Chapter 7
Direct Model Reference Adaptive Control: Motivation and Introduction

7.1 Model Reference Control: Motivational Example

In the design of flight control systems, it is essential to provide closed-loop stability, adequate command tracking performance, as well as robustness to model uncertainties, control failures, and environmental disturbances. In the previous chapters, we considered optimal linear quadratic regulator (LQR) control design techniques that were suitable for flight control of aerial systems. These design methods relied on the inherent robustness properties of LQR optimal controllers. It was shown that with a proper selection of the LQR design tuning parameters (Q and R matrices), we could achieve 6 dB gain margin, and at least 60° phase margin, at the system control input break points.

It is also possible to show that LQR optimal controllers can tolerate time-state-dependent nonlinear uncertainties that might be present in the system control channels. These uncertainties are called "matched" since they appear only where control inputs exist in the system dynamics. The matching conditions imply that if the system uncertainties were known, a controller would have the ability to cancel them out.

In the presence of matched uncertainties, a deterioration of the system baseline closed-loop performance is inevitable. This is to be expected since the LQR controllers are designed to be robust to the entire class of matched uncertainties. However, they are not tuned to handle any specific uncertainty from this class. In other words, these LQR controllers may become overly conservative.

We pose the question: "Can we restore a given baseline closed-loop performance of the system, while operating under matched uncertainties?" The answer is "yes." This is the area where adaptive controllers are highly effective.

Throughout the chapters of Part II, we shall utilize the concept of a reference model for specifying the desired closed-loop tracking performance. Fixed-gain controllers, as well as adaptive systems, can be constructed using the reference model-based design concept. We shall begin our discussions with a motivational example.

E. Lavretsky and K.A. Wise, *Robust and Adaptive Control*, Advanced Textbooks in Control and Signal Processing, DOI 10.1007/978-1-4471-4396-3_7,

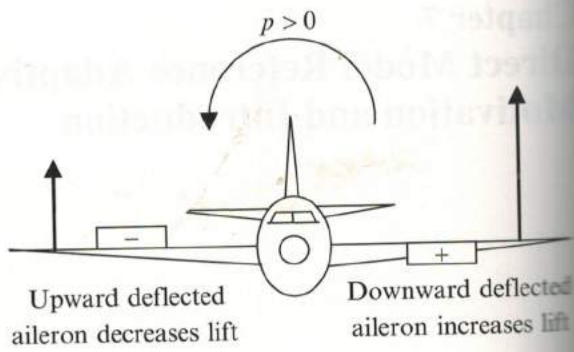

Fig. 7.1 Lift forces arising from positive differential aileron deflection cause aircraft to roll counterclockwise (positive roll rate)

Example 7.1 Fixed-Gain Model Reference Control of Aircraft Roll Dynamics. The roll dynamics of a conventional aircraft are controlled using differential motion of ailerons and spoilers. Ailerons are movable surfaces that are mounted outboards on the trailing edge of the wing, where they are placed symmetrically on each side of the wing, with respect to the aircraft centerline (Fig 7.1).

Deflected differentially (e.g., downward on one side and upward on the other), ailerons have the ability to increase the lift force on the downward deflected portion of the wing and to decrease it on the other side. The two distinct lift forces will create a rolling moment around the aircraft velocity vector placed at the aircraft center of gravity. While ailerons can move up and down, spoilers can only be deflected upward above the trailing edge of the wing to reduce the lift force and thus to aid ailerons in providing roll control. As a result, the aircraft rotates around its velocity vector. In this case, the aircraft roll dynamics can be approximated by a scalar (first-order) ordinary differential equation (ODE) in the form

$$\dot{p} = L_p\, p + L_{\delta_a}\, \delta_a \tag{7.1}$$

where p is the aircraft roll rate in stability axes (radians/s), δ_a is the total differential aileron-spoiler deflection (radians), L_p is the roll damping derivative, and L_{δ_a} is the dimensional rolling moment derivative with respect to differential aileron-spoiler deflection, (the aileron-to-roll control effectiveness). For a conventional open-loop-stable aircraft, the roll damping derivative L_p is negative, unless portions of the wing are stalled, in which case the roll damping may become positive. Positive differential aileron-spoiler deflection is defined to produce positive rolling moment, and as such, the aileron-to-roll control effectiveness L_{δ_a} typically has positive values.

