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# DIFFERENTIAL EQUATIONS MATLAB INVESTIGATION

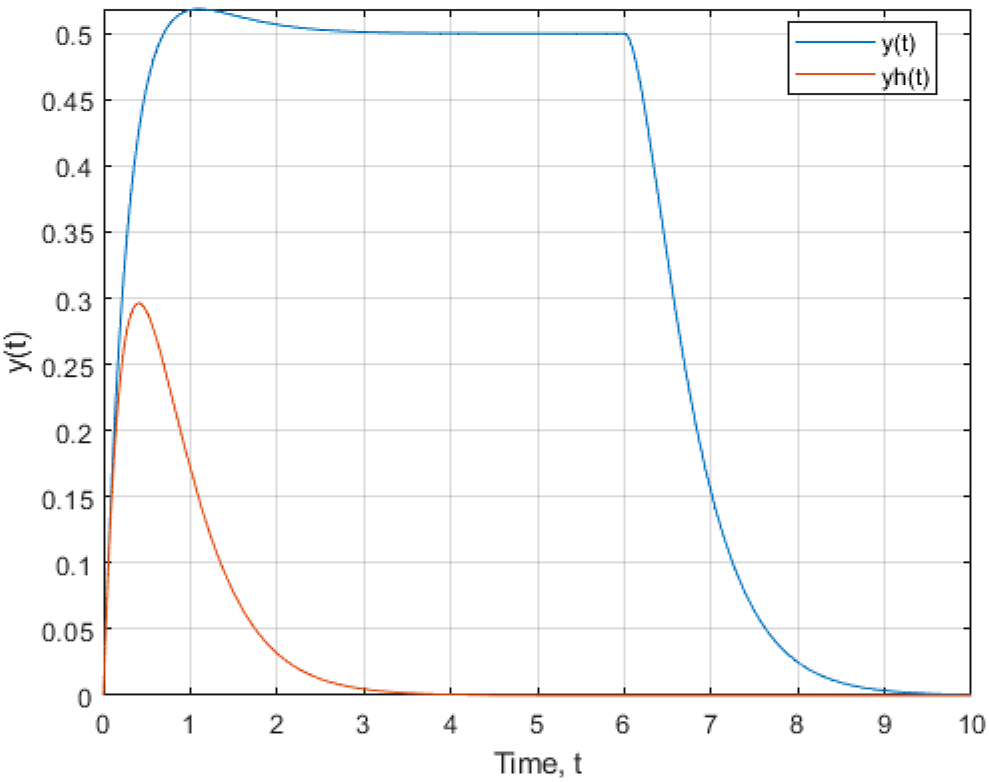
```
%a similar example with 5.40/404 glynjames
%x"+4x'+x=f(t) differential equation with an impulse external force
%f(t)=1 , T<=3, 0 for t >=3 the external force is a step function
```

## 5a. 1-2

```
clear; % clear variables
clc; % clear command window
close all

% Define symbolic variables
syms s t y(t) Y(s) yh(t) Yh(t)
D1y = diff(y);
D2y = diff(D1y);

% Dynamical system : define it
Eqn = D2y+5*D1y + 6*y == 3*heaviside(t)- 3*heaviside(t-6)
% Homogeneous equation
Eqnh = D2y+5*D1y + 6*y == 0

% Perform Laplace Transform of Eqn
LEqn = laplace(Eqn);
LEqnh = laplace(Eqnh);
% Substitute Laplace transform of y(t) for Y(s), and y(0) for the initial
 condition
LEqn = subs(LEqn, {laplace(y(t), t, s), y(0),D1y(0)}, {Y(s), 0,2});
LEqnh = subs(LEqnh, {laplace(y(t), t, s), y(0),D1y(0)}, {Yh(s), 0,2});
% Isolate Y(s) in LEqn
LEqn = isolate(LEqn, Y(s));
LEqnh = isolate(LEqnh, Yh(s));
```

```
% Perform inverse Laplace transform on the right-hand side of LEqn
y(t) = ilaplace(rhs(LEqn))
yh(t) = ilaplace(rhs(LEqnh))

% Plot the solution
fplot(@(t) y(t), [0 10])
hold on
grid on
fplot(@(t) yh(t), [0 10])
xlabel('Time, t')
ylabel('y(t)')
grid on
legend('y(t)', 'yh(t)','Location','best')
```

```
Eqn(t) =

6*y(t) + 5*diff(y(t), t) + diff(y(t), t, t) == 3*heaviside(t) - 3*heaviside(t
 - 6)


Eqnh(t) =

6*y(t) + 5*diff(y(t), t) + diff(y(t), t, t) == 0


y(t) =

exp(-2*t)/2 - exp(-3*t) - 3*heaviside(t - 6)*(exp(18 - 3*t)/3 - exp(12 -
 2*t)/2 + 1/6) + 1/2


yh(t) =

2*exp(-2*t) - 2*exp(-3*t)
```




