Analysis : Case 1

Uniform load, w:	10 N/mm
Length, L:	360 mm
Omit beam weight for this case	

C1 : Final Meshing

This mesh had two levels of refinement. For the larger blocks, the max element size was 16mm. For the smaller sections, the max element size was 8mm.

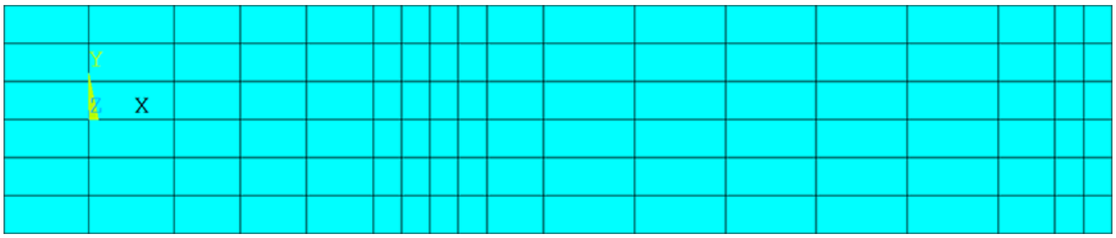


Fig. C1.1

C1 : Deflected shape of beam

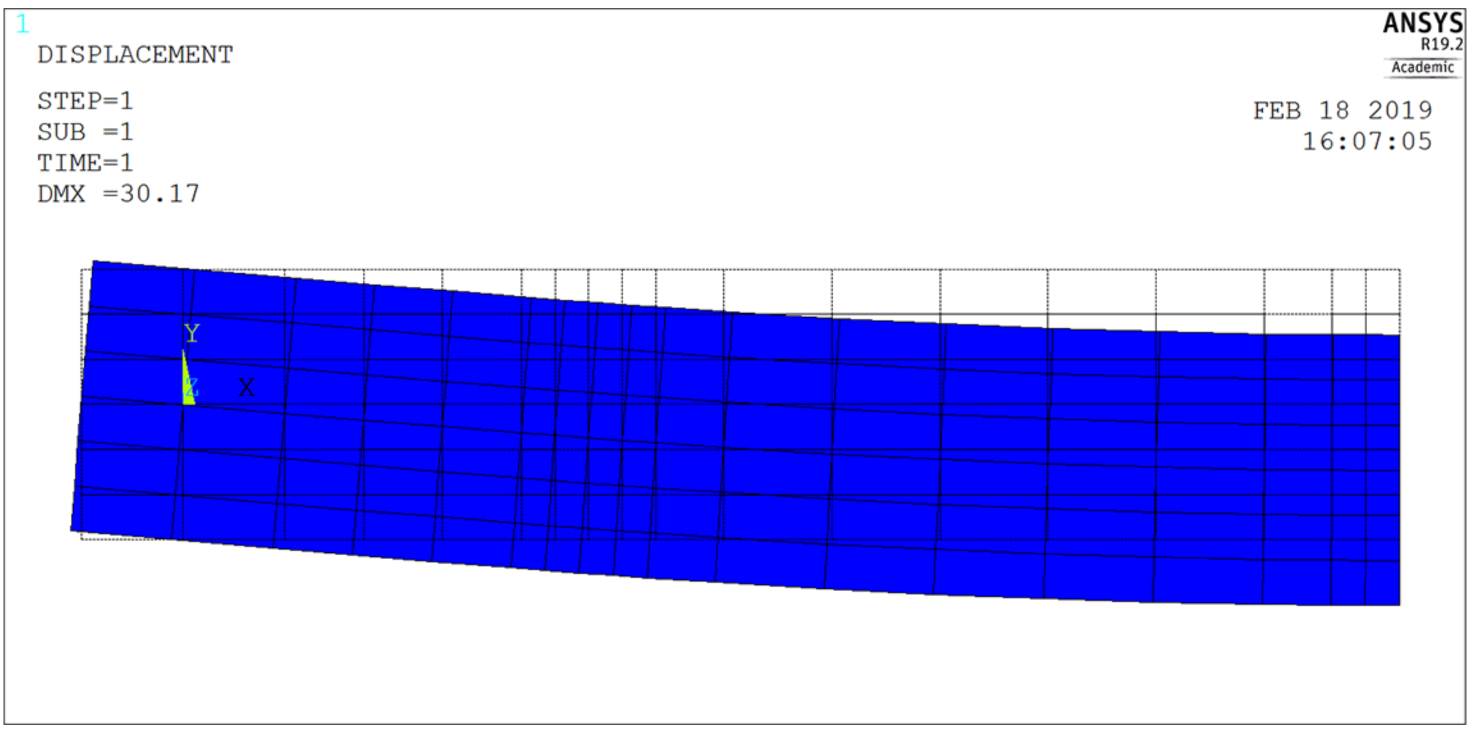


Fig. C1.2

C1 : Contour Plot of Normal Bending Stresses (σ_x)

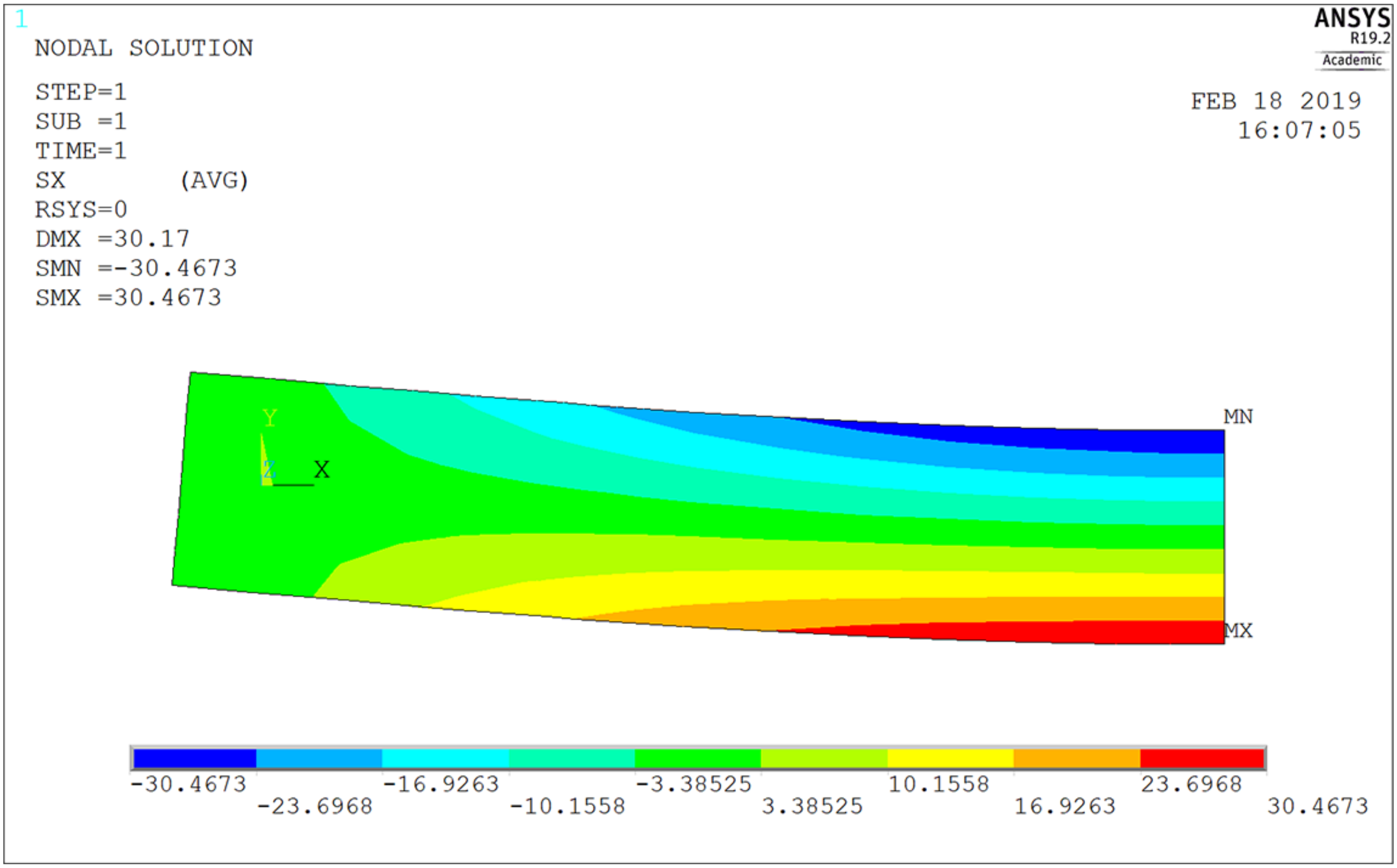


Fig. C1.3

C1 : Vertical Deflection at midspan from ANSYS Model

In order to easily review the nodes at location Q and the bottom of Cut 1, I created components that would allow me to call them. Fig. C1.4 shows the list of the components which allows me to identify which label corresponds to the nodes in Fig. C1.5.

```
LIST SELECTED COMPONENTS
ENTITY TYPE = NODE
   NAME        TYPE       SUBCOMPONENTS
 NODE_CUT_1    NODE        167

 NODE_Q        NODE        19
```

Fig. C1.4

```
PRINT U    NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE              UY
     19   -30.0155815879
    167   -15.2429198449

MAXIMUM ABSOLUTE VALUES
NODE          19
VALUE    -30.0155815879
```

