PART I (70 Marks)

1. Which of the following is used to measure the efficiency of an algorithm? **2.5**

   [A] Number of lines of its pseudo code
   [B] The running time of the algorithm on a machine
   [C] The number of important operations and space used by the algorithm
   [D] The running time of the algorithm on iphone 6 plus.
   [E] The size of the algorithm

2. What is the time complexity of the following algorithm of computing the sum of the first n non-zero natural numbers? **2.5**

   input n
   sum = n*(n+1)/2
   output sum

   [A] $O(n)$
   [B] $O(n^2)$
   [C] O(c), where c is a constant.
   [D] $O(n^3)$
   [E] $O(n*(n+1)/2)$

3. Two algorithms A1, A2 solve a problem with running times of $f_1(n)$ and $f_2(n)$, respectively. Then $f_1(n) \in O(f_2(n))$ means which of the following statements is true **2.5**

   [A] For all n, f1(n) $\leqslant$ f2(n)
   [B] There exist n0, such that for all n > n0, f1(n) $\leqslant$ f2(n)
   [C] There exist n0 and a constant c, such that for all n > n0, f1(n) $\leqslant$ cf2(n)
   [D] A1 is running fast in all cases.
   [E] None of the above.

4. Five algorithms A1, A2, A3, A4, A5 solve a problem with order $f_1(n)$ = 50log(log n) + 20, $f_2(n)$=10nlog2n + 100, $f_3(n)=10(\log n)^2 + 100$, $f_4(n)=100n^2 - 3n + 6$, $f_5(n)= n^2/8 - n/4 + 2$, respectively. The algorithm(s) with highest time complexity is (are) **2.5**

   [A] A1
   [B] A2, A3
   [C] A4
   [D] A4, A5
   [E] A5

Questions 5 to 9 refer to the following algorithm.

Algorithm: F($A[l\ldots r]$)

//Input: an array with a position p ($l \leq p \leq r$), such that $A[l] < A[l+1] < \ldots < A[p]$ and $A[p] > A[p+1] > \ldots > A[r]$.

```
Begin
  if l == r then
    return l
  else
    m= [(l + r)/2]
    if A[m] < A[m+ 1] then
      return F(A[m+1…r])
    else
      return F(A[l, m])
End
```

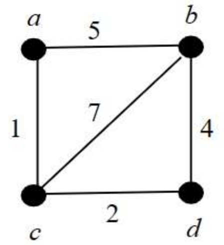
5. Which algorithm design technique is employed in the above algorithm?	***2.5***
[A] Brute Force technique [B] Greedy technique [C] Divide- and-Conquer [D] Dynamic Programming [E] Ad hoc technique	
6. The output of the algorithm is	***2.5***
[A] The largest element in the array [B] A position of the largest element in the array [C] The smallest element in the array [D] A position of the smallest element in the array [E] The element in the middle of the array	
7. What is the number of comparisons to return the output for the input $A[0..7]$ = [12, 13, 14, 15,14, 13, 12, 11]?	***2.5***
[A] 1 [B] 2 [C] 3 [D] 4 [E] 5	

8. If the size n of the array is greater than 1, then the time complexity of the algorithm can be expressed by the recurrence	*2.5*
[A] T(n)=2T(n/2) + 1 [B] T(n)=2T(n/2) + n [C] T(n)=T(n/2) + n [D] T(n)=T(n/2) + 1 [E] T(n)=T(n-1) + T(n-2)	
9. The time complexity of the algorithm is	*2.5*
[A] $O(2n)$ [B] $O(\log n)$ [C] $O(2n^2)$ [D] $O(n \log n)$ [E] None of the above	
Questions 10 to 12 refer to the graph G represented by the following adjacency matrix	
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$	
10. The total degree of the graph G is	*2.5*
[A] 21 [B] 18 [C] 19 [D] 20 [E] 10	
11. Starting at the vertex a and resolving ties by the vertex alphabetical order, traverse the graph by breadth-first-search (BFS). Then, the 6th vertex being visited is	*2.5*
[A] g [B] h [C] e [D] f [E] c	
12. Starting at the vertex a and resolving ties by the vertex alphabetical order, traverse the graph by depth-first-search (DFS). Then, the last vertex being visited is	*2.5*

