

Lecture 01 (A)



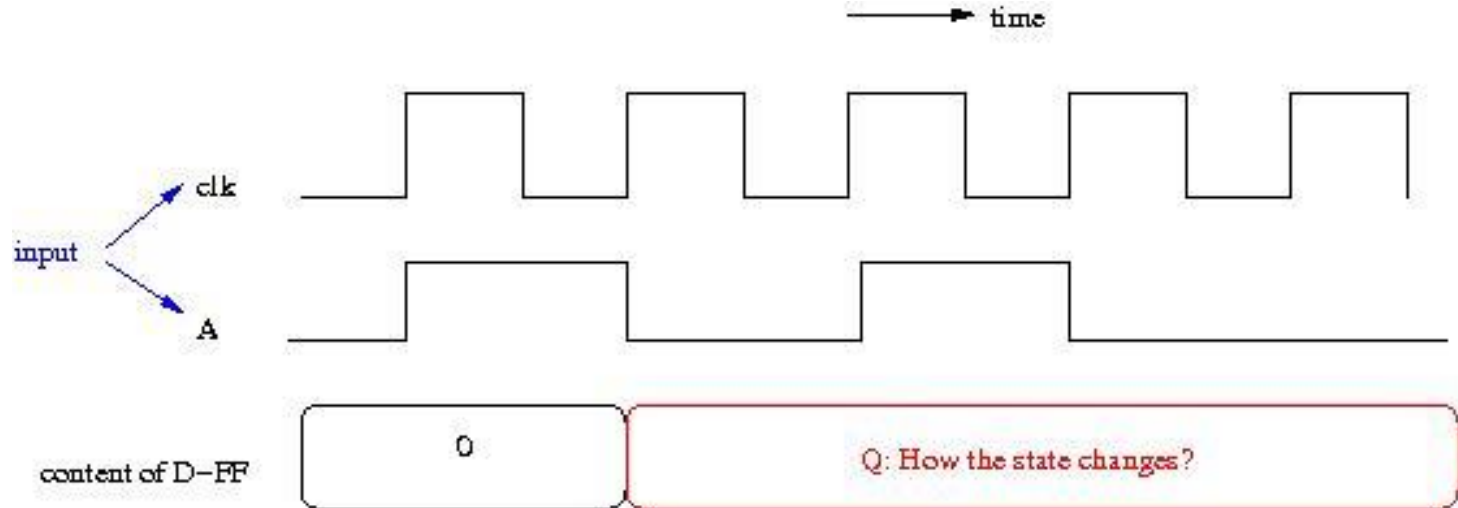
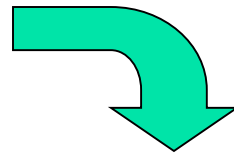
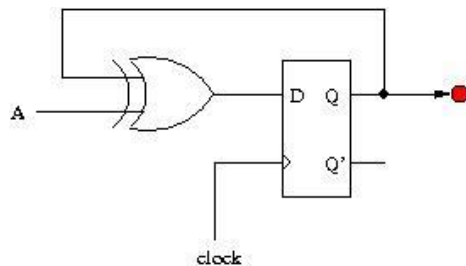
Sequential Circuit Analysis

Draw the timing diagram from the circuit

Section 5.4

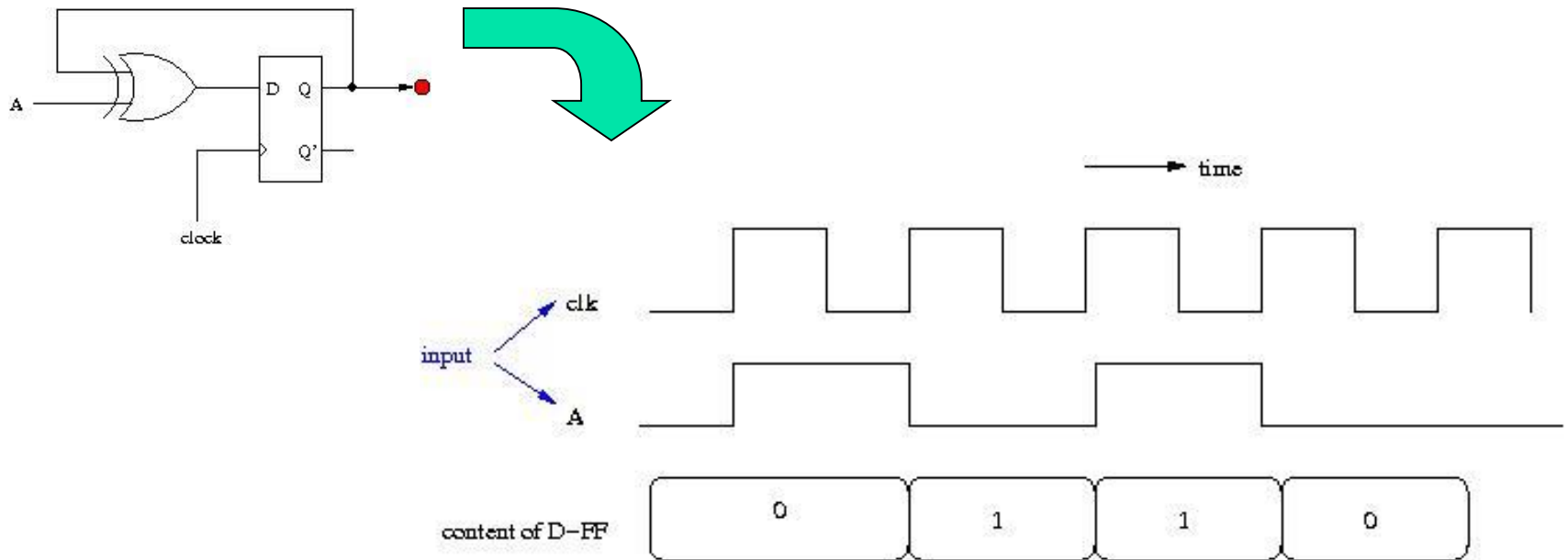
Today's Goal

- establish the ability to **imagine** how a circuit works
 - draw the timing waveform from a circuit diagram



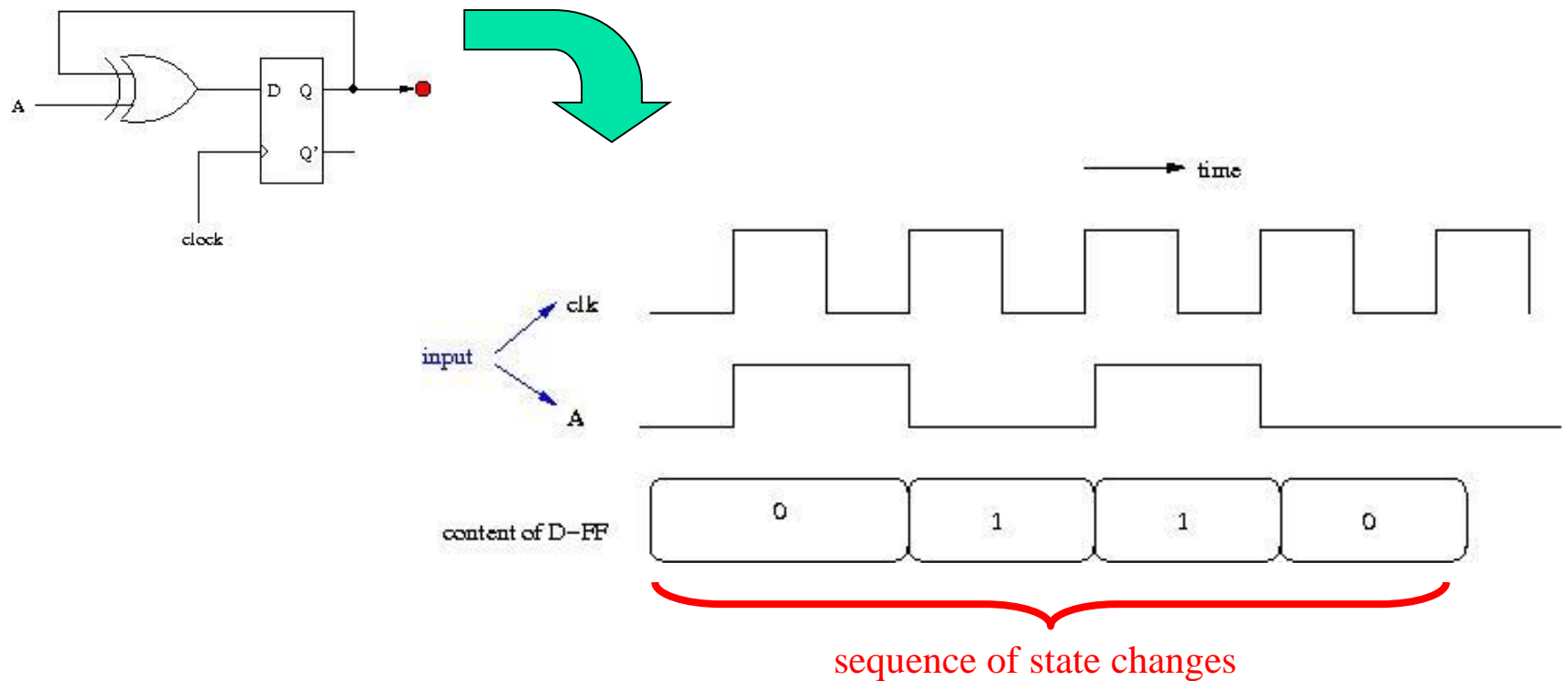
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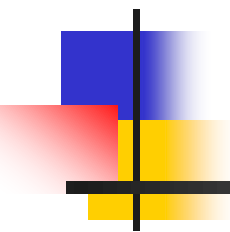
- That's why we call it **sequential circuit**





Materials coming from

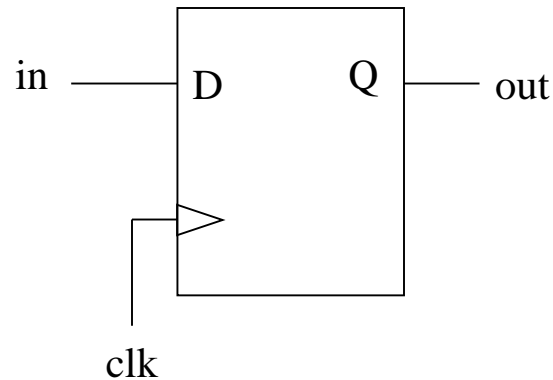
- Section 5.4



Preliminary: timing of logic elements (1)

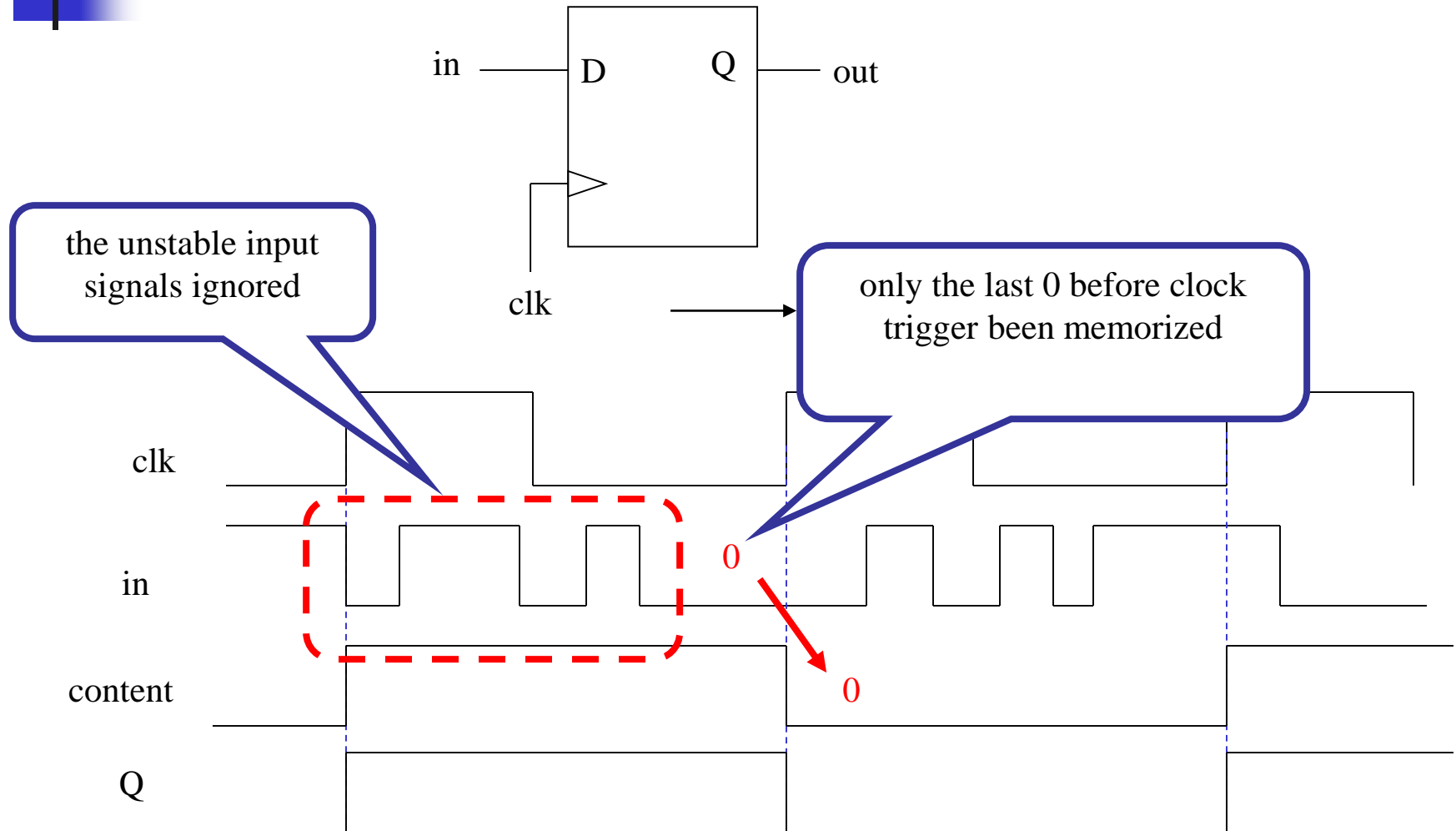
storage element: the D flip-flop

The D Flip-Flop

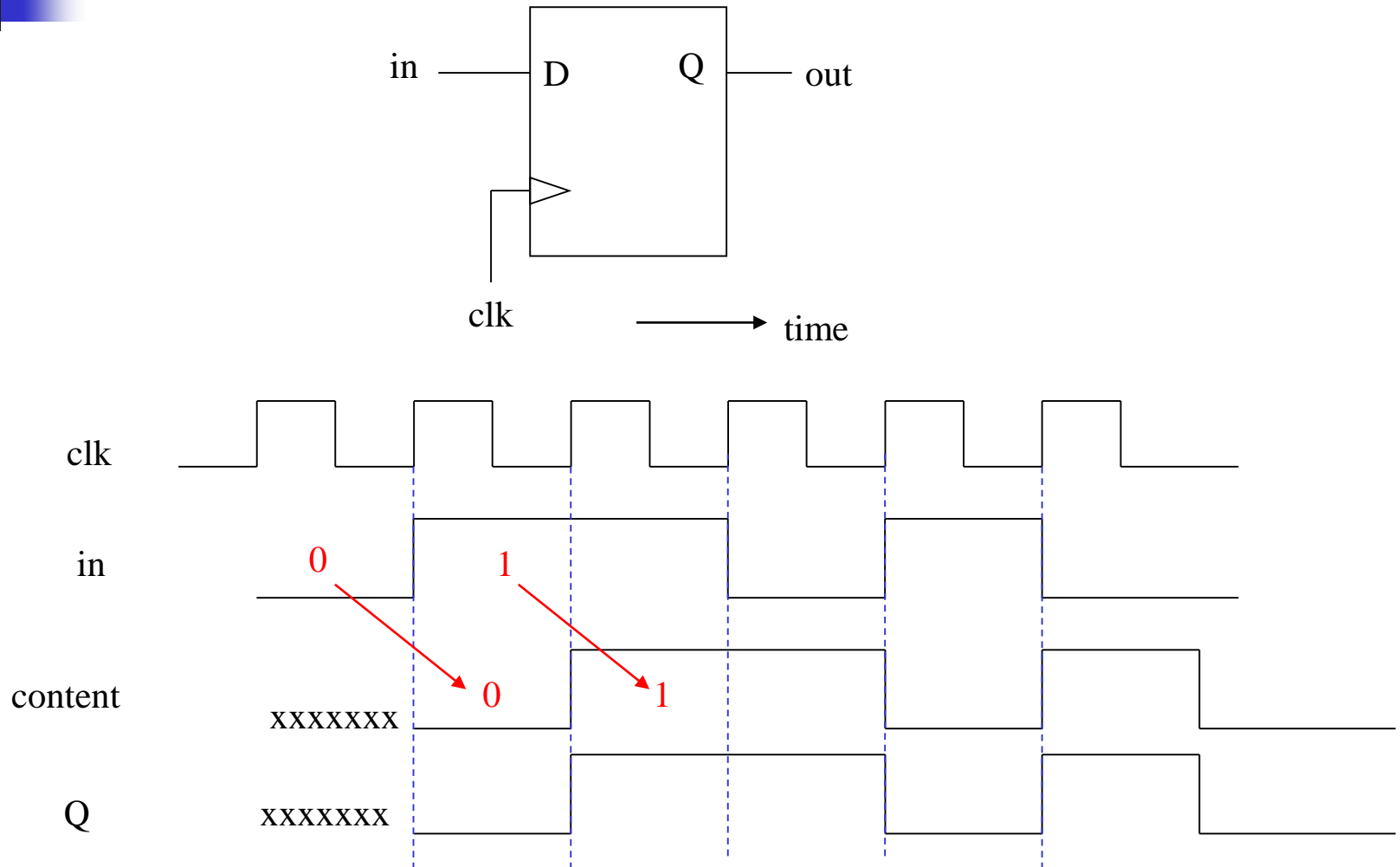


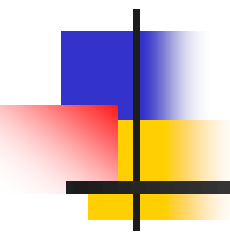
- Imagine that a D flip-flop is a box of 1-bit memory
- When the D flip-flop memorize the input?
 - at (positive) edge trigger of the clock (clk) signal
 - like a snapshot of a camera!