# 5a. 3 Explination

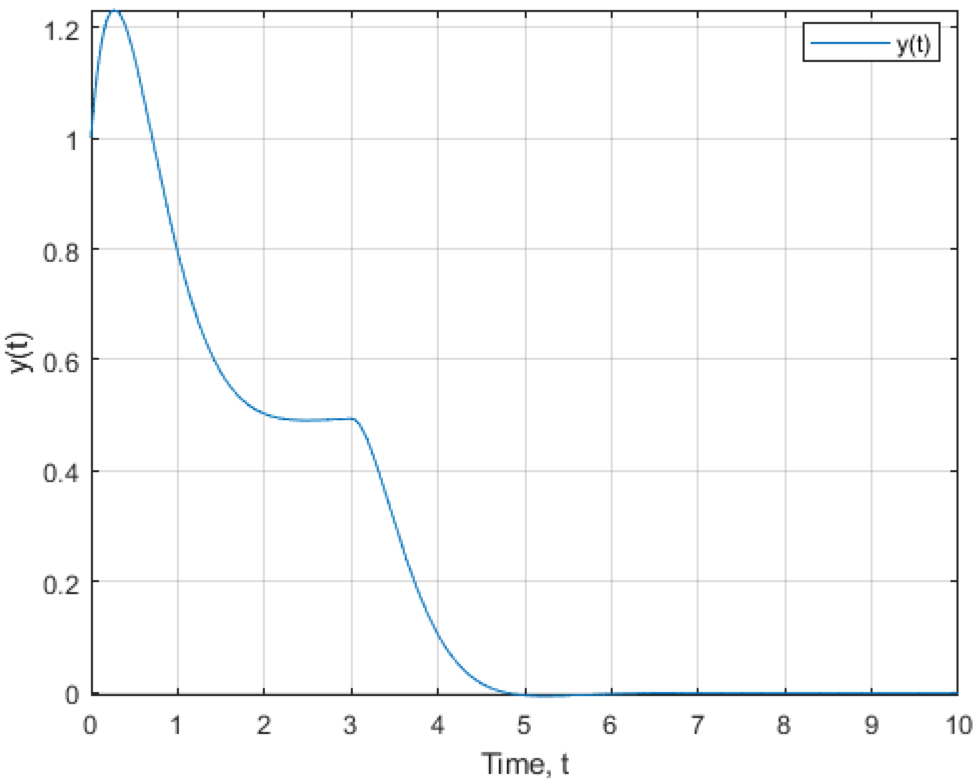
The homogeneous solution is an exponential function with lambda = -2 and -3. The solution with the forcing function still contains exponentials but its magnitude is larger and between t = 4 and t = 6, the function is constant at 0.5.

# 5a. 4

```
clear; % clear variables
clc; % clear command window
close all

syms s t y(t) Y(s)
D1y = diff(y);
D2y = diff(D1y);

% Dynamical system : define it
Eqn = D2y+4*D1y + 6*y == 3*heaviside(t)-3*heaviside(t-3);

% Perform Laplace Transform of Eqn
LEqn = laplace(Eqn);
% Substitute Laplace transform of y(t) for Y(s), and y(0) for the initial
 condition
LEqn = subs(LEqn, {laplace(y(t), t, s), y(0),D1y(0)}, {Y(s), 1,2});
% Isolate Y(s) in LEqn
LEqn = isolate(LEqn, Y(s));
```

```
% Perform inverse Laplace transform on the right-hand side of LEqn
y(t) = ilaplace(rhs(LEqn))

% Plot the solution
fplot(@(t) y(t), [0 10])
grid on

xlabel('Time, t')
ylabel('y(t)')
grid on
legend('y(t)','Location','best')
```

```
y(t) =

3*heaviside(t - 3)*((exp(6 - 2*t)*(cos(2^(1/2)*(t - 3)) +
 2^(1/2)*sin(2^(1/2)*(t - 3))))/6 - 1/6) - (exp(-2*t)*(cos(2^(1/2)*t)
 + 2^(1/2)*sin(2^(1/2)*t)))/2 + exp(-2*t)*(cos(2^(1/2)*t) -
 2^(1/2)*sin(2^(1/2)*t)) + 3*2^(1/2)*exp(-2*t)*sin(2^(1/2)*t) + 1/2
```




# 5a. 4 Explination

Compared to the previous solution, the fuction cap is now lower down the equation. This solution also decreases faster than the above solution.

