令 $z_{00} = a_{00} w_{00} + a_{01} w_{01} + \cdots + a_{33} w_{32}$

$z_{01} = a_{01} w_{00} + a_{02} w_{01} + \cdots + a_{23} w_{32}$

$z_{10} = a_{10} w_{00} + a_{11} w_{01} + \cdots + a_{32} w_{32}$

$z_{11} = a_{11} w_{00} + a_{12} w_{01} + \cdots + a_{33} w_{32}$

使得

$O_{00} = \sigma(z_{00})$

$O_{01} = \sigma(z_{01})$

$O_{10} = \sigma(z_{10})$

$O_{11} = \sigma(z_{11})$

令 $y = O_{00} w_a + O_{01} w_b + O_{10} w_c + O_{11} w_d$

$L = (yy - y)^2$

求 $\dfrac{dL}{dw_{11}} = \dfrac{dL}{dy} \cdot \dfrac{dy}{dw_{11}}$

$= \dfrac{dL}{dy} \left[\dfrac{dy}{dO_{00}} \cdot \dfrac{dO_{00}}{dw_{11}} + \dfrac{dy}{dO_{01}} \cdot \dfrac{dO_{01}}{dw_{11}} + \dfrac{dy}{dO_{10}} \cdot \dfrac{dO_{10}}{dw_{11}} + \dfrac{dy}{dO_{11}} \cdot \dfrac{dO_{11}}{dw_{11}} \right]$

$= \dfrac{dL}{dy} \left[\dfrac{dy}{dO_{00}} \dfrac{dO_{00}}{dz_{00}} \cdot \dfrac{dz_{00}}{dw_{11}} + \dfrac{dy}{dO_{01}} \cdot \dfrac{dO_{01}}{dz_{01}} \cdot \dfrac{dz_{01}}{dw_{11}} + \right.$

$\left. \dfrac{dy}{dO_{10}} \cdot \dfrac{dO_{10}}{dz_{10}} \cdot \dfrac{dz_{10}}{dw_{11}} + \dfrac{dy}{dO_{11}} \cdot \dfrac{dO_{11}}{dz_{11}} \cdot \dfrac{dz_{11}}{dw_{11}} \right]$

$= 2 \cdot (y - yy) \left[w_a \cdot (\sigma(z_{00}))(1 - \sigma(z_{00})) \cdot a_{11} + \right.$

$w_b \cdot (\sigma(z_{01})) \cdot (1 - \sigma(z_{01})) \cdot a_{12}$

$w_c \cdot (\sigma(z_{10})) \cdot (1 - \sigma(z_{10})) \cdot a_{21}$

$\left. w_d \cdot (\sigma(z_{11})) \cdot (1 - \sigma(z_{11})) \cdot a_{22} \right]$

$= 2 \cdot (O_{00} w_a + O_{01} w_b + O_{10} w_c + O_{11} w_d - yy) \times$

$\left[w_a \cdot O_{00} (1 - O_{00}) \cdot a_{11} + \right.$

$w_b \cdot O_{01} (1 - O_{00}) \cdot a_{12} +$

$w_c \cdot O_{10} (1 - O_{10}) \cdot a_{21} +$

$\left. w_d \cdot O_{11} (1 - O_{11}) \cdot a_{22} \right]$

求 $\dfrac{dL}{da_{00}} = \dfrac{dL}{dy} \cdot \dfrac{dy}{da_{00}}$

$= \dfrac{dL}{dy} \cdot \dfrac{dy}{dO_{00}} \cdot \dfrac{dO_{00}}{da_{00}}$

$= \dfrac{dL}{dy} \cdot \dfrac{dy}{dO_{00}} \cdot \dfrac{dO_{00}}{dz_{00}} \cdot \dfrac{dz_{00}}{da_{00}}$

$= 2(y - yy) \cdot w_a \cdot (\sigma(z_{00})(1 - \sigma(z_{00})) \cdot w_{00}$

$= 2(O_{00} w_a + O_{01} w_b + O_{10} w_c + O_{11} w_d - yy) \cdot w_a \cdot O_{00} (1 - O_{00}) \cdot w_{00}$