[A] *e*
[B] *d*
[C] *g*
[D] *f*
[E] *h*

13. Let G be a weighted connected graph	***2.5***
I. If e is a minimum-weight edge in G, it must be contained in a MST.	
II. If e is a minimum-weight edge in G, it must be contained in each MST.	
III. If e is a maximum-weight edge in G, it must not be contained in any MST.	
Which one of the following is correct?	
[A] I and III are true, II is false [B] I and II and III are true [C] I and II and III are false [D] II and III are true but I is false [E] I is true but II and III are false	
14. Let G be a weighted connected graph	***2.5***
I. If the edge weights are all different G must have exactly one MST	
II. If the edge weights are not all different G must have more than one MST	
III. If the edge weights are all same, every spanning tree of G is a MST	
Which one of the following is correct?	
[A] I is true, II and III are false [B] I and III are true but II is false [C] I and II and III are true [D] I and II and III are false [E] II is true but I and III are false	

Note: If a weighted graph is represented by its adjacency matrix, then its element $A[i, j]$ will simply contain the weight of the edge from the ith to the jth vertex if there is such an edge and a special symbol ∞, if there is no such edge. For example, in the following, the left side is a weighted graph and the right side is its weight matrix



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} \infty & 5 & 1 & \infty \\ 5 & \infty & 7 & 4 \\ 1 & 7 & \infty & 2 \\ \infty & 4 & 2 & \infty \end{bmatrix} \end{array}$$

Questions 15 to 18 refer to the following weighted graph represented by the following weight matrix:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} \infty & 2 & 4 & 4 & 18 \\ 2 & \infty & 4 & 3 & 4 \\ 4 & 4 & \infty & 1 & \infty \\ 4 & 3 & 1 & \infty & 9 \\ 18 & 4 & \infty & 9 & \infty \end{bmatrix} \end{array}$$

15. Let T be a minimum spanning tree of the graph computed using Kruskal's algorithm. The order of edges selected by Kruskal's algorithm is **2.5**

[A] *(c,d) (a,b) (b,d) (a,c)*
[B] *(c,d) (a,b) (b,d) (d,e)*
[C] *(c,d) (a,b) (b,d) (b,e)*
[D] *(c,d) (a,b) (b,d) (a,d)*
[E] *(c,d) (a,b) (b,d) (b,c)*

16. Let T be a minimum spanning tree of the graph computed using the Prim's algorithm: Assume vertex a is selected first, then the order of vertices selected by Prim's algorithm is **2.5**

[A] *a, b, d, e, d*
[B] *a, b, c, d, e*
[C] *a, c, d, b, e*
[D] *a, d, c, e, b*
[E] *a, b, d, c, e*

17. Assume the source vertex is a. Running Dijkstra's algorithm for the graph, after the termination, the label for vertex d is **2.5**

[A] *d(4,b),* [B] *d(6,b),* [C] *d(4,a),* [D] *d(6,a),* [E] *d(6,c),*	
18. Assume the source vertex is *a*. Running Dijkstra's algorithm for the graph, after termination, which one of the following could be an order of vertices selected by Dijkstra's algorithm? [A] *a, b, e, c, d* [B] *a, b, d, e, c* [C] *a, b, d, c, e* [D] *a, b, e, d, c* [E] None of the above	***2.5***
19. For the three statements below,	***2.5***
I. A problem in the class P can be solved in worst-case by a polynomial time algorithm. II. A problem in the class NP can be solved by a non-polynomial time algorithms III. A problem in the class NP can be verified in polynomial time Which one of the following is correct?	
[A] *I is true, II and III are false* [B] *I and II are true but III is false* [C] *I and II are false but III is true* [D] *II is true but I and III is false* [E] *None of the above*	
20. For the following problems	***2.5***
I. Vertex Cover Problem. II. Finding minimum spanning tree (MST) in a weighted undirected graph. III. 0/1 Knapsack problem. IV. Traveling Salesman problem. Which one of the following is correct?	
[A] I , II are NP-Complete Problems, III and IV are P-Problems [B] I , II are P-Problems, III and IV are NP-Complete Problems [C] I , III are NP-Complete Problems, II and IV are P-Problems [D] I , III are P-Problems, II and IV are NP-Problems [E] I, III and IV are NP-Complete Problems, II is a P-Problem.	