Fig. C1.5

C1 : Distribution of Bending Stress at Cut 1 and Cut 2

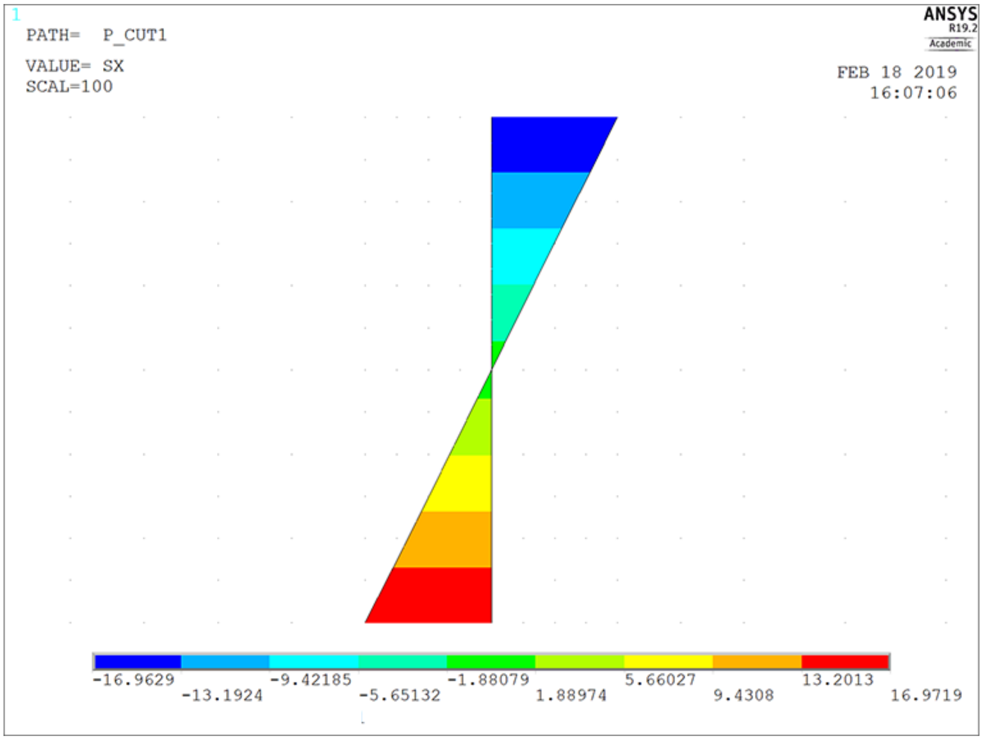


Fig. C1.6

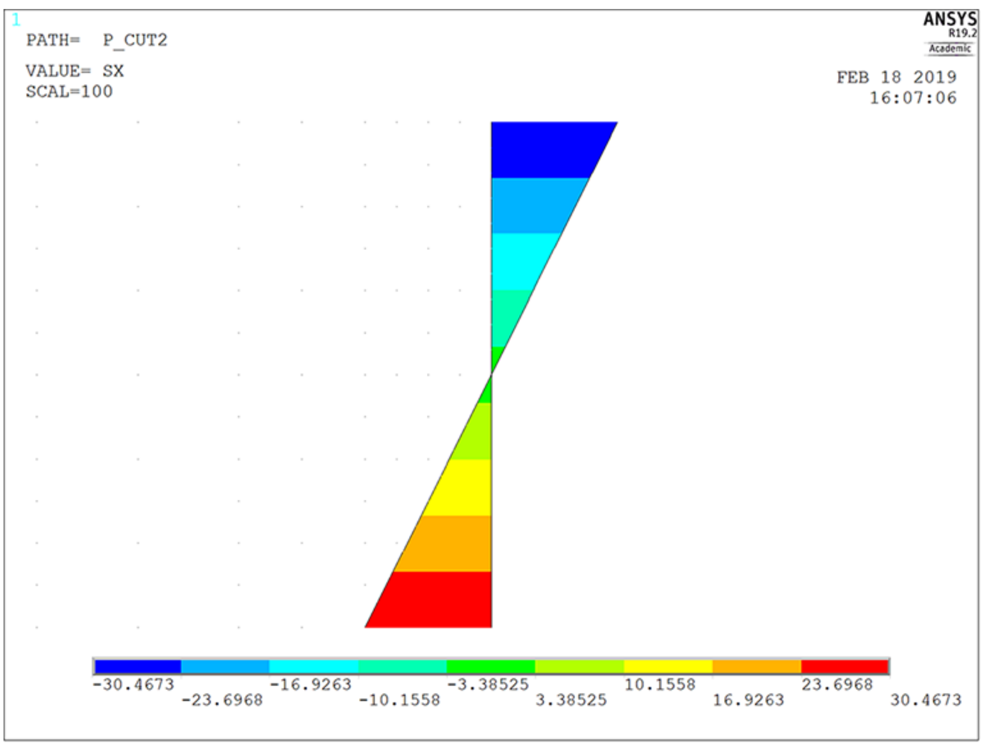


Fig. C1.7

C1 : Calculation of Deflection and Flexural Stress using Elementary Beam Theory

Common Properties			
Width	b	20	mm
Depth	h	40	mm
Modulus of Elasticity	E	700	N/mm
Moment of Inertia $= bh^3/12$	I	106667	Pa
Distance from N.A.	y	20	mm

Case 1 Unique Properties			
Uniform Load	ω	10	N/mm
Length	L	360	mm
Moment $=\omega L^2/8$	M	162000	Nm
Deflection at Midspan $= 5wL^4 /384EI$	δ	29.290	mm
Flexural Stress $=My/I$	σ	30.375	Pa

C1 : Comparison of Calcuations to ANSYS Results

	Calculation	ANSYS	Accuracy
Deflection at Midspan	29.290	30.0156	97.58%
Flexural Stress	30.375	30.4673	99.70%

Analysis : Case 2

Uniform load, w: .... 30 N/mm
Length, L: .... 120 mm
Omit beam weight for this case

C2 : Final Meshing

This mesh had two levels of refinement. For the larger blocks, the max element size was 4mm. For the smaller sections, the max element size was 2mm.

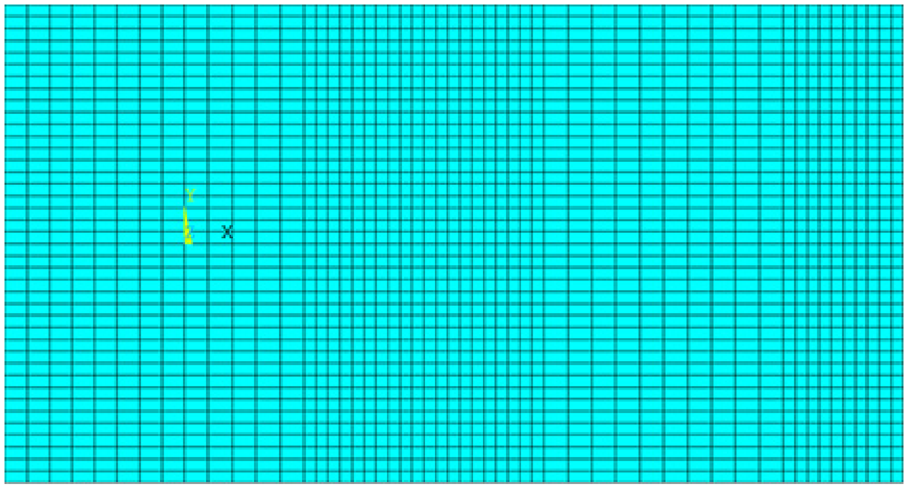


Fig. C2.1

C2 : Deflected shape of beam

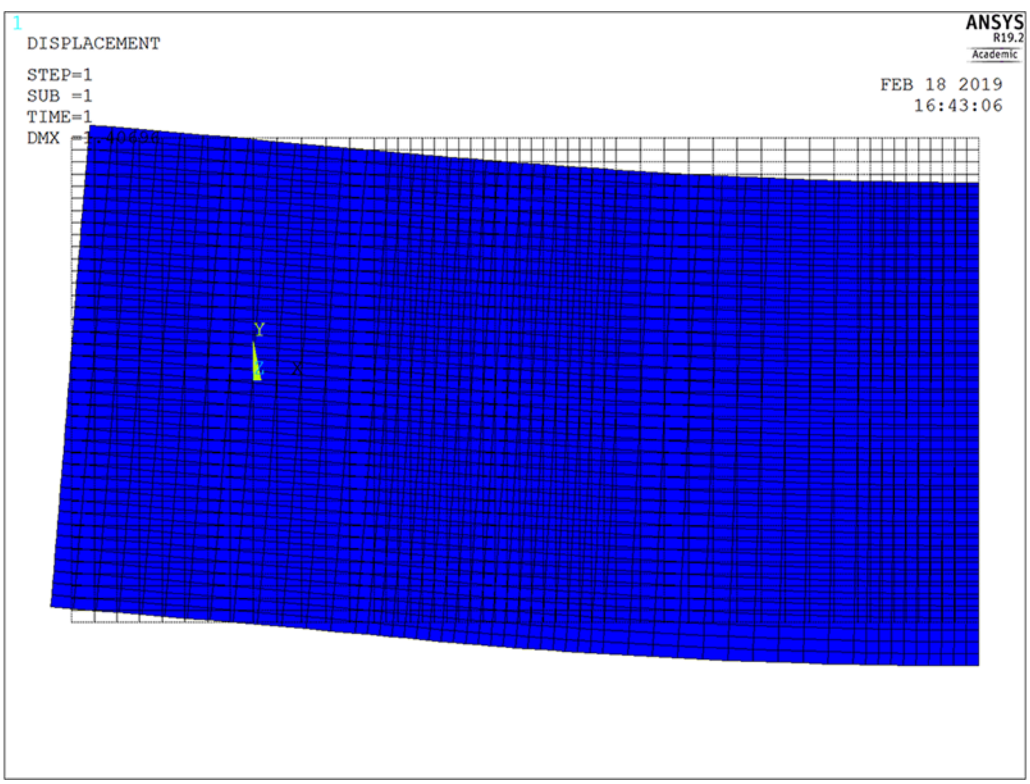


Fig. C2.2

C2 : Contour Plot of Normal Bending Stresses (σ_x)

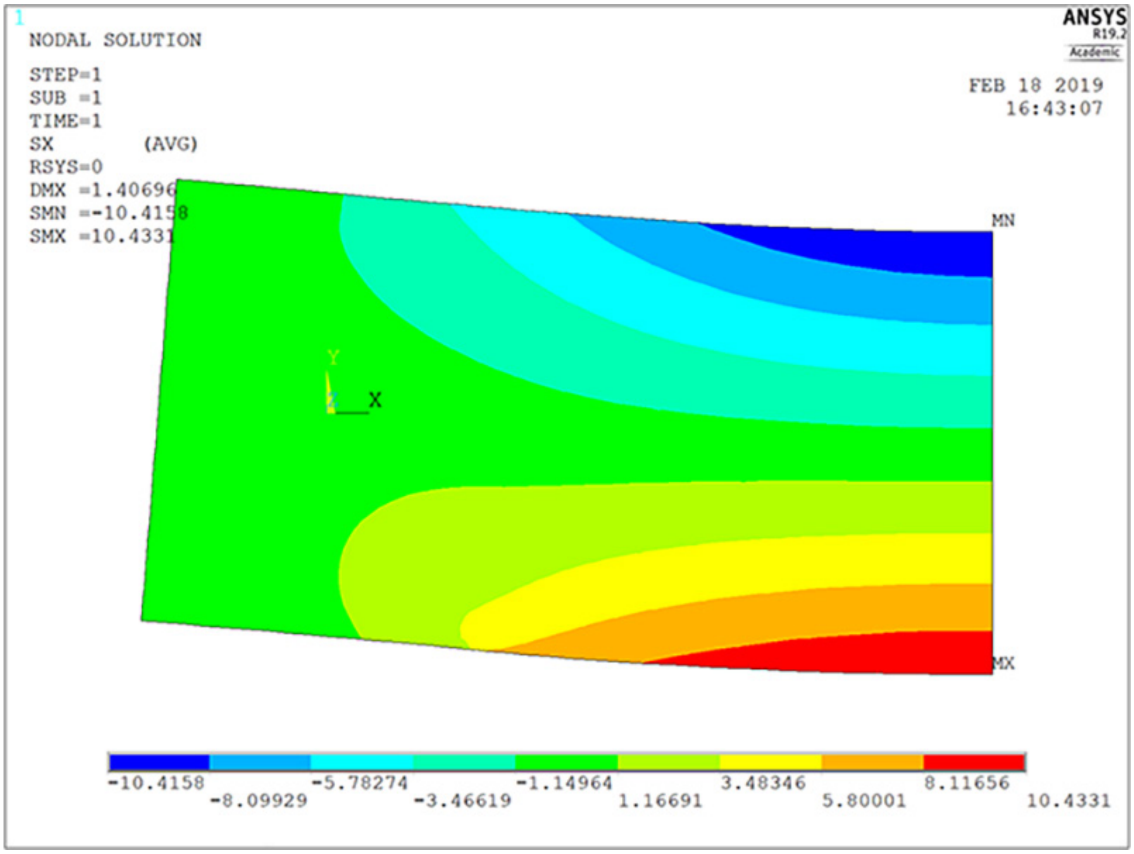


Fig. C2.3

C2 : Vertical Deflection at midspan from ANSYS Model

Fig. C2.4 shows the list of the components which allows me to identify which label corresponds to the nodes in Fig. C2.5.

```
LIST SELECTED COMPONENTS
ENTITY TYPE = NODE
   NAME        TYPE       SUBCOMPONENTS
 NODE_CUT_1    NODE        2970

 NODE_Q        NODE        538
```