Timing Waveform of the D Flip-Flop



Timing Waveform of the D Flip-Flop



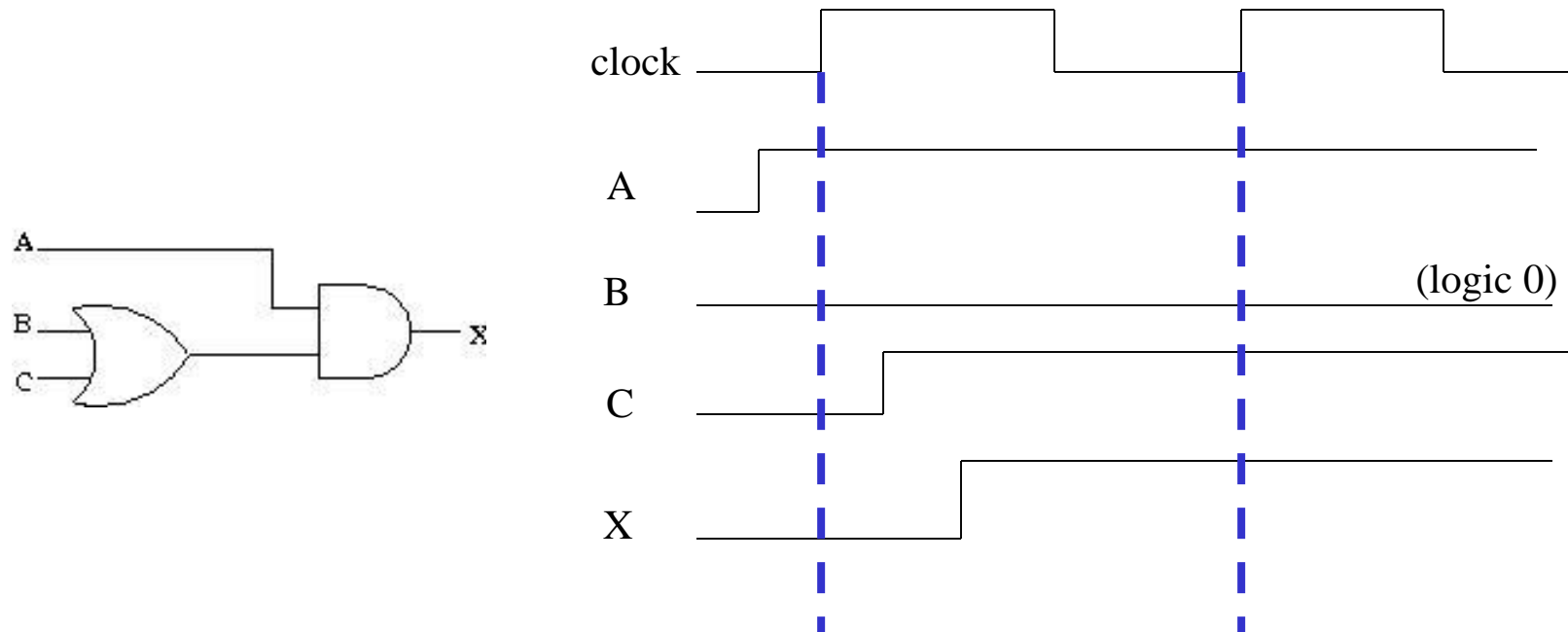


Preliminary: timing of logic elements (2)

elements for combinational circuit

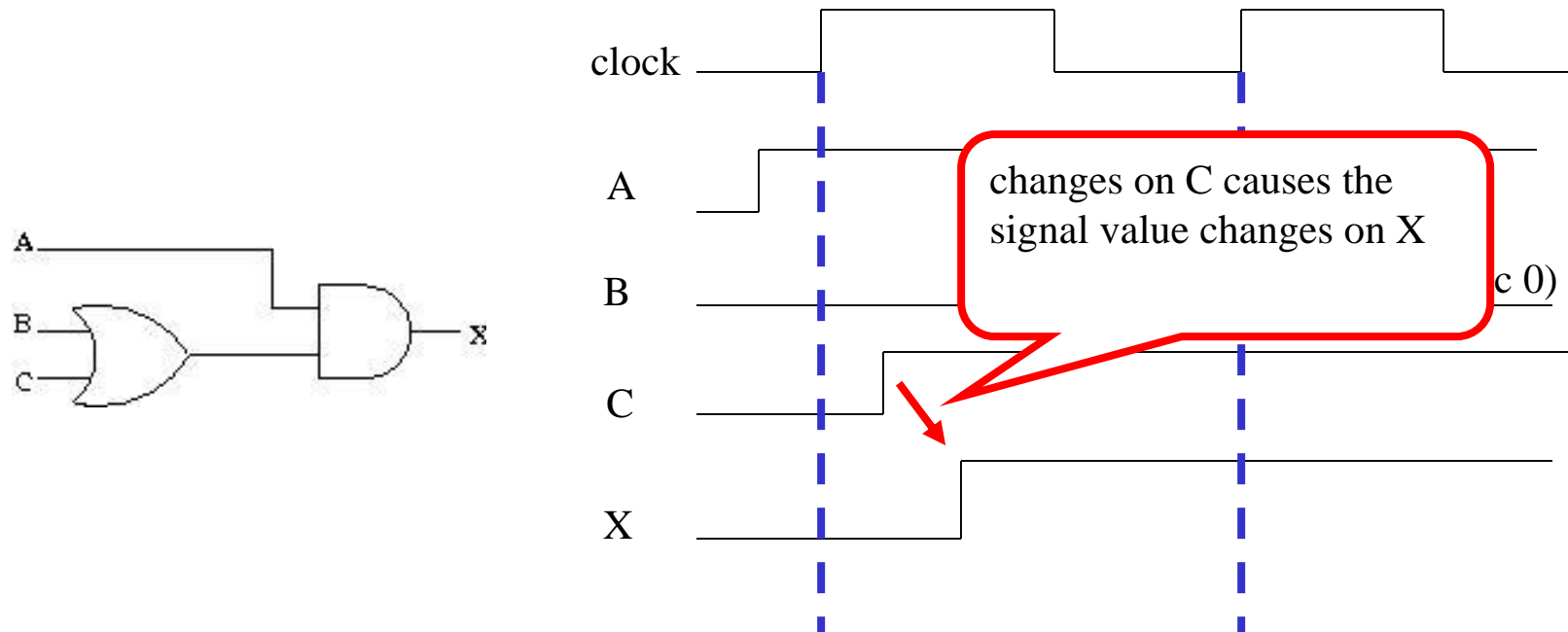
Timing of combinational circuit

- signal value changes immediately without waiting for the clock trigger



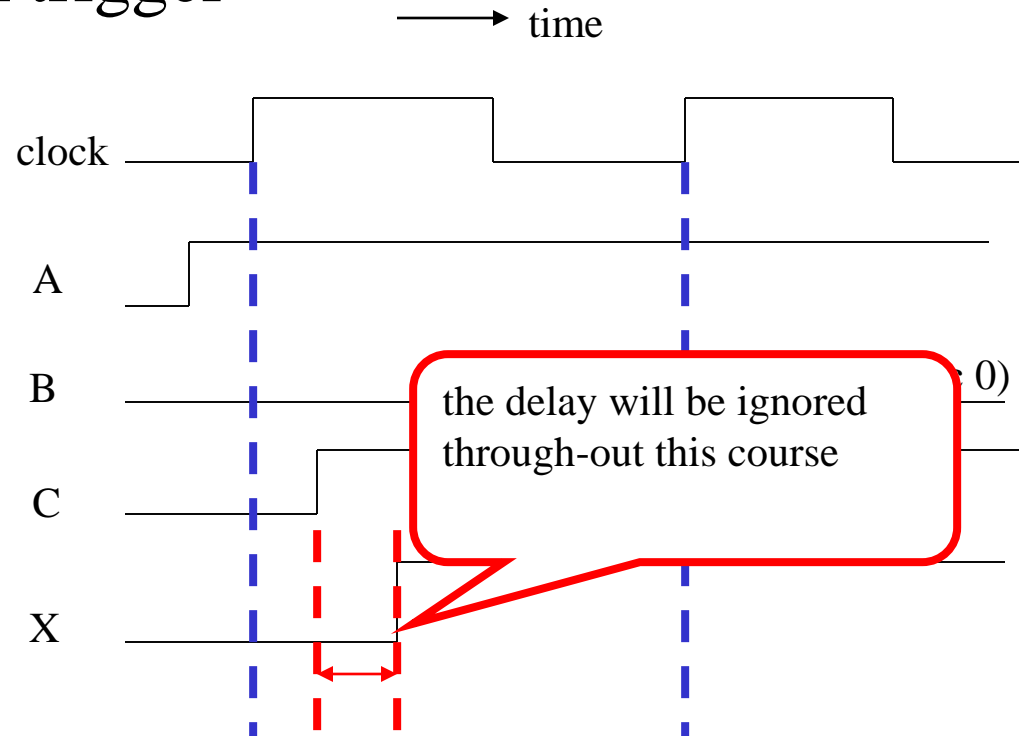
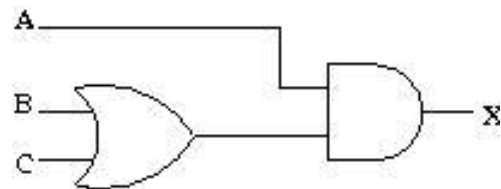
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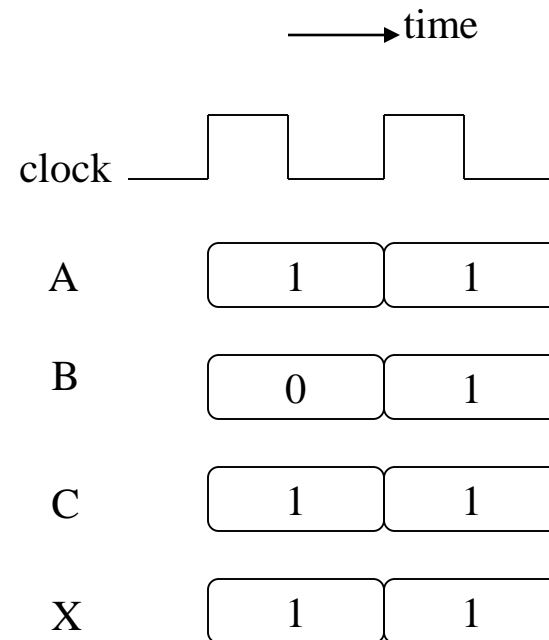
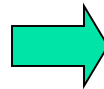
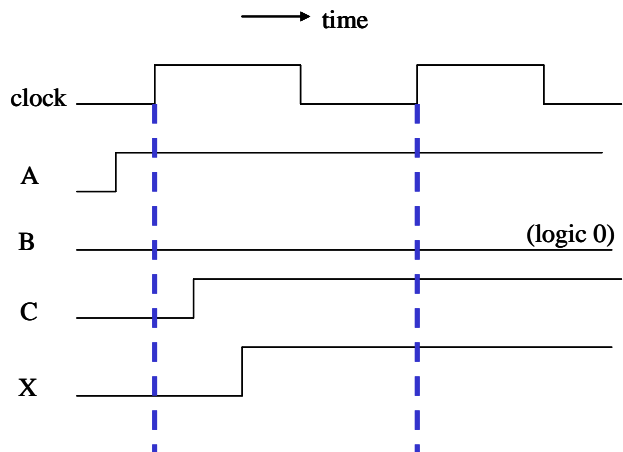
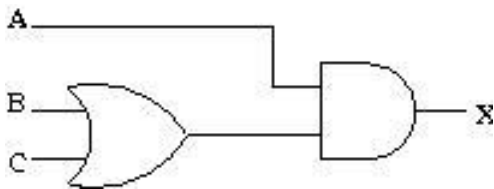
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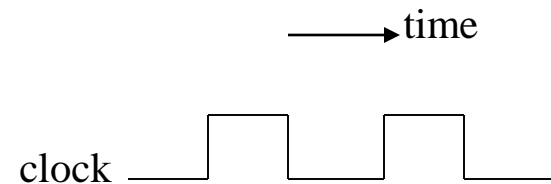
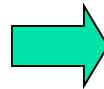
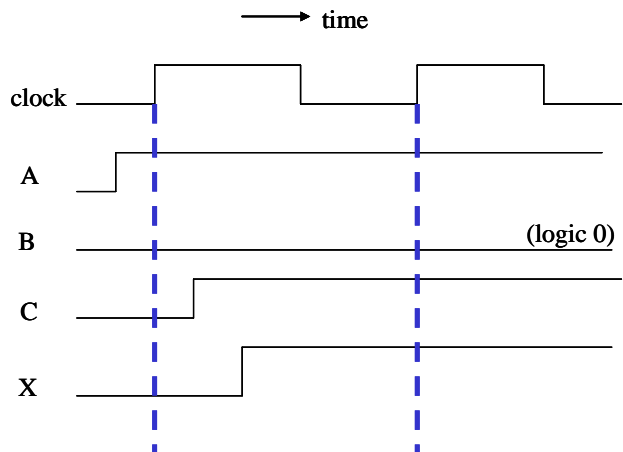
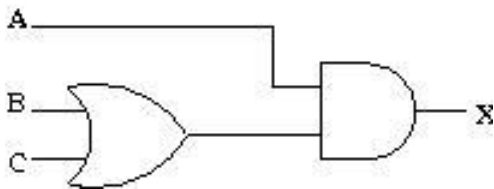
Timing of combinational circuit

- we simply express the timing waveform **cycle by cycle**



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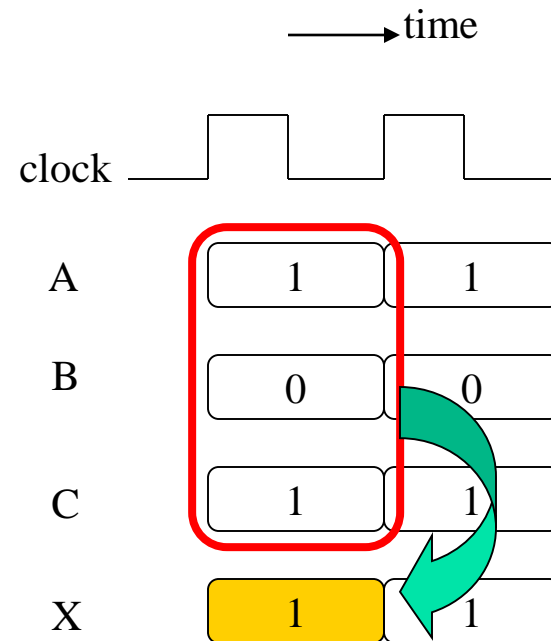
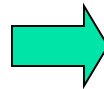
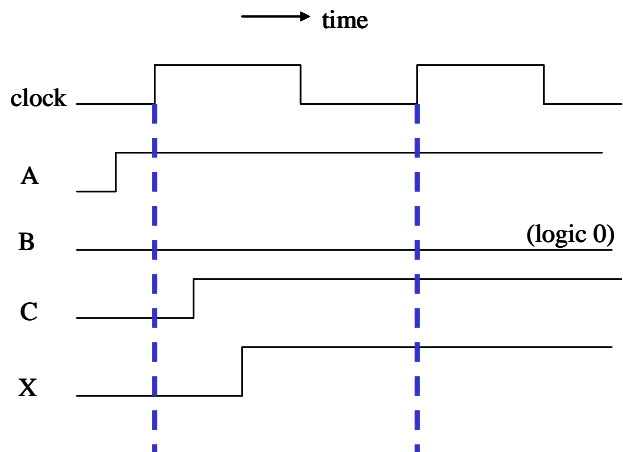
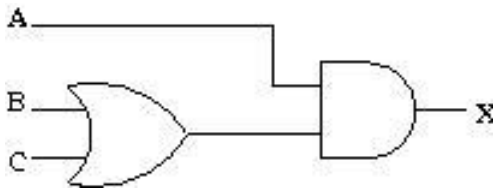


A	1	1
B	0	0
C	1	1
X	1	1

the signal value of C is stable at 1 at the end of this cycle

Timing of combinational circuit

- combinational circuit always gets the output **at the same cycle!**



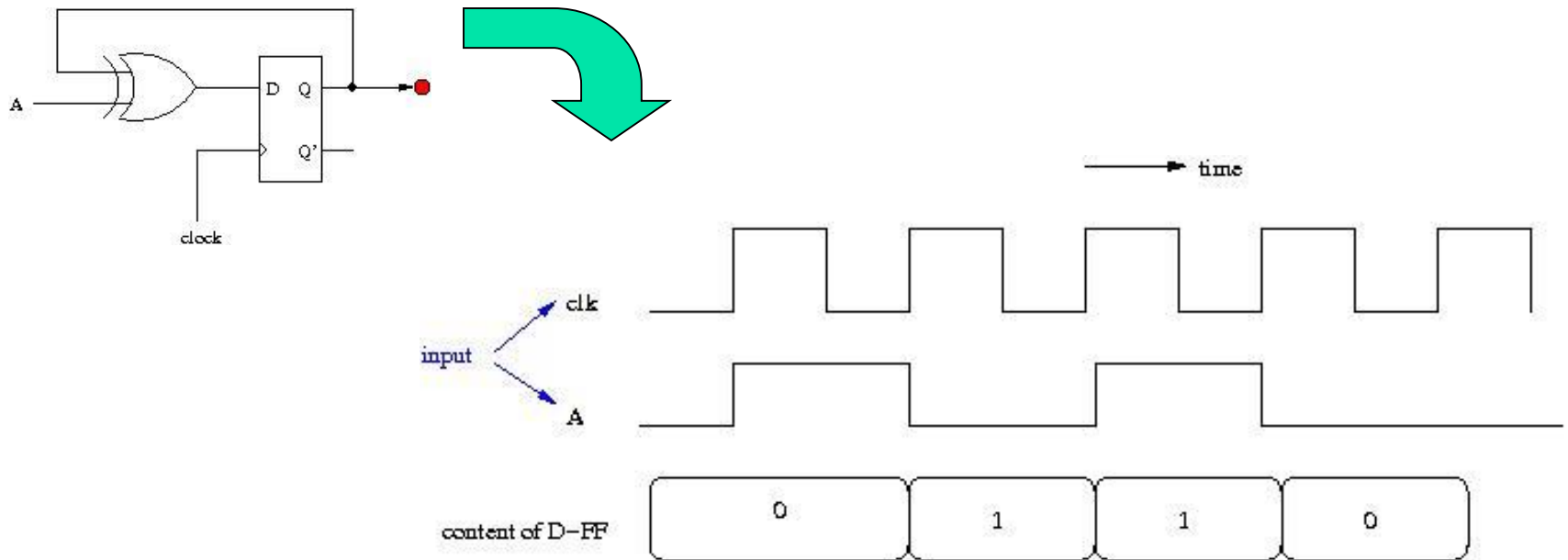


Now we begin

to draw the timing diagram from a
sequential circuit

Recall your goal

- establish the ability to **imagine** how a circuit works
 - draw the timing waveform from a circuit diagram





Method to analyze a sequential circuit

- Step 1: derive **input equations** to D flip-flops
- Step 2: derive the **state table**
- Step 3: draw the **state-diagram**

