

## A sliding mode throttle controller for drive-by-wire operation of a racing motorcycle engine

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**Abstract**—In this paper, the problem of designing a drive-by-wire throttle control system for a high performance motorcycle engine is considered. Given the characteristics of the electro-mechanical system to be controlled, an observer-based, Discrete-Time Sliding Mode (DTSM) servo controller with integral action is adopted. The controller is designed on the basis of an identified model of the system, whose states are estimated by means of a state observer. Different state observer structures are analysed, and it is shown that the main characteristics of standard state-feedback based sliding mode control systems are preserved. A theoretical analysis is also presented showing that a separation principle holds, so that the state observer and the DTSM controller can be designed independently. The performances of the designed controller are illustrated by reporting the results of tests performed on both an experimental setup and a racing motorcycle.

### I. INTRODUCTION

Throttle-by-wire systems were among the first drive-by-wire applications in the automotive industry (see e.g. [1] and the references therein). The use of drive-by-wire systems in the motorcycle industry is instead rather new, and it is so far mainly limited to motorcycle used in racing competitions. In this paper, we study the problem of designing a drive-by-wire throttle control system for a high-performance motorcycle engine. In the situation at hand, the main difference with respect to the standard situation that is found in automobiles is that the throttle body is not directly actuated by the electrical motor, but instead it is linked to the motor by a set of leverages, due to constraints on the available space. This makes the mathematical model of the mechanical device much more complicated with respect to the automobile engine case, and adds a further degree of nonlinearity to the system. Requirements on the control system performance are also much stricter than those typically encountered in standard situations, since the difference between the required and the actual throttle command may limit the driver capability of performing aggressive maneuvers.

A control technique that has been widely used to address the problems associated with throttle-by-wire systems is sliding mode control [2]-[4], essentially due to its properties

of robustness against large variations of the system parameters and disturbances. To avoid the difficulties of explicitly modeling the electromechanical system, we propose to use a state observer to estimate the states of an identified linear model of the system, and then feed such estimates to a Discrete-Time Sliding Mode (DTSM) servo controller with integral action. The integral action is designed according to a recently proposed approach [16], [18], that is here further developed, showing in particular how the tuning of the integral action can be done by using root-locus techniques. Two different state observers schemes, namely a Kalman Filter and a Sliding Mode Observer, are analysed and compared. A theoretical analysis is also presented showing that a separation principle holds, so that the state observer, the reference model, and the DTSM controller can be designed independently. The performances of the designed controller are illustrated by reporting the results of tests performed on both an experimental setup and a racing motorcycle.

The paper is organized as follows. In Section II some basic results of DTSM control theory are reviewed. In Section III the model reference controller structure is described, and the introduction of an integral action is discussed. In Section IV the use of state observers is discussed. Experimental results are presented in Section V, and some conclusions are drawn in Section VI. In consideration of the available space for the final version of the paper, Propositions proofs and some mathematical details are omitted, as well as some figures on experimental results. Such material will be reported in an extended version of the paper, presently in preparation.

### II. THE DISCRETE-TIME SLIDING MODE SERVO CONTROL SCHEME

The plant is described by a discrete-time, SISO, LTI model. Grouping together disturbances and uncertainties into a single term,  $k$ , the plant equation is:

$$x_{k+1} = A x_k + B u_k + k \quad (1)$$

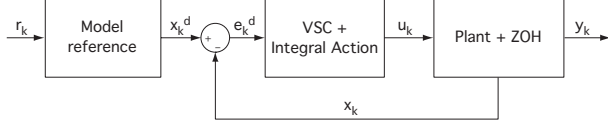


Fig. 1. Closed-loop system with SMC and reference model

Let the *sliding surface* be defined by

$$s_k = \begin{cases} 0, & k = 0, \\ \Lambda y_k - \epsilon_k, & k = 1, 2, \dots \end{cases} \quad (2)$$

Then the *discrete-time equivalent control* [5]  $u_k^{eq} = \begin{pmatrix} u_k^{eq} \\ u_k^{eq} \end{pmatrix}$  is defined by solving

$$s_{k+1} = 0 \quad k = 0, 1, 2, \dots \quad (3)$$

A convenient way to give specifications for a servo system is to use a two degree-of-freedom structure, sometimes referred to as *model reference control structure* (a block diagram of such a control system is shown in Fig. 1).

