Chapter 4 Homework, 7th edition, Fall 2016

4.

Using voltage division, $\dfrac{V_C(s)}{V_i(s)} = \dfrac{1/RC}{s+\dfrac{1}{RC}} = \dfrac{1}{s+1}$. Since $V_i(s) = \dfrac{5}{s}$

$$V_C(s) = \frac{5}{s}\left(\frac{1}{s+1}\right) = \frac{5}{s} - \frac{5}{s+1}.$$

Therefore: $v_c(t) = 5 - 5e^{-t}$.

Also, $T = \dfrac{1}{1} = 1\,\text{sec}; T_r = \dfrac{2.2}{1} = 2.2\,\text{sec}; T_s = \dfrac{4}{1} = 4\,\text{sec}$.

25.

a. Adding impedances $(5s^2 + 2s + 20)X(s) = F(s)$. So the transfer function is

$$\frac{X(s)}{F(s)} = \frac{1}{5s^2 + 2s + 20} = \frac{0.2}{s^2 + 0.4s + 4}.$$

b. Follows that $\omega_n^2 = 4$, or $\omega_n = 2$. $2\zeta\omega_n = 0.4$, so $\xi = 0.1$. $\%OS = 100e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 72.93\%$. $T_s = \dfrac{4}{\xi\omega_n} = 20$ sec. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1.58$ sec. To obtain the rise time Figure 4.16 is used $\omega_n T_r = 1.104$, or $T_r = 0.552$ sec. The dc gain of the system is $c_{max} = \dfrac{0.2}{4} = 0.05$.

35. (a) Does not meet the x5 rule (page 184): 4 < 5*(3)

Residue is not negligible (section 4.8):

$0.1 * \sqrt{0.001976^2 + 0.0005427^2} = 0.0002$ (using 10% as cutoff)

0.001524 > 0.0002

2nd order approximation is not valid

(b) Does not meet the x5 rule: 8 < 5*(6)

Residue is not negligible (section 4.8):

$0.1 * \sqrt{0.007647^2 + 0.01309^2} = 0.0015$ (using 10% as cutoff)

0.007 > 0.0015

2nd order approximation is not valid

(c) Does not meet the x5 rule (5.1 < 5*2)

Residue is negligible

$$0.1 * \sqrt{0.009990^2 + 0.001942^2} = 0.0022$$ (using 10% as cutoff)

0.00018 < 0.0022 (actually is less than 1%)

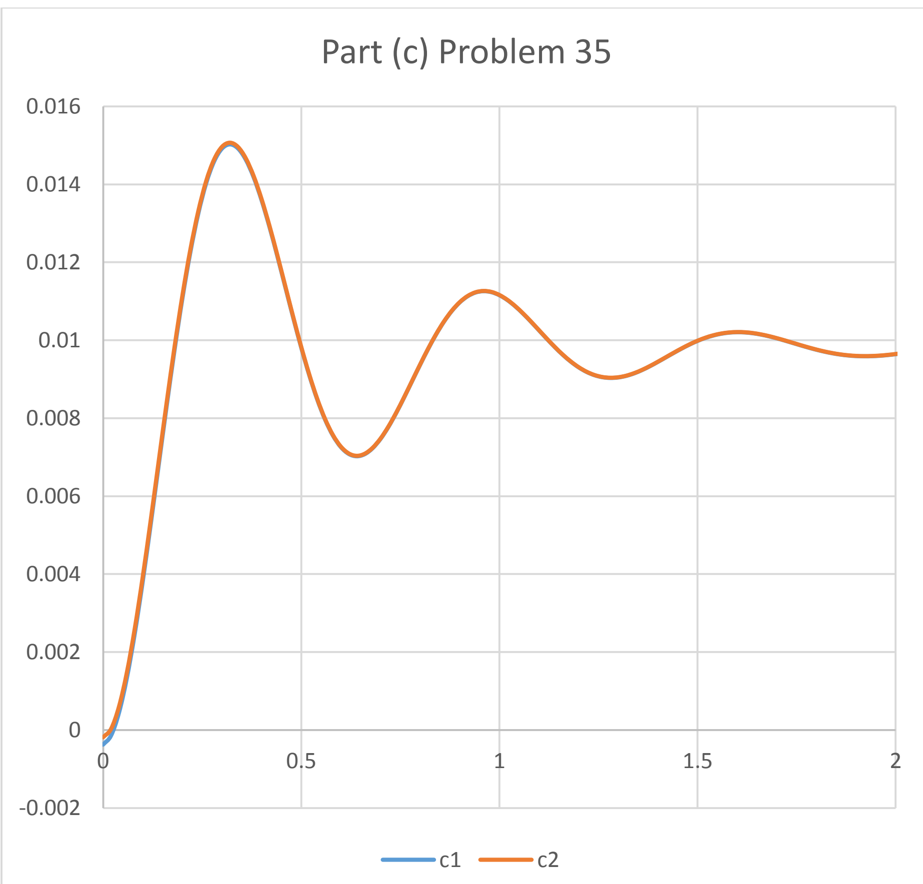
From the oscillatory part of c(t), the dominant poles are $s = -2 \pm j9.796$