Then you can draw the timing waveform from a state-diagram



Input equations to storage elements

for D flip-flops only

Example Circuit

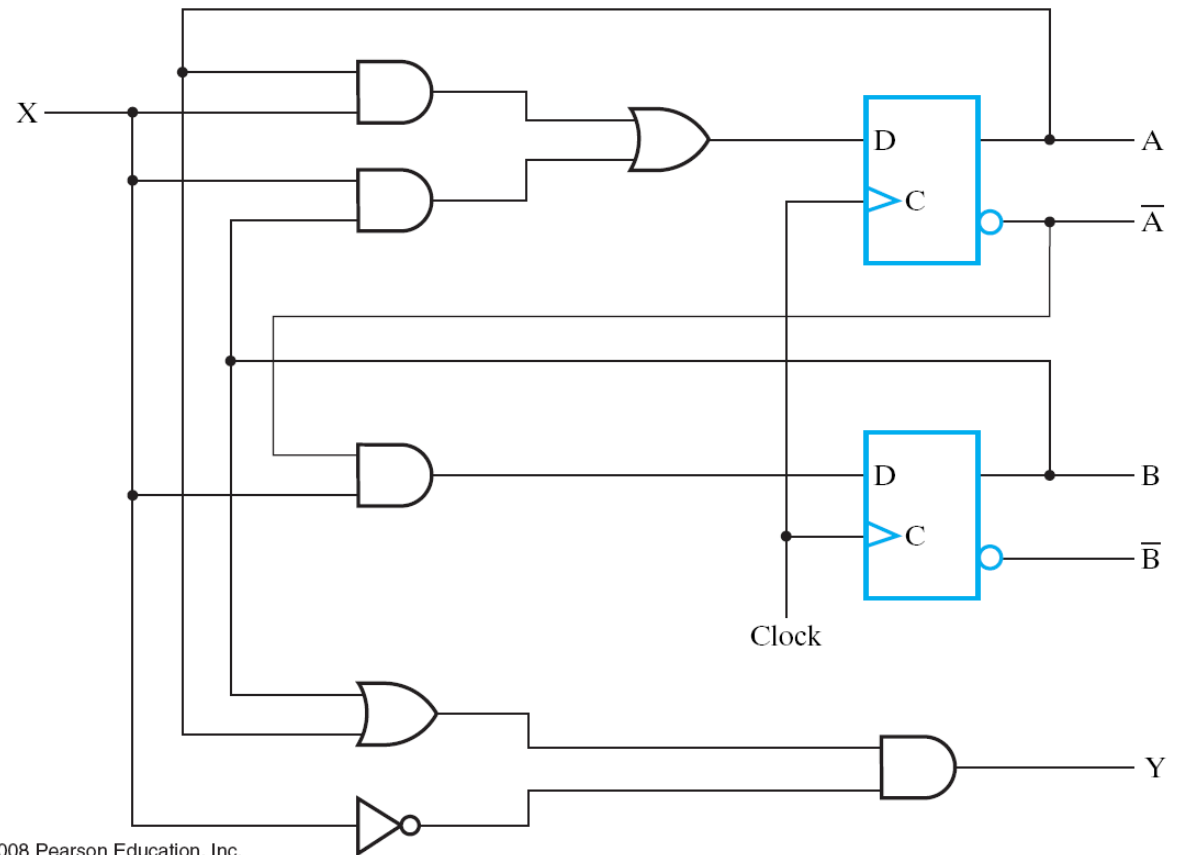
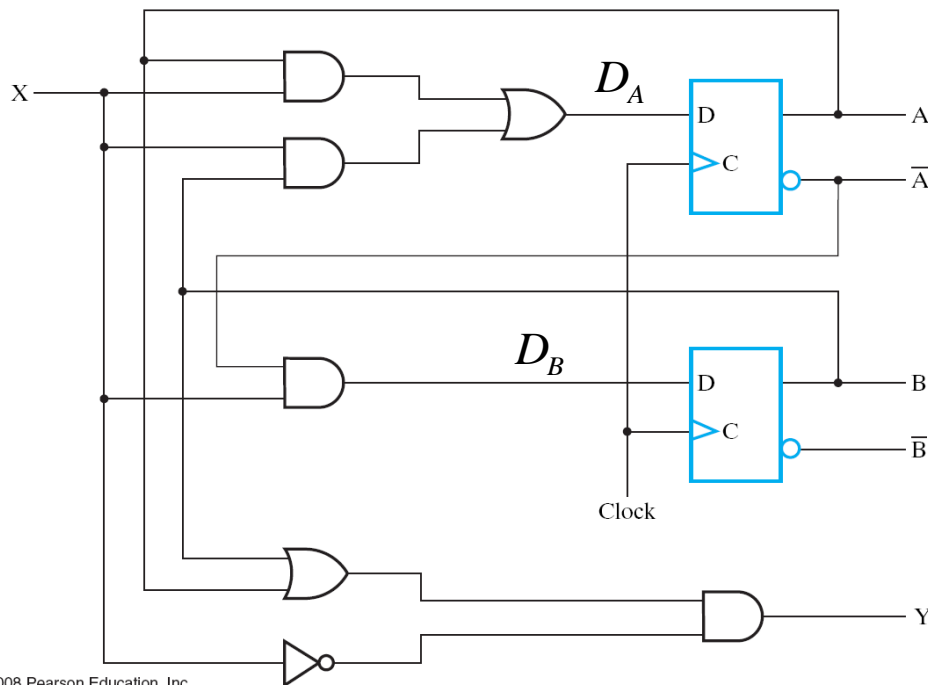


Figure 5-15 at page 241

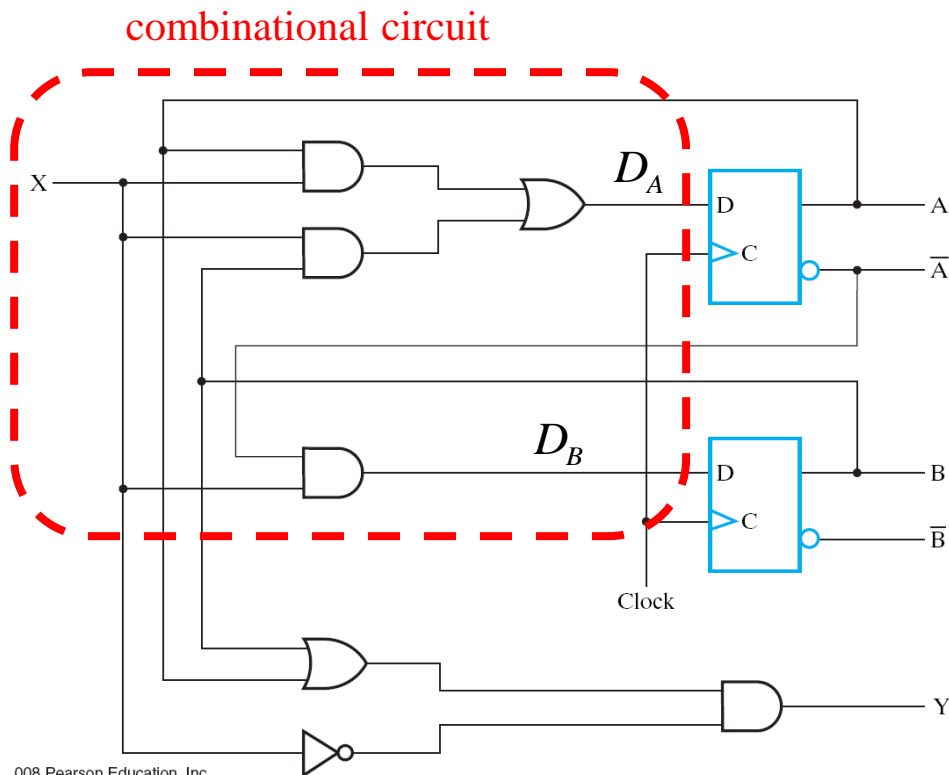
Input equation to D-FFs

- Boolean function to
 - determine the input to D-FFs
 - from current state (cycle t) and input signals



How to derive the input equations?

- ignore the D-FFs and consider the combinational circuit only
- simply write-down the Boolean equation of the combinational circuit

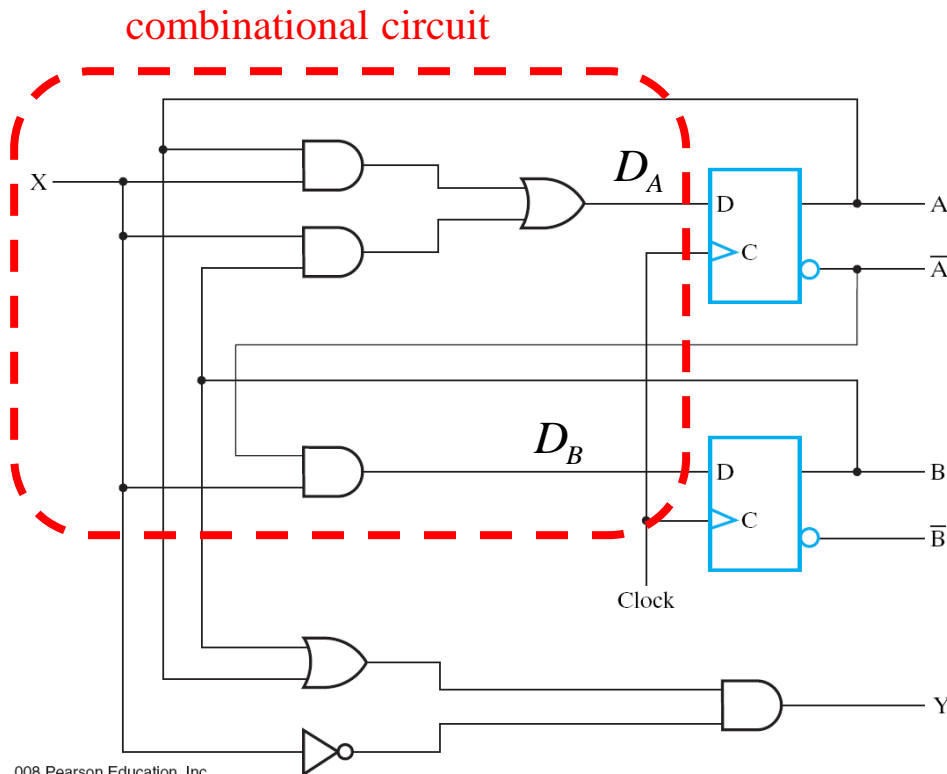


$$D_A = AX + BX$$

$$D_B = \bar{A}X$$

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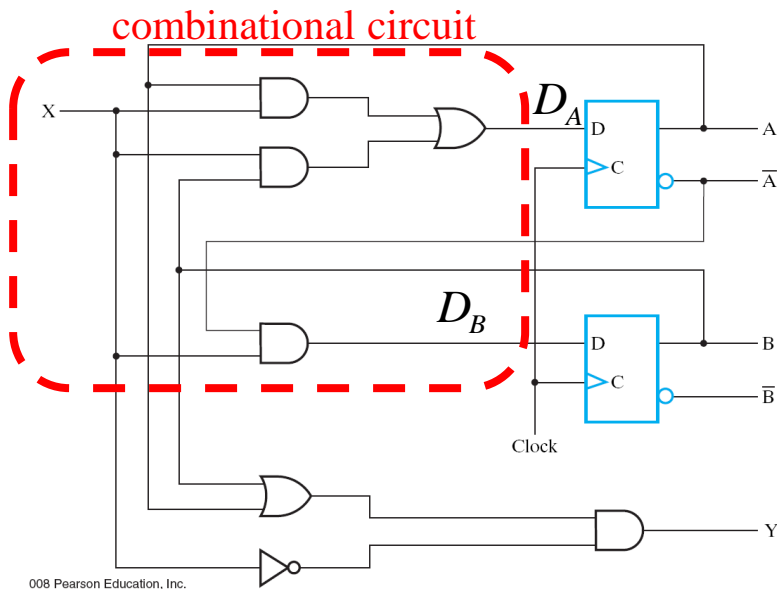
the input equation we want

Meaning of the input equations to D-FFs



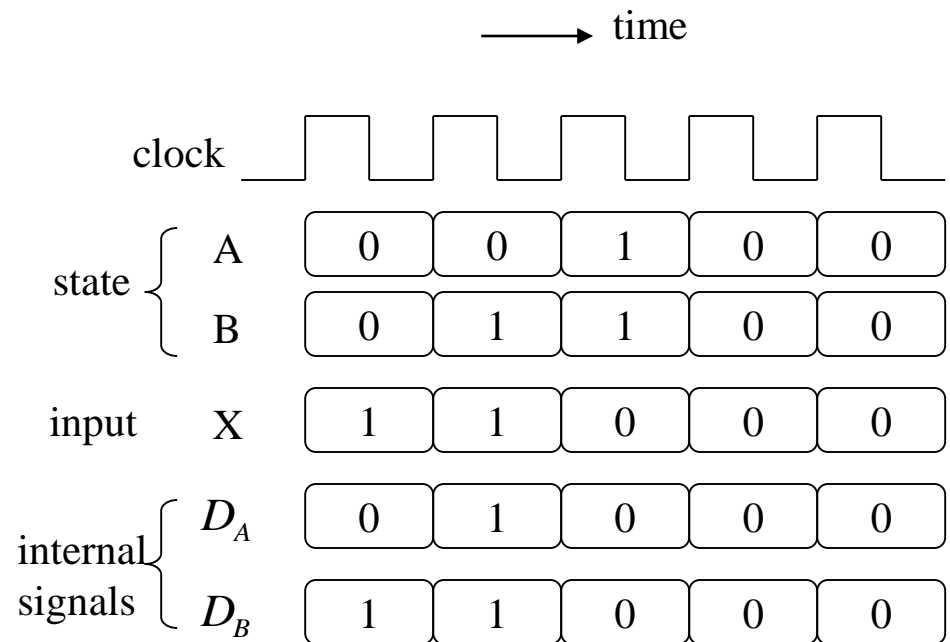
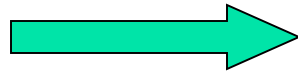
Meaning of the input equations