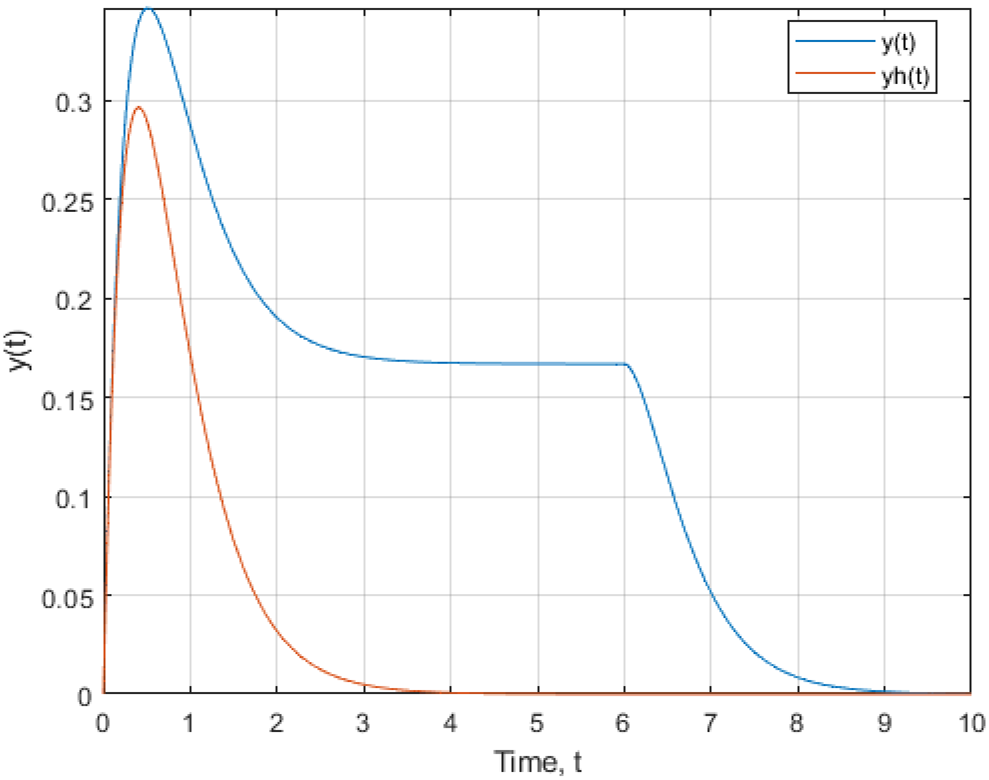
# 5a. 6

```
clear; % clear variables
clc; % clear command window
close all

% Define symbolic variables
syms s t y(t) Y(s) yh(t) Yh(t)
D1y = diff(y);
D2y = diff(D1y);

% Dynamical system : define it
Eqn = D2y+5*D1y + 6*y == heaviside(t)- heaviside(t-6)
% Homogeneous equation
Eqnh = D2y+5*D1y + 6*y == 0

% Perform Laplace Transform of Eqn
LEqn = laplace(Eqn);
LEqnh = laplace(Eqnh);
% Substitute Laplace transform of y(t) for Y(s), and y(0) for the initial
 condition
LEqn = subs(LEqn, {laplace(y(t), t, s), y(0),D1y(0)}, {Y(s), 0,2});
LEqnh = subs(LEqnh, {laplace(y(t), t, s), y(0),D1y(0)}, {Yh(s), 0,2});
% Isolate Y(s) in LEqn
LEqn = isolate(LEqn, Y(s));
LEqnh = isolate(LEqnh, Yh(s));

% Perform inverse Laplace transform on the right-hand side of LEqn
y(t) = ilaplace(rhs(LEqn))
yh(t) = ilaplace(rhs(LEqnh))

% Plot the solution
fplot(@(t) y(t), [0 10])
hold on
grid on
fplot(@(t) yh(t), [0 10])
xlabel('Time, t')
ylabel('y(t)')
grid on
legend('y(t)', 'yh(t)','Location','best')
```

```
Eqn(t) =

6*y(t) + 5*diff(y(t), t) + diff(y(t), t, t) == heaviside(t) - heaviside(t - 6)


Eqnh(t) =

6*y(t) + 5*diff(y(t), t) + diff(y(t), t, t) == 0


y(t) =
```

```
(3*exp(-2*t))/2 - (5*exp(-3*t))/3 - heaviside(t - 6)*(exp(18 - 3*t)/3 - exp(12
 - 2*t)/2 + 1/6) + 1/6
```

```
yh(t) =

2*exp(-2*t) - 2*exp(-3*t)
```




# 5a. 6 Explination

The homogeneous solution is the same homogeneous solution as in part 1, with different constants. The overall solution is similar to the one in part 1 but with a lower magnitude.