Fig. C2.4

```
PRINT U    NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE              UY
    538   -1.34568286451
   2970  -0.697286570642

MAXIMUM ABSOLUTE VALUES
NODE         538
VALUE    -1.34568286451
```

Fig. C2.5

C2 : Distribution of Bending Stress at Cut 1 and Cut 2

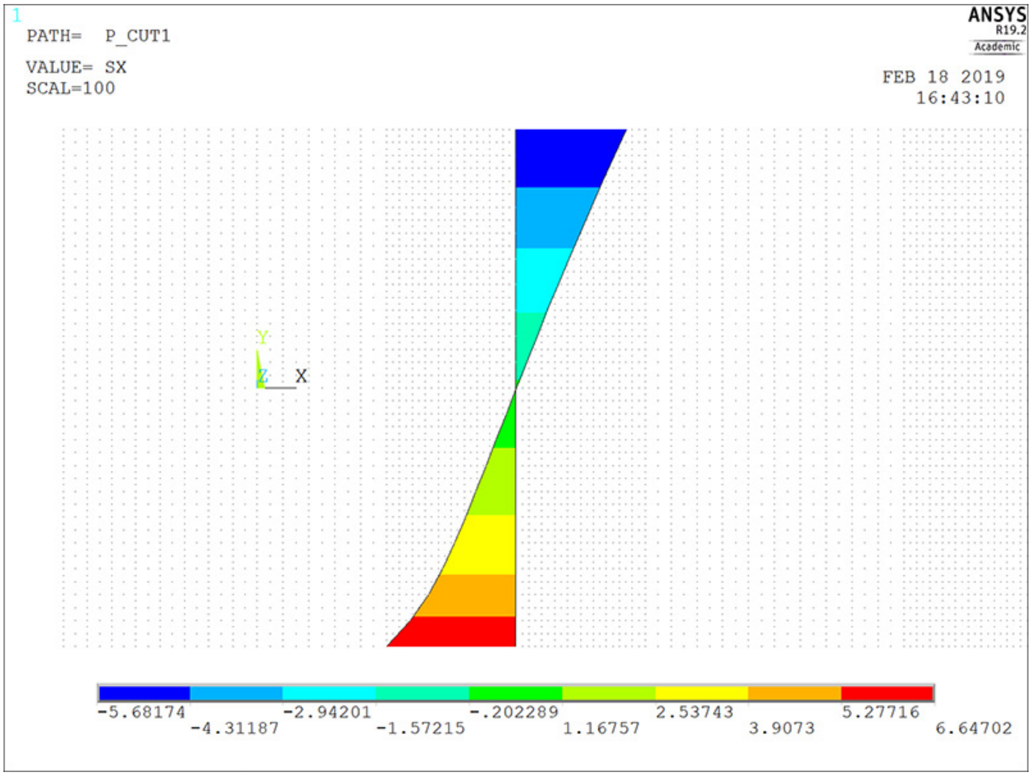


Fig. C2.6

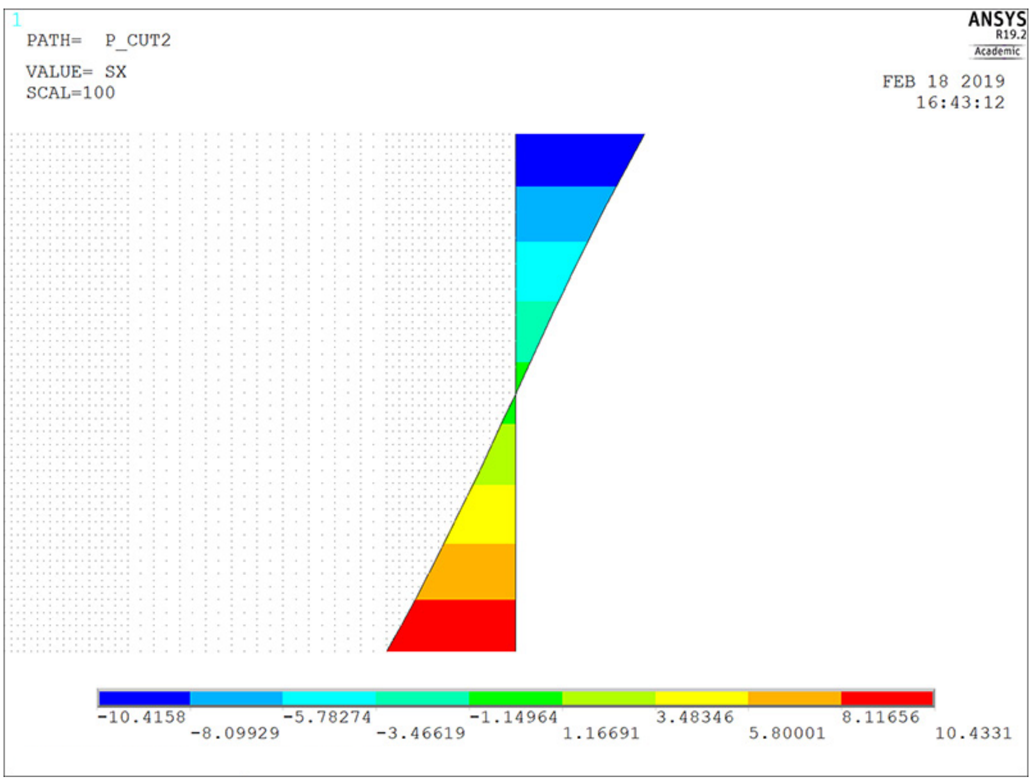


Fig. C2.7

C2 : Calculation of Deflection and Flexural Stress using Elementary Beam Theory

Common Properties			
Width	b	20	mm
Depth	h	40	mm
Modulus of Elasticity	E	700	N/mm
Moment of Inertia $= bh^3/12$	I	106667	Pa
Distance from N.A.	y	20	mm

Case 1 Unique Properties			
Uniform Load	ω	30	N/mm
Length	L	120	mm
Moment $=\omega L^2/8$	M	54000	Nm
Deflection at Midspan $= 5wL^4/384EI$	δ	1.085	mm
Flexural Stress $=My/I$	σ	10.125	Pa

C2 : Comparison of Calcuations to ANSYS Results

	Calculation	ANSYS	Accuracy
Deflection at Midspan	1.085	1.3457	80.61%
Flexural Stress	10.125	10.4331	97.05%

Analysis : Case 3

Length, L:	360 mm
Mass Density, ρ:	7.68E-05 N/mm3

C3 : Fundamental Natural Frequencies of the Beam System

Through four different substeps you are abble to see the different displacements and frequencies of the beam. Fig. C3.5 provides a table combining all of the numbers.