- the signal value at the end of the current cycle t



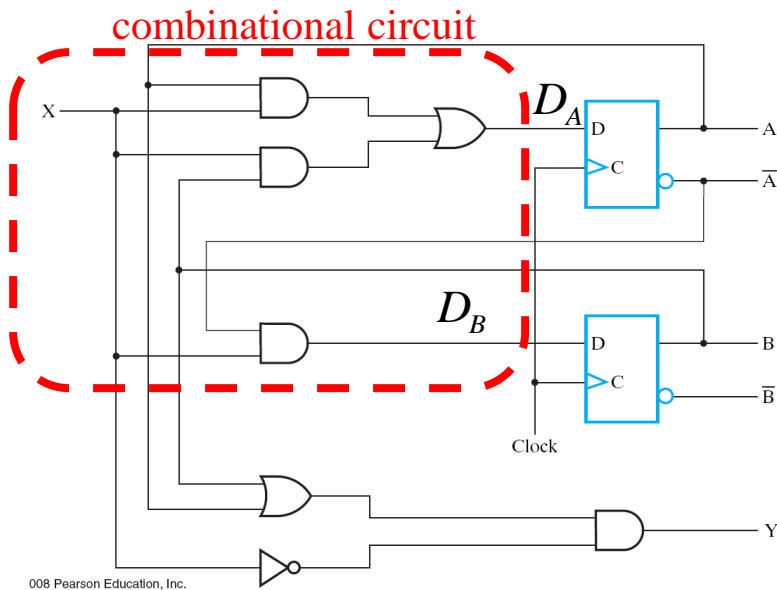
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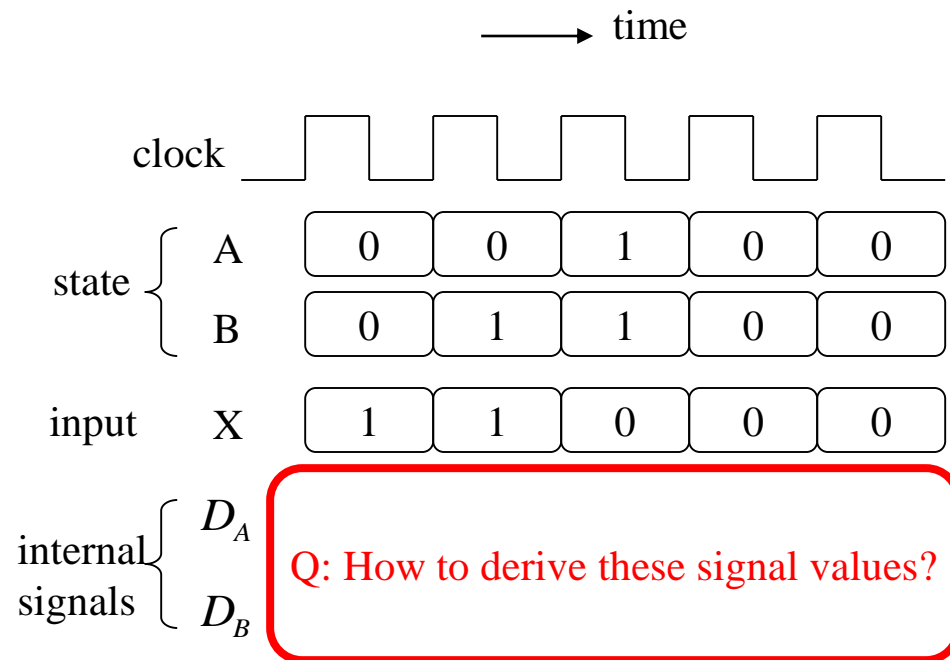
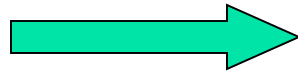
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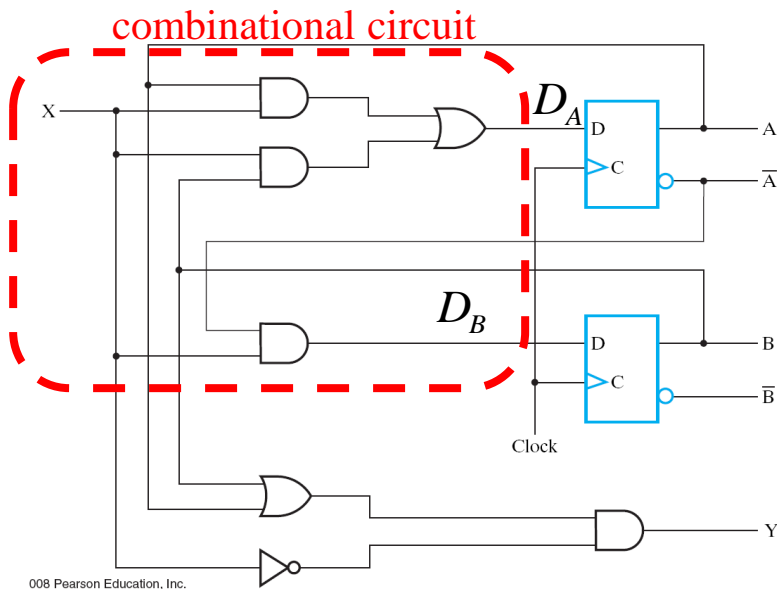
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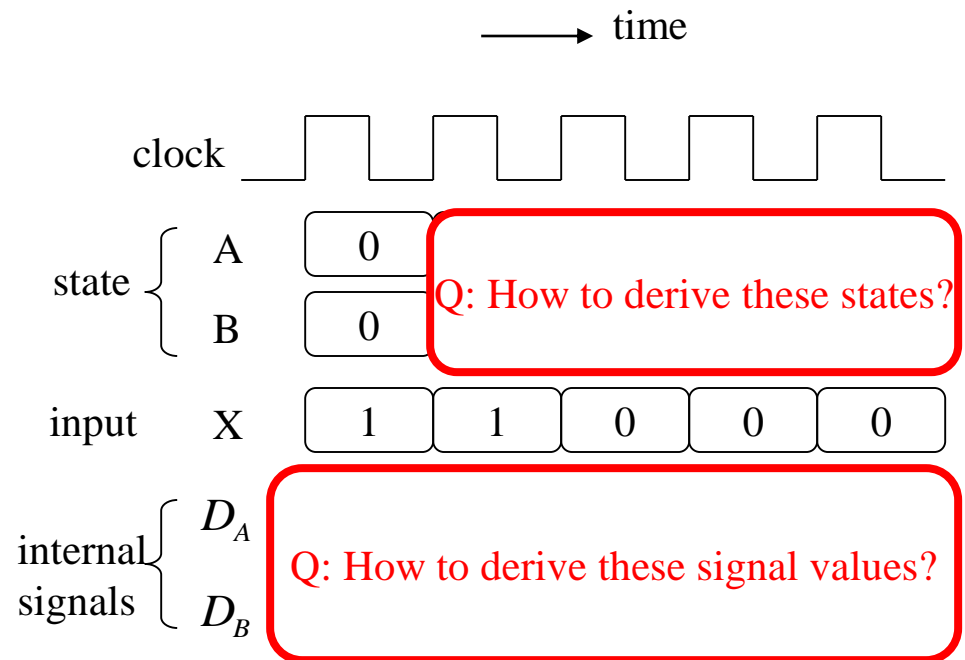
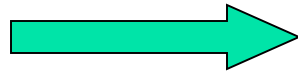
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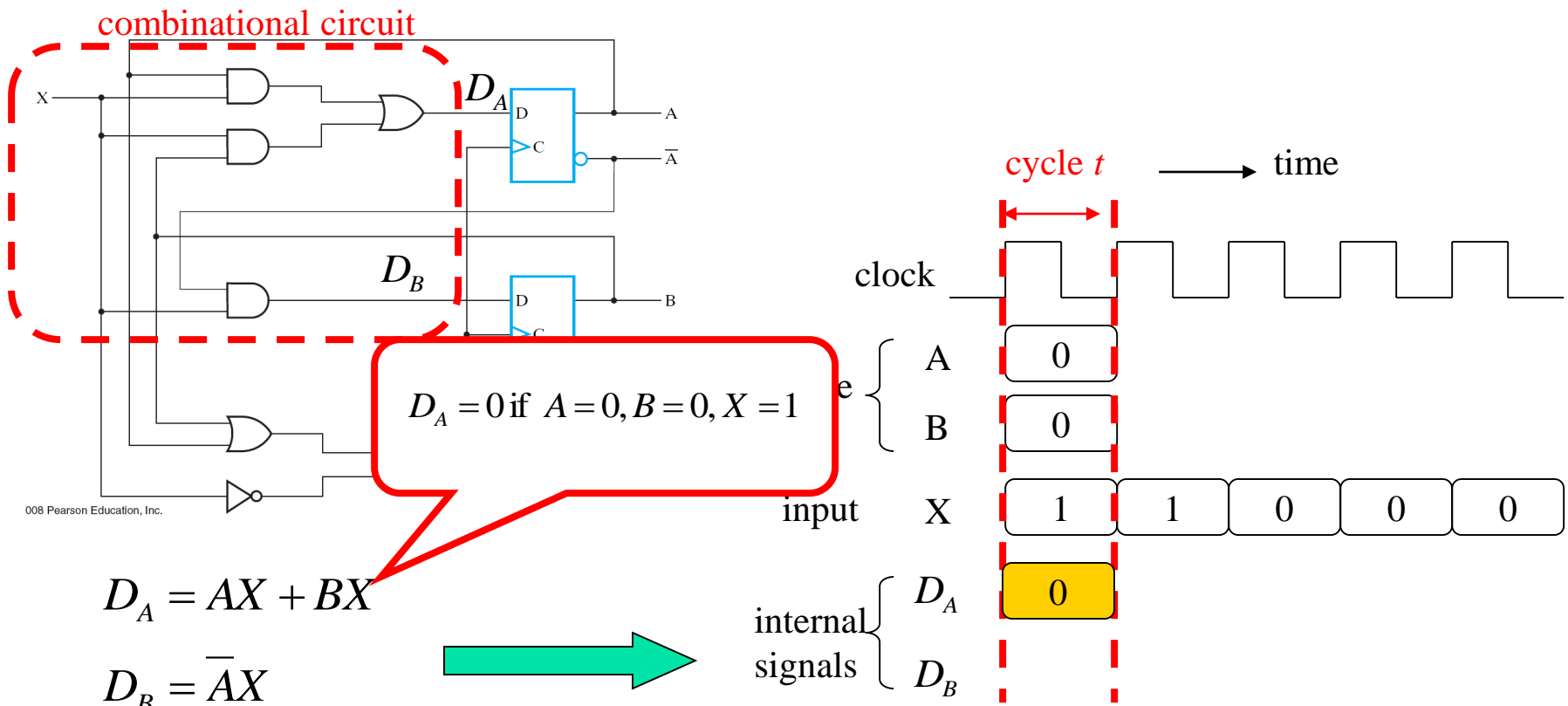
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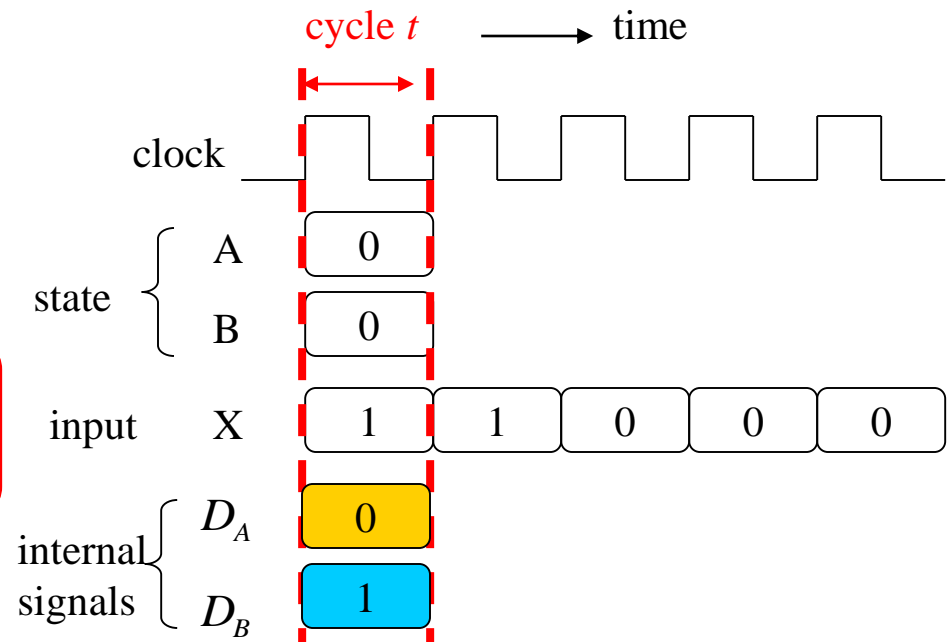
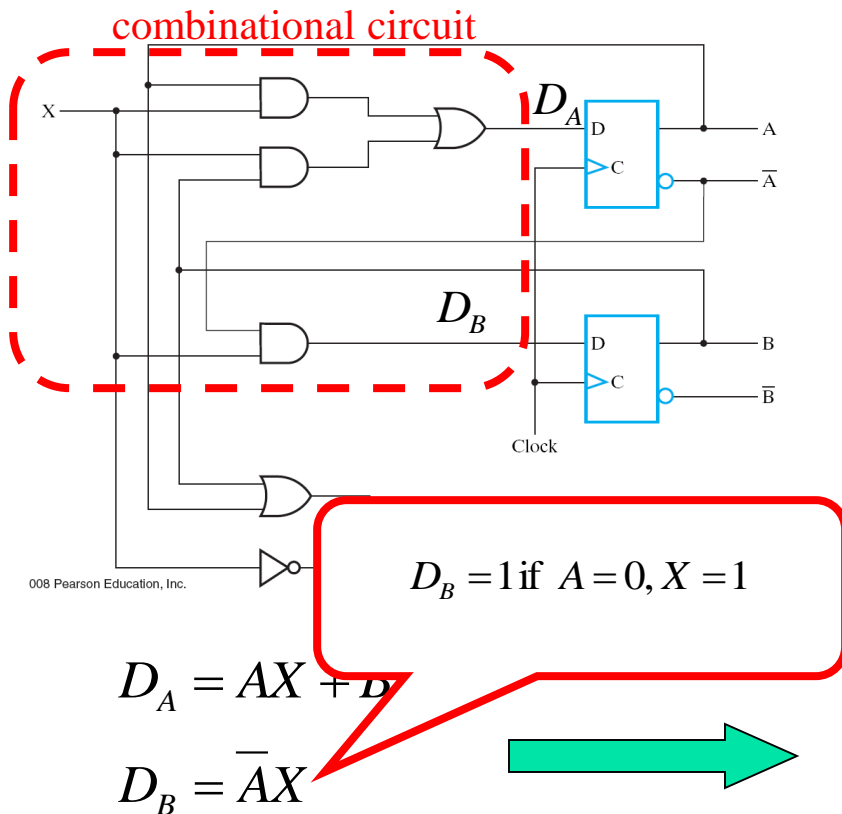
Meaning of the input equations

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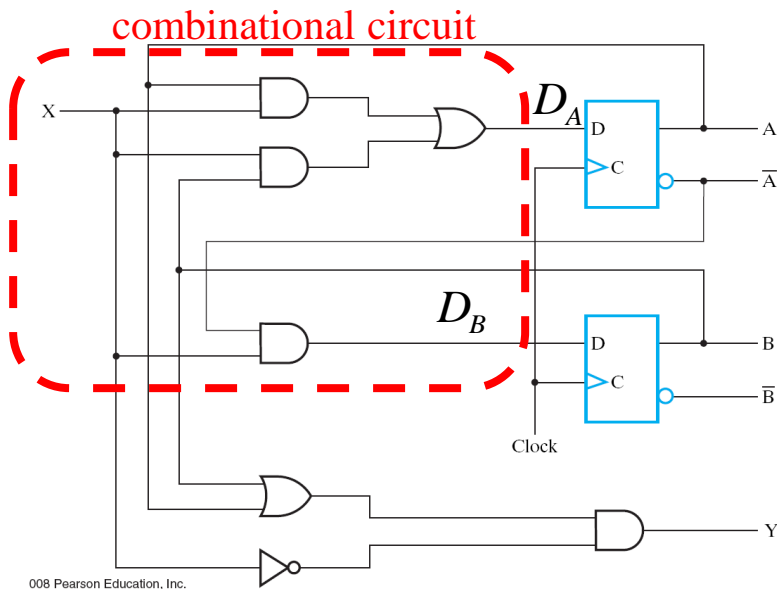
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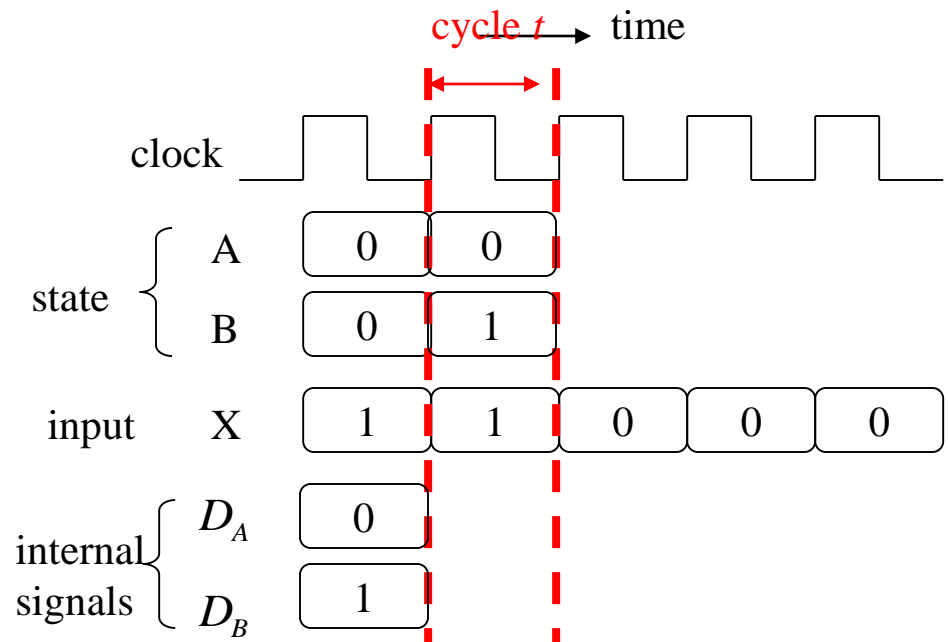
Meaning of the input equations

- the signal value at the end of the current cycle t



$$D_A = AX + BX$$

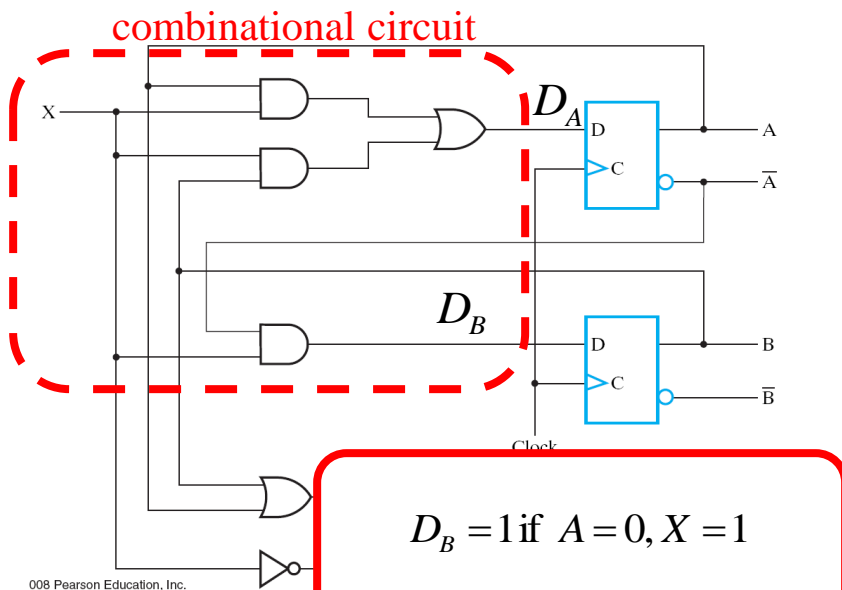
$$D_B = \bar{A}X$$



$$D_B = \overline{AX}$$

Meaning of the input equations

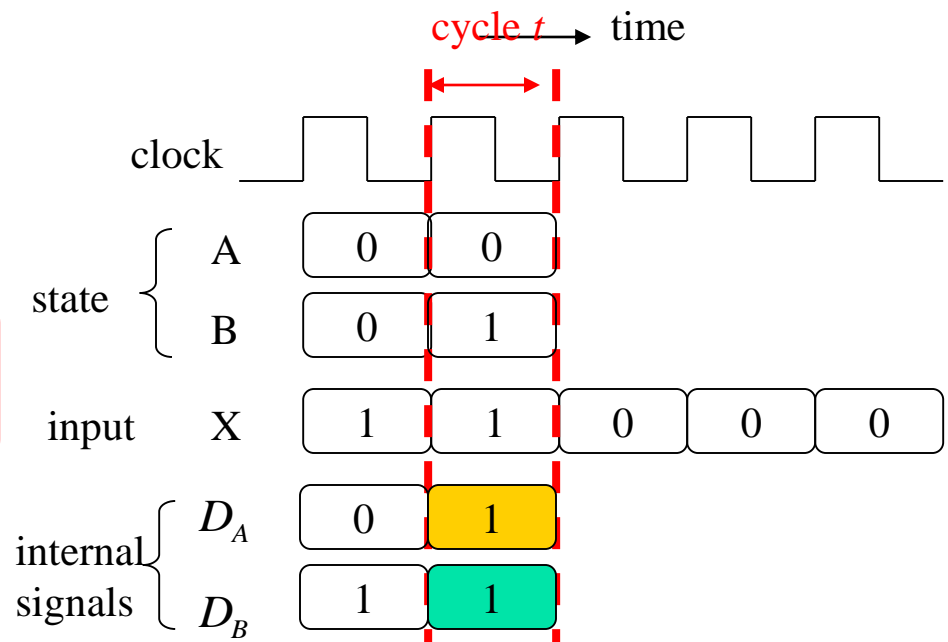
- the signal value at the end of the current cycle t