Questions 21 to 24 refer to the following Longest Common Subsequence problem	
Let c[i,j] be the length of the Longest Common Subsequence of Xi = x1, x2,…, xi and Yj= y1, y2,…,yj . Then c[i,j] can be recursively defined as following：	
$c[i,j]=\begin{cases}0 & \text{if } i=0 \text{ or } j=0\\ c[i-1,j-1]+1 & \text{if } i,j>0 \text{ and } x_i=y_j\\ \max\{c[i-1,j],c[i,j-1]\} & \text{if } i,j>0 \text{ and } x_i\neq y_j\end{cases}$	
The following is an incomplete table for the sequences of AATGTT and AGCT.	

		A	A	T	G	T	T
	0	0	0	0	0	0	0
A	0		1	1	1	1	1
G	0	1	1	1			
C	0	1					
T	0	1	1				

21. The value of c[3, 4] is **2.5**

[A] 1
[B] 2
[C] 3
[D] 4
[E] 5

22. The length of the longest common subsequence of AATGTT and AGCT is **2.5**

[A] 1
[B] 2
[C] 3
[D] 4
[E] 5

23. The longest common subsequence of AATGT and AGC is **2.5**

[A] AGC
[B] ATG
[C] AAT
[D] AGC
[E] AG

24. The longest common subsequence of AATGTT and AGCT is **2.5**

[A] AGCT
[B] ATGT
[C] AATG
[D] AGC

[E] AGT

Questions 25 to 28 refer to the following Knapsack problem: given the following instance of the 0/1 Knapsack problem.

item	weight	value
1	2	\$12
2	1	\$10
3	3	\$20

The Knapsack Capacity W=4

Let V[i, j] be the value of the most valuable subset of the first i items that fit into the Knapsack of capacity j. Then V[i, j] can be recursively defined as follows:

$$V[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ \max\{V[i-1, j], \quad v_i + V[i-1, j-w_i]\} & \text{if } j - w_i \geq 0 \\ V[i-1, j] & \text{if } j - w_i < 0 \end{cases}$$

For the above instance, the following is an incomplete table for V[i, j] (i=0, 1, 2, 3; j=0, 1, 2, 3,4)

		capacity j				
Item	i	0	1	2	3	4
	0	0	0	0	0	0
w_1=2, v_1=12	1	0	0	12	12	12
w_2=1, v_2=10	2	0	10	12	22	22
w_3=3, v_3=20	3	0	10	12	22	30

25. The value of V[0, 4] is **2.5**

[A] 12
[B] 10
[C] 22
[D] 0
[E] 24

26. What is the value of the most valuable subset that can fit into the knapsack? **2.5**

[A] 12
[B] 10
[C] 30
[D] 0
[E] 24

27. Which of the following is an optimal subset of the instance based on the table if the item3 is removed and the capacity of the knapsack is 4? **2.5**

[A] {item1, item2}
[B] {Item3}
[C] {Item1, item2, item3}
[D] {Item1, item3}

[E] {Item2, item3}	
28. Which of the following is an optimal subset of the instance based on the table if the capacity of the knapsack is 3 and item3 is removed?	***2.5***
[A] {item1, item2} [B] {Item3} [C] {Item1, item2, item3} [D] {Item1, item3} [E] {Item2, item3}	

PART II (30 Marks)	
Question I (12 marks)	
1. Briefly describe the idea of the divide-and-conquer technique.	*3*
2. Briefly describe the idea of the dynamic programming technique.	*3*
3. Given any two decision problems A and B, what is a polynomial time reduction from A to B? Briefly explain how this technique can be used to prove certain problems are NP- hard.	*3*
Question II (18 marks)	
1. There is a row of n coins whose values are some positive integers c1, c2, . . ., c_n, not necessarily distinct. Let F(n) be the maximum amount of money that can be picked up from the row subject to the constraint that no two coins adjacent in the initial row can be picked up.	
a) Set up a recurrence relation for F(n) that can be used by a dynamic programming algorithm. (hint: to derive a recurrence for F(n), you can partition all the allowed coin selections into two groups: those that include the last coin and those without it.	*6*
b) For coin row 5, 1, 2, 10, 6, 2 solve the coin row problem using the relation set in a).	*6*
c) Write pseudocode of the dynamic programming algorithm for solving this problem and determine its time complexity.	*6*

END OF THE PAPER