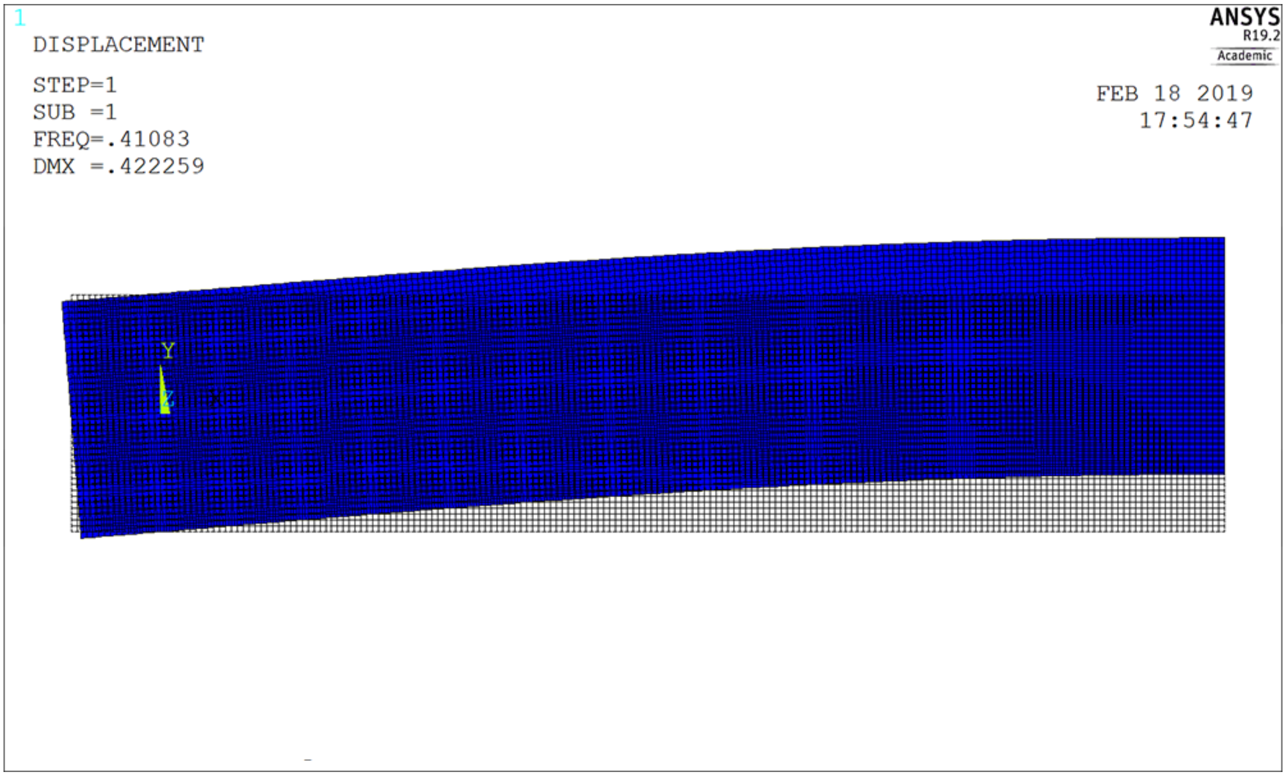


Fig. C3.1

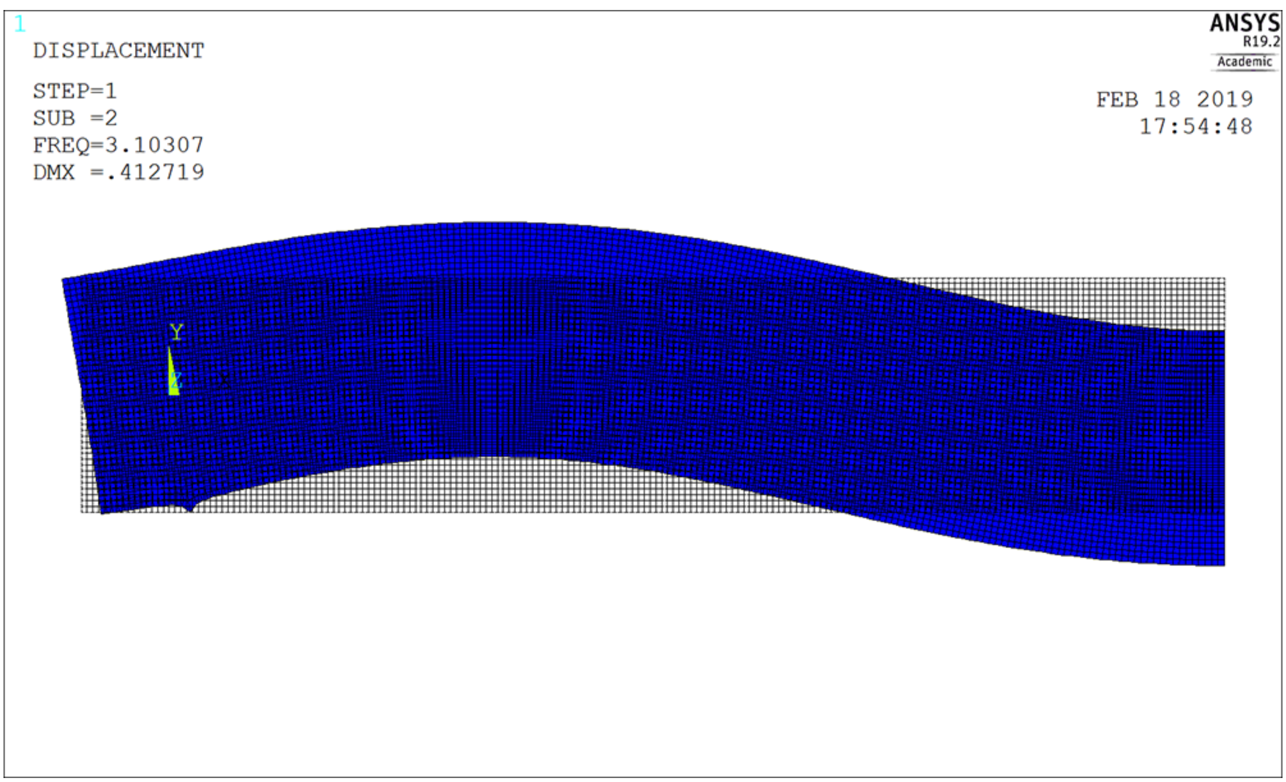


Fig. C3.2

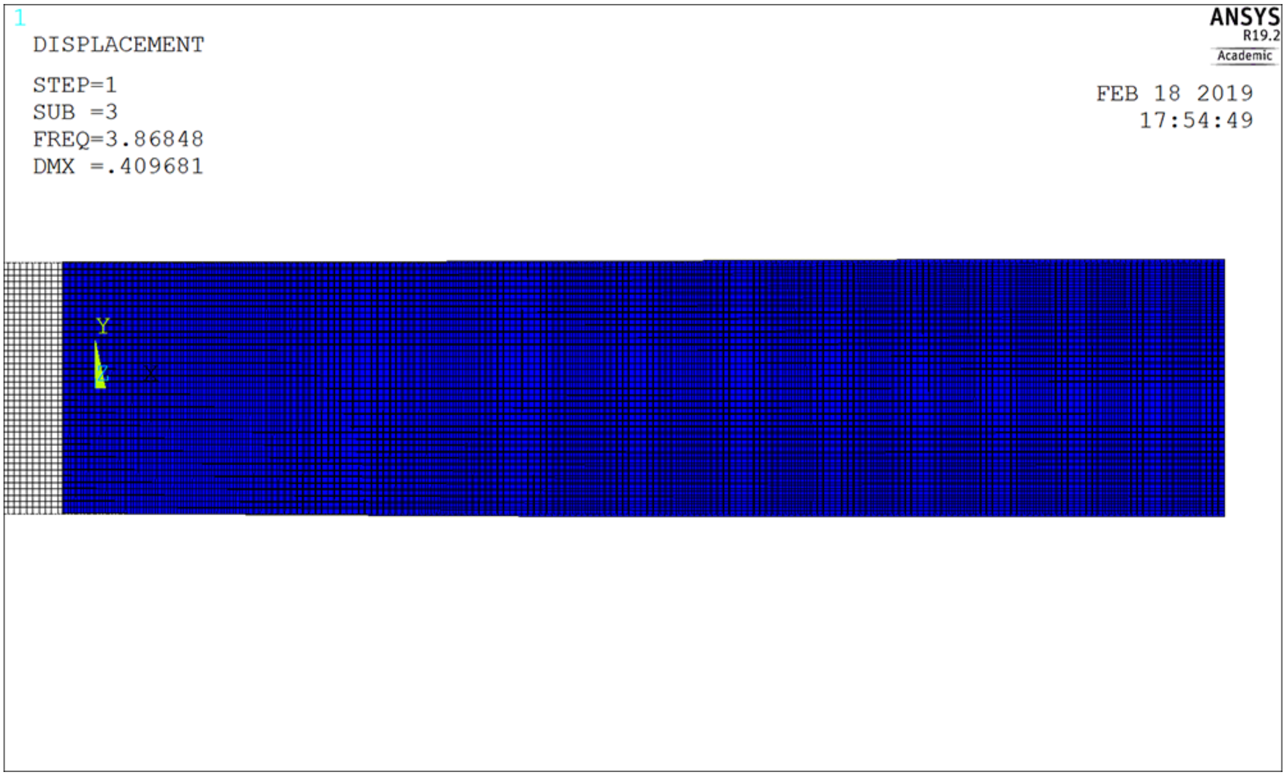


Fig. C3.3

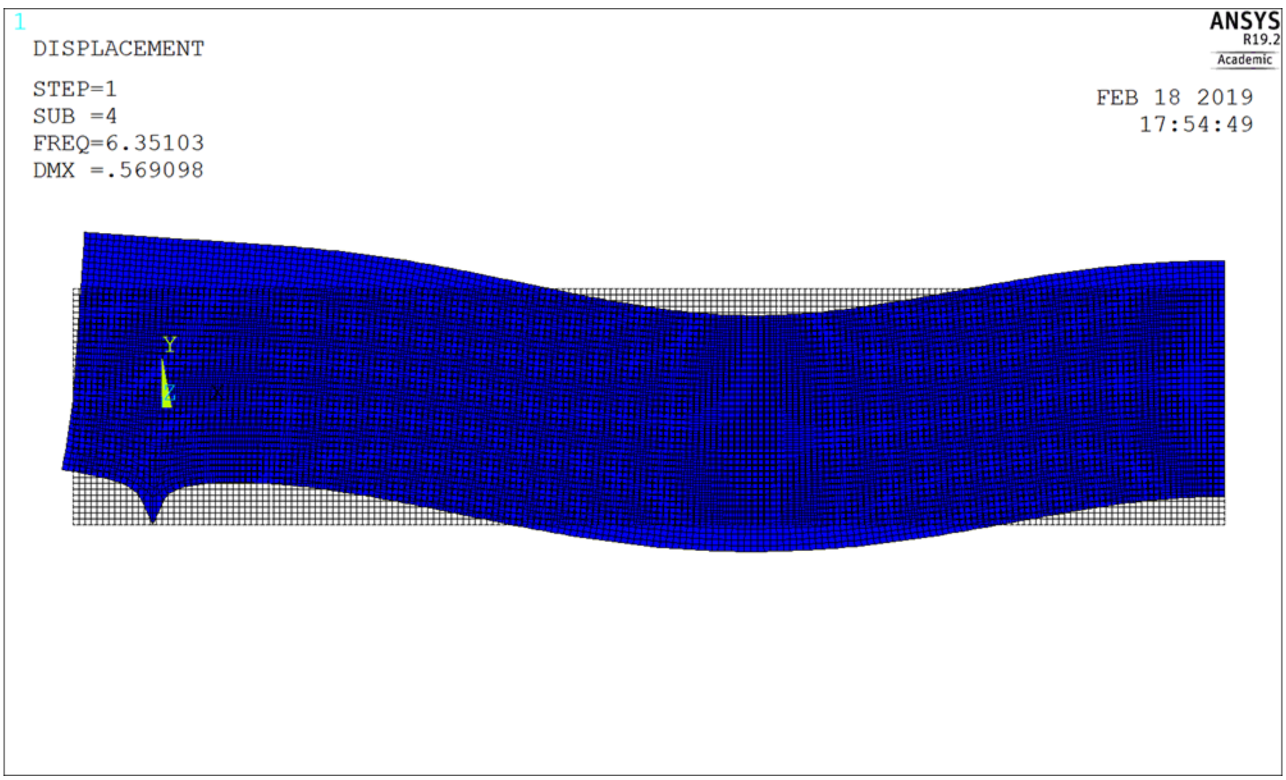


Fig. C3.4

***** INDEX OF DATA SETS ON RESULTS FILE *****

SET	TIME/FREQ	LOAD STEP	SUBSTEP	CUMULATIVE
1	0.41083	1	1	1
2	3.1031	1	2	2
3	3.8685	1	3	3
4	6.3510	1	4	4

Fig. C3.5

Analysis : Case 4

Uniform load, w:	7.5 N/mm
Length, L:	480 mm
Omit beam weight for this case	

C4 : Bilinear Material Model with Kiematic Hardening

Fig. C4.1 is the preview of the relationships that will be used for modeling the loads and deflections.