To design such a reference model, let us consider the discrete-time plant (1), where  $\epsilon_k$  is set to zero, and in which the input is obtained via state-feedback:

$$\begin{cases} d_{k+1} = d_k + \frac{d}{k} \\ \frac{d}{k} = -\frac{d}{k} + \frac{d}{k} \\ \frac{d}{k} = \frac{d}{k} \end{cases} \quad (4)$$

The tracking performance can be improved by adding an *integral action*. This is a topic that has been discussed at length in the literature, for both continuous-time systems ([11], [14], [15]) and discrete-time systems ([10], [12], [13]). Here we follow and further develop the approach recently proposed in [16],[18].

Starting from the discrete-time plant (1) and the reference-model (4), we can define the closed-loop error

$$d_k = d_k - \epsilon_k, \quad (5)$$

which has the following dynamics

$$d_{k+1} = d_k + \left( \frac{d}{k} - \epsilon_k \right) - \epsilon_k \quad (6)$$

If, to simplify the notation, we omit the control signal feed-forward action  $\frac{d}{k}$ , then (6) becomes

$$d_{k+1} = d_k - \epsilon_k - \epsilon_k \quad (7)$$

The tracking error is  $d_{y,k} = \frac{d}{k}$ , therefore a suitable integral action is introduced by augmenting the system state with a new state variable,  $\epsilon_k$ , with dynamics given by

$$\epsilon_{k+1} = \epsilon_k - \frac{d}{k} \quad (8)$$

The *sliding surface* is then defined as follows

$$\mathcal{S} = \left\{ \begin{bmatrix} d_k \\ \epsilon_k \end{bmatrix}^T \mid \Lambda \begin{bmatrix} d_k \\ \epsilon_k \end{bmatrix} = 0 \right\}, \quad (9)$$

where  $\Lambda$  determines the sliding surface when no integral action is applied. Therefore,

$$\begin{aligned} \epsilon_{k+1} &= \Lambda \frac{d}{k+1} + \epsilon_{k+1} \\ &= (\Lambda - \epsilon_k) \frac{d}{k} + \epsilon_k - \Lambda \epsilon_k - \Lambda \epsilon_k \end{aligned}$$

Recalling (3), and choosing  $\Lambda$  so that  $\Lambda \neq 0$ , we obtain the following equivalent control:

$$\begin{aligned} u_k^{eq} &= u_k = (\Lambda)^{-1} (\Lambda - \epsilon_k) \frac{d}{k} \\ &+ (\Lambda)^{-1} \epsilon_k - (\Lambda)^{-1} \Lambda \epsilon_k \end{aligned} \quad (10)$$

In the actual implementation of the control law (10), a one-step delayed estimate  $\hat{\epsilon}_k = \epsilon_{k-1}$  is used instead of the unknown disturbance  $\epsilon_k$  [6], which can be computed once  $\epsilon_k$ ,  $\epsilon_{k-1}$ , and  $\epsilon_{k-1}$  are known. The equivalent control then becomes

$$\begin{aligned} u_k^{eq} &= u_k = (\Lambda)^{-1} (\Lambda - \epsilon_{k-1}) \frac{d}{k} \\ &+ (\Lambda)^{-1} \epsilon_{k-1} - (\Lambda)^{-1} \Lambda \epsilon_{k-1} \end{aligned} \quad (11)$$

To analyze the closed-loop behaviour of the system, we compute the value of the equivalent-control at time  $(k+1)$ :

$$\begin{aligned} u_{k+1}^{eq} &= (\Lambda)^{-1} (\Lambda^2 - \epsilon_{k-1} - \epsilon_{k-1}) \frac{d}{k} \\ &+ (\Lambda)^{-1} \epsilon_{k-1} - (\Lambda)^{-1} (\Lambda - \epsilon_{k-1}) \epsilon_{k-1} \\ &- (\Lambda)^{-1} (\Lambda - \epsilon_{k-1} - \Lambda) \epsilon_{k-1} \end{aligned}$$