Strictly speaking, the roll dynamics approximation above is valid only for sufficiently small values of p and δ_a. In addition, it is assumed that the aircraft yawing motion is suppressed by the rudder – a vertical tail mounted surface. Readers who might be unfamiliar with the flight mechanics nomenclature may consider (7.1) as a scalar ODE $\dot{x} = a\,x + b\,u$, with two constant parameters $a = L_p$, $b = L_{\delta_a}$, whose state and control input are $x = p$ and $u = \delta_a$, respectively.

The control task of interest is to force the aircraft to roll like the reference model,

$$\dot{p}_{ref} = a_{ref}\, p_{ref} + b_{ref}\, p_{cmd} \tag{7.2}$$

with the prescribed values of $a_{ref} < 0$ (the desired inverse time constant) and $b_{ref} > 0$ (the desired DC gain). The reference model (7.2) is driven by the commanded roll rate p_{cmd} and it calculates the reference roll rate p_{ref}. In essence, the reference model (7.2) imbeds and defines the desired closed-loop command tracking performance. The control task amounts to finding δ_a that would force the aircraft roll rate p track any bounded, possibly time-varying, reference command p_{ref}. This is the model reference control design task. Sometimes, it is also referred to as the model following control. Using this concept allows the designer to create controllers whose main task is to asymptotically match a given reference model behavior. Let us now explore details of the model reference control design.

Comparing the roll dynamics (7.1) to that of the reference model (7.2), it is easy to see that a control solution can be formulated in the feedback-feedforward form

$$\delta_a = \left(\frac{a_{ref} - L_p}{L_{\delta_a}}\right) p + \left(\frac{b_{ref}}{L_{\delta_a}}\right) p_{cmd} \tag{7.3}$$

where $k_p = \left(\frac{a_{ref} - L_p}{L_{\delta_a}}\right)$ is the roll rate feedback gain, and $k_{p_{cmd}} = \left(\frac{b_{ref}}{L_{\delta_a}}\right)$ is the command feedforward gain. In fact, substituting the controller (7.3) into the roll dynamics (7.1), gives the desired closed-loop system dynamics.

$$\dot{p} = a_{ref}\, p + b_{ref}\, p_{cmd} \tag{7.4}$$

In order to formally assess if (7.4) indeed converges to (7.2), we first define the roll rate tracking error,

$$e = p - p_{ref} \tag{7.5}$$

and then compute the tracking error dynamics by differentiating e with respect to time, while substituting (7.4) and (7.2).

$$\dot{e} = \dot{p} - \dot{p}_{ref} = a_{ref}\left(p - p_{ref}\right) = a_{ref}\, e \tag{7.6}$$

Since by definition $a_{ref} < 0$ (e.g., the reference model is exponentially stable), the error dynamics (7.6) are globally exponentially stable. Therefore, given any initial values $p(0)$ and $p_{ref}(0)$, the tracking error $e(t)$ will converge to the origin exponentially fast,

$$e(t) = \exp\left(a_{ref}\, t\right) e(0) \tag{7.7}$$

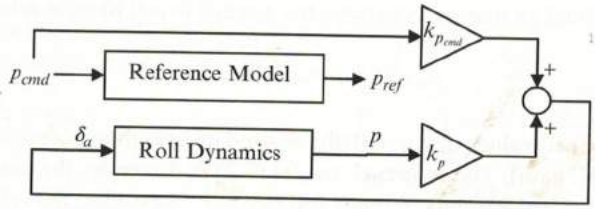

Fig. 7.2 Block diagram of the closed-loop roll dynamics with fixed-gain model reference controller obtained in Example 7.1

starting at any initial tracking error value $e(0) = p(0) - p_{ref}(0)$. So, the aircraft roll rate $p(t)$ will track the reference roll rate $p_{ref}(t)$, with the exponentially fast decaying tracking error $e(t)$,

$$p(t) = p_{ref}(t) + \exp\left(a_{ref}\, t\right)\left(p(0) - p_{ref}(0)\right) \tag{7.8}$$

and this closed-loop tracking performance is valid for any constant or bounded time-varying command $p_{cmd} = p_{cmd}(t)$. The command tracking problem is solved. The corresponding closed-loop system block diagram with the fixed-gain model reference controller (7.3) is shown in Fig. 7.2.