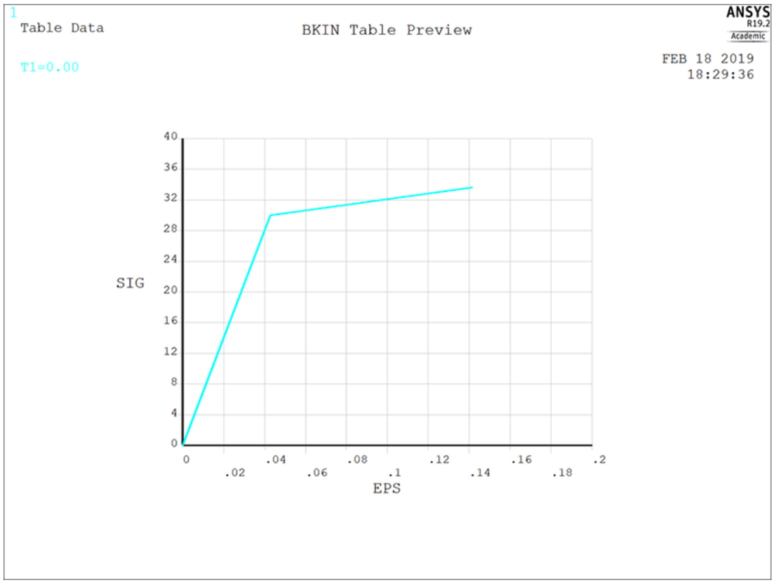


Fig. C4.1

C4 : Deflected Shape of Beam

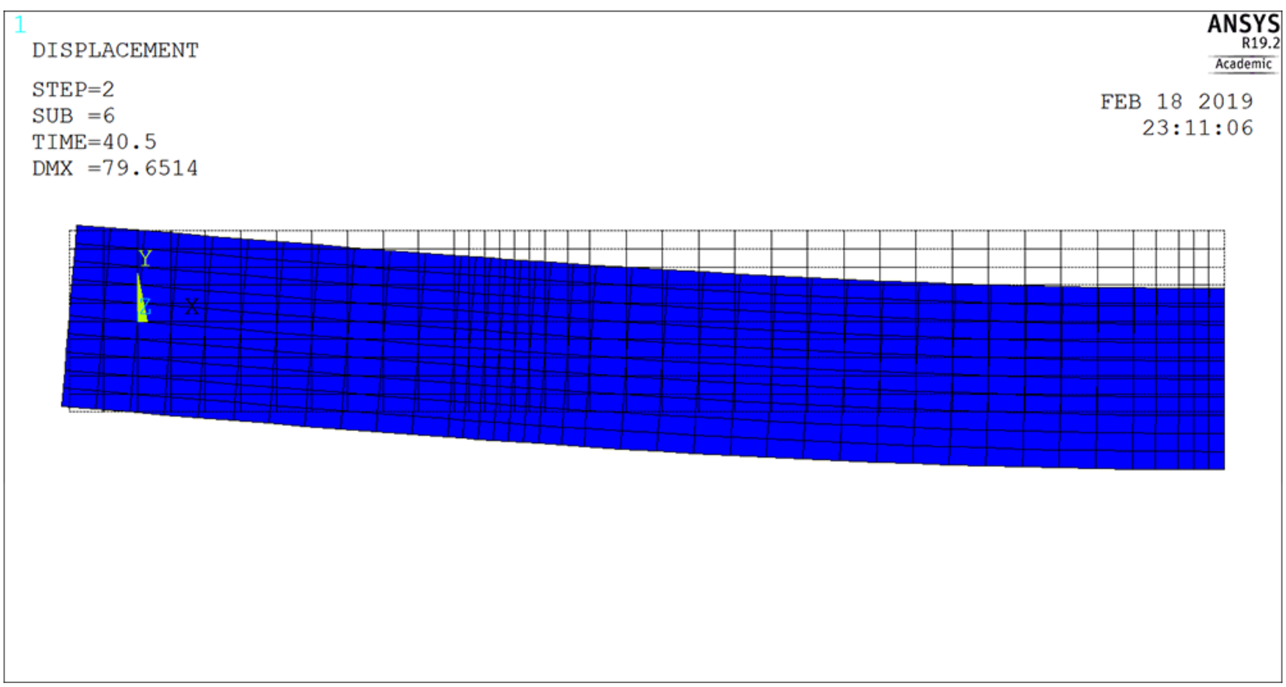


Fig. C4.2

C4 : Contour Plot of Plastic Strains (ε_x)

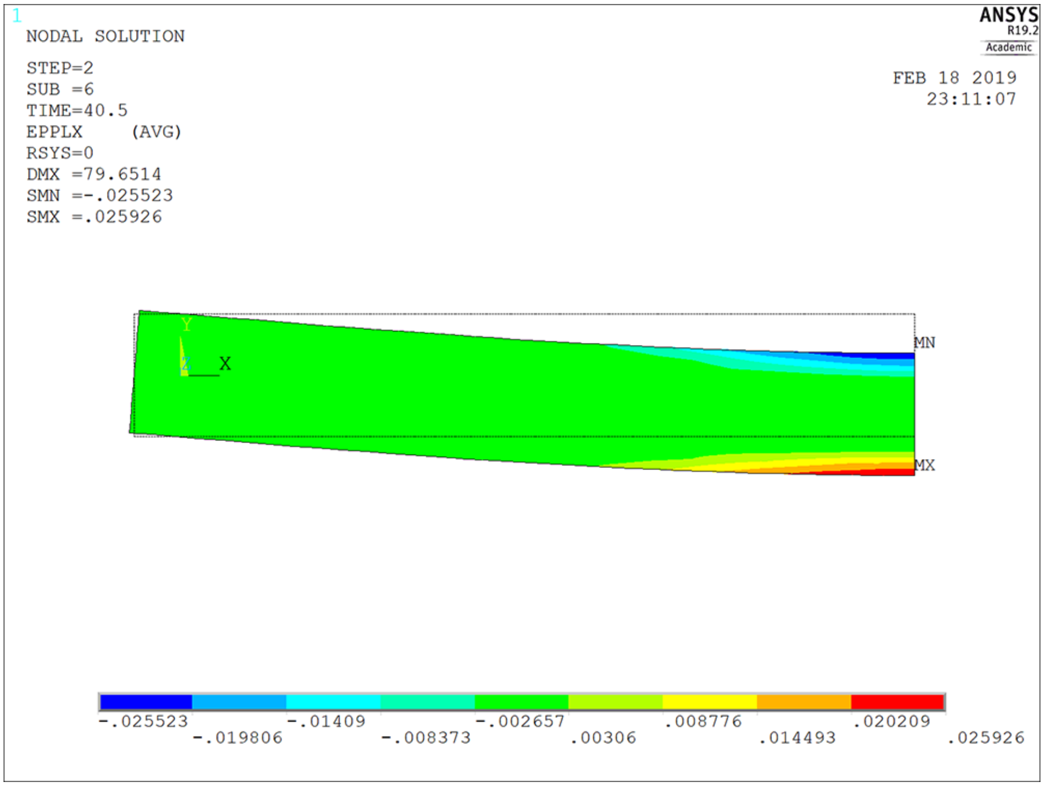


Fig. C4.3

C4 : Distribution of Total Strain at Midspan

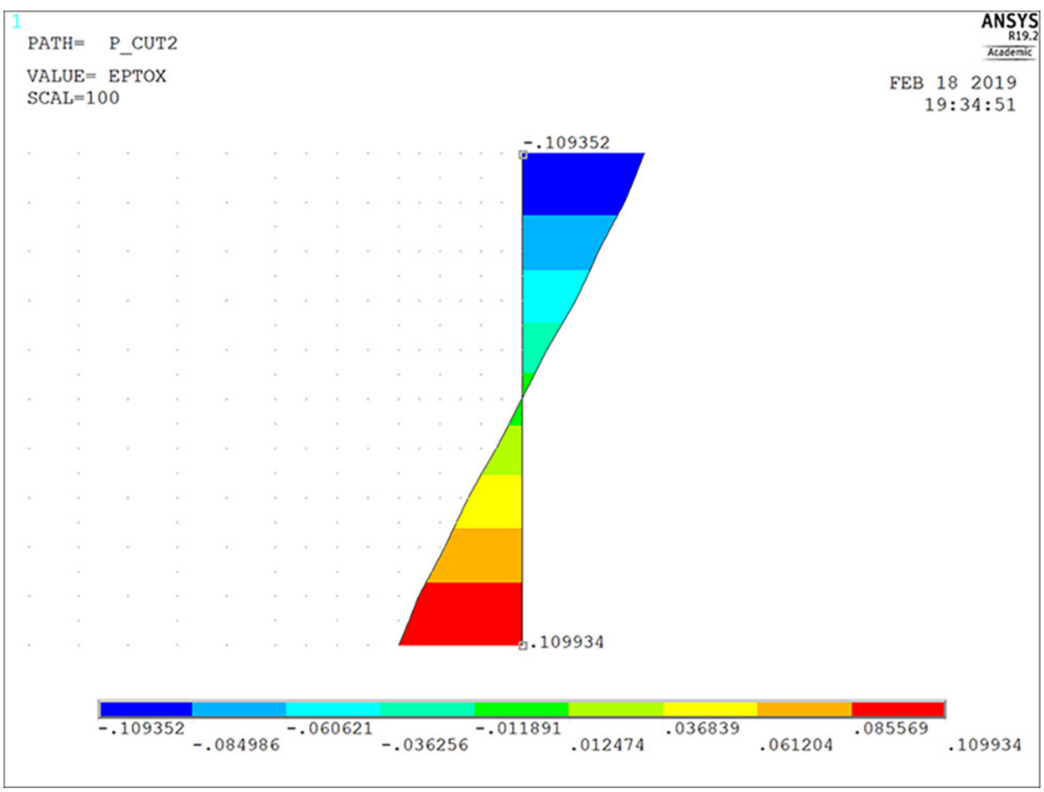


Fig. C4.4

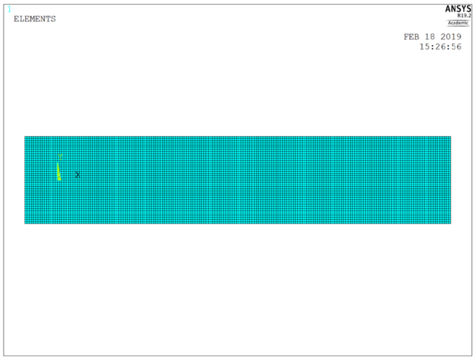
Appendix : Convergence Studies - Case 1

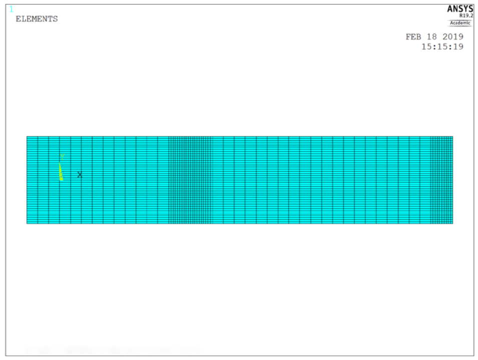
The initial mesh that was used during the building of the model was with everything sized at 1mm. When you refine a mesh by level 1 it reduces the size by 1/2. I applied this methodology and just rebuilt the models each time and increasing or reducing the size by 1/2. Since It was too many nodes to refine smaller than 1, I created a multi-zone area for the model in order to refine around the cuts more. Once I determined that there was minimal impact by refining the mesh smaller, I decided to take a different approach and increase the mesh size until there was a substantial difference. After I found this new baseline, I then proceeded to reduce the mesh size again until there was less than .01% change.