The state-space description of the closed-loop system is then

$$\begin{bmatrix} d_{k+1} \\ \epsilon_{k+1} \\ \epsilon_{k+1} \end{bmatrix} = \begin{bmatrix} \frac{d}{k} \\ \epsilon_k \\ \epsilon_k \end{bmatrix} + \begin{bmatrix} - \\ - \\ - \end{bmatrix} \epsilon_k, \quad (12)$$

where

$$\begin{bmatrix} - \\ - \\ - \end{bmatrix} = \begin{bmatrix} 0 & - & - \\ - & \frac{1}{31} & 0 \\ - & - & - \end{bmatrix}$$

with

$$\begin{aligned} -_{31} &= (\Lambda)^{-1} (\Lambda^2 - \epsilon_{k-1} - \epsilon_{k-1}) \\ -_{32} &= (\Lambda)^{-1} \epsilon_{k-1} \\ -_{33} &= -(\Lambda)^{-1} (\Lambda - \epsilon_{k-1}) \end{aligned}$$

and

$$\begin{bmatrix} - \\ - \\ - \end{bmatrix} = \begin{bmatrix} 0 \\ -(\Lambda)^{-1} (\Lambda - \epsilon_{k-1} - \Lambda) \end{bmatrix}$$

As observed in [18], care must be taken in choosing the integral action term so that closed loop stability is assured. In fact, in controller tuning phase, the DTSMC dynamics are first set by choosing an appropriate sliding surface  $\mathcal{S}$  via  $\Lambda$ , then the integral action is added, by setting a value for the integral action coefficient  $I$ , defined as

$$I = (\Lambda)^{-1} \epsilon_k \quad (13)$$

Clearly, any choice of  $I \neq 0$  causes a perturbation of the previously set closed loop dynamics, and there is no *a priori* guarantee that the system remains stable. It is therefore important to provide tools that can help in the choice of values of  $I$  that preserve stability. To this aim, we report in the following an analysis of the system closed-loop

dynamics, where the role of the integral action coefficient  $I$  is highlighted.

Let us first replace in (12) expression (10) for the equivalent control and (13). We obtain

$$\begin{aligned} \begin{bmatrix} d_{k+1} \\ k+1 \end{bmatrix} &= \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} d_k \\ k \end{bmatrix} + \begin{bmatrix} - \\ 0 \end{bmatrix} k + \begin{bmatrix} - \\ 0 \end{bmatrix} k \\ &= \begin{bmatrix} -(\Lambda)^{-1}\Lambda + I & -I \\ - & 1 \end{bmatrix} \begin{bmatrix} d_k \\ k \end{bmatrix} \\ &+ \begin{bmatrix} (\Lambda)^{-1}\Lambda \\ 0 \end{bmatrix} k_{-1} + \begin{bmatrix} - \\ 0 \end{bmatrix} k \end{aligned} \quad (14)$$

Therefore, the closed-loop dynamics are determined by the closed loop system matrix  $C_{CL}$ , where

$$C_{CL} = \begin{bmatrix} -(\Lambda)^{-1}\Lambda + I & -I \\ - & 1 \end{bmatrix} \quad (15)$$

Indeed, the following result holds, relating the spectrum of  $C_{CL}$  and  $\bar{\Lambda}$  in (12).

*Proposition 1:* The spectrum of  $\bar{\Lambda}$  is

$$\sigma(\bar{\Lambda}) = \{0\} \cup \sigma(C_{CL})$$

We suggest now an approach to study how the closed-loop system eigenvalues move as a function of the integral action coefficient  $I$ . We split the analysis in two parts, namely the study of the eigenvalues of the matrix  $C_{CL}$ , where

$$C_{CL} = -(\Lambda)^{-1}\Lambda + I, \quad (16)$$

describing the dynamics of the state tracking error  $d_k$ , and the study of the eigenvalues of  $C_{CL}$ .

Without loss of generality, we suppose that the discrete-time plant is in control canonical form, and  $\Lambda$  and  $\bar{\Lambda}$  are given as follows

$$\Lambda = \begin{bmatrix} 1 & 2 & \cdots & n-1 & n \end{bmatrix}, \quad \bar{\Lambda} = \begin{bmatrix} 1 & 2 & \cdots & n-1 & n \end{bmatrix}$$

We obtain that  $C_{CL}$  is still in companion form and its eigenvalues can be obtained by solving

$$\begin{aligned} &n + \left( n-1 \frac{-1}{n} - I \right) n^{-1} + \\ &\cdots + \left( 1 \frac{-1}{n} - I \right) 2 - I 1 = 0, \end{aligned} \quad (17)$$

that is

$$\begin{aligned} &\left( n^{-1} + n-1 \frac{-1}{n} n^{-2} + \cdots + 1 \frac{-1}{n} \right) \\ &+ I \left( -n \frac{n-1}{n} - \cdots - 2 - 1 \right) = 0 \end{aligned} \quad (18)$$