The model reference controller (7.3) is by no means unique in solving the command tracking problem of interest. Other solutions can be found. For example, any controller in the form

$$\delta_a = k_p\, p + k_{p_{cmd}}\, p_{cmd} - k_e\left(p - p_{ref}\right) \tag{7.9}$$

solves the same tracking problem, where $k_e \geq 0$ represents the error feedback gain.

However, does the error feedback in (7.9) give any advantage over the original controller (7.3)? In order to answer that question, let us calculate the error dynamics obtained using the modified controller (7.9).

$$\dot{e} = \left(a_{ref} - k_e\right) e \tag{7.10}$$

Consequently,

$$p(t) = p_{ref}(t) + \exp\left(\left(a_{ref} - k_e\right) t\right)\left(p(0) - p_{ref}(0)\right) \tag{7.11}$$

By definition, the error dynamics (7.10) define the transients that are incurred by the system while tracking a given reference command $p_{ref}(t)$. It is now evident that choosing $k_e > 0$ sufficiently large will allow the designer to obtain any desired (fast) transient dynamics. This constitutes the primary advantage of using an error feedback gain in the fixed-gain model reference controller (7.9). Figure 7.3 shows the resulting closed-loop system diagram.

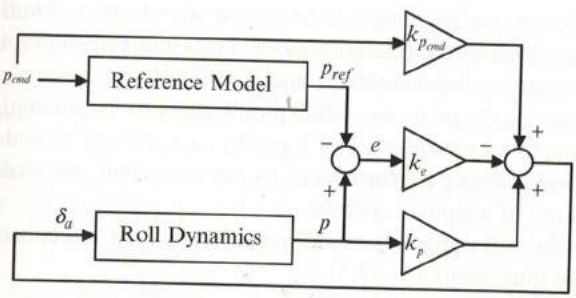

Fig. 7.3 Closed-loop system block diagram with fixed-gain model reference controller and error feedback obtained in Example 7.1

Of course, practical limitations, as well as stability robustness considerations, will place upper and lower limits on the selection of the controller gains. Eventually, these restrictions will dictate the trade-off between achievable transients in the closed-loop system and adequate stability robustness margins. □

7.2 Introduction to Direct Model Reference Adaptive Control

In the roll control example above, we have assumed that the system dynamics (7.1) (defined by the aircraft aerodynamics) were completely known. Then, we utilized the roll damping L_p and the aileron control effectiveness L_{δ_a} to design the two fixed-gain model reference controllers, (7.3) and (7.9).

In reality, aerodynamic parameters are rarely known exactly. This type of uncertainty is called parametric. If the true parameters are substantially different from their assumed constant values, controllers such as (7.9) can lead to instabilities in the system. Even when the system remains stable in the presence of parametric uncertainties, its closed-loop tracking performance may deteriorate to a point of becoming unacceptable.

Robustness considerations may not always solve the parameter sensitivity problem. Often, robust controllers will have a conservatism built into their design, and as such, they may not be able to provide adequate tracking performance, when operating under specific parametric uncertainties. This leads to the idea of adding a gain adaptation mechanism and arriving at model reference adaptive controllers.

Example 7.2 Model Reference Adaptive Control of Aircraft Roll Dynamics Suppose that the two aerodynamic parameters, L_p and L_{δ_a}, in the roll dynamics (7.1) are constant but otherwise completely unknown, with the exception that we do know the sign of the aileron control effectiveness L_{δ_a} (it is positive for a conventional aircraft). The control task remains the same as in Example 7.1 – we need to find δ_a such that p tracks p_{ref}, which in turn is driven by a bounded possibly time-varying command p_{cmd}.

The main control challenge here is to achieve the desired closed-loop tracking performance, specified by the reference model (7.2) while operating in the presence of constant parametric uncertainties L_p and L_{δ_a}.

In the forthcoming chapters, we will exploit Lyapunov-based methods that allow us to design adaptive controllers with formal guarantees of closed-loop stability, boundedness, and tracking performance. In the meantime, we shall outline main ideas in the design of adaptive systems.