$D_B = 1$ if $A = 0, X = 1$

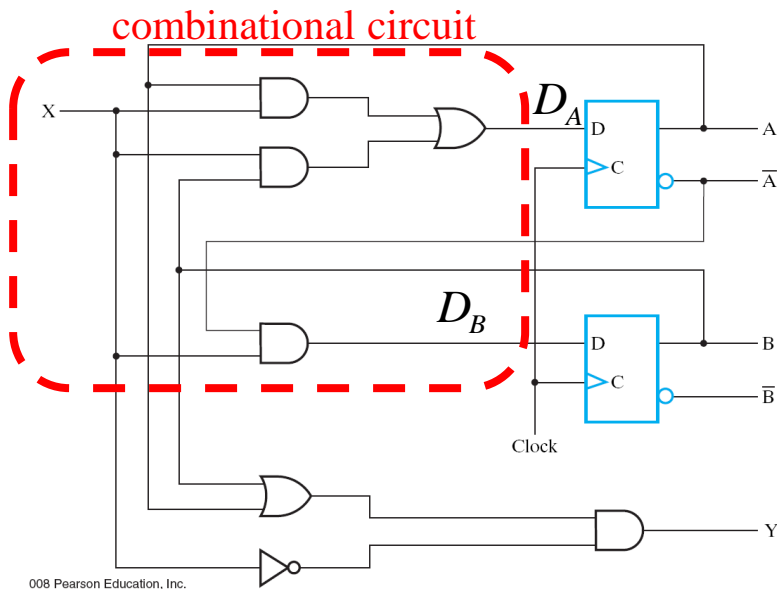
$$D_A = AX + \bar{A}\bar{X}$$

$$D_B = \bar{A}X$$



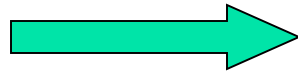
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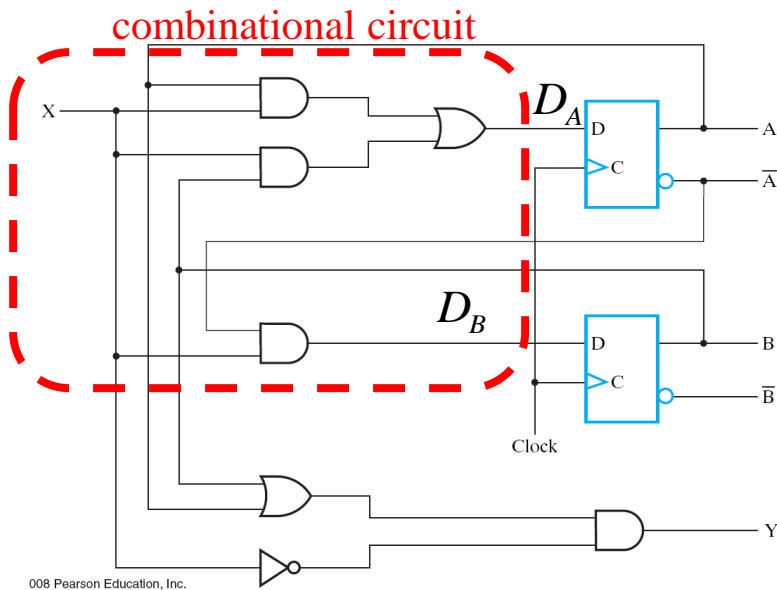


Derive remaining part by yourself!

clock						
state	A	0	0	1	0	0
	B	0	1	1	0	0
input	X	1	1	0	0	0
internal signals	D_A	0	1	0	0	0
	D_B	1	1	0	0	0

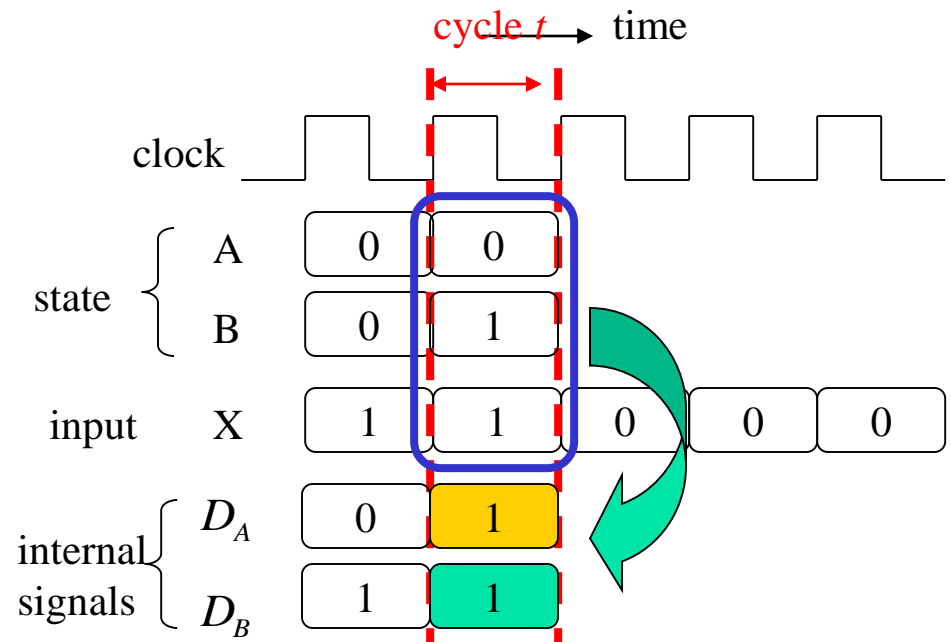
Rule of thumb

- a combinational circuit always generates the outputs at the same cycle



$$D_A = AX + BX$$

$$D_B = \bar{A}X$$





State table of a sequential circuit



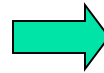
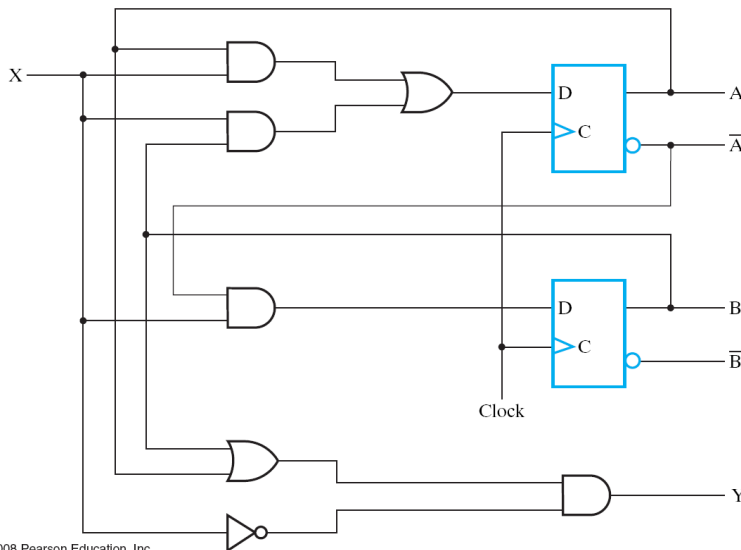
Method to analyze a sequential circuit

- Step 1: derive input equations to D flip-flops
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- Step 3: draw the state-diagram

Then you can draw the timing waveform from a state-diagram

What is a state table?

- like a truth table
- rules to determine the **state** at cycle $t+1$ from
 - (1) state at cycle t , and
 - (2) input signals at cycle t
- state: content of D-FFs

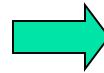
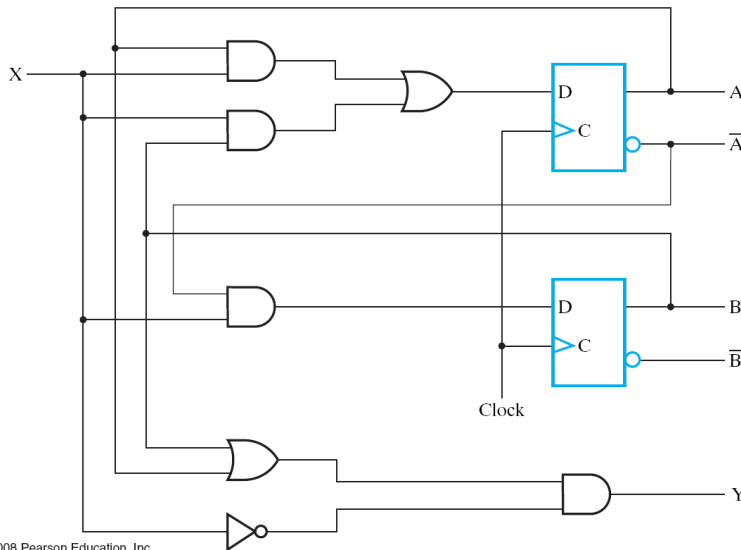


■ **TABLE 5-1**
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

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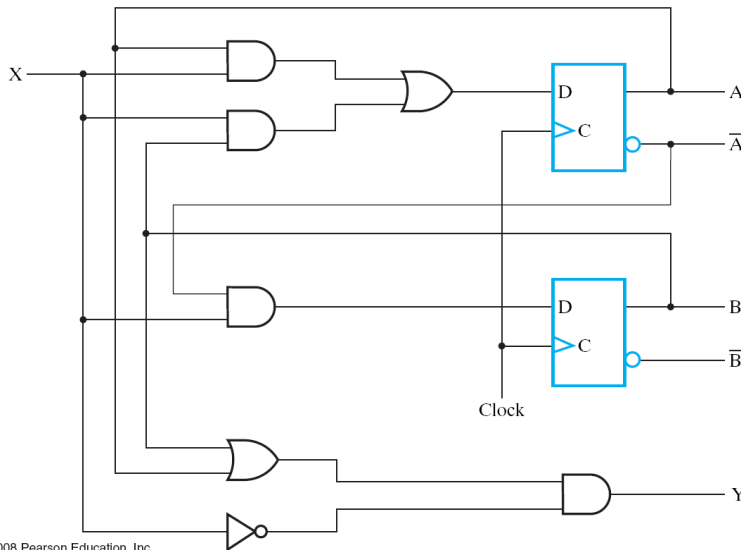
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1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

enumerate all
possible conditions

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state at cycle t

state at cycle $t+1$

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A	B		X	A	B	Y
0	0		0	0	0	0
0	0		1	0	1	0
0	1		0	0	0	1
0	1		1	1	1	0
1	0		0	0	0	1
1	0		1	1	0	0
1	1		0	0	0	1
1	1		1	1	0	0

enumerate all
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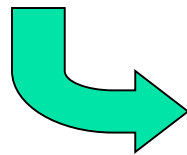
Meaning of a state-table

- timing waveform can be drawn from the state table

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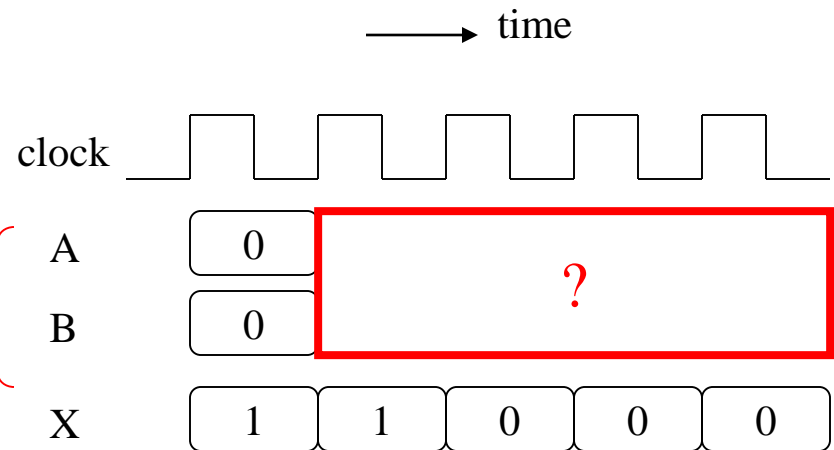
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0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



state

input



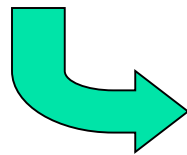
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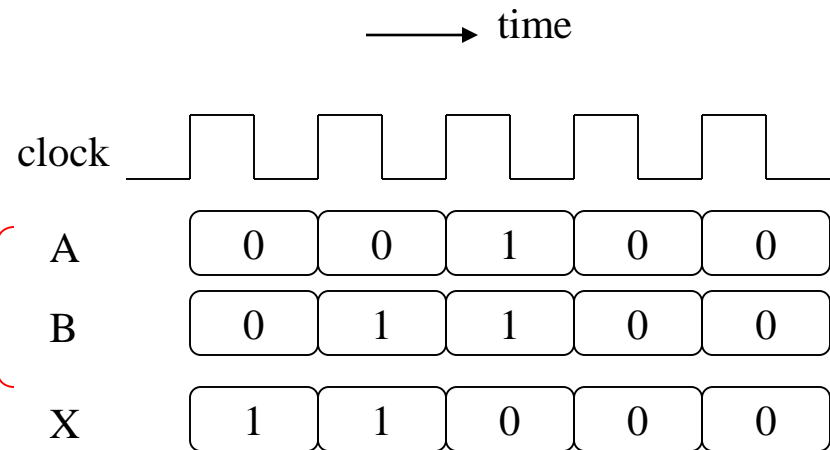
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0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



state

input



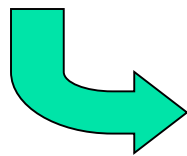
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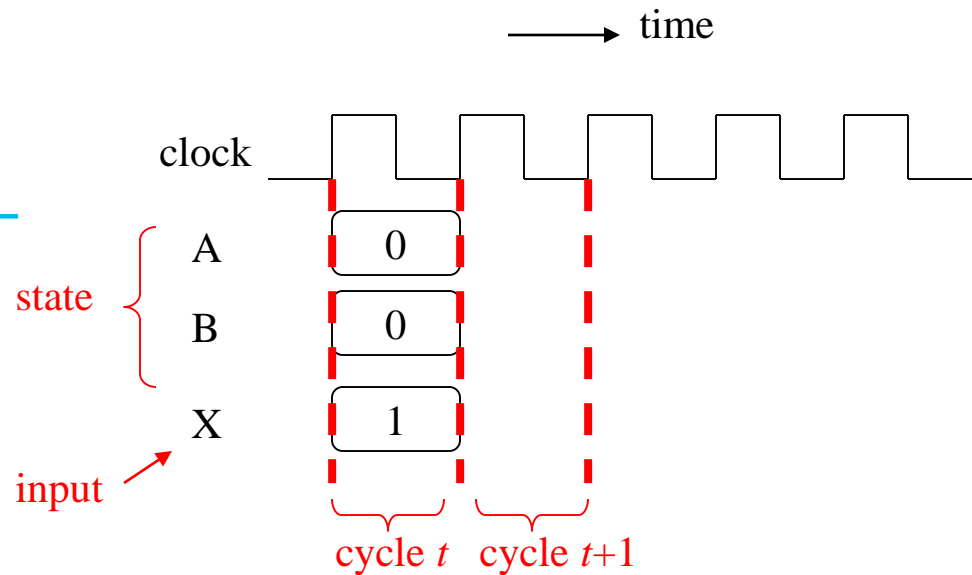
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1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

cycle t

cycle $t+1$



- rules to determine the state at cycle $t+1$ from
 - state at cycle t , and
 - input signals at cycle t



Meaning of a state-table

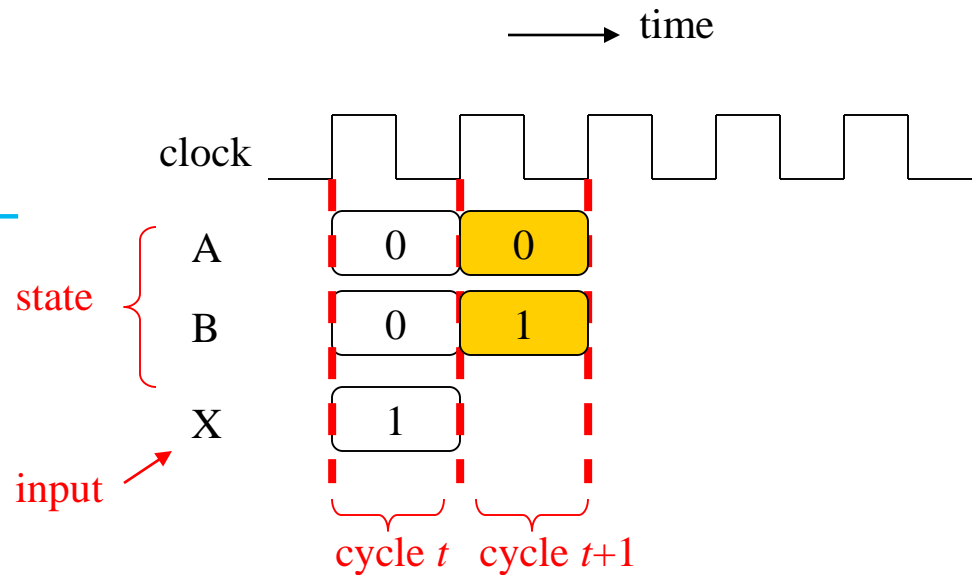
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1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
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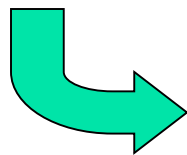
Meaning of a state-table

□ TABLE 5-1
State Table for Circuit of Figure 5-15

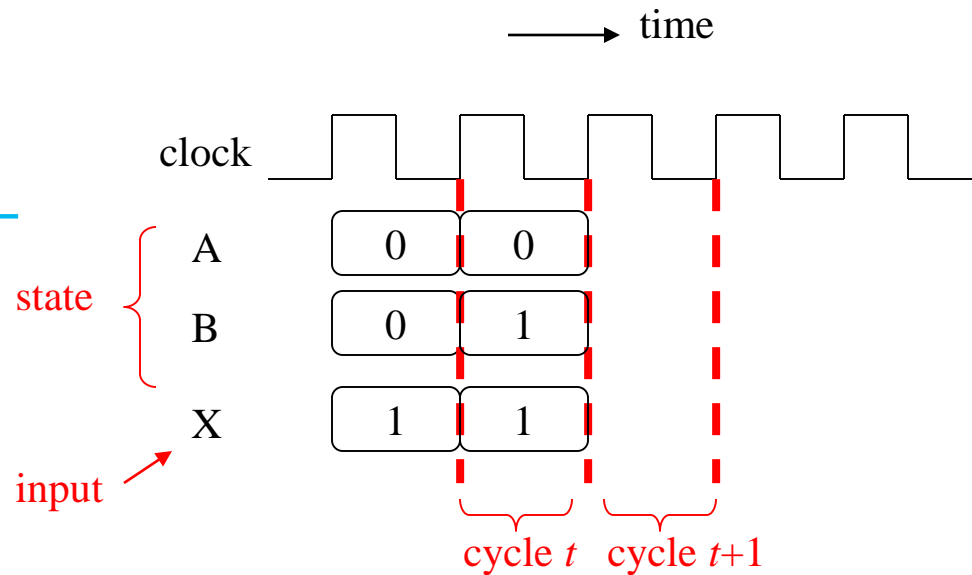
Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

cycle t

cycle $t+1$



- rules to determine the state at cycle $t+1$ from
 - state at cycle t , and
 - input signals at cycle t



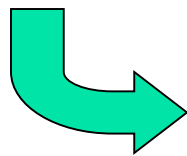
Meaning of a state-table

TABLE 5-1
State Table for Circuit of Figure 5-15

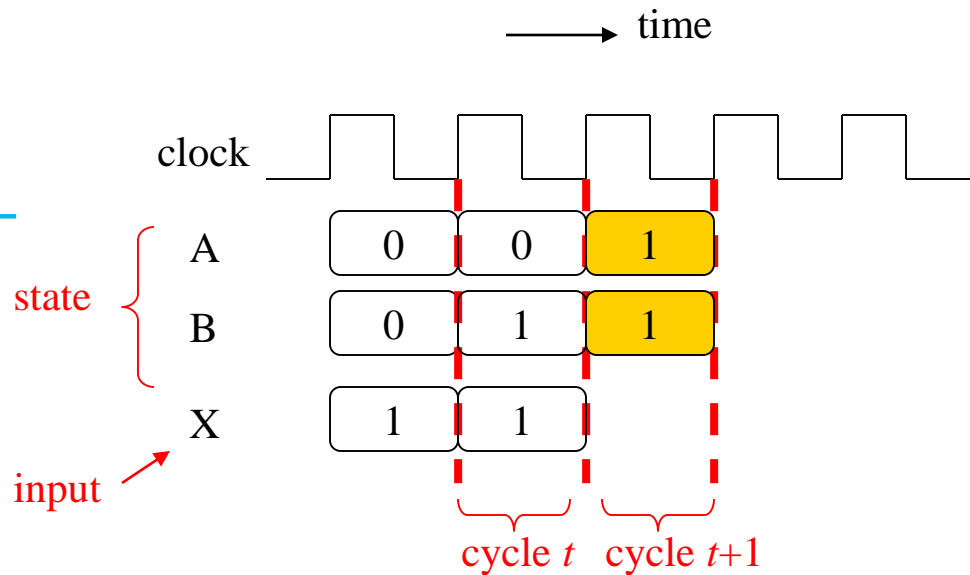
Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

cycle t

cycle $t+1$



- rules to determine the state at cycle $t+1$ from
 - state at cycle t , and
 - input signals at cycle t



Meaning of a state-table

- timing waveform can be drawn from the state table

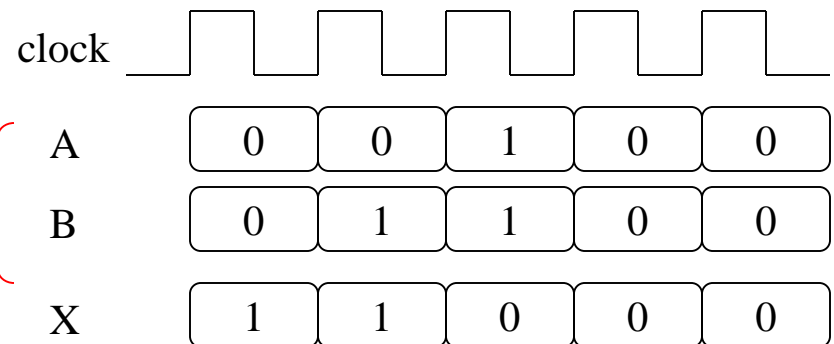
□ TABLE 5-1

State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

complete the timing waveform
by yourself!