The dependency of the solution of (18) on  $I$  can be studied with the root-locus method, by taking as the transfer function to be placed in a unitary negative feedback loop  $I^{-1}(\cdot)$ , where

$$I^{-1}(\cdot) = \frac{1(\cdot)}{1(\cdot)}$$

$$1(\cdot) = - \left( n \frac{n-1}{n} + \cdots + 2 + 1 \right)$$

$$1(\cdot) = n + n-1 \frac{-1}{n} n^{-1} + \cdots + 1 \frac{-1}{n}$$

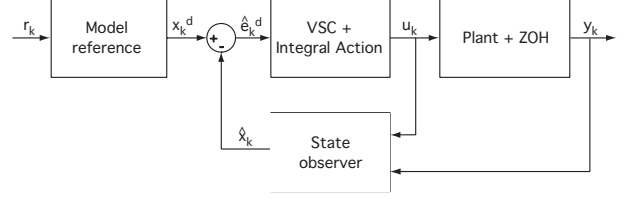


Fig. 2. Closed-loop system with SMC, model reference and state observer

The same approach can be used to study the spectrum of  $C_{CL}$ , whose characteristic polynomial is given in the following proposition.

*Proposition 2:* The characteristic polynomial of  $C_{CL}$  is

$$\begin{aligned} \det(-C_{CL}) &= \left\{ \left[ n + \left( n-1 \frac{-1}{n} - 1 \right) n^{-1} \right. \right. \\ &+ \left. \left( n-2 \frac{-1}{n} - n-1 \frac{-1}{n} \right) n^{-2} + \cdots + 1 \frac{-1}{n} \right] \\ &- I \left( n \frac{n-1}{n} + n-1 \frac{n-2}{n} + \cdots + 2 + 1 \right) \Big\} \end{aligned} \quad (19)$$

It is easy to see from (19) that, independently of the value of  $I$ , one of the eigenvalues of  $C_{CL}$  is in the origin of the complex plane. The behaviour of the remaining eigenvalues can be studied again via the root-locus method. Observe that, if  $I = 0$ , we have that the eigenvalues of  $C_{CL}$  are  $= 1$  (due to the presence of the integrator) and those that we would obtain without integral action using the same sliding surface.

In conclusion, the suggested approach provide an easy to use numerical tool to asses the effect of the introduction of the integral action on the closed loop dynamics, and to determine the interval of values of  $I$  that ensure that the closed loop stability is preserved.

### III. INTRODUCING A STATE OBSERVER

We turn now to the problem of obtaining state estimates to be used for feedback control when actual measurements of the state variables are not available, as is the case in the application reported in this paper. Since the proposed controller is a DTSMC, it seems appealing to remain in the same framework and use a *sliding mode observer* (SMO). As is known, sliding mode control theory can be successfully used to design state observers (see [20], [21], [22], [23], [24], [25]), that share some of the properties of sliding mode controllers, such as robustness in the face of disturbances and unmodelled dynamics. On the other hand, one of the most favoured control engineer's tools for state estimation is the Kalman Filter [19], that can be tuned on the field by means of statistical tests on the innovation process. We therefore implemented both a Kalman Filter and a SMO, and compared their performance when used in combination with the DTSMC described in the previous Sections. We refer the reader to standard books such as [19] for details on Kalman Filtering. The structure of the SMO that has been implemented is described in [21]. The resulting overall control scheme is reported in Fig. 2.

We analyse now the effect of the introduction of a state observer on the closed-loop system dynamics. To this aim,

we define the following error variables

$$\hat{e}_k = \hat{d}_k - d_k \quad (20)$$

$$\hat{e}_k^d = \hat{d}_k^d - d_k^d \quad (21)$$

that, together with (5), give

$$\hat{e}_k^d = \hat{d}_k^d - \hat{d}_k \quad (22)$$

Consider the expression (11) for the equivalent control. Clearly, if  $d_k$  is not available,  $\hat{d}_k^d$  cannot be computed. However, we can replace  $d_k$  with its estimate  $\hat{d}_k$ , or, equivalently, we can replace  $\hat{d}_k^d$  with  $\hat{e}_k^d$  in the expression for  $\hat{e}_k^{eq}$ , thus obtaining:

$$\hat{e}_k = (\Lambda)^{-1} (\Lambda - \epsilon) \hat{e}_k^d + I_k - (\Lambda)^{-1} \Lambda \hat{e}_{k-1}, \quad (23)$$

where  $\hat{e}_{k-1}$  denotes the one-step delayed estimate of the disturbance  $d_k$ , obtained by using  $\hat{d}_k$  and  $\hat{d}_{k-1}$  instead of  $d_k$  and  $d_{k-1}$ , respectively. Observe that  $\hat{e}_k$  can still be computed, since  $\hat{d}_k^d = \hat{d}_k - d_k = \hat{d}_k - d_k$ .