# 5a. 7 Explination

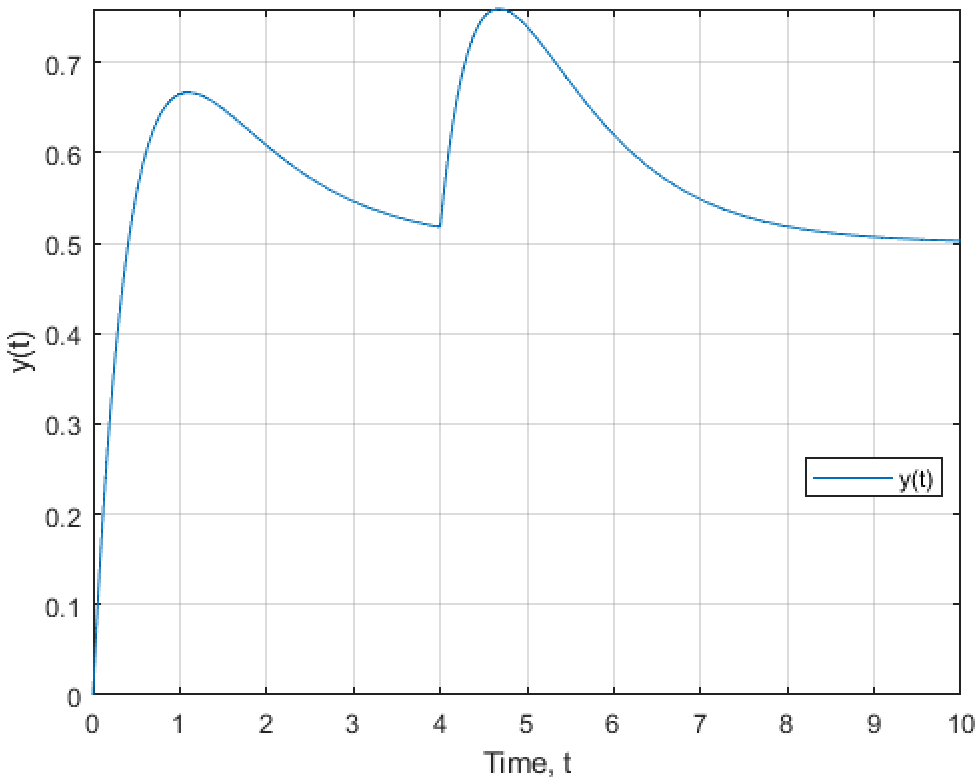
for all solutions, as t approaches infinity the solutions will approach 0.

# 5b. 1

```
clear; % clear variables
clc; % clear command window
close all
```

```
syms s t y(t) Y(s)
D1y = diff(y);
D2y = diff(D1y);

% Dynamical system : define it
Eqn = D2y+3*D1y + 2*y == 1+dirac(t-4)

% Perform Laplace Transform of Eqn
LEqn = laplace(Eqn);
% Substitute Laplace transform of y(t) for Y(s), and y(0) for the initial
 condition
LEqn = subs(LEqn, {laplace(y(t), t, s), y(0),D1y(0)}, {Y(s), 0,2});
% Isolate Y(s) in LEqn
LEqn = isolate(LEqn, Y(s))

% Perform inverse Laplace transform on the right-hand side of LEqn
y(t) = ilaplace(rhs(LEqn))

% Plot the solution
fplot(@(t) y(t), [0 10])
grid on

xlabel('Time, t')
ylabel('y(t)')
grid on
legend('y(t)','Location','best')
```

```
Eqn(t) =

2*y(t) + 3*diff(y(t), t) + diff(y(t), t, t) == dirac(t - 4) + 1


LEqn =

Y(s) == (exp(-4*s) + 1/s + 2)/(s^2 + 3*s + 2)


y(t) =

exp(-t) - (3*exp(-2*t))/2 + heaviside(t - 4)*(exp(4 - t) - exp(8 - 2*t)) + 1/2
```




# 5b. 2 Explination

Before t = 4, the system follows a solution determined by the homogeneous part of the equation, and at t = 4, the system experiences a sudden change in its behavior due to the impulse function.

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