→ time





How to derive the state table



Method to analyze a sequential circuit

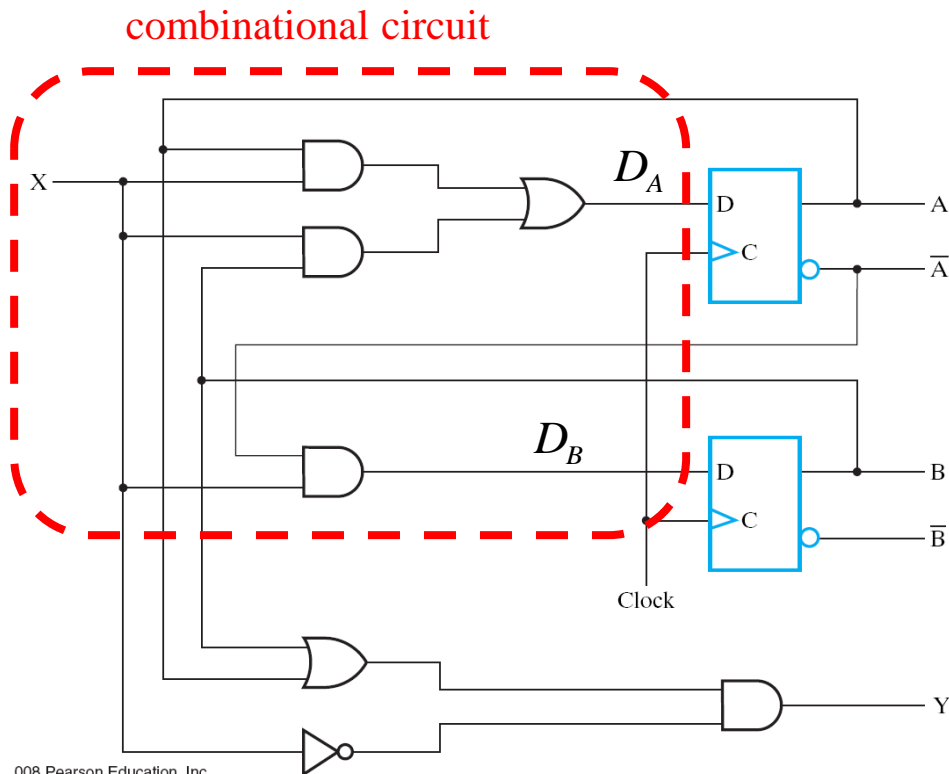
- Step 1: derive input equations to D flip-flops
- Step 2: derive the **state table**
- Step 3: draw the state-diagram

Then you can draw the timing waveform from a state-diagram

How to derive the state-table

- Rule of thumb:

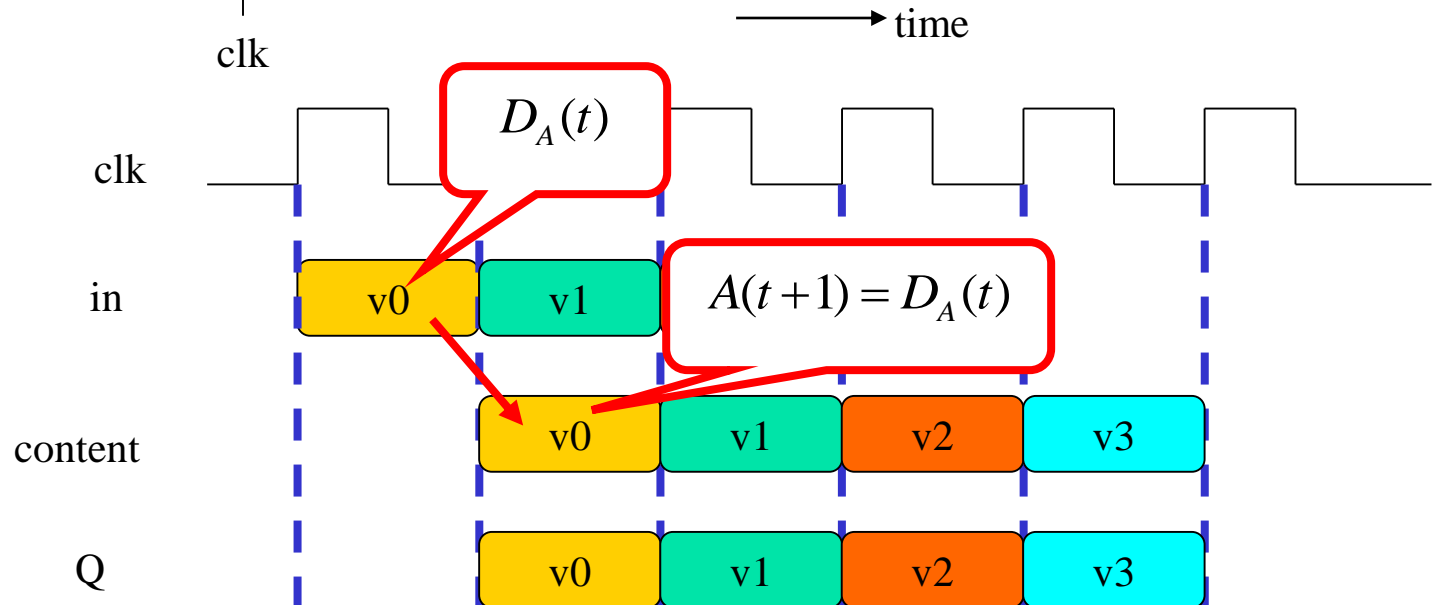
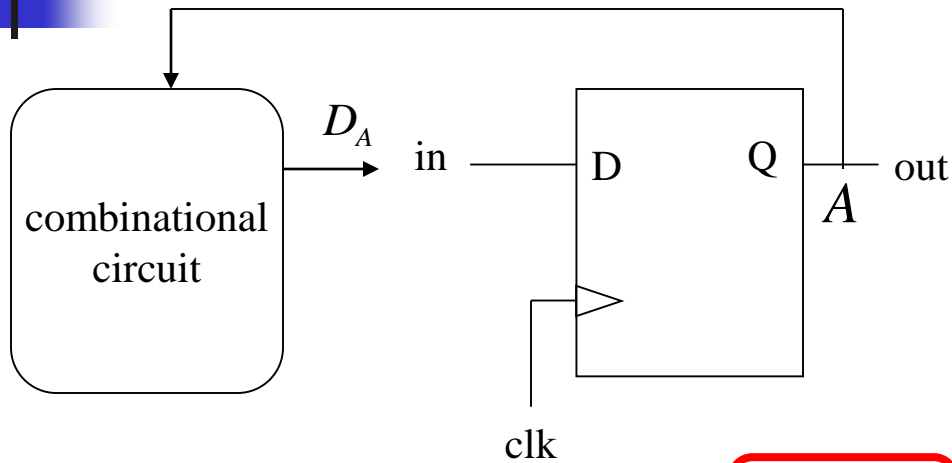
- the input to a D-FF at cycle t equals the state of D-FF at cycle $t+1$



$$A(t+1) = D_A(t) = A(t)X(t) + B(t)X(t)$$

$$B(t+1) = D_B(t) = \overline{A(t)}X(t)$$

Timing Waveform of the D Flip-Flop



How to derive the state-table

- check the rules and the state-table

$$D_A(t)=0$$

$$A(t+1) = D_A(t) = A(t)X(t) + B(t)X(t)$$

$$B(t+1) = D_B(t) = \overline{A(t)}X(t)$$

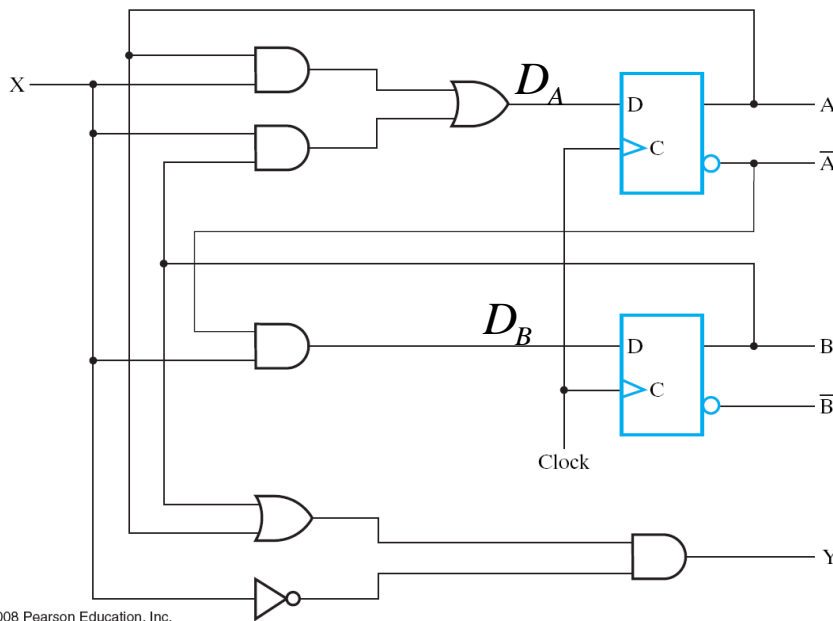
$$D_B(t)=1$$

state at cycle t

state at cycle $t+1$

State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

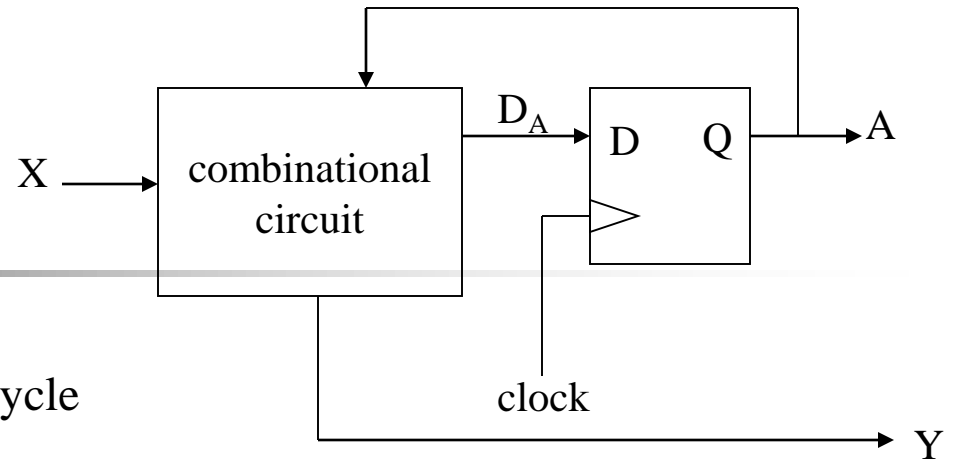




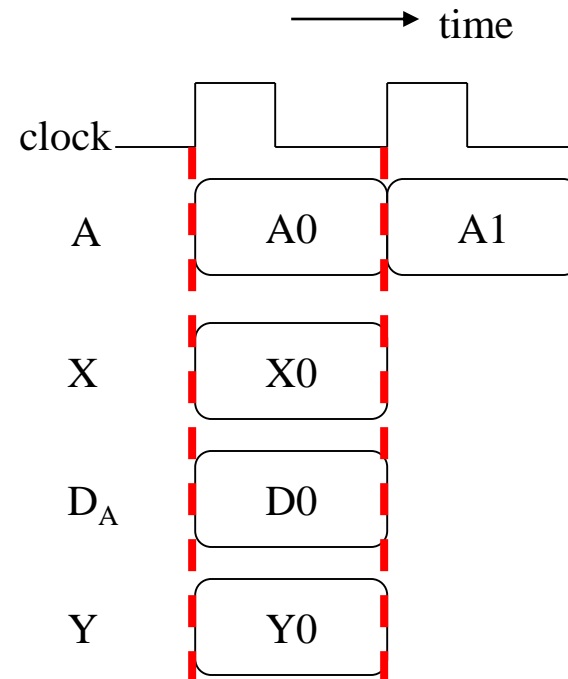
Put it all together

derive the state table and the timing
waveform

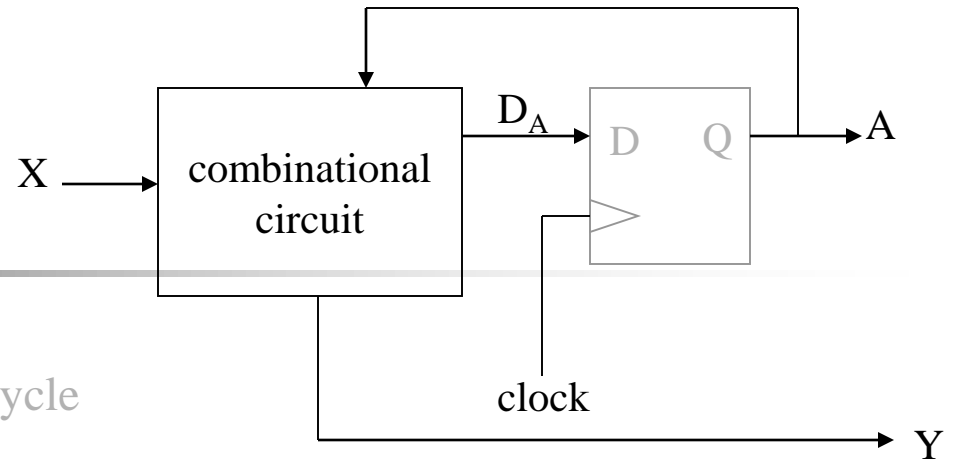
General rule



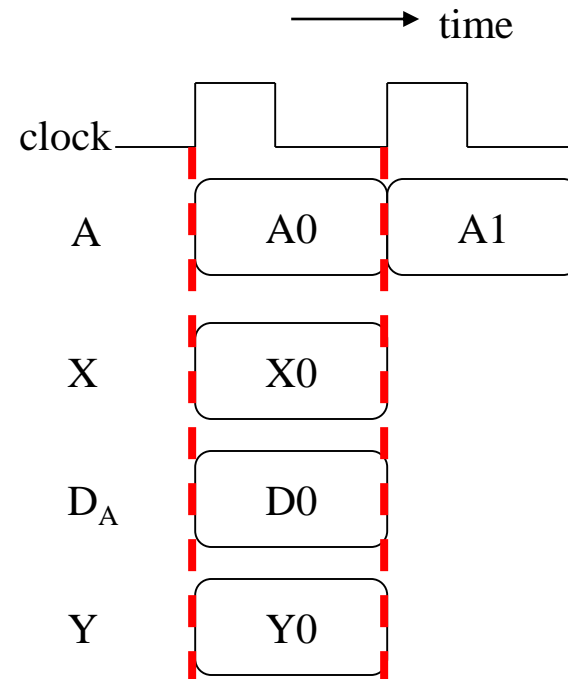
- draw the timing waveform cycle-by-cycle
- Step 1: remove D-FFs and derive Boolean equations for the combinational circuit
 - state A is an input to the combinational circuit
- Step 2: derive outputs of the combinational circuit at the same cycle
 - inputs to D-FFs are outputs of the combinational circuit
 - the combinational circuit generates output at the same cycle
- Step 3: derive the next state of D-FFs
 - $A(t+1) = D_A(t)$