The performance of the closed-loop system can be studied by analysing the behavior of the error  $\hat{e}_k^d$ . We consider the case when a Kalman Filter based observer is used, whose dynamics are given by

$$\hat{e}_{k+1}^d = \hat{e}_k^d + \Pi (d_k - \hat{d}_k),$$

where  $\Pi$  is the filter gain. A similar analysis can be carried out when the SMO is used. Some computations show that:

$$\hat{e}_{k+1}^d = (-\Pi) \hat{e}_k^d - d_k + \Pi \hat{d}_k = \Gamma \hat{e}_k^d - d_k + \Pi \hat{d}_k \quad (24)$$

At time  $(k+1)$ , the control is

$$\begin{aligned} u_{k+1} &= (\Lambda)^{-1} (\Lambda - \Gamma - \epsilon) \hat{e}_k^d \\ &+ (\Lambda)^{-1} (\Lambda - \Pi - \epsilon) \hat{e}_k^d \\ &+ I_k - (\Lambda)^{-1} (\Lambda - \epsilon) \hat{e}_k - (\Lambda)^{-1} \Lambda \hat{e}_k \end{aligned} \quad (25)$$

Consider now the system obtained by augmenting the state vector of (12) with  $\hat{e}_k^d$ . Using (25) we get:

$$\begin{bmatrix} \hat{d}_{k+1}^d \\ \hat{d}_{k+1} \\ \hat{e}_{k+1}^d \\ \hat{e}_{k+1} \end{bmatrix} = \tilde{A} \begin{bmatrix} \hat{d}_k^d \\ \hat{d}_k \\ \hat{e}_k^d \\ \hat{e}_k \end{bmatrix} + \tilde{B}_1 d_k + \tilde{B}_2 \hat{e}_k, \quad (26)$$

where

$$\tilde{A} = \begin{bmatrix} 0 & - & 0 \\ - & 1 & 0 \\ \tilde{A}_{31} & I & \tilde{A}_{33} \\ \Pi & 0 & - & \Gamma \end{bmatrix},$$

$$\tilde{B}_1 = \begin{bmatrix} - \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} 0 \\ 0 \\ -(\Lambda)^{-1} \Lambda \\ 0 \end{bmatrix},$$

where

$$\begin{aligned} \tilde{A}_{31} &= (\Lambda)^{-1} (\Lambda - \Pi - \epsilon) \\ \tilde{A}_{33} &= -(\Lambda)^{-1} (\Lambda - \epsilon) \\ \tilde{A}_{34} &= (\Lambda)^{-1} (\Lambda - \Gamma - \epsilon) \end{aligned} \quad (27)$$

It is possible to prove the following result.

*Proposition 3:* The spectrum of  $\tilde{A}$  is given by

$$\sigma(\tilde{A}) = \{0\} \cup \sigma(\Lambda - \epsilon) \cup \sigma(\Gamma) \quad (28)$$

Proposition 3 shows that the dynamics of the DTSMC with integral action and the dynamics of the observer are *decoupled*, with obvious advantages in the tuning phase of the control system.

#### IV. EXPERIMENTAL RESULTS

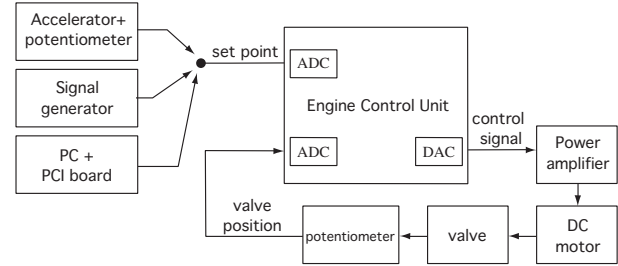


Fig. 3. Architecture of the experimental setup.