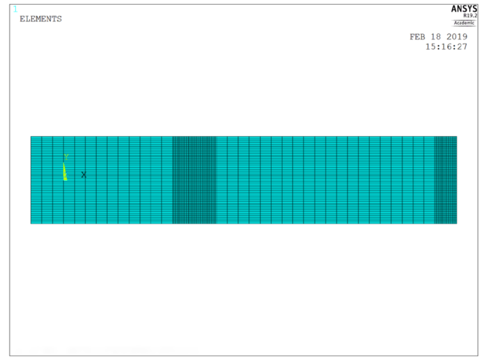
Case 1	Mesh 0	Mesh 1	Mesh 2				
	ALL=1	LG=5, SM=1	LG=5,SM=.5				
δ_y at Q	-30.01732	-30.01716	-30.01702				
δ_y at Cut 1	-15.24462	-15.24446	-15.24433				
δ_{max}	30.1717	30.1716	30.1715				
σ_{max}	30.4749	30.4749	30.475				
		M1/M0	M2/M1				
δ_y at Q Diff		-0.00052%	-0.00046%				
δ_y at C1 Diff		-0.00102%	-0.00090%				
δ_{max} Diff		-0.00033%	-0.00033%				
σ_{max} Diff		0.00000%	0.00033%				

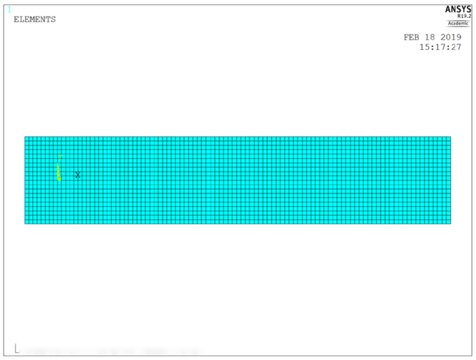
Case 1	Mesh 0	Mesh 3	Mesh 4	Mesh 5	Mesh 6	Mesh 7	Mesh 8
	ALL=1	LG=2,SM=2	LG=4,SM=4	LG=8,SM=8	LG=16,SM=16	ALL=16	ALL=32
δ_y at Q	-30.01732	-30.01732	-30.01730	-30.01710	-30.01594	-30.01594	-30.01011
δ_y at Cut 1	-15.24462	-15.24462	-15.24461	-15.24444	-15.24343	-15.24342	-15.24018
δ_{max}	30.1717	30.1717	30.1717	30.1715	30.1704	30.1704	30.1645
σ_{max}	30.4749	30.4744	30.4722	30.4673	30.4668	30.4863	30.5469
		M3/M0	M4/M3	M5/M4	M6/M5	M7/M6	M8/M7
δ_y at Q Diff		0.00000%	-0.00005%	-0.00069%	-0.00386%	0.00001%	-0.01942%
δ_y at C1 Diff		0.00000%	-0.00006%	-0.00113%	-0.00660%	-0.00007%	-0.02123%
δ_{max} Diff		0.00000%	0.00000%	-0.00066%	-0.00365%	0.00000%	-0.01956%
σ_{max} Diff		-0.00164%	-0.00722%	-0.01608%	-0.00164%	0.06400%	0.19878%

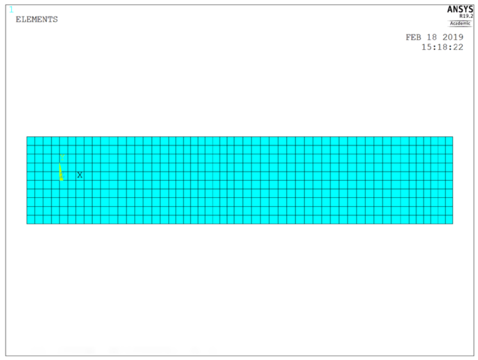
Case 1	Mesh 8	Mesh 9	Mesh 10				
	ALL=32	LG=32, SM=16	LG=16, SM=8				
δ_y at Q	-30.01011	-30.01593	-30.01558				
δ_y at Cut 1	-15.24018	-15.24342	-15.24292				
δ_{max}	30.1645	30.1704	30.17				
σ_{max}	30.5469	30.4666	30.4673				
		M9/M8	M10/M9				
δ_y at Q Diff		0.01940%	-0.00118%				
δ_y at C1 Diff		0.02124%	-0.00327%				
δ_{max} Diff		0.01956%	-0.00133%				
σ_{max} Diff		-0.26287%	0.00230%				

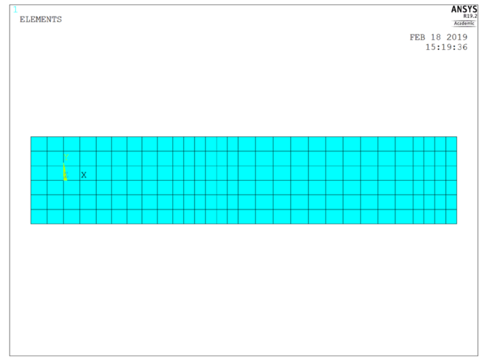

ELEMENTS
ANSYS
FEB 18 2019
15:26:56

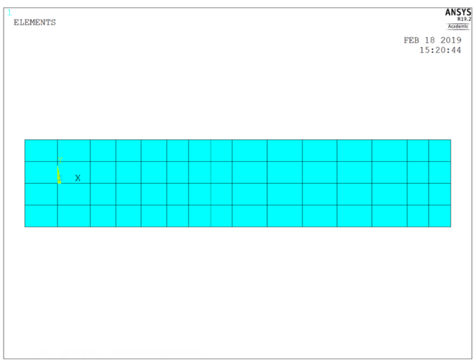

ELEMENTS
ANSYS
FEB 18 2019
15:15:19


ELEMENTS
ANSYS
FEB 18 2019
15:16:27

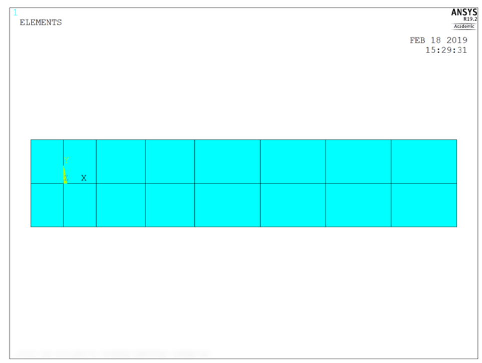

ELEMENTS
ANSYS
FEB 18 2019
15:17:27

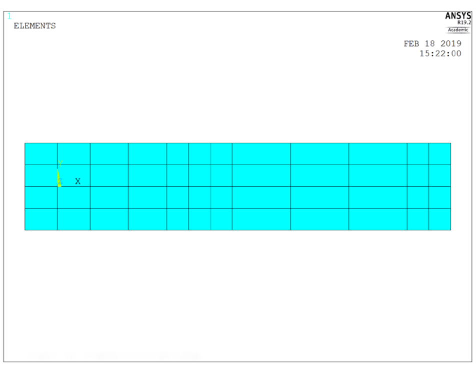

ELEMENTS
ANSYS
FEB 18 2019
15:18:22

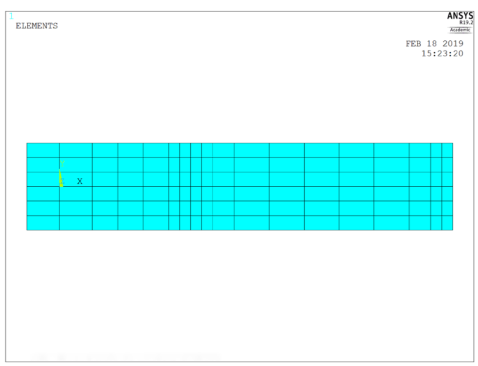

ELEMENTS
ANSYS
FEB 18 2019
15:19:36


ELEMENTS
ANSYS
FEB 18 2019
15:20:44


ELEMENTS
ANSYS
FEB 18 2019
15:28:19


ELEMENTS
ANSYS
FEB 18 2019
15:29:31


ELEMENTS
ANSYS
FEB 18 2019
15:22:00

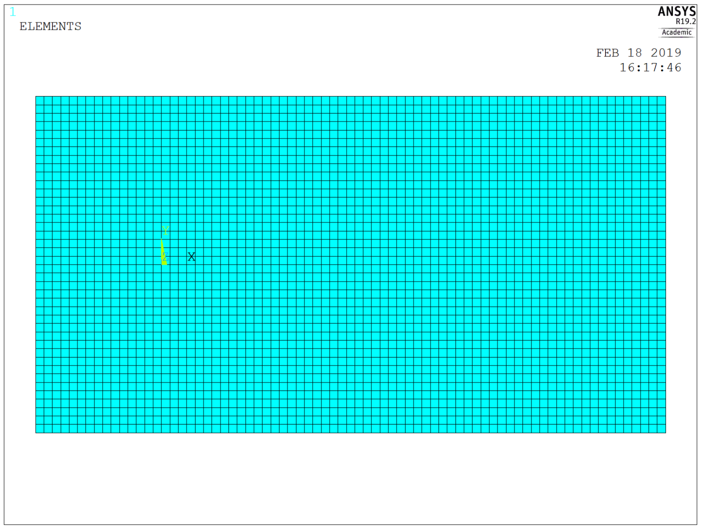

ELEMENTS
ANSYS
FEB 18 2019
15:23:20

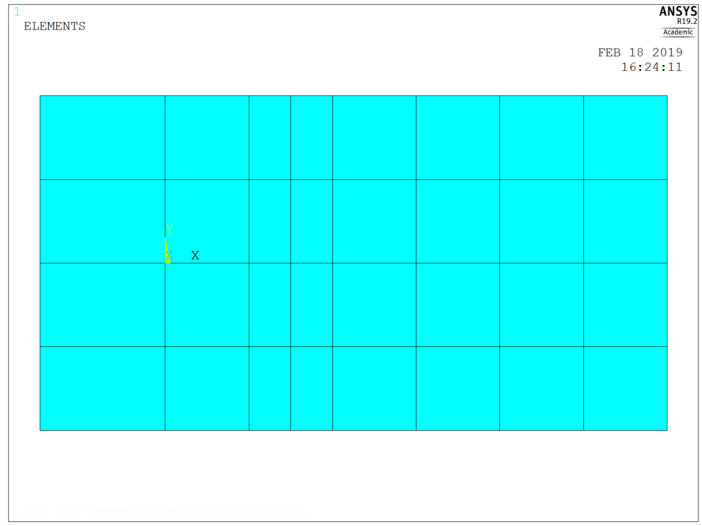
Appendix : Convergence Studies - Case 2

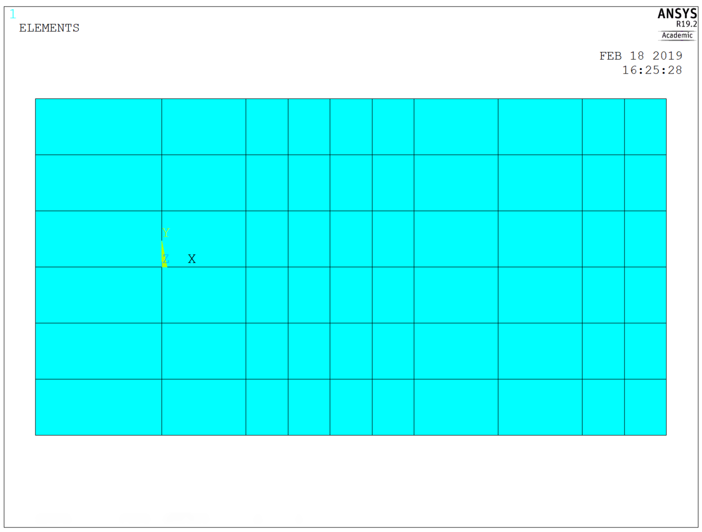
The first thing I checked with Case 2 was to see if the difference between the mesh used with Case 1 baseline and final was a comparable difference if the same two were used for Case 2. Unfortunately they were not close enough for me to accept. I stepped the outcome up to one step above what was used with Case 1, and then started refining from there. I ended up chosing Mesh 4, which now looking at the numbers I have determined it was not adequately descritized.

