_pcnt = 25;
t_settle = 2;

zeta = (-log(os_pcnt/100)) / sqrt(pi^2 + log(os_pcnt/100)^2);
sigma = 4/t_settle;
w_d = sigma * tan(acos(zeta));
s_12 = (-sigma + w_d*1i);

ang_con = -(180 + (-angle(s_12) - angle(s_12 + 4) - angle(s_12 + 6) -
angle(s_12 + 10))*(180/pi));
p_comp = (w_d / (tand(ang_con)) + sigma);

gain_pd = -real(s_12 * (s_12 + 4) * (s_12 + 6) * (s_12 + 10) / (s_12 +
2.95));

% G = zpk([],[0 -4 -6 -10],1);
% Gc = zpk([-p_comp],[],[gain_pd]);
% Tc = feedback(Gc*G, 1);
% step(Tc)

gain_pid = -real(s_12^2 * (s_12 + 4) * (s_12 + 6) * (s_12 + 10) / ((s_12 +
2.95) * (s_12 + 0.1)));

sprintf('G_c(s) = %1.1f (s + 0.1)(s + %1.2f) / s',gain_pid, p_comp)

G = zpk([],[0 -4 -6 -10],1);
Gc = zpk([-0.1 -p_comp],[0],[gain_pid]);
Tc = feedback(Gc*G, 1);
step(Tc)
figure
t = 0:0.1:15;
```

```
u = t;
y = lsim(Tc,u,t);
plot(t,u,t,y,'r--');
legend('Ramp Input','Ramp Response')
```

Prob 42

$$G_1(s) = \frac{239.51}{s(s+16)}$$

```
% Characteristic eqn for closed loop system
% s^2 + 2s + 25 = 0

w_n_old = sqrt(25);
zeta_old = 2/(2*w_n_old);

os_pcnt = exp(-zeta_old*pi/sqrt(1-zeta_old^2))*100;
t_s = 4/(zeta_old*w_n_old);

os_des = 15; % desired overshoot percent
t_s_des = 0.5; % desired settling time
zeta = (-log(os_des/100)) / sqrt(pi^2 + log(os_des/100)^2);
sigma = 4/t_s_des;
w_n = sigma/zeta;

% T(s) = C(s) / R (s) = (25 * K_1) / (s^2 + (2 + 25 * K_f) * s + 25 * K_1)

K_f = (2*sigma - 2)/ 25;
K_1 = (w_n^2)/25;

e_ss = 1/25/2;

G1_num = 25*K_1;
G1_den2 = 2 + 25*K_f;

e_ss2 = G1_num / G1_den2;

sprintf('G_1(s) = %1.2f / s (s + %d)',G1_num, G1_den2)
```