If we knew the roll dynamics model parameters, then a feedback-feedforward controller in the form similar to (7.3)

$$\delta_a = k_p\, p + k_{p_{cmd}}\, p_{cmd} \tag{7.12}$$

would have solved the tracking problem. Since the system parameters are unknown, the ideal controller gains, k_p and $k_{p_{cmd}}$, cannot be computed directly as in Example 7.1. Instead, we consider an adaptive controller in the form

$$\delta_a = \hat{k}_p\, p + \hat{k}_{p_{cmd}}\, p_{cmd} \tag{7.13}$$

where $\left(\hat{k}_p,\ \hat{k}_{p_{cmd}}\right)$ represent the estimated feedback and feedforward gains, in that order. Substituting (7.13) into (7.1) gives the closed-loop system.

$$\dot{p} = \left(L_p + L_{\delta_a}\, \hat{k}_p\right) p + \left(L_{\delta_a}\, \hat{k}_{p_{cmd}}\right) p_{cmd} \tag{7.14}$$

Using parameterization (7.3), the reference model dynamics (7.2) can be equivalently written in terms of the ideal unknown gains as

$$\dot{p}_{\text{ref}} = \underbrace{\left(L_p + L_{\delta_a}\, k_p\right)}_{a_{ref}} p_{ref} + \underbrace{\left(L_{\delta_a}\, k_{p_{cmd}}\right)}_{b_{ref}} p_{cmd} \tag{7.15}$$

We now define the gain estimation errors,

$$\Delta k_p = \hat{k}_p - k_p, \quad \Delta k_{p_{cmd}} = \hat{k}_{p_{cmd}} - k_{p_{cmd}} \tag{7.16}$$

and rewrite the closed-loop system (7.14) in the following form:

$$\dot{p} = \underbrace{\left(L_p + L_{\delta_a}\, k_p\right)}_{a_{ref}} p + \underbrace{\left(L_{\delta_a}\, k_{p_{cmd}}\right)}_{b_{ref}} p_{cmd} + L_{\delta_a}\left(\Delta k_p\, p + \Delta k_{p_{cmd}}\, p_{cmd}\right) \tag{7.17}$$

Subtracting (7.15) from (7.17) gives the tracking error dynamics.

$$\dot{e} = a_{ref}\, e + L_{\delta_a}\left(\Delta k_p\, p + \Delta k_{p_{cmd}}\, p_{cmd}\right) \tag{7.18}$$

There are three error signals in the error dynamics (7.18): (1) the roll rate tracking error e, (2) the feedback gain estimation error Δk_p, and (3) the feedforward gain estimation error $\Delta k_{p_{cmd}}$. We are going to devise adaptive laws for changing the gains $\left(\hat{k}_p,\ \hat{k}_{p_{cmd}}\right)$, such that all these three errors tend to zero, globally and asymptotically.

In order to do that, we first define a scalar function V, representative of the total "kinetic energy" of all the errors in the system.

$$V\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = \frac{e^2}{2} + \frac{|L_{\delta_a}|}{2\gamma_p}\,\Delta k_p^2 + \frac{|L_{\delta_a}|}{2\,\gamma_{p_{cmd}}}\,\Delta k_{p_{cmd}}^2 \tag{7.19}$$

The "energy" function represents a weighted sum of squares of all the errors in the system. This is the so-called Lyapunov function candidate, and the positive constant scalar weights $\left(\gamma_p,\ \gamma_{p_{cmd}}\right)$ will eventually become the rates of adaptation. We can easily evaluate the time derivative of V.

$$\dot{V}\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = e\,\dot{e} + \frac{|L_{\delta_a}|}{\gamma_p}\,\Delta k_p\,\dot{\hat{k}}_p + \frac{|L_{\delta_a}|}{\gamma_{p_{cmd}}}\,\Delta k_{p_{cmd}}\,\dot{\hat{k}}_{p_{cmd}} \tag{7.20}$$

This is the system "power." Substituting (7.18) into (7.20) yields the time derivative of V, along the trajectories of the error dynamics (7.18) but without explicit knowledge of these trajectories.

$$\begin{aligned}
&\dot{V}\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = a_{ref}\,e^2 \\
&\quad + e\,L_{\delta_a}\left(\Delta k_p\,p + \Delta k_{p_{cmd}}\,p_{cmd}\right) + \frac{|L_{\delta_a}|}{\gamma_p}\,\Delta k_p\,\dot{\hat{k}}_p + \frac{|L_{\delta_a}|}{\gamma_{p_{cmd}}}\,\Delta k_{p_{cmd}}\,\dot{\hat{k}}_{p_{cmd}}
\end{aligned} \tag{7.21}$$