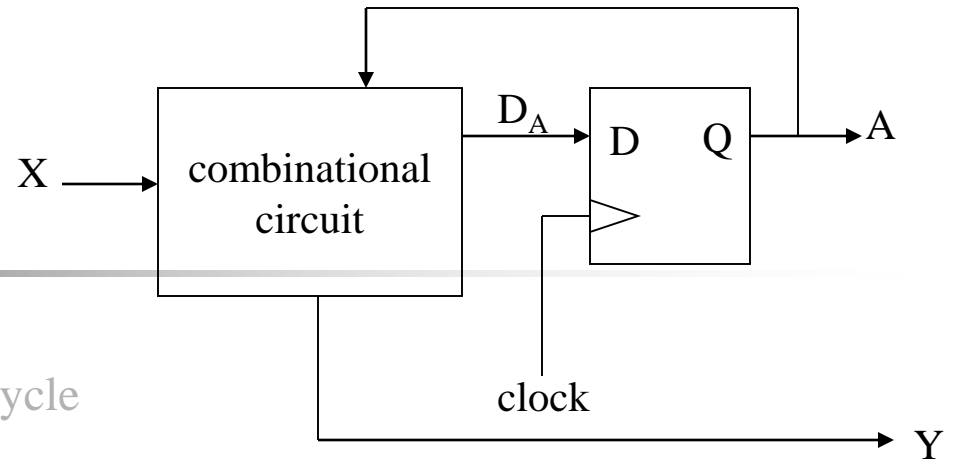
General rule



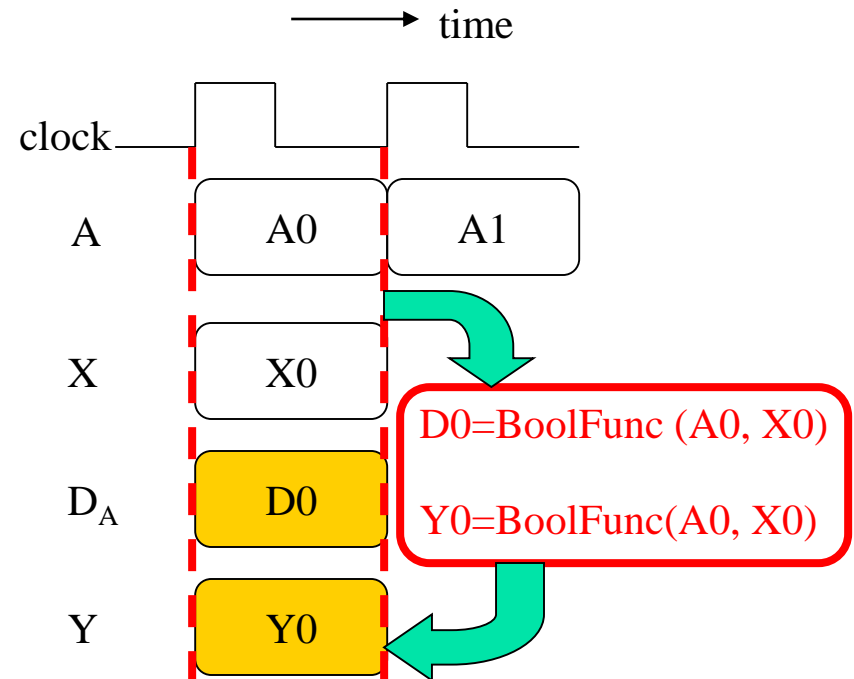
- draw the timing waveform cycle-by-cycle
- Step 1: remove D-FFs and derive Boolean equations for the combinational circuit
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- Step 2: derive outputs of the combinational circuit at the same cycle
 - inputs to D-FFs are outputs of the combinational circuit
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 - $A(t+1) = D_A(t)$



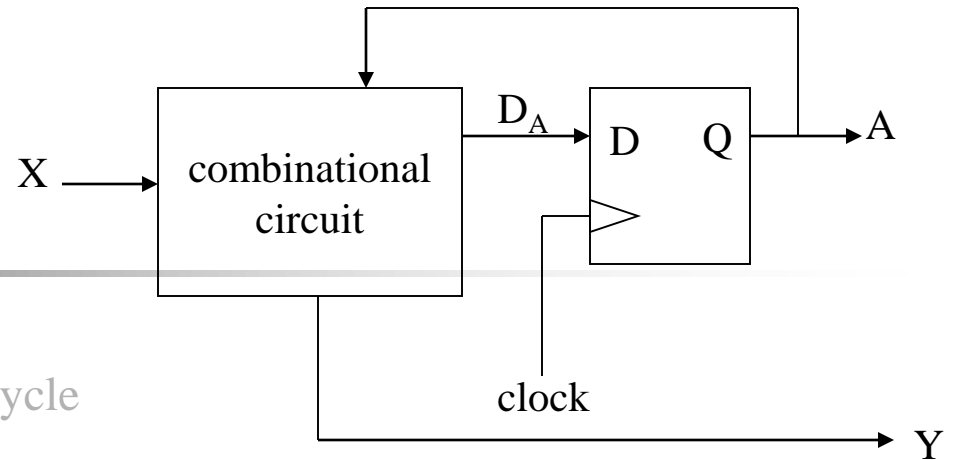
General rule



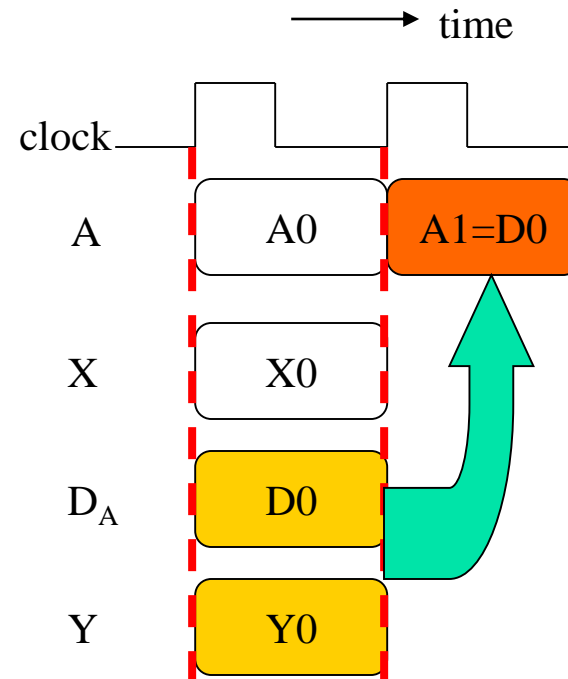
- draw the timing waveform cycle-by-cycle
- Step 1: remove D-FFs and derive Boolean equations for the combinational circuit
 - state A is an input to the combinational circuit
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 - $A(t+1) = D_A(t)$



General rule



- draw the timing waveform cycle-by-cycle
- Step 1: remove D-FFs and derive Boolean equations for the combinational circuit
 - state A is an input to the combinational circuit
- Step 2: derive outputs of the combinational circuit at the same cycle
 - inputs to D-FFs are outputs of the combinational circuit
 - the combinational circuit generates output at the same cycle
- Step 3: derive the next state of D-FFs
 - $A(t+1) = D_A(t)$

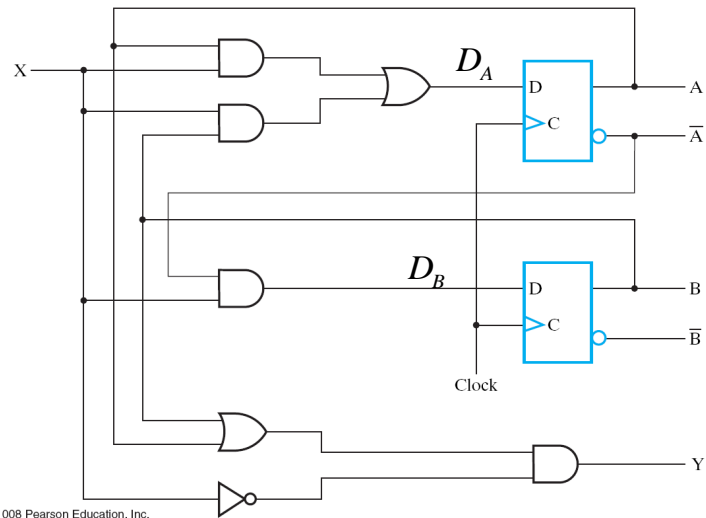




Rule of Thumb

- Combinational Circuit: generate outputs at the same cycle to inputs
- D Flip-Flop: $A(t+1) = D_A(t)$
 - state changes at the next cycle

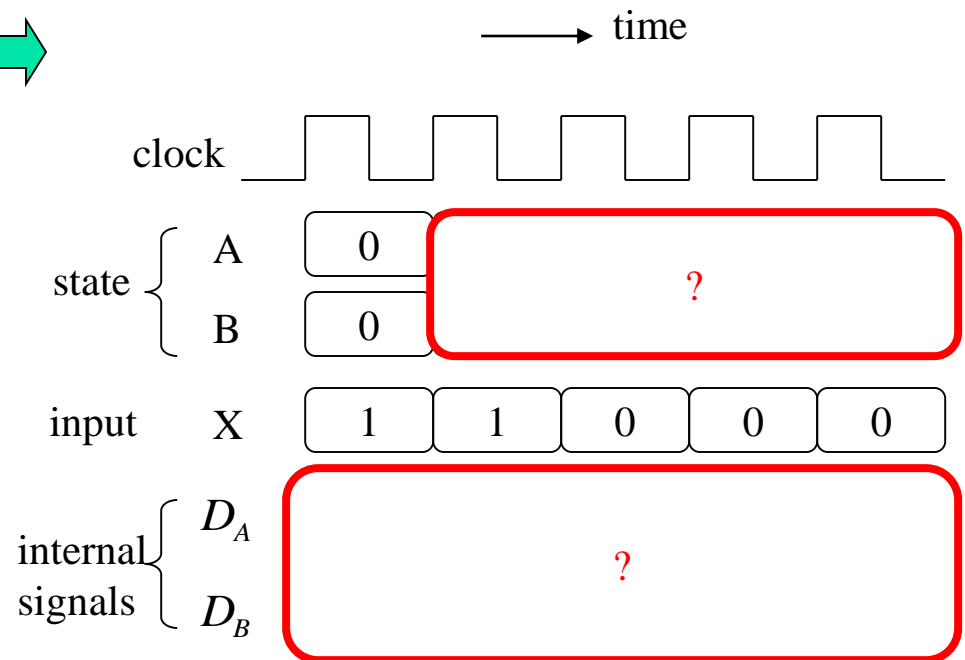
Example



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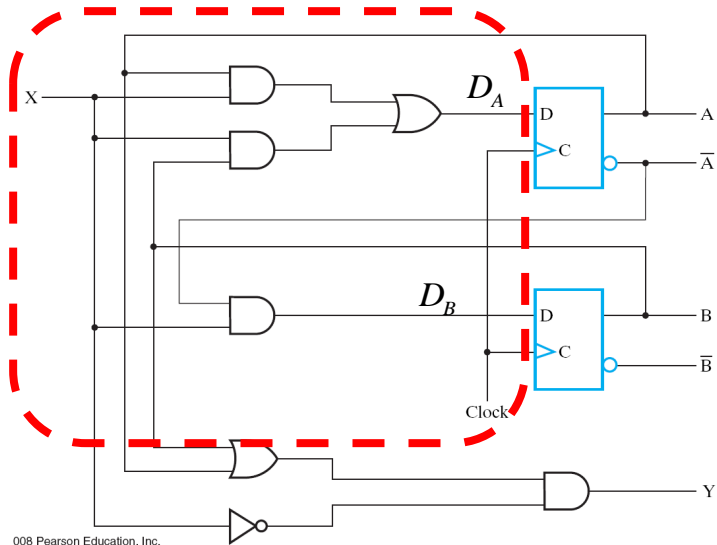
TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	?		
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



Example

combinational circuit



$$D_A = AX + BX$$

$$D_B = \bar{A}X$$

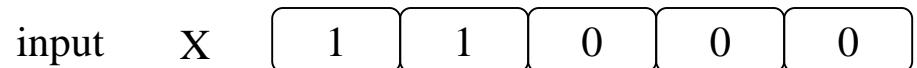
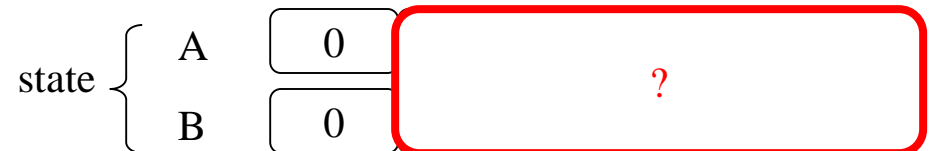
TABLE 5-1

State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	?		
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

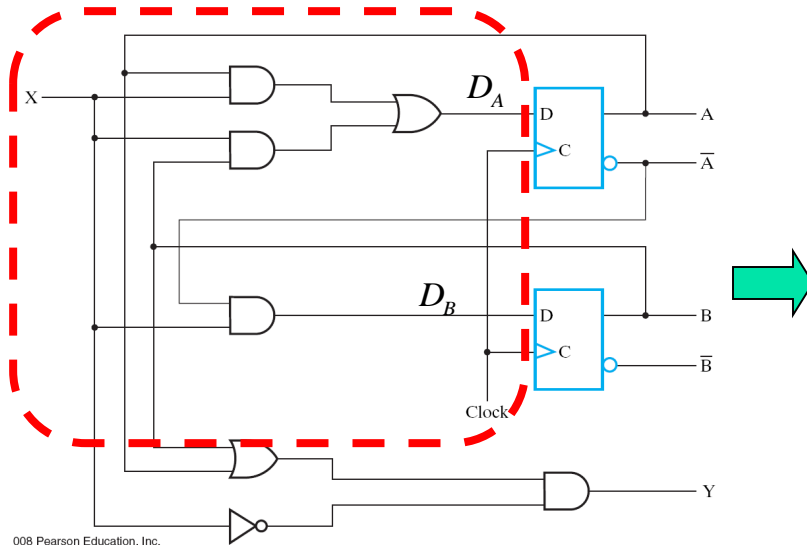


time



Example

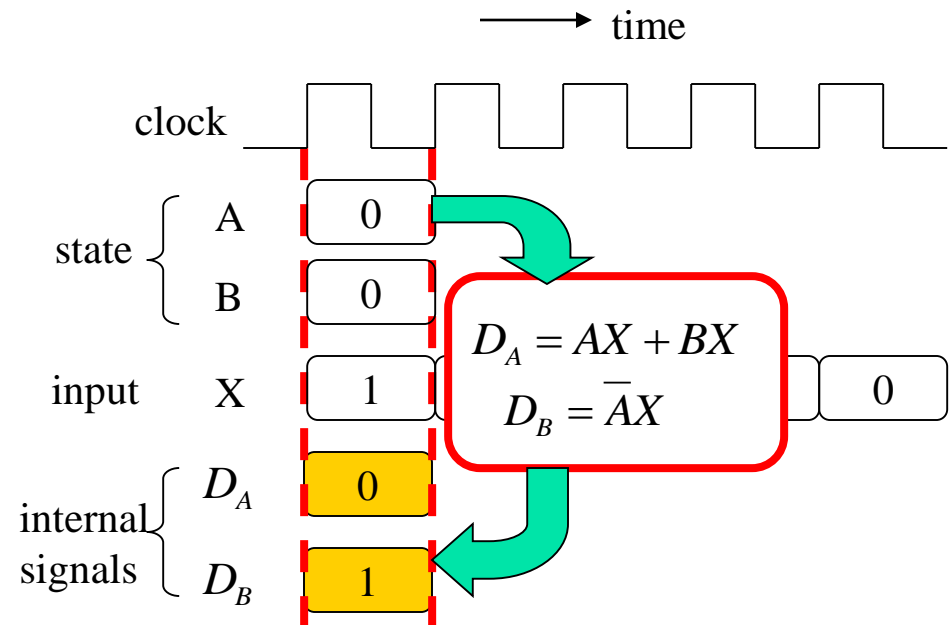
combinational circuit



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TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input X	Next State		Output Y
A	B		A	B	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



combinational circuit

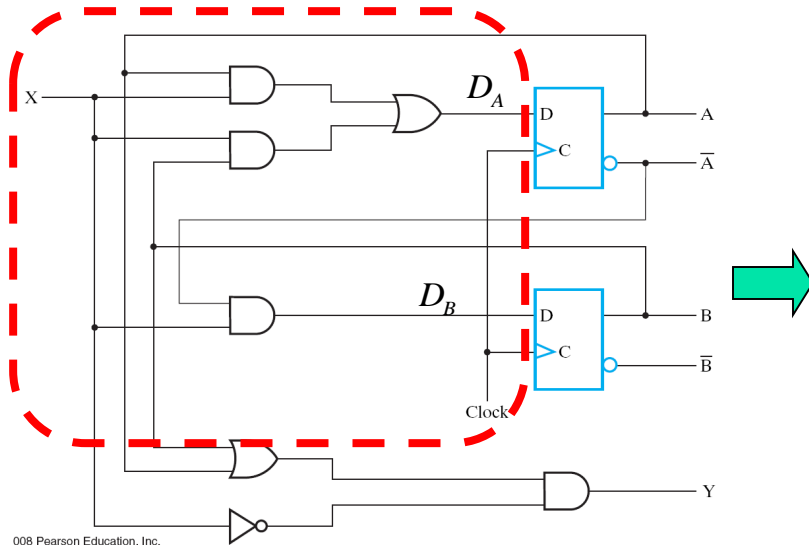
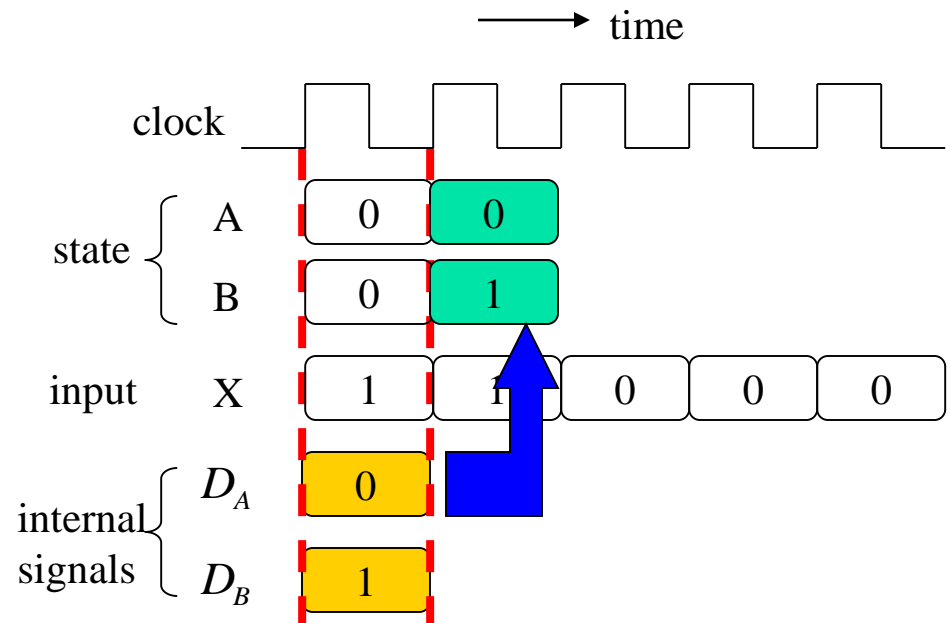