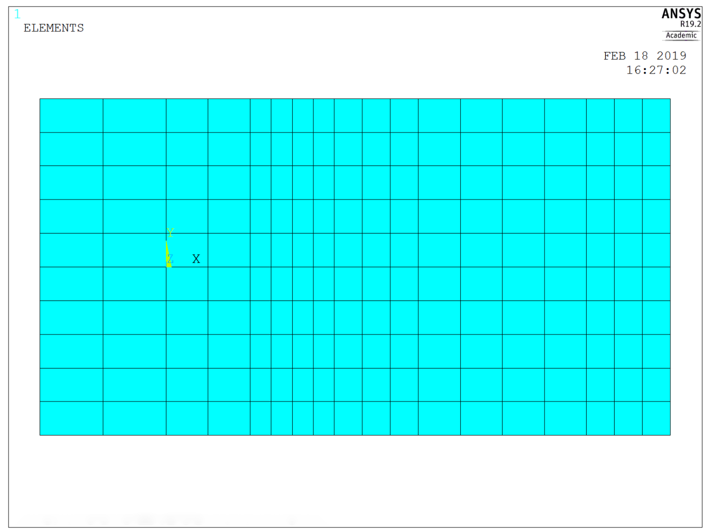
Case 2	Mesh 0	Mesh 10					
	All=1	LG=16, SM=8					
δ_y at Q	-1.34568	-1.34581					
δ_y at Cut 1	-0.69729	-0.69747					
δ_{max}	1.40696	1.40708					
σ_{max}	10.4331	10.4113					
		M10/M0	Case 1 M10/M0	Difference C1 -> C2			
δ_y at Q Diff		0.00927%	-0.00578%	-0.00349%			
δ_y at C1 Diff		0.02639%	-0.01114%	-0.01526%			
δ_{max} Diff		0.00853%	-0.00563%	-0.00289%			
σ_{max} Diff		-0.20895%	-0.02494%	-0.18401%			

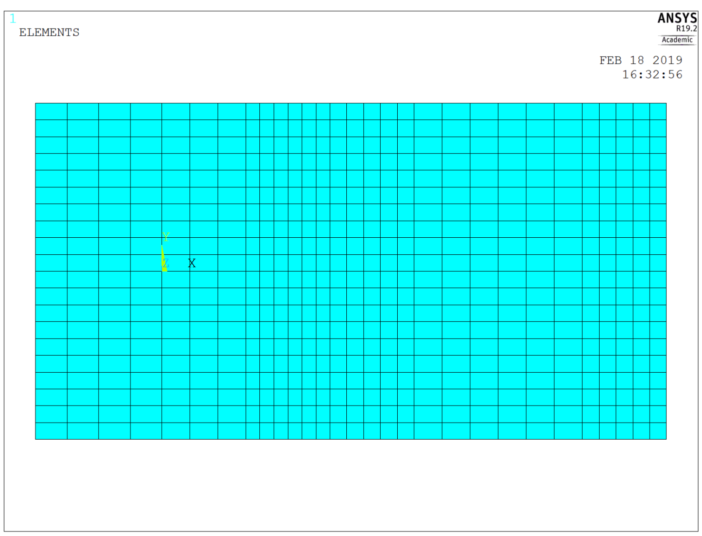
Case 2	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5		
	LG=32, SM=16	LG=16, SM=8	LG=8, SM=4	LG=4, SM=2	LG=2, SM=1		
δ_y at Q	-1.34569473	-1.345807623	-1.345680846	-1.345682763	-1.345682865		
δ_y at Cut 1	-0.6973392	-0.697470642	-0.697227077	-0.697292698	-0.697286571		
δ_{max}	1.40575	1.40708	1.40671	1.40696	1.40696		
σ_{max}	10.4083	10.4113	10.4256	10.4319	10.4331		
		M2/M1	M3/M2	M4/M3	M5/M4		
δ_y at Q Diff		0.00839%	-0.00942%	0.00014%	0.00001%		
δ_y at C1 Diff		0.01885%	-0.03492%	0.00941%	-0.00088%		
δ_{max} Diff		0.09461%	-0.02630%	0.01777%	0.00000%		
σ_{max} Diff		0.02882%	0.13735%	0.06043%	0.01150%		

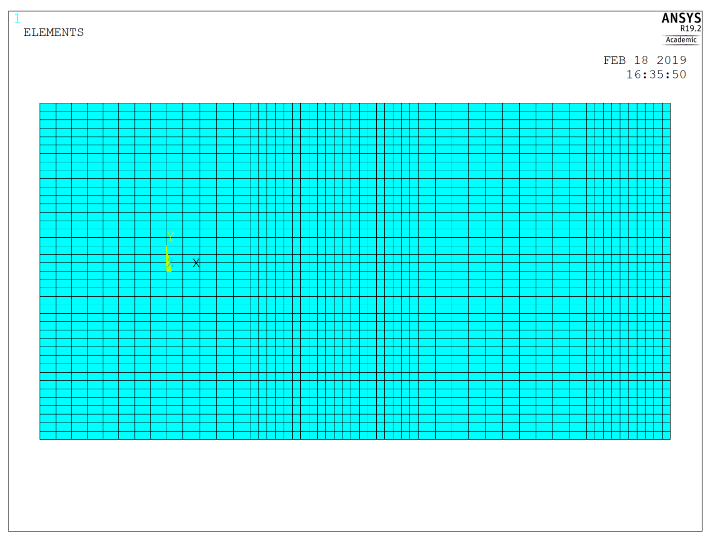

1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:17:46
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:24:11
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:25:28
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:27:02
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:32:56
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
16:35:50
Y
X

Appendix : Convergence Studies - Case 3

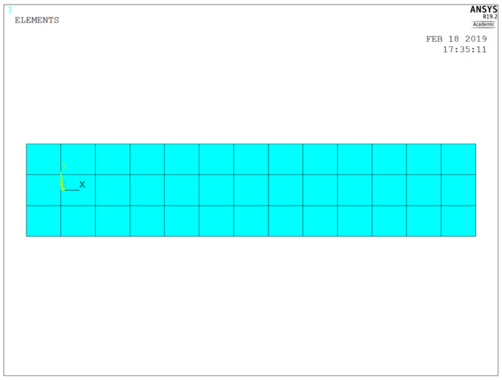
For Case 3 I did not try and compare to Case 1 differences since the measurements and procedures were too different. I started with the same baseline mesh as I did for Case 2. I decided to use substep 4 for the refinement since it was the last step. After a futile effort, I determined substep 4 was too volitile and switched to substep 1. While substep 1 still seem to be too volitile, I relaxed my tolerace and settled for the first refinement that was within .1%.

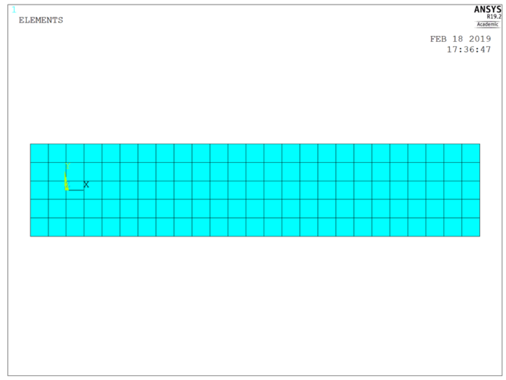
Case 3 Sub=4	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5		
	LG=32, SM=16	LG=16, SM=8	LG=8, SM=4	LG=4, SM=2	LG=2, SM=1		
Frequency	7.13497	6.99014	6.87927	6.64242	6.3915		
δ_{max}	0.41743	0.424802	0.422222	0.452673	0.555384		
		M2/M1	M3/M2	M4/M3	M5/M4		
Frequency Diff		-2.02986%	-1.58609%	-3.44295%	-3.77754%		
δ_{max} Diff		1.76531%	-0.60734%	7.21208%	22.68989%		

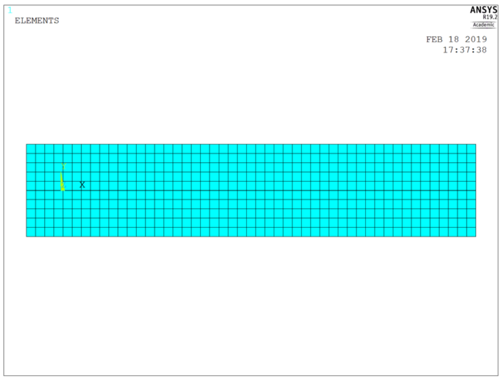
Case 3 Sub=4	Mesh 1a	Mesh 2a	Mesh 3a	Mesh 4a	Mesh 5a		
	ALL=16	ALL=8	ALL=4	ALL=2	ALL=1		
Frequency	7.19015	7.08105	6.85946	6.80675	6.35103		
δ_{max}	0.419254	0.418829	0.424583	0.423935	0.56910		
		M2/M1	M3/M2	M4/M3	M5/M4		
Frequency Diff		-1.51735%	-3.12934%	-0.76843%	-6.69512%		
δ_{max} Diff		-0.10137%	1.37383%	-0.15262%	34.24181%		

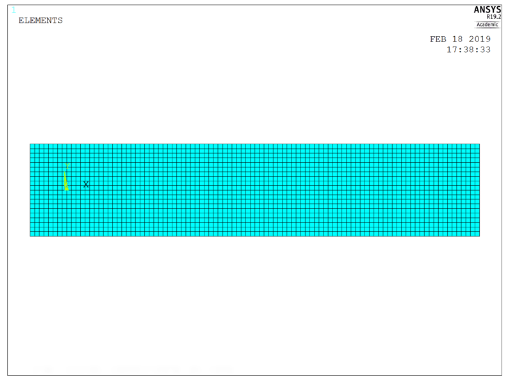
Case 3 Sub=1	Mesh 1b	Mesh 2b	Mesh 3b	Mesh 4b	Mesh 5b	Mesh 6b	
	ALL=16	ALL=8	ALL=4	ALL=2	ALL=1	LG=1, SM=1	
Frequency	0.41226	0.414618	0.41565	0.411193	0.41083	0.41083	
δ_{max}	0.422626	0.423305	0.423576	0.422473	0.42226	0.422259	
		M2/M1	M3/M2	M4/M3	M5/M4		
Frequency Diff		0.57197%	0.24890%	-1.07230%	-0.08828%		
δ_{max} Diff		0.16066%	0.06402%	-0.26040%	-0.05065%		