The time constant is the reciprocal of the magnitude of the real part, ½ second. The settling time is four time constants: 4*(1/2) = **2 seconds = T_s**. The damped frequency is 9.796 rad/s. The peak time is one half of the period: Tp = π/ω_d = **0.3207 seconds = T_p**.

To find damping constant, use $\zeta\omega_n = 2$ *and* $\omega_n\sqrt{1-\zeta^2} = 9.796$. Rearranging yields

$$\zeta = \sqrt{2^2/(2^2 + 9.796^2)} = 0.200$$

Then %OS = $e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$x100 = 52.7%



Approximate curve in matlab is:

```
c2 = 0.009804-0.009990*exp(-2*t).*cos(9.796*t)
        -0.001942 *exp(  -2*t).*sin(9.796*t);
```

(d) Exactly meets the x5 rule: 10 = 5*2

That term also has a smaller residue, but only about five times smaller, so it does not meet the 10% cutoff.

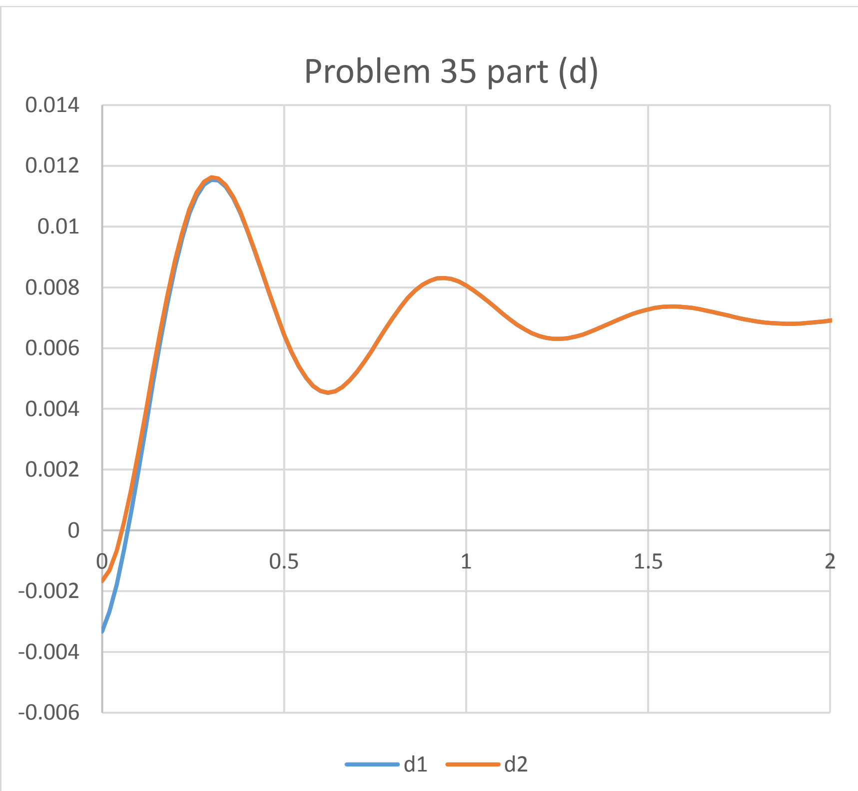
From the oscillatory part of c(t), the dominant poles are $s = -2 \pm j9.951$

The time constant is the reciprocal of the magnitude of the real part, ½ second. The settling time is four time constants: 4*(1/2) = **2 seconds = T_s**. The damped frequency is 9.951 rad/s. The peak time is one half of the period: Tp = π/ω_d = **0.3157 seconds = T_p**.

To find damping constant, use $\zeta\omega_n = 2 \; and \; \omega_n\sqrt{1-\zeta^2} = 9.951$. Rearranging yields

$$\zeta = \sqrt{2^2/(2^2 + 9.951^2)} = 0.197$$

Then %OS = $e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$x100 = 53.1%



As in part (c), dropping the term with a small residue and rapid decay rate.

