1.

a.

PROOF:

b.

c. probability of drawing red ball on the 1st consecutive draws.

Logic:

d)





2a.



2b. let

3.

Let

By Bayes theorem,

and

­

Let

Exhibits that when priors = each other, likelihood = posterior….?

4. Gamma distribution:

Poisson likelihood function:

Kernel of gamma distribution:

Normalizing constant of gamma distribution:

(integral of normalizing constant \* kernel = 1)

Kernel of poisson:

Normalizing constant of poisson:

Observe:

=

=

Integral of kernels of likelihood\*prior not equal to one (without normalizing constant – which is the integral of the distribution’s kernel with respect to mu.

FACT:

5.

Multinomial model:

Dirichlet distribution:

6.

Normal distribution:

Entropy:

Intuitively, this makes sense since entropy, or uncertainty, should only be reliant on variance and thus the model becomes more imprecise as variance increases.

R exercises:

1. A bank has made 100 mortgages of a new type (say it’s 2005 and they are subprime mortgages), and all have been outstanding 5 years. Of these 100, 5 of them have de- faulted. The bank would like to estimate the probability θ of default in the first five years for this type of mortgage, and get some idea of how much uncertainty there is about the probability, given the observed data. These being a new type of mortgage, the bank assigns a uniform prior over θ.
   1. What is the likelihood *p*[*x*|θ]? Binomial distribution
   2. Plot the likelihood in R and indicate on the plot (e.g. use the abline() function) the location of the maximum likelihood value of θ as well as the expected value of θ.

> optimize(log.post,interval=c(0,1),n=100,k=5,maximum=T)

$maximum $objective

[1] 0.05000354 [1] -1.714699



* 1. Using the quantile function *qbeta*() calculate and indicate asymmetric 95% confidence interval (cut off 2.5% of the left and right tail). Does this look like a reasonable confidence interval?
     1. Yes, since the highest likelihood is contained and peaks within . =.897
     2. Plotted with mle



* 1. The shortest interval with 95% probability will have the likelihood the same height at each end. Using the package “TeachingDemos" use the HPD function to find the shortest interval.

Shortest interval:

* 1. HPD is indeed the shortest interval

2.

a)

b) 

c) neither since the order of the balls being drawn does not change the probability of the outcome.

d)IP

e)IP