Rearranging terms, we further get

$$\begin{aligned}
&\dot{V}\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = a_{ref}\,e^2 \\
&\quad + \Delta k_p\,|L_{\delta_a}|\left(\operatorname{sgn}(L_{\delta_a})\,p\,e + \frac{\dot{\hat{k}}_p}{\gamma_p}\right) + \Delta k_{p_{cmd}}\,|L_{\delta_a}|\left(\operatorname{sgn}(L_{\delta_a})\,p_{cmd}\,e + \frac{\dot{\hat{k}}_{p_{cmd}}}{\gamma_{p_{cmd}}}\right)
\end{aligned} \tag{7.22}$$

We want the energy function V to dissipate in time. It is then sufficient to require that its derivative $\dot{V}$ (the system power) be nonpositive, when evaluated along the system trajectories. The nonpositivity of $\dot{V}$ can be easily achieved if we select the following adaptive laws:

$$\begin{aligned}
\dot{\hat{k}}_p &= -\gamma_p\,p\;e\;\operatorname{sgn}(L_{\delta_a}) \\
\dot{\hat{k}}_{p_{cmd}} &= -\gamma_{p_{cmd}}\,p_{cmd}\;e\;\operatorname{sgn}(L_{\delta_a})
\end{aligned} \tag{7.23}$$

or, equivalently,

$$\begin{aligned} \dot{\hat{k}}_p &= -\gamma_p \, p \, e \\ \dot{\hat{k}}_{p_{cmd}} &= -\gamma_{p_{cmd}} \, p_{cmd} \, e \end{aligned} \tag{7.24}$$

thus making the second and the third terms in (7.22) disappear. Then,

$$\dot{V}\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = a_{ref}\, e^2 \le 0 \tag{7.25}$$

and consequently, the system kinetic energy V is a nonincreasing function of time. This fact immediately implies that all the signals in the error dynamics (7.18), such as $\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right)$, are bounded functions of time. Furthermore, since the ideal gains $\left(k_p,\ k_{p_{cmd}}\right)$ are constant, the adaptive gains $\left(\hat{k}_p,\ \hat{k}_{p_{cmd}}\right)$ are also bounded.

The stable (by design) reference model (7.2), when driven by a bounded command p_{cmd}, gives a bounded output p_{ref}. Also, e was proven to be bounded. Then, the roll rate p is bounded. Consequently, the control input δ_a in (7.13) and the roll acceleration $\dot{p}$ in the system dynamics (7.1) are bounded. Furthermore, since $\dot{p}_{ref}$ is bounded, then $\dot{e}$ is bounded, and so

$$\ddot{V}\left(e,\ \Delta k_p,\ \Delta k_{p_{cmd}}\right) = 2\, a_{ref}\, e\, \dot{e} \tag{7.26}$$

is a uniformly bounded function of time. The latter implies that $\dot{V}$ is a uniformly continuous function of time.

By definition (7.19), $V \ge 0$ and because of (7.25), V is a nonincreasing function of time. Therefore, V tends to a limit as $t \to \infty$, where the function limiting value may not necessarily be zero.

We have shown that $0 \le \lim\limits_{t\to\infty} V\left(e(t),\ \Delta k_p(t),\ \Delta k_{p_{cmd}}(t)\right) < \infty$ and $\dot{V}$ are uniformly continuous. According to Barbalat's lemma (see Chap. 8), these two facts imply that the system power $\dot{V}$ in (7.25) asymptotically tends to zero, which in turn means

$$\lim_{t\to\infty} e(t) = 0 \tag{7.27}$$

Thus, the adaptive controller (7.13), along with the adaptive laws (7.24), forces p track its reference signal p_{ref} asymptotically and for any initial conditions (globally). At the same time, all signals in the corresponding closed-loop system remain uniformly bounded. These arguments prove closed-loop stability and tracking performance of the closed-loop system with the adaptive controller. The corresponding block diagram is shown in Fig. 7.4.

As seen from the figure, the closed-loop system is comprised of the original roll dynamics (7.1) operating under the adaptive controller (7.13), with the reference model dynamics (7.2), and using the adaptive laws (7.24). Here, the external input is the roll rate command p_{cmd}.

