Vector Addition

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1 Vector Representation

What is a vector?, any mathematical object that has both magnitude and direction. Consider the number 5 on the real number line, here 5 is a vector that is five units from the origin in the positive direction, and is an example of a one dimensional vector. There are many ways by which we may represent vectors. Some of these ways are:

- $a\hat{i} + b\hat{j} + c\hat{k}$ where $a, b, c \in \mathbb{R}$ and $\hat{i}, \hat{j}, \hat{k}$ are the cardinal directions in 3-D, in 2-D $\hat{i}, \hat{j}$ are commonly used. Beyond three dimensions it is common to use $\hat{x_1}, \hat{x_2}, \ldots, \hat{x_1}$ where each represents a cardinal direction in the space.

- $\langle x_1, x_2, \ldots, x_n \rangle$ where $x_1, x_2, \ldots, x_n \in \mathbb{R}$

- $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ where $x_1, x_2, \ldots, x_n \in \mathbb{R}$

- $\vec{x}$ to denote an arbitrary vector. Note $\vec{0}$ is commonly used to denote the vector with zero magnitude.

- Graphically using an arrow in 2-D or 3-D, but not 4-D or higher because then it's hard to draw.

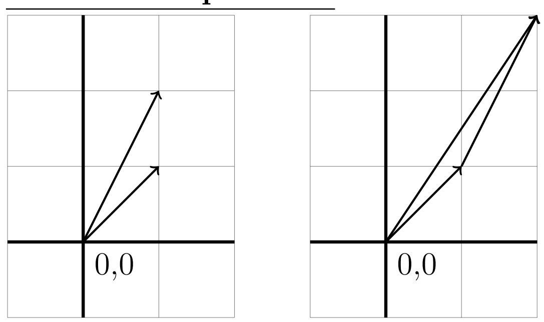
2 How To

We add vectors component-wise so given two n-dimensional vecors $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$ and $\vec{y} = \langle y_1, y_2, \ldots, y_n \rangle$,

$$\vec{x} + \vec{y} = \langle x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n \rangle$$

We can add vectors graphically via the **tail to tip** method. Note that two vectors that have the same magnitude and direction are equivalent no matter their position in the space.

The **tail to tip** method



3 Activity: Space Domination

Cut out the provided starships and torpedoes. Activity meant to be done in groups of 3.

The rules:

Play on the table top. 1 unit = 1 cm, adjust if needed. You can use rulers, meter sticks, tape measures or any other straight edge to represent vectors.

Each player's starship has 3 damage points. When a starship gets hit by a torpedo or another starship, it loses a damage point. If a starship reaches 0 damage points, flip it over and draw an 'X' on the back. Should a starship's movement cause it to fly off the edge of the table the starship's damage points is reduced to 0.

Each player's starship starts with 3 torpedoes

Roll a six sided die to determine turn order, highest roll goes first. On your turn you can either move your starship or lauch a torpedo.

When your starship moves it accelerates in the direction it is pointing. Where your ship moves to is defined by a vector, $\vec{v}$, where:

$$\vec{v} = \vec{a} + \vec{r} \tag{1}$$

calculated using the **tail to tip** method.

When your ship moves it produces an acceleration vector, $\vec{a}$, with magnitude in units equal to the value rolled on a six-sided die that is in the direction of your ship's acceleration. When the game begins $\vec{r} = \vec{0}$, so on your first move $\vec{v} = \vec{a}$. On your turn before you move $\vec{r}$ is set to the $\vec{v}$ of the previous turn. On your next move you can accelerate again producing a new vector, $\vec{v}$ via eq. (1).

While moving you may rotate your ship up to 180° (π radians) clockwise or counterclockwise.

When a torpedo is lauched the starship drifts along its $\vec{r}$. A topedo moves in the same fashion as a starship, but its initial direction is determined by the direction the launching starship is pointed, and the magnitude of the torpedo's $\vec{a}$ is always 6 units. A torpedo launched is not regained.

Win condition: Be the last starship standing.

Starships

Torpedoes

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