

Given: Random sample (x_1, \dots, x_n)

$$L(\theta_1, \theta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-(x_i - \mu)^2}{2\sigma^2}\right)}$$

Taking natural log of likelihood fnc

$$\ln(\theta_1, \theta_2) = \sum_{i=1}^n \left(\frac{-(x_i - \mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

To find MLE, diff log likelihood w.r.t θ_1, θ_2

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^n \frac{x_i - \mu}{\sigma^2} = 0$$

$$\sum_{i=1}^n x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^n x_i \quad \theta_1 = \frac{\sum x_i}{n}$$

For $\theta_2 \Rightarrow$

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^n \left(\frac{-(x_i - \theta_1)^2}{2\theta_2} + \frac{1}{2\theta_2} \right) = 0$$

$$\sum_{i=1}^n \left(\frac{(x_i - \theta_1)^2}{\theta_2} \right) = 0$$

$$\theta_2 = \frac{1}{n} \sum (x_i - \theta_1)^2$$

To find MLE of θ for Binomial dist $B(m, \theta)$ where m is a +ve integer.

$$L(\theta) = \prod_{i=1}^n \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking \ln

$$\ln(L(\theta)) = \sum_{i=1}^n \ln \binom{m}{x_i} + x_i \ln(\theta) + (m-x_i) \ln(1-\theta)$$

$$\frac{\partial}{\partial \theta} \ln(L(\theta)) = \sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^n \frac{x_i}{\theta} = \sum_{i=1}^n \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^n x_i (1-\theta) = \sum_{i=1}^n (m-x_i) \theta$$

$$\theta \sum_{i=1}^n x_i = m \sum_{i=1}^n \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^n x_i$$