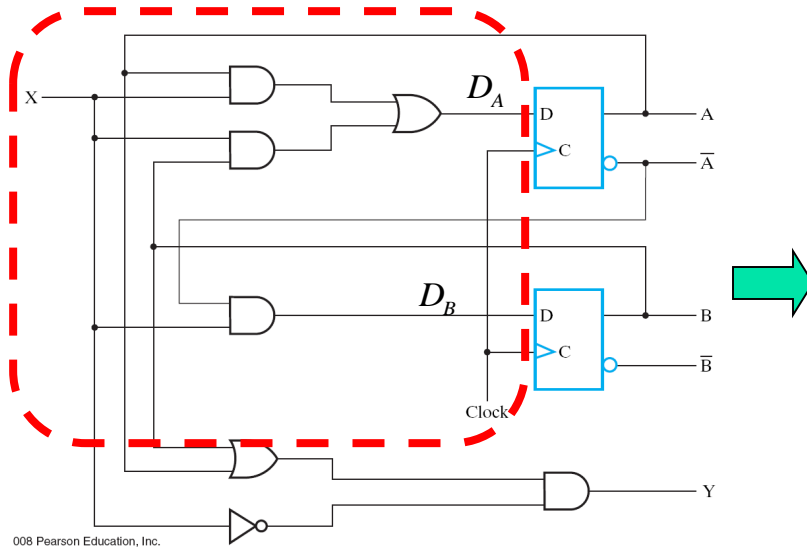
TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



Example

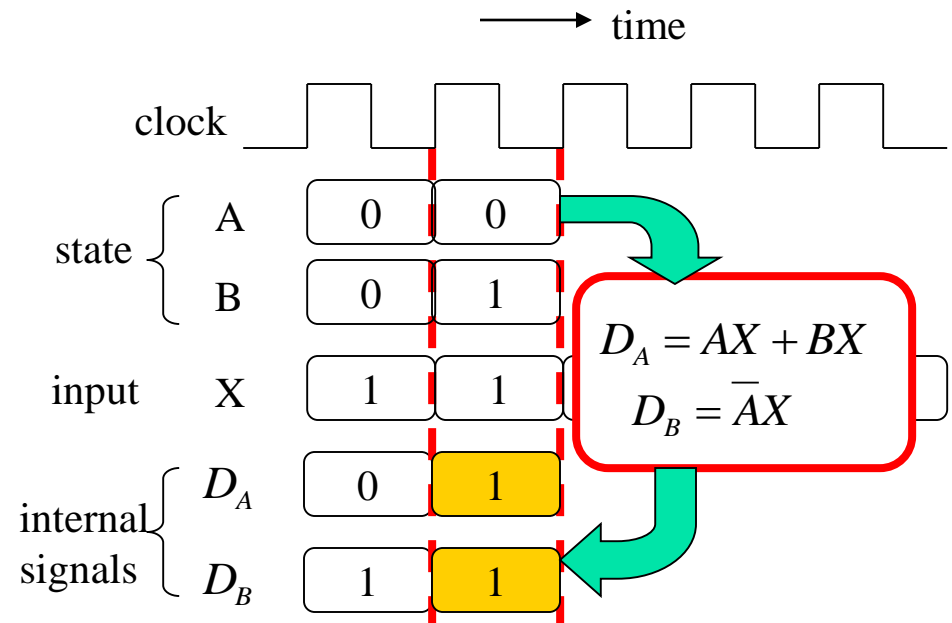
combinational circuit



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TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



Example

combinational circuit

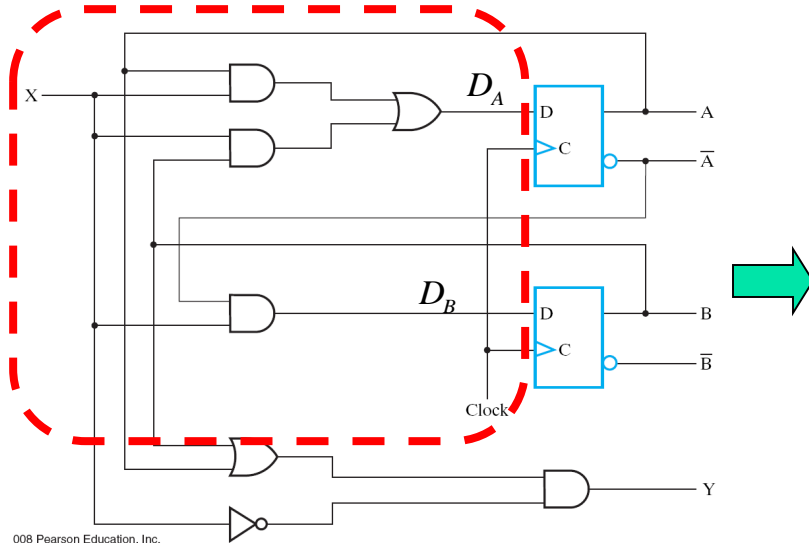
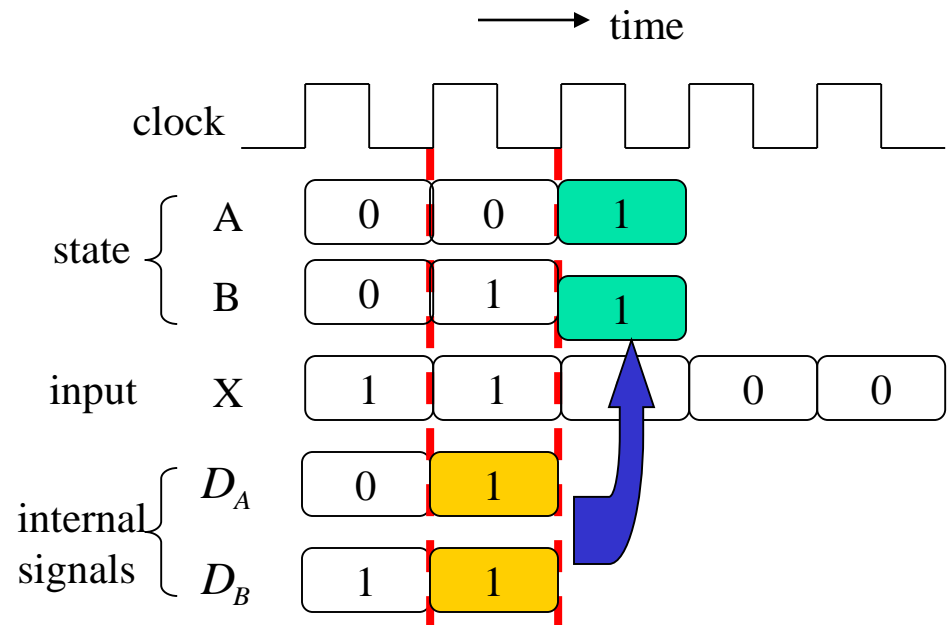


TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input X	Next State		Output Y
A	B		A	B	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



Example

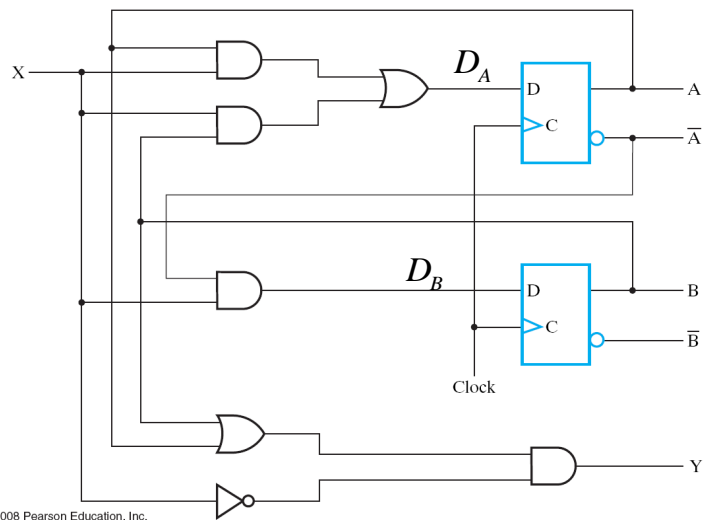
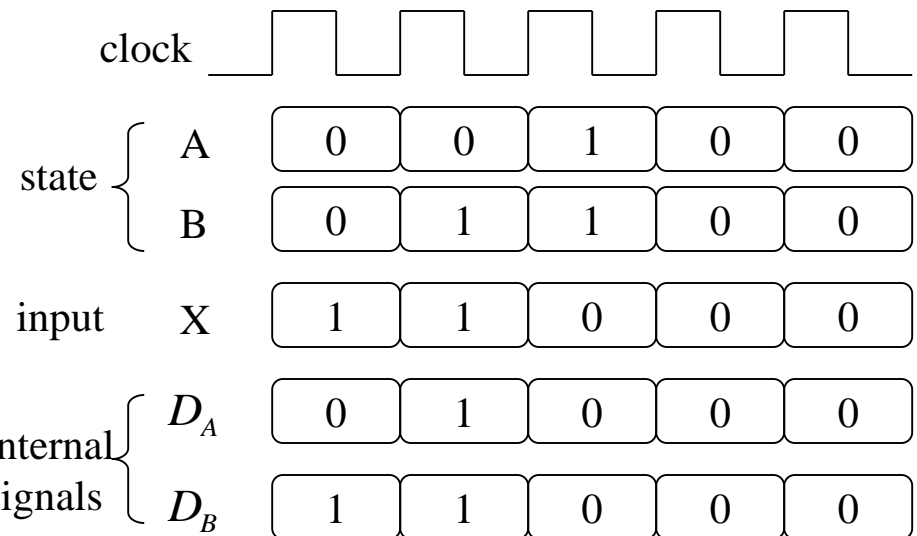


TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
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0	0	1	0	1	0
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0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

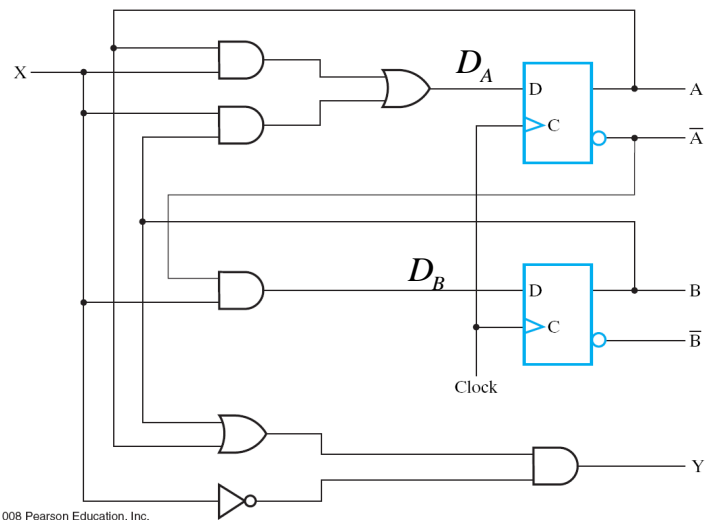


time



Complete by yourself!

Example



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TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



time

clock						
state	A	0	0	1	0	0
	B	0	1	1	0	0
input	X	1	1	0	0	0
internal signals	DA	0	1	0	0	0
	DB	1	1	0	0	0
output	Y	<div style="border: 2px solid red; padding: 5px; display: inline-block;">?</div>				

Q: How about output Y?



State diagram

finite-state machine model for digital
circuit



Method to analyze a sequential circuit

- Step 1: derive input equations to D flip-flops
- Step 2: derive the state table
- Step 3: draw the **state-diagram**

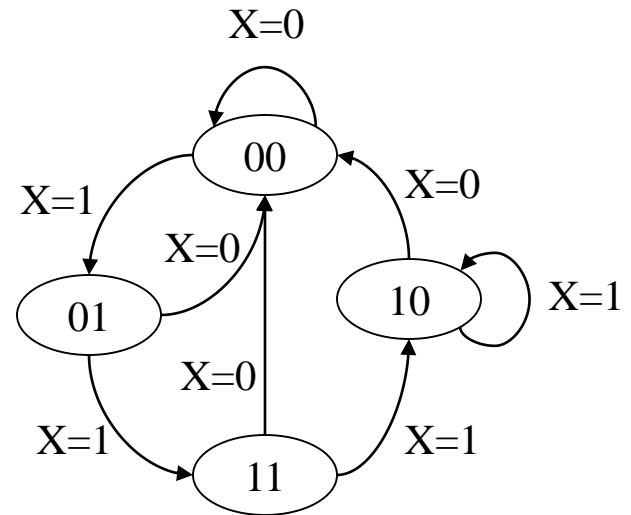
Then you can draw the timing waveform from a state-diagram

What is the state diagram?

- a graphical way to represent the state table

TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

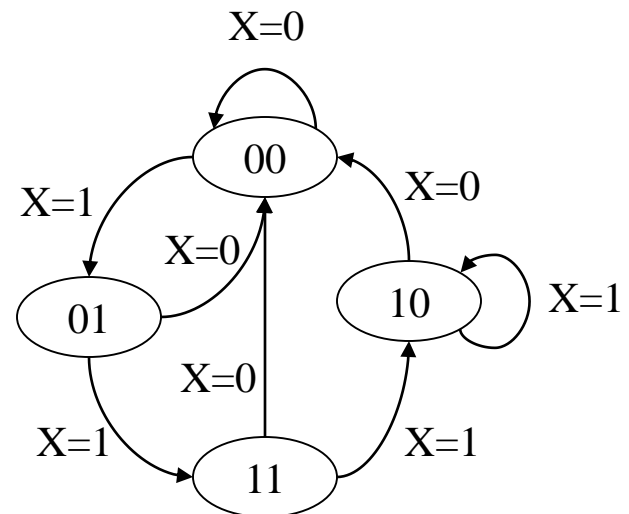


What is the state diagram?

- node (state): content of the D-FFs
- edge: state transition upon receiving input signal value

TABLE 5-1
State Table for Circuit of Figure 5-15

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

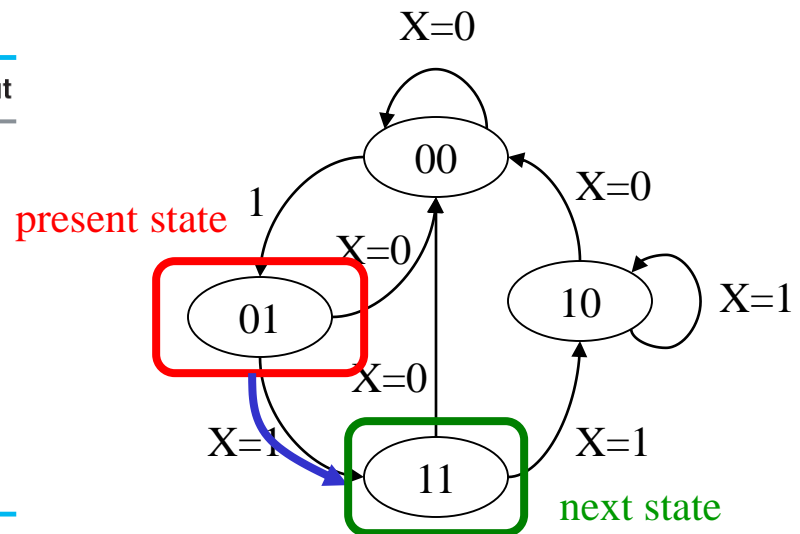


What is the state diagram?

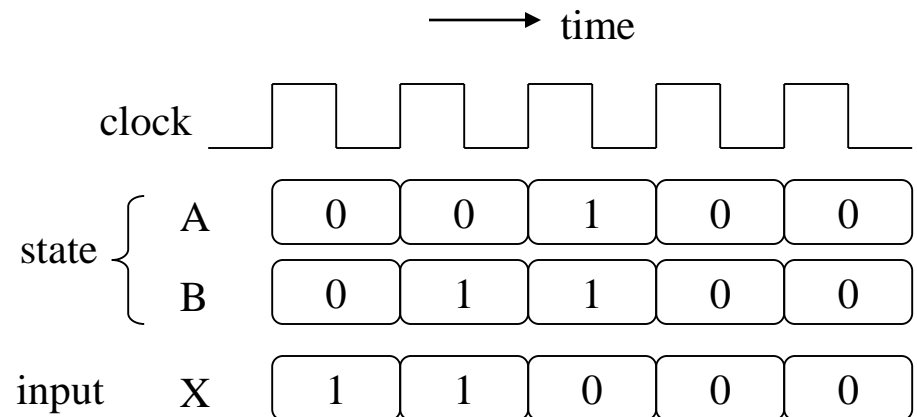
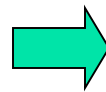
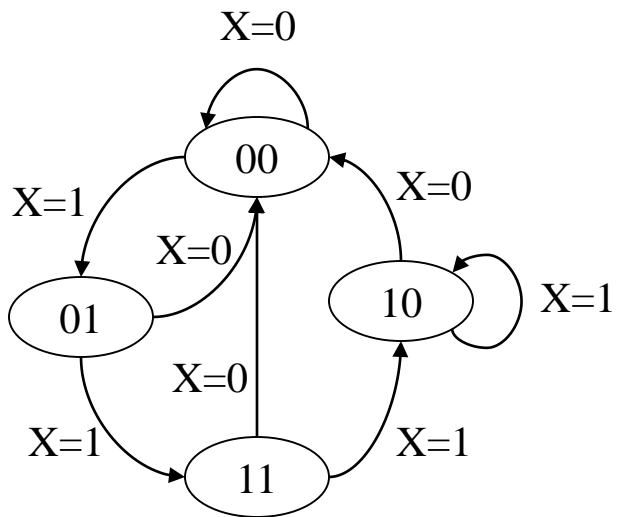
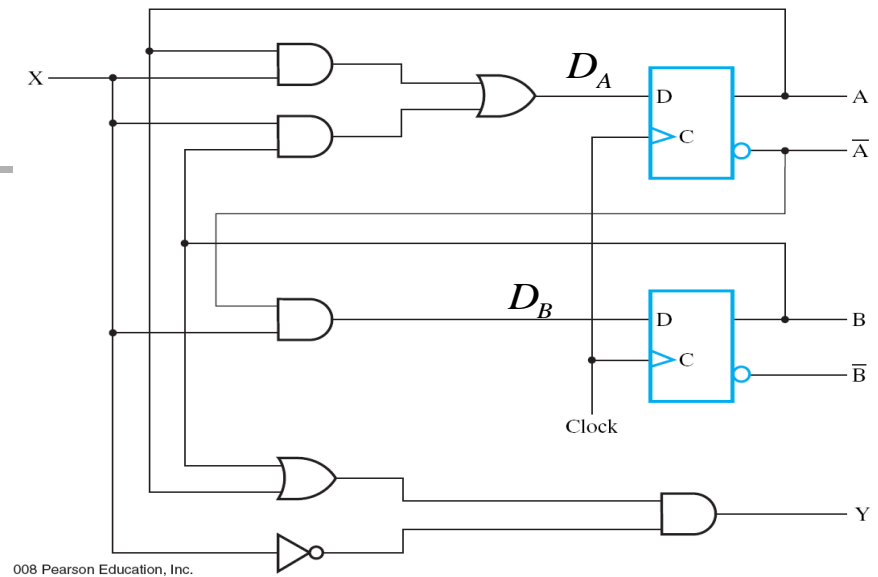
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0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



Draw timing waveform from state-diagram



Draw timing waveform from state-diagram