The architecture of the experimental platform is shown in Fig. 3. Observe that the gas request signal determining the valve opening setpoint can be generated in three different ways: An accelerator with a potentiometer can be used to manually reproduce the driver gas request; a signal generator is employed to produce standard inputs such as steps, ramps, and sinusoids; a PC with a PCI board is used to input generic signals, such as those coming from telemetry data. The DTSMC has been tested in the two configurations for the state observer described in Section IV.

Since the model-reference and DTSMC dynamics are decoupled, they can be set by choosing independently matrices  $\Lambda$  in (4) and  $\Lambda$  in (9). As far as the DTSMC tuning is concerned, the position in the complex plane of poles of  $\Lambda - \epsilon$  in (15) are first placed via  $\Lambda$ , then the effect on such poles of different choices for the integral action coefficient  $I$  is studied according to the root locus based procedure outlined in Section III. As for the state observer, the Kalman filter has been designed according to the LQG/LTR technique [26], whereas for the SMO, the gain matrix has been chosen so as to ensure dead-beat behavior [21]. Finally, the disturbance estimates  $\hat{d}_k \simeq \hat{d}_{k-1}$  have been low pass filtered, to avoid excitation of unmodeled dynamics [16].

Test results for the Kalman filter based DTSMC are summarized in Figs. 4-8. Fig. 4 shows the system response to a gas input signal reproducing the actual driver demand on a test track. The corresponding tracking error is shown in Fig. 5, whereas Fig. 6 shows the Kalman filter output estimation error. In Fig. 7, the system response to a step input of small amplitude is shown. It can be noticed that there is almost no overshoot in the response, showing the the DTSMC controller counteracts very successfully the effects

of friction, that are particularly relevant when small valve openings are requested. Also, there is no steady state error, due to the presence of the integral action. Fig. 8 shows the system response to ramp inputs. It can be noticed that the actual valve opening follows the ramp set point with a constant delay.

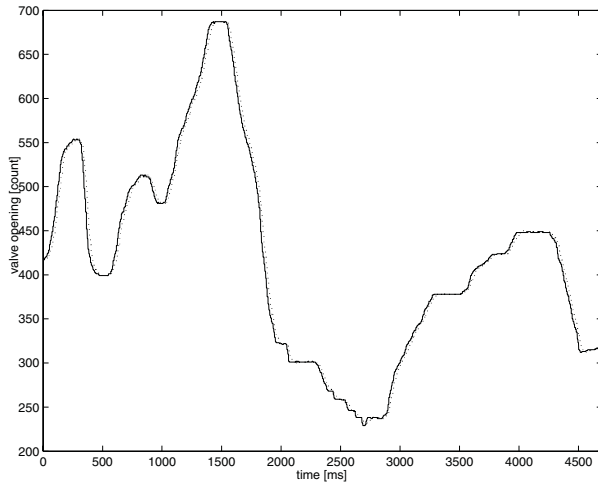


Fig. 4. System response to a generic gas input signal (continuous line: set point; dotted line: actual valve opening).

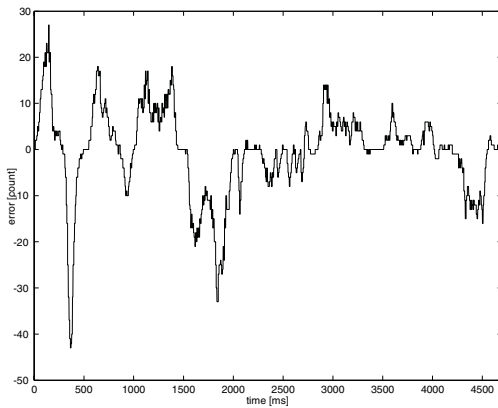


Fig. 5. Tracking error.

The same input signal of Fig. 4 has been used to test the performance of the DTSMC control system with SMO. The results, not reported here for space limitations, are quite similar, qualitatively, to those obtained with the Kalman filter, although the SMO exhibits better performance in terms of output estimation error.