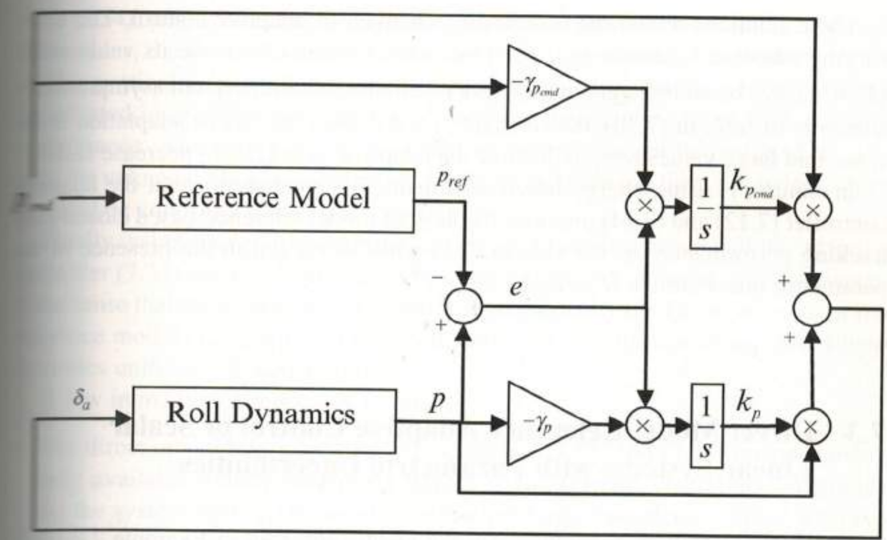

Fig. 7.4 Model reference adaptive controller obtained in Example 7.2

$$
\begin{aligned}
\dot{p} &= \left(L_p + L_{\delta_a}\hat{k}_p\right)p + L_{\delta_a}\hat{k}_{p_{cmd}}\,p_{cmd} \\
\dot{p}_{\text{ref}} &= a_{ref}\,p_{ref} + b_{ref}\,p_{cmd} \\
\dot{\hat{k}}_p &= -\gamma_p\,p\,\left(p - p_{ref}\right) \\
\dot{\hat{k}}_{p_{cmd}} &= -\gamma_{p_{cmd}}\,p_{cmd}\,\left(p - p_{ref}\right)
\end{aligned} \tag{7.28}
$$

Equivalently, this system can be written in terms of the tracking and parameter estimation errors.

$$
\begin{aligned}
\dot{e} &= \left(a_{ref} + L_{\delta_a}\Delta k_p\right)e + L_{\delta_a}\left(\Delta k_p\,p_{ref} + \Delta k_{p_{cmd}}\,p_{cmd}\right) \\
\frac{d}{dt}\left(\Delta k_p\right) &= -\gamma_p\left(e + p_{ref}\right)\,e \\
\frac{d}{dt}\left(\Delta k_{cmd}\right) &= -\gamma_{p_{cmd}}\,p_{cmd}\,e
\end{aligned} \tag{7.29}
$$

If instead of command tracking, the state regulation is of interest, then $p_{ref} = p_{cmd} = 0$, and so $\hat{k}_{p_{cmd}} = k_{p_{cmd}} = 0$. In this case, the closed-loop systems (7.28) and (7.29) simplify to the following time-invariant second-order inherently nonlinear dynamics,

$$
\begin{aligned}
\dot{p} &= \left(L_p + L_{\delta_a}\hat{k}_p\right)p \\
\dot{\hat{k}}_p &= -\gamma_p\,p^2
\end{aligned} \tag{7.30}
$$

These relations reveal the essential mechanism of adaptive control. The time-varying adaptive feedback gain $\hat{k}_p(t)$ will monotonically decrease its value until $(L_p + L_{\delta_a}\hat{k}_p)$ becomes negative, and as a result, the roll rate $p(t)$ will asymptotically converge to zero. In (7.30), the constant $\gamma_p > 0$ defines the rate of adaptation in the sense that large values of γ_p will force the adaptive gain $\hat{k}_p(t)$ to decrease faster.