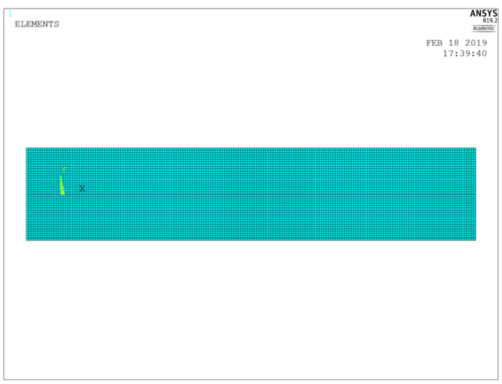
Case 3 Sub=1	Mesh 6b	Mesh 7b	Mesh 8b	Mesh 9b Free			
	LG=1, SM=1	LG=10, SM=1	LG=10, SM=.5	LG=10, SM=.5			
Frequency	0.41083	0.410613	0.409739	0.412041			
δ_{max}	0.422259	0.422167	0.421793	0.422767			
		M7/M6	M8/M7	M8/M9			
Frequency Diff		-0.05282%	-0.21285%	0.56182%			
δ_{max} Diff		-0.02179%	-0.08859%	0.23092%			

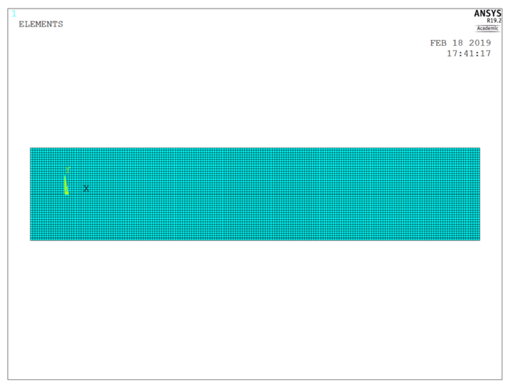

ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:35:11
X

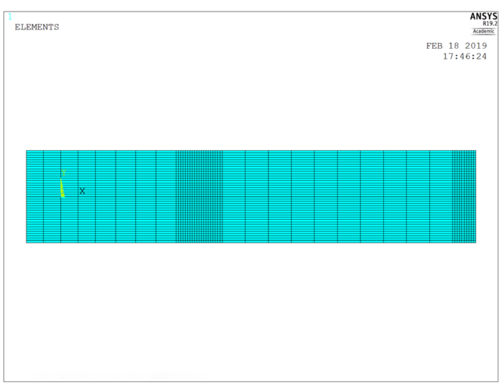

ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:36:47
X

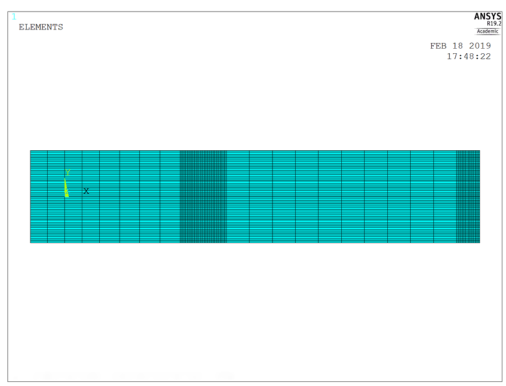

ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:37:38
X

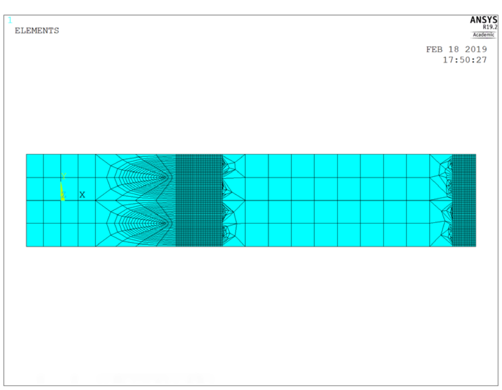

ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:38:33
X


ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:39:40
X


ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:41:17
X


ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:46:24
X


ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:48:22
X

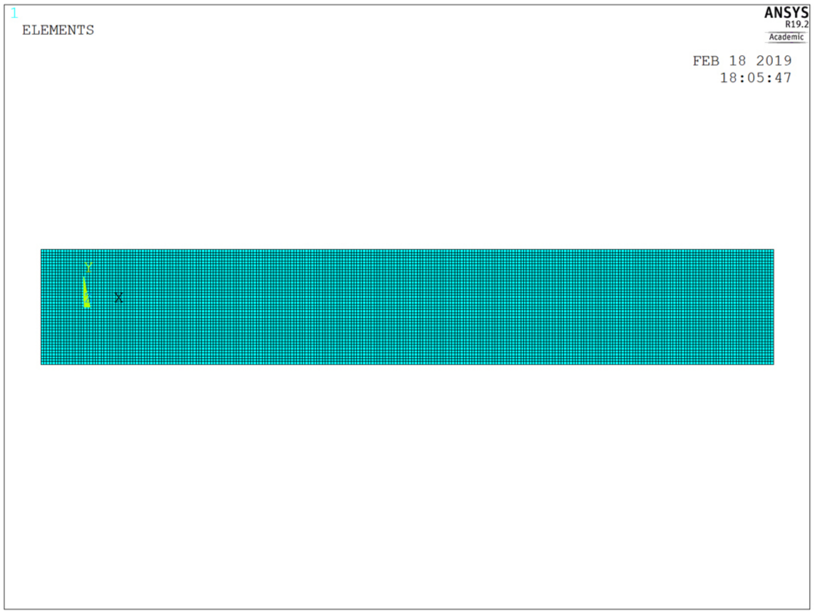

ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
17:50:27
X

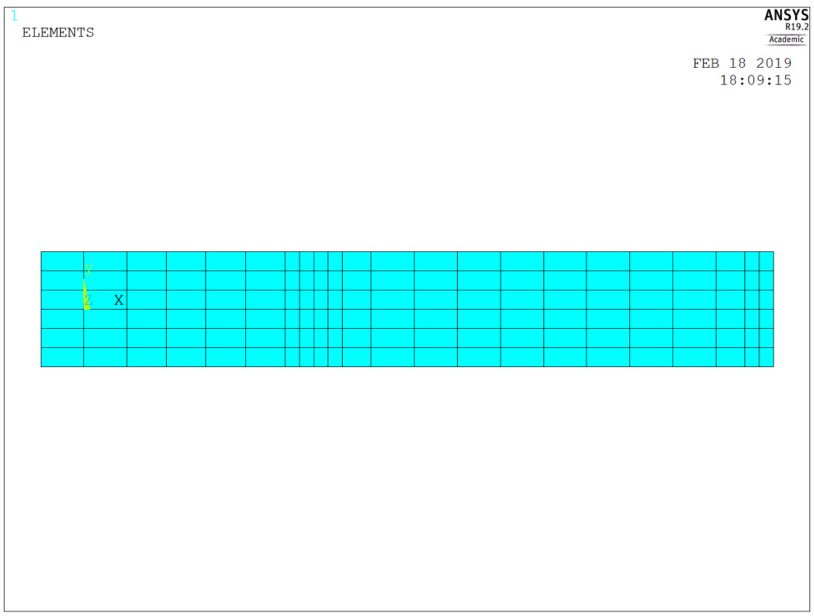
Appendix : Convergence Studies - Case 4

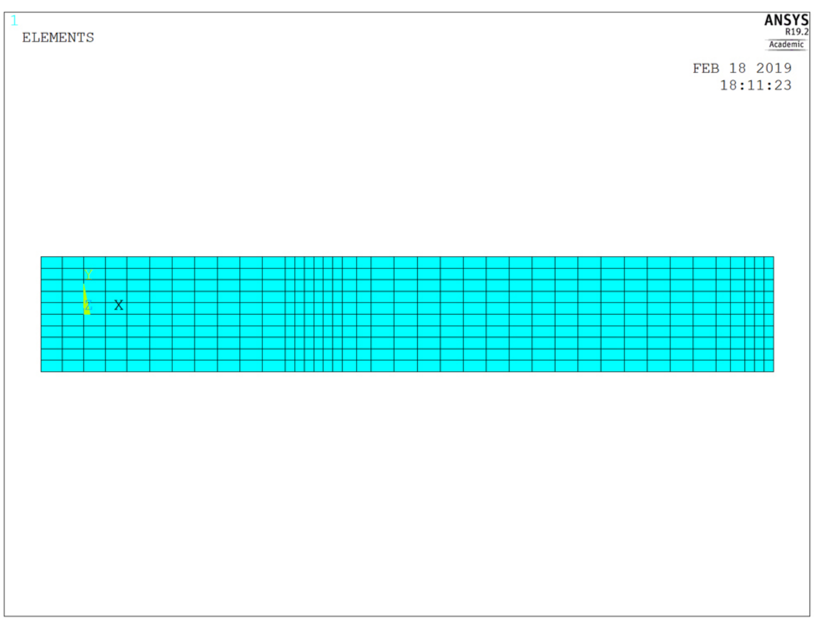
The first thing I checked with Case 4 was to see if the difference between the mesh used with Case 1 baseline and final was a comparable difference if the same two were used for Case 4. While it was very close, the difference was just over my ideal threshold of .01%. After refining the mesh once, it was within the tolerance.

Case 4	Mesh 0	Mesh 10					
	All=1	LG=16, SM=8					
δ_y at Q	-78.20845	-78.20649					
δ_y at Cut 1	-39.65608	-39.65418					
δ_{max}	78.4356	78.4336					
σ_{max}	45.0832	45.0769					
		M10/M0	Case 1 M10/M0	Difference C1 -> C2			
δ_y at Q Diff		-0.00250%	-0.00578%	0.00327%			
δ_y at C1 Diff		-0.00479%	-0.01114%	0.00634%			
δ_{max} Diff		-0.00255%	-0.00563%	0.00308%			
σ_{max} Diff		-0.01397%	-0.02494%	0.01096%			

Case 4	Mesh 10	Mesh 2					
	LG=16, SM=8	LG=8, SM=4					
δ_y at Q	-78.20649	-78.20819					
δ_y at Cut 1	-39.65418	-39.65583					
δ_{max}	78.4336	78.4353					
σ_{max}	45.0769	45.081					
		M10/M0					
δ_y at Q Diff		0.00218%					
δ_y at C1 Diff		0.00416%					
δ_{max} Diff		0.00217%					
σ_{max} Diff		0.00910%					


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
18:05:47
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
18:09:15
Y
X


1
ELEMENTS
ANSYS
R19.2
Academic
FEB 18 2019
18:11:23
Y
X