42.

$$\boldsymbol{X}(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\left(\boldsymbol{x}(\boldsymbol{0}) + \boldsymbol{B}u(t)\right) = \begin{bmatrix} s+3 & 0 \\ 1 & s+1 \end{bmatrix}^{-1} \left[\begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \frac{1}{s} \right]$$

$$= \begin{bmatrix} \dfrac{1}{s+3} & 0 \\ -\dfrac{1}{(s+1)(s+3)} & \dfrac{1}{s+1} \end{bmatrix} \begin{bmatrix} \dfrac{2s+2}{s} \\ \dfrac{1}{s} \end{bmatrix} = \begin{bmatrix} \dfrac{2(s+1)}{s(s+3)} \\ -\dfrac{s-1}{s(s+1)(s+3)} \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{2(s+1)}{s(s+3)} \\ -\dfrac{s-1}{s(s+1)(s+3)} \end{bmatrix} = \frac{2(s+1)}{s(s+3)} = \frac{2}{3}\frac{1}{s} + \frac{5}{3}\frac{1}{s+3}$$

Obtaining the inverse Laplace transform

$$y(t) = \frac{2}{3} + \frac{5}{3}e^{-3t}$$

79.

Let the rotation of the shaft with gear N_2 be $\theta_L(s)$. Assuming that all rotating load has been reflected to the N_2 shaft, $\left(J_{eqL}s^2+D_{eqL}s+K\right)\theta_L(s)+F(s)r=T_{eq}(s)$, where F(s) is the force from the translational system, $r = 2$ is the radius of the rotational member, J_{eqL} is the equivalent inertia at the N_2 shaft, and D_{eqL} is the equivalent damping at the N_2 shaft. Since $J_{eqL} = 1(2)^2 + 1 = 5$ and $D_{eqL} = 1(2)^2 = 4$, the equation of motion becomes, $\left(5s^2+4s+K\right)\theta_L(s)+2F(s)=T_{eq}(s)$. For the translational system $(Ms^2+s)X(s)=F(s)$. Substituting $F(s)$ into the rotational equation of motion, $\left(5s^2+4s+K\right)\theta_L(s)+\left(Ms^2+s\right)2X(s)=T_{eq}(s)$. But,

$\theta_L(s)=\dfrac{X(s)}{r}=\dfrac{X(s)}{2}$ and $T_{eq}(s)=2T(s)$. Substituting these quantities in the equation above yields $\left((5+4M)s^2+8s+K\right)\dfrac{X(s)}{4}=T(s)$.

Thus, the transfer function is $\dfrac{X(s)}{T(s)}=\dfrac{4/(5+4M)}{s^2+\dfrac{8}{(5+4M)}s+\dfrac{K}{(5+4M)}}$.

Now, $T_s=20=\dfrac{4}{R_e}=\dfrac{4}{\dfrac{8}{2(5+4M)}}=(5+4M)$. Hence, $M = 15/4$.

For 16% overshoot, $\zeta = 0.504$ from Eq. (4.39).

Therefore, $2\zeta\omega_n=\dfrac{8}{(5+4M)}=0.4$. Solving for ω_n yields $\omega_n = 0.3968$.

But, $\omega_n=\sqrt{\dfrac{K}{(5+4M)}}=\sqrt{\dfrac{K}{20}}=0.3968$. Thus, $K = 3.15$.

85.

a.

Substituting $\Delta\ F(s) = \dfrac{2650}{s}$ into the transfer function and solving for $\Delta V(s)$ gives:

$$\Delta V(s) = \frac{\Delta F(s)}{1908 \cdot s} = \frac{2650}{s(1908 \cdot s + 10)} = \frac{A}{s} + \frac{B}{(1908 \cdot s + 10)}$$

Here: $A = \left.\dfrac{2650}{(1908 \cdot s + 10)}\right|_{s=0} = 265$ and $B = \left.\dfrac{2650}{s}\right|_{s = -1/190.8} = -505{,}620$

Substituting we have:

$$\Delta V(s) = \frac{265}{s} - \frac{505620}{(1908 \cdot s + 10)} = 265\left(\frac{1}{s} - \frac{1}{(s + 5.24 \times 10^{-3})}\right)$$

Taking the inverse Laplace transform, we have:

$$\Delta v(t) = 265(1 - e^{-5.24 \times 10^{-3} t}) \cdot u(t), \text{ in m/s}$$

b.

```
>> s=tf('s');
>> G=1/(1908*s+10);
>> t=0:0.1:1000;
>> y1=2650*step(G,t);
>> y2=265*(1-exp(-5.24e-3.*t));
>> plot(t,y1,t,y2)
>> xlabel('sec')
>> ylabel('m/s')
```