In summary, using energy-based arguments, we have shown that the adaptive controller (7.12) and (7.24) provides the desired model reference-based closed-loop tracking performance for the system (7.1) while operating in the presence of the parametric uncertainties $(L_p,\ L_{\delta_a})$. □

7.3 Direct Model Reference Adaptive Control of Scalar Linear Systems with Parametric Uncertainties

Let us now generalize and summarize the results obtained in Example 7.2 while restating them for a generic class of scalar linear-time-invariant uncertain systems in the form

$$\dot{x} = a\,x + b\,u \tag{7.31}$$

where $x \in R$ is the systems state, $u \in R$ is the control input, and $(a,\ b)$ represent the parametric uncertainties, (constant and unknown), with the known sgn$b$.

First, we choose the desired reference model,

$$\dot{x}_{ref} = a_{ref}\,x_{ref} + b_{ref}\,r \tag{7.32}$$

with $a_{ref} < 0$. This model is driven by any bounded, possibly time-varying reference command r. The model parameters $(a_{ref},\ b_{ref})$ must be chosen such that x_{ref} tracks r, with the designer specified criteria. For example, one might set $b_{ref} = -a_{ref}$ in order to enforce the unity DC gain from r to x_{ref}. Also, the value of $|a_{ref}|$ can be chosen such that the desired inverse time constant of the reference model is achieved.

Second, we define the model reference adaptive controller as a linear combination of feedback and feedforward terms,

$$u = \hat{k}_x\,x + \hat{k}_r\,r \tag{7.33}$$

where $(\hat{k}_x,\ \hat{k}_r)$ are the two adaptive gains, whose adaptive law dynamics are constructed similar to (7.24).

$$\begin{aligned}\dot{\hat{k}}_x &= -\gamma_x\,x\,\left(x - x_{ref}\right)\,\mathrm{sgn}(b)\\ \dot{\hat{k}}_r &= -\gamma_r\,r\,\left(x - x_{ref}\right)\,\mathrm{sgn}(b)\end{aligned} \tag{7.34}$$

In (7.34), positive scalars $(\gamma_x,\ \gamma_r)$ are called the rates of adaptation. The larger their values, the faster the system will adapt to the parametric uncertainties.

This particular controller is called "direct" to indicate that the controller gains are adapted in (7.34) directly in order to enforce the desired closed-loop tracking performance. Alternatively, indirect adaptive controllers can be designed to estimate the unknown plant parameters $(a,\ b)$ online and then use their estimated values to calculate controller gains.

Finally, using energy-based arguments, we can formally prove that the adaptive controller (7.33) and (7.34) provides the desired closed-loop tracking performance, in the sense that the system state x globally asymptotically tracks the state x_{ref} of the reference model (7.32) while keeping all signals in the corresponding closed-loop dynamics uniformly bounded in time.

A few immediate remarks are in order:

- The direct model reference adaptive controller (7.33) and (7.34) operates using only available (online measured) signals in the system. The latter consists of: (a) the system state x, (b) the state of the reference model x_{ref}, (c) the tracking error $e = x - x_{ref}$, and (d) the sign of the control effectiveness sgn$b$.
- All signals in the closed-loop system remain uniformly bounded in time.
- The system state x tracks the state of the reference model $x_{ref,}$ globally and asymptotically. However, a characterization of the system transient dynamics in model reference adaptive control remains an open problem.
- The adaptive parameters $(\hat{k}_x,\ \hat{k}_r)$ are not guaranteed to converge to their true unknown values $(k_x,\ k_r)$ nor are they assured to converge to constant values in any way. All that is known is that these parameters remain uniformly bounded in time. Sufficient conditions for parameter convergence are known as persistency of excitation [1, 2]. It turns out that for a first-order linear system such as (7.1), persistent excitation is guaranteed if the commanded signal $r(t)$ contains at least one sinusoidal component. In this case, the two adaptive gains $(\hat{k}_x,\ \hat{k}_r)$ will converge to their true constant unknown values, exponentially fast.

7.4 Historical Roots and Foundations of Model Reference Adaptive Control

The adaptive control development was largely motivated in the early 1950s by the design of autopilots for aircraft that operated in a wide flight envelope, with a large range of speeds and altitudes. Different flight conditions caused the aircraft dynamics to change significantly. This phenomenon called for flight controllers that could accommodate drastic changes in the aircraft aerodynamic and propulsive forces and moments. Adaptive control was proposed as one of the design approaches to solving the flight control problem.

The concept of a model-reference adaptive system (MRAS) was originally proposed in 1958 by Whitaker et al. at MIT [3, 4]. The main idea behind this