Test have also been performed using the same controller on different throttle bodies, to evaluate its robustness. The results have been fully satisfactory, proving the validity of the adopted approach. Finally, the drive-by-wire throttle controller has been tested on track on an Aprilia RS Cube

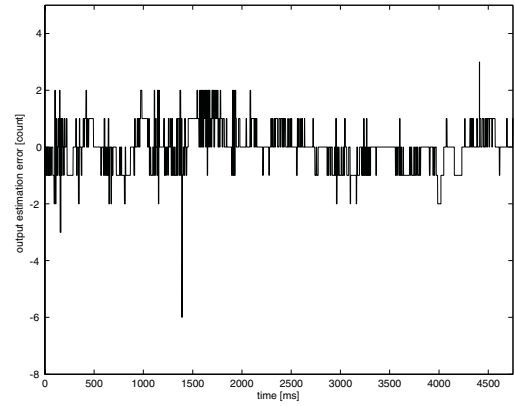


Fig. 6. Kalman filter output estimation error.

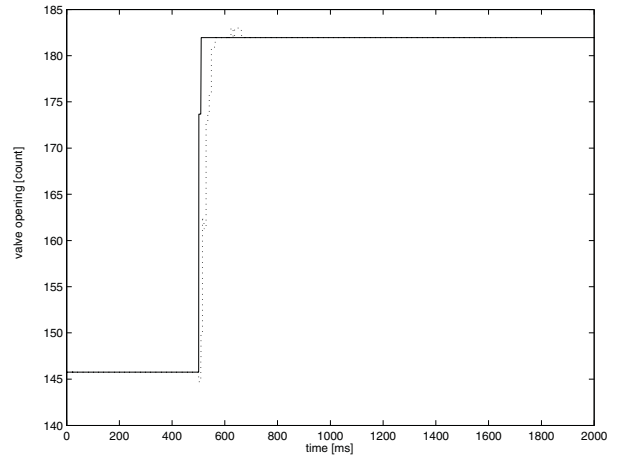


Fig. 7. System response to a step gas input signal (continuous line: set point; dotted line: actual valve opening).

racing motorcycle, taking part to the 2004 MotoGP World Championship, to verify its performance when subject to disturbances that were not considered in the experimental setup (in particular, engine induced vibrations). A few seconds of the system response to the driver demand are shown in Fig. 9. The track tests confirmed the good performance of the control system, in terms of both robustness and readiness.

## V. CONCLUSIONS

A discrete-time, observer-based sliding mode throttle controller with integral action has been designed and implemented on a racing motorcycle engine. Tests on both an experimental setup and a MotoGP racing motorcycle confirmed the good performance of the proposed control scheme, in particular in terms of robustness and readiness. The most critical disturbance, i.e. friction, is successfully dealt with, as is shown by analysing the system response to small valve opening requests, where a very small overshoot

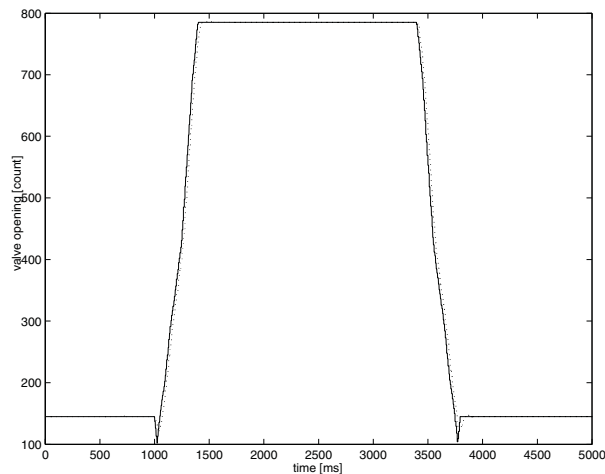


Fig. 8. System response to a ramp gas input signal (continuous line: set point; dotted line: actual valve opening).

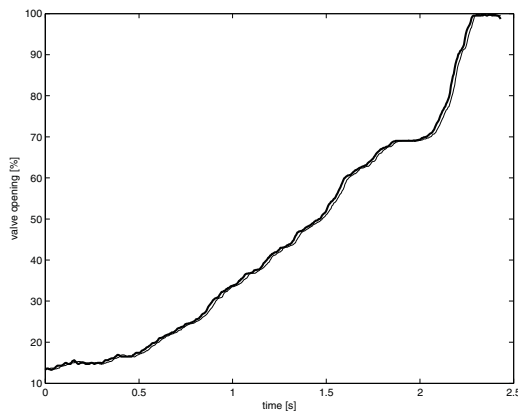


Fig. 9. Telemetry data from the track test with the Aprilia RS Cube

can be achieved, without affecting the system readiness. A theoretical analysis has also been proposed, clarifying the interplay among the DTSMC dynamics, the observer dynamics, and the effect of the integral action.

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