Método de diferencias finitas para problemas no lineales

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Problema de frontera de segundo orden

$$y'' = f(x, y, y'), \quad \text{for } a \leq x \leq b, \quad y(a) = \alpha \quad \text{and} \quad y(b) = \beta.$$

Condiciones para la existencia y unicidad de una solución

- f and the partial derivatives f_y and $f_{y'}$ are all continuous on

$$D = \{\, (x, y, y') \mid \; a \leq x \leq b, \text{ with } -\infty < y < \infty \text{ and } -\infty < y' < \infty \,\};$$

- $f_y(x, y, y') \geq \delta$ on D, for some $\delta > 0$;
- Constants k and L exist, with

$$k = \max_{(x,y,y') \in D} |f_y(x, y, y')| \quad \text{and} \quad L = \max_{(x,y,y') \in D} |f_{y'}(x, y, y')|.$$

Problema lineal

$$y'' = p(x)y' + q(x)y + r(x)$$

for $a \leq x \leq b$, with $y(a) = \alpha$ and $y(b) = \beta$

Problema no lineal

$$y''(x_i) = f(x_i, y(x_i), y'(x_i))$$

$$i = 1, 2, \ldots, N$$

Diferencias centrales

$$y'(x_i) = \frac{1}{2h}[y(x_{i+1}) - y(x_{i-1})] - \frac{h^2}{6}y'''(\eta_i),$$

$$y''(x_i) = \frac{1}{h^2}[y(x_{i+1}) - 2y(x_i) + y(x_{i-1})] - \frac{h^2}{12}y^{(4)}(\xi_i),$$

$$\frac{y(x_{i+1}) - 2y(x_i) + y(x_{i-1})}{h^2} = f\left(x_i, y(x_i), \frac{y(x_{i+1}) - y(x_{i-1})}{2h} - \frac{h^2}{6}y'''(\eta_i)\right) + \frac{h^2}{12}y^{(4)}(\xi_i),$$

$$-\frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} + f\left(x_i, w_i, \frac{w_{i+1} - w_{i-1}}{2h}\right) = 0,$$

Se busca entonces la solución a un sistema NxN de la forma:

$$F_1(w_1, w_2, \ldots, w_N) = 0$$
$$F_2(w_1, w_2, \ldots, w_N) = 0$$
$$\vdots$$
$$F_N(w_1, w_2, \ldots, w_N) = 0$$

Ejemplo 1

$$y'' = -(y')^2 - y + \ln x, \quad 1 \le x \le 2, \quad y(1) = 0, \quad y(2) = \ln 2.$$

solución: $y = \ln x.$

scipy.optimize.newton_krylov(*F*, *xin*, **...**)

Find a root of a function, using Krylov approximation for inverse Jacobian.

F : *function(x) -> f*

Function whose root to find; should take and return an array-like object.

x0 : *array_like*

Initial guess for the solution

Ejemplo 2

$$y'' = \frac{1}{8}(32 + 2x^3 - yy'),$$

for $1 \le x \le 3$, with $y(1) = 17$ and $y(3) = \frac{43}{3}$,