A   Computation problems

1. (10 points) The table below shows a training set with 10 examples that is used for training a **3-nearest-neighbors** classifier that uses Manhattan distance, i.e., the distance between two points at coordinates p and q is $|p–q|$. The only attribute, X, is real-valued, and the label Y has two possible classes, 0 and 1. The first fold contains the first 5 examples, and the second fold contains that last 5 examples. In case of ties in distance, use the example with smallest X value as the neighbor. Please compute the 2-fold cross validation accuracy (percentage correct classification).

X	0	1	2	3	4	5	6	7	8	9
Y	0	1	1	1	0	1	0	1	0	1

2. (10 points) You want to cluster 7 points into 3 clusters using **the k-means clustering algorithm**. Suppose after the first iteration, clusters C_1, C_2 and C_3 contain the following two-dimensional points:

   C_1 contains the 2 points: $\{(12,6),(6,0)\}$

   C_2 contains the 3 points: $\{(2,5),(5,4),(5,6)\}$

   C_3 contains the 2 points: $\{(5,6),(7,8)\}$

   Please compute the coordinates of **cluster centers** for these 3 clusters.

3. (20 points) The following dataset as in the table is provided to build a naive Bayes classifier, where $\{x_1, x_2, x_3, x_4\}$ and l are the features and the label, respectively. Please give the process of building the classifier and predict the label of the unknown instance $\boldsymbol{x} = [1,0,1,0]^T$.

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & l \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

B   Essay questions

4. (20 points) Perceptron is a function that maps input $\boldsymbol{x}$ to a label as follows

$$f(\boldsymbol{x}) = \begin{cases} 1, & w \cdot x + b > 0 \\ 0, & \text{otherwise} \end{cases}$$

1) (4 points) Now consider solving the logical **AND** problems (as shown in two tables) with the perceptron model.

$$y = f(\boldsymbol{x}) = \begin{cases} 1, & w_1 x_1 + w_2 x_2 + b > 0 \\ 0, & \text{otherwise} \end{cases}$$

Table 1: Logical AND

x_1	x_2	y
0	1	0
1	1	1
1	0	0
0	0	0

Please draw all datapoints of the tables in the two-dimensional space for logical AND problems, respectively, where different classes are marked with different shapes.

2) (4 points) Please explain whether the perceptron can mimic the output of logical AND or not. If so, please give an example of function $f(\boldsymbol{x})$; if not, please prove that there is no such function $f(\boldsymbol{x})$.

3) (12 points) Assuming our dataset contains the following examples:

class 0: $\{(0,0),(1/2,1/2),(1,1)\}$,

class 1: $\{(1,0),(0,1)\}$,

please explain whether the perceptron can classify all examples of the dataset correctly or not. If so, give an example of function $f(\boldsymbol{x})$; if not, prove that there is no such function $f(\boldsymbol{x})$.